

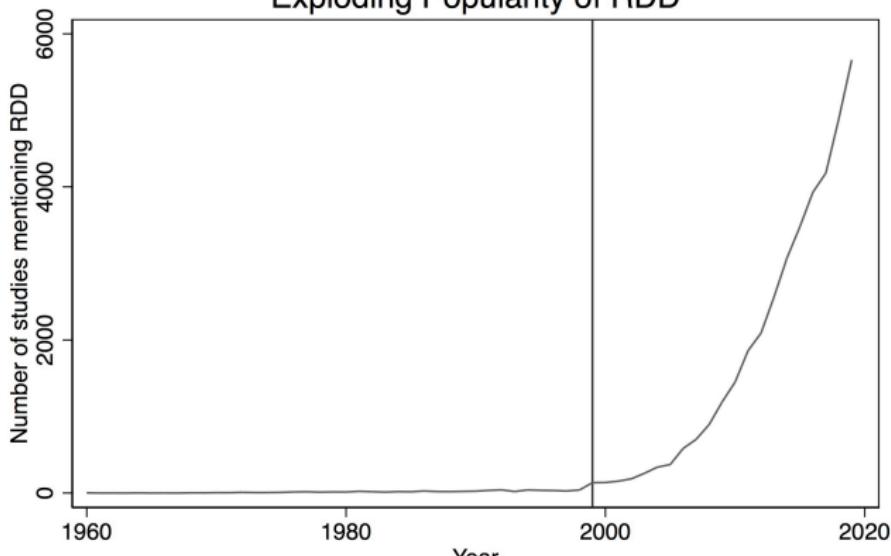
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What is regression discontinuity design?

Very popular particular type of research design known as *regression discontinuity design* (RDD). Cook (2008) has a fascinating history of thought on how and why.

- Donald Campbell, educational psychologist, invented regression discontinuity design (Thistlethwaite and Campbell, 1960), but then it went dormant for decades (Cook 2008).
- Angrist and Lavy (1999) and Black (1999) independently rediscover it. It's become incredibly popular in economics

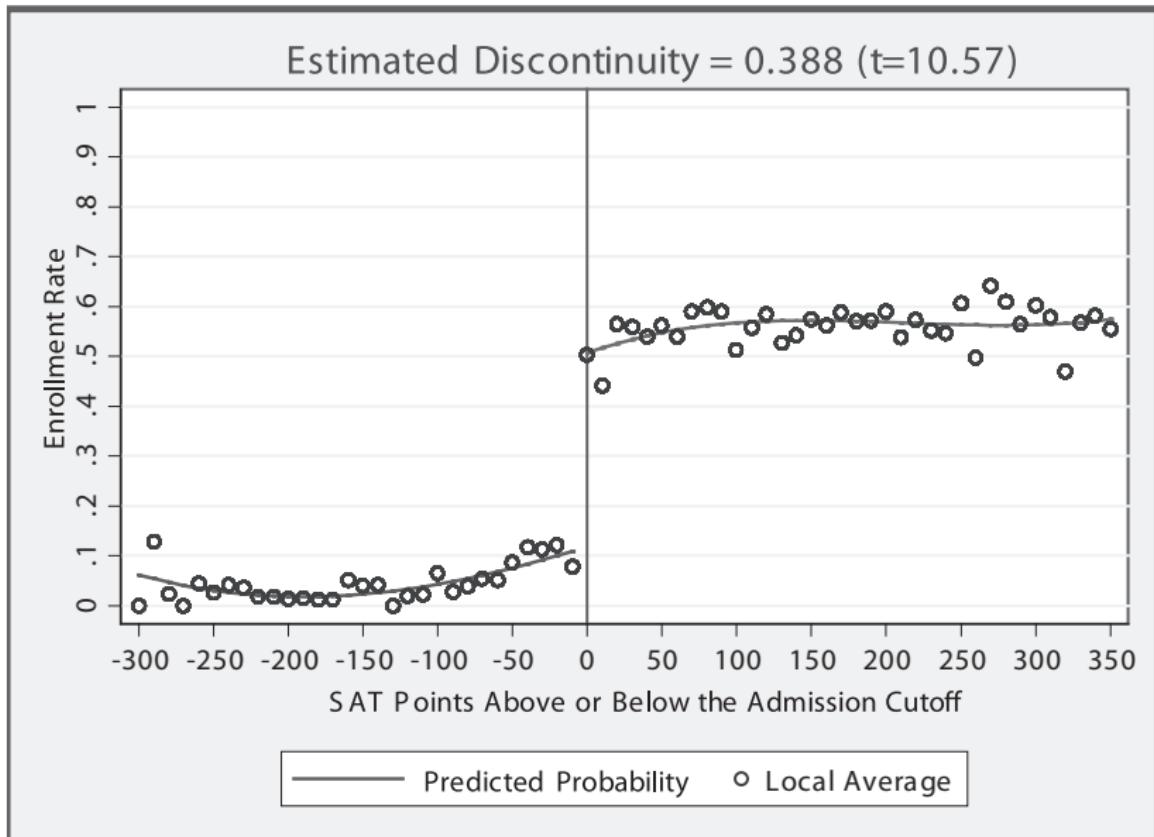
Exploding Popularity of RDD



Vertical bar is Angrist and Lavy (1999) and Black (1999)

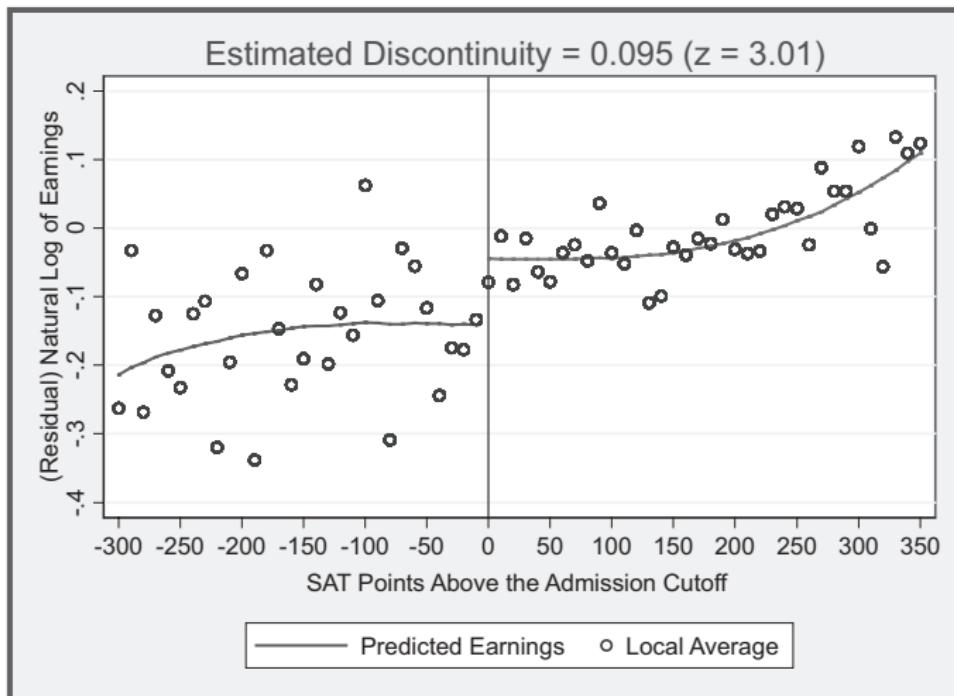
Tell me what you think is happening

FIGURE 1.—FRACTION ENROLLED AT THE FLAGSHIP STATE UNIVERSITY



Tell me what you think is happening

FIGURE 2.—NATURAL LOG OF ANNUAL EARNINGS FOR WHITE MEN TEN TO FIFTEEN YEARS AFTER HIGH SCHOOL GRADUATION (FIT WITH A CUBIC POLYNOMIAL OF ADJUSTED SAT SCORE)



What is a regression discontinuity design?

- We want to estimate some causal effect of a treatment on some outcome, but we're worried about selection bias

$$E[Y^0|D = 1] \neq E[Y^0|D = 0]$$

due to self-selection into treatment

- RDD is based on a idea: if treatment assignment occurs abruptly when some underlying variable X called the “running variable” passes a cutoff c_0 , then we can use that to estimate the causal effect *even of a self-selected treatment*

Running and jumping

- Firms, schools and govt agencies have running variables that are used to assign treatments in their rules
- And consequently, probabilities of treatment will “jump” when that running variable exceeds a known threshold
- Most effective RDD studies involve programs where running variables assign treatments based on a “hair trigger”
- Good reasons; inexplicable reasons; arbitrary rules; a choice made by necessity and resource constraints; natural experiments

Selection examples and solutions from the literature

Think of these in light of a treatment where

$$E[Y^0|D = 1] \neq E[Y^0|D = 0]$$

- Yelp rounded a continuous score of ratings to generate stars which Anderson and Magruder 2011 used to study firm revenue
- US targeted air strikes in Vietnam using rounded risk scores which Dell and Querubin 2018 used to study the military and political activities of the communist state
- Card, Dobkin, and Maekas 2008 studied the effect of universal healthcare on mortality and healthcare usage exploiting jumps at age 65
- Almond, et al. 2010 studied the effect of intensive medical attention on health outcomes when a newborn's birthweight fell just below 1,500 grams

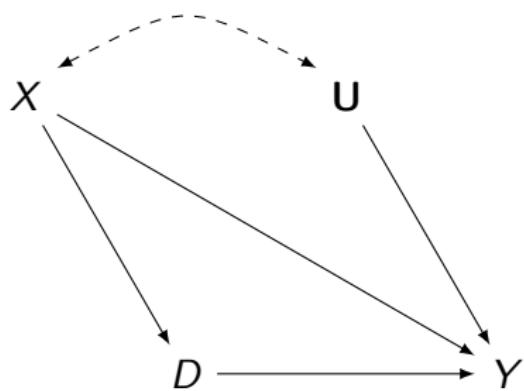
Hungry, hungry hippo

- Data requirements can be substantial. Large sample sizes are characteristic features of the RDD
 - If there are strong trends, one typically needs a lot of data for reasons I'll explain soon
 - Researchers are typically using administrative data or settings such as birth records where there are **many** observations
- Might explain why the method never caught on until the 00's

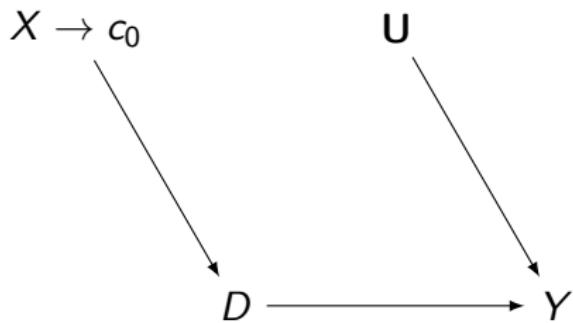
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(A) Data generating graph



