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Fast Cooperative Trajectory Optimization for Close-Range Satellite Formation Using Bezier Shape-based Method

ZICHEN FAN[1], MINGYING HUO[1], SONG XU[1], JUN ZHAO[1], AND NAIMING QI.[1]

[1]School of Astronautics, Harbin Institute of Technology, Harbin 150001, China

Corresponding author: Mingying Huo (e-mail: huomingying@hit.edu.cn).

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ABSTRACT Close-range formation flying of multiple satellites is an important technology for future space missions, where formation reconfiguration is a very important field and needs to design suitable flight paths and consider the risk of collisions between satellites. This paper uses the Bezier shape-based (SB) method to rapidly generate the low-thrust collision-avoidance flight paths in the formation reconfiguration. The reconfiguration process of two satellite formations is considered and compared with the finite Fourier series (FFS) method. The simulation results show that the Bezier method spends less computation time to obtain better performance index than the FFS method. In order to verify the applicability of the results obtained by the Bezier method to a direct solver, the results are applied to the Gauss pseudospectral method (GPM) solver as an initial guess. The results show that the Bezier method can take a short time to design suitable initial collision-avoidance flight paths for an direct optimization solver. This is very important for the rapid feasibility evaluation of numerous flight scenarios at the stage of initial mission design and onboard flight paths generation.

INDEX TERMS Bezier shape-based method, Close-range satellites formation, Collision-avoidance, Low-thrust trajectory approximation, Preliminary mission analysis

I. INTRODUCTION

In recent years, the scientific community has shown great interest in interplanetary missions [1]- [3]. Therefore, the requirements for satellite function and performance are getting higher and higher [4], [5]. Formation flying of multiple satellites [6]- [11] can complete complex tasks with less fuel and higher efficiently than a single large satellite [12], just like aircrafts [13]. This paper mainly studies the formation reconfiguration of close-range formation flying of multiple satellites. In previous studies, Robertson et al. [14] researched formation initialization and formation reconfiguration of the formation satellites. Mbede et al. [15] used the virtual potential function method in the formation reconfiguration problem. Godard et al. [16] used adaptive variable structure control strategy to propose two satellite formation reconfiguration planning methods. In the formation reconfiguration and initialization process, Richards et al. [17] used collision avoidance as a constraint criterion to make path planning. In view of the collision problem in the formation reconstruction, this paper uses SB method to rapidly generate three-dimensional low-thrust collision-avoidance flight paths.

Flight paths and mission cost of low-thrust satellites need a method to approximate [18], [19]. To optimize propellant consumption, the main problem is to determine the direction and magnitude of the thrust and satisfy the equations of motion (EoM), boundary conditions (BCs) and constraints to avoid collisions in the flight paths, which makes the mission design more challenging computationally. For indirect or direct optimization method, they all need a reasonable initial guess. Therefore, rapid initial trajectory design is very important for trajectory optimization. And SB methods are presented in initial low-thrust trajectory design due to the rapidity of their calculations [20]- [30]. Recently, the FFS approximation of the trajectory shape was developed [31]- [35]. Huo et al. designed the trajectory of Electric Solar Wind

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Sail (E-sail) by the Bezier method [36].

This paper used the Bezier method to rapidly design the three-dimensional flight paths of close-range satellite formation and considered the Clohessy-Wiltshire (C-W) equation at the same time. The reconfiguration process of two satellite formations is considered and compared with the FFS method to verify the validity of the proposed method. The simulation results show that the Bezier method spends less computation time to obtain better performance index than the FFS method. The main reason for this is that for the time-free problem the coefficient matrices of the Bezier method don't need to be repeatedly calculated, but the partial coefficient matrices of the FFS method need to be calculated repeatedly. When calculating a single flight path, the influence of this difference is small. However, when calculating multiple flight paths at the same time, the influence of this difference on the computation time will be magnified many times. And the results obtained with the Bezier method are then used as a first guess for the GPM solver. The simulation results show that the proposed method can rapidly generate appropriate initial flight paths in a short time, which is very important for satellite formation reconfiguration and enables satellites to calculate the flight paths of reconstructed formation everywhen.

This paper is organized as follows. Section II introduces the coordinate systems, EoM, and BCs. In section III, the Bezier approximation method for satellite formation reconfiguration is presented. In section IV, the effectiveness of the Bezier method is checked by the simulation of the reconfiguration process of two satellite formations and compared with FFS and GPM. Section V presents the result analysis. Finally, in the last section the conclusion is described.

II. PROBLEM DESCRIPTION

When studying satellite formation, a satellite is chosen as the reference satellite. The relative motion between satellites should be described in the reference satellite's orbit coordinate system, so the reference satellite's orbit coordinate system is established $Oxyz$. As illustrated in Figure. 1, the origin O is defined at the center-of-mass of the reference satellite. The x-axis points from the earth center to the center-of-mass of the reference satellite, the y-axis is perpendicular to the x-axis and points to the forward direction of the reference satellite and the z-axis is perpendicular to Oxy plane.

As shown in Figure. 1, the radius vector (in the inertial frame) of the center-of-mass of the reference satellite is $\boldsymbol{r_1}$, the radius vector (in the inertial frame) of the center-of-mass of a following satellite is $\boldsymbol{r_2}$, and the relative motion radius vector [37], $\boldsymbol{\rho}$, is

$$\boldsymbol{\rho} = \boldsymbol{r_2} - \boldsymbol{r_1} \tag{1}$$

The equation of motion (in the inertial frame) for each satellite is

$$\ddot{\boldsymbol{r}}_i = -\frac{\mu}{r_i^3}\boldsymbol{r}_i + \boldsymbol{f}_i(t) \qquad (i = 1, 2) \tag{2}$$

