An Adaptive Gaussian Filter For Noise Reduction and Edge Detection

G. Deng and L. W. Cahill
Department of Electronic Engineering, La Trobe University
Bundoora Victoria 3083 Australia

Abstract

Gaussian filtering has been intensively studied in image processing and computer vision. Using Gaussian filter for noise suppression, the noise is smoothed out, at the same time the signal is also distorted. The use of Gaussian filter as pre-processing for edge detection will also give rise to edge position displacement, edges vanishing, and phantom edges. In this paper, we first review various techniques for these problems. We then propose an adaptive Gaussian filtering algorithm in which the filter variance is adapted to both the noise characteristics and the local variance of the signal.

I. INTRODUCTION

Gaussian filtering [1], [5] has been intensively studied in image processing and computer vision. Although it is regarded as the optimal filter in certain sense, it has a number of problems. Using Gaussian filter for noise suppression, the noise is smoothed out, at the same time the signal is also distorted. The use of Gaussian filter as pre-processing for edge detection will also give rise to displacement of edge position, edges vanishing, and phantom edges [3-4]. These problems will eventually effect the accuracy of the interpretation of an image by either man or machine. In this paper, we first review the present techniques to overcome these problems. We then propose a new adaptive Gaussian filtering algorithm on the basis of Hodson's work [6].

II. ADAPTIVE GAUSSIAN FILTERING TECHNIQUES

The two-dimensional digital Gaussian filter can be expressed as:

$$G(x,y)=\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-(x^2+y^2)/2\sigma^2\right), \quad (1)$$

where σ^2 is the variance of Gaussian filter, and the size of the filter kernel l $(-l \le x, y \le l)$ is often determined by omitting values lower than five percent of the maximum value of the kernel. The one-dimensional Gaussian filter is expressed as:

$$G(x)=\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-x^2/2\sigma^2\right). \quad (2)$$

When the Gaussian filter is used for noise suppression, a large filter variance is effective in smoothing out noise, but at the same time it distorts those parts of the image where there are abrupt changes in pixel brightness. The Gaussian filter also gives rise to edge position displacement, the vanishing of edges, and phantom edges [1-3], [4].

There are two basic methods to solve these problems. The first method pre-processes an image using different filters with different filter variances, and then detects edges in these filtered images. The final edge detection result is a synthesis of the edge images by some pre-defined rules [1], [5].

The second method makes the filter variance adapt to the local characteristics of an image. This is based on the strategy that different parts of an image should be smoothed differently, depending on the degree of noisiness and the type of edges. Hodson *et al.* [6] have shown that Gaussian filtering of a signal $F(x)$, denoted as $F_g(x)$, can be expressed as:

$$F_g(x)=F(x)+\frac{F''(x)}{2}\sigma^2+\ldots+\frac{F^{(2m)}(x)}{\prod_{p=1}^{m}2p}\sigma^{2m}+\ldots, \quad (3)$$

where $m=1,2,\ldots,$ $F''(x)$ is the second derivative of $F(x)$, and $F^{(2m)}(x)$ is the $2m$th order derivatives of $F(x)$. Omitting the higher order terms, the above equation can be approximated by:

$$F_g(x)\cong F(x)+\frac{F''(x)}{2}\sigma^2. \quad (4)$$

They thus defined the adaptive filter variance as:

$$\sigma^2(x)=\frac{2\varepsilon}{|F''(x)|}, \quad (5)$$

where ε is the amount of pre-defined error due to Gaussian filtering. Using this algorithm, the error due to filtering is approximately less than ε, i.e., $|F_g(x)-F(x)|\le\varepsilon$. A complicated regression based method has been proposed to estimated $F''(x)$ from the noisy corrupted signal.

A more complicated method has been proposed by Jeong and Kim [7]. They treated the adaptive Gaussian filtering problem as the minimisation of the mean square error under the condition that the filter variance should not change abruptly from pixel to pixel. Let G and F denote the two dimensional Gaussian filter and the image, respectively, then the optimal filter variance is obtained by minimizing $E(\sigma)$ in the following equation:

0-7803-1487-5/94$04.00©1994IEEE

$$E(\sigma)=\iint\left((F-G*F)^2+\lambda\left|\nabla\sigma^{-1}\right|^2\right)dxdy \qquad (6)$$

where "*" denotes convolution.