(B) Limiting graph



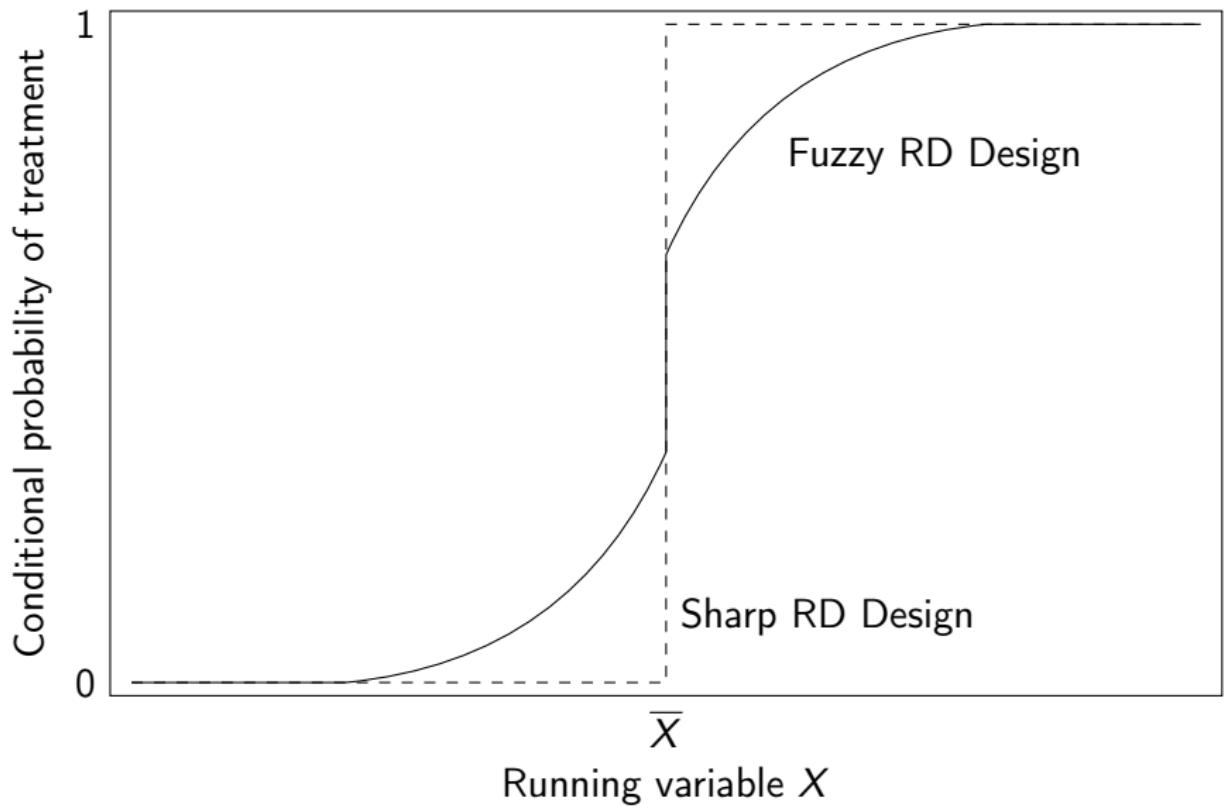


Figure: Sharp vs. Fuzzy RDD

Sharp vs. Fuzzy RDD

- There's traditionally thought to be two kinds of RD designs:
 - ① Sharp RDD: Treatment is a deterministic function of running variable, X . Example: Medicare benefits.
 - ② Fuzzy RDD: Discontinuous “jump” in the *probability* of treatment when $X > c_0$. Cutoff is used as an instrumental variable for treatment. Example: attending state flagship
- Fuzzy is a type of IV strategy and requires explicit IV estimators like 2SLS; sharp is reduced form IV and doesn't require IV-like estimators

Overlap

- Independence implies an equal distribution of characteristics across two groups guaranteeing overlap
 - In an RCT you can find 65 year olds treated and untreated
- But RDD doesn't have this feature bc you don't have groups with the same value of X in each group, so no overlap
 - 64 years olds are control, not treatment. 66 years olds are in treatment not control
- Some methods require overlap and therefore are off the table without it; but RDD has a workaround using extrapolation

Treatment assignment in the sharp RDD

Deterministic treatment assignment ("sharp RDD")

In Sharp RDD, treatment status is a deterministic and discontinuous function of a covariate, X_i :

$$D_i = \begin{cases} 1 & \text{if } X_i \geq c_0 \\ 0 & \text{if } X_i < c_0 \end{cases}$$

where c_0 is a known threshold or cutoff. In other words, if you know the value of X_i for a unit i , you know treatment assignment for unit i with certainty.

Universal health insurance: Americans aged 64 are not eligible for Medicare, but Americans aged 65 ($X \geq c_{65}$) are eligible for Medicare (ignoring disability exemptions)

Treatment effect definition and estimation

Definition of treatment effect

The treatment effect parameter, δ , is the discontinuity in the conditional expectation function:

$$\begin{aligned}\delta &= \lim_{X_i \rightarrow c_0} E[Y_i^1 | X_i = c_0] - \lim_{c_0 \leftarrow X_i} E[Y_i^0 | X_i = c_0] \\ &= \lim_{X_i \rightarrow c_0} E[Y_i | X_i = c_0] - \lim_{c_0 \leftarrow X_i} E[Y_i | X_i = c_0]\end{aligned}$$

The sharp RDD estimation is interpreted as an average causal effect of the treatment at the discontinuity

$$\delta_{SRD} = E[Y_i^1 - Y_i^0 | X_i = c_0]$$

D is correlated with X and deterministic function of X ; overlap only occurs in the limit and thus the treatment effect is in the limit as X approaches c_0

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Extrapolation

- In RDD, the counterfactuals are conditional on X .
- We use *extrapolation* in estimating treatment effects with the sharp RDD bc we do not have overlap
 - Left of cutoff, only non-treated observations, $D_i = 0$ for $X < c_0$
 - Right of cutoff, only treated observations, $D_i = 1$ for $X \geq c_0$
- The extrapolation is to a counterfactual

Extrapolation

Estimation methods attempt to approximate the limiting parameter using units left and right of the cutoff

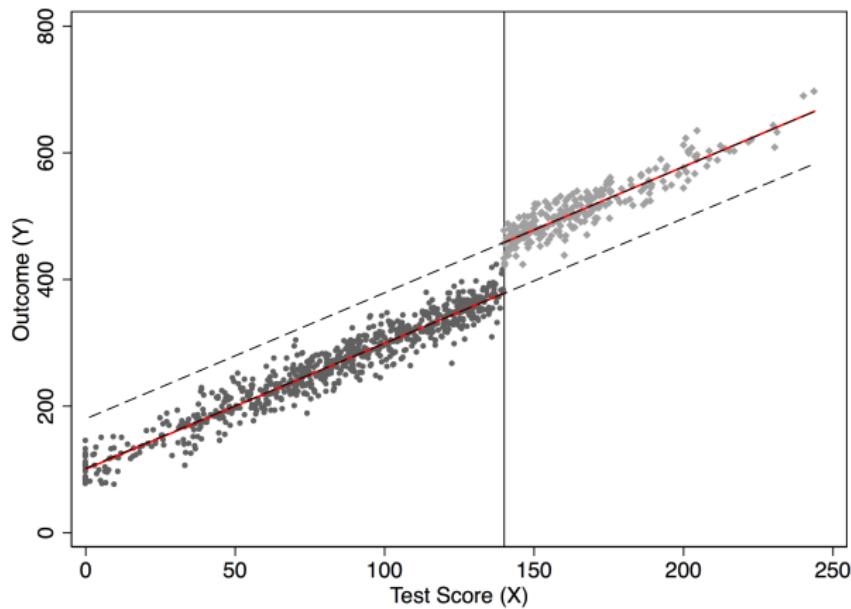


Figure: Dashed lines are extrapolations

Key identifying assumption

Smoothness (or continuity) of conditional expectation functions
(Hahn, Todd and Van der Klaauw 2001)

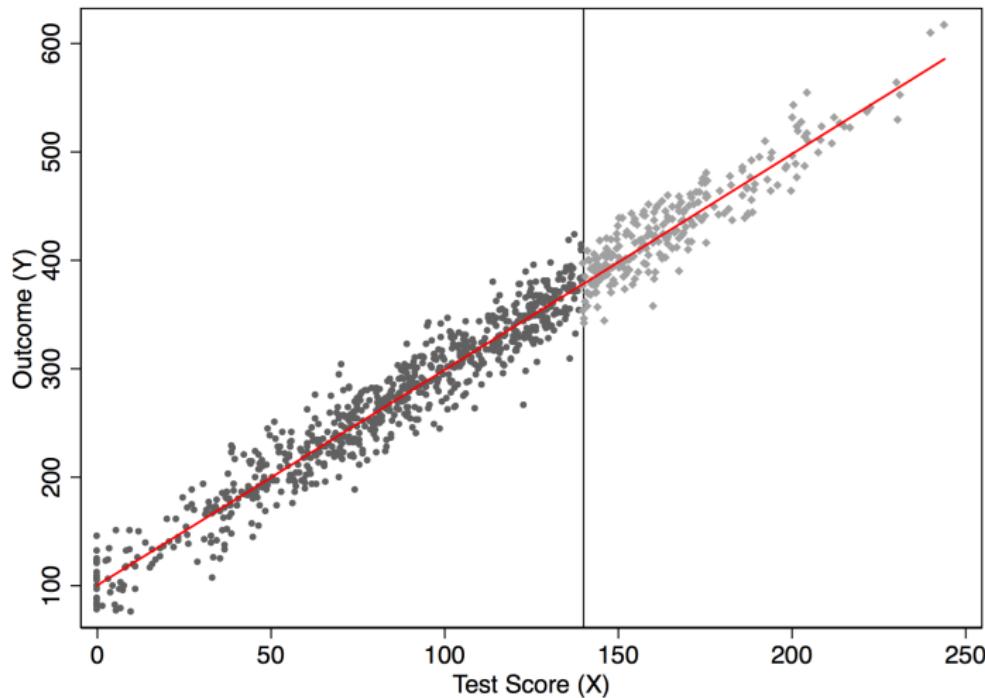
$E[Y_i^0|X = c_0]$ and $E[Y_i^1|X = c_0]$ are continuous (smooth) in X at c_0 .

