Addressing the Hubble tension with scalar fields

In collaboration with Ed Copeland, Adam Moss and Jade White

Sergio Sevillano

Talk prepared for UNAM April 2024


Glasgow
Edinburgh
United Kingdom
Isle of Man
Ireland
Dublin
Manchester
Birmingham
London

Durham, UK


Glasgow
Edinburgh
United Kingdom
Isle of Man
Manchester
Dublin
Ireland
Birmingham
London

Durham, UK

Nottingham, UK

Scaling solutions as Early Dark Energy resolutions to the Hubble tension

Edmund J. Copeland[a] Adam Moss[a] Sergio Sevillano Muñoz[a] Jade M. M. White[a]

[a]School of Physics and Astronomy, University of Nottingham, Nottingham NG7 2RD, United Kingdom

E-mail: ed.copeland@nottingham.ac.uk, adam.moss@nottingham.ac.uk, s.sevillano@nottingham.ac.uk, jade.white@nottingham.ac.uk

Nottingham, UK

Hubble Tension?

What is the Hubble Tension?

Disagreement between measurements of the current expansion of the universe, H_0

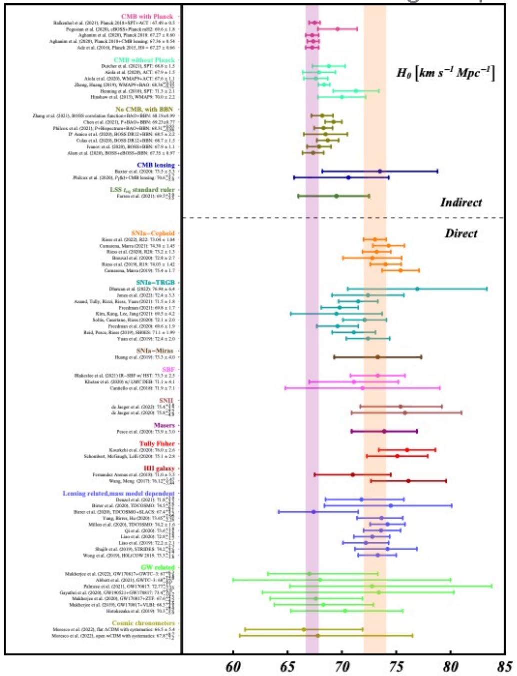
Direct: $73.0\ \pm 1.0\ km\ \sec^{-1} Mpc^{-1}$

Observation from cepheids and supernovae

Indirect: $67.4\ \pm\ 0.5\ km\ \sec^{-1} Mpc^{-1}$

Using CMB data and ΛCDM (plus SM)

Abdalla et al. 2203.06142



Hubble Tension?

Direct Measurement SH0ES Team

Just need redshift and distance from standard candles

$$D = \frac{cz}{H_0}\left\{1 - \left[1 + \frac{q_0}{2}\right]z + \left[1 + q_0 + \frac{q_0^2}{2} - \frac{j_0}{6}\right]z^2 + O(z^3)\right\}$$

D: Distance

z: redshift

q(t)= $-\frac{d^2a}{dt^2}\frac{H(t)^{-2}}{a}$

j(t)= $-\frac{d^3a}{dt^3}\frac{H(t)^{-3}}{a}$

Distance Ladder:

Geometry → Cepheids → (~40 Mpc) Supernovae Type Ia

Geometry: Parallax

Cepheids: Very luminous stars, ~100,000 solar lum.

Supernovae Type Ia: Detonation of white dwarf, ~1B solar lum.

Hubble Tension?

Direct Measurement SH0ES Team

Distance Ladder: Cepheids

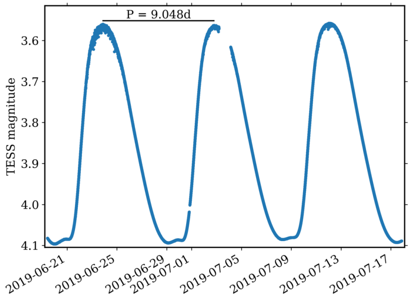
Most consistently calibrated standard candle

Easy to identify due to their oscillatory brightness

The periodicity of the oscillations is closely related to the brightness of the star

Using geometry (parallax) we can find that relation

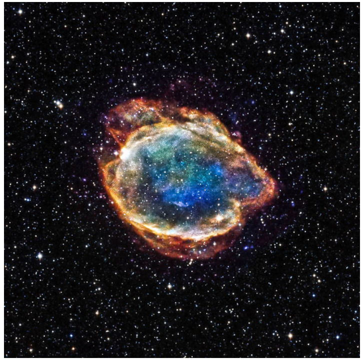
κ Pavonis



Hubble Tension?

Direct Measurement SH0ES Team

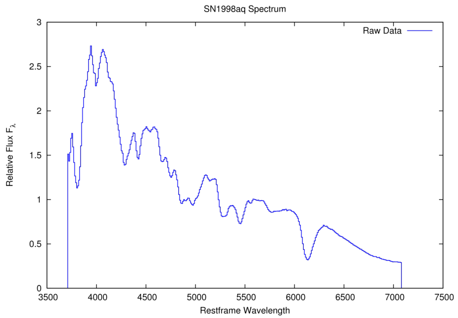
Distance Ladder: SN Ia



Not very abundant, so cannot use parallax

Brightness-to-distance ratio being defined with nearby cepheids

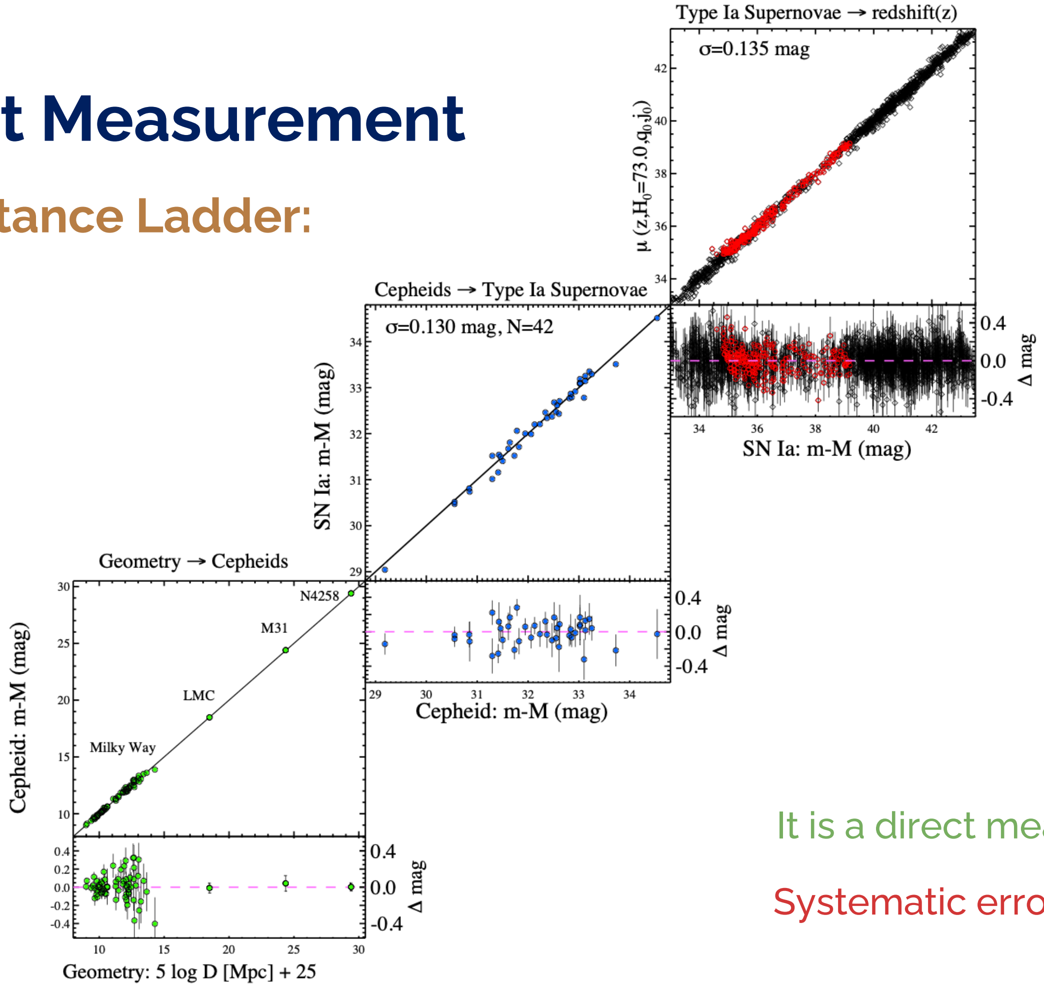
Close relation between the speed of luminosity evolution and total brightness



Direct Measurement

Based on 2309.15295

Distance Ladder:

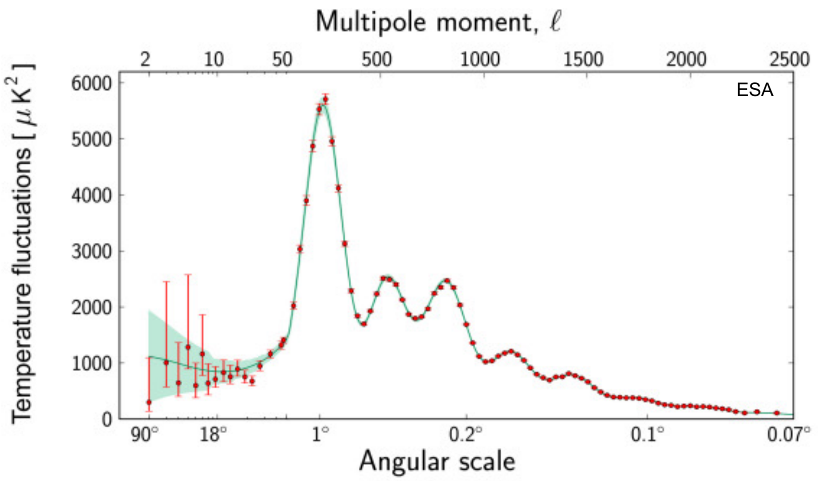


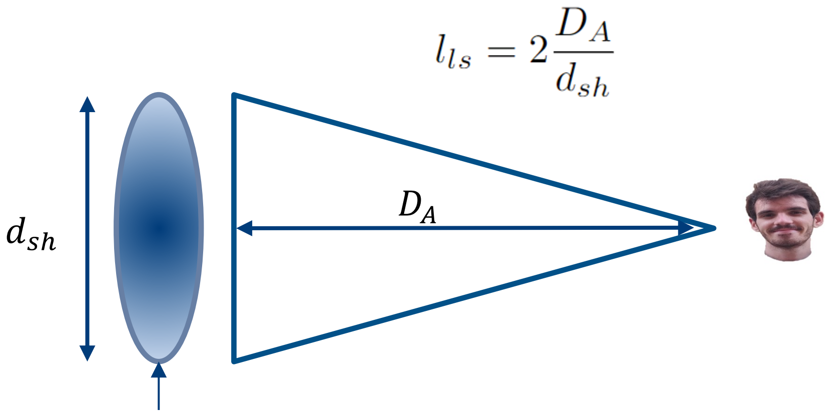
It is a direct measurement

Systematic errors when calibrating SN Ia?

Indirect Measurement

To infer H_0 from the CMB we need to take the first peak of its power spectrum





Density perturbation at the surface of last scattering

Hubble Tension?

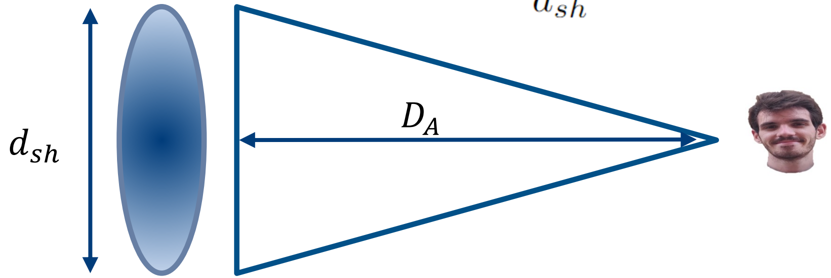
Indirect Measurement

Distance travelled by light from CMB

$$D_A = \int_0^{z_{ls}} \frac{c\,dz}{H(z)} = \frac{c}{H_0} \int_0^{z_{ls}} \frac{dz}{\sqrt{\rho(z)/\rho_0}}$$

Density perturbation at the surface of last scattering

$$l_{ls} = 2\frac{D_A}{d_{sh}}$$



Last scattering (ls)

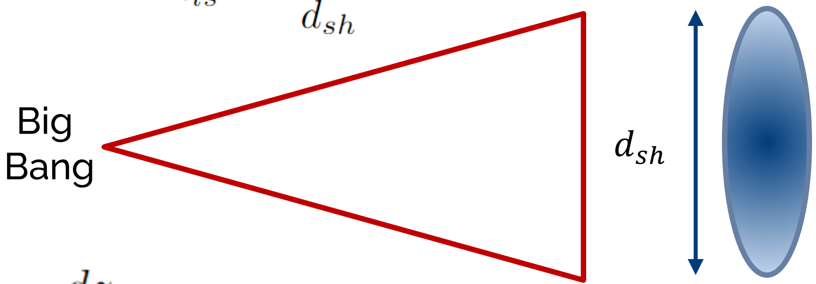
Hubble Tension?

Indirect Measurement

Growth of a point-like perturbation up to last scattering (sound horizon)

$$l_{ls} = 2\frac{D_A}{d_{sh}}$$

Density perturbation at the surface of last scattering



Last scattering (ls)

$$d_{sh} = \int_{z_{ls}}^{\infty} \frac{c_s(z)\, dz}{H(z)} = \frac{c}{\sqrt{3}H_{ls}} \int_{z_{ls}}^{\infty} \frac{dz}{\sqrt{\rho(z)/\rho(z_{lh})}\sqrt{1+R}}$$

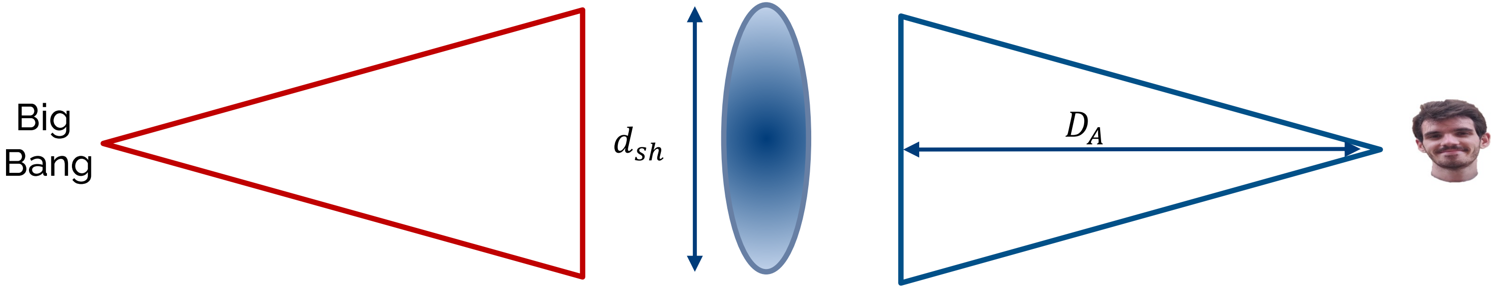
Hubble Tension?