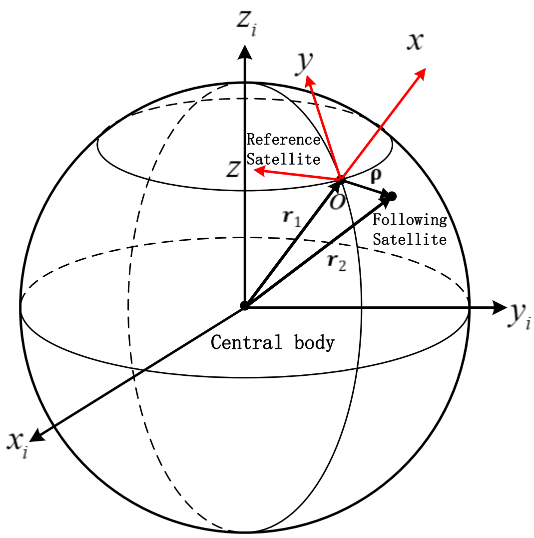

FIGURE 1. The reference satellite's orbit coordinate system.

where $\mu = 3.986 \times 10^5 km^3/s^2$ is the Earth (the Central body) gravitational constant and $\boldsymbol{f}_i$ is the active control of each satellite. When the reference satellite's orbit is circular, the following satellite's orbit is near circular, and $\boldsymbol{r_2} \approx \boldsymbol{r_1} \gg \boldsymbol{\rho}$, the linearization equation of the relative motion of two satellites in the reference satellite's orbit coordinate system can be obtained, which is called C-W equation, and its concrete form is as follows:

$$\begin{aligned} &\ddot{x} - 2n\dot{y} - 3n^2x = a_x \\ &\ddot{y} + 2n\dot{x} = a_y \\ &\ddot{z} + n^2z = a_z \end{aligned} \tag{3}$$

where $n = \sqrt{\mu/a_0^3}$ is the average orbital angular velocity of the reference satellite, a_0 is the semi-major axis of the reference satellite, x, y, z are the projection of the relative motion radius vector $\boldsymbol{\rho}$ in the orbit coordinate system of the reference satellite, and a_x, a_y and a_z are the thrust acceleration components of active control (regardless of perturbative force). So, the total thrust acceleration, a, is

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} \tag{4}$$

The propellant consumed by all satellites in low-thrust collision-avoidance flight paths is the performance index in this work, which is represented by ΔV, and it can be computed as

$$\Delta V = \Delta V_1 + \Delta V_2 + \cdots + \Delta V_n = \int_0^T a_1\,dt + \int_0^T a_2\,dt + \cdots + \int_0^T a_n\,dt \tag{5}$$

where T is the total flight time for each satellite, ΔV_1, ΔV_2, $\cdots$, ΔV_n correspond to the propellant consumed by satellites in flying, and a_1, a_2, $\cdots$, a_n are the corresponding satellite thrust acceleration.

When a satellite moves between two positions, 12 BCs are:

$$\begin{gathered} x(\tau=0)=x_i, x(\tau=1)=x_f, x'(\tau=0)=T\dot{x}_i, x'(\tau=1)=T\dot{x}_f \\ y(\tau=0)=y_i, y(\tau=1)=y_f, y'(\tau=0)=T\dot{y}_i, y'(\tau=1)=T\dot{y}_f \\ z(\tau=0)=z_i, z(\tau=1)=z_f, z'(\tau=0)=T\dot{z}_i, z'(\tau=1)=T\dot{z}_f \end{gathered} \tag{6}$$

where $0 \leq \tau = t/T \leq 1$ is the scaled time, t is the flight time, and the symbol $\dot{}$ and the superscript $'$ denote the derivative of time (t) and scaling time (τ), respectively. The subscript "i" denotes the initial condition, and the subscript "f" denotes the final condition. Each satellite contains the above 12 BCs.

III. BEZIER APPROXIMATION

A. STATES APPROXIMATION

In the proposed method, the orbital coordinates of satellites has the form of Bezier curve function [38], [39]. According to [36], the approximation of orbit coordinate x (as an example) is

$$x(\tau) = \sum_{j=0}^{n_x} B_{x,j}(\tau) P_{x,j} \tag{7}$$

where n_x is the the order of Bezier function, $P_{x,j}$ is the unknown Bezier coefficients, and $B_{x,j}(\tau)$ is the Bezier basis functions.

$$B_{x,j}(\tau) = \frac{n_x!}{j!(n_x-j)!}\tau^j(1-\tau)^{n_x-j} \qquad j \in [0, n_x] \tag{8}$$

Hence, $x'(\tau)$, $x''(\tau)$, $B'_{x,j}(\tau)$ and $B''_{x,j}(\tau)$ are easy to get(see [36] for details). Substituting ($\tau = 0, 1$) into (8), we can get

$$\begin{aligned} B_{x,j}(\tau=0) &= \begin{cases} 1 & j=0 \\ 0 & j\in[1,n_x] \end{cases} \\ B_{x,j}(\tau=1) &= \begin{cases} 0 & j\in[0,n_x-1] \\ 1 & j=n_x \end{cases} \\ B'_{x,j}(\tau=0) &= \begin{cases} -n_x & j=0 \\ n_x & j=1 \\ 0 & j\in[2,n_x] \end{cases} \\ B'_{x,j}(\tau=1) &= \begin{cases} 0 & j\in[0,n_x-2] \\ -n_x & j=n_x-1 \\ n_x & j=n_x \end{cases} \end{aligned} \tag{9}$$

Considering (6), (7) and (9), we can get

$$\begin{aligned} x_i &= x(\tau=0) = P_{x,0} \\ x_f &= x(\tau=1) = P_{x,n_x} \\ T\dot{x}_i &= x'(\tau=0) = n_x(P_{x,1}-P_{x,0}) \\ T\dot{x}_f &= x'(\tau=1) = n_x(P_{x,n_x}-P_{x,n_x-1}) \end{aligned} \tag{10}$$

The Bezier coefficients $P_{x,0}$, $P_{x,1}$, P_{x,n_x-1} and P_{x,n_x} can be obtained

$$\begin{aligned} P_{x,0} &= x_i & P_{x,1} &= x_i + T\dot{x}_i/n_x \\ P_{x,n_x-1} &= x_f - T\dot{x}_f/n_x & P_{x,n_x} &= x_f \end{aligned} \tag{11}$$

The roots of the mth-degree Legendre polynomial (Legendre-Gauss distribution of discretization points) is adopted.

