AHP Presentation

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Introduction

The Analytic Hierarchy Process (AHP) is a structured decision-making methodology developed by Thomas L. Saaty. It helps break down complex problems into a hierarchy of simpler, more manageable components. By systematically evaluating options through pairwise comparisons and assigning weights, AHP enables decision-makers to prioritize alternatives based on both quantitative and qualitative criteria.

Real-World Example

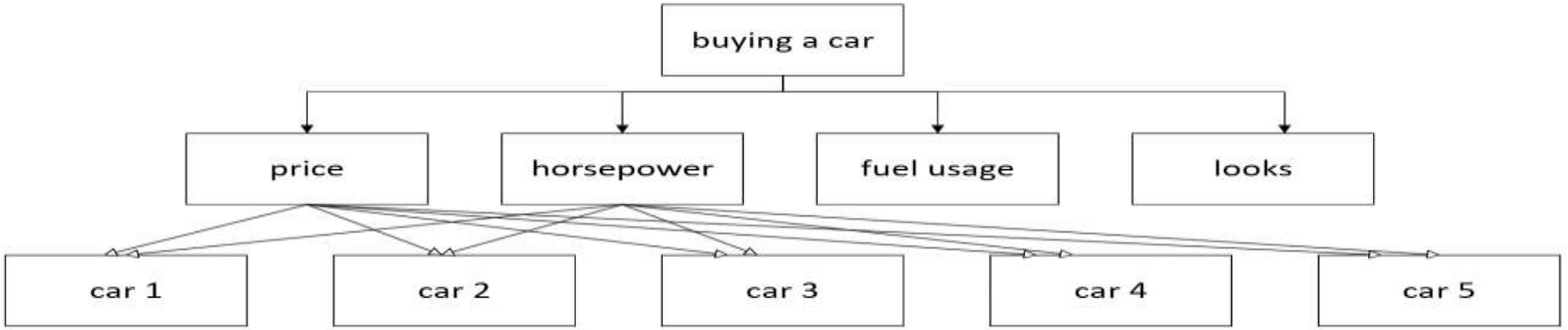
Alternatives

Criteria→	Price	HorsePower	Fuel Usage	Look Rate/5
Car 1	25000$	170hp	180km/20L	5
Car 2	20000$	170hp	150km/20L	3
Car 3	30000$	240hp	220km/20L	4
Car 4	27500$	230hp	140km/20L	4
Car 5	22500$	180hp	210km/20L	2

Hierarchical Structure

	1	2	3	4
Criteria→	Price	HorsePower	Fuel Usage	Look Rate/5

Step 1: Develop a hierarchical structure with a goal at the top, followed by criteria then the alternatives at last.



Scaling

Step 2: Define a scale(e.g:how important is price with respect to horsepower) (scale down below) then construct the matrix (next page) then fill the matrix based on pairwise comparison matrix.

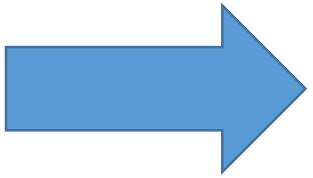
1: equal importance
3: moderate importance
5: strong importance
7: very strong importance
9: extreme importance
2,4,6,8: intermediate values
1/3, 1/5, 1/7, 1/9: values for inverse comparison

Pair-wise matrix

-Price have a strong importance than horsepower.
-Price have an equal to moderate importance than fuel usage.
-Price have a very strong importance than look rate.
-Horsepower have a strong importance than look rate.
-Fuel usage have a moderate importance than horsepower.
-Fuel usage have a very strong importance than look rate.

