Shape-based low-thrust trajectory optimization enhanced via orthogonal functions, collocation and regularization

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Abstract

This work investigates novel aspects and approaches to previously-developed shape-based methods for optimal control and navigation problems in astrodynamics. The main contributions of our research refer to the introduction of new functional descriptions of the orbital motion, along with sampling point locations compliant with collocation theory, and hybridized with classical regularization techniques, in order to enhance the numerical performance of the proposed algorithms.

1 Introduction

Shape-based methods have gained increasing attention within the astronautical community in recent times, with extensive applications within optimization problems. The rationale behind such methodologies lies in exploiting particular functions to represent the orbital motion of the system, typically a spacecraft. Such analytical expressions, usually given by direct application of the boundary conditions of the problem, enable a quick and fast generation of preliminary mission design trajectories or initial guesses, which may undergo further refinement within more complex optimization solvers. Clearly, the function in-use is a key element of the methodology, whose fundamental features define the radius of convergence of the algorithm together with the intrinsic capabilities of the method to represent complex dynamics and associated solutions.

Shape-based methods for trajectory design were first introduced by Petropoulos and Longuski [1], by selecting an exponential sinusoid function to describe the trajectory of a low-thrust accelerated spacecraft. Thereafter, sinusoids have been a traditional choice to represent spacecraft dynamics for other applications [3]. Wall and Conway [6] presented inverse polynomials to match the spacecraft boundary conditions and its intrinsic dynamics. More recently, Xie et al. [7] suggested a rapid shaping method based on the radial coordinate form of the initial and target orbits, and Roa et al. [4] introduced the concept of generalized logarithmic spirals in a series of works.

Orthogonal polynomial bases, as well as general spirals, have been used massively in direct transcription optimal control solvers and numerical approximation across a wide range of fields, from Fluid Mechanics to Astrodynamics. However, they are not often selected to construct shape-based methods for orbital mechanics applications. More recently, Taheri [5] introduced a shape-based formulation to describe spacecraft trajectories based on a finite Fourier series. Based on these latter results, Hou et al. [8, 2] presented a shape-based method to design the 3D trajectories of electric solar wind sails, relying on a Bézier curves approximation, which builds upon the family of Bernstein polynomials. Despite their successful application to this low-thrust trajectory design optimization, their mathematical formulation has not been completely explored yet to its full potential, nor their numerical and computational advantages have been exploited. In addition, despite some recent work on optimal control [9], general shape-based methods have not been employed as a core optimization engines, but as a low-cost, fast technique to generate dynamically-compliant trajectories to be refined afterwards in more detail design phases.

This work presents a novel approach to tackle optimization problems in astrodynamics using enhanced shape-based methods, together with a real assessment of the viability of these algorithms as general optimization solvers. Two specific problems are mainly studied: the design of orbital transfers for active debris removal missions and orbit determination for propelled spacecraft. In particular, we introduce new functional representations of the system's time evolution by means of an orthogonal version of the Bernstein polynomials family, in order to enhance the algorithm's numerical behaviour and improve on it con-

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vergence properties. Additionally, a direct performance comparison is performed and presented against classical orthogonal bases, which have still not been employed in this methodology. The optimization-associated collocation problem is then reformulated, initially on the natural nodes of the selected functional bases and then in regularized coordinates, to explore the solution's intrinsic features and dynamics. Research on the effect of the collocation mesh on the final optimal solution is also conducted and presented. Finally, several benchmark missions are introduced and solved by the proposed techniques for demonstration purposes.

2 Shape-based methods

A great variety of methods have been developed for numerically solving optimal control problems. Due to their numerical performance and ease of pose, direct transcription methods, despite not being the most accurate, are among the most extensively used. Direct transcription methods project the state evolution and the control input onto given functional bases and pose a discretized Non Linear Programming (NLP) Problem to solve the given general Bolza problem of interest [10].

Shape-based methods, which show different formulations, are closely related to direct transcription solvers, as they also project the state of the system onto given selected functional bases. However, when compared against each other, all shape-based methods impose boundary conditions into the solution by quasi-analytically selecting appropriate constants or weights in the state functional expansion. This allows for a quick generation of boundary-compliant initial guesses for optimization solvers (notably, for direct transcription ones) as long as the selected functional base is able to capture the problem's intrinsic dynamical features.

In our particular case, additionally to imposing boundary conditions as already explained, when used as a general NLP optimization core, shape-based methods minimize the problem's cost function by selecting the optimal weights for each of the function of the base used in the state expansion. Such cost function shall be expressed as a residual of the state functional series and the problem's dynamics, therefore intrinsically imposing dynamic and path constraints at prescribed, discrete sampling points on the independent variable in the equations of motion.