- Potential outcomes not actual outcomes
- If population average *potential outcomes*, Y^1 and Y^0 , are smooth functions of X through the cutoff, c_0 , then potential average outcomes *won't jump* at c_0 .
- Implies the cutoff is exogenous – i.e., nothing else changes related to potential outcomes at c_0
- Unobservables are evolving smoothly, too, through the cutoff

Smoothness is the identifying assumption and untestable

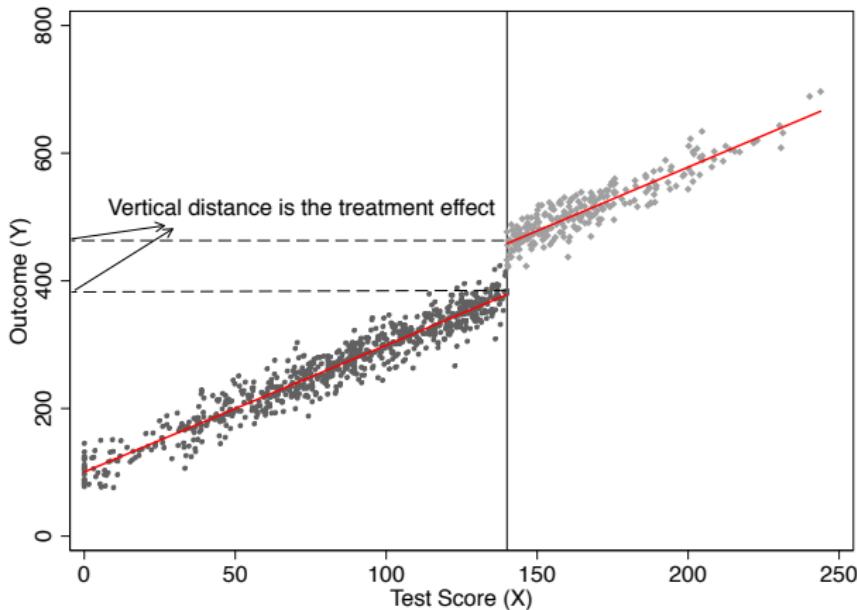
- The smoothness assumption allows us to use average outcome of units right below the cutoff as a valid counterfactual for units right above the cutoff.
- In other words, extrapolation is allowed if smoothness is credible, and extrapolation is nonsensical if smoothing isn't credible
- The causal effect of the treatment will be based on **extrapolation** from the trend, $E[Y_i^0|X < c_0]$, to those values of $X > c_0$ for the $E[Y_i^0|X > c_0]$.
- Means you have to think long and hard about smoothness and what violations mean in your context
- Why then is it not directly testable? Because potential outcomes are counterfactual

Graphical example of the smoothness assumption



Note these are *potential* not *actual* outcomes

Graphical example of the treatment effect, not the smoothness assumption



Note that these are *actual*, not *potential* outcomes

Re-centering the data

- It is common for authors to transform X by “centering” at c_0 :

$$Y_i = \alpha + \beta(X_i - c_0) + \delta D_i + \varepsilon_i$$

- This doesn’t change the interpretation of the treatment effect – only the interpretation of the intercept.

Re-centering the data

- Example: Medicare and age 65. Center the running variable (age) by subtracting 65:

$$\begin{aligned}Y &= \beta_0 + \beta_1(Age - 65) + \beta_2Edu \\&= \beta_0 + \beta_1Age - \beta_165 + \beta_2Edu \\&= \alpha + \beta_1Age + \beta_2Edu\end{aligned}$$

where $\alpha = \beta_0 - \beta_165$.

- All other coefficients, notice, have the same interpretation, except for the intercept.

Regression without re-centering

```
reg y D x
```

Source	SS	df	MS	Number of obs =	999
Model	15842893.9	2	7921446.97	F(2, 996) =	19988.47
Residual	394715.557	996	396.30076	Prob > F =	0.0000
Total	16237609.5	998	16270.1498	R-squared =	0.9757

y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
D	80.01418	2.144779	37.31	0.000	75.80537 84.22298
x	1.986975	0.186779	106.38	0.000	1.950322 2.023627
_cons	100.3885	1.70944	58.73	0.000	97.03397 103.743

Regression with centering

```
gen x_c = x - 140
```

```
reg y D x_c
```

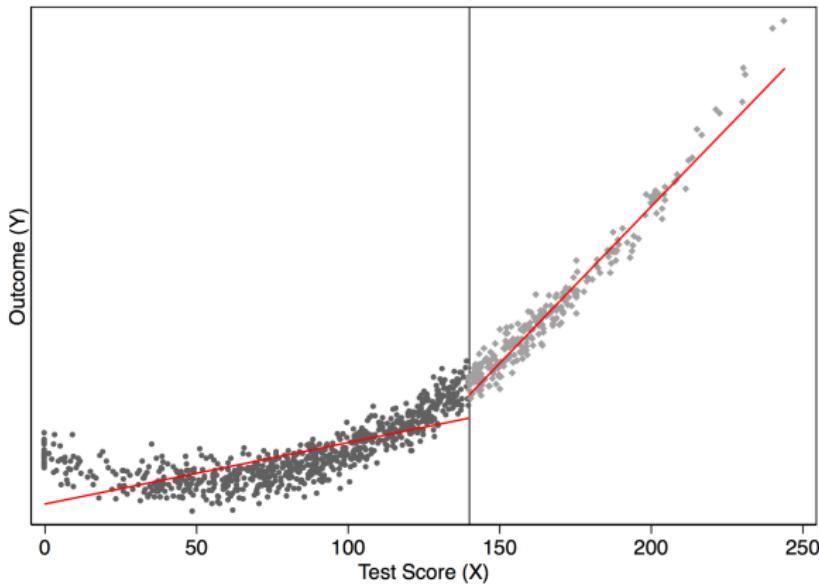
Source	SS	df	MS	Number of obs =	999
Model	15842893.9	2	7921446.97	F(2, 996) =	19988.47
Residual	394715.554	996	396.300757	Prob > F =	0.0000
Total	16237609.5	998	16270.1498	R-squared =	0.9757
				Adj R-squared =	0.9756
				Root MSE =	19.907

y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
D	80.01418	2.144779	37.31	0.000	75.80537 84.22298
x_c	1.986975	.0186779	106.38	0.000	1.950322 2.023627
cons	378.565	1.290755	293.29	0.000	376.032 381.0979

Nonlinearity bias

- Smoothness and *linearity* are different things.
- What if the trend relation $E[Y_i^0|X_i]$ does not jump at c_0 but rather is simply nonlinear?
- Then your linear model will identify a treatment effect when there isn't because the functional form had poor predictive properties beyond the cutoff
- Let's look at a simulation

```
gen x2 = x*x  
  
gen x3 = x*x*x  
  
gen y = 10000 + 0*D    100*x +x2 + rnormal(0, 1000)  
  
scatter y x if D==0, msize(vsmall) || scatter y x /  
if D==1, msize(vsmall) legend(off) xline(140, ///  
lstyle(foreground)) ylabel(none) || lfit y x ///  
if D ==0, color(red) || lfit y x if D ==1, ///  
color(red) xtitle("Test Score (X)") ///  
ytitle("Outcome (Y)")
```



See how the two lines don't touch at c_0 but empirically should?
That's bc the linear fit is the wrong functional form – we know this
from the simulation that it's the wrong functional form.

Sharp RDD: Nonlinear Case

- Suppose the nonlinear relationship is $E[Y_i^0|X_i] = f(X_i)$ for some reasonably smooth function $f(X_i)$ (drumroll – like a cubic!)
- In that case we'd fit the regression model:

$$Y_i = f(X_i) + \delta D_i + \eta_i$$

- Since $f(X_i)$ is counterfactual for values of $X_i > c_0$, how will we model the nonlinearity?
- There are 2 common ways of approximating $f(X_i)$

Nonlinearities

People until Gelman and Imbens 2018 favored “higher order polynomials” but this is problematic due to overfitting. Gelman and Imbens 2018 recommend at best a quadratic

- ① Use global and local regressions with $f(X_i)$ equalling a p^{th} order polynomial

$$Y_i = \alpha + \delta D_i + \beta_1 x_i + \beta_2 x_i^2 + \cdots + \beta_p x_i^p + \eta_i$$

- ② Or use some nonparametric kernel method which I'll cover later

Different polynomials on the 2 sides of the discontinuity

- We can generalize the function, $f(x_i)$, by allowing it to differ on both sides of the cutoff by including them both individually and interacting them with D_i .
- In that case we have:

$$E[Y_i^0|X_i] = \alpha + \beta_{01}\tilde{X}_i + \beta_{02}\tilde{X}_i^2 + \cdots + \beta_{0p}\tilde{X}_i^p$$

$$E[Y_i^1|X_i] = \alpha + \delta + \beta_{11}\tilde{X}_i + \beta_{12}\tilde{X}_i^2 + \cdots + \beta_{1p}\tilde{X}_i^p$$

where \tilde{X}_i is the centered running variable (i.e., $X_i - c_0$).

Lines to the left, lines to the right of the cutoff

- Re-centering at c_0 ensures that the treatment effect at $X_i = c_0$ is the coefficient on D_i in a regression model with interaction terms
- As Lee and Lemieux (2010) note, allowing different functions on both sides of the discontinuity should be the main results in an RDD paper

Different polynomials on the 2 sides of the discontinuity

- To derive a regression model, first note that the observed values must be used in place of the potential outcomes:

$$E[Y|X] = E[Y^0|X] + (E[Y^1|X] - E[Y^0|X]) D$$

which is the switching equation from earlier expressed in terms of conditional expectation functions

- Regression model you estimate is:

$$\begin{aligned} Y_i &= \alpha + \beta_{01}\tilde{x}_i + \beta_{02}\tilde{x}_i^2 + \cdots + \beta_{0p}\tilde{x}_i^p \\ &\quad + \delta D_i + \beta_1^* D_i \tilde{x}_i + \beta_2^* D_i \tilde{x}_i^2 + \cdots + \beta_p^* D_i \tilde{x}_i^p + \varepsilon_i \end{aligned}$$

where $\beta_1^* = \beta_{11} - \beta_{01}$, $\beta_2^* = \beta_{21} - \beta_{20}$ and $\beta_p^* = \beta_{1p} - \beta_{0p}$

- The treatment effect at c_0 is δ

Polynomial simulation example

```
capture drop y x2 x3

gen x2 = x*x
gen x3 = x*x*x
gen y = 10000 + 0*D    100*x +x2 + rnormal(0, 1000)

reg y D x x2 x3
predict yhat

scatter y x if D==0, msize(vsmall) || scatter y x
if D==1, msize(vsmall) legend(off) xline(140,
lstyle(foreground)) ylabel(none) || line yhat x
if D ==0, color(red) sort || line yhat x if D==1,
sort color(red) xtitle("Test Score (X)")
ytitle("Outcome (Y)")
```

Polynomial simulation example

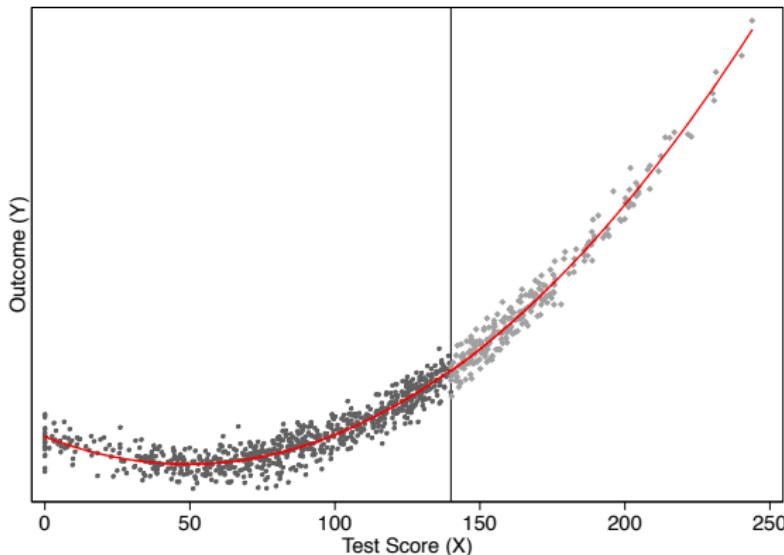


Figure: Third degree polynomial. Actual model second degree polynomial.

