Mathematics 24/25 Group Project – Group 40

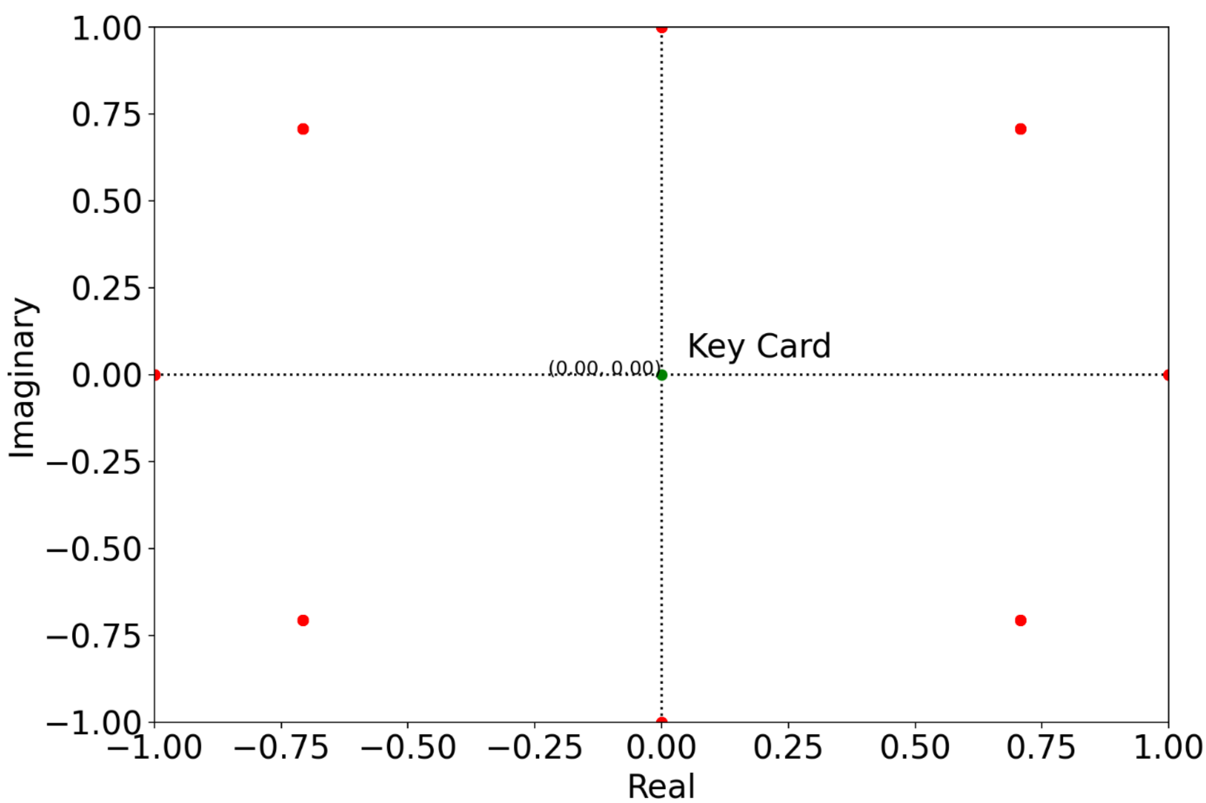
Task 1

Task 1.1 - Code:

```
import math
import cmath
import numpy as np
import matplotlib as mpl
import matplotlib.pyplot as plt

mpl.rcParams['font.size'] = 20
plt.xlim(-1, 1)
plt.ylim(-1, 1)
plt.axhline(c='k', ls=":")
plt.axvline(c='k', ls=":")
plt.xlabel("Real")
plt.ylabel("Imaginary")

# --- Task 1 ---
# -- 1.1 --

# Setting up the starting complex number
z_num = (1 + 1j)  # Defining the numerator
z_den = math.sqrt(2) * (1j**4) * ((1 + 1j) / abs((1 + 1j)))**40 # Defining the
denominator
z_initial = z_num / z_den

# This is the complex number to calculate movement
u = math.sqrt(2) / (1 + 1j)

# Plot the initial point on the graph
plt.plot(z_initial.real, z_initial.imag, 'o')  # Mark it with a dot

total = 0
z_position = z_initial

# We loop 64 times, updating the position each time
for i in range(64):
```

```
    z_position /= u  # Move to the next position by dividing by u
    total += z_position  # Add the new position to the total
    plt.plot(z_position.real, z_position.imag, 'o', c='r')  # Plot it as a red
dot

# After all the moves, calculate the average (mean) position
mean = total / 64

# Plot the mean position on the graph
plt.plot(mean.real, mean.imag, 'o', c='g')  # Mark it with a green dot

# Show the coordinates of the mean point
plt.text(mean.real, mean.imag, f"({mean.real:.2f}, {mean.imag:.2f})",
fontsize=12, ha='right')

plt.text(0.05, 0.05, "Key Card")  # Label it as "Key Card"

plt.show()
# Answer: The key card is at (0, 0)
```

Graph Plot:



Task 1.2 – Maths:

The first part of solving Task 1.2 is to simplify the $Z_{initial}$. To do this, we take α (our group number: 40) along with *j* (the imaginary number $\sqrt{-1}$) and apply it to the below equation:

$$Z_{initial} = \frac{(1+j)}{\sqrt{2}\cdot j^4 \cdot \left(\frac{1+j}{|1+j|}\right)^{40}}$$