Indirect Measurement

$$l_{ls} = 2\frac{D_A}{d_{sh}}$$

Combining everything…

Density perturbation at the surface of last scattering



$$H_0 = \sqrt{3} H_{\rm ls} \theta_s \frac{\int_0^{z_{\rm ls}} dz\ [\rho(z)/\rho_0]^{-1/2}}{\int_{z_{\rm ls}}^{\infty} dz\ [\rho(z)/\rho(z_{\rm ls})]^{-1/2} (1+R)^{-1/2}}$$

Hubble Tension?

Possible solution:

Late-time solution: Requires exotic fields such as phantom fields

$$H_0 = \sqrt{3} H_{\rm ls} \theta_s \frac{\int_0^{z_{\rm ls}} dz\ [\rho(z)/\rho_0]^{-1/2}}{\int_{z_{\rm ls}}^{\infty} dz\ [\rho(z)/\rho(z_{\rm ls})]^{-1/2} (1+R)^{-1/2}}$$

Hubble Tension?

Possible solution:

Late-time solution: Requires exotic fields such as phantom fields

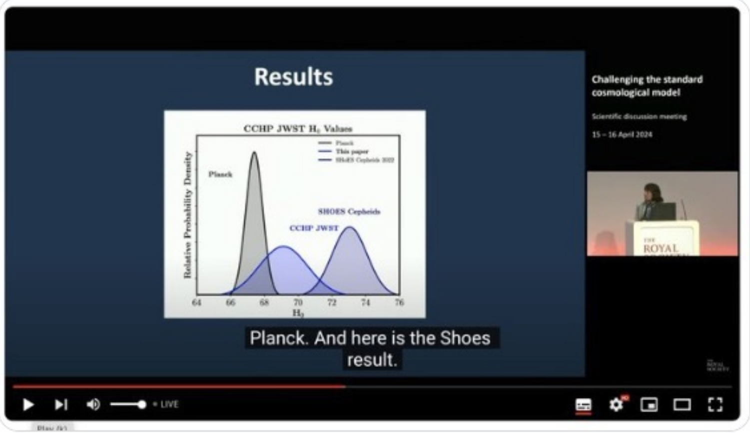
$$H_0 = \sqrt{3} H_{\mathrm{ls}} \theta_s \frac{\int_0^{z_{\mathrm{ls}}} dz \, [\rho(z)/\rho_0]^{-1/2}}{\int_{z_{\mathrm{ls}}}^{\infty} dz \, [\rho(z)/\rho(z_{\mathrm{ls}})]^{-1/2} (1+R)^{-1/2}}$$

Early-time solution: Early Dark Energy, but highly constraint (clustering + additional perturbations)

Hubble Tension?

Eugene Lim
@tukohbin

From Wendy Freedman talk at the Royal Society this morning. H0 tension dead?

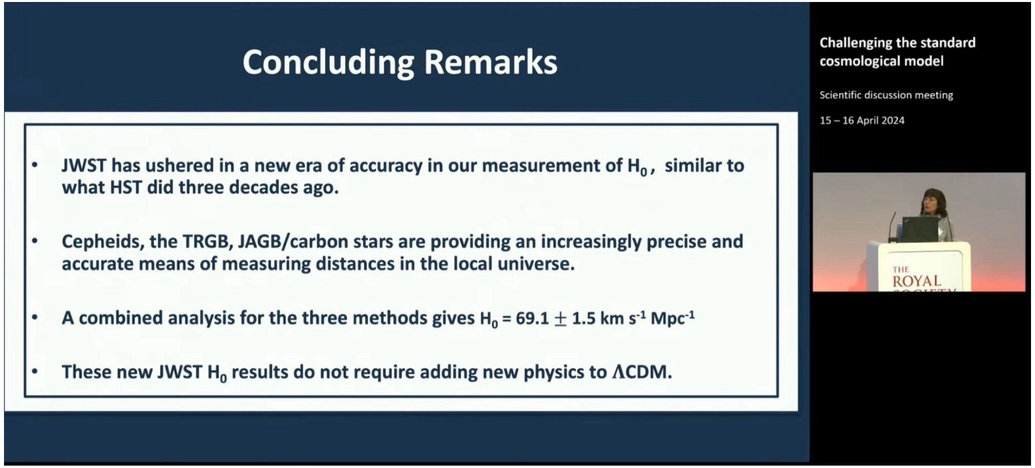


Still waiting for a paper…

Niko
@NikoSarcevic

H0 👀👀👀👀👀

5:43 AM · Apr 15, 2024 from Dunston, England · **2,948** Views



UNAM

on 2309.15295

Small Magellanic Cloud Cepheids Observed with the Hubble Space Telescope Provide a New Anchor for the SH0ES Distance Ladder

Louise Breuval,[1] Adam G. Riess,[1,2] Stefano Casertano,[2] Wenlong Yuan,[1] Lucas M. Macri,[3] Martino Romaniello,[4] Yukei S. Murakami,[1] Daniel Scolnic,[5] Gagandeep S. Anand,[2] and Igor Soszyński[6]

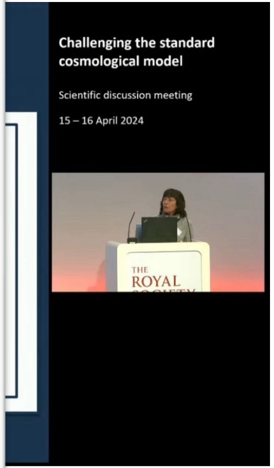
[1] Department of Physics and Astronomy, Johns Hopkins University, Baltimore, MD 21218, USA
[2] Space Telescope Science Institute, 3700 San Martin Drive, Baltimore, MD 21218, USA
[3] NSF's NOIRLab, 950 N Cherry Ave, Tucson AZ 85719, USA
[4] European Southern Observatory, Karl-Schwarzschild-Strasse 2, 85478 Garching bei München, Germany
[5] Department of Physics, Duke University, Durham, NC 27708, USA
[6] Astronomical Observatory, University of Warsaw, Al. Ujazdowskie 4, 00-478 Warszawa, Poland

ABSTRACT

We present photometric measurements of 88 Cepheid variables in the core of the Small Magellanic Cloud (SMC), the first sample obtained with the *Hubble* Space Telescope (*HST*) and Wide Field Camera 3, in the same homogeneous photometric system as past measurements of all Cepheids on the SH0ES distance ladder. We limit the sample to the inner core and model the geometry to reduce errors in prior studies due to the non-trivial depth of this Cloud. Without crowding present in ground-based studies, we obtain an unprecedentedly low dispersion of 0.102 mag for a Period-Luminosity relation in the SMC, approaching the width of the Cepheid instability strip. The new geometric distance to 15 late-type detached eclipsing binaries in the SMC offers a rare opportunity to improve the foundation of the distance ladder, increasing the number of calibrating galaxies from three to four. With the SMC as the only anchor, we find $H_0 = 74.1 \pm 2.1$ km s^{-1} Mpc^{-1}. Combining these four geometric distances with our *HST* photometry of SMC Cepheids, we obtain $H_0 = 73.17 \pm 0.86$ km s^{-1} Mpc^{-1}. By including the SMC in the distance ladder, we also double the range where the metallicity ([Fe/H]) dependence of the Cepheid Period-Luminosity relation can be calibrated, and we find $\gamma = -0.22 \pm 0.05$ mag dex^{-1}. Our local measurement of H_0 based on Cepheids and Type Ia supernovae shows a 5.8σ tension with the value inferred from the CMB assuming a ΛCDM cosmology, reinforcing the possibility of physics beyond ΛCDM.

Eugene Lim
@tukohbin

From Wendy Freedma
tension dead?



Still wait

Sergio Sevillano

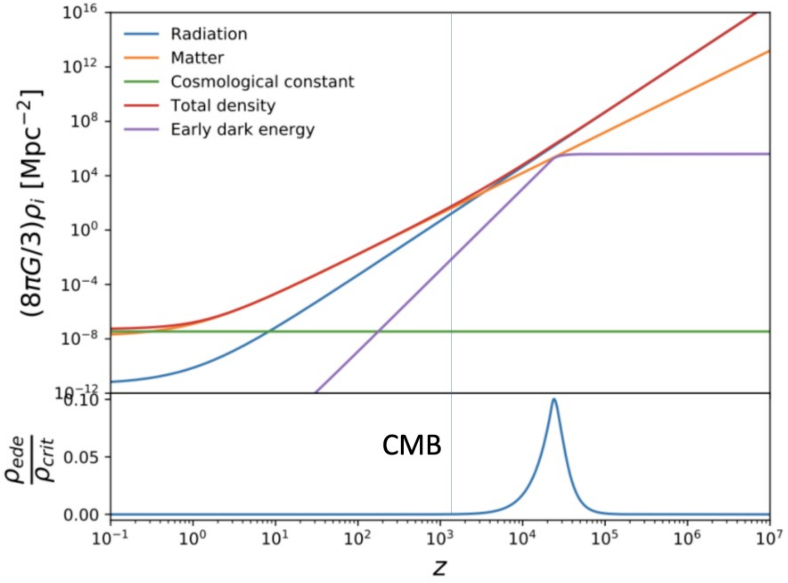
Hubble Tension?

Early Dark Energy (EDE):

EDE must create a peak in the energy density of the universe just before recombination

There are ways to induce a kick into the field (Sakstein and Trodden)

Figure courtesy T. Karwal.



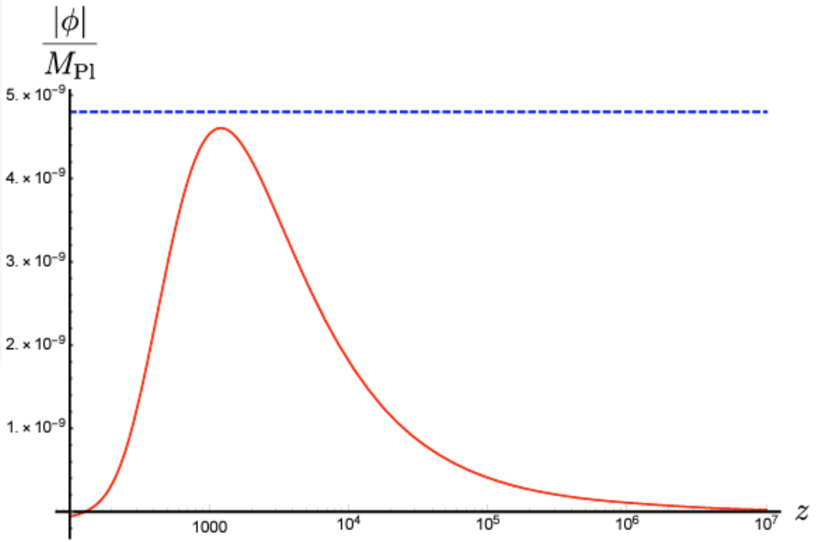
Hubble Tension?

Early dark energy from massive neutrinos — a natural resolution of the Hubble tension

Jeremy Sakstein* and Mark Trodden†
Center for Particle Cosmology, Department of Physics and Astronomy, University of Pennsylvania 209 S. 33rd St., Philadelphia, PA 19104, USA

The Hubble tension can be significantly eased if there is an early component of dark energy that becomes active around the time of matter-radiation equality. Early dark energy models suffer from a coincidence problem—the physics of matter-radiation equality and early dark energy are completely disconnected, so some degree of fine-tuning is needed in order for them to occur nearly simultaneously. In this paper we propose a natural explanation for this coincidence. If the early dark energy scalar couples to neutrinos then it receives a large injection of energy around the time that neutrinos become non-relativistic. This is precisely when their temperature is of order their mass, which, coincidentally, occurs around the time of matter-radiation equality. Neutrino decoupling therefore provides a natural trigger for early dark energy by displacing the field just before matter-radiation equality. We discuss various theoretical aspects of this proposal, potential observational signatures, and future directions for its study.

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{{M_{\rm pl}}^2 R(g)}{2} - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - V(\phi) + i\bar{\nu}\gamma^\mu\nabla_\mu\nu + m_\nu\left(1+\beta\frac{\phi}{M_{\rm pl}}+\cdots\right)\bar{\nu}\nu\right],$$



Hubble Tension?

Today's plan:

I will show that using standard Scalar fields we can generate such a peak

-Quintessence (Exponential potentials)

-Quintessence (Any potential)

-K-essence (Exponential potentials)

How well can we relax the Hubble tension??

Quintessence

Exponential case:

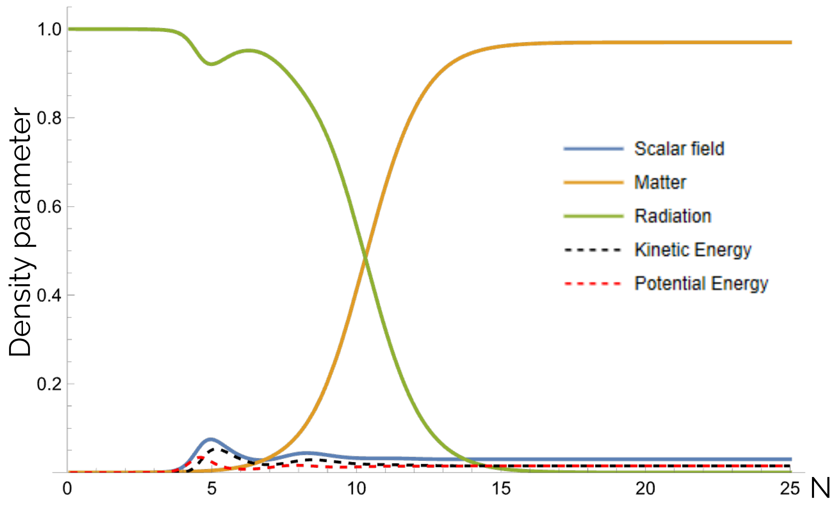
$$\mathcal{L} = \frac{1}{2}\dot{\phi}^2 - V(\phi) \qquad V(\phi) = A_0 e^{-\kappa\lambda\phi}$$

One finds the following equations of motion:

$$H^2 = \frac{\kappa^2}{3}\left(\rho_\gamma + \rho_m + \frac{\dot{\phi}^2}{2} + V(\phi)\right)$$

$$\dot{\rho}_\gamma = -3H\gamma_r\rho_\gamma$$

$$\dot{\rho}_m = -3H\gamma_m\rho_m$$



Quintessence

Exponential case:

To study the evolution of each density, it is useful to parametrise them as

$$x = \frac{\kappa\dot{\phi}}{\sqrt{6}H} \qquad y = \frac{\kappa\sqrt{V(\phi)}}{\sqrt{3}H} \qquad z = \frac{\kappa\sqrt{\rho_\gamma}}{\sqrt{3}H}$$

