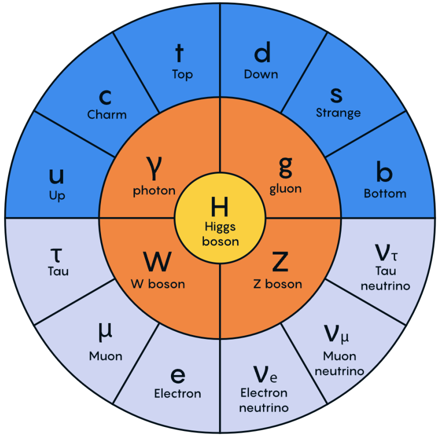
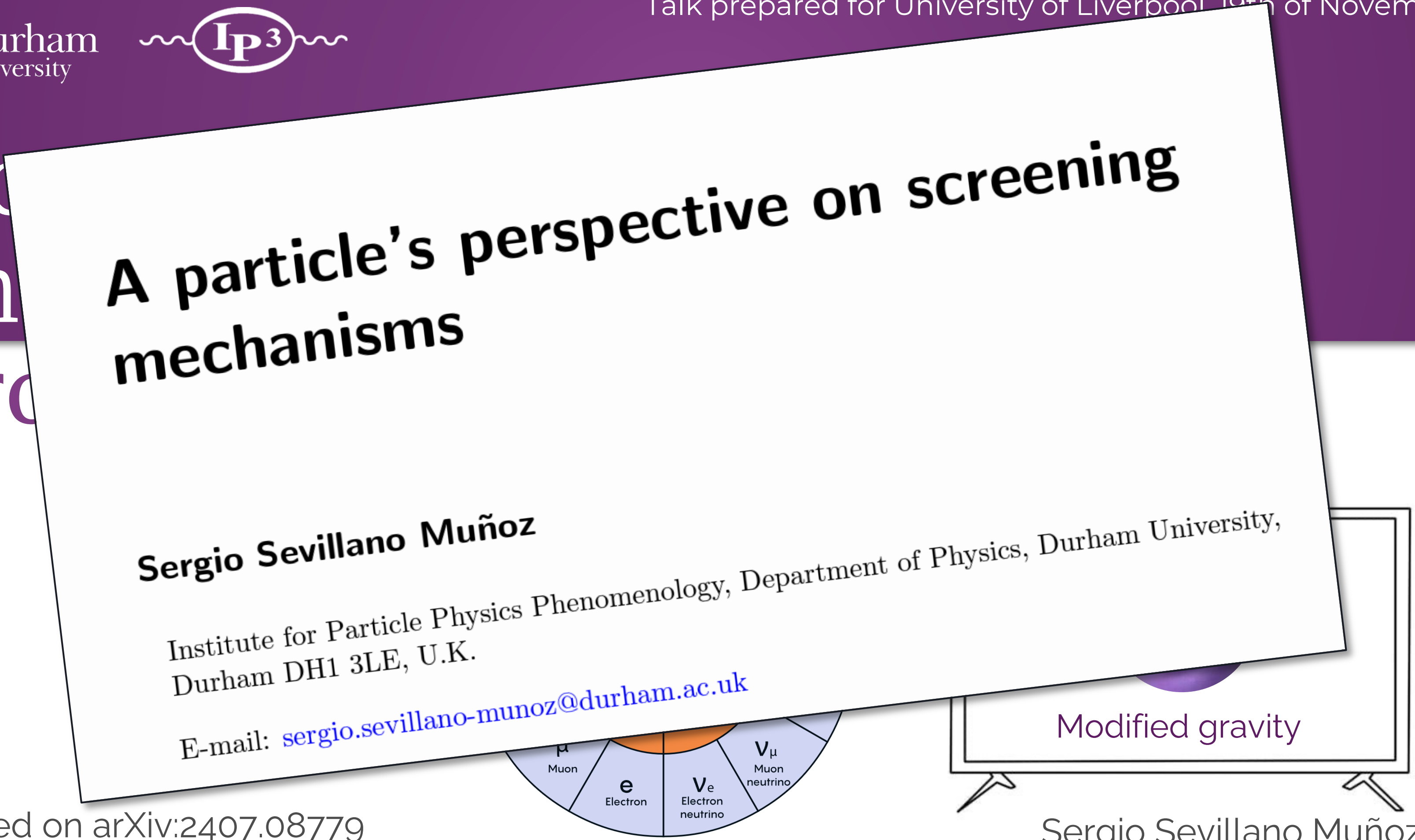
Screening mechanisms in scalar-tensor theories from a particle's perspective



Based on arXiv:2407.08779

Sergio Sevillano Muñoz

Talk prepared for University of Liverpool 19th of November



A particle's perspective on screening mechanisms

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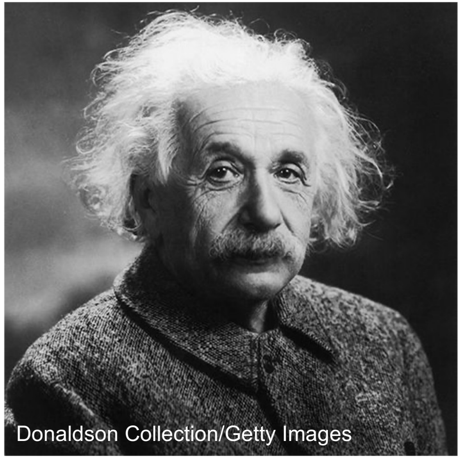
Based on arXiv:2407.08779

Sergio Sevillano Muñoz

Why would we need to modify gravity?

We all know and love Einstein's General Relativity, which
Is usually described by the Einstein-Hilbert action

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[-\underbrace{\frac{M_{\mathrm{pl}}^2}{2}R}_{\text{Curvature}} + \underbrace{\mathcal{L}_{\mathrm{m}}\{\psi_i, \phi_i, g_{\mu\nu}, ...\}}_{\text{Matter action}}\right]$$



However, we already know that this is not the whole story to the theory of gravity

Why would we need to modify gravity?

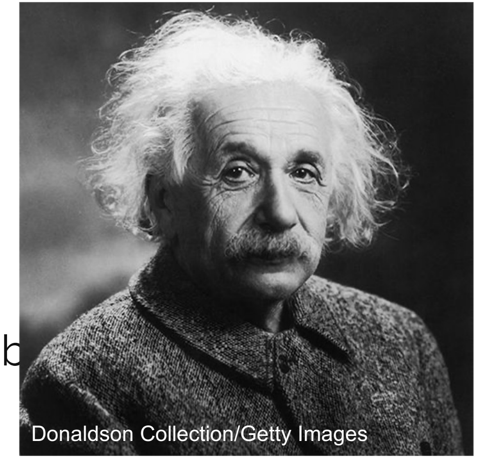
There are many problems to choose from with GR and the Standard Model

-The cosmological constant.

-What is dark matter.

-The early universe (Inflation, phase transitions)

-The coincidence problem, Hubble tension, Strong CP prob mass



Pick your favourite!

Why would we need to modify gravity?