We can simplify this equation. Take j^4. We know that $j = \sqrt{-1}$, so j^4 can be shown as:
$j^4 = \sqrt{-1} \cdot \sqrt{-1} \cdot \sqrt{-1} \cdot \sqrt{-1} = -1 \cdot -1 = 1$

This then makes our $Z_{initial}$ equation:

$$Z_{initial} = \frac{(1+j)}{\sqrt{2}\cdot \left(\frac{1+j}{|1+j|}\right)^{40}}$$

Next, we take $|1+j|$. The complex number $1+j$ can be expressed as $a+bj$. In this case, a and b are both 1. We can then calculate the magnitude of the complex number as shown below:

$$\sqrt{1^2+1^2} = \sqrt{2}$$

Our equation is then:

$$Z_{initial} = \frac{(1+j)}{\sqrt{2}\cdot \left(\frac{1+j}{\sqrt{2}}\right)^{40}}$$

$$= \frac{(1+j)}{\sqrt{2}\cdot \frac{(1+j)^{40}}{\left(\sqrt{2}\right)^{40}}}$$

The $\sqrt{2}$ can be canceled out to give us:

$$= \frac{(1+j)}{\frac{(1+j)^{40}}{\left(\sqrt{2}\right)^{39}}}$$

From this point we can perform fraction division (flip and multiply):

$$= (1+j) \cdot \frac{\left(\sqrt{2}\right)^{39}}{(1+j)^{40}}$$

Which in turn can be simplified to:

$$= \frac{\left(\sqrt{2}\right)^{39}}{(1+j)^{39}}$$

$$Z_{initial} = \left(\frac{\sqrt{2}}{1+j}\right)^{39}$$

The resulting equation from the above simplification is a power of u, where $u = \frac{\sqrt{2}}{1+j}$, the complex number to calculate the next position from the previous. The formula to calculate the next positions is as follows:

$$Z_{position\ 1} = \frac{Z_{initial}}{u} = \left(\frac{\sqrt{2}}{1+j}\right)^{38}$$

$$Z_{position\ 2} = \frac{Z_{position\ 1}}{u} = \left(\frac{\sqrt{2}}{1+j}\right)^{37}$$

$$Z_{position\ 3} = \frac{Z_{position\ 2}}{u} = \left(\frac{\sqrt{2}}{1+j}\right)^{36}$$

…

$$Z_{position\ 64} = \frac{Z_{position\ 63}}{u} = \left(\frac{\sqrt{2}}{1+j}\right)^{-25} = \left(\frac{1+j}{\sqrt{2}}\right)^{25}$$

We see that with each movement, the power decreases by 1. The division by the complex number u, is what causes the circular pattern we see when we graph the results. We can take this even further by calculating the angle it is being rotated by.

Using,

$$\cos\theta = \frac{a}{d} \; where\; d = \sqrt{a^2 + b^2} \; and\; a + bj \; is\; equal\; to\; 1 + j$$

$$\cos\theta = \frac{1}{\sqrt{1^2 + 1^2}}$$

$$= \frac{1}{\sqrt{2}}$$

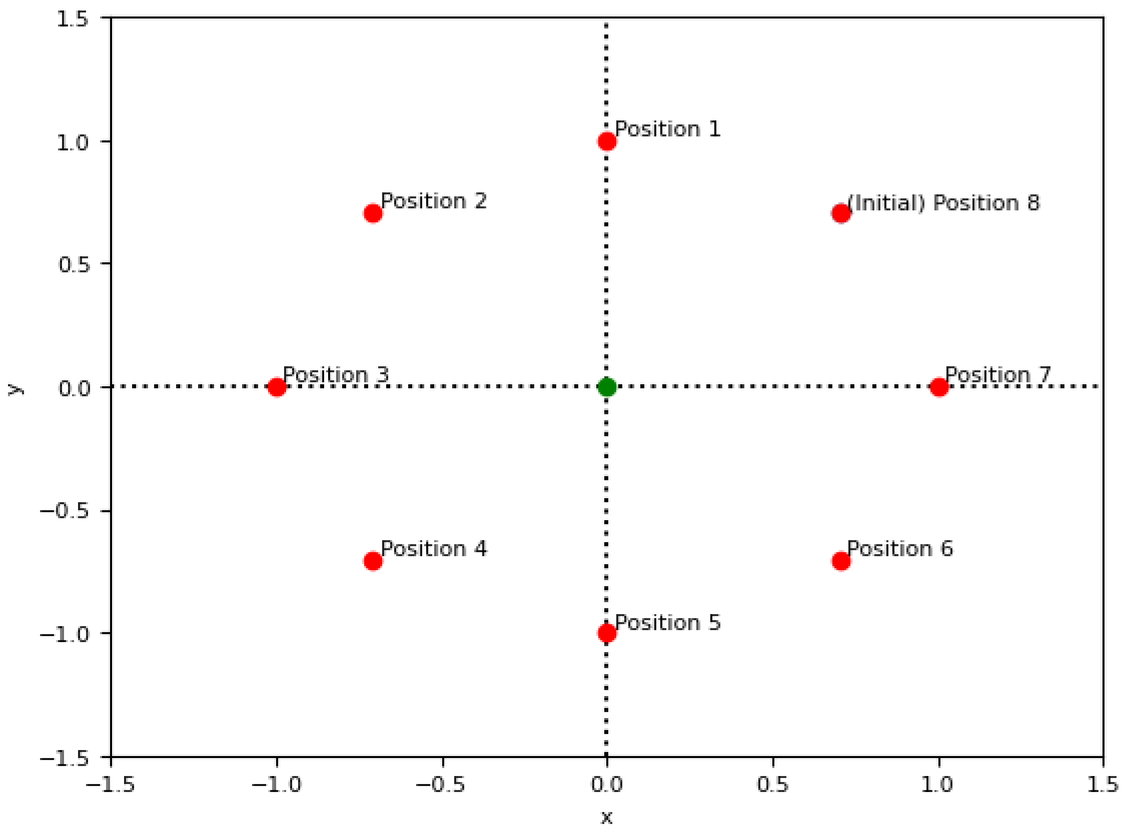
$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$\theta = 45°$$

Every time we divide by the complex number u , we rotate 45° around the origin.