Which constraints one of the densities through the Friedman equation

$$\Omega_m \equiv \frac{\kappa^2\rho_m}{3H^2} = 1 - (x^2 + y^2 + z^2)$$

Quintessence

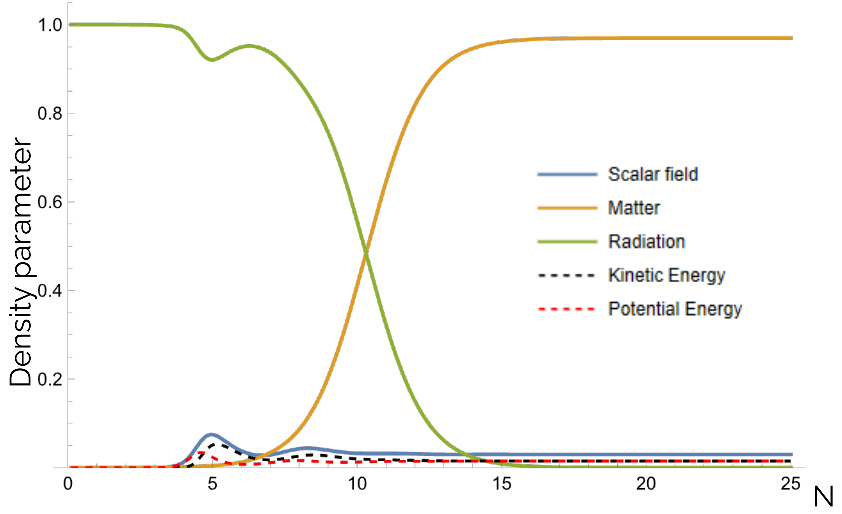
Exponential case:

$$x = \frac{\kappa\dot{\phi}}{\sqrt{6}H} \qquad y = \frac{\kappa\sqrt{V(\phi)}}{\sqrt{3}H} \qquad z = \frac{\kappa\sqrt{\rho_\gamma}}{\sqrt{3}H}$$

Leading to the following evolution of each parameter

$$\begin{aligned} x' &= +\sqrt{\frac{3}{2}}\lambda y^2 - \frac{x}{2}(3 - 3x^2 + 3y^2 - z^2), \\ y' &= -\sqrt{\frac{3}{2}}\lambda xy + \frac{y}{2}(3 + 3x^2 - 3y^2 + z^2), \\ z' &= -\frac{z}{2}(1 - 3x^2 + 3y^2 - z^2). \end{aligned}$$

(using $\gamma_r = 4/3, \gamma_m = 1$)



Quintessence

Exponential case:

$$x = \frac{\kappa\dot{\phi}}{\sqrt{6}H} \qquad y = \frac{\kappa\sqrt{V(\phi)}}{\sqrt{3}H} \qquad z = \frac{\kappa\sqrt{\rho_\gamma}}{\sqrt{3}H}$$

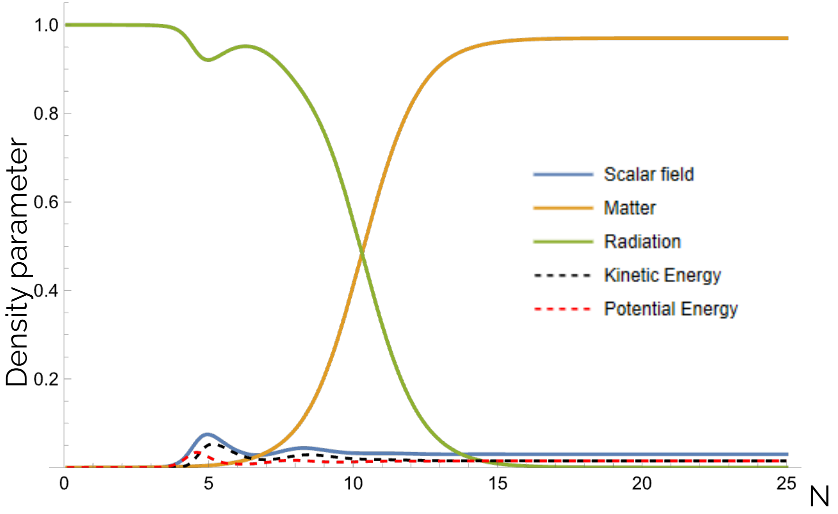
To make the equations more intuitive, use instead of z

$$\gamma_{\text{eff}} = 1 + \frac{p_\gamma + p_m}{\rho_\gamma + \rho_m} = 1 + \frac{1}{3}\left(\frac{z^2}{1 - x^2 - y^2}\right)$$

$$x' = \sqrt{\frac{3}{2}}\lambda y^2 + \frac{3x}{2}(-2 + 2x^2 + \gamma_{\text{eff}}(1 - x^2 - y^2)),$$

$$y' = -\sqrt{\frac{3}{2}}\lambda xy + \frac{3y}{2}(2x^2 + \gamma_{\text{eff}}(1 - x^2 - y^2)),$$

$$\gamma_{\text{eff}}' = (\gamma_{\text{eff}} - 1)(3\gamma_{\text{eff}} - 4).$$



Quintessence

Exponential case:

$$x = \frac{\kappa\dot{\phi}}{\sqrt{6}H} \qquad y = \frac{\kappa\sqrt{V(\phi)}}{\sqrt{3}H}$$

Since (Copeland, Liddle and Wands) we know the late time attractors of these equations

$$\begin{aligned}
x' &= \sqrt{\frac{3}{2}}\lambda y^2 + \frac{3x}{2}(-2 + 2x^2 + \gamma_{\rm eff}(1 - x^2 - y^2)),\\
y' &= -\sqrt{\frac{3}{2}}\lambda xy + \frac{3y}{2}(2x^2 + \gamma_{\rm eff}(1 - x^2 - y^2)),\\
\gamma_{\rm eff}' &= (\gamma_{\rm eff} - 1)(3\gamma_{\rm eff} - 4).
\end{aligned}$$

$\lambda \geq \sqrt{3\gamma_{eff}}$

$$\begin{aligned}
x_{\rm sc} &= \sqrt{\frac{3}{2}}\frac{\gamma_{\rm eff}}{\lambda}\\
y_{\rm sc} &= \left(\frac{3}{2}\frac{\gamma_{\rm eff}(2-\gamma_{\rm eff})}{\lambda^2}\right)^{1/2}\\
\Omega_{\phi_{\rm sc}} &= x_{\rm sc}^2 + y_{\rm sc}^2 = \frac{3\gamma_{\rm eff}}{\lambda^2}\\
\gamma_\phi &= \gamma_{\rm eff}
\end{aligned}$$

Scaling solution

$\lambda \leq \sqrt{3\gamma_{eff}}$

$$\begin{aligned}
x_{\rm d} &= \frac{\lambda}{\sqrt{6}}\\
y_{\rm d} &= \left(1 - \frac{\lambda^2}{6}\right)^{1/2}\\
\Omega_{\phi_{\rm d}} &= x_{\rm d}^2 + y_{\rm d}^2 = 1\\
\gamma_\phi &= \frac{\lambda^2}{3}
\end{aligned}$$

Scalar domination

Quintessence

Exponential case:

$$x = \frac{\kappa\dot{\phi}}{\sqrt{6}H} \qquad y = \frac{\kappa\sqrt{V(\phi)}}{\sqrt{3}H}$$

Since (Copeland, Liddle and Wands) we know the late time attractors of these equations

$$\lambda \geq \sqrt{3\gamma_{eff}}$$

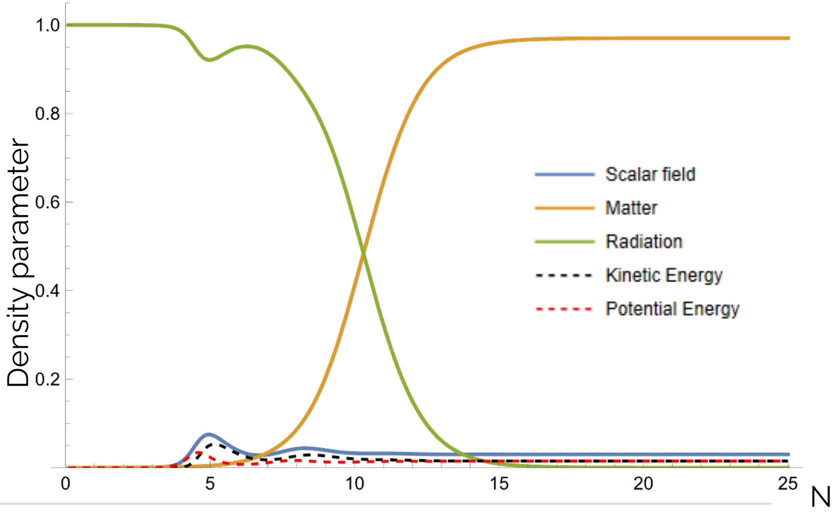
$$x_{\rm sc} = \sqrt{\frac{3}{2}}\frac{\gamma_{\rm eff}}{\lambda}$$

$$y_{\rm sc} = \left(\frac{3}{2}\frac{\gamma_{\rm eff}(2-\gamma_{\rm eff})}{\lambda^2}\right)^{1/2}$$

$$\Omega_{\phi_{\rm sc}} = x_{\rm sc}^2 + y_{\rm sc}^2 = \frac{3\gamma_{\rm eff}}{\lambda^2}$$

$$\gamma_\phi = \gamma_{\rm eff}$$

Scaling solution



Quintessence

Exponential case:

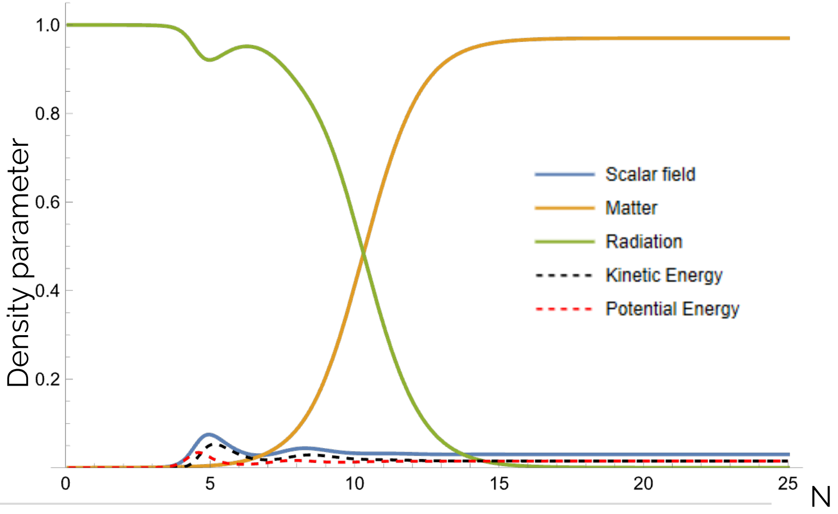
$$x = \frac{\kappa \dot{\phi}}{\sqrt{6}H} \qquad y = \frac{\kappa \sqrt{V(\phi)}}{\sqrt{3}H}$$

However, we want to understand the evolution, not the final attractor

For that, we'll take two approximations:

1) Given that $\Omega_\phi^{sc} = \frac{3\gamma_{\rm eff}}{\lambda^2}$, we can assume that $x, y \ll 1$

$$\boxed{\begin{aligned} x' &= \sqrt{\frac{3}{2}}\lambda y^2 + \frac{3x}{2}(-2 + 2x^2 + \gamma_{\rm eff}(1 - x^2 - y^2)), \\ y' &= -\sqrt{\frac{3}{2}}\lambda x y + \frac{3y}{2}(2x^2 + \gamma_{\rm eff}(1 - x^2 - y^2)), \\ \gamma_{\rm eff}' &= (\gamma_{\rm eff} - 1)(3\gamma_{\rm eff} - 4). \end{aligned}}$$



Quintessence

Exponential case:

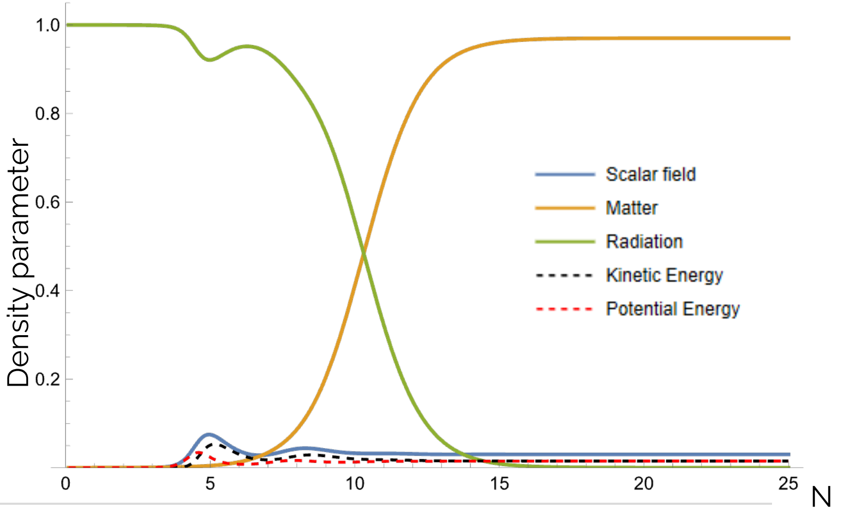
$$x = \frac{\kappa\dot{\phi}}{\sqrt{6}H} \qquad y = \frac{\kappa\sqrt{V(\phi)}}{\sqrt{3}H}$$

However, we want to understand the evolution, not the final attractor

For that, we'll take two approximations:

1) Given that $\Omega_\phi^{sc} = \frac{3\gamma_{\rm eff}}{\lambda^2}$, we can assume that $x, y \ll 1$

$$\boxed{\begin{aligned} x' &\approx \left(\frac{3}{2}\gamma_{\rm eff} - 3\right) x + \sqrt{\frac{3}{2}}\lambda y^2, \\ y' &\approx \frac{3}{2}\gamma_{\rm eff} y - \sqrt{\frac{3}{2}}\lambda x y, \\ \gamma_{\rm eff}' &\approx (\gamma_{\rm eff} - 1)(3\gamma_{\rm eff} - 4)\,. \end{aligned}}$$

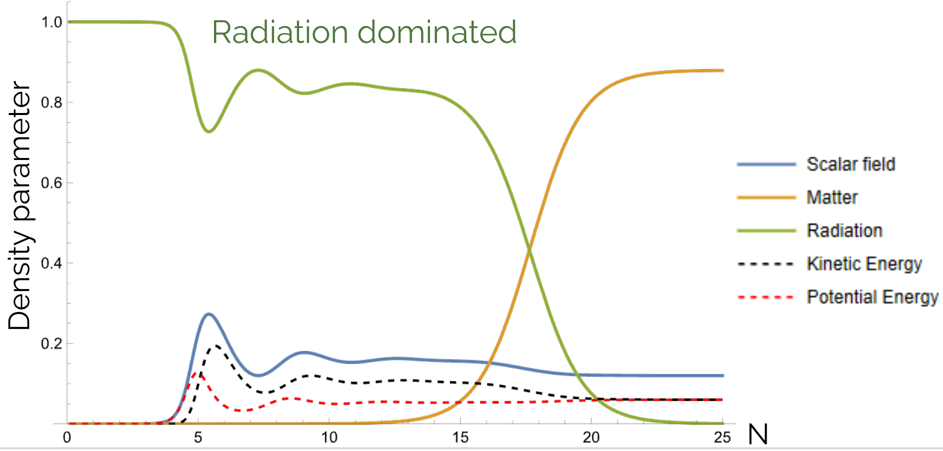


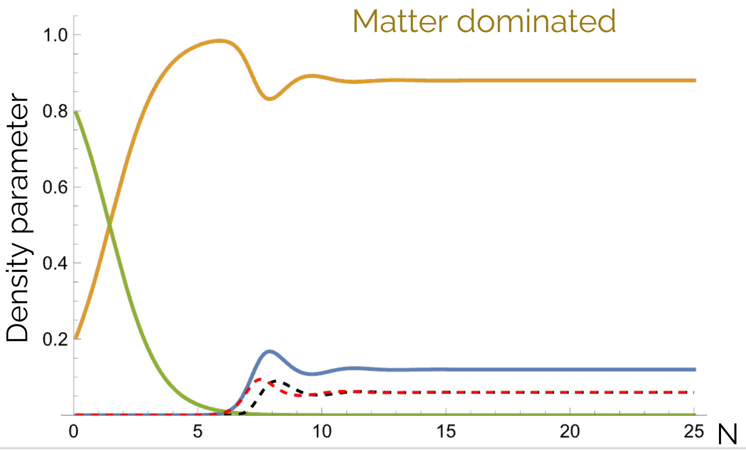
Quintessence

Exponential case:

$$x = \frac{\kappa\dot{\phi}}{\sqrt{6}H} \qquad y = \frac{\kappa\sqrt{V(\phi)}}{\sqrt{3}H}$$