$$\tau_1 = 0 < \tau_2 < \cdots < \tau_{m-1} < \tau_m = 1 \tag{12}$$

Coordinates (x, y, z) and their associated τ-derivatives can be presented in a compact matrix notation form. Coordinate x is taken as an example.

$$\begin{aligned} [x]_{m\times1} &= [B_x]_{m\times(n_x+1)}[P_x]_{(n_x+1)\times1} \\ [x']_{m\times1} &= [B'_x]_{m\times(n_x+1)}[P_x]_{(n_x+1)\times1} \\ [x'']_{m\times1} &= [B''_x]_{m\times(n_x+1)}[P_x]_{(n_x+1)\times1} \end{aligned} \tag{13}$$

where $[P_x] = \begin{bmatrix} P_{x,0} & P_{x,1} & [X_x]^T_{(n_x-3)\times1} & P_{x,n_x-1} & P_{x,n_x} \end{bmatrix}^T$ are the Bezier coefficients. $P_{x,0}$, $P_{x,1}$, P_{x,n_x-1} and P_{x,n_x} are the known coefficients, and $[X_x]_{(n_x-3)\times1} = \begin{bmatrix} P_{x,2} & \cdots & P_{x,n_x-2} \end{bmatrix}^T$ are the unknown coefficients. $[B_x], [B'_x], [B''_x]$ can be calculated by substituting τ_i into (8) and (9), which are constant matrices and doesn't need to recomputed at each iteration when the number of Bezier orders and discretization points are given.

$$[B_x]_{m\times(n_x+1)} = \begin{bmatrix} B_{x,0}(\tau_1) & \cdots & B_{x,n_x}(\tau_1) \\ \vdots & \ddots & \vdots \\ B_{x,0}(\tau_m) & \cdots & B_{x,n_x}(\tau_m) \end{bmatrix} \tag{14}$$

$[B'_x]$ and $[B''_x]$ have similar structure, not repetition.

When $n_x = n_y = n_z = 3$, the shape of the flight path is determined by the BCs, which can be used to generate initial value of iteration. When $n_x > 3, n_y > 3, n_z > 3$, there are unknown coefficients ($[X_x]_{(n_x-3)\times1}, [X_y]_{(n_y-3)\times1}, [X_z]_{(n_z-3)\times1}$) to be optimized to satisfy constraints.

Using the matrix form of the coordinates, the equations of thrust acceleration are

$$\begin{aligned} [a_x]_{m\times1} &= a_x\left([x]_{m\times1}, [z]_{m\times1}, [y']_{m\times1}, [x'']_{m\times1}\right) \\ [a_y]_{m\times1} &= a_y\left([x]_{m\times1}, [x']_{m\times1}, [y']_{m\times1}, [y'']_{m\times1}\right) \\ [a_z]_{m\times1} &= a_z\left([x]_{m\times1}, [z]_{m\times1}, [z'']_{m\times1}\right) \end{aligned} \tag{15}$$

and (4) can be written as

$$[a] = \sqrt{[a_x]^2 + [a_y]^2 + [a_z]^2} \leq a_{max} \tag{16}$$

where a_{max} is the maximum limit of the thrust acceleration value.

All satellites maneuver at the same time. Thus, the nonlinear programming (NLP) problem is

$$\begin{aligned} & \min_{[X_{x1}],[X_{y1}],[X_{z1}],\cdots,[X_{xn}],[X_{yn}],[X_{zn}],[T]} \quad \Delta V \\ s.t. \quad & [a_1] \leq a_{max}, \cdots, [a_n] \leq a_{max}, \\ & [d_{12}] \geq d_s, \cdots, \left[d_{(n-1)n}\right] \geq d_s, \\ & [d_{01}] \geq d_s, \cdots, [d_{0n}] \geq d_s \end{aligned} \tag{17}$$

where n is the number of satellites, X_{xn}, X_{yn} and X_{zn} are the unknown Bezier coefficients, d_s is the safe distance between two satellites, $d_{12}, \cdots, d_{(n-1)n}$ are the distance between two following satellites respectively, and $d_{01}, \cdots, d_{0n}$ are the distance between following satellites and the reference satellite respectively. For the time-free problem, the number of variables needed to be optimized is $n(n_x + n_y + n_z - 9) + 1$.

B. INITIALIZATION OF UNKNOWN COEFFICIENTS

An approximation of the coordinates (x, y, z) at n_{APP} Legendre-Gauss discretization points is adopted, where n_{APP} is larger than m. All satellites move and arrive at the same time, and the approximated flight time T_{APP} is estimated under the following assumptions: randomly selecting a satellite to use maximum thrust acceleration to fly from the start to the end point, with the start and end speeds equalling zero.

$$T_{APP} = \sqrt{\frac{2S}{a_{max}}} \tag{18}$$

where S is the distance between the start and end points of the selected satellite.

By using $n_x = n_y = n_z = 3$, the approximations of x_{APP}, y_{APP} and z_{APP} can be written as follows

$$\begin{aligned}
x_{APP}(\tau) &= (1-\tau)^3 P_{x,0} + 3\tau(1-\tau)^2 P_{x,1} + 3\tau^2(1-\tau)P_{x,2} + \tau^3 P_{x,3} \\
y_{APP}(\tau) &= (1-\tau)^3 P_{y,0} + 3\tau(1-\tau)^2 P_{y,1} + 3\tau^2(1-\tau)P_{y,2} + \tau^3 P_{y,3} \\
z_{APP}(\tau) &= (1-\tau)^3 P_{z,0} + 3\tau(1-\tau)^2 P_{z,1} + 3\tau^2(1-\tau)P_{z,2} + \tau^3 P_{z,3}
\end{aligned} \tag{19}$$

Using the BCs, these coefficients can be obtained

$$\begin{aligned}
&P_{x,0} = x_i \quad P_{x,1} = x_i + T_{APP}\dot{x}_i/3 \quad P_{x,2} = x_f - T_{APP}\dot{x}_f/3 \quad P_{x,3} = x_f \\
&P_{y,0} = y_i \quad P_{y,1} = y_i + T_{APP}\dot{y}_i/3 \quad P_{y,2} = y_f - T_{APP}\dot{y}_f/3 \quad P_{y,3} = y_f \\
&P_{z,0} = z_i \quad P_{z,1} = z_i + T_{APP}\dot{z}_i/3 \quad P_{z,2} = z_f - T_{APP}\dot{z}_f/3 \quad P_{z,3} = z_f
\end{aligned} \tag{20}$$