A rotation of 45° means it takes $\frac{360}{45} = 8\; movements$ to get back to the initial position, and since she moves 64 times, $\frac{64}{8} = 8\; laps$ to get back to the initial position.

Using 8 movements as an example (64 would yield the same results):



Since dividing a position by the complex number u gives us the next position, dividing by it 8 times, multiple times will give us an equivalent position, but one where we can do feasible calculations to get the (Re, Ima) coordinates for each movement:

$$Z_{initial} = Z_{position\,8} = Z_{position\,16} = Z_{position\,24} = Z_{position\,32} = Z_{position\,40}$$

$$Z_{initial} = \left(\frac{\sqrt{2}}{1+j}\right)^{31} = \left(\frac{\sqrt{2}}{1+j}\right)^{23} = \left(\frac{\sqrt{2}}{1+j}\right)^{15} = \left(\frac{\sqrt{2}}{1+j}\right)^{7} = \left(\frac{\sqrt{2}}{1+j}\right)^{-1}$$

$$Z_{initial} = \left(\frac{\sqrt{2}}{1+j}\right)^{-1} = \frac{1+j}{\sqrt{2}} = \frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}} = 0.7071 + 0.7071j$$

$$Z_{position\,1} = Z_{position\,9} = Z_{position\,17} = Z_{position\,25} = Z_{position\,33} = Z_{position\,41}$$

$$Z_{position\,1} = \left(\frac{\sqrt{2}}{1+j}\right)^{30} = \left(\frac{\sqrt{2}}{1+j}\right)^{22} = \left(\frac{\sqrt{2}}{1+j}\right)^{14} = \left(\frac{\sqrt{2}}{1+j}\right)^{6} = \left(\frac{\sqrt{2}}{1+j}\right)^{-2}$$

$$Z_{position\,1} = \left(\frac{\sqrt{2}}{1+j}\right)^{-2} = \frac{(1+j)^2}{\left(\sqrt{2}\right)^2} = \frac{2j}{2} = 0\ +\ 1j$$

$$Z_{position\,7} = Z_{position\,15} = Z_{position\,23} = Z_{position\,31} = Z_{position\,39}$$

$$Z_{position\,7} = \left(\frac{\sqrt{2}}{1+j}\right)^{24} = \left(\frac{\sqrt{2}}{1+j}\right)^{16} = \left(\frac{\sqrt{2}}{1+j}\right)^{8} = \left(\frac{\sqrt{2}}{1+j}\right)^{0}$$

$$Z_{position\,7} = \left(\frac{\sqrt{2}}{1+j}\right)^{0} = 1 + 0j$$

$$Z_{position\,6} = Z_{position\,14} = Z_{position\,22} = Z_{position\,30} = Z_{position\,38}$$

$$Z_{position\,6} = \left(\frac{\sqrt{2}}{1+j}\right)^{25} = \left(\frac{\sqrt{2}}{1+j}\right)^{17} = \left(\frac{\sqrt{2}}{1+j}\right)^{9} = \left(\frac{\sqrt{2}}{1+j}\right)^{1}$$

$$Z_{position\,6} = \left(\frac{\sqrt{2}}{1+j}\right)^{1} = \frac{\sqrt{2}}{1+j}\cdot\frac{1-j}{1-j} = \frac{\sqrt{2}-\sqrt{2}j}{2} = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}j}{2} = 0.7071\ -\ 0.7071j$$

$$Z_{position\,5} = Z_{position\,13} = Z_{position\,21} = Z_{position\,29} = Z_{position\,37}$$

$$Z_{position\,5} = \left(\frac{\sqrt{2}}{1+j}\right)^{26} = \left(\frac{\sqrt{2}}{1+j}\right)^{18} = \left(\frac{\sqrt{2}}{1+j}\right)^{10} = \left(\frac{\sqrt{2}}{1+j}\right)^{2}$$

$$Z_{position\,5} = \left(\frac{\sqrt{2}}{1+j}\right)^{2} = \frac{2}{2j} = \frac{1}{j} = \frac{\bar{j}}{j\cdot\bar{j}} = \frac{0-j}{(0+j)(0-j)} = \frac{-j}{1} = 0 - 1j$$

$Positions\ 2,\ 3,\ 4\ are\ mirrors\ of\ 8,\ 7,\ 6$

When it comes to adding up all her movements, since she is moving in a circular motion, the total will give us a coordinate at the origin (0,0) and dividing it by the total amount of moves gives us the same answer.