Some of these problems require an extension of the Standard Model:

Standard Model $\longrightarrow$ **Beyond the Standard Model theories (BSM)**

Axion, inflation, quintessence…

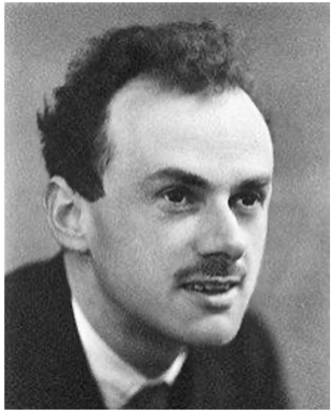
But in this talk we will focus on theories that instead modify the gravitational action

Standard Einstein-Hilbert action $\longrightarrow$ **New Modified gravity (?)**

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{M_{\rm pl}^2}{2}R + \mathcal{L}_{\rm m}\{\psi_i, \phi_i, g_{\mu\nu}, \dots\}\right]$$

An example:

One of the first motivations for modified gravity started with Dirac, and its **large number hypothesis**



He noticed that both dimensionless ratios were of the same order

$$\frac{ct}{r_e} \approx 3.47 \cdot 10^{41} \approx 10^{42}$$

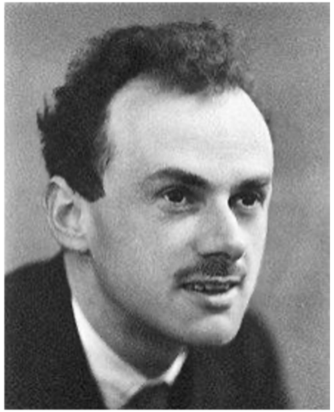
$$\frac{e^2}{4\pi\epsilon_0 G m_p m_e} \approx 10^{40}$$

From which he hypothesised that the strength of gravity might be changing with time!

$$G \sim \frac{1}{t}$$

An example:

Time-dependent gravitational constant? Quite crazy... $G \sim \frac{1}{t}$



However, of course, we cannot just insert some function of time as:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{M_{\mathrm{pl}}(t)^2}{2}R + \mathcal{L}_{\mathrm{m}}\{\psi_i, \phi_i, g_{\mu\nu}, \ldots\}\right]$$

Types of modified gravity:

Work within General Relativity
and choose different action

Ignore General Relativity
and change some classical mechanic definitions

Modified gravity
is not exclusively MOND

We'll choose theories within General Relativity but have a non-canonical gravitational action

How do we modify gravity consistently?

How *general* is the Einstein–Hilbert action?

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{M_{\mathrm{pl}}^2}{2}R + \mathcal{L}_{\mathrm{m}}\{\psi_i, \phi_i, g_{\mu\nu}, ...\}\right]$$

or the Einstein equation:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{2}{M_{\mathrm{pl}}^2}T_{\mu\nu}$$

Can we build any new modified gravity action?

How do we modify gravity consistently?

Lovelock's theorem:

In 4-D the only divergence-free rank-2 tensor constructed from only the metric and its derivatives up to second order, and preserving diffeomorphism invariance, is the Einstein tensor with a cosmological constant term.

How do we modify gravity consistently?

Lovelock's theorem:

In 4-D the only divergence-free rank-2 tensor constructed from only the metric and its derivatives up to second order, and preserving diffeomorphism invariance, is the Einstein tensor with a cosmological constant term.

Fancy words saying that if we have a rank-2 tensor *L* satisfying

i) $L^{\mu\nu} = L^{\mu\nu}(g_{\alpha\beta}, \partial_\sigma g_{\alpha\beta}, \partial_\sigma \partial_\rho g_{\alpha\beta});$

ii) $\nabla_\mu L^{\mu\nu} = 0,$

$$\longrightarrow \quad L_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

How do we modify gravity consistently?

Therefore, it will necessarily satisfy the Einstein equation

$$L_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \qquad\qquad R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{2}{M_{\mathrm{pl}}^2} T_{\mu\nu}$$

and come from the Einstein-Hilbert action

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left[-\frac{M_{\mathrm{pl}}^2}{2} R + \mathcal{L}_{\mathrm{m}} \{ \psi_i, \phi_i, g_{\mu\nu}, \ldots \} \right]$$

Implies that THIS is the most general action for such requirements

<u>How can we modify gravity?</u>

How do we modify gravity consistently?

Just break Lovelock's theorem:

In 4-D the only divergence-free rank-2 tensor constructed from only the metric and its derivatives up to second order, and preserving diffeomorphism invariance, is the Einstein tensor with a cosmological constant term.

We will choose a theory that allows for couplings to a scalar field.

How do we modify gravity consistently?

Either from an **effective field theory** standpoint or more **fundamental theories** of gravity (such as compactifications of extra dimensions) it is natural to extend the gravitational action:

Different functions of gravity

$$S_{\mathrm{grav}} = \int \mathrm{d}^4x\sqrt{-g}\left[-aR - F(\varphi)R - bR^2 - cR_{\mu\nu}R^{\mu\nu} + \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \ldots\right]$$

Non-minimal couplings to a scalar field

These extensions of gravity are called **Scalar-Tensor theories**

How do we modify gravity consistently?

Horndeski theory:

$$S_{\mathrm{H}} = \int \mathrm{d}^4x \sqrt{-g}\,[\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5]$$

$$\mathcal{L}_2 = G_2(\varphi, X)$$

$$\mathcal{L}_3 = G_3(\varphi, X)\Box\varphi$$

$$\mathcal{L}_4 = G_4(\varphi, X)R + G_{4,X}(\varphi, X)\left[(\Box\varphi)^2 - (\nabla_\mu\nabla_\nu\varphi)^2\right]$$

$$\mathcal{L}_5 = G_5(\varphi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\varphi$$
$$- \frac{1}{6}G_{5,X}(\varphi, X)\left[(\Box\varphi)^3 - 3\Box\varphi(\nabla_\mu\nabla_\nu\varphi)^2 + 2(\nabla_\mu\nabla_\nu\varphi)^3\right]$$

$$G^{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R$$
$$X = \tfrac{1}{2}\partial_\mu\varphi\partial^\mu\varphi$$

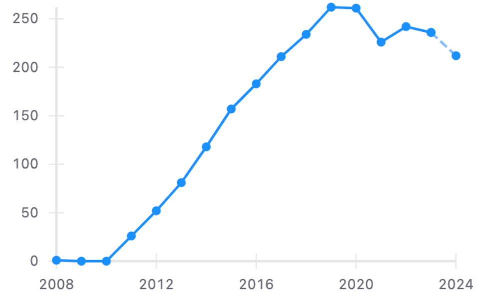
Second-order scalar-tensor field equations in a four-dimensional space

Gregory Walter Horndeski (Waterloo U.)
1974

21 pages
Published in: *Int.J.Theor.Phys.* 10 (1974) 363-384
DOI: 10.1007/BF01807638
View in: AMS MathSciNet

cite claim reference search 2,502 citations

Citations per year



$$\mathcal{L}_3 = G_3(\varphi, X)\Box\varphi$$

$$\mathcal{L}_4 = G_4(\varphi, X)R + G_{4,X}(\varphi,$$

$$\mathcal{L}_5 = G_5(\varphi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\varphi$$

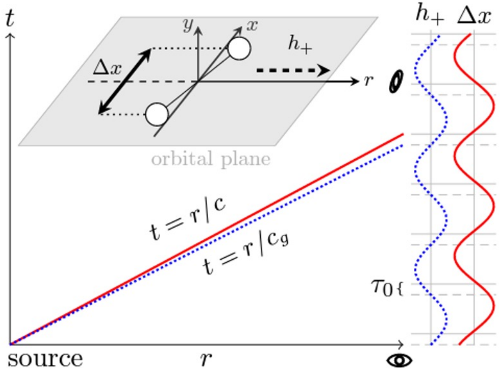
$$-\frac{1}{6}G_{5,X}(\varphi, X)\left[(\Box\varphi)^3 -\right.$$

Bounds on Scalar-Tensor theories

Speed of gravitational waves:

$$\boxed{-3 \cdot 10^{-15} \leq c_T/c - 1 \leq 7 \cdot 10^{-16}}$$

$$\frac{c_T^2}{c^2} = \frac{G_4 - XG_{5,\varphi} - \ddot{\varphi}G_{5,\varphi}}{G_4 - 2XG_{4,\varphi} + XG_{5,\varphi} - \dot{\varphi}HXG_{5,X}}$$



Bettoni, Ezquiaga et al. 2017

Horndeski theory can be simplified to

$$\mathcal{L}_{\mathrm{H}}^{(c)} = G_4(\varphi)R + G_2(\varphi, X) - G_3(\varphi, X)\Box\varphi$$

Brans and Dicke (1961)

Bounds on Scalar-Tensor theories

However, Brans, Dicke and Jordan had already been thinking about something similar, but using a scalar field

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{F(\varphi)}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - U(\varphi) + \mathcal{L}_\mathrm{m}\{\psi_i, \phi_i, g_{\mu\nu}, ...\}\right]$$

Such that at late times, when the scalar stabilizes

$$F(v_\varphi) \equiv M_\mathrm{pl}^2$$

This is a consistent theory with changing gravitational force!

