Algorithmics	Student information	Date	Number of session
	UO: 296503	03/03/2025	3
	Surname: Mulet Alonso		
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Activity 1. Divide and Conquer by subtraction

- Subtraction1:

It stops giving times at n = 8192, due to stackOverflowError as it is mentioned in the pdf of the practice "Subtraction1.java and Subtraction2.java classes have an approach by subtraction with a=1, which involves a large expenditure of stack memory", it overflows.

- Subtraction2:

It stops exactly at the same size, it makes sense since it has the same problem as Susbtraction1.

- How many years will take for Subtraction3 complete n=80?

For calculating this we have to make the following operation: T(n) = a * T(n-1)

The last measurable value I got for Susbtraction3 is n=30 t=38705ms.

T(80) = 38705 * $2^{(80-30)}$ = 4.3578*10^{19} ms = 1.380.931.845 years

- Substraction4

We have to implement a divide and conquer by subtraction algorithm with complexity O(n3), in my case I implemented an algorithm with a=1; b=1; k=2.

As a=1 the complexity is O(n^{k+1}) which is O(n^3)

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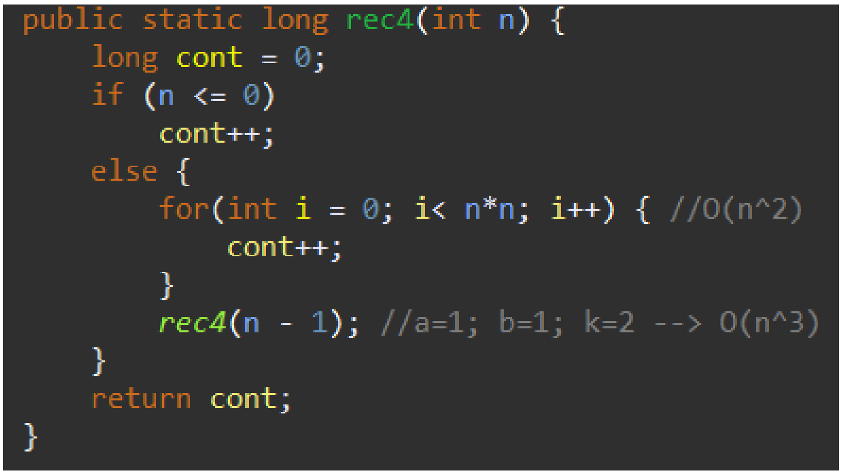

```
public static long rec4(int n) {
    long cont = 0;
    if (n <= 0)
        cont++;
    else {
        for(int i = 0; i< n*n; i++) { //O(n^2)
            cont++;
        }
        rec4(n - 1); //a=1; b=1; k=2 --> O(n^3)
    }
    return cont;
}
```

Times I got:

n	t(ms)
100	1
200	11
400	86
800	713
1600	5845
3200	47420
6400	OoT

Times match the complexity, let’s take for instance n=1600, with this size the algorithm finished at 5845milliseconds, by multiplying the size by 2, I got a time which is more or less 2^3 times bigger (n=3200; t = 47420), 47420/5845 = 8,11.

- Subtraction5:

We have to implement a divide and conquer by subtraction algorithm with complexity O($3^{n/2}$), for that, we have to implement an algorithm with a=3 and b=2 (in my case k=0) As a > 1 the complexity is O($a^{n/b}$).

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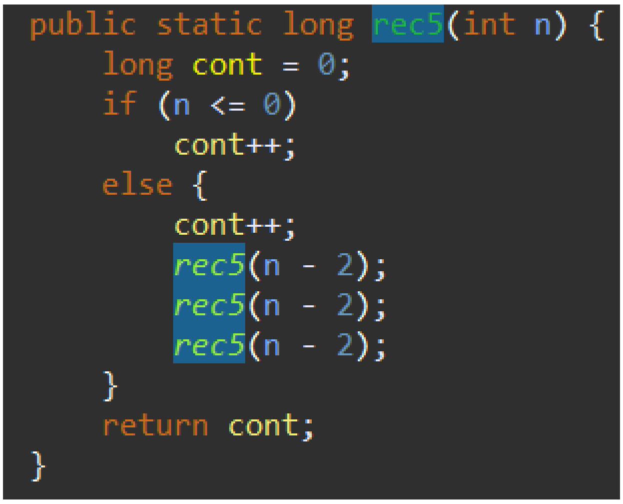

```java
public static long rec5(int n) {
    long cont = 0;
    if (n <= 0)
        cont++;
    else {
        cont++;
        rec5(n - 2);
        rec5(n - 2);
        rec5(n - 2);
    }
    return cont;
}
```

Times I got:

n	t(ms)
30	399
32	1181
34	5535
36	10763
38	32119
40	OoT

For n=30 we get a time of 399ms, this match the complexity since as we increase n by 2, times are multiplied more or less by 3, and this is because n is divided by b, which is 2, so it is growing by one and then being multiplied by 3 make sense.