0 + 1j + 0.7071 + 0.7071j + 1 + 0j + 0.7071 - 0.7071j + 0 – 1j - 0.7071 - 0.7071j - 1 + 0j - 0.7071 + 0.7071j

= 0/8 (Mean) = 0

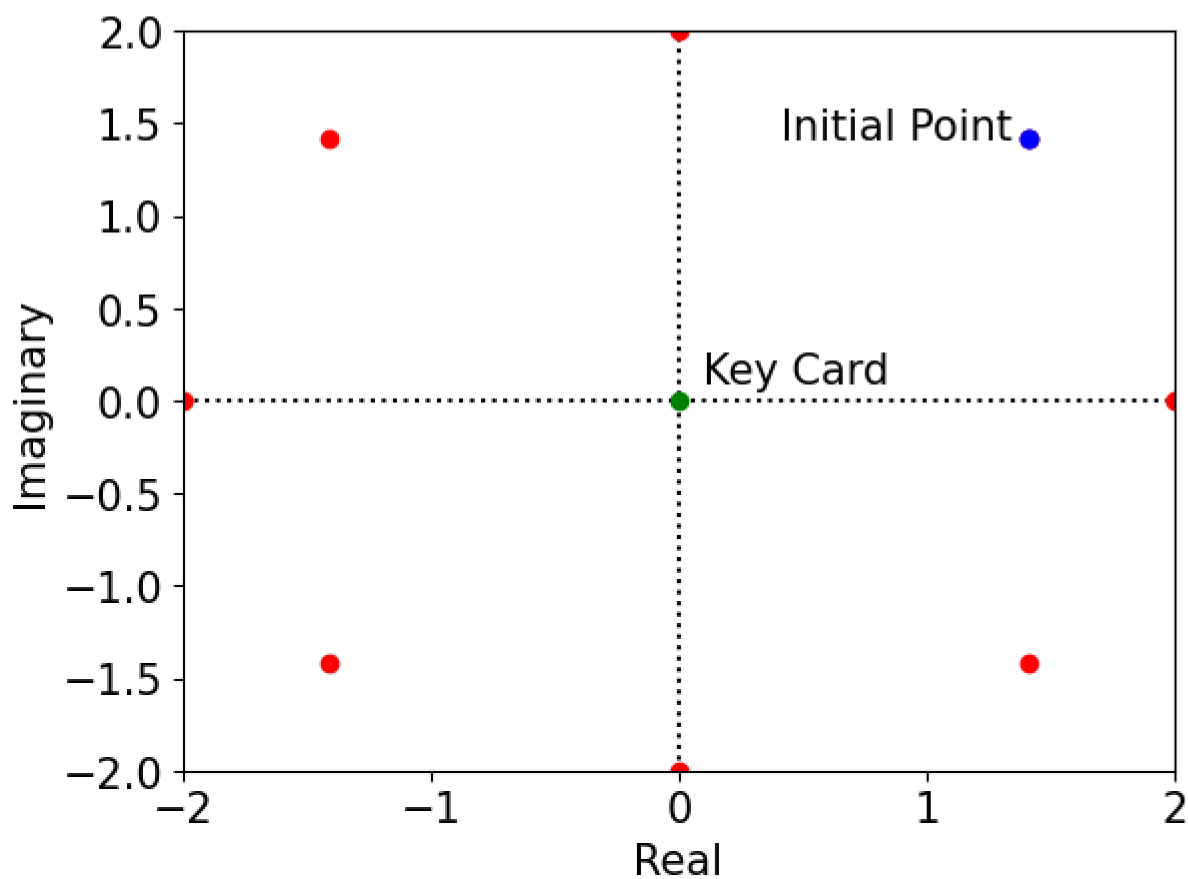
Task 2:

Task 2.1 – Code:

```
import math
import cmath
import numpy as np
import matplotlib as mpl
import matplotlib.pyplot as plt

mpl.rcParams['font.size'] = 15
plt.xlim(-2, 2)
plt.ylim(-2, 2)
plt.axhline(c='k', ls=":")
plt.axvline(c='k', ls=":")
plt.xlabel("Real")
plt.ylabel("Imaginary")

# ---Task 2---
# --2.1--

#Setting up the starting complex number
z_num = math.sqrt(2) * (1 + 1j) # Defining the numerator
z_den = ((1 + 1j) / (math.sqrt(2))) ** 40 # Defining the denominator
z_initial = z_num / z_den

u = math.sqrt(2) / (1 + 1j) # The complex number to calculate movement

plt.plot(z_initial.real, z_initial.imag, 'o', c='b') # Plotting initial point
on graph as a blue dot

z_position = z_initial
moves = 0
total = 0

# Indefinite loop to determine the amount of moves
while True:
    z_position /= u  # Move the point
    plt.plot(z_position.real, z_position.imag, 'o', c='r')  # Plot each move as
a red dot
    moves += 1  # Increment move counter
    total += z_position  # Add current position to total
```

```
    mean = total / moves  # Calculate mean position

    # Check if mean position is at origin
    if abs(mean.real * 10).__trunc__() == 0 and abs(mean.imag * 10).__trunc__()
== 0:
        plt.plot(z_initial.real, z_initial.imag, 'o', c='b')  # Clarify initial
point in blue
        plt.text(z_initial.real - 1, z_initial.imag, "Initial Point")  # Label
initial point
        plt.plot(mean.real, mean.imag, 'o', c='g')  # Plot keycard in green
        plt.text(0.1, 0.1, "Key Card")  # Label keycard
        break  # Exit loop

print(moves) #Outputs 8
#Answer = The minimum number of movements is 8
```

