

Session 2 Exercises

Estimating a Proportion with a Discrete Prior

Consider a population of "successes" and "failures" where the proportion of successes is p . Suppose p takes on the discrete set of values 0, .01, ..., .99, 1 and one assigns a uniform prior on these 101 values, meaning that the probability of any one of the 101 possible outcomes (remember, elements of the \mathbf{p} vector include both 0 and 1) is equally likely. Use R to enter the values of \mathbf{p} and the associated probabilities into the vectors \mathbf{p} and \mathbf{prior} , respectively. Plot the resulting \mathbf{prior} distribution with the values of \mathbf{p} on the horizontal axis and the values of \mathbf{prior} on the vertical axis (hint: use `plot()` function with argument `type = "h"`). What family of random variable does this prior distribution look like?

Posterior Distribution

Suppose one takes a random sample from the population without replacement and observes 20 successes and 12 failures. The function `pdisc()` in the **LearnBayes** package computes the associated posterior probabilities for p . After loading the **LearnBayes** package, call up help for the `pdisc()` function with the R command `?pdisc`. The inputs to `pdisc()` are the prior (vector of values of \mathbf{p} and vector of prior probabilities) and a vector containing the number of successes and failures. Create a variable named `post` which captures the posterior distribution output by `pdisc()`. Plot the resulting `post` distribution with the values of \mathbf{p} on the horizontal axis and the values of the posterior distribution on the vertical axis (hint: again use `plot()` function with argument `type = "h"`). What family of random variable does the posterior distribution look like?

Highest Probability

A highest probability interval for a discrete distribution is obtained using the `discint()` function. Call up the help screen for the `discint()` function using the R command `?discint`. This function has two inputs: the probability distribution matrix where the first column contains the values and the second column contains the probabilities, and the desired probability content. Use `discint()` to separately compute 90, 95 and 99 percent probability intervals for p from the posterior distribution. What are the upper and lower bounds of the interval that contains \mathbf{p} with a 90% probability? With a 95% probability? With a 99% probability interval?