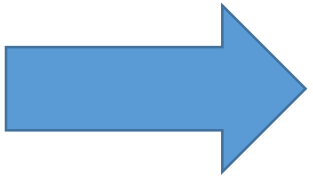
	Price	HorsePower	Fuel Usage	Look Rate/5
Price	1	5	2	7
HorsePower	1/5	1	1/3	5
Fuel Usage	1/2	3	1	7
Look Rate/5	1/7	1/5	1/7	1

Pair-wise matrix:



	Price	HorsePower	Fuel Usage	Look Rate/5
Price	1	5	2	7
HorsePower	0.2	1	0.33	5
Fuel Usage	0.5	3	1	7
Look Rate/5	0.14	0.2	0.14	1
Sum	1.84	9.2	3.47	20

Normalised pair-wise matrix



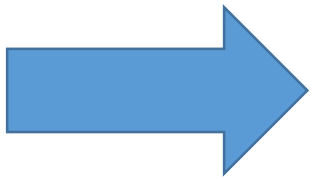
	Price	HorsePower	Fuel Usage	Look Rate/5
Price	0.543	0.543	0.576	0.35
HorsePower	0.109	0.109	0.095	0.25
Fuel Usage	0.272	0.326	0.288	0.35
Look Rate/5	0.076	0.022	0.04	0.05

This matrix is calculated by dividing each element with the sum obtained at the same column.

Criteria Weights

Criteria Weights are calculated by averaging elements from each row:

Criteria Weights
2.012 / 4
0.563 / 4
1.236 / 4
0.279 / 4



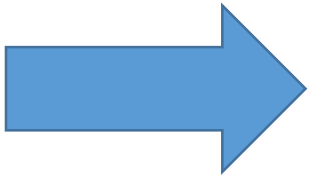
Criteria Weights
0.503
0.14
0.309
0.047

Consistency Calculations

Step 3: Calculate the consistency to check if the calculated values are correct.

First multiply columns by the corresponding criteria weight (not normalized matrix):

Criteria weights	0.503	0.14	0.309	0.07
	Price	HorsePower	Fuel Usage	Look Rate/5
Price	1 x 0.503	5 x 0.14	2 x 0.309	7 x 0.047
HorsePower	0.2 x 0.503	1 x 0.14	0.33 x 0.309	5 x 0.047
Fuel Usage	0.5 x 0.503	3 x 0.14	1 x 0.309	7 x 0.047
Look Rate/5	0.14 x 0.503	0.2 x 0.14	0.14 x 0.309	1 x 0.047



Criteria weights	0.503	0.14	0.309	0.07
	Price	HorsePower	Fuel Usage	Look Rate/5
Price	0.503	0.7	0.618	0.329
HorsePower	0.1	0.14	0.102	0.235
Fuel Usage	0.251	0.42	0.309	0.329
Look Rate/5	0.07	0.028	0.043	0.047

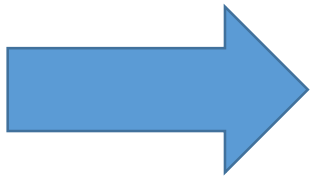
Weighted Sum Values

Weighted Sum Values are calculated by adding elements from each row:

	Price	HorsePower	Fuel Usage	Look Rate/5	Weighted Sum Values
Price	0.505	0.704	0.619	0.329	2.157
HorsePower	0.1	0.14	0.102	0.235	0.583
Fuel Usage	0.251	0.42	0.309	0.329	1.321
Look Rate/5	0.07	0.028	0.043	0.047	0.192

Calculate the ratio between weighted sum values and criteria weights:

Weighted Sum Values	Criteria Weights	
2.157	0.503	= 4.288
0.583	0.14	= 4.164
1.321	0.309	= 4.275
0.192	0.048	= 4



λmax

λmax is calculated by averaging the ratios calculated before:

= 4.288
= 4.164
= 4.275
= 4

$$\lambda max = \frac{4.288+4.164+4.275+4}{4}$$

$$\lambda max = 4.181$$

Consistency Index

Consistency Index (C.I.) = $\frac{\lambda \text{max} - n}{n-1}$

There are 4 criteria in this example so n=4

$$= \frac{4.181 - 4}{4 - 1}$$

$$= 0.06$$

Consistency Ratio

$$\text{Consistency Ratio} = \frac{\text{Consistency Index (C.I.)}}{\text{Random Index (R.I.)}}$$

n	1	2	3	4	5	6	7	8	9	10
Random Index	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