3 Orthogonal functional bases

Shape-based methods are primarily constructed upon the selection of an appropriate functional basis on which to decompose the orbital motion evolution. Such selection may be accomplished by balancing different criteria, from the computational cost and complexity of evaluating the function to its analytical expression and suitableness to express the problem dynamics. Despite the existence of a vast number of options, previous work have focused on a really narrow range of bases, whose mathematical and approximation properties are still to be objectively explored. In particular, in spite of showing clear advantages within general optimization algorithms, to the best of our knowledge, orthogonal polynomials have not been introduced into the topic, so their behavior and actual performance is still to be unveiled.

Firstly, this work presents a novel approach to the application of Bézier curves to these optimization problems using an alternative functional basis; in particular, an orthogonal version of their generating Bernstein polynomials are proposed, to enhance their numerical behaviour and improve on their convergence. Secondly, the use of additional orthogonal bases is investigated, such as that of Chebyshev or Legendre, due to the intrinsic benefits they may bring when constructing shape-based methods upon them. Finally and as further discussed, direct comparison is performed between the different functional bases considered, together with exploring the intrinsic applications and characteristics of each of them.

4 Nodal and regularized discretization grid

One of the main drivers of the performance of the algorithm is the trajectory sampling grid, given by the discretization of the independent variable of the problem. This work introduces two major novelties from which the overall performance of the algorithm may benefit.

Up to now, previous work has focused on functional discretization and interpolation by means of non-optimal node selection techniques. Specifically, shape-based method have relied on classical collocation nodes, such as those of Legendre-Gauss-Radau or Legendre-Gauss-Lobatto [2], to define the trajectory sampling points, i.e. the time instants at which both, the control and dynamic constraints are evaluated. While this may show advantages when the optimal solution is then used as an initial guess in an spectral optimization solver, based on such classical nodes, it does not exploit the intrinsic approximation capabilities of the selected functional bases, nor they minimize the approximation error as compared to using their natural collocation nodes [11]. By exploiting the natural formulation of these polynomials, the numerical performance of these curves, as used in shape-based methods, is increased: their interpolation accuracy is not only ensured but the problems-at-hands dimensionality is also reduced, with clear advantages within standard optimization NLP solving techniques.

However, the definition of this discretization grid by means of a time-like variable may not be the optimal choice, due to the intrinsic dynamics of the Keplerian gravitational problem. Although applied to perturbed trajectories, orbital regularization techniques may also provide a particularly interesting approach to express the problem's dynamic constraints. They enable the definition of linearly-spaced grid points in a fictitious time scale, while ensuring to capture the natural features of the optimal solution trajectory and associated orbital motion evolution.

5 Applications and benchmark missions

The proposed algorithms and techniques are demonstrated and validated within several end-to-end low-thrust mission design cases, as shown in Figure 1. Specifically: 1) low-thrust transfers within low Earth orbit are solved using low-order, orthogonal functions to illustrate the performance of the proposed enhancements in orbit raising problems and active debris removal missions for towing defunct satellites; and 2) preliminary orbit determination for low-thrust propelled spacecraft is achieved by re-purposing the proposed shape-based techniques, thus demonstrating their suitability to perform orbit determination with the presence of thrusting trajectory arcs. The computational benefits of this novel formulation are shown by confronting them in the aforementioned missions against other traditional approaches: the use of other mentioned orthogonal polynomial bases, and the already described Bézier functions. The effect of several regularization techniques and the resulting regularized sampling points is also studied, together with their benefits and possible added drawbacks.

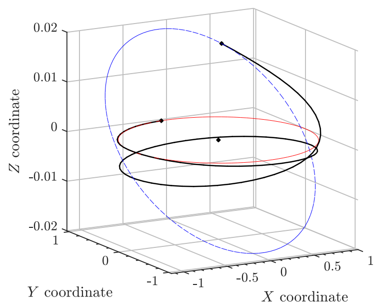

Figure 1: LEO low-thrust orbit transfer in normalized coordinates.

6 Conclusions

This work presents an extended formulations of shape-based methods for general astrodynamics optimization problems, with particular applications within low-thrust trajectory optimization. Such methodology enables a low-cost NLP optimization to replace classical direct transcription optimal control problems. New functional bases on which to express the system's motion are presented together with their performance comparison against classical orthogonal polynomials. Additionally, time regularization is employed to construct a sampling grid that exploits the natural features of orbital motion, thus enhancing the performance of shape-based methods. Several benchmark missions are presented and solved using the aforementioned techniques to demonstrate their capabilities in real mission scenarios. Finally, new ideas, extensions and follow-up work for the presented optimization techniques are introduced and discussed.

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