Other adaptive Gaussian filters such as the Intensity-Dependent-Spread (IDS) model [8] and the Contrast-Dependant Spread (CDS) [9] filter are proposed in an attempt to model the characteristics of the human visual system. The Intensity-Dependent-Spread model, proposed by Cornsweet and Yellot, uses the inverse of the pixel brightness $F(x,y)$ (intensity) as the filter variance $\sigma(x,y)$,

$$\sigma(x,y)=1/F(x,y). \qquad (7)$$

This model has been applied to image enhancement [8]. The Contrast-Dependant Spread filter, also known as the composite-scale Gaussian filter [9], has been studied by Vaezi and Bavarian. In CDS filter, the filter variance is given by:

$$\sigma(x,y)=\frac{\sqrt{2}C}{\sqrt{2+C}}, \qquad (8)$$

where C is the contrast defined as: $C=\frac{|F(x,y)-F_a(x,y)|}{\max(F(x,y),F_a(x,y))}$, and $F_a(x,y)$ is the local mean brightness value.

III. THE NEW ADAPTIVE ALGORITHM

Since the two-dimensional Gaussian filter can be regarded as a cascade of two one-dimensional Gaussian filters, then only the one-dimensional case is discussed. In noise filtering, a model is commonly assumed as follows:

$$S(x)=F(x)+N(x) \qquad (9)$$

where $S(x)$ is the observed signal, $F(x)$ is the original signal and $N(x)$ is the independent identical distribution Gaussian noise with zero mean and the variance σ_n^2. A further assumption is that $N(x)$ is not correlated with $F(x)$. Applying the Gaussian filter to this signal and using Hodson's result (equation (4)) yields:

$$\begin{aligned}\hat{S}(x)&=G(x)*S(x)\\ &\cong F(x)+\frac{F''(x)\sigma^2}{2}+G(x)*N(x)\end{aligned} \qquad (10)$$

where $F''(x)$ is the second derivative of $F(x)$. It can be seen in equation (10) that to minimize the distortion due to the second term the variance should be as small as possible. On the other hand, to smooth out noise, as indicated in the third term, the variance should be as large as possible. Therefore, the problem in Gaussian filtering can be formalized as adaptively choosing the variance to meet such conflicting requirements.

To solve this problem, it is noticed that images with sharp edges are more pleasing to the human eye, and that the human visual system is sensitive to the position of edges and noise in smooth areas. In an area with sharp edges the second derivatives are large and the filter variance should be small to preserve sharp edges and to keep the distortion small. In a smooth area the second derivatives are small and the variance should be large to filter out noise. These facts lead naturally to the idea that the variance should be proportional to the noise variance and reciprocal to the second derivative of the signal. Since the signal $F(x)$ is yet to be restored from $S(x)$, it is difficult to estimate the second derivative $F''(x)$ from $S(x)$. A possible solution is to use the local variance of the original signal denoted as $\sigma_f^2(x)$, which also needs to be estimated or assumed. Notice that the local variance of $S(x)$ denoted as $\sigma_s^2(x)$ can be easily estimated and the following relation holds:

$$\sigma_s^2(x)=\sigma_f^2(x)+\sigma_n^2. \qquad (11)$$

Therefore, it is proposed in this paper that the variance of Gaussian filter at location x is:

$$\sigma^2(x)=\frac{k\sigma_n^2}{\sigma_s^2(x)}=\frac{k\sigma_n^2}{\sigma_f^2(x)+\sigma_n^2}, \qquad (12)$$

where k is a scaling factor.

It can be seen that at locations where the signal changes very rapidly, i.e., $\sigma_f^2(x)>>\sigma_n^2$, then equation (12) can be approximated by: $\sigma^2(x)\cong k\sigma_n^2/\sigma_f^2(x)$; while on the other hand, at locations where the signal changes very slowly, i.e., $\sigma_f^2(x)<<\sigma_n^2$, then equation (12) can be approximated by: $\sigma^2(x)\cong k$. Therefore, in an edge area the filter variance is adapted to the local variances of signal and noise, while in the smooth area the signal variance is small and the filter variance can be replaced by a fixed value.

When the above adaptive Gaussian filter is applied to image processing, the two dimensional filter kernel is given by: $G(x,y)=uG(x)G(y)$, where u is the normalise factor to ensure that the sum of the filter kernel $G(x,y)$ is unity. The variance of the signal $\sigma_s^2(x)$ is estimated using a $(m\times m)$ window that centers at location x, where the window size m is an odd number.