Notice: no more gap at c_0 once we model the function $f(x)$

Stata simulation

```
gen x2_c = x2 - 140
```

```
gen x3_c = x3 - 140
```

```
reg y D x x2
```

```
reg y D x_c x2_c
```

Polynomial simulation example

Source	SS	df	MS	Number of obs =	999
Model	3.7863e+10	3	1.2621e+10	F(3, 995) =	13115.22
Residual	957507024	995	962318.617	Prob > F =	0.0000
Total	3.8821e+10	998	38898361.8	R-squared =	0.9753
				Adj R-squared =	0.9753
				Root MSE =	980.98

y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
D	-115.5381	127.4967	-0.91	0.365	-365.7314 134.6552
x	-98.57582	2.285769	-43.13	0.000	-103.0613 -94.09034
x2	1.000001	.0122767	81.45	0.000	.9759098 1.024092
_cons	9864.218	111.1206	88.77	0.000	9646.16 10082.28

Notice: no more gap at c_0 once we model the function $f(x)$ (e.g., D is insignificant once we include polynomials)

Polynomial simulation example

Source	SS	df	MS	Number of obs	=	999
Model	3.7863e+10	3	1.2621e+10	F(3, 995)	=	13115.22
Residual	957507020	995	962318.613	Prob > F	=	0.0000
Total	3.8821e+10	998	38898361.8	R-squared	=	0.9753

y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
D	-115.5381	127.4967	-0.91	0.365	-365.7315	134.6552
x_c	-98.57582	2.285769	-43.13	0.000	-103.0613	-94.09034
x2_c	1.000001	.0122767	81.45	0.000	.9759098	1.024092
_cons	-3796.397	227.7894	-16.67	0.000	-4243.4	-3349.394

And centering did nothing to the interpretation of the main results (D), only to the intercept.

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Robustness against what?

- Are you done now that you have your main results? No
- Your main results are only causal insofar as smoothness is a credible belief, and since smoothness isn't guaranteed by "the science" like an RCT, you have to build your case
- You must now scrutinize alternative hypotheses that are consistent with your main results through sensitivity checks, placebos and alternative approaches

Main Challenges

Classify your concern regarding smoothness violations into two categories:

- Manipulation on the running variable
- Endogeneity of the cutoff

Most robustness is aimed at building credibility around these,

Manipulation of your running variable score

- Treatment is not as good as randomly assigned around the cutoff, c_0 , when agents are able to manipulate their running variable scores. This happens when:
 - ① the assignment rule is known in advance
 - ② agents are interested in adjusting
 - ③ agents have time to adjust
 - ④ administrative quirks like nonrandom heaping along the running variable

Examples include re-taking an exam, self-reported income, certain types of non-random rounding.

- Since necessarily treatment assignment is no longer independent of potential outcomes, it's likely this implies smoothness has been violated

Test 1: Manipulation of the running variable

Manipulation of the running variable

Assume a desirable treatment, D , and an assignment rule $X \geq c_0$. If individuals sort into D by choosing X such that $X \geq c_0$, then we say individuals are manipulating the running variable.

Also can be called “sorting on the running variable” – same thing

A badly designed RCT

- Suppose a doctor randomly assigns heart patients to statin and placebo to study the effect of the statin on heart attacks within 10 years
- Patients are placed in two different waiting rooms, *A* and *B*, and plans to give those in *A* the statin and those in *B* the placebo.
- The doors are unlocked and movement between the two can happen
- Versions of this happened with HIV RCTs in the 1980s ironically in which medication from treatment group was given to the control group, but I'm talking about something a little different

McCrary Density Test

We would expect waiting room A to become *crowded*. In the RDD context, sorting on the running variable implies heaping on the “good side” of c_0

- McCrary (2008) suggests a formal test: under the null the *density* should be continuous at the cutoff point.
- Under the alternative hypothesis, the density should increase at the kink (where D is viewed as good)
 - ① Partition the assignment variable into bins and calculate frequencies (i.e., number of observations) in each bin
 - ② Treat those frequency counts as dependent variable in a local linear regression
- This is oftentimes visualized with confidence intervals illustrating the effect of the discontinuity on density - you need no jump to pass this test

McCrary density test

- The McCrary Density Test has become **mandatory** for every analysis using RDD.
 - If you can estimate the conditional expectations, you evidently have data on the running variable. So in principle you can always do a density test
 - You can download the (no longer supported) Stata ado package, DCdensity, to implement McCrary's density test (<http://eml.berkeley.edu/~jmccrary/DCdensity/>)
 - You can install `rdrobust` for Stata and R too, and it will implement the test

Caveats about McCrary Density Test

- For RDD to be useful, you already need to know something about the mechanism generating the assignment variable and how susceptible it could be to manipulation. Note the rationality of economic actors that this test is built on.
- A discontinuity in the density is “suspicious” – it *suggests* manipulation of X around the cutoff is probably going on. In principle one doesn’t need continuity.
- This is a high-powered test. You need a lot of observations at c_0 to distinguish a discontinuity in the density from noise.

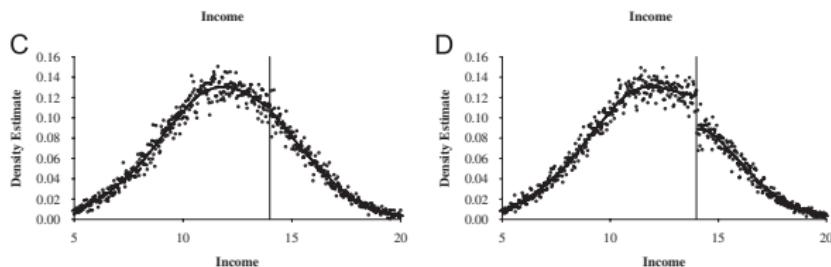
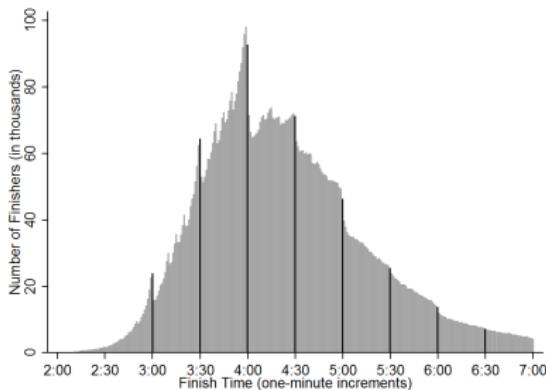


Figure: Panel C is density of income when there is no pre-announcement and no manipulation. Panel D is the density of income when there is pre-announcement and manipulation. From McCrary (2008).

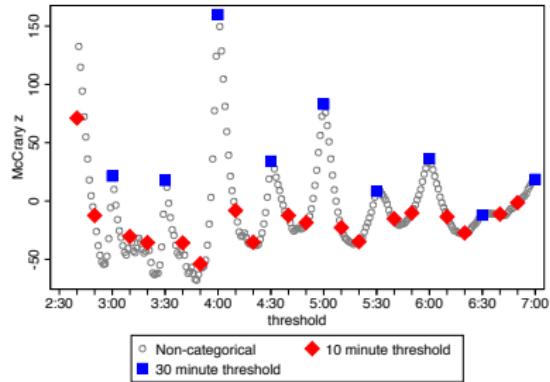
Visualizing manipulation

Figure 2: Distribution of marathon finishing times ($n = 9,378,546$)



NOTE: The dark bars highlight the density in the minute bin just prior to each 30 minute threshold.

Figure 3: Running McCrary z -statistic



NOTE: The McCrary test is run at each minute threshold from 2:40 to 7:00 to test whether there is a significant discontinuity in the density function at that threshold.

Figure: Figures 2 and 3 from Eric Allen, Patricia Dechow, Devin Pope and George Wu's (2013)
"Reference-Dependent Preferences: Evidence from Marathon Runners".

http://faculty.chicagobooth.edu/devin.pope/research/pdf/Website_Marathons.pdf

Newborn mortality and medical expenditure

- Almond, et al. 2010 attempted to estimate the causal effect of medical expenditures on health outcomes, which is ordinarily rife with selection bias due to endogenous physician behavior (independence is violated)
- In the US, newborns whose birthweight falls below 1500 grams receive heightened medical attention bc 1500 is the “very low birth weight” range and quite dangerous for infants
- Used RDD with hospital administrative records and found 1-year infant mortality decreased by 1pp just below 1500 grams compared to just above – medical expenditures are cost-effective

Heaping problem

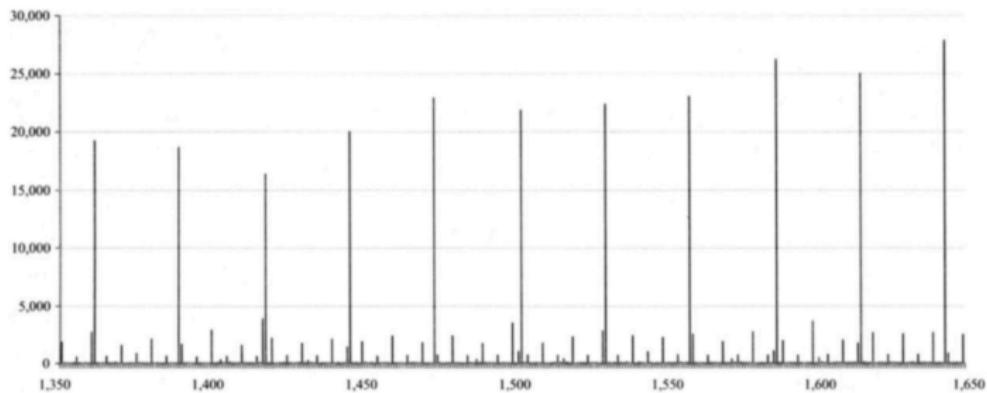


FIGURE I
Frequency of Births by Gram: Population of U.S. Births
between 1,350 and 1,650 g

NCHS birth cohort linked birth/infant death files, 1983–1991 and 1995–2003,
as described in the text.

Figure: Distribution of births by gram from Almond, et al. 2010

Heaping, Running and Jumping

- This picture shows “heaping” which is excess mass at certain points along the running variable
- Unlikely births actually heap at certain intervals; more likely someone is rounding
- Some scales may be less sophisticated, some practices may be more common in some types of hospitals than others, there could outright manipulation

Failure to reject

- Almond, et al. 2010 used the McCrary density test but found no evidence of manipulation
- Ironically, the McCrary density test may fail to reject in a heaping scenario
- In this scenario, the heaping is associated with high mortality children who are outliers compared to newborns both to the left and to the right

"This [heaping at 1500 grams] may be a signal that poor-quality hospitals have relatively high propensities to round birth weights but is also consistent with manipulation of recorded birth weights by doctors, nurses, or parents to obtain favorable treatment for their children. Barreca, et al. 2011 show that this nonrandom heaping leads one to conclude that it is "good" to be strictly less than any 100-g cutoff between 1,000 and 3,000 grams."