The scaled time vector becomes

$$\tau_{App,0} = 0 < \tau_{App,1} < \cdots < \tau_{App,(n_{App}-1)} = 1 \tag{21}$$

Therefore, $[x_{APP}]$, $[y_{APP}]$ and $[z_{APP}]$ can be obtained by substituting $\tau = \tau_{App}$ into (19). An initial guess for the unknown parameters in Bezier curves is

$$\begin{aligned}
[P_{xAPP}]_{(2n_x-3)\times 1} &= \left([B_{xAPP}]_{n_{APP}\times(2n_x-3)}\right)^{-1}[x_{APP}]_{n_{APP}\times 1} \\
[P_{yAPP}]_{(2n_y-3)\times 1} &= \left([B_{yAPP}]_{n_{APP}\times(2n_y-3)}\right)^{-1}[y_{APP}]_{n_{APP}\times 1} \\
[P_{zAPP}]_{(2n_z-3)\times 1} &= \left([B_{zAPP}]_{n_{APP}\times(2n_z-3)}\right)^{-1}[z_{APP}]_{n_{APP}\times 1}
\end{aligned} \tag{22}$$

IV. SIMULATION EXAMPLE

A. PLANAR CONFIGURATION TO PLANAR CONFIGURATION

A simulation example of three following satellites from planar configuration to planar configuration is given to verify the effectiveness of the proposed method. Choose the geosynchronous orbit satellite as the reference satellite. The thrust acceleration limit is set to $a_{max} = 6 \times 10^{-3}$ m/s^2. The Fourier terms are ($n_x = 6$, $n_y = 6$, $n_z = 8$), the orders of Bezier curve function are ($n_x = 12$, $n_y = 12$, $n_z = 16$), and two methods have the same number of unknown coefficients. m and n_{App} are 120 and 600, respectively [33]. In the trajectory optimization by using GPM, the number of Legendre-Gauss points is selected as 60, and the NLP is solved by the sequential quadratic programming. The initial positions (x_i, y_i, z_i) (units are meter) of the three following satellites are respectively (0, 2, 0), (0, 4, 0) and (0, 2, 3), the terminal positions (x_f, y_f, z_f) of the three satellites are respectively (0 ,-4 ,0), (0, -2, 0) and (0, -2, -3), and the initial and terminal velocities of the three satellites are 0. The diameter of the satellite is 1.0 m, and the safe distance between two satellites is 1.6 m. Small distance between satellites will make the design of collision-avoidance trajectories more difficult. Therefore, according to the initial position and terminal position of the satellites, this paper sets the safe distance between satellites. The initial flight paths are designed by using Bezier, the further optimized flight paths are obtained by using the GPM solver, and the comparisons with FFS method are also implemented. All tests were carried out on a i7 3.40 GHz processor with Windows 7 and run on MATLAB R2015b.

The results of the flight paths are illustrated in Fig. 2. The curves of black, red and green represent the results of Bezier, FFS and GPM methods, respectively. In this scenario, the total flying time obtained by using the proposed Bezier method is 299.952 s, and that obtained by using FFS and GPM is 299.834 s and 299.952 s respectively.

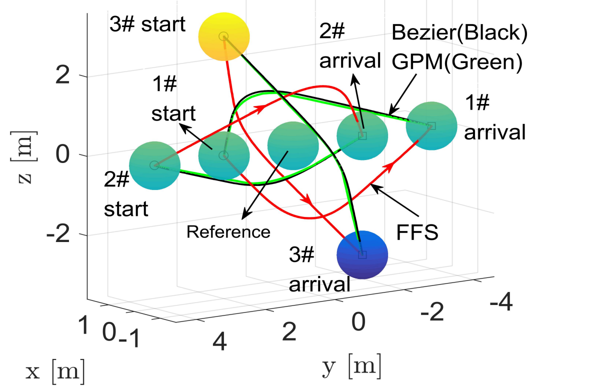

FIGURE 2. Formation reconfiguration of three following satellites with Bezier, FFS and GPM methods.

Fig. 3 is the thrust acceleration components of three following satellites and Fig. 4 shows the relative distance between two satellites, which is obtained by using Bezier. Fig. 5 is the thrust acceleration components of three following satellites and Fig. 6 shows the relative distance between two satellites, which is obtained by using FFS. The thrust acceleration components of three following satellites and the relative distance between two satellites obtained by using GPM are illustrated in Fig. 7 and Fig. 8. From these results, it can be seen that the three methods can satisfy the thrust

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acceleration constraint and the safety distance constraint very well.