IV. SIMULATION RESULTS

The above algorithm has been tested on a (256x256) chess board image with added Gaussian noise of zero mean and a variance $\sigma_n^2=2500$. This image is shown in Fig. 1. The brightness values of the original dark square and the bright square are 70 and 170, respectively. The size of each square is 32x32 pixels. Using Pratt's [10] definition of signal to noise ratio: $SNR=h^2/\sigma_n^2$, where h is the brightness jump of the edge, the SNR of this image is 4. Using Haralick's [11] definition: $SNR=10\log(h/\sigma_n)$ dB, the SNR of this image is 3 dB.

First, to show that the filter variance of the above algorithm is adapted to the changes of image brightness and to study the effects of different window sizes on estimating the filter variance, the average value of the filter variance over each column of the image using window sizes of 5, 9 and 13, has been calculated. The results are shown in Fig. 2 which shows that in the smooth area the values of the filter

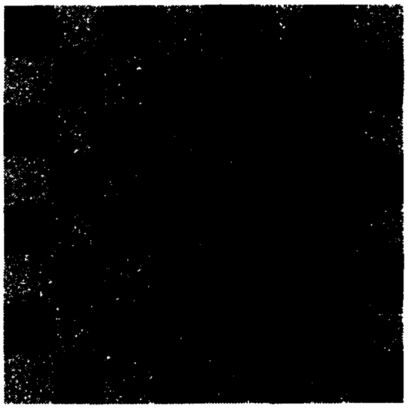


Fig. 1 The noisy chess board image.

variance are almost the same, and that they are drastically decreased in the edge area. The minimum variance is always located at the edge point. It can be also observed in Fig. 2 that the filter variance using a small widow size has a steeper decrease near the edge area than that using a large window size. Therefore, the small window size adapts to rapid signal changes better than large window size does. In the following experiments the (5x5) window is used to estimate the local signal variance: $\sigma_s^2(x)$.

Second, the mean square error (*MSE*) is used as a criterion to compare the filtered image using the above algorithm with that using the non-adaptive Gaussian filter. The *MSE* is defined as:

$$MSE = \frac{1}{N \times N} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} \left(F(x,y) - \hat{F}(x,y) \right)^2 , \qquad (13)$$

where $F(x,y)$ is the original noise free image, $\hat{F}(x,y)$ is the filtered noisy image, and N is the size of the image. The results are shown in Fig. 3. In Fig. 3, the horizontal axis stands for k (the scaling factor) for the results of adaptive algorithm, and the horizontal axis stands for σ (the filter variance) for the results of the non-adaptive Gaussian filter. It is noticed that using the adaptive algorithm the filtered image always has smaller mean square error than that using the non-adaptive Gaussian filter.

Third, the effect of the proposed adaptive algorithm on edge detection is compared with that of the non-adaptive algorithm. The noisy image is firstly filtered by using both the adaptive (k=2) and the non-adaptive algorithm ($\sigma^2 = 4$). The Sobel operator is then used to calculate the gradients of these two filtered images, and a thresholding process is used to extracted the edges. The thresholds are set such that the resultant images have the same number of pixels as edge pixel. The edge images are shown in Figs. 4(a) and (b). Fig. 4(c) is the result of using the (7x7) average filter for noise smoothing, then using the same edge detection method to extract edges. Fig. 4(c) has the same number of edge pixels as in Figs. 4(a) and (b). It can be observed from these results that the adaptive algorithm preserves sharper edges than the non-adaptive algorithm, and in the areas where two edges cross the result of the non-adaptive algorithm has larger gaps than that of the adaptive algorithm. It is also clear that the edge image Fig. 4(a), that is extracted from the adaptive algorithm filtered image, is the best of the three edge images.

V. CONCLUSION

In this paper, an adaptive Gaussian filter algorithm has been proposed. Simulations have shown that (1) the window size of (5x5) is suitable for calculating the local signal variance, (2) the minimum filter variance is always located at the edge point, thus the adaptive algorithm causes less distortion to the edges, (3) the image processed by the adaptive Gaussian filter always has smaller mean square error (MSE) than that processed the non-adaptive Gaussian filter has, and (4) the edges extracted from the image processed by the adaptive algorithm is better than that from the non-adaptive algorithm, especially at locations where two edges cross.