Bounds on Scalar-Tensor theories

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{F(\varphi)}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - U(\varphi) + \mathcal{L}_\mathrm{m}\{\psi_i, \phi_i, g_{\mu\nu}, \ldots\}\right]$$

In most cases, they introduce new dynamics into the matter sector (**fifth forces**)

The tightest constraints come from large scales:

-Solar system scales (Cassini spacecraft)

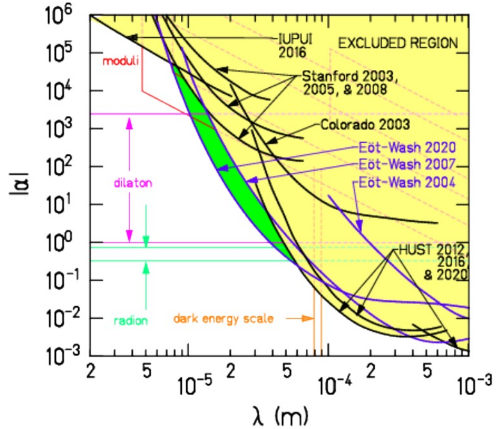
Bertotti et al. 2003

-Atomic scales (Atom interferometry)

YT talk by Clare Burrage in "Cosmology Talks"

Bounds on Scalar-Tensor theories

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{F(\varphi)}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - U(\varphi) + \mathcal{L}_\mathrm{m}\{\psi_i, \phi_i, g_{\mu\nu}, \ldots\}\right]$$



$$V(r) = \frac{Gm_1m_2}{r}\left(1 + \alpha e^{-r/\lambda}\right)$$

Screening mechanisms

Screening mechanisms naturally evade all bounds. Let's have a quick look on how they do it

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{F(\varphi)}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - U(\varphi) + \mathcal{L}_\mathrm{m}\{\psi_i, \phi_i, g_{\mu\nu}, ...\}\right]$$

The equations of motion are coupled:

$$G^{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R$$

$$\frac{1}{2}F(\varphi)G_{\mu\nu} + \nabla_\mu\nabla_\nu F(\varphi) - g_{\mu\nu}\Box F(\varphi) = \frac{1}{2}T^{(\mathrm{m})}_{\mu\nu} + \frac{1}{2}T^{(\varphi)}_{\mu\nu}$$

$$\Box\varphi + U'(\varphi) + \frac{F'(\varphi)}{2}R = 0$$

Screening mechanisms

Taking the trace of the modified Einstein's equation:

$$R = -\frac{1}{F(\varphi)}\rho_m$$

$$G^{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R$$

The field's equation of motion takes the following form:

$$\Box\varphi + U'(\varphi) + \frac{F'(\varphi)}{2}R = 0 \longrightarrow \Box\varphi + U'(\varphi) - \frac{F'(\varphi)}{2F(\varphi)}\rho_m = 0$$

Density dependent potential!!

$$F(\varphi) = \tilde{M}_{\rm Pl}^2/A^2(\varphi)$$

$$\boxed{V_{\rm eff}(\varphi) = U(\varphi) + \log(A(\varphi))\rho_m}$$

Screening mechanisms

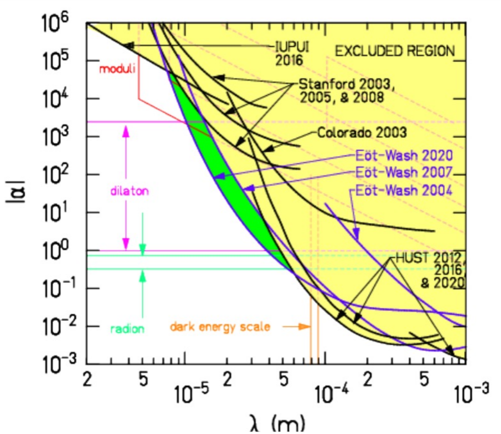
How can we use this? Two examples:

$$\boxed{V_{\text{eff}}(\varphi) = U(\varphi) + \log(A(\varphi))\rho_m}$$

Chameleon model:

The mediator's mass increases in high density environments

$$V(r) = \frac{Gm_1m_2}{r}\left(1+\alpha e^{-r/\lambda}\right)$$



Symmetron model:

The interaction strength vanishes in high density environments

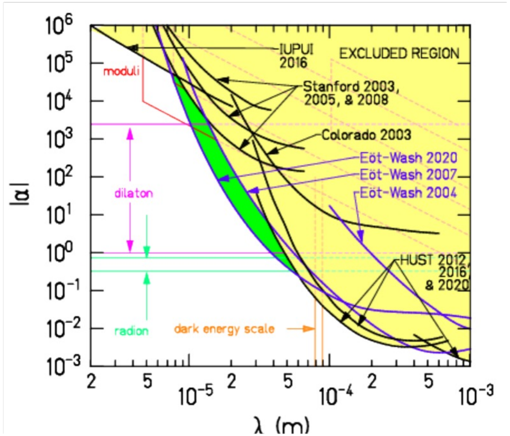
$$\Box\varphi + U'(\varphi) + \frac{F'(\varphi)}{2}R = 0$$

Based on arXiv:2211.14300

Screening mechanisms

For every observable, we may always be able to hide the effects due to screening mechanisms

Nice paradox: It may be that the very own experiments we build to test for fifth forces are screening them away



While current research tries to look away from screening mechanisms, I want to explore what happens when we look into them instead?

C. Burrage, E. Copeland, P. Millington and M. Spannowsky (arXiv:1804.07180)
E. Copeland, P. Millington and SSM (2111.06357)

Particle physics

These calculations are usually very complicated. However, modifications on the Standard Model can be described by a Beyond the Standard Model theory.

For that, instead of working with modified gravity, we will make the following transformation

Our plan:

Modified theory of gravity
+
Standard Model

$\xrightarrow{\text{Using Field theory}}$

General Relativity
+
Beyond the Standard Model

C. Burrage, E. Copeland, P. Millington and M. Spannowsky (arXiv:1804.07180)
E. Copeland, P. Millington and SSM (2111.06357)

Modified Gravity=BSM?

As an example, let's study the simplest modified theory of gravity

Brans-Dicke Action:

Matter action

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{F(\varphi)}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - U(\varphi) + \mathcal{L}_\mathrm{m}\{\psi_i, \phi_i, g_{\mu\nu}, \ldots\}\right]$$

To satisfy constraints, we need at late times: $F(v_\varphi) \equiv M_\mathrm{pl}^2$

$$g_{\mu\nu} \to \frac{\tilde{M}_\mathrm{Pl}^2}{F(\varphi)}\tilde{g}_{\mu\nu}, \qquad g^{\mu\nu} \to \frac{F(\varphi)}{\tilde{M}_\mathrm{Pl}^2}\tilde{g}^{\mu\nu},$$

Take a conformal transformation so that all modifications in gravity disappear!