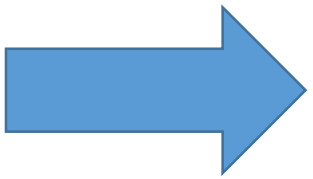
Consistency Ratio = $\frac{0.06}{0.9}$ = 0.067 < 0.1 → Conclusion: the matrix is consistent

Alternatives Calculations

Alternative 1, Criteria 1: 25000
Alternative 1, Criteria 2: 170
Alternative 1, Criteria 3: 180
Alternative 1, Criteria 4: 5
Alternative 2, Criteria 1: 20000
Alternative 2, Criteria 2: 170
Alternative 2, Criteria 3: 150
Alternative 2, Criteria 4: 3
Alternative 3, Criteria 1: 30000
Alternative 3, Criteria 2: 240
Alternative 3, Criteria 3: 220
Alternative 3, Criteria 4: 4
Alternative 4, Criteria 1: 27500
Alternative 4, Criteria 2: 230
Alternative 4, Criteria 3: 140
Alternative 4, Criteria 4: 4
Alternative 5, Criteria 1: 22500
Alternative 5, Criteria 2: 180
Alternative 5, Criteria 3: 210
Alternative 5, Criteria 4: 2

Criteria→	Price	HorsePower	Fuel Usage	Look Rate/5
Car 1	25000	170	180	5
Car 2	20000	170	150	3
Car 3	30000	240	220	4
Car 4	27500	230	140	4
Car 5	22500	180	210	2
sum	125000	990	900	18

Normalised matrix



Criteria→	Price	HorsePower	Fuel Usage	Look Rate/5
Car 1	0.2	1.172	0.2	0.278
Car 2	0.16	1.172	0.167	0.167
Car 3	0.24	0.242	0.244	0.222
Car 4	0.22	0.232	0.156	0.222
Car 5	0.18	0.182	0.233	0.111

This matrix is calculated by dividing each element with the sum obtained at the same column.

Alternatives Score Calculations

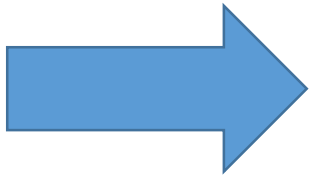
Criteria→	Price	HorsePower	Fuel Usage	Look Rate/5
Car 1	0.2	0.172	0.2	0.278
Car 2	0.16	0.172	0.167	0.167
Car 3	0.24	0.242	0.244	0.222
Car 4	0.22	0.232	0.156	0.222
Car 5	0.18	0.182	0.233	0.111

	Criteria Weights
Price	0.503
HorsePower	0.14
Fuel Usage	0.309
Look Rate	0.047

Car 1 Score : 0.2 x 0.503 + 0.172 x 0.14 + 0.2 x 0.309 + 0.278 x 0.047 = 0.2
Car 2 Score : 0.16 x 0.503 + 0.172 x 0.14 + 0.167 x 0.309 + 0.167 x 0.047 = 0.164
Car 3 Score : 0.24 x 0.503 + 0.242 x 0.14 + 0.244 x 0.309 + 0.222 x 0.047 = 0.24
Car 4 Score : 0.22 x 0.503 + 0.232 x 0.14 + 0.156 x 0.309 + 0.222 x 0.047 = 0.201
Car 5 Score : 0.18 x 0.503 + 0.182 x 0.14 + 0.233 x 0.309 + 0.111 x 0.047 = 0.193

Ranking Alternatives

Car 1 Score = 0.2
Car 2 Score = 0.164
Car 3 Score = 0.24
Car 4 Score = 0.201
Car 5 Score = 0.193