Donut holes

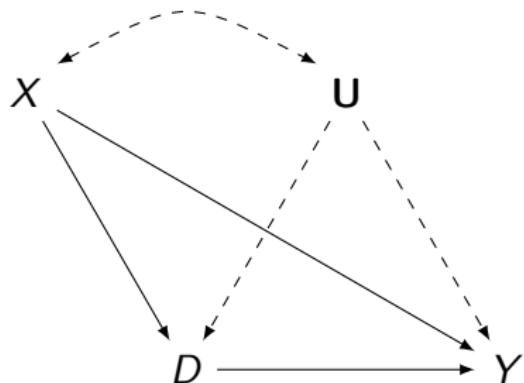
- RDD compares means as we approach c_0 from either direction along X
- Estimates should not logically be sensitive to the observations at the cutoff – if it is, then smoothness may be violated
- Through Monte Carlos, Barreca, et al. 2016 suggest an alternative strategy – drop the units in the vicinity of 1500 grams, and re-estimate the model
- They call this a “donut” RDD bc you drop the units at the cutoff (the “donut hole”) and estimate your model on the units in the neighborhood instead

Newborn mortality and medical expenditure

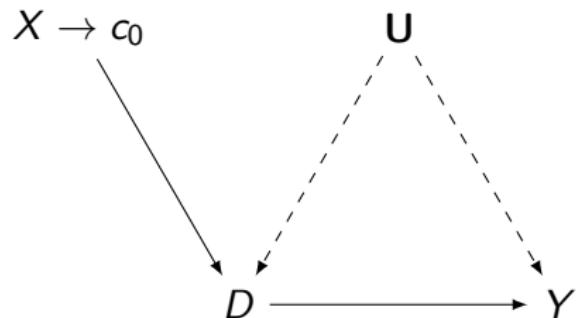
- Dropping units (e.g., trimming) always changes the parameter we're estimating – it's not the ATE, the ATT, not even the LATE except under strong assumptions
- In this case, dropping at the threshold reduced sample size by 2%
- But the strength of this practice is that it allows for the possibility that units at the heap differ markedly due to selection bias than those in the surrounding area
- Donut RDD analysis found effect sizes that were approximately 50% smaller than Almond, et al 2010
- Caution with heaping is a good attitude to have

Endogenous cutoffs

(A) Data generating graph



(B) Limiting graph



Endogenous cutoffs

- RCT randomization breaks all ordinary backdoor paths between D and Y because that's how "the science" of randomization works
- RDD blocks the backdoor path from $D \leftarrow X \leftarrow ? \rightarrow U \rightarrow Y$; it *assumes* away the backdoor path $D \leftarrow U \rightarrow Y$
- But if cutoffs are endogenous, then it is there, which means absent the treatment, smoothness would've been violated *anyway*
- Smoothness isn't guaranteed by an RDD unless $D \leftarrow U \rightarrow Y$ isn't present – which is why it is *the* critical identifying assumption

Endogenous cutoffs

- Examples of endogenous cutoffs
 - Age thresholds used for policy (i.e., person turns 18, and faces more severe penalties for crime) is correlated with other variables that affect the outcome (i.e., graduation, voting rights, etc.)
 - Age 65 is correlated with factors that directly affect healthcare expenditure and mortality such as retirement
- But some of these can be weakly defended with balance tests (observables), or may be directly testable through placebos assuming you have the data

Evaluating smoothness through balance

- Balance tests and placebo tests are related but distinct
- We can't directly test smoothness bc we are missing counterfactuals
- Ask yourself: why should average values of exogenous covariates jump if potential outcomes are smooth through the cutoff?
- If there are exogenous (non collider) covariates strongly associated with potential outcomes but exogenous to them, then they should be the same on either side of the cutoff if smoothness holds
- In this sense, balance tests are indirect searching for evidence supporting smoothness

Balance implementation

Don't make it hard – do what you did to Y , only to Z

- Choose other noncolliders associated with potential outcomes, Z
- Create similar graphical plots as you did for Y
- Could also conduct the parametric and nonparametric estimation on Z
- You do **not** want to see a jump around the cutoff, c_0

Visualizing Balance

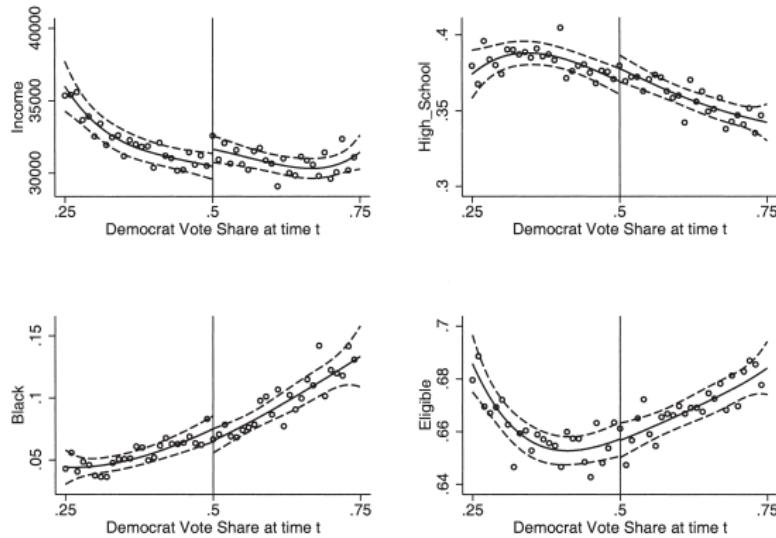


Figure: Figure 3 from Lee, Moretti and Butler (2004), "Do Voters Affect or Elect Policies?" *Quarterly Journal of Economics*. Panels refer to (top left to bottom right) the following district characteristics: real income, percentage with high-school degree, percentage black, percentage eligible to vote. Circles represent the average characteristic within intervals of 0.01 in Democratic vote share. The continuous line represents the predicted values from a fourth-order polynomial in vote share fitted separately for points above and below the 50 percent threshold. The dotted line represents the 95 percent confidence interval.

Placebos at non-discontinuous points

- Placebos in time are common with panels; placebo in running variables are their equivalent in RDD
- Imbens and Lemieux (2010) suggest we look at one side of the discontinuity (e.g., $X < c_0$), take the median value of the running variable in that section, and pretend it was a discontinuity, c'_0
- Then test whether in reality there is a discontinuity at c'_0 . You do **not** want to find anything.
- Remember though: smoothness at placebo points is neither necessary nor sufficient for smoothness in the potential outcomes at the cutoff
- So there are Type I and Type II risks of error with this

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Pictures, pictures and more pictures

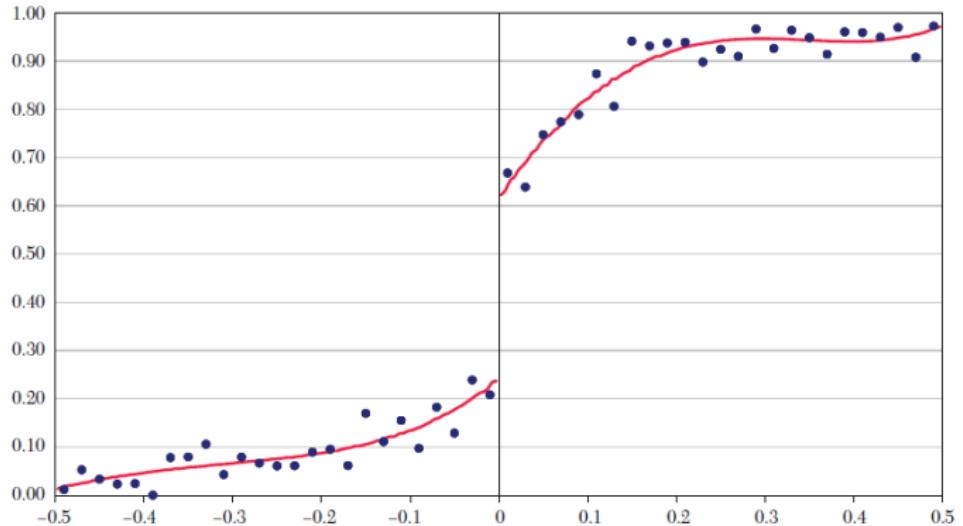
- Synthetic control and RDD are visually intense
- Eyeball tests are rampant (and deservedly) in RDD studies
- Even if your main results are all parametric, you'll still want to present at least some nonparametric style pictures according to Imbens and Lemieux (2010)
- Let's review some of the graphs you have to include

Outcomes

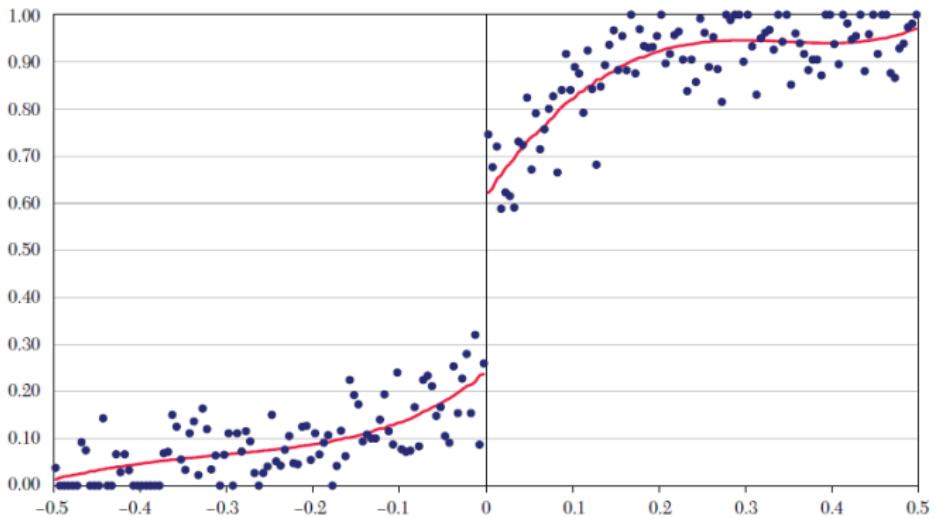
① Outcome by running variable, (X_i):

- Construct bins and average the outcome within bins on both sides of the cutoff
- Look at different bin sizes when constructing these graphs
- Plot the running variables, X_i , on the horizontal axis and the average of Y_i for each bin on the vertical axis
- Consider plotting a relatively flexible regression line on top of the bin means, but some readers prefer an eyeball test without the regression line to avoid “priming”