Matter sector:

As an example, let's see how this affects a QED+Higgs singlet model:

In the Jordan frame:

$$
\begin{aligned}
S_{\mathrm{m}}[g_{\mu\nu}] = \int \mathrm{d}^4x\sqrt{-g}\bigg[&\overbrace{-\frac{1}{4}g^{\alpha\mu}g^{\beta\nu}F_{\alpha\beta}F_{\mu\nu}}^{\text{Photon}} + \overbrace{\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}^{\text{Higgs}} \\
&\boxed{+ i\bar{\psi}e^\mu_a\gamma^a\partial_\mu\psi + \frac{1}{2}\bar{\psi}e^\mu_a\gamma^a\Omega_\mu\psi - q\bar{\psi}e^\mu_a\gamma^a A_\mu\psi} \\
&\underbrace{- y\bar{\psi}\phi\psi}_{\text{Fermion}} + \underbrace{\frac{1}{2}\mu^2\phi^2 - \frac{\lambda}{4!}\phi^4 - \frac{3\mu^4}{2\lambda}}_{\text{Higgs}}\bigg],
\end{aligned}
$$

Matter sector:

As an example, let's see how this affects a QED+Higgs singlet model:

$$A^2(\chi) = \tilde{M}_{\rm Pl}^2/F(\varphi(\chi))$$

In the Einstein frame

$$
\begin{aligned}
S[\tilde{g}_{\mu\nu}] = \int \mathrm{d}^4x\sqrt{-\tilde{g}}\,\Bigg[& -\frac{\tilde{M}_{\rm Pl}^2}{2}\tilde{R} + \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - V(\chi) \\
& \text{Photon} \quad - \frac{1}{4}\tilde{g}^{\alpha\mu}\tilde{g}^{\beta\nu}F_{\alpha\beta}F_{\mu\nu} + \frac{A^2(\chi)}{2}\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \quad \text{Higgs} \\
& + A^3(\chi)\left(i\bar{\psi}\tilde{e}_a^\mu\gamma^a\partial_\mu\psi + \frac{1}{2}\bar{\psi}\tilde{e}_a^\mu\gamma^a\tilde{\Omega}_\mu\psi - q\bar{\psi}\tilde{e}_a^\mu\gamma^a A_\mu\psi\right) \\
& \underbrace{- A^4(\chi)y\bar{\psi}\phi\psi}_{\text{Fermion}} + \underbrace{A^4(\chi)\left(\frac{1}{2}\mu^2\phi^2 - \frac{\lambda}{4!}\phi^4 - \frac{3\mu^4}{2\lambda}\right)}_{\text{Higgs}}\Bigg],
\end{aligned}
$$

Matter sector:

Rescaling the fields depending on their scaling dimension:

$$\phi \to A^{-1}(\chi)\tilde{\phi}, \qquad \psi \to A^{-3/2}(\chi)\tilde{\psi}$$

$$S_{\mathrm{m}}[\tilde{g}_{\mu\nu}] = \int \mathrm{d}^4x\sqrt{-\tilde{g}}\Bigg[-\frac{\tilde{M}_{\mathrm{Pl}}^2}{2}\tilde{R} + \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - V(\chi)$$

Photon

$$-\frac{1}{4}\tilde{g}^{\alpha\mu}\tilde{g}^{\beta\nu}F_{\alpha\beta}F_{\mu\nu} + \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\tilde{\phi}\partial_\nu\tilde{\phi}$$

$$-\frac{A'(\chi)}{A(\chi)}\tilde{\phi}\tilde{g}^{\mu\nu}\partial_\mu\chi\partial_\nu\tilde{\phi} + \frac{\tilde{\phi}^2}{2}\frac{A'(\chi)^2}{A^2(\chi)}\tilde{g}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi$$

Higgs

Fermion

$$+ i\bar{\tilde{\psi}}\tilde{e}^\mu_a\gamma^a\partial_\mu\tilde{\psi} + \frac{1}{2}\bar{\tilde{\psi}}\tilde{e}^\mu_a\gamma^a\tilde{\Omega}_\mu\tilde{\psi} - q\bar{\tilde{\psi}}\tilde{e}^\mu_a\gamma^a A_\mu\tilde{\psi}$$

$$-y\bar{\tilde{\psi}}\tilde{\phi}\tilde{\psi} + \left(\frac{1}{2}\mu^2A^2(\chi)\tilde{\phi}^2 - \frac{\lambda}{4!}\tilde{\phi}^4 - A^4(\chi)\frac{3\mu^4}{2\lambda}\right)\Bigg]$$

Higgs

Matter sector:

Making a final transformation, we obtain: $$\tilde{\chi} = \int \mathrm{d}\hat{\chi}\sqrt{1+\tilde{\phi}^2\left(\frac{A'(\hat{\chi})}{A(\hat{\chi})}\right)^2}$$

$$
\begin{aligned}
S_{\mathrm{m}}[\tilde{g}_{\mu\nu}] = \int \mathrm{d}^4x\sqrt{-\tilde{g}}\Bigg[&-\frac{\tilde{M}_{\mathrm{Pl}}^2}{2}\tilde{R} + \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\tilde{\chi}\partial_\nu\tilde{\chi} - \tilde{V}(\tilde{\chi}) \\
&- \frac{1}{4}\tilde{g}^{\alpha\mu}\tilde{g}^{\beta\nu}F_{\alpha\beta}F_{\mu\nu} + \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\tilde{\phi}\partial_\nu\tilde{\phi} - \frac{\tilde{A}'(\tilde{\chi})}{\tilde{A}(\tilde{\chi})}\tilde{\phi}\tilde{g}^{\mu\nu}\partial_\mu\tilde{\chi}\partial_\nu\tilde{\phi} \\
&+ i\bar{\tilde{\psi}}\tilde{e}_a^\mu\gamma^a\partial_\mu\tilde{\psi} + \frac{1}{2}\bar{\tilde{\psi}}\tilde{e}_a^\mu\gamma^a\tilde{\Omega}_\mu\tilde{\psi} - q\bar{\tilde{\psi}}\tilde{e}_a^\mu\gamma^a A_\mu\tilde{\psi} \\
&- y\bar{\tilde{\psi}}\tilde{\phi}\tilde{\psi} + \left(\frac{1}{2}\mu^2\tilde{A}^2(\tilde{\chi})\tilde{\phi}^2 - \frac{\lambda}{4!}\tilde{\phi}^4 - \tilde{A}^4(\tilde{\chi})\frac{3\mu^4}{2\lambda}\right)\Bigg],
\end{aligned}
$$

Higgs

Photon

Fermion

Higgs

Matter sector:

Making a final transformation, we obtain: $$\tilde{\chi} = \int \mathrm{d}\hat{\chi}\sqrt{1+\tilde{\phi}^2\left(\frac{A'(\hat{\chi})}{A(\hat{\chi})}\right)^2}$$

$$S_{\mathrm{m}}[\tilde{g}_{\mu\nu}] = \int \mathrm{d}^4x\sqrt{-\tilde{g}}\Bigg[-\frac{\tilde{M}_{\mathrm{Pl}}^2}{2}\tilde{R} + \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\tilde{\chi}\partial_\nu\tilde{\chi} - \tilde{V}(\tilde{\chi})$$

Higgs

Photon $$-\frac{1}{4}\tilde{g}^{\alpha\mu}\tilde{g}^{\beta\nu}F_{\alpha\beta}F_{\mu\nu} + \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\tilde{\phi}\partial_\nu\tilde{\phi} - \frac{\tilde{A}'(\tilde{\chi})}{\tilde{A}(\tilde{\chi})}\tilde{\phi}\tilde{g}^{\mu\nu}\partial_\mu\tilde{\chi}\partial_\nu\tilde{\phi}$$

Fermion $$+ i\bar{\tilde{\psi}}\tilde{e}^\mu_a\gamma^a\partial_\mu\tilde{\psi} + \frac{1}{2}\bar{\tilde{\psi}}\tilde{e}^\mu_a\gamma^a\tilde{\Omega}_\mu\tilde{\psi} - q\bar{\tilde{\psi}}\tilde{e}^\mu_a\gamma^a A_\mu\tilde{\psi}$$

$$- y\bar{\tilde{\psi}}\tilde{\phi}\tilde{\psi} + \boxed{\left(\frac{1}{2}\mu^2\tilde{A}^2(\tilde{\chi})\tilde{\phi}^2 - \frac{\lambda}{4!}\tilde{\phi}^4 - \tilde{A}^4(\tilde{\chi})\frac{3\mu^4}{2\lambda}\right)}\Bigg],$$ Higgs

Brans-Dicke = Higgs portal terms!