Graph Plot:



Task 2.2 – Maths:

First, we simplify this equation:

$$Z_{initial} = \frac{\sqrt{2} \cdot (1+j)}{\left(\frac{1+j}{\sqrt{2}}\right)^{40}}$$

$$= \sqrt{2} \cdot (1+j) \cdot \frac{\left(\sqrt{2}\right)^{40}}{(1+j)^{40}}$$

$$Z_{initial} = \frac{\left(\sqrt{2}\right)^{41}}{(1+j)^{39}}$$

Again, we can get the next position by dividing by the complex number $u = \frac{\sqrt{2}}{1+j}$,

$$Z_{position\ 1} = \frac{Z_{initial}}{u} = \frac{\left(\sqrt{2}\right)^{41}}{(1+j)^{39}} \cdot \frac{(1+j)}{\sqrt{2}} = \frac{\left(\sqrt{2}\right)^{40}}{(1+j)^{38}}$$

$$Z_{position\ 2} = \frac{Z_{position\ 1}}{u} = \frac{\left(\sqrt{2}\right)^{40}}{(1+j)^{38}} \cdot \frac{(1+j)}{\sqrt{2}} = \frac{\left(\sqrt{2}\right)^{39}}{(1+j)^{37}}$$

$$Z_{position\ 3} = \frac{Z_{position\ 2}}{u} = \frac{\left(\sqrt{2}\right)^{39}}{(1+j)^{37}} \cdot \frac{(1+j)}{\sqrt{2}} = \frac{\left(\sqrt{2}\right)^{38}}{(1+j)^{36}}$$

…

$$Z_{position\ 8} = \frac{Z_{position\ 7}}{u} = \frac{\left(\sqrt{2}\right)^{34}}{(1+j)^{32}} \cdot \frac{(1+j)}{\sqrt{2}} = \frac{\left(\sqrt{2}\right)^{33}}{(1+j)^{31}}$$

Here we see that every time we divide by u, the exponents of $\sqrt{2}$ and $(1+j)$ go down by 1.

Since she moves in the exact same way as **Task 1**, we know that she rotates 45° around the origin every move.

$$\cos\theta = \frac{1}{\sqrt{1^2 + 1^2}}$$

$$= \frac{1}{\sqrt{2}}$$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$\theta = 45°$

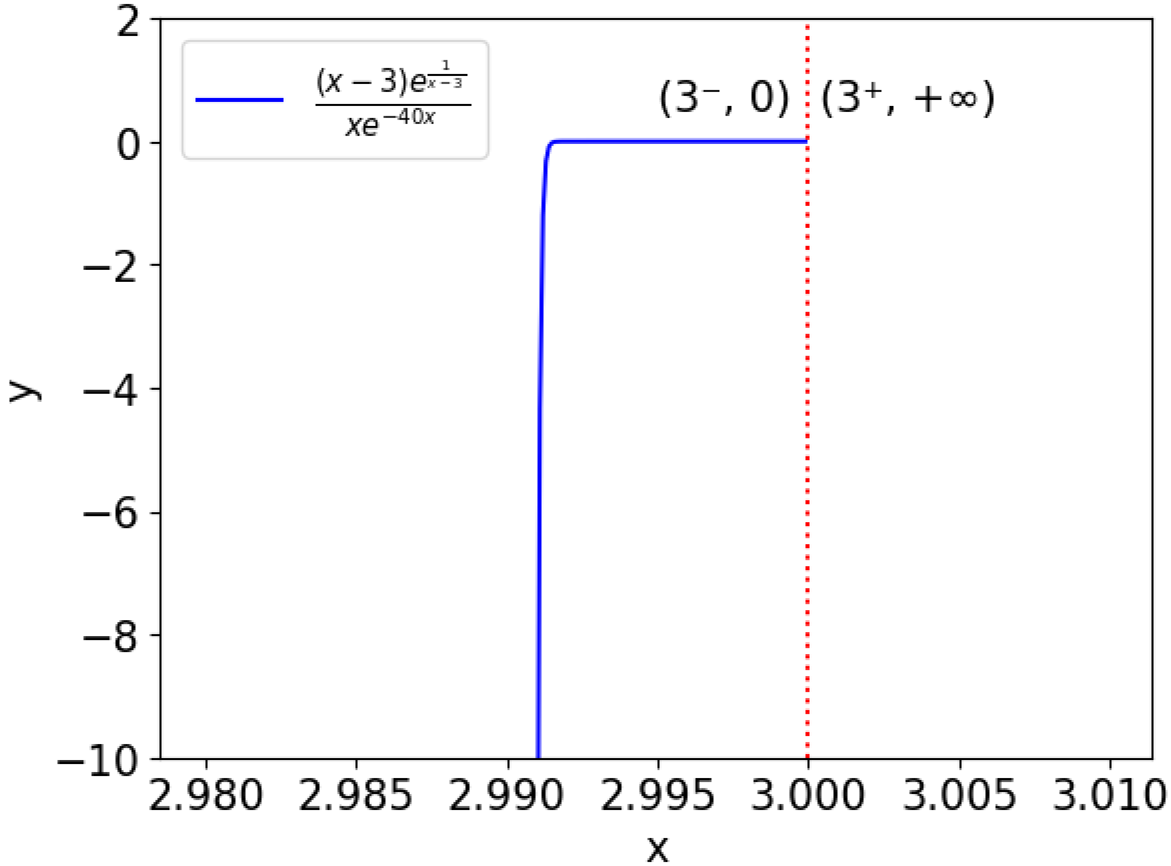
Which means to get to the same coordinates as in **Task 1**, she needs to move $\frac{360}{45} = 8\ times$ until the mean of all her movements gives us the coordinate at the origin.