Example: Outcomes by Running Variables



Example: Outcomes by Running Variables with smaller bins



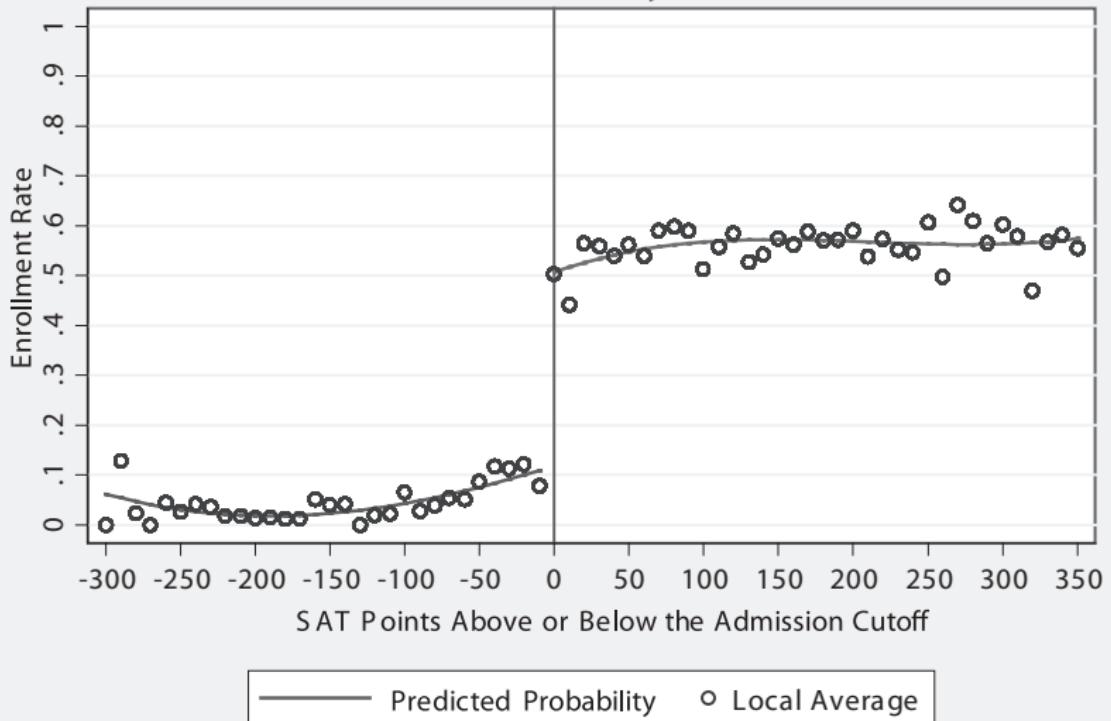
Probability of treatment

② Probability of treatment by running variable if fuzzy RDD

- In a fuzzy RDD, you also want to see that the treatment variable jumps at c_0
- This tells you whether you have a first stage ("bite")
- Let's look at that again from earlier Hoekstra (2008) and enrollment at the flagship

FIGURE 1.—FRACTION ENROLLED AT THE FLAGSHIP STATE UNIVERSITY

Estimated Discontinuity = 0.388 ($t=10.57$)



McCrary Density

③ Density of the running variable

- One should plot the number of observations in each bin.
- This plot allows to investigate whether there is a discontinuity or heaping in the distribution of the running variable at the threshold
- Heaping or discontinuities in the density suggest that people can manipulate their running variable score
- This is an indirect test of the identifying assumption that each individual has imprecise control over the assignment variable, which may violate smoothness

Density of the running variable

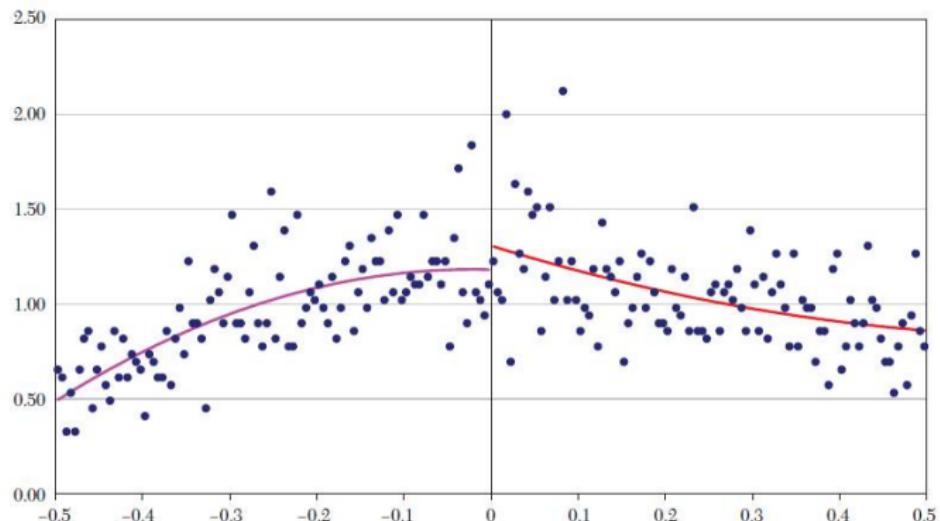


Figure 16. Density of the Forcing Variable (Vote Share in Previous Election)

Balance pictures

④ Covariates by a running variable

- Construct a similar graph to the outcomes graph but use a noncollider covariate as the “outcome”
- Balance implies smoothness through the cutoff, c_0 .
- If noncollider covariates jump at the cutoff, one is probably justified to reject that potential outcomes aren’t also probably jumping there

Example: Covariates by Running Variable

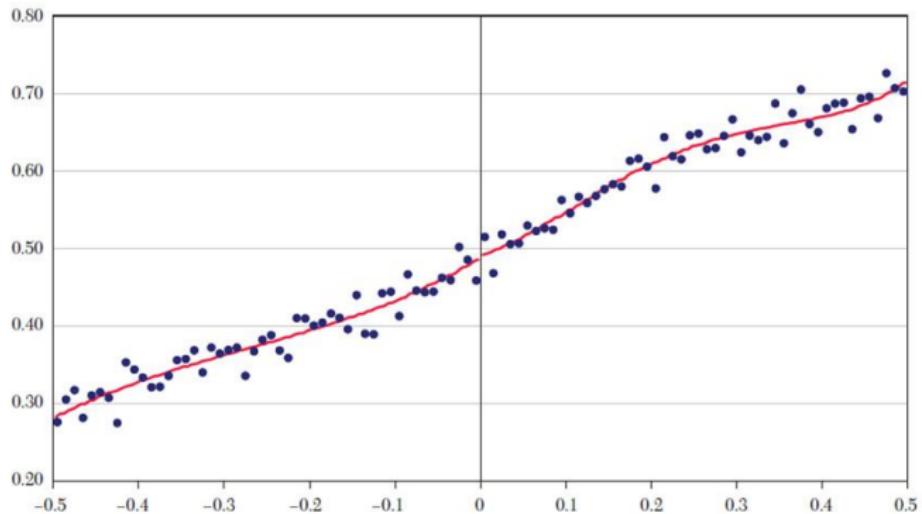


Figure 17. Discontinuity in Baseline Covariate (Share of Vote in Prior Election)

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Inference – honesty

- Lee and Card (2008) and Lee and Lemieux (2010) recommend clustering standard errors on the running variable
- Kolesár and Rothe (2018) provide extensive theoretical and simulation-based evidence that this is not good; you'd be better off just with heteroskedastic robust
- They propose two alternative confidence intervals that achieve correct coverage in large samples – called “honest” (great intro! Still studying this procedure)
- Unavailable in Stata, but is available in R – RDHonest – at <https://github.com/kolesarm/RDHonest>

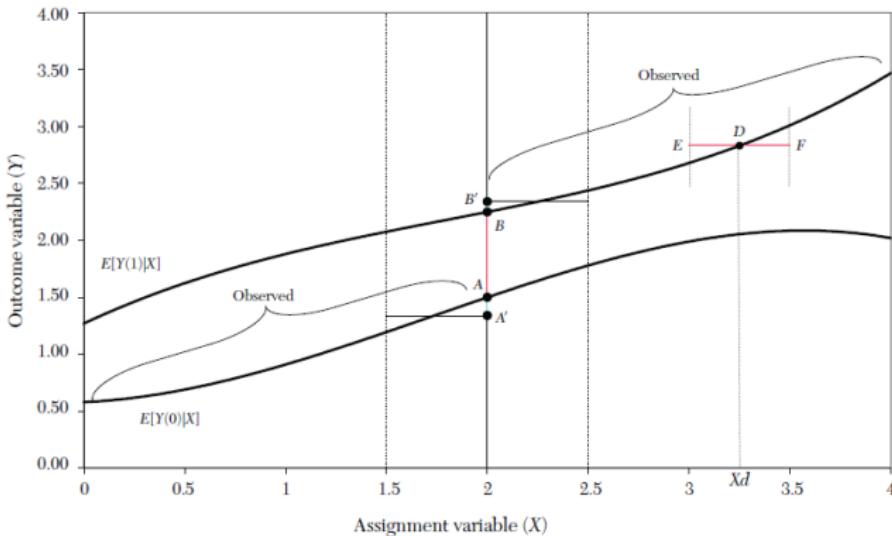
Inference – randomization inference

- Cattaneo, et al. (2015) say to consider that the cutoff is a randomized experiment
- Use randomization inference which is a test of the null of no individual unit level treatment effect at the cutoff

Parametric vs. nonparametric approaches

- Least squares approaches, because it models the counterfactual using functional forms, is parametric
- As a result, it can have poor predictive properties on counterfactuals above/below the cutoff
- Another way of approximating $f(X_i)$ is to use a nonparametric kernel which has its own problems; just not that one.

Kernel regression



- While the “true” effect is AB , with a certain bandwidth a rectangular kernel would estimate the effect as $A'B'$
- There is therefore systematic bias with the kernel method if the $f(X)$ is upwards or downwards sloping

Kernel weighted local polynomial regression

- The nonparametric one-sided kernel estimation problems are called “boundary problems” at the cutoff (Hahn, Todd and Van der Klaauw 2001)
- Kernel estimation (such as lowess) may have poor properties because the point of interest is at a boundary
- They proposed to use “local linear nonparametric regressions” instead

Local linear regression with weights

- Local linear nonparametric regression substantially reduces the bias
- Think of it as a weighted regression restricted to a window – kernel provides the weights to that regression.

$$(\hat{a}, \hat{b}) \equiv_{a,b} \sum_{i=1}^n (y_i - a - b(x_i - c_0))^2 K\left(\frac{x_i - c_0}{h}\right) 1(x_i > c_0)$$

where x_i is the value of the running variable, c_0 is the cutoff, K is a kernel function and $h > 0$ is a suitable bandwidth

Animation of a local linear regression

https://twitter.com/page_eco/status/958687180104245248

Estimation

- Stata's `poly` estimates kernel-weighted local polynomial regressions.
- A rectangular kernel would give the same result as $E[Y]$ at a given bin on X . The triangular kernel gives more importance to observations close to the center.
- This method will be sensitive to how large the bandwidth (window) you choose

Optimal bandwidths

- A rectangular kernel would give the same result as taking $E[Y]$ at a given bin on X whereas the triangular kernel gives more importance to the observations closer to the center.
- While estimating this in a given window of width h around the cutoff is straightforward, it's more difficult to choose this bandwidth (or window), and the method is sensitive to the choice of bandwidth.

Bandwidths

- Several methods for choosing the optimal bandwidth (window), but it's always a trade off between bias and variance
- In practical applications, you want to check for balance around that window
- Standard error of the treatment effects can be bootstrapped but there are also other alternatives
- You could add other variables to nonparametric methods.