SSM (2407.08779)

Screening mechanisms:

We can take different directions from here: $$- y\bar{\psi}\tilde{\phi}\psi + \left(\frac{1}{2}\mu^2\tilde{A}^2(\tilde{\chi})\tilde{\phi}^2 - \frac{\lambda}{4!}\tilde{\phi}^4 - \tilde{A}^4(\tilde{\chi})\frac{3\mu^4}{2\lambda}\right)$$

1) Expanding $\tilde{A}(v_{\tilde{\chi}}) \approx 1$ and perturbatively and find new forces.

Burrage and Millington (2310.12071)

2) Don't assume that $\tilde{A}(v_{\tilde{\chi}}) \approx 1$ and find the effect on the Higgs vacuum: $\rho_\psi = y v_{\tilde{\phi}}\bar{\psi}\psi$

$$\frac{\lambda}{6}v_{\tilde{\phi}}^4 - \mu^2 v_{\tilde{\phi}}^2\tilde{A}^2(v_{\tilde{\chi}}) + \rho_\psi = 0,$$

$$\tilde{A}'(v_{\tilde{\chi}})\tilde{A}^3(v_{\tilde{\chi}})\frac{6\mu^4}{\lambda} - \mu^2 v_{\tilde{\phi}}^2\tilde{A}'(v_{\tilde{\chi}})\tilde{A}(v_{\tilde{\chi}}) + \tilde{V}'(v_{\tilde{\chi}}) = 0,$$

Screening mechanisms:

While ρ_ψ does not affect the Higgs' vev, it does affect $\tilde{\chi}$

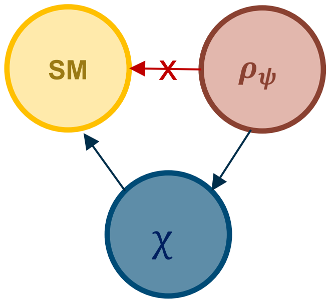
$$- y\bar{\psi}\tilde{\phi}\psi + \left(\frac{1}{2}\mu^2 \tilde{A}^2(\tilde{\chi})\tilde{\phi}^2 - \frac{\lambda}{4!}\tilde{\phi}^4 - \tilde{A}^4(\tilde{\chi})\frac{3\mu^4}{2\lambda}\right)$$

$$v^2 = 6\mu^2/\lambda.$$

$$v_{\tilde{\phi}}^2 = \frac{v^2}{2}\tilde{A}^2(v_{\tilde{\chi}})\left(1 + \sqrt{1 - \frac{4\rho_\psi}{v^2\mu^2\tilde{A}^4(v_{\tilde{\chi}})}}\right)$$

$$\tilde{V}'(v_{\tilde{\chi}}) = -\frac{\tilde{A}'(v_{\tilde{\chi}})}{\tilde{A}(v_{\tilde{\chi}})}\rho_\psi,$$

We therefore recover the usual screening equation for χ.



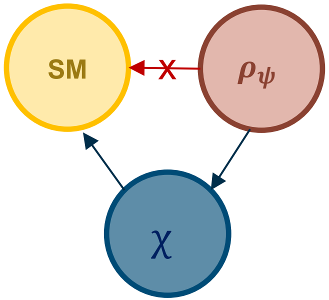
The Higgs does not modify the equation, only enables it!

Screening mechanisms:

From this, we can already see an important implication of screening mechanisms

Assuming

$$\rho_\psi \ll v^2\mu^2\tilde{A}^4(v_{\tilde{\chi}})$$



This means that the fermion's masses now rescale by:

$$m_\psi = yv_{\tilde{\phi}} \approx y\sqrt{\frac{6\mu^2}{\lambda}}\tilde{A}(v_{\tilde{\chi}})$$

Which from the vev equations we can see that it is density dependent!

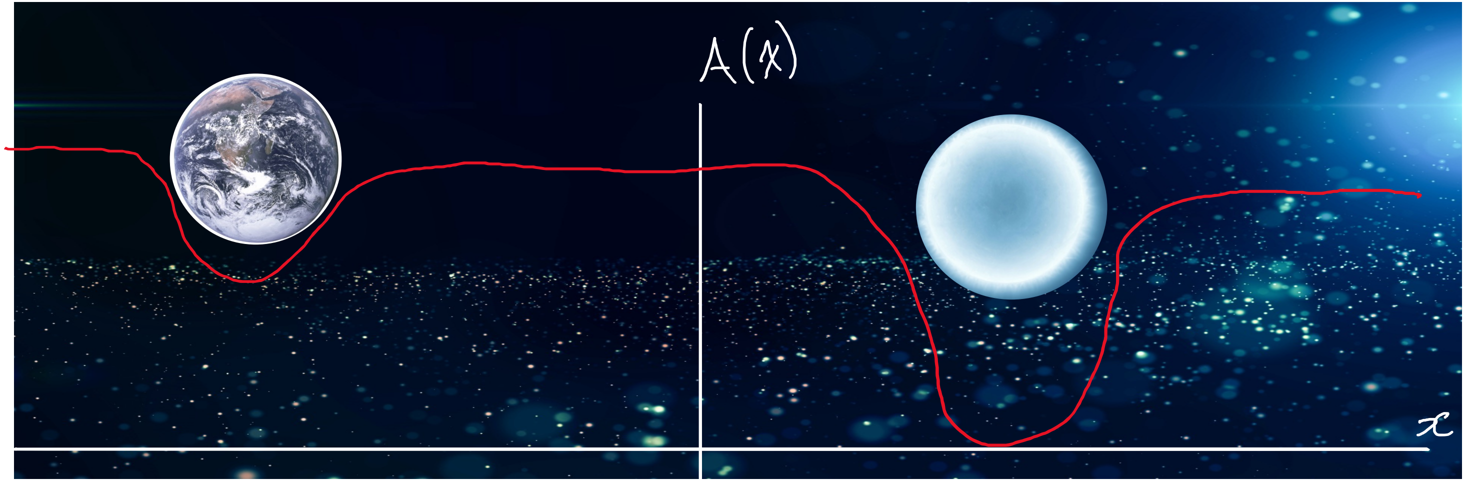
$$\rho = yv\bar{\tilde{\psi}}\tilde{\psi}$$

$$\tilde{V}'(v_{\tilde{\chi}}) = -\frac{\tilde{A}'(v_{\tilde{\chi}})}{\tilde{A}(v_{\tilde{\chi}})}\rho_\psi \qquad \tilde{V}'(v_{\tilde{\chi}}) = -\tilde{A}'(v_{\tilde{\chi}})\rho$$

Is this physical? YES!