Task 3:

Task 3.1 – Code:

```
import math
import cmath
import numpy as np
import matplotlib as mpl
import matplotlib.pyplot as plt

# Setup initial graph plot
mpl.rcParams['font.size'] = 15
plt.xlabel("x")
plt.ylabel("y")
plt.ylim(-10, 2)

# ---Task 3---
# --3.1--

# Define the range for x values from 2.98 to 3.01, with step size of 1e-4
x = np.arange(3 - 2e-2, 3 + 1e-2, 0.0001)

#Setting up the limit
y_num = (x - 3) * np.exp(1 / (x - 3)) # Defining the numerator
y_den = x * np.exp(-40 * x) # Defining the Denominator
y = y_num / y_den

# Plot the function in blue
plt.plot(x, y, 'b')

# Draw a vertical line at x = 3 in dotted red
plt.axvline(3, c='r', ls=':')

plt.text(3 - 5e-3, 0.5, "(3⁻, 0)")  # Adding left limit text
plt.text(3 + 4e-4, 0.5, "(3⁺, +∞)")  # Adding right limit text

plt.legend([r'$\frac{(x-3)e^{\frac{1}{x-3}}}{xe^{-40x}}$']) # Function legend
for the plot

#Answer = Limit as x approaches 3 does not exist (NaN, as the left-sided limit
is zero and the right-sided limit is +∞, which differ from each other),
therefore push a Big Red Button
```

Limit Plot:



Task 3.2 – Maths:

$y = \lim_{n \to 3} \frac{(x-3)e^{\frac{1}{x-3}}}{xe^{-40x}}$

Looking at the limit from both sides we get:

$y = \lim_{n \to 3^+} \frac{(x-3)e^{\frac{1}{x-3}}}{xe^{-40x}}$

$= \frac{(3^+ - 3)e^{\frac{1}{3^+-3}}}{3^+ e^{-40 \cdot 3^+}}$, e dominates, so we can ignore everything besides it

$= \frac{e^{+\infty}}{e^{-120}}$

$= e^{+\infty}$

$= +\infty$

$$y = \lim_{n \to 3^-} \frac{(x-3)e^{\frac{1}{x-3}}}{xe^{-40x}}$$

$= \frac{(3^- - 3)e^{\frac{1}{3^- - 3}}}{3^- e^{-40 \cdot 3^-}}$, e dominates, so we can ignore everything besides it

$$= \frac{e^{-\infty}}{e^{-120}}$$

$$= \frac{1}{e^{\infty}} \cdot \frac{1}{e^{-120}}$$

$$= -\frac{1}{e^{\infty}}$$

$$= 0$$

Since the limits vary from both sides, that means the limit as x approaches 3 does not exist (NaN), therefore you should press the Big Red Button.

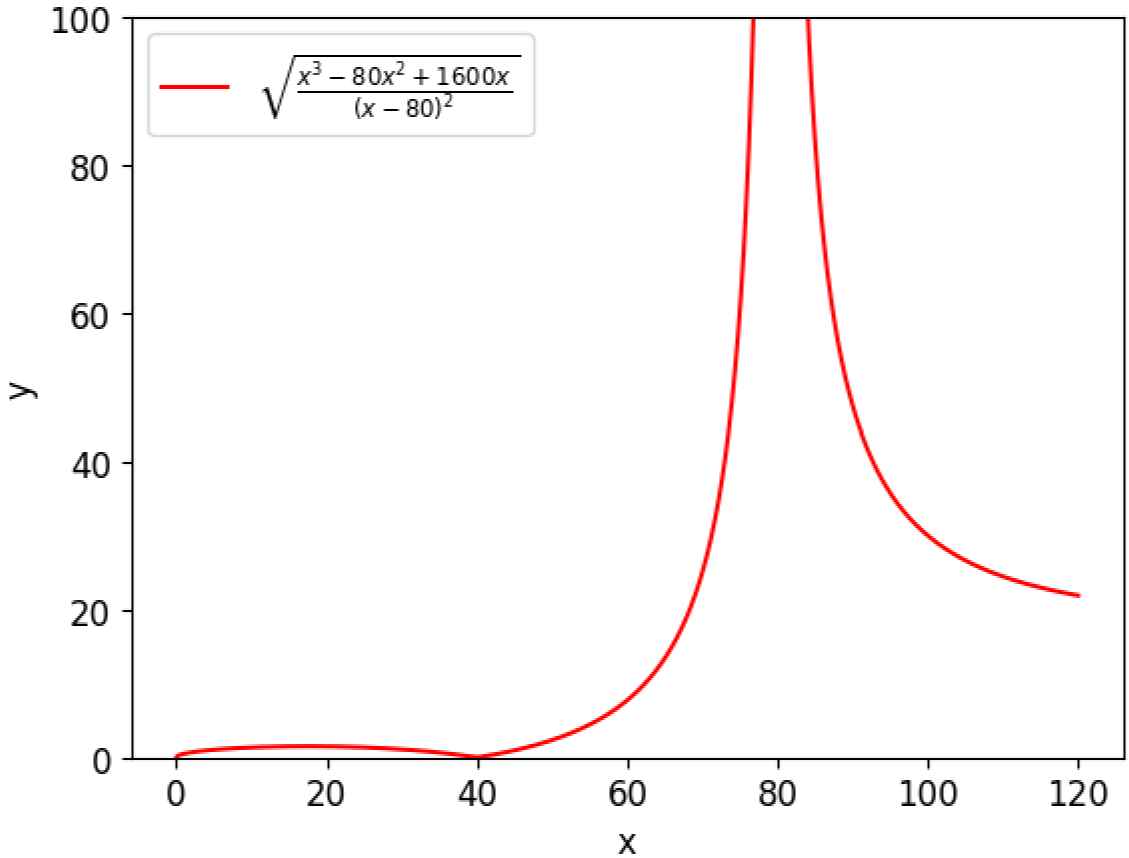
Task 4:

Task 4.1 - Code:

```
import math
import cmath
import numpy as np
import matplotlib as mpl
import matplotlib.pyplot as plt

#Setup graph
mpl.rcParams['font.size'] = 12
plt.xlabel("x")
plt.ylabel("y")
plt.ylim(0,100)

#---Task 4---
#--4.1--

x_initial = -3 * 40  # Initial x value = -120
x_final = 3 * 40  # Final x value = 120
x = np.arange(x_initial, x_final, 0.001)  # Values of x from initial x to final
x with a step of 0.001

#Setting up the function
y_num = (x**3) - (2 * 40 * (x**2)) + ((40**2) * x) # Defining the numerator
y_den = (x - 2 * 40)**2 # Defining the Denominator
y = np.sqrt(y_num / y_den)

# Plot the function in red on the graph
plt.plot(x, y, 'r')

# Adding a legend for the function
plt.legend([r'$\sqrt{\frac{x^3 - 80x^2 + 1600x}{(x-80)^2}}$'])

# This calculation below is to find the x-value that makes the function head
towards +inf
x = np.arange(0, x_final, 0.001)  # Values of x from 0 to final with a step of
0.001
y_num = (x**3) - (2 * 40 * (x**2)) + ((40**2) * x)
y_den = (x - 2 * 40)**2
y = np.sqrt(y_num / y_den) # Calculate the y values
```

```
# Iterate through the coordinates to find where y is +inf
for coord in zip(x, y):
    if coord[1] == np.inf:
        print(f"y({coord[0]}) = +inf")  # Print the x value where y is +inf
        # Prints y(80.0) = +inf

#Answer = Safe positions: x ∈ [0, 80) ∪ (80, +∞)
```

Function Plot:



Task 4.2 - Maths:

First, we turn this implicit equation into a function of $y(x)$:

$$y^2 \cdot (x - 2 \cdot \alpha)^2 = x^3 - 2 \cdot \alpha \cdot x^2 + \alpha^2 \cdot x$$

$$y^2 \cdot \left(x - 2(40)\right)^2 = x^3 - 2(40)x^2 + (40)^2 x$$

$$y^2 = \frac{x^3 - 80x^2 + 1600x}{(x - 80)^2}$$

$$y = \sqrt{\frac{x^3 - 80x^2 + 1600x}{(x - 80)^2}}$$

To determine the safe positions on the x-axis, we set the expression inside the square root to be non-negative:

$$\frac{x^3 - 80x^2 + 1600x}{(x - 80)^2} \geq 0$$

Meaning the safe positions are values > 0, otherwise they will go into complex numbers.

Moreover, we can solve the numerator,

$$x^3 - 80x^2 + 1600x \; = \; 0$$

$$x(x - 40)^2 = 0$$

$$x = 0, \; x = \; 40$$

The numerator indicates that the graph intersects the x-axis at 0 and 40.

We then solve the denominator,

$$(x - 80)^2 = 0$$

$$x = 80$$

The denominator indicates that when x equals 80, a divide by 0 occurs; the graph heads towards +inf, therefore not being a safe position.

From this we can say the domain of the function is $x \, \in \, [0,80) \, \cup \, (80, \, +\infty)$.