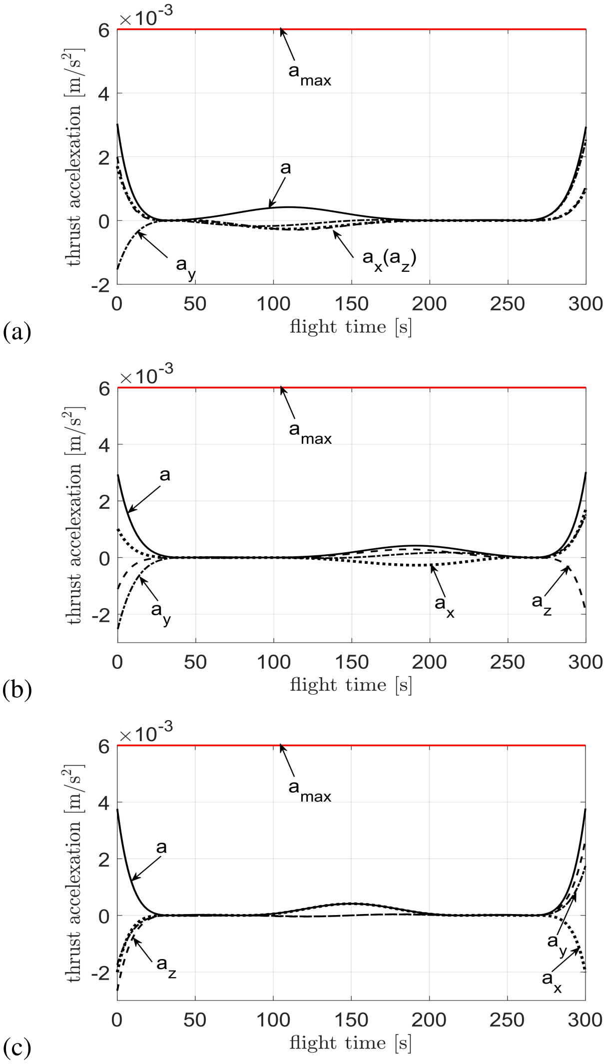

FIGURE 3. The thrust acceleration of three following satellites with Bezier method: (a) No.1 satellite (b) No.2 satellite (c) No.3 satellite.

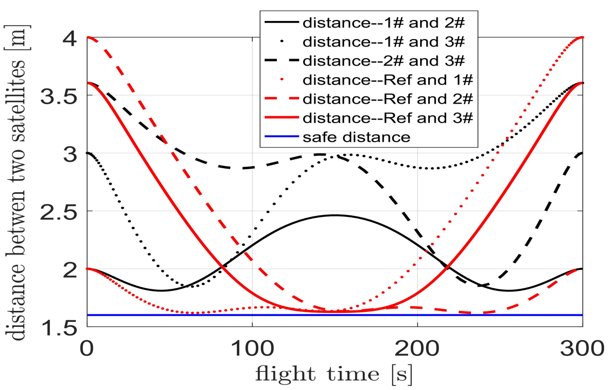

FIGURE 4. Distance between satellites with Bezier method.

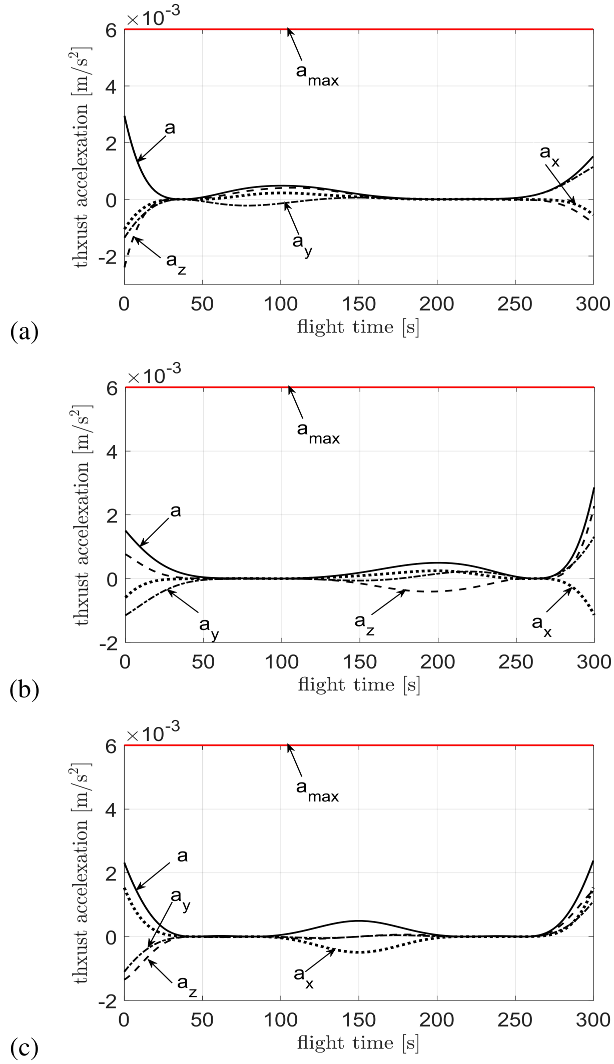

FIGURE 5. The thrust acceleration of three following satellites with FFS method: (a) No.1 satellite (b) No.2 satellite (c) No.3 satellite.

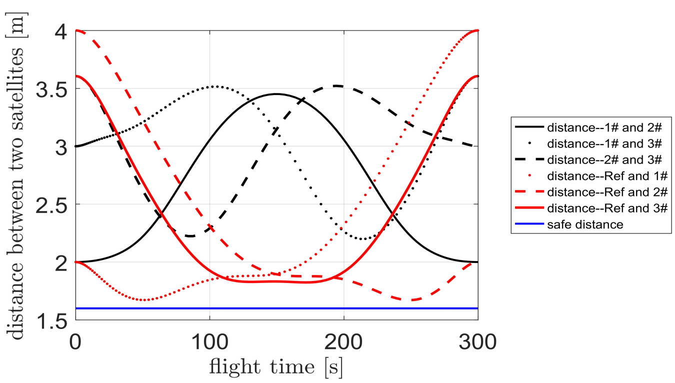

FIGURE 6. Distance between satellites with FFS method.