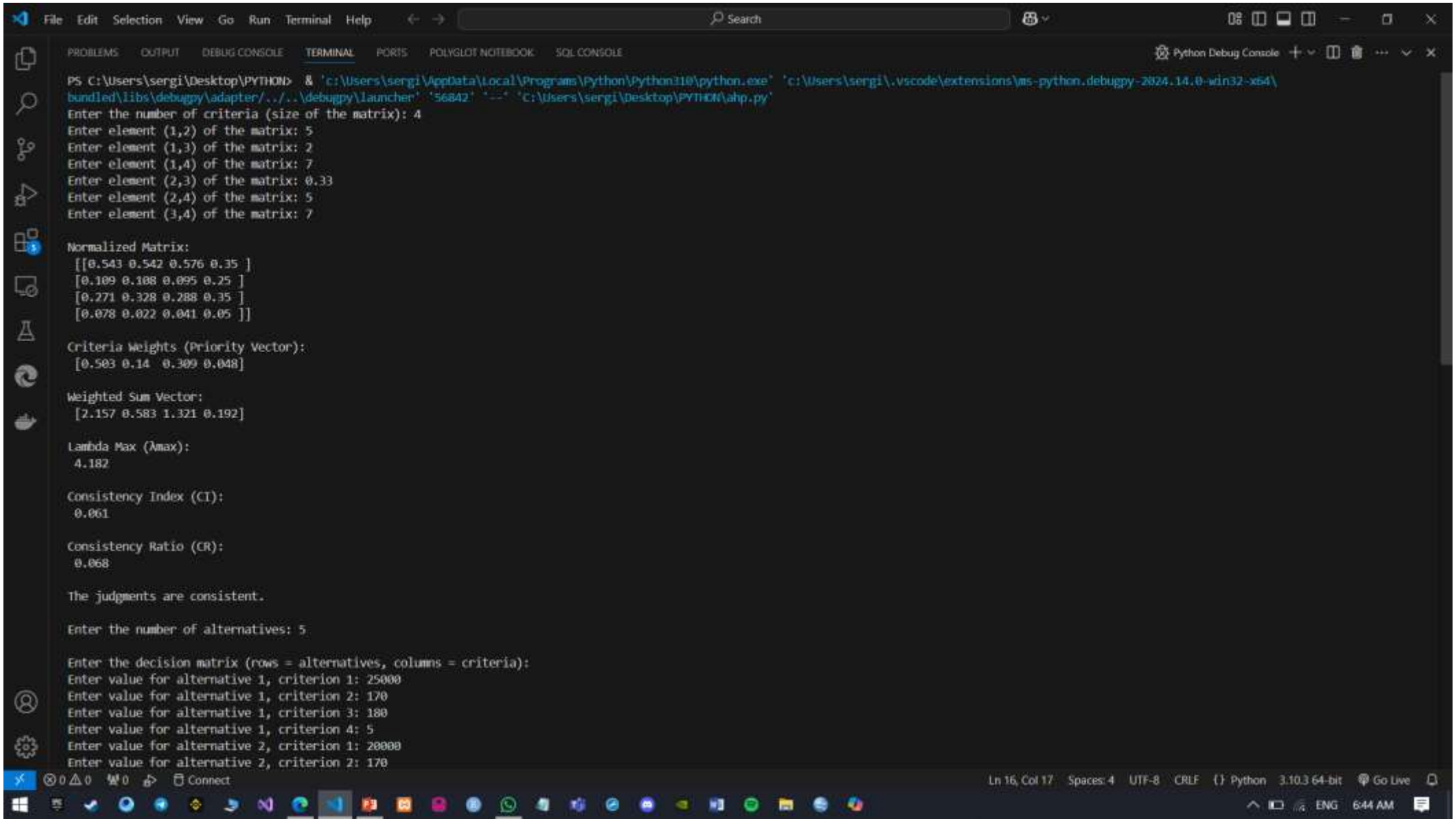
1- Car 3
2- Car 4
3- Car 1
4- Car 5
5- Car 2

The best alternative is Car 3 And the worst alternative is Car 2, based on criteria priorities and alternatives characteristics

AHP Using Python

It will be a lot faster and easier to compute all the numbers we got (Criteria weights, weighted sum values, consistency index (CI), and consistency ratio).

The code is provided (in a file aside) with its results to check if the numbers calculated before are true.