Einstein frame: $$\tilde{V}'(v_{\tilde{\chi}}) = -\frac{\tilde{A}'(v_{\tilde{\chi}})}{\tilde{A}(v_{\tilde{\chi}})}\rho_\psi \qquad m_\psi = y v_{\tilde{\phi}} \approx y\sqrt{\frac{6\mu^2}{\lambda}}\tilde{A}(v_{\tilde{\chi}})$$

Not to scale*



Jordan frame: $$\Box\varphi + U'(\varphi) - \frac{F'(\varphi)}{2F(\varphi)}\rho_m = 0 \qquad G_{\mu\nu} = \frac{2}{F(\varphi)}\left(\frac{1}{2}T^{(\mathrm{m})}_{\mu\nu} + \frac{1}{2}T^{(\varphi)}_{\mu\nu} - \nabla_\mu\nabla_\nu F(\varphi) + g_{\mu\nu}\Box F(\varphi)\right)$$

Is this physical? YES!

The mass of elementary particles can change in space!

While this has been previously considered in time evolution:

Growing neutrinos and cosmological selection

C. Wetterich
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Philosophenweg 16, D-69120 Heidelberg

Cosmology of Mass-Varying Neutrinos Driven by Quintessence: Theory and Observations

A. W. Brookfield,[1] C. van de Bruck,[2] D. F. Mota,[3] and D. Tocchini-Valentini[4]

Early dark energy from massive neutrinos — a natural resolution of the Hubble tension

Jeremy Sakstein* and Mark Trodden†
Center for Particle Cosmology, Department of Physics and Astronomy,
University of Pennsylvania 209 S. 33rd St., Philadelphia, PA 19104, USA

Coupled Quintessence

Luca Amendola
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Viale Parco Mellini 84,
00136 Roma, Italy
amendola@oarhp1.rm.astro.it

We have more independent observations of physical spaces

Screening mechanisms:

What about other physics? Do they get modified?

Assuming $\tilde{A}(\tilde{\chi}+v_{\tilde{\chi}}) = \tilde{A}(v_{\tilde{\chi}}) + \tilde{A}'(v_{\tilde{\chi}})\chi + \tilde{A}''(v_{\tilde{\chi}})\frac{\chi^2}{2} + \dots .$

The matter Lagrangian shifts to $\tilde{\phi} \to \tilde{\bar{\phi}} + v_{\tilde{\phi}}$ and $\tilde{\chi} \to \tilde{\chi} + v_{\tilde{\chi}}$

$$S_{\rm m}[\tilde{g}_{\mu\nu}] = \int \mathrm{d}^4x\sqrt{-\tilde{g}}\left[-\frac{\tilde{M}_{\rm Pl}^2}{2}\tilde{R} + \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\tilde{\chi}\partial_\nu\tilde{\chi} - \tilde{V}(\tilde{\chi}+v_{\tilde{\chi}}) - \frac{1}{4}\tilde{g}^{\alpha\mu}\tilde{g}^{\beta\nu}F_{\alpha\beta}F_{\mu\nu} + \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\tilde{\phi}\partial_\nu\tilde{\phi} - \frac{\tilde{A}'(\tilde{\chi})}{\tilde{A}(\tilde{\chi})}\tilde{\phi}\tilde{g}^{\mu\nu}\partial_\mu\tilde{\chi}\partial_\nu\tilde{\phi} + i\tilde{\bar{\psi}}\tilde{e}_a^\mu\gamma^a\tilde{\nabla}_\mu\tilde{\psi} - y\bar{\tilde{\psi}}\tilde{\phi}\psi - yv_{\tilde{\phi}}\bar{\tilde{\psi}}\psi + H(\tilde{\phi},\tilde{\chi})\right],$$

Screening mechanisms:

What about other physics? Do they get modified?

Assuming $$\tilde{A}(\tilde{\chi}+v_{\tilde{\chi}}) = \tilde{A}(v_{\tilde{\chi}}) + \tilde{A}'(v_{\tilde{\chi}})\chi + \tilde{A}''(v_{\tilde{\chi}})\frac{\chi^2}{2} + \dots .$$

The matter Lagrangian shifts to $\tilde{\phi} \to \tilde{\bar{\phi}} + v_{\tilde{\phi}}$ and $\tilde{\chi} \to \tilde{\chi} + v_{\tilde{\chi}}$

$$S_{\rm m}[\tilde{g}_{\mu\nu}] = \int \mathrm{d}^4x\sqrt{-\tilde{g}}\Bigg[-\frac{\tilde{M}_{\rm Pl}^2}{2}\tilde{R} + \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\tilde{\chi}\partial_\nu\tilde{\chi} - \boxed{\tilde{V}(\tilde{\chi}+v_{\tilde{\chi}})}$$
$$- \frac{1}{4}\tilde{g}^{\alpha\mu}\tilde{g}^{\beta\nu}F_{\alpha\beta}F_{\mu\nu} + \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\tilde{\phi}\partial_\nu\tilde{\phi} - \frac{\tilde{A}'(\tilde{\chi})}{\tilde{A}(\tilde{\chi})}\tilde{\phi}\tilde{g}^{\mu\nu}\partial_\mu\tilde{\chi}\partial_\nu\tilde{\phi}$$
$$+ i\tilde{\bar{\psi}}\tilde{e}^\mu_a\gamma^a\tilde{\nabla}_\mu\tilde{\psi} - y\bar{\psi}\tilde{\phi}\psi - yv_{\tilde{\phi}}\bar{\psi}\psi + \boxed{H(\tilde{\phi},\tilde{\chi})}\Bigg],$$

Screening mechanisms:

What about other physics? Do they get modified?

To make it simpler, let's substitute $\tilde{A}(\tilde{\chi} + v_{\tilde{\chi}}) = a + b\chi + \frac{c}{2}\chi^2 + \cdots$

Then, the potential is given by:

$$
\begin{aligned}
H(\tilde{\phi}, \tilde{\chi}) =& \frac{\mu^2}{2}\left(a^2\tilde{\phi}^2 + 2abv_{\tilde{\phi}}\tilde{\phi}\tilde{\chi} + (b^2 + ac)v_{\tilde{\phi}}^2\chi^2\right) \\
& - \frac{\lambda}{4}v_{\tilde{\phi}}^2\tilde{\phi}^2 - \frac{3\mu^4}{2\lambda}(2a^3c + 6a^2b^2)\tilde{\chi}^2 + \ldots,
\end{aligned}
$$

To find any new Yukawa interactions and the mass of the propagator we must diagonalise it

Screening mechanisms:

What about other physics? Do they get modified?

To find any new Yukawa interactions and the mass of the propagator we must diagonalise it

$$m_{\tilde\phi}^2 = -a^2\mu^2 + \frac{\lambda}{2}v_{\tilde\phi}^2$$

$$m_{\tilde\chi}^2 = -\mu^2(b^2+ac)v_{\tilde\phi}^2\chi^2 + \frac{v^2\mu^2}{2}(2a^3c+6a^2b^2) - \tilde V''(v_{\tilde\chi})$$

$$\alpha_{\tilde\phi\tilde\chi} = \mu^2 ab v_{\tilde\phi}.$$

$$M = \begin{pmatrix} m_{\tilde\phi}^2 & \alpha_{\tilde\phi\tilde\chi} \\ \alpha_{\tilde\phi\tilde\chi} & m_{\tilde\chi}^2 \end{pmatrix}$$

Screening mechanisms:

What about other physics? Do they get modified?

$$\tilde{A}(\tilde{\chi} + v_{\tilde{\chi}}) = a + b\chi + \frac{c}{2}\chi^2 + \cdots$$

The masses of the diagonalised modes is given by

$$m_h^2 = 2\mu^2 a^2,$$
$$m_\sigma^2 = \tilde{V}''(v_{\tilde{\chi}}) + \frac{c}{a}\rho_\psi$$

$$m_\sigma^2 = V''_{\text{eff}}(v_{\tilde{\chi}})$$

$$V_{\text{eff}}(\tilde{\chi}) = V(\tilde{\chi}) + \tilde{A}'(\tilde{\chi})\rho_\psi / \tilde{A}(\tilde{\chi})$$

And the coupling to fermions

$$\mathcal{L} \supset -yv\tilde{A}'(v_{\tilde{\chi}})\bar{\psi}\sigma\psi.$$

$$\mathcal{L} \supset -\frac{\tilde{A}'(v_{\tilde{\chi}})}{\tilde{A}(v_{\tilde{\chi}})}\sigma\rho_\psi$$

Which agrees with the standard result!