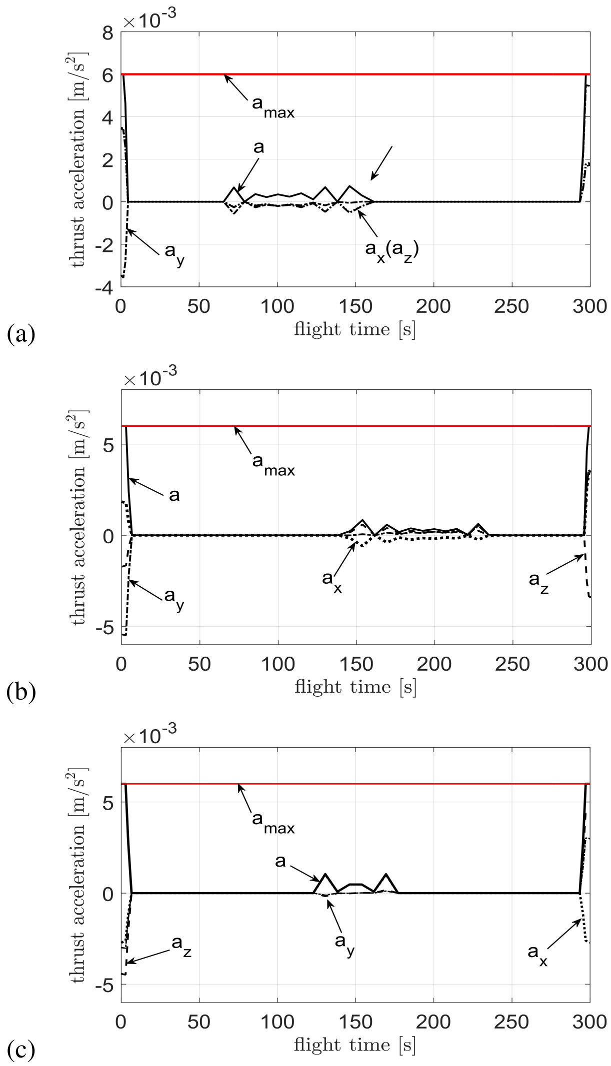

FIGURE 7. The thrust acceleration of three following satellites with GPM method: (a) No.1 satellite (b) No.2 satellite (c) No.3 satellite.

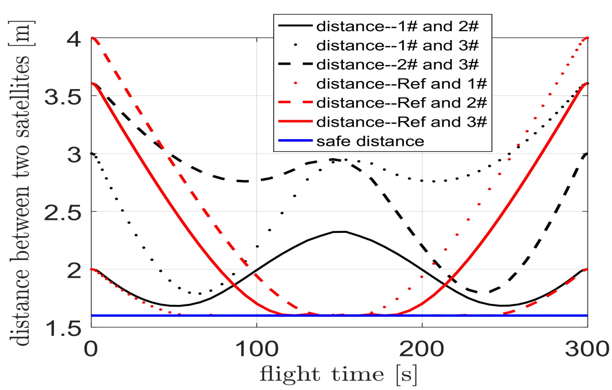

FIGURE 8. Distance between satellites with GPM method.

B. TETRAHEDRAL TO TETRAHEDRAL CONFIGURATION

A simulation example of three following satellites from tetrahedral to tetrahedral configuration is carried out. The design parameters are in agreement with the above example. The initial positions (x_i, y_i, z_i) (units are meter) of the three following satellites are respectively (2, -3, 0), (-1.5, -3, 3) and (-1.5, -3, -3), the terminal positions (x_f, y_f, z_f) of the three satellites are respectively (-1 ,2 ,0), (1, 2, -1.5) and (1, 2, 1.5), and the initial and terminal velocities of the three satellites are 0. The initial flight paths are designed by using Bezier, the further optimized flight paths are obtained by using the GPM solver, and the comparisions with FFS method are also implemented.

The results of the flight paths are illustrated in Fig. 9. The curves of black, red and green represent the results of Bezier, FFS and GPM methods, respectively. In this scenario, the total flying time obtained by using the proposed Bezier method is 299.992 s, and that obtained by using FFS and GPM is 299.568 s and 300.000 s respectively.

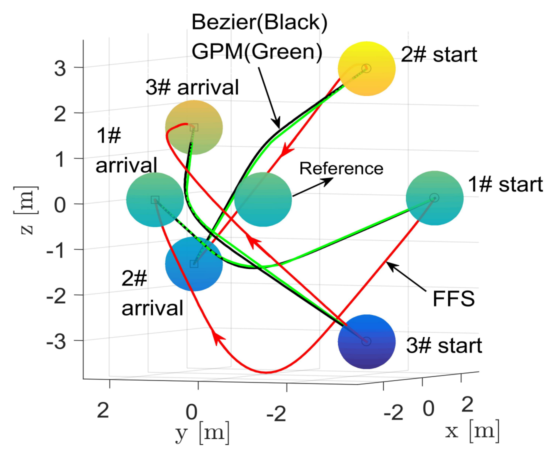

FIGURE 9. Formation reconfiguration of three following satellites with Bezier, FFS and GPM methods.

Fig. 10 is the thrust acceleration components of three following satellites and Fig. 11 shows the relative distance between two satellites, which is obtained by using Bezier. Fig. 12 is the thrust acceleration components of three following satellites and Fig. 13 shows the relative distance between two satellites, which is obtained by using FFS. The thrust acceleration components of three following satellites and the relative distance between two satellites obtained by using GPM are illustrated in Fig. 14 and Fig. 15.

V. RESULT ANALYSIS

In the simulation of the formation reconstruction of planar triangles, the initial flight paths are designed by using Bezier and FFS methods, and the solution obtained by Bezier method is used as an initial guess for the GPM solver. The results obtained by three methods are shown in Table 1. As shown in Table 1, the ΔV obtained by using the Bezier, FFS and GPM methods are 0.246 km/s, 0.274 km/s and

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Methods	ΔV of No.1 /(m/s)	ΔV of No.2 /(m/s)	ΔV of No.3 /(m/s)	Total ΔV /(m/s)	Total flight time/(s)	Computation time/(s)
Bezier	0.081	0.081	0.083	0.246	299.952	138.848
FFS	0.092	0.092	0.089	0.274	299.834	147.533
GPM	0.076	0.076	0.079	0.231	299.952	8573.984

TABLE 1. Planar Configuration to Planar Configuration: Comparisons of the results of Bezier, FFS and GPM methods.

Methods	ΔV of No.1 /(m/s)	ΔV of No.2 /(m/s)	ΔV of No.3 /(m/s)	Total ΔV /(m/s)	Total flight time/(s)	Computation time/(s)
Bezier	0.076	0.079	0.083	0.238	299.992	233.338
FFS	0.150	0.148	0.152	0.449	299.568	393.850
GPM	0.073	0.075	0.078	0.226	300.000	8882.182

TABLE 2. Tetrahedral to tetrahedral configuration: Comparisons of the results of Bezier, FFS and GPM methods.