File Edit Selection View Go Run Terminal Help
Search
PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL PORTS POLYGLOT NOTEBOOK SQL CONSOLE
Python Debug Console
PS C:\Users\sergi\Desktop\PYTHON> & 'c:\Users\sergi\AppData\Local\Programs\Python\Python310\python.exe' 'c:\Users\sergi\.vscode\extensions\ms-python.debugpy-2024.14.0-win32-x64\bundled\libs\debugpy\adapter/../..\debugpy\launcher' '56842' '--' 'C:\Users\sergi\Desktop\PYTHON\ahp.py'
Enter the number of criteria (size of the matrix): 4
Enter element (1,2) of the matrix: 5
Enter element (1,3) of the matrix: 2
Enter element (1,4) of the matrix: 7
Enter element (2,3) of the matrix: 0.33
Enter element (2,4) of the matrix: 5
Enter element (3,4) of the matrix: 7
Normalized Matrix:
[[0.543 0.542 0.576 0.35]
[0.109 0.108 0.095 0.25]
[0.271 0.328 0.288 0.35]
[0.078 0.022 0.041 0.05]]
Criteria Weights (Priority Vector):
[0.503 0.14 0.309 0.048]
Weighted Sum Vector:
[2.157 0.583 1.321 0.192]
Lambda Max (λmax):
4.182
Consistency Index (CI):
0.061
Consistency Ratio (CR):
0.068
The judgments are consistent.
Enter the number of alternatives: 5
Enter the decision matrix (rows = alternatives, columns = criteria):
Enter value for alternative 1, criterion 1: 25000
Enter value for alternative 1, criterion 2: 170
Enter value for alternative 1, criterion 3: 180
Enter value for alternative 1, criterion 4: 5
Enter value for alternative 2, criterion 1: 20000
Enter value for alternative 2, criterion 2: 170
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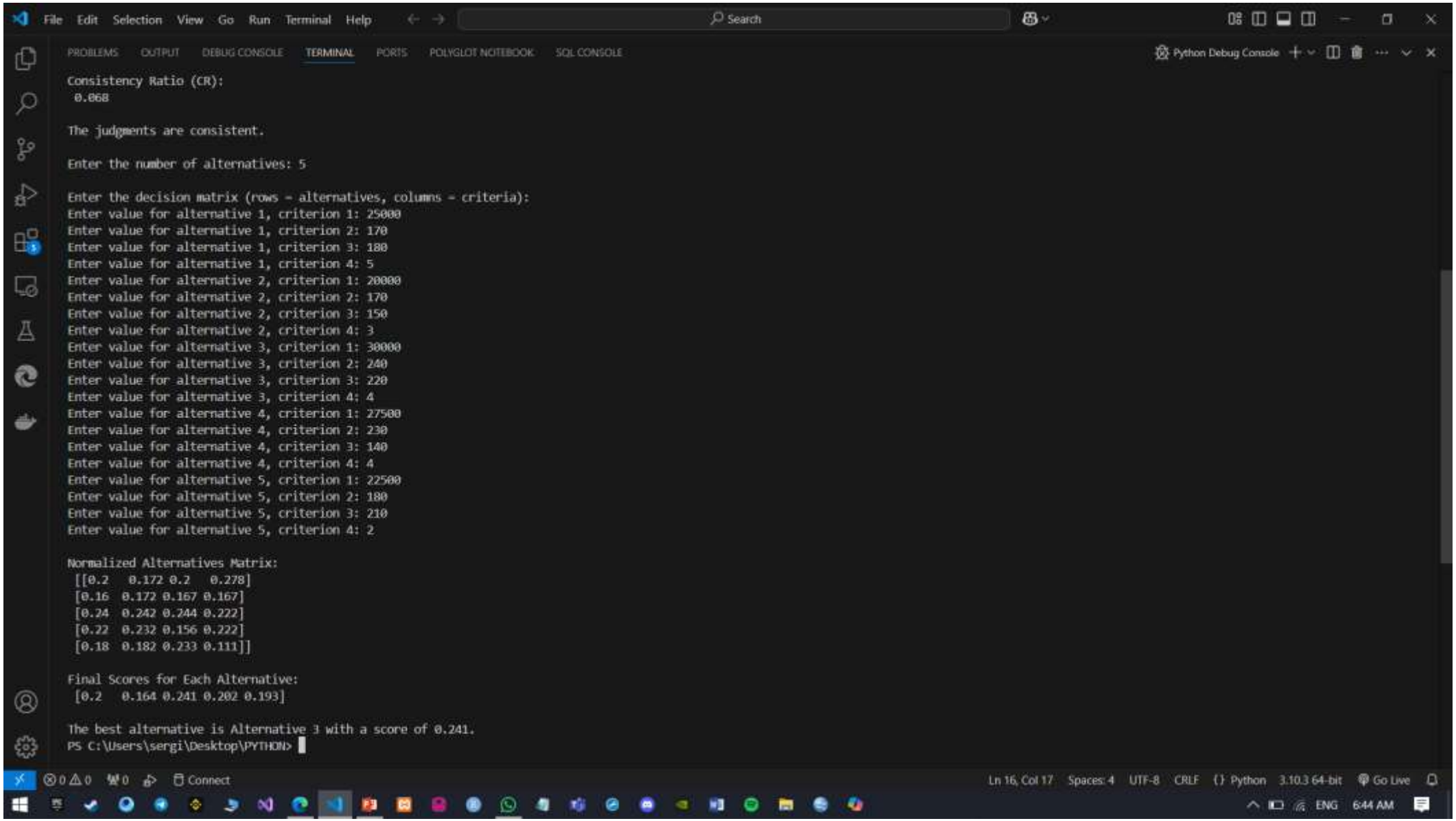
File Edit Selection View Go Run Terminal Help
Search
PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL PORTS POLYGLOT NOTEBOOK SQL CONSOLE
Python Debug Console
Consistency Ratio (CR):
0.068
The judgments are consistent.
Enter the number of alternatives: 5