Short recap:

-The scale-breaking of the Higgs introduces new couplings to the new scalar

-This new scalar will be sensitive to the local background's density

-While this dependence is used to screen the field, it is also able to rescale the Higgs vev!

$$\boxed{\begin{aligned} v_{\tilde{\phi}}^2 &= \frac{v^2}{2}\tilde{A}^2(v_{\tilde{\chi}})\left(1+\sqrt{1-\frac{4\rho_\psi}{v^2\mu^2\tilde{A}^4(v_{\tilde{\chi}})}}\right) \\ \tilde{V}'(v_{\tilde{\chi}}) &= -\frac{\tilde{A}'(v_{\tilde{\chi}})}{\tilde{A}(v_{\tilde{\chi}})}\rho_\psi, \end{aligned}}$$

Model dependence!

$$m_\psi = yv_{\tilde{\phi}} \approx y\sqrt{\frac{6\mu^2}{\lambda}}\tilde{A}(v_{\tilde{\chi}})$$

$$\mathcal{L} \supset -yv\tilde{A}'(v_{\tilde{\chi}})\bar{\psi}\sigma\psi$$

$$m_\sigma^2 = \tilde{V}''(v_{\tilde{\chi}}) + \frac{c}{a}\rho_\psi$$

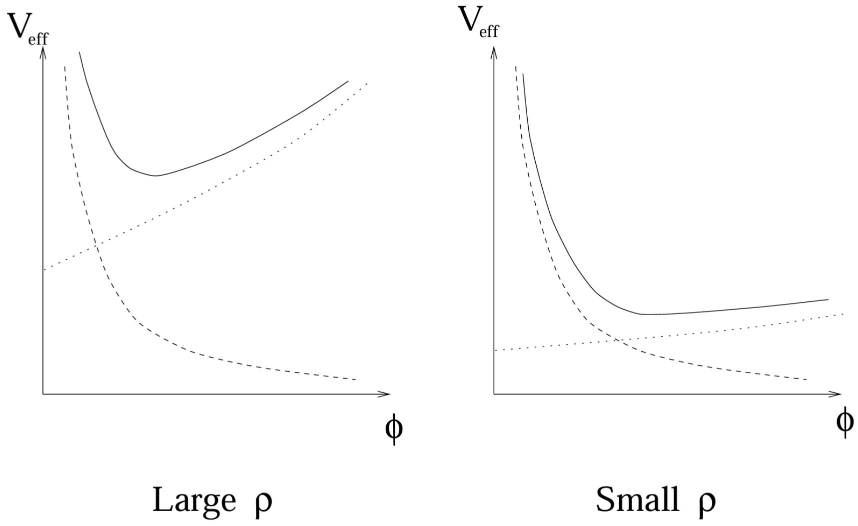
Implications:

Chameleon: It increases the propagator's mass with the background's density

$$\tilde{A}(\tilde{\chi}) = e^{\frac{\tilde{\chi}}{M}}, \quad \tilde{V}(\tilde{\chi}) = V_0 e^{\frac{-n\tilde{\chi}}{M}},$$

$$\boxed{\frac{V_0 n}{M} e^{\frac{-n v_{\tilde{\chi}}}{M}} = \frac{1}{M} e^{\frac{v_{\tilde{\chi}}}{M}} \rho,}$$

$$\frac{v_{\tilde{\chi}}(\rho_\psi)}{M} = (1+n)\log\left(\frac{n V_0}{\rho}\right)$$



Khoury and Weltman, arXiv:astro-ph/0309411

Implications:

Chameleon: It increases the propagator's mass with the background's density

$$\boxed{m = yv\tilde{A}(v_{\tilde{\chi}}) = m_0 \left(\frac{nV_0}{\rho}\right)^{1+n}}$$

Is not M suppressed

$$m_\sigma^2 = V''(\tilde{\chi}) + \frac{\tilde{A}''(v_{\tilde{\chi}})}{\tilde{A}(v_{\tilde{\chi}})}\rho_\psi = V_0 \left(\frac{\rho}{nV_0}\right)^{n(1+n)} \frac{n^2}{M^2} + \frac{1}{M^2}(nV_0)^{1-n}\rho^{-n}$$

$$\mathcal{L} \supset -yv\tilde{A}'(v_{\tilde{\chi}})\sigma\bar{\tilde{\psi}}\tilde{\psi} = -\frac{yv}{M}e^{v_{\tilde{\chi}}/M}\sigma\bar{\tilde{\psi}}\tilde{\psi}.$$

Implications:

Chameleon: It increases the propagator's mass with the background's density

Let's say that we have a galaxy shaped structure

$$\rho_{\text{disk},0}(r,z) = \frac{\Sigma_{\text{disk}}}{2z_{\text{disk}}} e^{-r/r_{\text{disk}}} e^{-|z|/z_{\text{disk}}}$$

That gets rescaled into

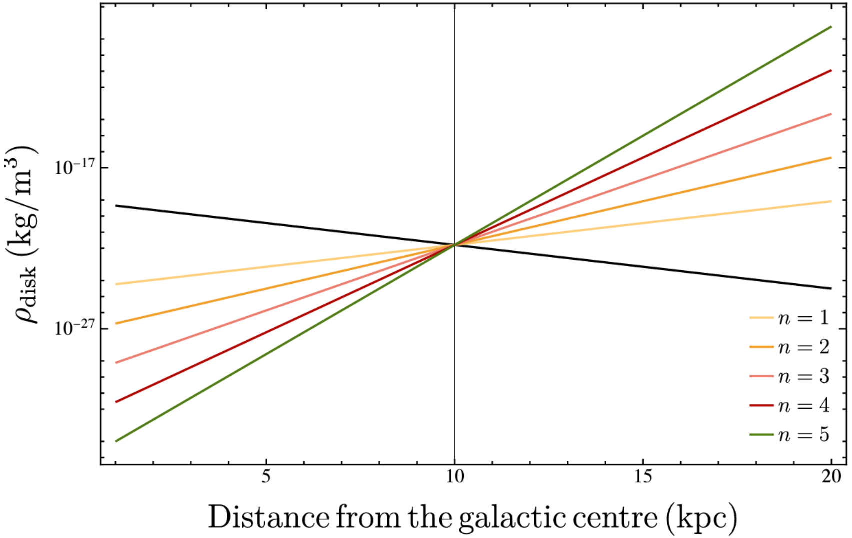
$$\rho_{\text{disk}}(r,z) = \rho_{\text{disk},0}(r,z)\tilde{A}(v_{\tilde{\chi}}(\rho_{\text{disk},0}(r,z))$$

*Toy model, not real computation

Implications:

Chameleon: It increases the propagator's mass with the background's density

$$m = yv\tilde{A}(v_{\tilde{\chi}}) = m_0 \left(\frac{nV_0}{\rho}\right)^{1+n}$$



Very sensitive to local density:

Higher density leads to smaller fifth forces, but the effects on particles become significant.