The second approximation has to do with the effective global attractor:





Quintessence

Exponential case:

$$x = \frac{\kappa\dot{\phi}}{\sqrt{6}H} \qquad y = \frac{\kappa\sqrt{V(\phi)}}{\sqrt{3}H}$$

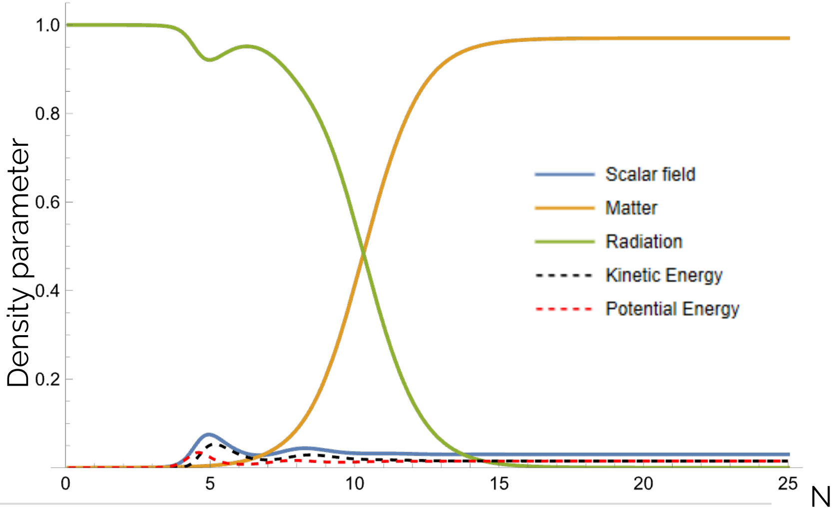
However, we want to understand the evolution, not the final attractor

For that, we'll take two approximations:

1) Given that $\Omega_\phi^{sc} = \frac{3\gamma_{\rm eff}}{\lambda^2}$, we can assume that $x, y \ll 1$

2) For the local evolution of the field, we can take $\gamma_{\rm eff}$ constant

$$\boxed{\begin{aligned} x' &\approx \left(\frac{3}{2}\gamma_{\rm eff} - 3\right)x + \sqrt{\frac{3}{2}}\lambda y^2, \\ y' &\approx \frac{3}{2}\gamma_{\rm eff}\, y - \sqrt{\frac{3}{2}}\lambda x y, \end{aligned}}$$

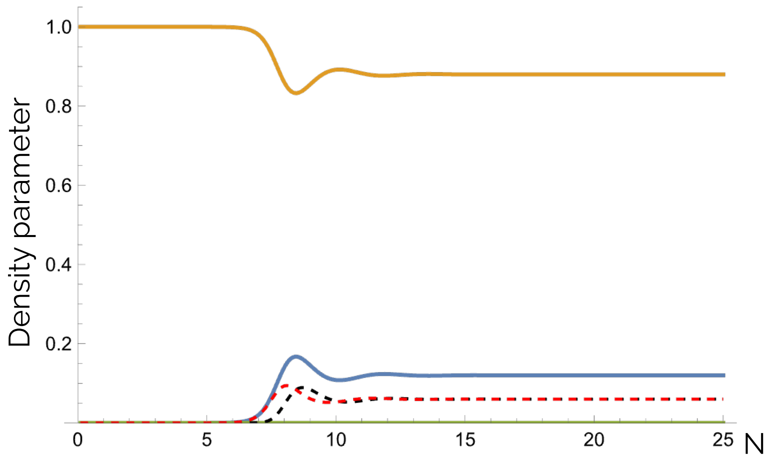


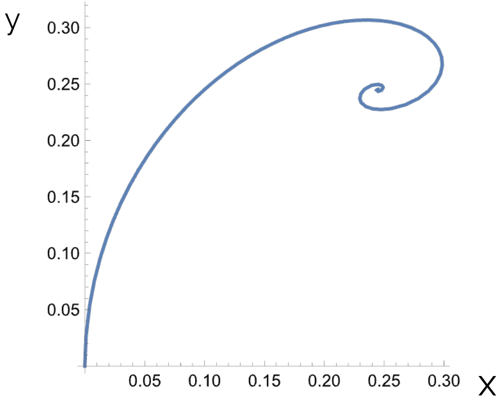
Quintessence

Exponential case: Why?

$$x = \frac{\kappa \dot{\phi}}{\sqrt{6}H} \qquad y = \frac{\kappa \sqrt{V(\phi)}}{\sqrt{3}H}$$

After these approximations, only the scaling fixed point remains!



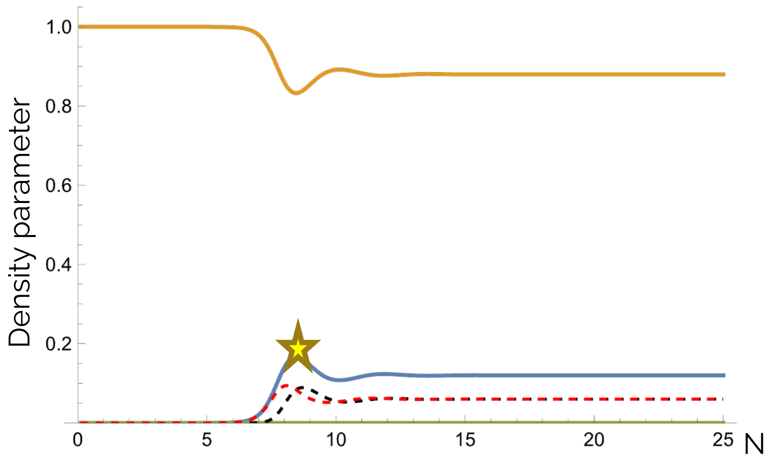


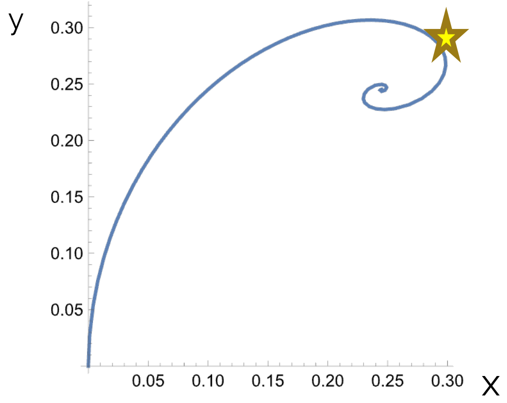
Quintessence

Exponential case: Why?

$$x = \frac{\kappa \dot{\phi}}{\sqrt{6}H} \qquad y = \frac{\kappa \sqrt{V(\phi)}}{\sqrt{3}H}$$

After these approximations, only the scaling fixed point remains!





Quintessence

Exponential case: Why?

$$x = \frac{\kappa \dot{\phi}}{\sqrt{6}H} \qquad y = \frac{\kappa \sqrt{V(\phi)}}{\sqrt{3}H}$$

Analysing the stability of the **scaling** fixed point we find the eigenvalues:

$$E_{\pm} = -\frac{3(2-\gamma_{\rm eff})}{4}\left(1 \pm \sqrt{1 - \frac{8\gamma_{\rm eff}}{2-\gamma_{\rm eff}}}\right)$$

Which is complex for both matter and radiation!

Quintessence

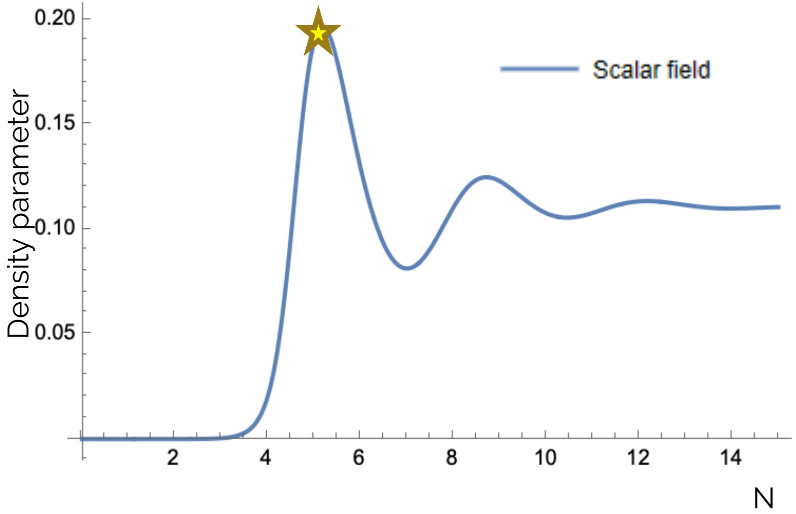
Exponential case: When?

$$x = \frac{\kappa\dot{\phi}}{\sqrt{6}H} \qquad y = \frac{\kappa\sqrt{V(\phi)}}{\sqrt{3}H}$$

At early times, when $x \ll \lambda, \quad y \ll \lambda$

$$x' \approx \left(\frac{3}{2}\gamma_{\rm eff} - 3\right)x + \cancel{\sqrt{\frac{3}{2}}\lambda y^2},$$

$$y' \approx \frac{3}{2}\gamma_{\rm eff}\, y - \cancel{\sqrt{\frac{3}{2}}\lambda x y},$$



Quintessence

Exponential case: When?

$$x = \frac{\kappa\dot{\phi}}{\sqrt{6}H} \qquad y = \frac{\kappa\sqrt{V(\phi)}}{\sqrt{3}H}$$

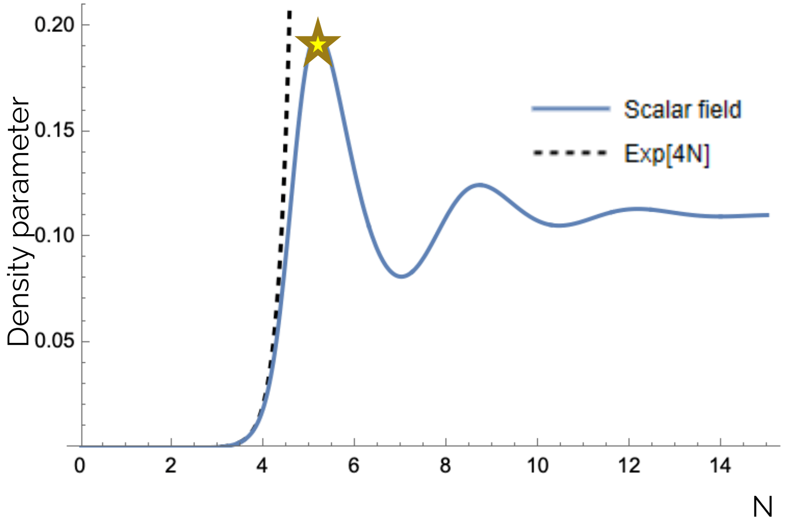
At early times, when $x \ll \lambda, \quad y \ll \lambda$

$$x' \approx \left(\frac{3}{2}\gamma_{\rm eff} - 3\right)x + \cancel{\sqrt{\frac{3}{2}}\lambda y^2},$$

$$y' \approx \frac{3}{2}\gamma_{\rm eff}\, y - \cancel{\sqrt{\frac{3}{2}}\lambda x y},$$

$$x_{early}(N) \approx (x_i - a_i)e^{-\Delta N_i} + a_i e^{4\Delta N_i},$$

$$y_{early}(N) \approx y_i e^{2\Delta N_i},$$



Quintessence

Exponential case: When?

$$x = \frac{\kappa\dot{\phi}}{\sqrt{6}H} \qquad y = \frac{\kappa\sqrt{V(\phi)}}{\sqrt{3}H}$$

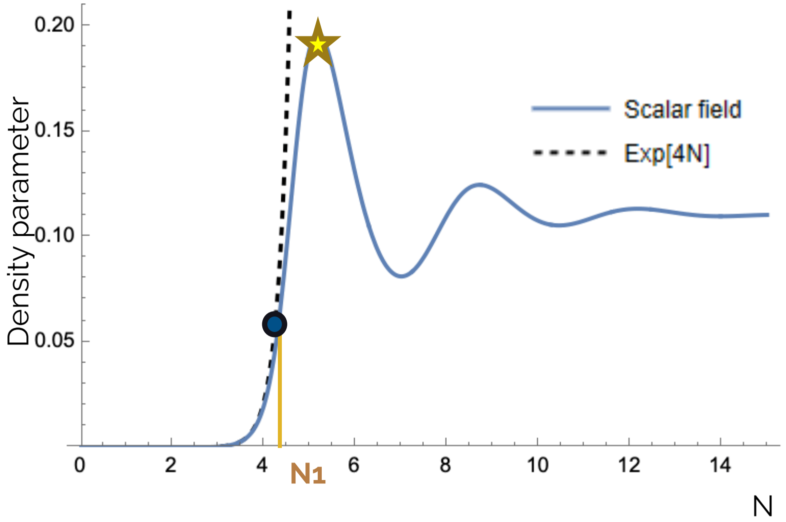
At early times, when $x \ll \lambda, \quad y \ll \lambda$

$$x_{early}(N) \approx (x_i - a_i)e^{-\Delta N_i} + a_i e^{4\Delta N_i},$$
$$y_{early}(N) \approx y_i e^{2\Delta N_i},$$

Because of Hubble friction y will lead the peak

$$y_{early}(N_1) \approx \sqrt{\Omega_\phi^{(\mathrm{sc})}/2}.$$

$$N_1 \approx N_i + \frac{1}{2}\log\left(\frac{\sqrt{3}\gamma_{\mathrm{eff}}}{y_i\lambda\sqrt{2}}\right)$$

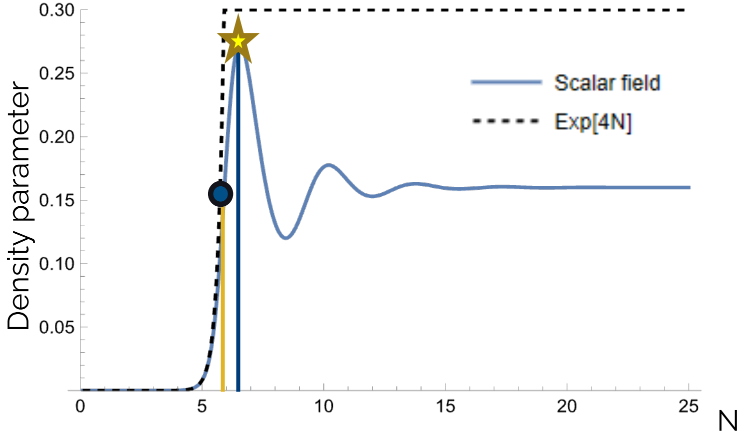


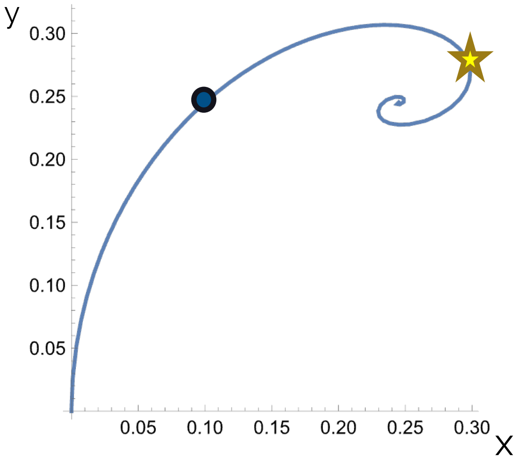
Quintessence

Exponential case: Height

$$x = \frac{\kappa\dot{\phi}}{\sqrt{6}H} \qquad y = \frac{\kappa\sqrt{V(\phi)}}{\sqrt{3}H}$$

We know that the peak cannot be bigger than twice the height of the late time attractor





Can we do better?