We noticed that the assumption of the proposed filter is that noise is Gaussian with known variance. In practical situations, noise variance has to be estimated. Further investigation is yet to be done to reduce the computational complexity of the proposed algorithm.

VI. REFERENCES

[1] F. Bergholm, "Edge focusing," *IEEE Trans. Patt. Anal. Mach. Intellig.*, vol. PAMI-9, pp. 726-741, Nov. 1987.

[2] D. J. Williams and M. Shah, "Normalized edge detector," in *Proc. of 1990 IEEE Int. Conf. on Pattern Recognition*, pp. 942-946, 1990.

[3] Y. Lu and R. C. Jain, "Behavior of edges in scale space," *IEEE Trans. Patt. Anal. Mach. Intellig.*, vol. PAMI-11, pp. 337-356, Apr. 1989.

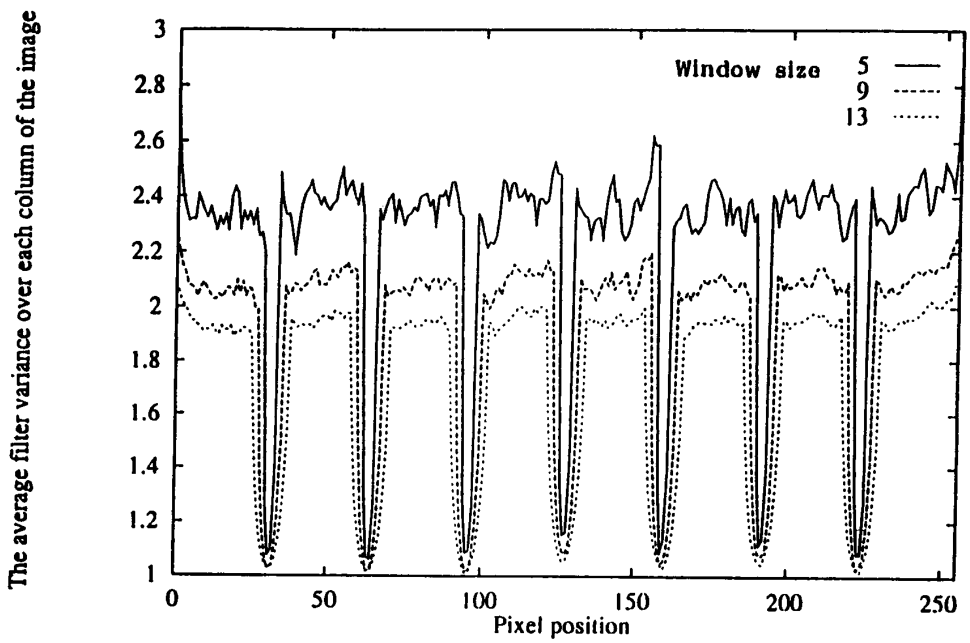


Fig. 2 The average filter variances calculated by using the window sizes of 5, 9, and 13