Bandwidths

- Imbens and Kalyanaraman (2012), and more recently Calonico, et al. (2017), have proposed methods for estimating “optimal” bandwidths which may differ on either side of the cutoff.
- Calonico, et al (2017) propose local-polynomial regression discontinuity estimators with robust confidence intervals
- Stata ado package and R package are both called `rdrobust`

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DO VOTERS AFFECT OR ELECT POLICIES? EVIDENCE FROM THE U. S. HOUSE*

DAVID S. LEE
ENRICO MORETTI
MATTHEW J. BUTLER

There are two fundamentally different views of the role of elections in policy formation. In one view, voters can *affect* candidates' policy choices: competition for votes induces politicians to move toward the center. In this view, elections have the effect of bringing about some degree of policy compromise. In the alternative view, voters merely *elect* policies: politicians cannot make credible promises to moderate their policies, and elections are merely a means to decide which one of two opposing policy views will be implemented. We assess which of these contrasting perspectives is more empirically relevant for the U. S. House. Focusing on elections decided by a narrow margin allows us to generate quasi-experimental estimates of the impact of a "randomized" change in electoral strength on subsequent representatives' roll-call voting records. We find that voters merely *elect* policies: the degree of electoral strength has no effect on a legislator's voting behavior. For example, a large *exogenous* increase in electoral strength for the

Implementation

- The following paper is a seminal paper in public choice both scientifically and methodologically – the close election RDD
- I call the close election RDD a type of sub-RDD in that it's widely used in political science and economics to the point that it's taken on a life of its own
- Let's take everything we've done and apply it by replicating this paper using programs I've provided

Public choice

There are two fundamentally different views of the role of voters in a representative democracy.

- ① **Convergence:** Voters force candidates to become relatively moderate depending on their size in the distribution (Downs 1957).

“Competition for votes can force even the most partisan Republicans and Democrats to moderate their policy choices. In the extreme case, competition may be so strong that it leads to ‘full policy convergence’: opposing parties are forced to adopt identical policies” – Lee, Moretti, and Butler 2004.

- ② **Divergence:** Voters pick the official and after taking office, she pursues her most-preferred policy.

Falsification of either hypothesis had been hard

- Very difficult to test either one of these since you don't observe the counterfactual votes of the loser for the same district/time
- Winners in a district are selected based on their policy's conforming to unobserved voter preferences, too
- Lee, Moretti and Butler (2004) develop the "close election RDD" which has the aim of determining whether convergence, while theoretically appealing, has any explanatory power in Congress
- The metaphor of the RCT is useful here: maybe close elections are being determined by coin flips (e.g., a few votes here, a few votes there)

Outcome is Congress person's liberal voting score

- **Liberal voting score** is a report card from the Americans for Democratic Action (ADA) for the House election results 1946-1995
 - Authors use the ADA score for all US House Representatives from 1946 to 1995 as their voting record index
 - For each Congress, ADA chooses about twenty high-profile roll-call votes and creates an index varying 0 and 100 for each Representative of the House measuring liberal voting record

Democratic “voteshare” is the running variable

- **Voteshare** from the same races
 - The running variable is voteshare which is the share of all votes that went to a Democrat.
 - They use a close Democratic victory to check whether convergence or divergence is correct (what's smoothness here?)
 - Discontinuity in the running variable occurs at $\text{voteshare} = 0.5$. When $\text{voteshare} > 0.5$, the Democratic candidate wins.
- I'll show `lmb1.do` to `lmb10.do` (and R) at times just so we can all see the simple estimation methods ourselves.

Remember these results

TABLE I
RESULTS BASED ON ADA SCORES—CLOSE ELECTIONS SAMPLE

Variable	Total effect			Elect component	Affect component
	γ	$\pi_1(P_{t+1}^D - P_{t+1}^R)$	$\pi_1[(P_{t+1}^D - P_{t+1}^R)]$	$\pi_0[P_{t+1}^{SD} - P_{t+1}^{SR}]$	
	ADA_{t+1}	ADA_t	DEM_{t+1}	(col. (2) + col. (3))	(col. (1)) – (col. (4))
	(1)	(2)	(3)	(4)	(5)
Estimated gap	21.2 (1.9)	47.6 (1.3)	0.48 (0.02)		
				22.84 (2.2)	-1.64 (2.0)

Standard errors are in parentheses. The unit of observation is a district-congressional session. The sample includes only observations where the Democrat vote share at time t is strictly between 48 percent and 52 percent. The estimated gap is the difference in the average of the relevant variable for observations for which the Democrat vote share at time t is strictly between 50 percent and 52 percent and observations for which the Democrat vote share at time t is strictly between 48 percent and 50 percent. Time t and $t + 1$ refer to congressional sessions. ADA_t is the adjusted ADA voting score. Higher ADA scores correspond to more liberal roll-call voting records. Sample size is 915.

Figure: Lee, Moretti, and Butler 2004, Table 1.

Nonparametric estimation

- Hahn, Todd and Van der Klaauw (2001) emphasized using local polynomial regressions
- Estimate $E[Y|X]$ in such a way that doesn't require committing to a functional form
- That model would be something general like

$$Y = f(X) + \varepsilon$$

Nonparametric estimation (cont.)

- We'll do this estimation just rolling $E[ADA]$ across the running variable *voteshare* visually
- Stata has an option to do this called `cmmogram` and it has a lot of useful options, though many people prefer to graph it themselves bc it gives more flexibility.
- We can recreate Figures I, IIA and IIB using it

Future liberal voting score

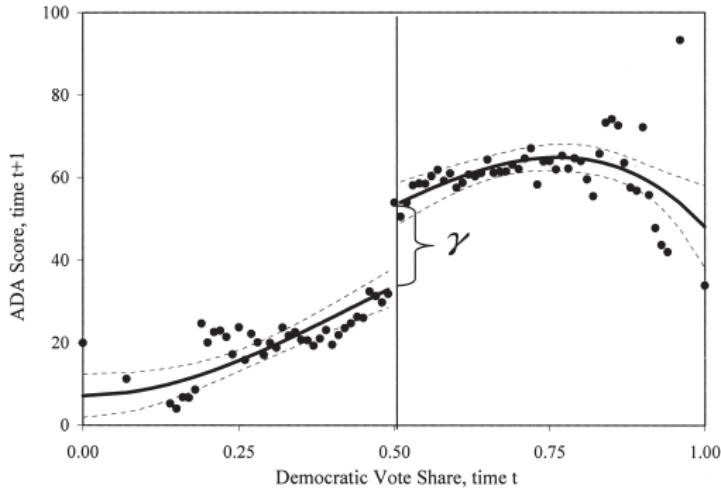


FIGURE I

Total Effect of Initial Win on Future ADA Scores: γ

This figure plots ADA scores after the election at time $t + 1$ against the Democrat vote share, time t . Each circle is the average ADA score within 0.01 intervals of the Democrat vote share. Solid lines are fitted values from fourth-order polynomial regressions on either side of the discontinuity. Dotted lines are pointwise 95 percent confidence intervals. The discontinuity gap estimates

$$\gamma = \underbrace{\pi_0(P_{t+1}^{*D} - P_{t+1}^{*R})}_{\text{"Affect"}} + \underbrace{\pi_1(P_{t+1}^{*D} - P_{t+1}^{*R})}_{\text{"Elect"}}$$

Figure: Lee, Moretti, and Butler 2004, Figure I. $\gamma \approx 20$

Contemporaneous liberal voting score

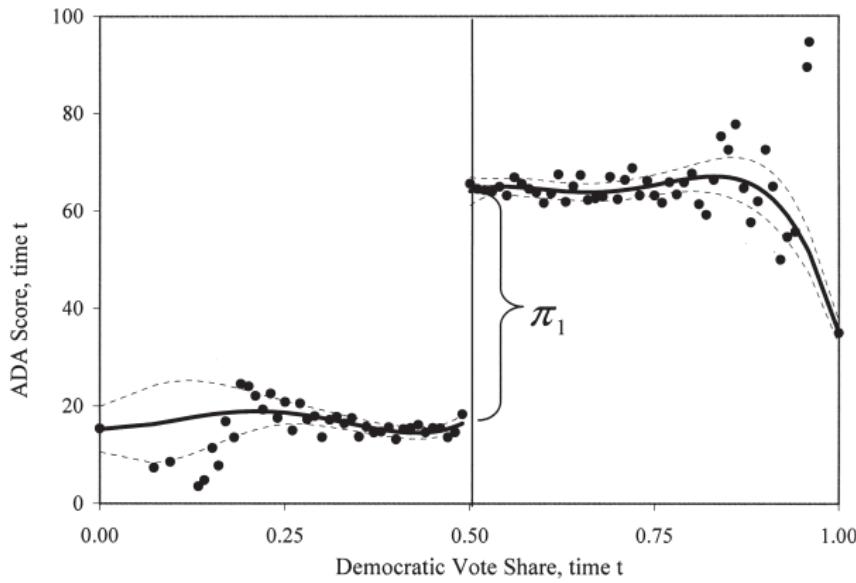


FIGURE IIa
Effect of Party Affiliation: π_1

Figure: Lee, Moretti, and Butler 2004, Figure IIa. $\pi_1 \approx 45$

Inc incumbency advantage

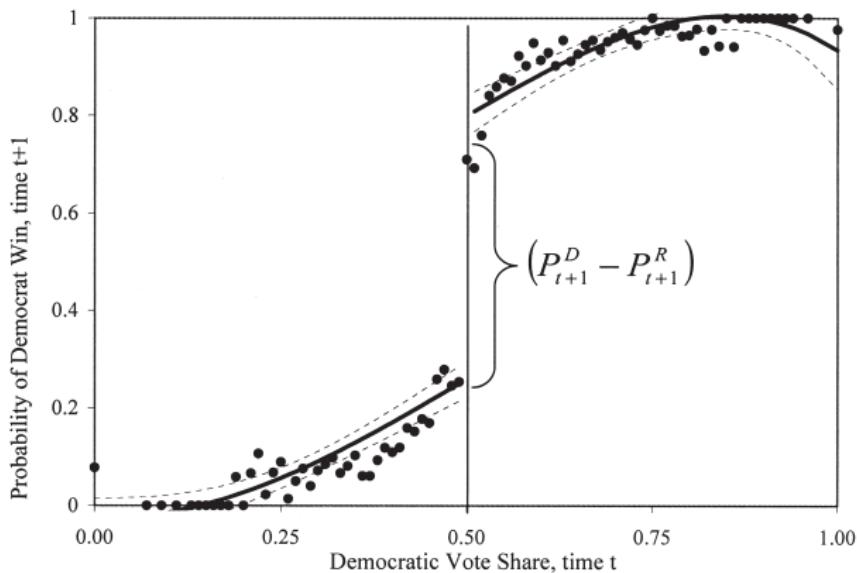


FIGURE IIb
Effect of Initial Win on Winning Next Election: $(P_{t+1}^D - P_{t+1}^R)$

Figure: Lee, Moretti, and Butler 2004, Figure IIb. $(P_{t+1}^D - P_{t+1}^R) \approx 0.50$

Concluding remarks

- Caughey and Sekhon (2011) questioned the finding (not the design per se) saying that bare winners and bare losers in the US House elections differed considerably on pretreatment covariates (imbalance), which got worse in the closest elections
- Eggers, et al. (2014) evaluated 40,000 close elections including the House in other time periods, mayor races, and other types of US races including nine other countries
- They couldn't find another instance where Caughey and Sekhon's critique applied
- Assumptions behind close election design therefore probably holds and is one of the best RD designs we have

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Fuzzy RDD, IV and ITT

- Fuzzy RDD is an IV estimator, and requires those assumptions
- You may be more comfortable with presenting the intent-to-treat (ITT) parameter which is just the reduced form regression of Y on Z , therefore
- Many papers will not present an IV-style parameter, but rather a blizzard of ITT parameters, out of a “fear” that the exclusion restrictions may not hold
- But let’s review the IV approach anyway for completeness (more IV to come!)