Quintessence

Exponential case: Height

$$x = \frac{\kappa\dot{\phi}}{\sqrt{6}H} \qquad y = \frac{\kappa\sqrt{V(\phi)}}{\sqrt{3}H}$$

Using the eigenvalues, we can get a perturbative equation around the peak

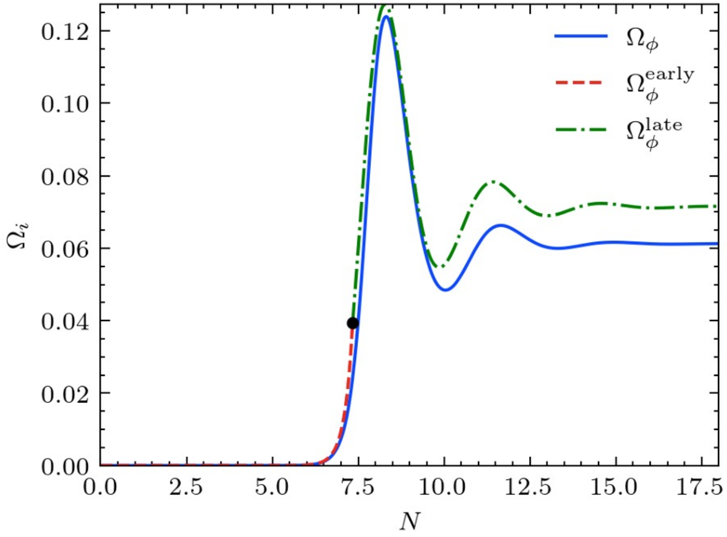
$$\begin{aligned} x(N) &= x_{early}(N_1) + A_x \exp(E_+\Delta N_1) + B_x \exp(E_-\Delta N_1) \\ y(N) &= y_{early}(N_1) + A_y \exp(E_+\Delta N_1) + B_y \exp(E_-\Delta N_1) \end{aligned}$$

Assuming the peak to be at the other side of the orbit, we can get the height

Quintessence

Exponential case: Height

$$x = \frac{\kappa\dot{\phi}}{\sqrt{6}H} \qquad y = \frac{\kappa\sqrt{V(\phi)}}{\sqrt{3}H}$$



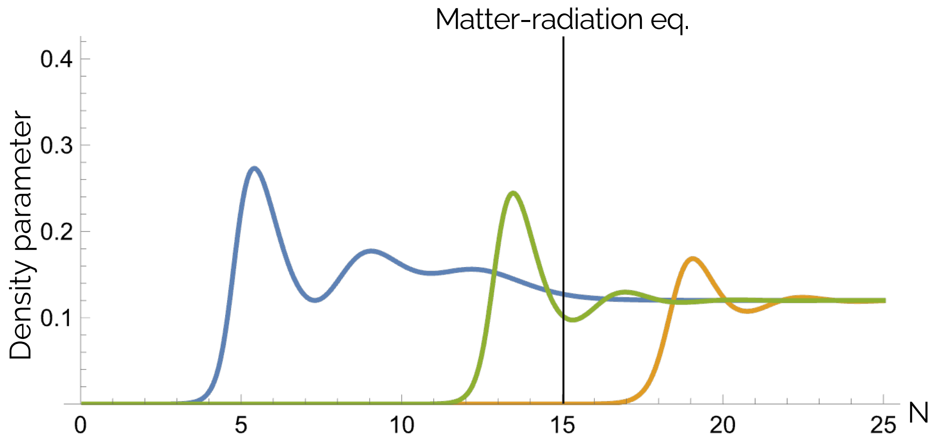
Quintessence

Exponential case: Height?

Depending on the background, we will have different peak heights

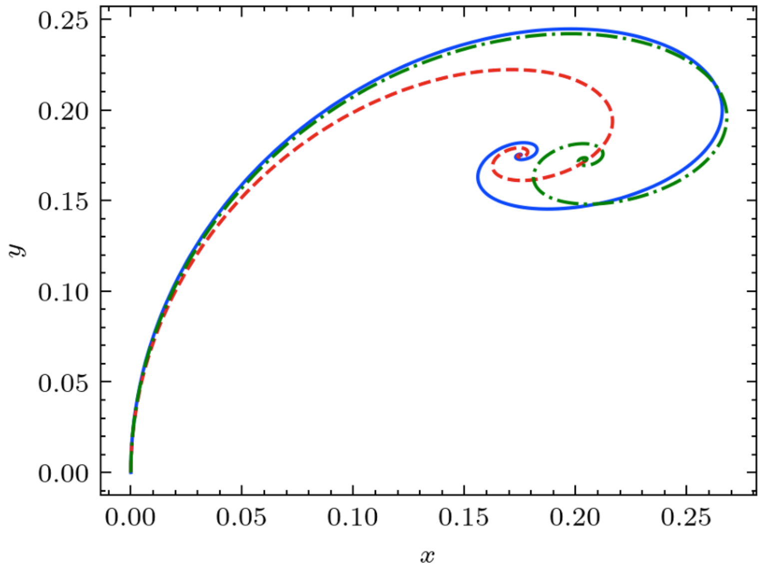
For matter-radiation equality, we have

$$\gamma_{\rm eff} = 7/6$$



Quintessence

Exponential case:



What we have learnt:

Why? Spiral nature of **scaling solution**

When? When the exponential increase of y matches the effective stable fixed-point height

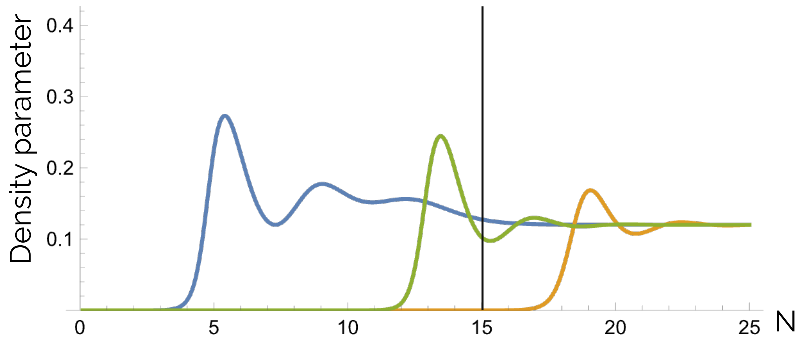
Height? Given by the linearising around the effective stable fixed-point at the time of the peak

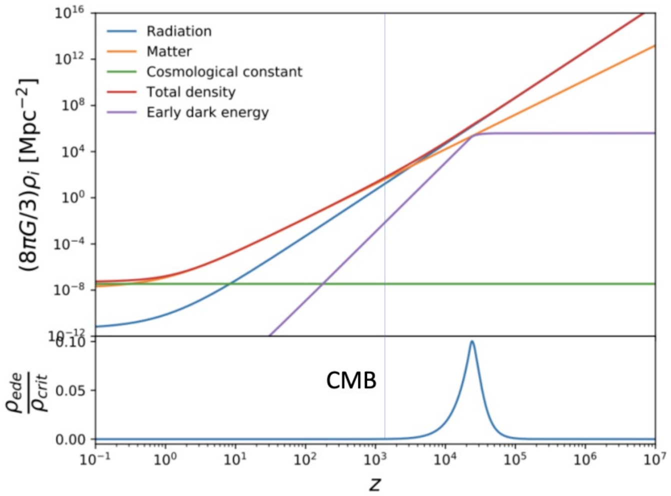
Late time? The system will end up in the true global attractor eventually

Quintessence

Exponential case: Drama

However, even for that case, to have a 0.1 peak, we will get a 0.05 late dark energy with matter equation of state



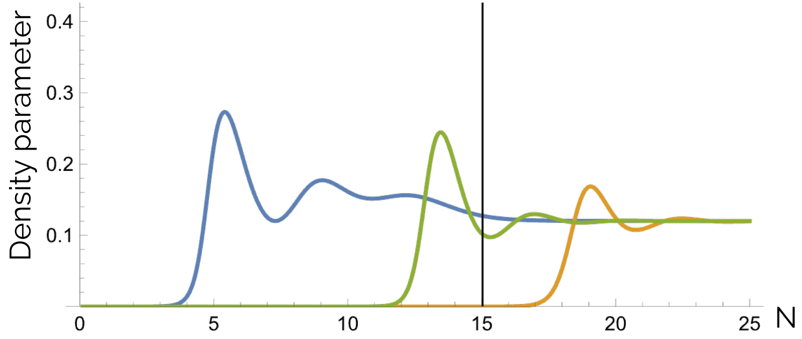


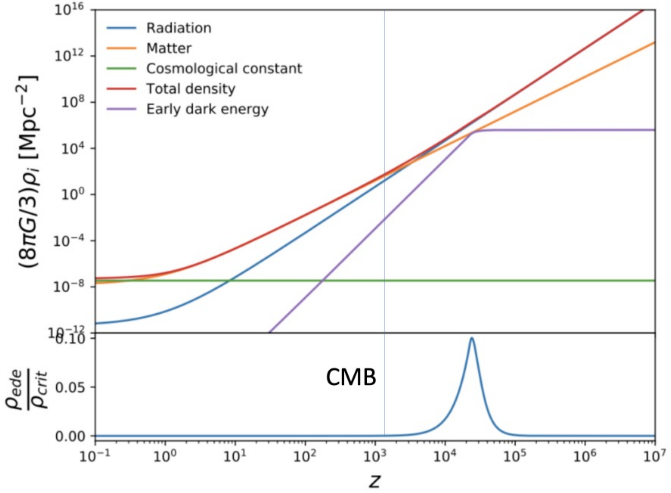
Is there a way of having the peak and then vanishing energy density?

Quintessence

Exponential case: Drama

However, even for that case, to have a 0.1 peak, we will get a 0.05 late dark energy with matter equation of state





Is there a way of having the peak and then vanishing energy density? YES!

Quintessence

Generic potentials!

$$\mathcal{L} = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

Equations of motion:

$$H^2 = \frac{\kappa^2}{3}\left(\rho_\gamma + \rho_m + \frac{\dot{\phi}^2}{2} + V(\phi)\right)$$

$$\dot{\rho}_\gamma = -3H\gamma_r\rho_\gamma$$

$$\dot{\rho}_m = -3H\gamma_m\rho_m$$

Reparametrisation: $x = \frac{\kappa\dot{\phi}}{\sqrt{6}H}$ $\quad y = \frac{\kappa\sqrt{V(\phi)}}{\sqrt{3}H}$ $\quad z = \frac{\kappa\sqrt{\rho_\gamma}}{\sqrt{3}H}$

$$\Omega_m \equiv \frac{\kappa^2\rho_m}{3H^2} = 1 - (x^2 + y^2 + z^2)$$

Quintessence

$$\Gamma = \frac{V_{,\phi\phi} V}{V_{,\phi}^2} \qquad \tilde{\lambda} = -\frac{V_{,\phi}(\phi)}{\kappa V(\phi)}$$

$$\boxed{\begin{aligned} x' &= \sqrt{\frac{3}{2}}\tilde{\lambda} y^2 - \frac{x}{2}(3 - 3x^2 + 3y^2 - z^2), \\ y' &= -\sqrt{\frac{3}{2}}\tilde{\lambda} x y + \frac{y}{2}(3 + 3x^2 - 3y^2 + z^2), \\ z' &= -\frac{z}{2}(1 - 3x^2 + 3y^2 - z^2), \\ \tilde{\lambda}' &= -\sqrt{6}\tilde{\lambda}^2(\Gamma - 1)x, \end{aligned}}$$

Quintessence

$$\Gamma = \frac{V_{,\phi\phi} V}{V_{,\phi}^2} \qquad \tilde{\lambda} = -\frac{V_{,\phi}(\phi)}{\kappa V(\phi)}$$

$$\begin{aligned}
x' &= \sqrt{\frac{3}{2}}\tilde{\lambda} y^2 - \frac{x}{2}(3 - 3x^2 + 3y^2 - z^2), \\
y' &= -\sqrt{\frac{3}{2}}\tilde{\lambda} x y + \frac{y}{2}(3 + 3x^2 - 3y^2 + z^2), \\
z' &= -\frac{z}{2}(1 - 3x^2 + 3y^2 - z^2), \\
\tilde{\lambda}' &= -\sqrt{6}\tilde{\lambda}^2(\Gamma - 1)x,
\end{aligned}$$

$$\begin{aligned}
x' &= +\sqrt{\frac{3}{2}}\lambda y^2 - \frac{x}{2}(3 - 3x^2 + 3y^2 - z^2), \\
y' &= -\sqrt{\frac{3}{2}}\lambda x y + \frac{y}{2}(3 + 3x^2 - 3y^2 + z^2), \\
z' &= -\frac{z}{2}(1 - 3x^2 + 3y^2 - z^2).
\end{aligned}$$

Generic quintessence is equivalent to exponential potentials with a time-varying Lambda!