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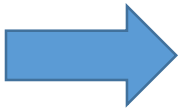
Enter value for alternative 2, criterion 2: 170
Enter value for alternative 2, criterion 3: 150
Enter value for alternative 2, criterion 4: 3
Enter value for alternative 3, criterion 1: 30000
Enter value for alternative 3, criterion 2: 240
Enter value for alternative 3, criterion 3: 220
Enter value for alternative 3, criterion 4: 4
Enter value for alternative 4, criterion 1: 27500
Enter value for alternative 4, criterion 2: 230
Enter value for alternative 4, criterion 3: 140
Enter value for alternative 4, criterion 4: 4
Enter value for alternative 5, criterion 1: 22500
Enter value for alternative 5, criterion 2: 180
Enter value for alternative 5, criterion 3: 210
Enter value for alternative 5, criterion 4: 2
Normalized Alternatives Matrix:
[[0.2 0.172 0.2 0.278]
[0.16 0.172 0.167 0.167]
[0.24 0.242 0.244 0.222]
[0.22 0.232 0.156 0.222]
[0.18 0.182 0.233 0.111]]
Final Scores for Each Alternative:
[0.2 0.164 0.241 0.202 0.193]
The best alternative is Alternative 3 with a score of 0.241.
PS C:\Users\sergi\Desktop\PYTHON>
Connect
Ln 16, Col 17 Spaces: 4 UTF-8 CRLF {} Python 3.10.3 64-bit Go Live
ENG 6:44 AM

Conclusion

- It’s a decision-making framework.
- Used to solve complex problems by structuring them hierarchically.
- Involves priority ranking to solve a problem with multi-criteria.
- It turns qualitative criteria into quantitative ones so its easier to compare between them.
- Breaks down complex problems into smaller ones.
- Provides clear and reasonable decisions.
- Assign weights to criteria based on its importance.



So these are some benefits:

- Structured Decision-Making
- Flexibility
- Transparency
- Consistency Checking
- Prioritization
- Simplifies Complexity