- How many years will take for Subtraction5 complete n=80?

For calculating this we have to make the following operation: T(n) = a * T(n-1)

The last measurable value I got for Susbtraction5 is n=38 t=32119ms.

T(80) = 32119 * $2^{(80-38)}$ = 1,4126*10^{17} ms = 4.476.351,201472 years

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Activity 2. Divide and conquer by division

- Division1

It has O(n) complexity, we don't get big times for a real measure because it is too fast, but we can appreciate how by multiplying the size by two, times get multiplied by two also, for instance, when n = 262144 t = 3 and when n = 524288 t = 6.

At the end of execution, we get n = 4194304 t = 37 and then n = 8388608 t = 74. This is the best case in which we see how times follow the complexity.

- Division2

It has O(nlogn) complexity, in this case times grow more or less as in Division1, maybe this is due to the waste of stack, which is O(logn).

For instance, n = 4194304 t = 1059 and n = 8388608 t = 2155, which is slightly more than the doble.

- Division3

It has O(n) complexity, and it is clearly shown by times.

For instance, n = 524288 t = 66 and n = 1048576 t = 137. By increasing the size by two, the time increases also by two, then it follows the linear complexity.

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- Division4

n	t(ms)
1000	6
2000	27
4000	94
8000	349
16000	1447
32000	5858
64000	

Times follow the complexity $O(n^2)$ clearly

- Division5

n	t(ms)
1000	26
2000	106
4000	427
8000	1750
16000	7090
32000	27632
64000	OoT

In my case, I have implemented an algorithm with a=4, b =2 and k=1, so a >bk, then the complexity is $O(n^{\log_b a})$ => $O(n^2)$

Times follow quadratic complexity.

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Activity 3. Two basic examples.

- Vector

All the measurements are with 1000000 as number of repetitions.

Sum 1 (iterative)

n	t(ns)
3	40
6	64
12	93
24	142
48	245
96	438
192	817
384	1575
768	3111
1536	6156
3072	12289
6144	24575
12288	49382
24576	OoT

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Sum2 (D&C by subtraction)

n	t(ns)
3	84
6	137
12	227
24	398
48	745
96	1461
192	2881
384	5976
768	13259
1536	27890
3072	55784
6144	OoT

Sum3 (D&C by division)

n	t(ms)
3	90
6	195
12	404
24	825
48	1721
96	3403
192	7046
384	14269
768	28780
1536	53457

The three approaches follow the linear complexity, however it seems that for this specific problem, the iterative approach is the best of them. This could be because since it is not a big problem, using divide and conquer may be useless (in fact it is worse).

- Fibonacci

Fib1 (iterative solution O(n))

n	t(ns)
10	93
11	87
12	94
13	98
14	106
15	108
20	134
40	229
59	300

Fib2(Iterative solution using a vector, dynamic programming)

n	t(ms)
10	111
11	122
12	130
13	132
14	145
15	149
20	184
40	370
59	482

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- Fib3(D&C by subtraction)

n	t(ns)
10	202
11	218
12	234
13	251
14	256
15	270
20	358
40	640
59	928

Fib4(D&C by subtraction)

n	t(ns)
10	2436
11	3970
12	7093
13	11013
14	17788
15	29146
16	45632
17	OoT

For fib1, fib2 and fib3 is hard to see the linear complexity, as we are increasing size one by one and they are small values, we can see that times follows the complexity more or less from n=20 to n=40, for instance fib2 n=20 t=184 and n=20 t =370.

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For this problem, as it happened with vector, the iterative solution is the best option. Dynamic programming is faster than D&C by subtraction.

Fib4 is the worst case, as it follows an exponential complexity 1,6^n and times show this perfectly, by increasing just by 1 the problem size, we are multiplexing the times by 1.6, for instance, n=11 t = 3970 and n=12 t = 7093. We must try to avoid this type of complexity.

4. Petanque championship organization