Quintessence

Fang Model:

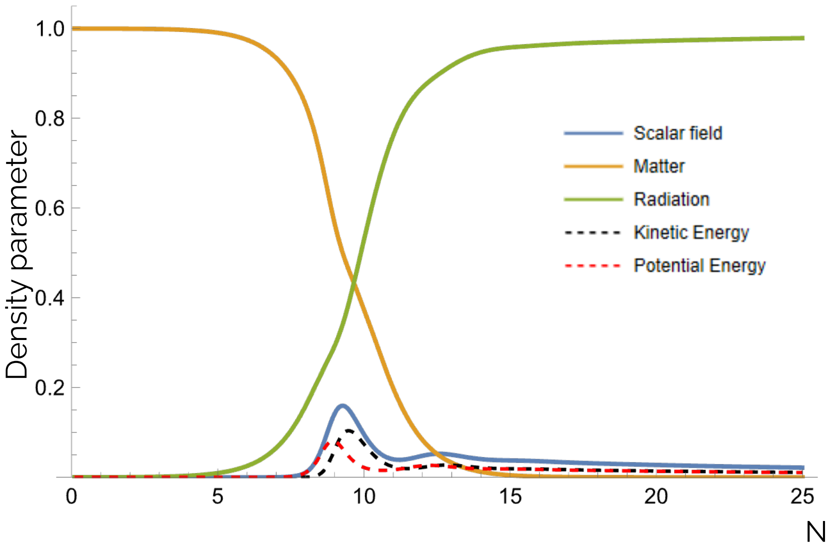
$$x = \frac{\kappa\dot{\phi}}{\sqrt{6}H} \qquad y = \frac{\kappa\sqrt{V(\phi)}}{\sqrt{3}H}$$

The Fang model has the following potential

$$V = V_0 e^{\alpha\phi(\phi+\beta)/2}$$

$$\lambda = -\frac{V_{,\phi}(\phi)}{\kappa V(\phi)} \qquad \Gamma = \frac{V_{,\phi\phi}V}{V_{,\phi}^2}$$

$$\lambda = \alpha(2\phi+\beta)/2, \quad \Gamma = 1 + \frac{\alpha}{\lambda^2},$$



Quintessence

Fang Model:

$$x = \frac{\kappa\dot{\phi}}{\sqrt{6}H} \qquad y = \frac{\kappa\sqrt{V(\phi)}}{\sqrt{3}H}$$

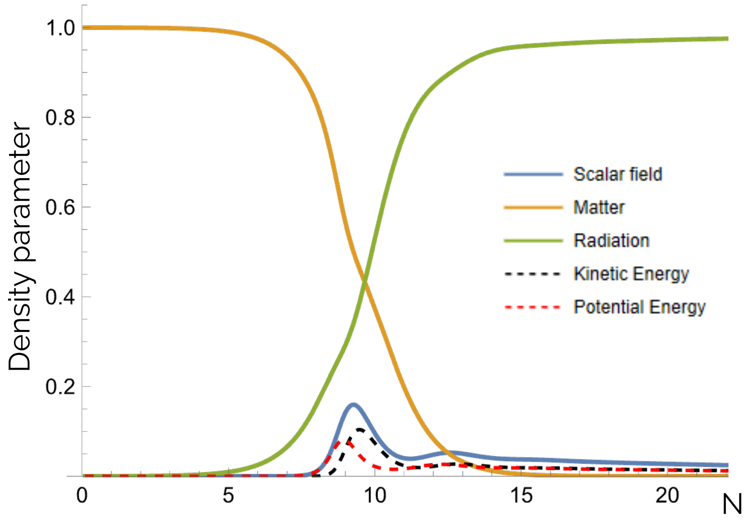
Leading to the equations

$$x' = \sqrt{\frac{3}{2}}\tilde{\lambda}y^2 + \frac{3x}{2}(-2 + 2x^2 + \gamma_{\text{eff}}(1 - x^2 - y^2)),$$

$$y' = -\sqrt{\frac{3}{2}}\tilde{\lambda}xy + \frac{3y}{2}(2x^2 + \gamma_{\text{eff}}(1 - x^2 - y^2)),$$

$$\gamma_{\text{eff}}' = (\gamma_{\text{eff}} - 1)(3\gamma_{\text{eff}} - 4),$$

$$\tilde{\lambda}' = -\sqrt{6}\tilde{\alpha}x$$



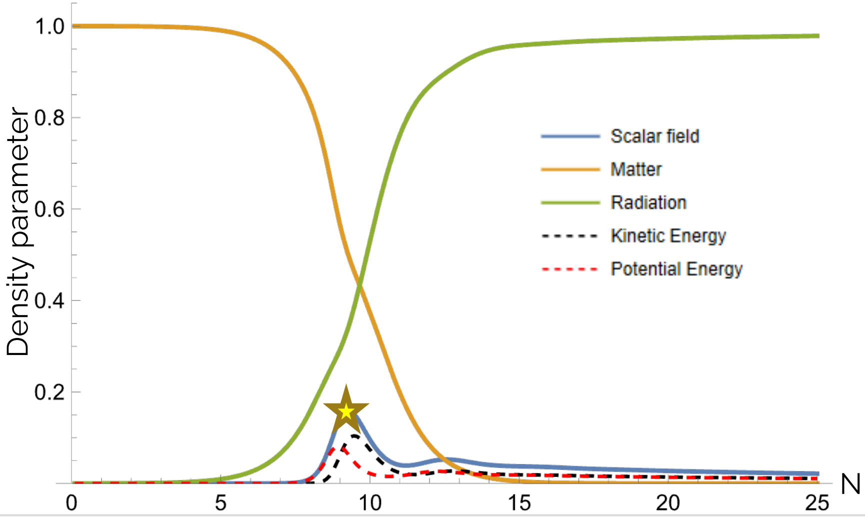
With fixed points at:

$$x_{\text{sc}} = 0, \qquad y_{\text{sc}} = 0, \qquad \tilde{\lambda}_{\text{sc}} \to \infty$$

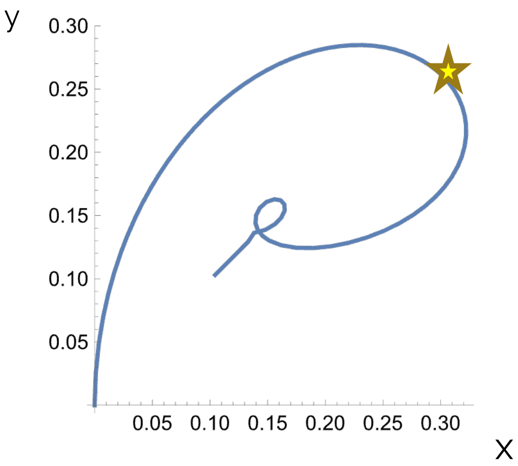
Quintessence

Fang Model: $x_{\mathrm{sc}} = 0, \quad y_{\mathrm{sc}} = 0, \quad \tilde{\lambda}_{\mathrm{sc}} \to \infty$

How can this give a peak if the stable point is at zero?



$\alpha < 0$



Quintessence

Fang Model:

$$x = \frac{\kappa \dot{\phi}}{\sqrt{6} H} \qquad y = \frac{\kappa \sqrt{V(\phi)}}{\sqrt{3} H}$$

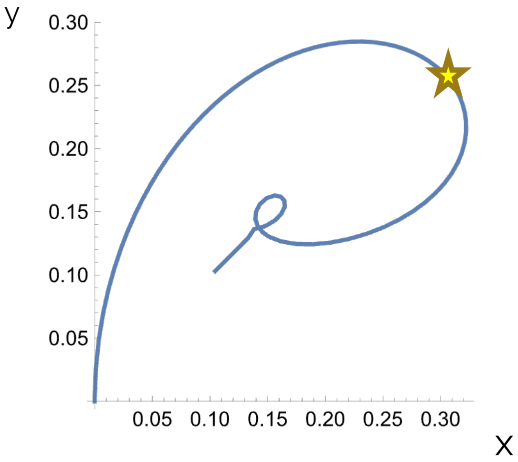
Before the peak, lambda is frozen, so the evolution of the system should be described by the exponential case

$$x' = \sqrt{\frac{3}{2}}\tilde{\lambda} y^2 + \frac{3x}{2}(-2 + 2x^2 + \gamma_{\rm eff}(1 - x^2 - y^2)),$$

$$y' = -\sqrt{\frac{3}{2}}\tilde{\lambda} x y + \frac{3y}{2}(2x^2 + \gamma_{\rm eff}(1 - x^2 - y^2)),$$

$$\gamma_{\rm eff}' = (\gamma_{\rm eff} - 1)(3\gamma_{\rm eff} - 4),$$

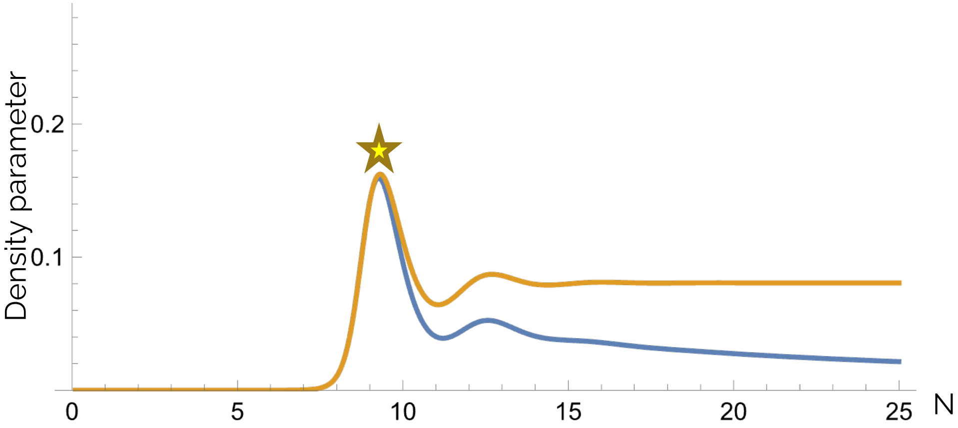
$$\tilde{\lambda}' = -\sqrt{6}\tilde{\alpha} x$$

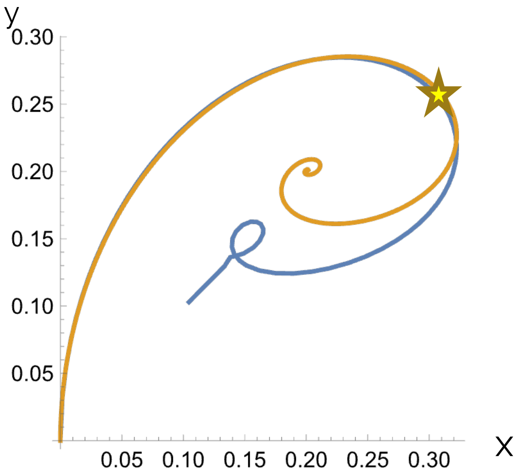


Quintessence

Fang Model:

We should always find an exponential case that matches the peak (as long as it doesn't take place too quickly)





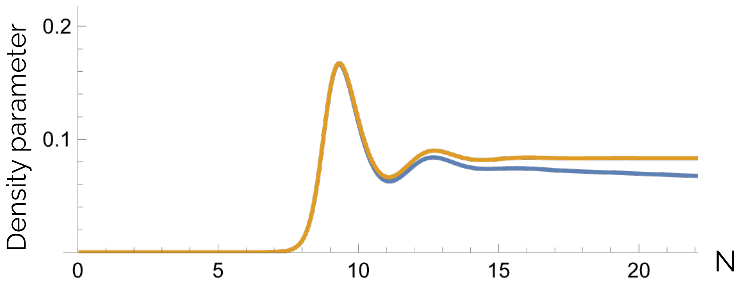
Quintessence

Fang Model:

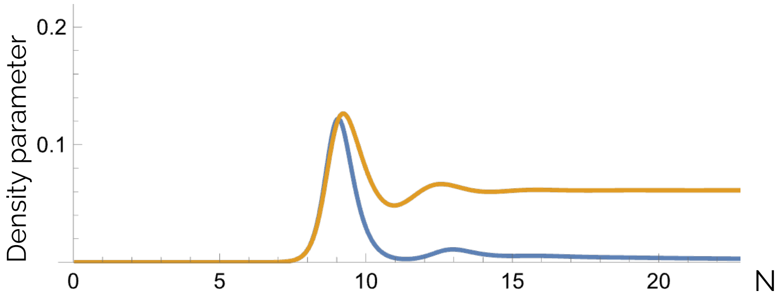
It gets worse for higher lambda but gives us still a good estimate if we find the correct lambda!

$$\tilde{\lambda}' = -\sqrt{6}\tilde{\alpha}x$$

$$\alpha = -0.1, \quad \lambda_i = 6, \quad \lambda_p = 6$$



$$\alpha = -10, \quad \lambda_i = 6, \quad \lambda_p = 7$$



Quintessence

Fang Model:

We can understand the Fang model by finding its corresponding exponential case!

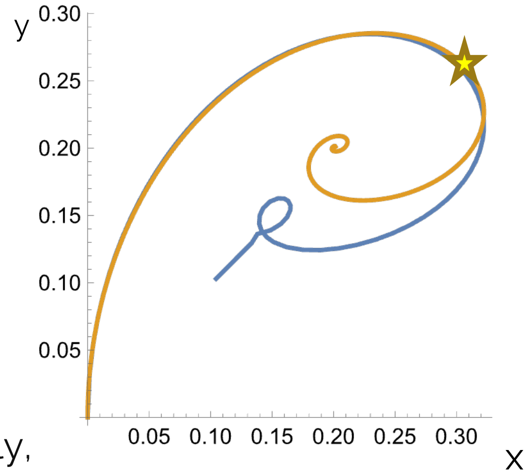
What we have learnt:

Why? Spiral nature of **scaling solution** of the frozen exponential case

When? When the exponential increase of y matches the effective stable fixed-point height of the frozen exponential case

Height? Use the frozen exponential case analytics

Late time? The system will end up in the true global attractor eventually, given by the unfrozen lambda



Quintessence

A good peak?

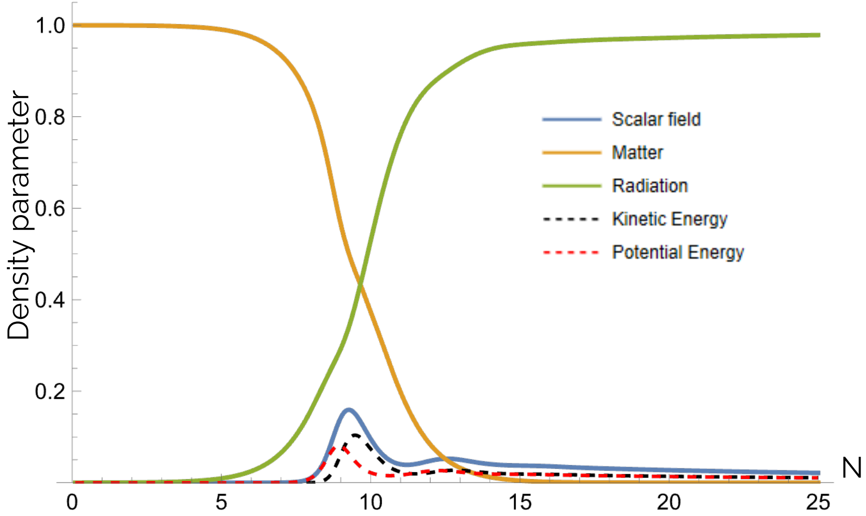
To have generic Quintessence providing a good peak, we need:

Before the peak: A frozen lambda of order 10

During the peak: Lambda not changing too rapidly

After the peak: Lambda goes to infinity

$$x_{\rm sc} = 0, \qquad y_{\rm sc} = 0, \qquad \tilde{\lambda}_{\rm sc} \to \infty$$



K-essence

We can extend this even further! Let's consider non-canonical fields such as

$$\mathcal{L} = \frac{X(\dot{\phi})^n}{M^{4(n-1)}} - V(\phi)$$

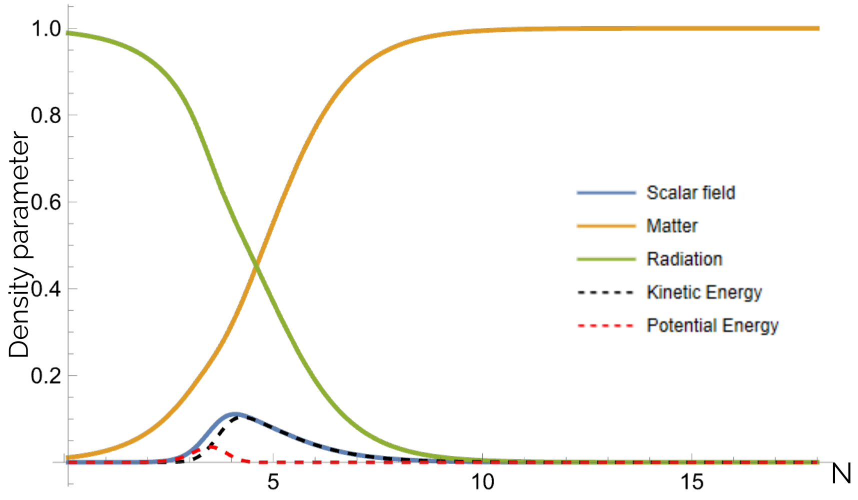
$X(\dot{\phi}) \equiv \frac{1}{2}\dot{\phi}^2$