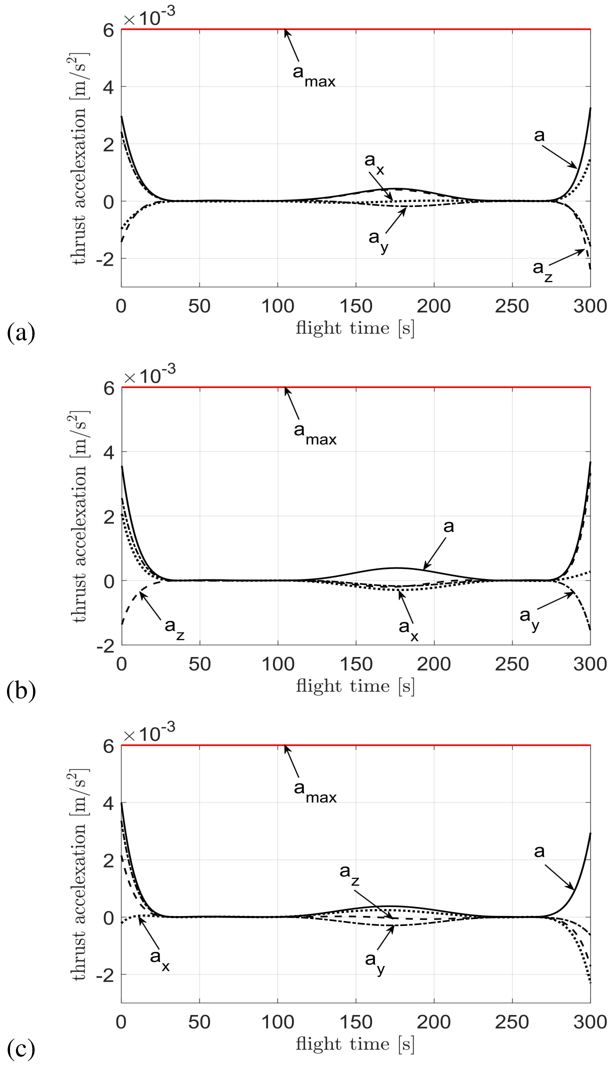

FIGURE 10. The thrust acceleration of three following satellites with Bezier method: (a) No.1 satellite (b) No.2 satellite (c) No.3 satellite.

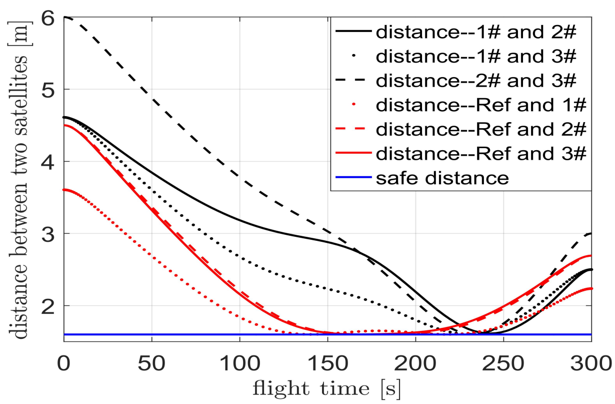

FIGURE 11. Distance between satellites with Bezier method.

0.231 km/s respectively. By comparison we can see that the difference between the performance index of the Bezier and GPM is only approximately 6.45 %, but Bezier only use 1.62% of the computational time used to further optimize and generate the flight paths by using GPM, even if the Bezier provided an initial guess for the GPM. Moreover, the numerical results show that the proposed Bezier method can get better performance-index solution in shorter computation time compared with the FFS method. The Bezier method uses 94.1% of the computational time obtained by FFS to get better flight paths, which only consumes 89.90% of ΔV obtained by FFS.

In the simulation of the formation reconstruction of space tetrahedron, the initial flight paths are designed by using Bezier and FFS methods, and the solution obtained by Bezier method is used as an initial guess for the GPM solver. The results obtained by three methods are shown in Table 2. As shown in Table 2, the ΔV obtained by using the Bezier, FFS and GPM methods are 0.238 km/s, 0.449 km/s and 0.226 km/s respectively. By comparison we can see that the difference between the performance index of the Bezier and GPM is only approximately 5.33 %, but Bezier only use 2.63% of the computational time by using GPM, even if the Bezier provided an initial guess for the GPM. Moreover, the Bezier

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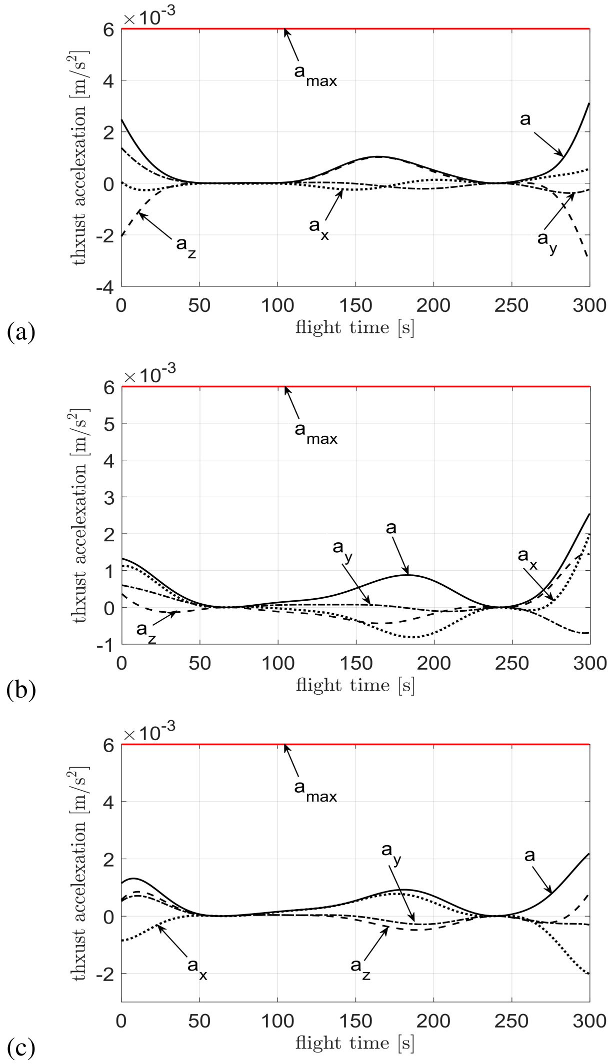

FIGURE 12. The thrust acceleration of three following satellites with FFS method: (a) No.1 satellite (b) No.2 satellite (c) No.3 satellite.