Probability of treatment jumps at discontinuity

Probabilistic treatment assignment (i.e. “fuzzy RDD”)

The probability of receiving treatment changes discontinuously at the cutoff, c_0 , but need not go from 0 to 1

$$\lim_{X_i \rightarrow c_0} Pr(D_i = 1 | X_i = c_0) \neq \lim_{c_0 \leftarrow X_i} Pr(D_i = 1 | X_i = c_0)$$

Examples: Incentives to participate in some program may change discontinuously at the cutoff but are not powerful enough to move everyone from non participation to participation.

Deterministic (sharp) vs. probabilistic (fuzzy)

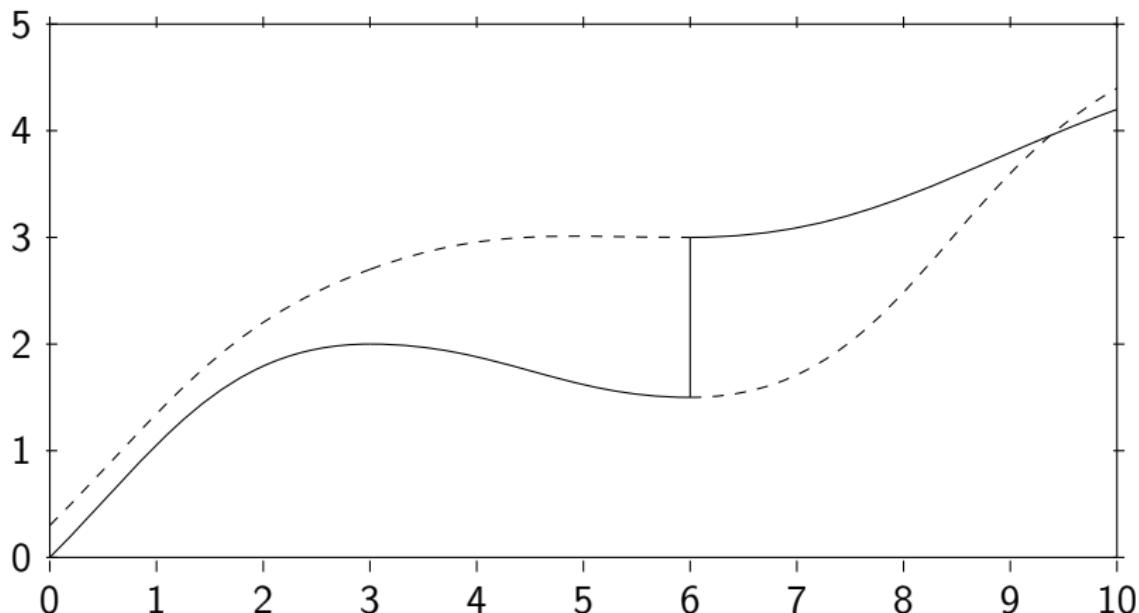
- In the sharp RDD, D_i was *determined* by $X_i \geq c_0$
- In the fuzzy RDD, the *conditional probability* of treatment *jumps* at c_0 .
- The relationship between the conditional probability of treatment and X_i can be written as:

$$P[D_i = 1|X_i] = g_0(X_i) + [g_1(X_i) - g_0(X_i)]Z_i$$

where $Z_i = 1$ if $(X_i \geq c_0)$ and 0 otherwise.

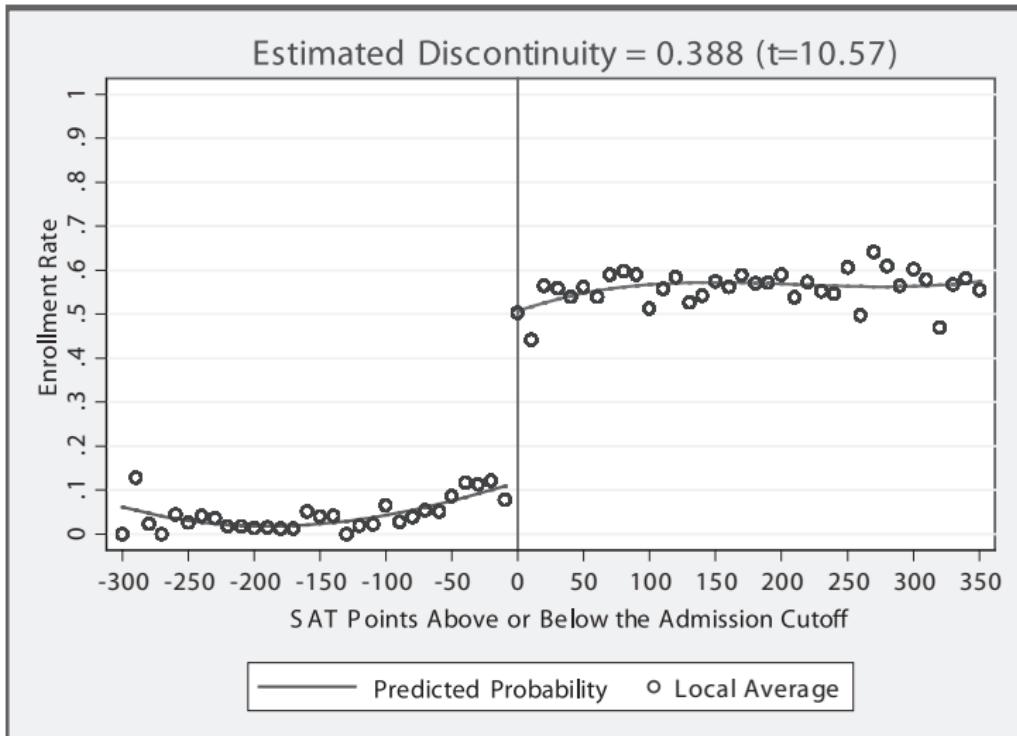
Visualization of identification strategy (i.e. smoothness)

- $E[Y^0|X]$ and $E[Y^1|X]$ for $D = 0, 1$ are the dashed/solid continuous functions
- $E[Y|X]$ is the solid which jumps at $X = 6$



Hoekstra flagship school

FIGURE 1.—FRACTION ENROLLED AT THE FLAGSHIP STATE UNIVERSITY



Instrumental variables

- As said, fuzzy designs are numerically equivalent and conceptually similar to IV
 - Reduced form Numerator: “jump” in the regression of the outcome on the running variable, X .
 - First stage Denominator: “jump” in the regression of the treatment indicator on the running variable X .
- Same IV assumptions, caveats about compliers vs. defiers, and statistical tests that we will discuss in next lecture with instrumental variables apply here – e.g., check for weak instruments using F test on instrument in first stage, etc.

Wald estimator

Wald estimator of treatment effect under Fuzzy RDD

Average causal effect of the treatment is the Wald IV parameter

$$\delta_{\text{Fuzzy RDD}} = \frac{\lim_{X \rightarrow c_0} E[Y|X = c_0] - \lim_{c_0 \leftarrow X} E[Y|X = c_0]}{\lim_{X \rightarrow c_0} E[D|X = c_0] - \lim_{c_0 \leftarrow X} E[D|X = c_0]}$$

RDD's Relationship to IV

- Center X it's equal to zero at c_0 and define $Z = 1(X \geq 0)$
- The coefficient on Z in a regression like

```
. reg Y Z X X2 X3
```

is the reduced form discontinuity, and

```
. reg D Z X X2 X3
```

is the first stage discontinuity

- Ratio of discontinuities is estimate of $\delta_{\text{Fuzzy RDD}}$
- Simple way to implement is IV

```
. ivregress 2sls Y (D=Z) X X2 X3
```

First stage relationship between X and D

- One can use both Z_i as well as the interaction terms as instruments for D_i .
- If one uses only Z_i as IV, then it is a “just identified” model which usually has good finite sample properties.
- In the just identified case, the first stage would be:

$$D_i = \gamma_0 + \gamma_1 X_i + \gamma_2 X_i^2 + \cdots + \gamma_p X_i^p + \pi Z_i + \varepsilon_{1i}$$

where π is the causal effect of Z on the conditional probability of treatment.

- The fuzzy RD reduced form is:

$$Y_i = \mu + \kappa_1 X_i + \kappa_2 X_i^2 + \cdots + \kappa_p X_i^p + \rho \pi Z_i + \varepsilon_{2i}$$

Fuzzy RDD with varying Treatment Effects - Second Stage

- As in the sharp RDD case one can allow the smooth function to be different on both sides of the discontinuity.
- The second stage model with interaction terms would be the same as before:

$$Y_i = \alpha + \beta_{01}\tilde{x}_i + \beta_{02}\tilde{x}_i^2 + \cdots + \beta_{0p}\tilde{x}_i^p + \rho D_i + \beta_1^* D_i \tilde{x}_i + \beta_2^* D_i \tilde{x}_i^2 + \cdots + \beta_p^* D_i \tilde{x}_i^p + \eta_i$$

- Where \tilde{x} are now not only normalized with respect to c_0 but are also fitted values obtained from the first stage regression.

Fuzzy RDD with Varying Treatment Effects - First Stages

- Again one can use both Z_i as well as the interaction terms as instruments for D_i ;
- Only using Z the estimated first stages would be:

$$D_i = \gamma_{00} + \gamma_{01}\tilde{X}_i + \gamma_{02}\tilde{X}_i^2 + \cdots + \gamma_{0p}\tilde{X}_i^p + \pi Z_i + \gamma_1^* \tilde{X}_i Z_i + \gamma_2^* \tilde{X}_i^2 Z_i + \cdots + \gamma_p^* Z_i + \varepsilon_{1i}$$

- We would also construct analogous first stages for $\tilde{X}_i D_i$, $\tilde{X}_i^2 D_i$, \dots , $\tilde{X}_i^p D_i$.

Limitations of the LATE

- Fuzzy RDD has assumptions of all standard IV framework (exclusion, independence, nonzero first stage, and monotonicity)
- As with other binary IVs, the fuzzy RDD is estimating LATE: the local average treatment effect for the group of *compliers*
- In RDD, the compliers are those whose treatment status changed as we moved the value of x_i from just to the left of c_0 to just to the right of c_0
- Means we can use Medicare age cutoff to estimate the effect of public insurance on mortality (LATE) and still not know the effect of public insurance on mortality (ATE)