With an exponential potential

$$V(\phi) = A_0 e^{-\kappa\lambda\phi}$$

Sound speed:

$$c_s^2 = \frac{1}{2n-1}$$



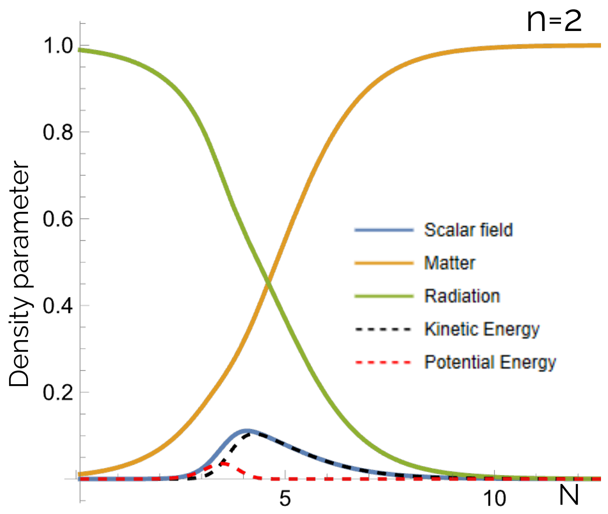
K-essence

In this case, we have the following equations of motion:

$$H^2 = \frac{\kappa^2}{3}\left(\rho_r + \rho_m + \frac{2n-1}{2^n M^{4(n-1)}}(\dot{\phi}^2)^n + V(\phi)\right)$$

The evolution equations are

$$\dot{\rho}_r = -3H\gamma_r\rho_r,$$
$$\dot{\rho}_m = -3H\gamma_m\rho_m,$$
$$\frac{n(2n-1)}{2^{n-1}M^{4(n-1)}}\dot{\phi}^{2n-2}\ddot{\phi} + \frac{3Hn}{2^{n-1}M^{4(n-1)}}\dot{\phi}^{2n-1} + V_{,\phi}(\phi) = 0$$



K-essence

Making the following reparametrisation:

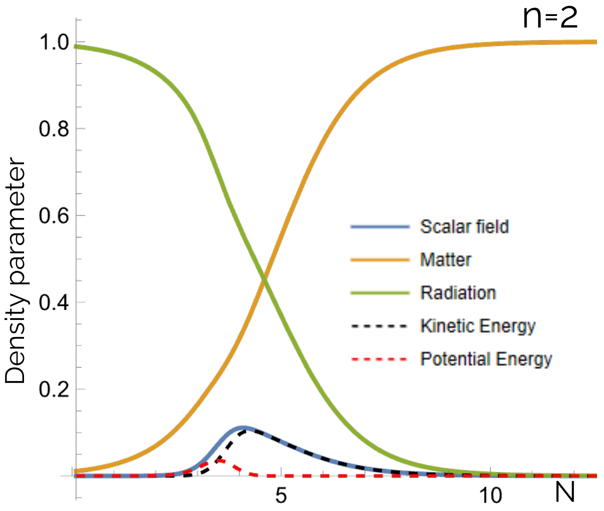
$$x = \frac{\kappa\sqrt{2n-1}}{2^{n/2}M^{4(n-1)}}\frac{\dot{\phi}^n}{\sqrt{3}H}, \qquad y = \frac{\kappa\sqrt{V}}{\sqrt{3}H}, \qquad z = \frac{\kappa\sqrt{\rho_r}}{\sqrt{3}H},$$

We find:

$$x' = \sqrt{\frac{3}{2}\frac{M^{4(n-1)}\lambda y^2}{(2n-1)^{\frac{1}{2n}}}}\underline{\left(\frac{\kappa}{\sqrt{3}Hx}\right)^{1-\frac{1}{n}}} + \frac{x}{2}\left(\frac{3x^2-3}{2n-1} - 3y^2 + z^2\right),$$

$$y' = -\sqrt{\frac{3}{2}\frac{M^{4(n-1)}\lambda xy}{(2n-1)^{\frac{1}{2n}}}}\underline{\left(\frac{\kappa}{\sqrt{3}Hx}\right)^{1-\frac{1}{n}}} + \frac{y}{2}\left(3 + \frac{3x^2}{2n-1} - 3y^2 + z^2\right),$$

$$z' = \frac{z}{2}\left(-1 + \frac{3x^2}{2n-1} - 3y^2 + z^2\right),$$



K-essence

K-essence

$$x' = \sqrt{\frac{3}{2}}\frac{M^{4(n-1)}\lambda y^2}{(2n-1)^{\frac{1}{2n}}}\left(\frac{\kappa}{\sqrt{3}Hx}\right)^{1-\frac{1}{n}} + \frac{3x}{2}\left(\frac{2n(x^2-1)}{2n-1} + \gamma_{\rm eff}(1-x^2-y^2)\right),$$

$$y' = -\sqrt{\frac{3}{2}}\frac{M^{4(n-1)}\lambda xy}{(2n-1)^{\frac{1}{2n}}}\left(\frac{\kappa}{\sqrt{3}Hx}\right)^{1-\frac{1}{n}} + \frac{3y}{2}\left(\frac{2nx^2}{(2n-1)} + \gamma_{\rm eff}(1-x^2-y^2)\right),$$

Quintessence (exp)

$$x' = \sqrt{\frac{3}{2}}\tilde{\lambda}y^2 + \frac{3x}{2}(-2+2x^2+\gamma_{\rm eff}(1-x^2-y^2)),$$

$$y' = -\sqrt{\frac{3}{2}}\tilde{\lambda}xy + \frac{3y}{2}(2x^2+\gamma_{\rm eff}(1-x^2-y^2)),$$

K-essence

K-essence

$$\boxed{\eta = \frac{M^{4(n-1)}\lambda}{(2n-1)^{\frac{1}{2n}}}\left(\frac{\kappa}{\sqrt{3}Hx}\right)^{1-\frac{1}{n}}}$$

$$x' = \sqrt{\frac{3}{2}}\eta y^2 + \frac{3x}{2}\left(\frac{2n(x^2-1)}{2n-1} + \gamma_{\rm eff}(1-x^2-y^2)\right)$$

$$y' = -\sqrt{\frac{3}{2}}\eta xy + \frac{3y}{2}\left(\frac{2nx^2}{(2n-1)} + \gamma_{\rm eff}(1-x^2-y^2)\right)$$

$$\eta' = \eta(n-1)\left(\frac{3}{2n-1} - \sqrt{\frac{3}{2}}\frac{\eta y^2}{nx}\right)$$

Quintessence

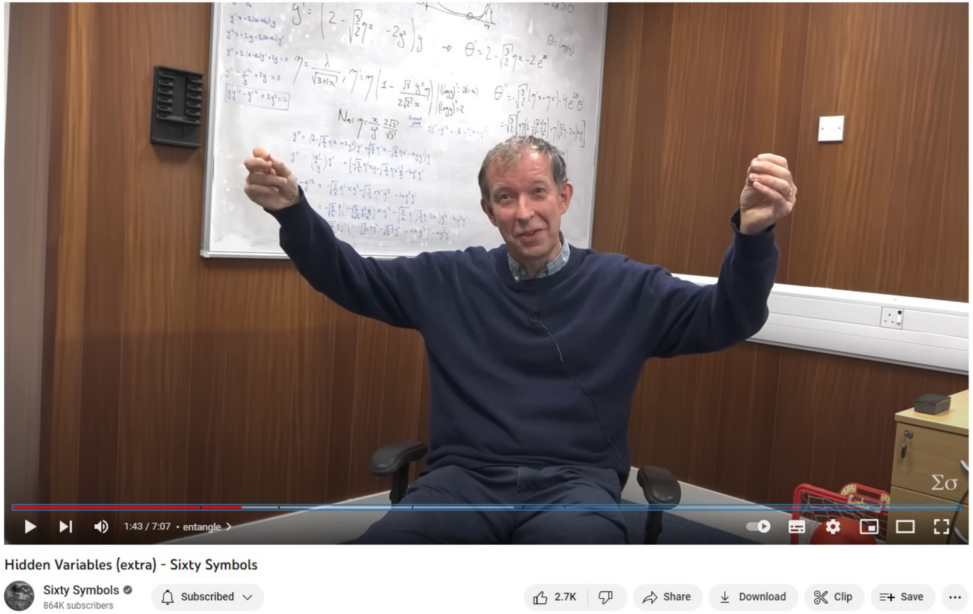
$$x' = \sqrt{\frac{3}{2}}\tilde{\lambda}y^2 + \frac{3x}{2}(-2+2x^2+\gamma_{\rm eff}(1-x^2-y^2)),$$

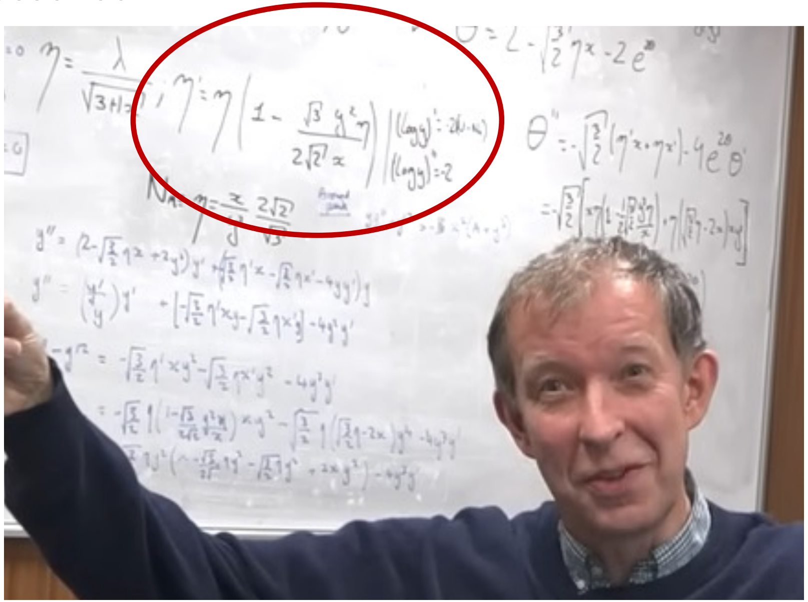
$$y' = -\sqrt{\frac{3}{2}}\tilde{\lambda}xy + \frac{3y}{2}(2x^2+\gamma_{\rm eff}(1-x^2-y^2)),$$

$$\tilde{\lambda}' = -\sqrt{6}\tilde{\lambda}^2(\Gamma-1)x$$

K-essence

We can use the same quintessence techniques on K-essence!





K-essence

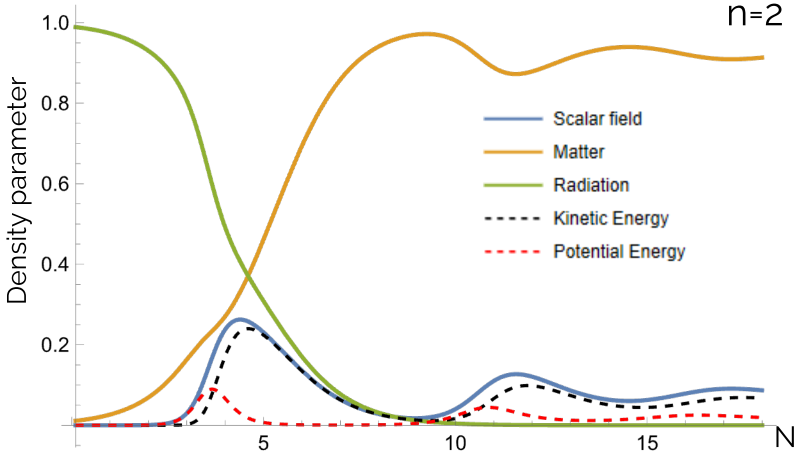
$$x = \frac{\kappa\sqrt{2n-1}}{2^{n/2}M^{4(n-1)}}\frac{\dot{\phi}^n}{\sqrt{3}H}, \qquad y = \frac{\kappa\sqrt{V}}{\sqrt{3}H},$$

Let's take for a moment the constant η case. It is not phenomenologically relevant, but interesting!

$$x' = (\frac{3}{2}\tau - 2)x + \sqrt{\frac{3}{2}}\eta y^2,$$
$$y' = \frac{3}{2}\tau y - \sqrt{\frac{3}{2}}\eta x y,$$

The stable points are now:

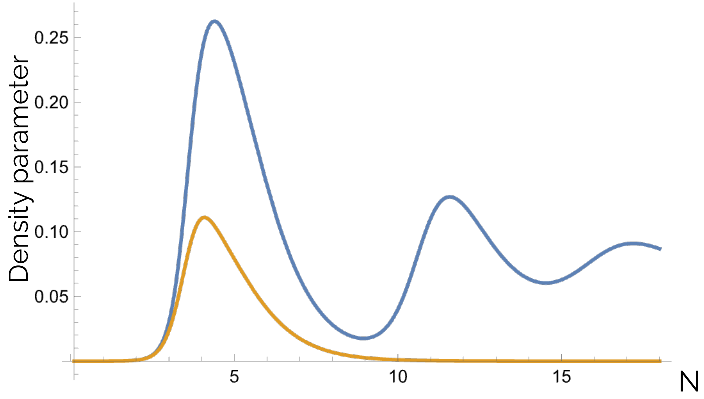
$$x_{(\mathrm{sc})} = \sqrt{\frac{3}{2}}\frac{\gamma_{\mathrm{eff}}}{\eta_c}, \qquad y_{(\mathrm{sc})} = \sqrt{\frac{3}{2}}\frac{\sqrt{\gamma_{\mathrm{eff}}}}{\eta_c}\sqrt{\frac{2n}{(2n-1)} - \gamma_{\mathrm{eff}}},$$

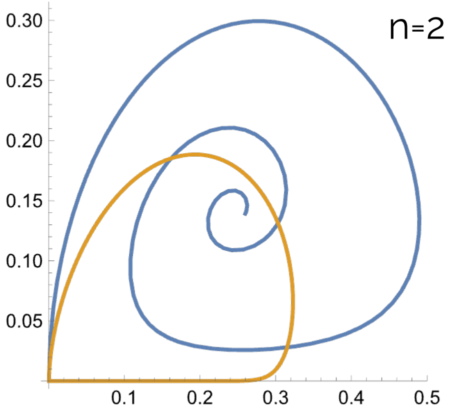


K-essence

$$x = \frac{\kappa\sqrt{2n-1}}{2^{n/2}M^{4(n-1)}}\frac{\dot{\phi}^n}{\sqrt{3}H}, \qquad y = \frac{\kappa\sqrt{V}}{\sqrt{3}H},$$

