SI1001 Theory of Computation
Homework 2
Barendregt's Encoding of Natural Numbers

Andrés Sicard-Ramírez

7th April 2025, version 0.2

1 Deadline

See the course homepage.

2 Barendregt's Encoding of Natural Numbers

Let M and N be λ-terms. Barendregt's encoding of natural numbers is given by the following combinators:

$$
\begin{aligned}
\mathsf{true} &:= \lambda\, x.\, \lambda\, y.\, x, && \text{(Booleans)}\\
\mathsf{false} &:= \lambda\, x.\, \lambda\, y.\, y, && \\
[\, M, N\,] &:= \lambda z.\, z\, M\, N, && \text{(order pair)}\\
\mathsf{fst} &:= \lambda p.\, p\, \mathsf{true}, && \\
\mathsf{snd} &:= \lambda p.\, p\, \mathsf{false}, && \\
\overline{0} &:= \lambda x.x, && \text{(numerals)}\\
\overline{n+1} &:= [\, \mathsf{false}, \overline{n}\,], && \\
\mathsf{S}^{+} &:= \lambda x.[\, \mathsf{false}, \overline{x}\,], && \text{(successor)}\\
\mathsf{P}^{-} &:= \lambda x.\, x\, \mathsf{false}, && \text{(predecessor)}\\
\mathsf{Zero} &:= \lambda x.\, x\, \mathsf{true}. && \text{(test for zero)}
\end{aligned}
$$

The above combinators satisfy the following theorems:

$$
\begin{aligned}
\lambda \vdash \mathsf{true}\, M\, N &=_{\text{conv}} M,\\
\lambda \vdash \mathsf{false}\, M\, N &=_{\text{conv}} N,\\
\lambda \vdash \mathsf{fst}\, [\, M, N\,] &=_{\text{conv}} M,\\
\lambda \vdash \mathsf{snd}\, [\, M, N\,] &=_{\text{conv}} N,\\
\lambda \vdash \mathsf{S}^{+}\, \overline{n} &=_{\text{conv}} \overline{n+1},\\
\lambda \vdash \mathsf{P}^{-}\, \overline{0} &=_{\text{conv}} \mathsf{false},\\
\lambda \vdash \mathsf{P}^{-}\, \overline{n+1} &=_{\text{conv}} \overline{n},\\
\lambda \vdash \mathsf{Zero}\, \overline{0} &=_{\text{conv}} \mathsf{true},\\
\lambda \vdash \mathsf{Zero}\, \overline{n+1} &=_{\text{conv}} \mathsf{false}.
\end{aligned}
$$

3 Assignment (95%)

(i) (20%) For each of the combinators S^{+}, P^{-}, and Zero, write two examples showing its behaviour using the Lambda Shell program[1].

(ii) (5%) To document (in English) your source code. The documentation should explain your solution.

(iii) (70%) Oral explanation of your solution.

4 Requirements (5%)

(i) The homework should be solved with other student taking the course.

(ii) To add to the repository a `README.md` file (Markdown format) in English containing the following information:

- Your(s) full name(s).
- Versions used of operating system, compiler and tools in your implementation.
- Detailed instructions for running your solution.
- Any information (books, articles, videos, AIs, repositories, etc.) you did use for the homework.

(iii) Do not include unnecessary files or directories in the repository.

[1] See instructions for its installation in the course homepage.

5 Clean code

Before submitting your code, which includes your `README.md` file, clean it up:

- Does not have long lines (at most 80 columns).
- Has an uniformly indentation (we recommended two characters).
- Has a consistent layout.
- Has good comments.
- Has no junk (unused code, commented code, unnecessary code).
- Has no overly complicated function definitions.
- Does not contain any repetitive code.
- Has no tabs.
- Has no unnecessary spaces at the end of a lines, or empty lines at the end of a file.
- Has spell-checked comments.

6 Delivery

I shall send the GitHub Education link to the final project via EAFIT Interactiva.

7 From the coordination

El control de versiones no es solamente un herramienta que facilitará la comunicación entre los miembros del grupo y la administración de los cambios al código. El control de versiones también ayudará al profesor a llevar un control sobre el desarrollo de la práctica. Se espera que las diferentes registros dentro del control de versiones sean cambios graduales. En caso contrario, se procederá a realizar un escrutinio con el objetivo de evitar fraudes.