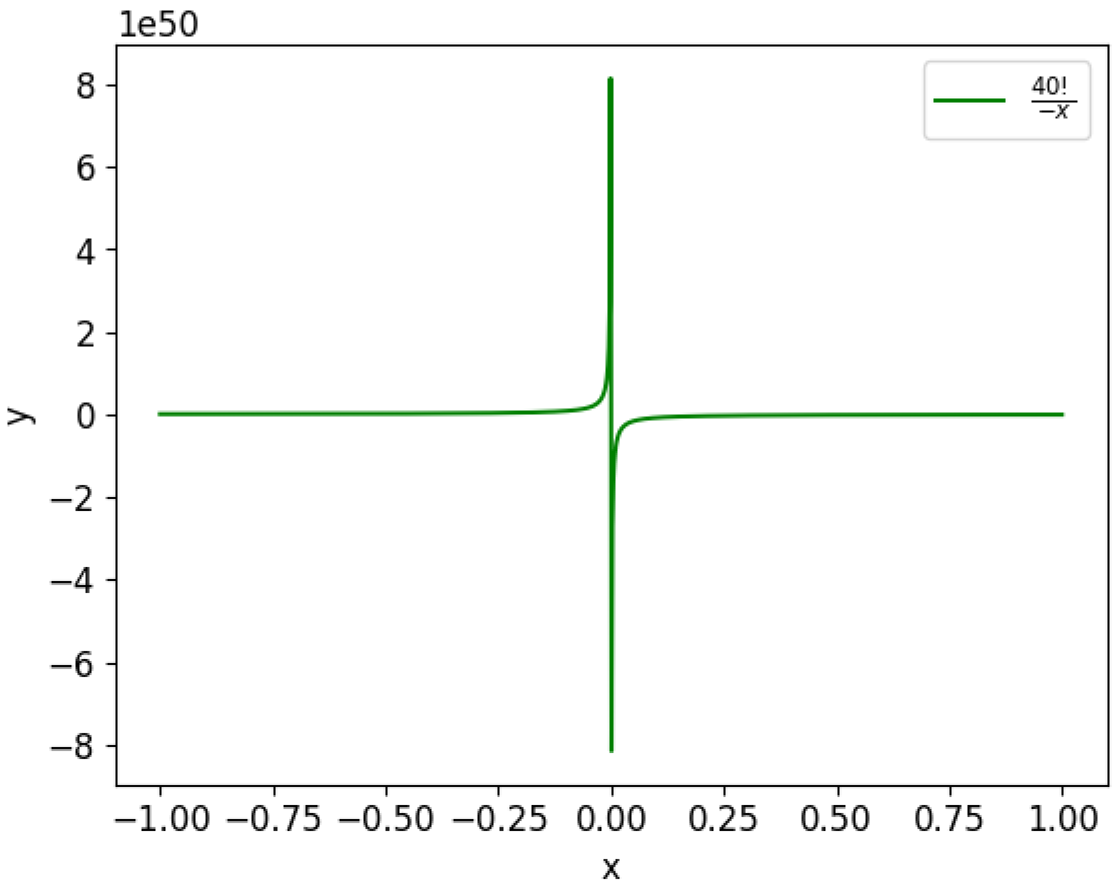
Task 5:

Task 5.1 - Code:

```
import math
import cmath
import numpy as np
import matplotlib as mpl
import matplotlib.pyplot as plt
from scipy.special import comb, perm

#Setup graph
mpl.rcParams['font.size'] = 12
plt.xlabel("x")
plt.ylabel("y")

#---Task 5---
#--5.1--

#Combination
c = comb(40,1) # Equals 40

#Permutation
p = perm(40,1) # Equals 40

#Limit
x = np.linspace(-1,1,1000)
y_num = math.factorial(40)
y_den = x
y = -(y_num/y_den)
plt.plot(x,y,'g')
plt.legend([r'$\frac{40!}{-x}$'])
#+inf and -inf, therefore not a number

S = {40, 40, np.nan, np.nan} #Despite the set removing duplicates, the set operations still provide the correct results

#Evaluating the states of p, q and r
p = S.intersection({40,0}) == {40}
q = {-np.inf}.issubset(S)
r = S.union({0}).issubset(S)
```

```
#Outputting their logical states
print(f"p: {p}, q: {q}, r: {r}")

#Evaluating the statement
outcome = (q^p)^r == (not(not(q)) or not(r)) # p: True, q: False, r: False
print("Outcome:",outcome) # Outputs True

#Answer = True, therefore choose the left lift
```

Limit Graph:



Task 5.2 - Maths:

To start, we must fill in the spots in the multiset (allows duplicate values):

$$S = \left\{\binom{\alpha}{1}, P(\alpha,\ 1),\ \lim_{x\to 0} -\frac{\alpha!}{x},\ T_3\right\}$$

First, the combinations:

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

$$\binom{\alpha}{1} = \binom{40}{1} = \frac{40!}{1!\,(40-1)!} = \frac{40!}{39!} = 40$$

Second, the permutations:

$$P(n,r) = \frac{n!}{(n-r)!}$$

$$P(\alpha,\ 1) = P(40,\ 1) = \frac{40!}{(40-1)!} = \frac{40!}{39!} = 40$$

Third, the limit:

$$\lim_{x\to 0} -\frac{\alpha!}{x} = \lim_{x\to 0} -\frac{40!}{x}$$

$$\lim_{x\to 0^+} -\frac{40!}{x}$$

$$= -\frac{40!}{0^+}$$

$$= -\infty$$

$$\lim_{x\to 0^-} -\frac{40!}{x}$$

$$= -\frac{40!}{0^-}$$

$$= +\infty$$

Different limits on both sides, therefore limit does not exist (NaN).

Task 3's answer is NaN, so the final multiset is now:

$S = \{40,\ 40,\ NaN,\ NaN\}$

Now, we find the states of p , q and r :

p: $S \cap \{\alpha,\ 0\} \equiv \{\alpha\}$

$S \cap \{40,\ 0\} \equiv \{40\}$

$\{40,\ 40,\ NaN,\ NaN\} \cap \{40,\ 0\} \equiv \{40\}$

$\{40\} \equiv \{40\}$

$= True$

q: $\{-\infty\} \subset S$

$= False$

r: $S \cup \{0\} \subset S$

$\{40,\ 40,\ NaN,\ NaN,\ 0\} \subset S$

$= False$

Now we can evaluate $(q \oplus p) \oplus r \Leftrightarrow (\neg q \rightarrow \neg r)$,

0: $False$ 1: $True$

$(0 \oplus 1) \oplus 0 \Leftrightarrow (\neg 0 \rightarrow \neg 0)$

$1 \oplus 0 \Leftrightarrow (1 \rightarrow 1)$

$1 \Leftrightarrow (0 \vee 1)$

$1 \Leftrightarrow 1$

$1\ (True)$

True, therefore choose the left lift.