Very similar to Quintessence, where constant η corresponds to exponential case

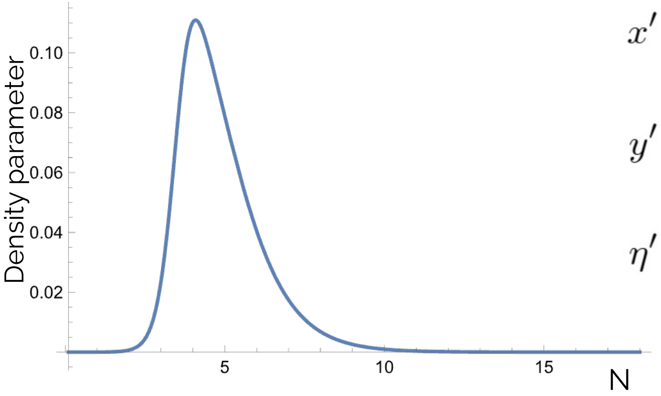




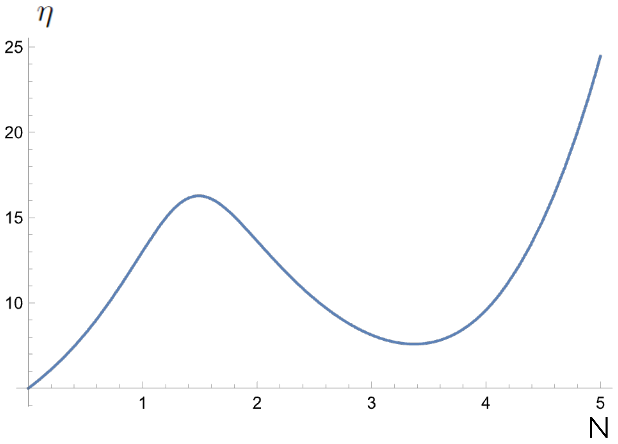
However, we need to know the value for η at the time of the peak, which is difficult to get for the equation

K-essence

Given that x and y are much smaller than η up to the peak, it won't affect their dynamics



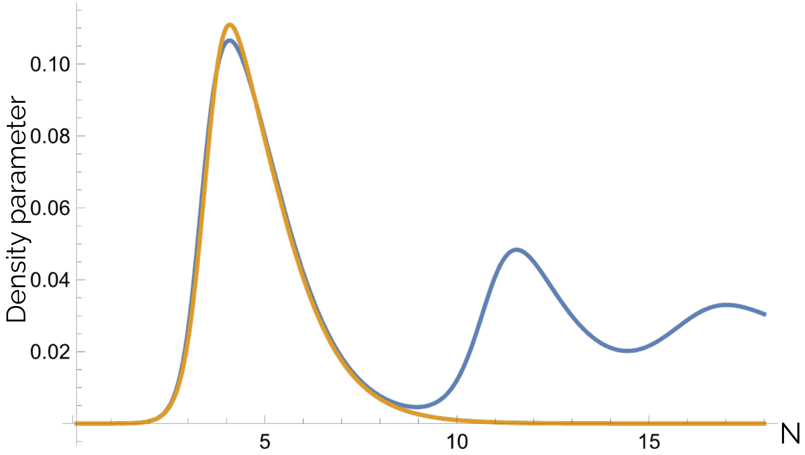
$$x' = \left(2 - \frac{3n}{2n-1}\right)x + \sqrt{\frac{3}{2}}\eta y^2,$$
$$y' = 2y - \sqrt{\frac{3}{2}}\eta x y,$$
$$\eta' = \eta(n-1)\left(\frac{3}{2n-1} - \sqrt{\frac{3}{2}}\frac{\eta y^2}{nx}\right),$$

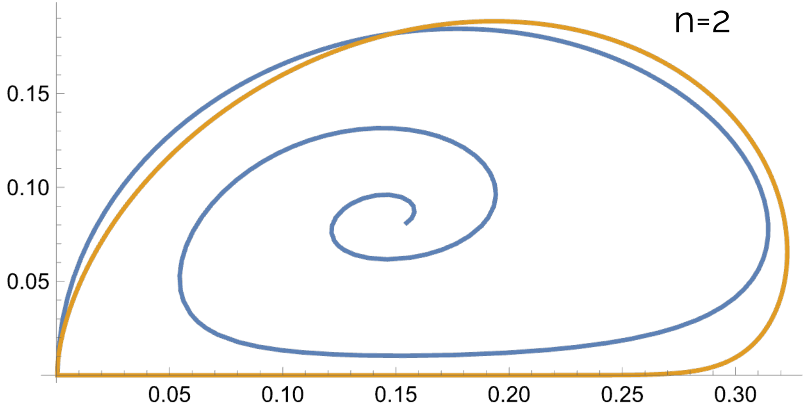


η can vary a lot until the peak, is there a way of finding this value?

K-essence

$$\eta_1 = \eta_i \left(\frac{x_i}{y_i}\right)^{\frac{n-1}{n}} \left(\frac{10}{3}\sqrt{\frac{n}{\gamma_{\rm eff}(2n-1)}}\right)^{\frac{n-1}{n}}$$





K-essence

So far, we have kept n constant, can we constrain that?

$$\mathcal{L} = \frac{X(\dot{\phi})^n}{M^{4(n-1)}} - V(\phi)$$

The equation of state is

$$\gamma_\phi = \frac{2nx^2}{(2n-1)(x^2+y^2)}$$

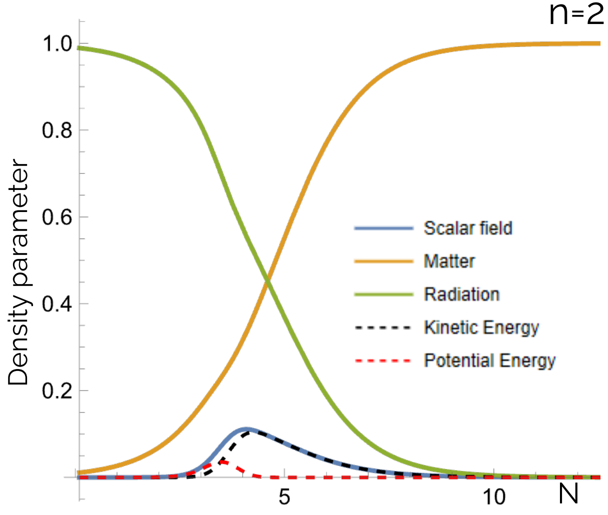
Meaning that after the peak takes place, it decays as (y=0)

$$\gamma_\phi = \frac{2n}{(2n-1)}$$

Therefore,

Canonical quintessence limit $1 \leq n \leq 2$ Decays slower than radiation

n=2



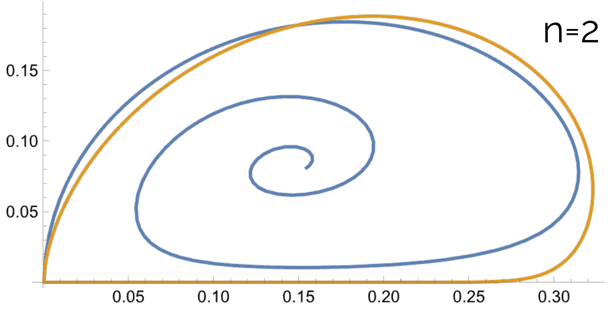
K-essence

Reparametrizing H(t) into η and setting it to a constant allows us to find the stable fixed points that the system tracks when giving the peak

What we have learnt:

Why? Spiral nature of **scaling solution** of the constant η case

When? When the exponential increase of y matches the effective stable fixed-point height of the constant η case



Height? Given by equivalent arguments to the ones applied in quintessence (perturbing around fixed point)

Late time? The system will end up vanishing due to H(t) increasing exponentially

Check against data

We used CAMB, which solves for the perturbed field equations

$$\frac{d\rho_\phi}{d\tau} = -3\mathcal{H}\gamma_\phi\rho_\phi\,,$$
$$\frac{d\delta_\phi}{d\tau} = -\left[ku_\phi + \frac{\gamma_\phi}{2}\frac{dh}{d\tau}\right] - 3\mathcal{H}(c_\mathrm{s}^2 - \gamma_\phi - 1)\left(\delta_\phi + 3\mathcal{H}\frac{u_\phi}{k}\right) - 3\mathcal{H}\frac{1}{\gamma_\phi}\frac{d\gamma_\phi}{d\tau}\frac{u_\phi}{k}\,,$$
$$\frac{du_\phi}{d\tau} = -(1-3c_\mathrm{s}^2)\mathcal{H}u_\phi + \frac{1}{\gamma_\phi}\frac{d\gamma_\phi}{d\tau}u_\phi + kc_\mathrm{s}^2\,\delta_\phi\,,$$

Where

$$c_s^2 = \frac{1}{2n-1} \qquad \gamma_\phi = \frac{2nx^2}{(2n-1)(x^2+y^2)}$$

From Q: $\left\{x_i, y_i, \tilde{\lambda}_i, \tilde{\alpha}\right\}$

K: $\{x_i, y_i, \eta_i, n\}$

Check against data

Using Planck 2018 + BAO (BOSS DR12, 6dFGS and SDSS-MGS)

Parameter	ΛCDM	K-essence	Fang
H_0	68.19 ± 0.37 (68.09)	$69.7^{+1.3}_{-1.6}$ (70.45)	68.20 ± 0.37 (68.29)
$\Omega_b h^2$	$0.02248^{+0.00010}_{-0.00012}$ (0.02247)	$0.02251^{+0.00016}_{-0.00018}$ (0.02251)	0.02248 ± 0.00012 (0.02250)
$\Omega_c h^2$	0.11824 ± 0.00086 (0.1185)	$0.1240^{+0.0051}_{-0.0062}$ (0.1278)	$0.11822^{+0.00078}_{-0.00088}$ (0.1180)
n_s	0.9721 ± 0.0034 (0.9711)	$0.9806^{+0.0092}_{-0.0077}$ (0.9873)	0.9717 ± 0.0033 (0.9722)
$\log(10^{10} A_s)$	3.056 ± 0.013 (3.055)	$3.063^{+0.014}_{-0.016}$ (3.058)	3.056 ± 0.013 (3.057)
τ_{reio}	0.0583 ± 0.0068 (0.05784)	$0.0570^{+0.0064}_{-0.0072}$ (0.05122)	0.0585 ± 0.0070 (0.05884)
$r_d h$	$100.54^{+0.60}_{-0.68}$ (100.4)	$100.65^{+0.64}_{-0.73}$ (100.5)	100.55 ± 0.64 (100.7)
$S_8 \equiv \sigma_8(\Omega_m/0.3)^{0.5}$	0.8175 ± 0.0094 (0.8195)	$0.830^{+0.012}_{-0.017}$ (0.8378)	$0.8171^{+0.0091}_{-0.010}$ (0.8153)
$\chi^2_{H0.riess2020}$	15.5	4.5 (**-11.0**)	14.2 (-1.2)
χ^2_{Planck}	1014.2	1017.1 (2.8)	1015.3 (1.1)
χ^2_{ACT}	240.4	234.4 (**-6.0**)	240.1 (-0.3)
χ^2_{data}	2310.1	2296.0 (**-14.2**)	2309.7 (-0.5)

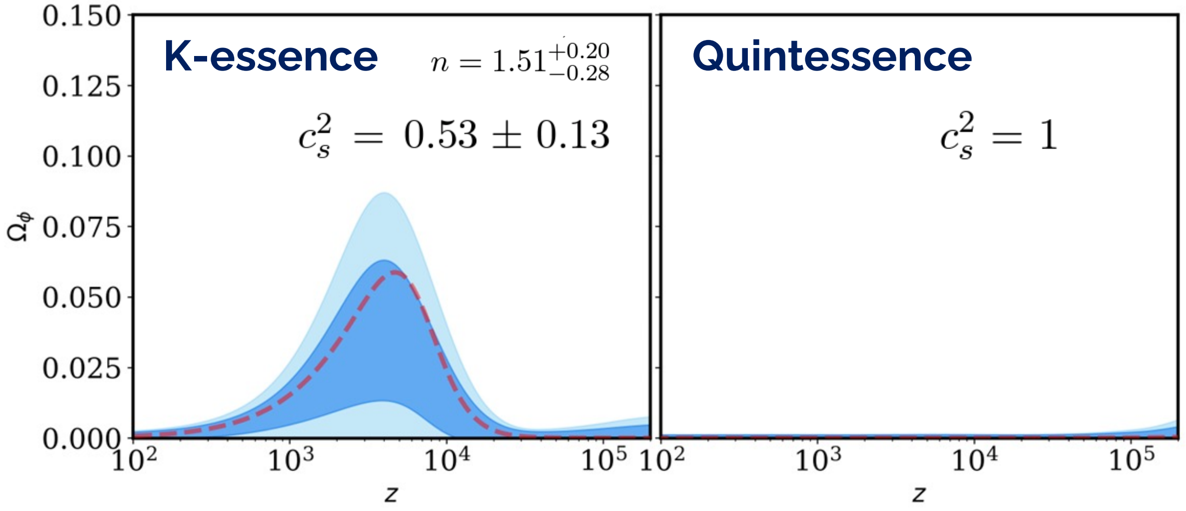
Check against data

Using Planck 2018 + BAO (BOSS DR12, 6dFGS and SDSS-MGS)

Parameter	ΛCDM	K-essence	Fang
H_0	68.19 ± 0.37 (68.09)	$69.7^{+1.3}_{-1.6}$ (70.45)	68.20 ± 0.37 (68.29)
$\Omega_b h^2$	$0.02248^{+0.00010}_{-0.00012}$ (0.02247)	$0.02251^{+0.00016}_{-0.00018}$ (0.02251)	0.02248 ± 0.00012 (0.02250)
$\Omega_c h^2$	0.11824 ± 0.00086 (0.1185)	$0.1240^{+0.0051}_{-0.0062}$ (0.1278)	$0.11822^{+0.00078}_{-0.00088}$ (0.1180)
n_s	0.9721 ± 0.0034 (0.9711)	$0.9806^{+0.0092}_{-0.0077}$ (0.9873)	0.9717 ± 0.0033 (0.9722)
$\log(10^{10} A_s)$	3.056 ± 0.013 (3.055)	$3.063^{+0.014}_{-0.016}$ (3.058)	3.056 ± 0.013 (3.057)
τ_{reio}	0.0583 ± 0.0068 (0.05784)	$0.0570^{+0.0064}_{-0.0072}$ (0.05122)	0.0585 ± 0.0070 (0.05884)
$r_d h$	$100.54^{+0.60}_{-0.68}$ (100.4)	$100.65^{+0.64}_{-0.73}$ (100.5)	100.55 ± 0.64 (100.7)
$S_8 \equiv \sigma_8(\Omega_m/0.3)^{0.5}$	0.8175 ± 0.0094 (0.8195)	$0.830^{+0.012}_{-0.017}$ (0.8378)	$0.8171^{+0.0091}_{-0.010}$ (0.8153)
$\chi^2_{H0.riess2020}$	15.5	4.5 (**-11.0**)	14.2 (-1.2)
χ^2_{Planck}	1014.2	1017.1 (2.8)	1015.3 (1.1)
χ^2_{ACT}	240.4	234.4 (**-6.0**)	240.1 (-0.3)
χ^2_{data}	2310.1	2296.0 (**-14.2**)	2309.7 (-0.5)

Check against data

Best fit for scalar fields



Data shows the importance of sound speed when addressing the Hubble tension

Summary

EDE can relax the Hubble tension as long as it doesn't have an impact at other epochs

Scalar Fields can relax the tension due to their oscillating behaviour around the scaling fixed point

We can approximate Quintessence and K-Essence to a corresponding exponential case

In particular, we found that no Quintessence can solve the Hubble tension in this procedure

However, K-essence is a good candidate as it provides $c_s^2 = 0.53 \pm 0.13$

Thank you for listening!

Any questions?

Sergio.sevillano-munoz@durham.ac.uk