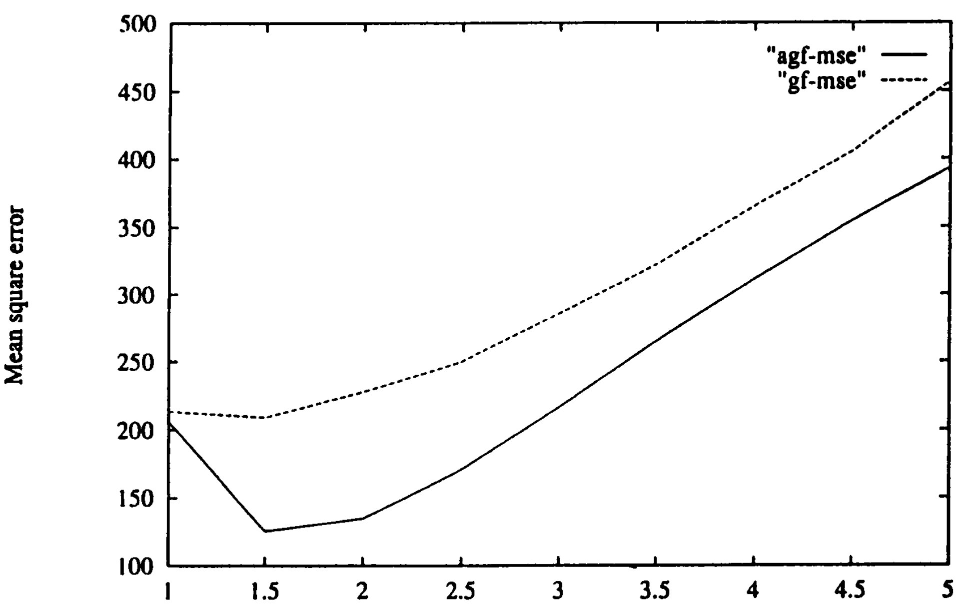


Fig. 3 The mean square errors of the images using adaptive Gaussian filter and non-adaptive Gaussian filter. The horizontal axis represents the scaling factor for the results of adaptive algorithm, and the filter variance for the results of the non-adaptive Gaussian filter.

[4] J. J. Clark, "Authenticating edges produced by zero-crossing algorithms," *IEEE Trans. Patt. Anal. Mach. Intellig.*, vol. PAMI-11, pp. 43-57, Jan. 1989.

[5] D. Marr and E. Hildreth, " Theory of edge detection," *Proc. of the Royal Society London B*, vol. 207, pp. 187-217, 1980.

[6] E. K. Hodson, D. R. Thayer and C. Franklin, "Adaptive Gaussian filtering and local frequency estimates using local curvature analysis," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-29, pp. 854-859, Aug. 1981.

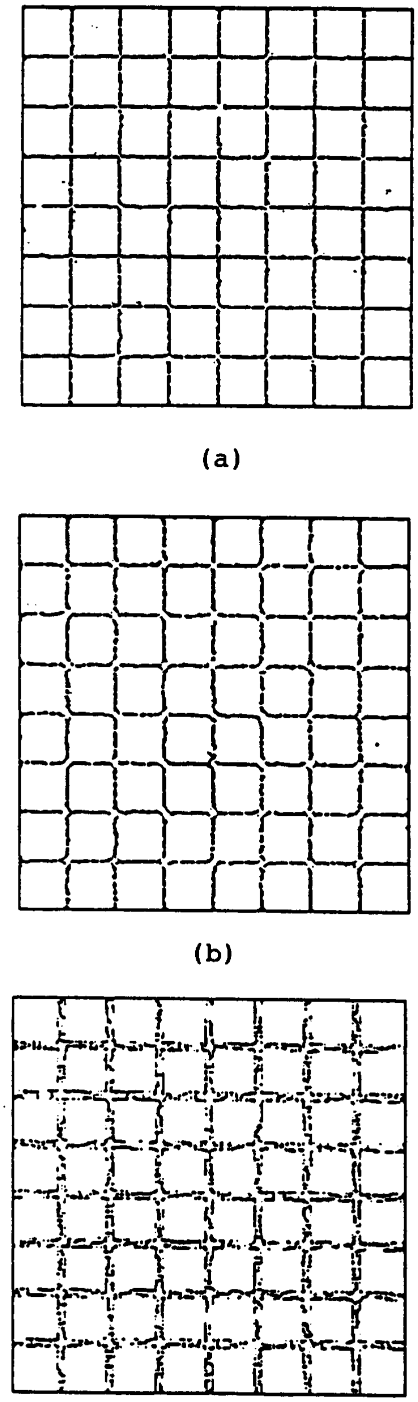
[7] H. Jeong and C. I. Kim, "Adaptive determination of filter scales for edge detection," *IEEE Trans. Pattern anal. Mach. Intell.*, vol. PAMI-14, pp. 579-585, May 1992.

[8] T. N. Cornsweet and J. I. Yellot, Jr., "Intensity-dependant spatial summation," *J. Opt. Soc. Am., Part A*, vol. 2, pp. 1769-1786, Oct. 1985.

[9] M. Vaezi and B. Bavarian, " Contrast-dependant spread filters," *Proc. SPIE: Image Processing Algorithms and Techniques*, vol. 1244, pp. 100-107, 1990.

[10] W. K. Pratt, *Digital Image Processing*. New York: Wiley, 1978.

[11] R. M. Haralick, "Digital step edges from zero-crossings of second directional derivatives," *IEEE Trans. Patt. Anal. Mach. Intellig.*, vol. PAMI-6, pp. 58-68, Jan. 1984.



(a)

(b)

(c)

Fig. 4 Edge detection results. (a) Adaptive Gaussian filter + Sobel operator. (b) Non-adaptive Gaussian filter + Sobel operator. (c) (7x7) mean filter + Sobel operator.