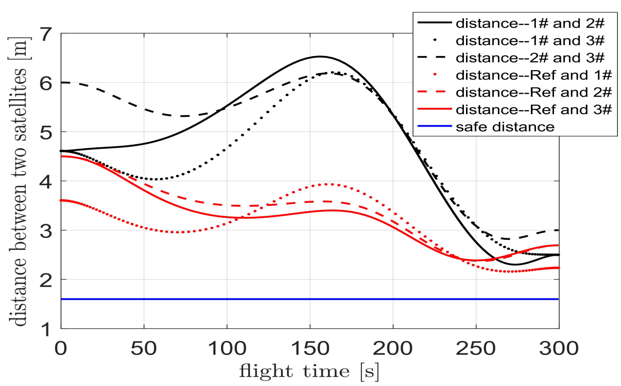

FIGURE 13. Distance between satellites with FFS method.

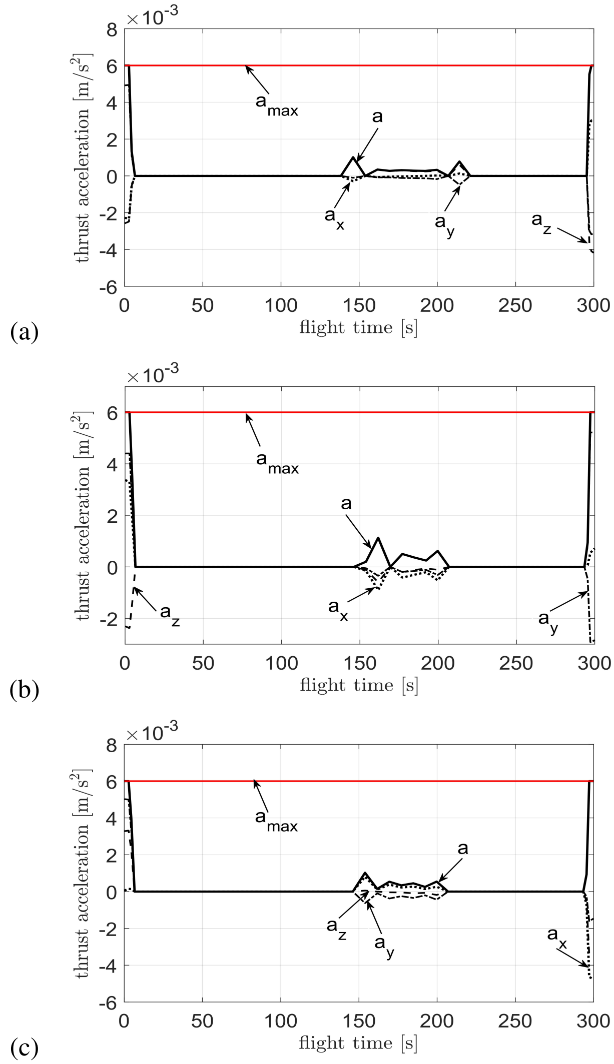

FIGURE 14. The thrust acceleration of three following satellites with GPM method: (a) No.1 satellite (b) No.2 satellite (c) No.3 satellite.

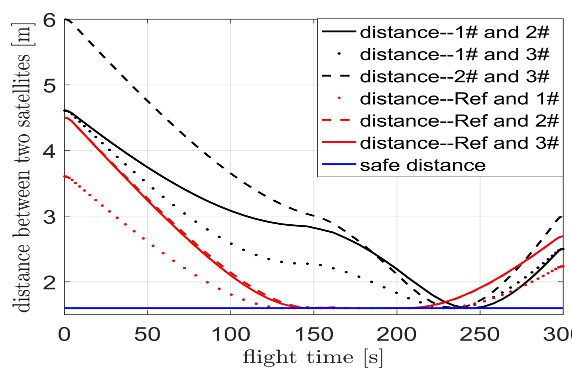

FIGURE 15. Distance between satellites with GPM method.

method uses 59.25% of the computational time obtained by FFS to get better flight paths, which only consumes 52.96% of ΔV obtained by FFS.

From the two examples of the formation reconstruction, the Bezier method can use shorter computation time to obtain a reasonable three-dimensional initial guess with better performance index. This is mainly because the Bezier method does not need to repeatedly calculate the coefficient matrices regardless of the time-fixed and time-free problem. In contrast, some of the matrices in the FFS method needs to be calculated within each iteration for the time-free rendezvous problem. When calculating a single flight path, the difference in computing time caused by the matrices calculation repeatability may be only a few seconds. However, when multiple flight paths are generated simultaneously, the repeatability of the matrices calculation will be magnified to many times. When the formation changes more complex, the time superiority of the Bezier method is more obvious than the FFS method. Through the results, we can see that the Bezier method can design a reasonable three-dimensional initial collision-avoidance flight paths for optimization solver in a short time. This is very important for the rapid generation of initial solutions for numerous flight scenarios at the stage of initial task design and onboard satellite formation reconfiguration.

VI. CONCLUSION

This paper presents a method to rapidly generate the low-thrust collision-avoidance three-dimensional trajectories in the formation reconfiguration by using the Bezier Curve. In order to verify the progressiveness of the proposed method, the comparisons with the FFS method are implemented through numerical simulation. The numerical results show that the proposed Bezier method can rapidly generate flight paths, effectively reduce the risk of collision between satellites and get better performance-index solution in shorter computation time than the FFS method. When formation changes more complex, the time superiority of the Bezier is more obvious than the FFS method. The paper made the obtained solutions by the Bezier method as an initial guess for a GPM solver to evaluate the suitability of the solutions. Compared with the GPM, the Bezier method only takes very little computation time to get the result that the performance index has very few difference. The numerical results indicate that the proposed Bezier method can design reasonable collision-avoidance three-dimensional initial flight paths for an optimization solver in a short time. This is very important for the rapid generation of initial solutions for a lot of flight scenarios at the stage of preliminary mission design and onboard satellite formation reconfiguration.

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