I call it over-screening

*Toy model, not real computation

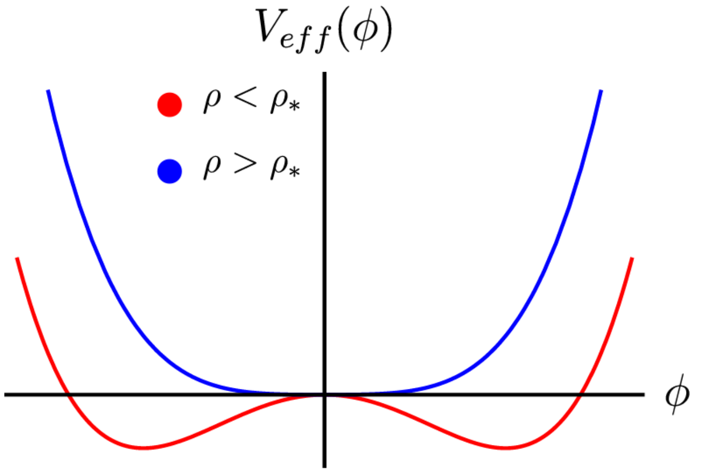
Implications:

Symmetron: It decreases the coupling strength with the background's density

$$\tilde{V}(\tilde{\chi}) = -\frac{1}{2}\mu_\chi^2\tilde{\chi}^2 + \frac{\lambda_\chi}{4!}\tilde{\chi}^4 \qquad \tilde{A}(\tilde{\chi}) = 1 + \frac{\tilde{\chi}^2}{2M_{\rm sym}^2}$$

$$\boxed{V_{\rm eff}(\tilde{\chi}) = \frac{1}{2}\left(\frac{\rho}{M_{\rm sym}^2} - \mu_\chi^2\right)\tilde{\chi}^2 + \frac{\lambda_\chi}{4!}\tilde{\chi}^4}$$

$$\mathcal{L} \supset -yv\tilde{A}'(v_{\tilde{\chi}})\sigma\bar{\tilde{\psi}}\tilde{\psi} = -yv\frac{v_{\tilde{\chi}}}{M_{\rm sym}^2}\sigma\bar{\tilde{\psi}}\tilde{\psi}$$



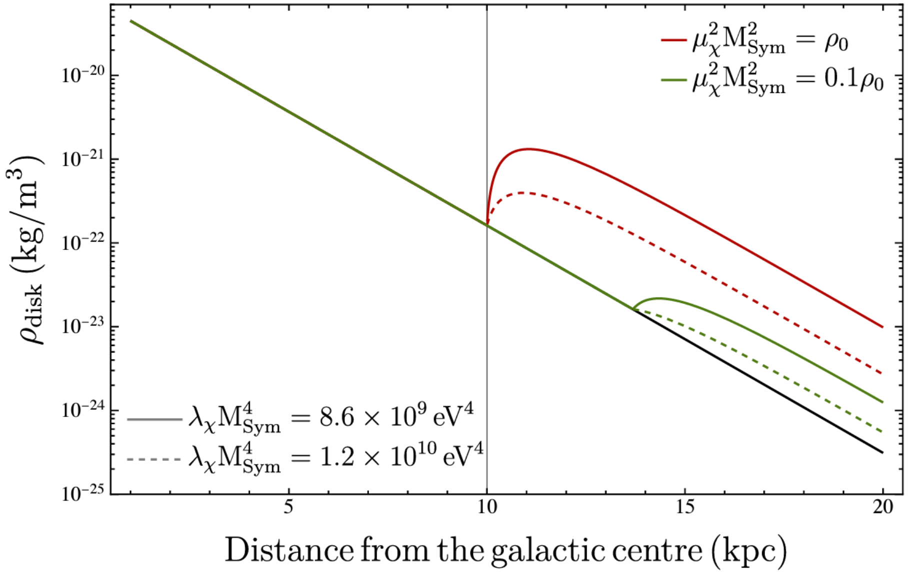
Burrage and Sakstein. arXiv:1709.09071

Implications:

Symmetron: It decreases the coupling strength with the background's density

$$v_{\tilde{\chi}}^2 = \begin{cases} \frac{6}{\lambda_\chi}\left(\mu_\chi^2 - \frac{\rho}{M_{\rm sym}^2}\right) & \text{for } \rho < \mu_\chi^2 M_{\rm sym}^2 \\ 0 & \text{for } \rho > \mu_\chi^2 M_{\rm sym}^2 \end{cases}$$

$$m = yv\tilde{A}(v_{\tilde{\chi}}) = m_0\left(1 + \frac{v_{\tilde{\chi}}^2}{\tilde{M}_{\rm pl}^2}\right)$$

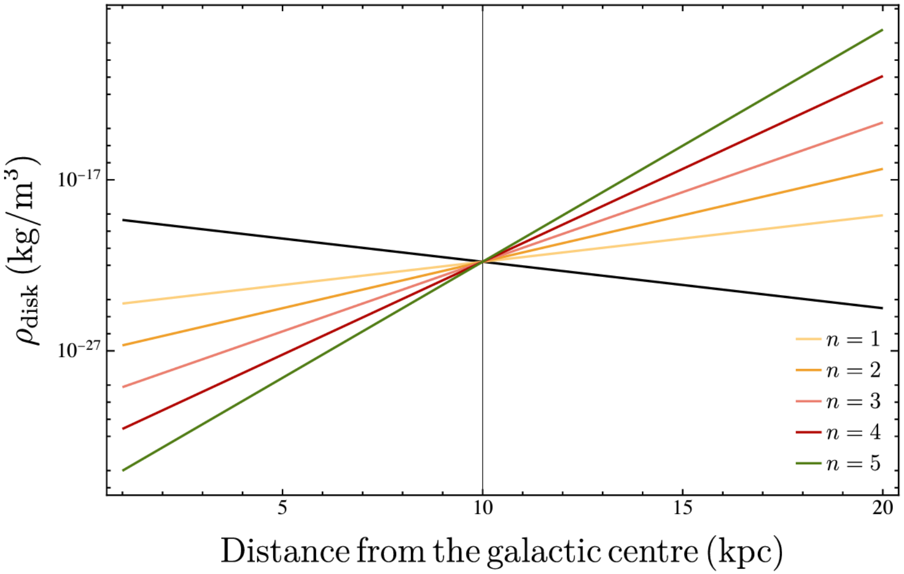


Mass variance is limited:

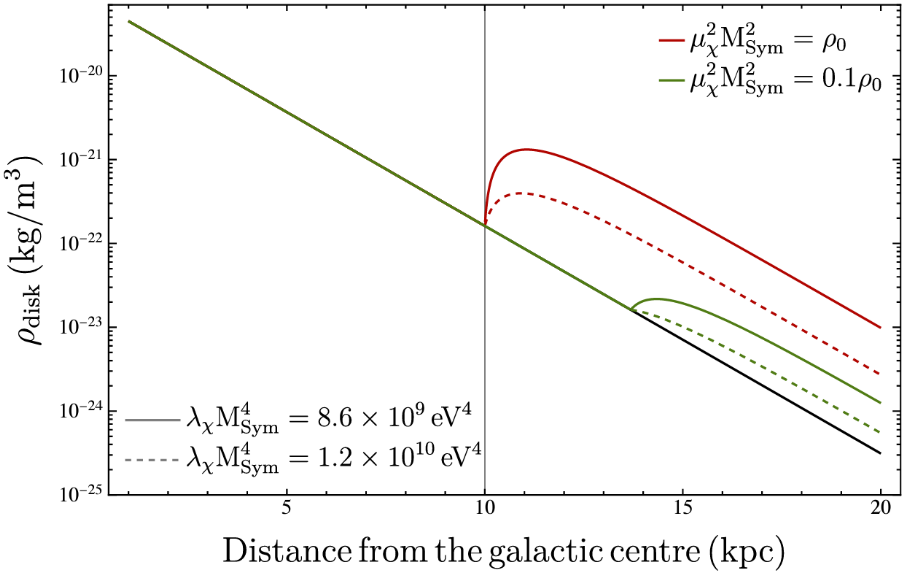
Due to the Z2 symmetry of the potential with its stable minimum in the broken phase.

The symmetron does not over-screen

Chameleon

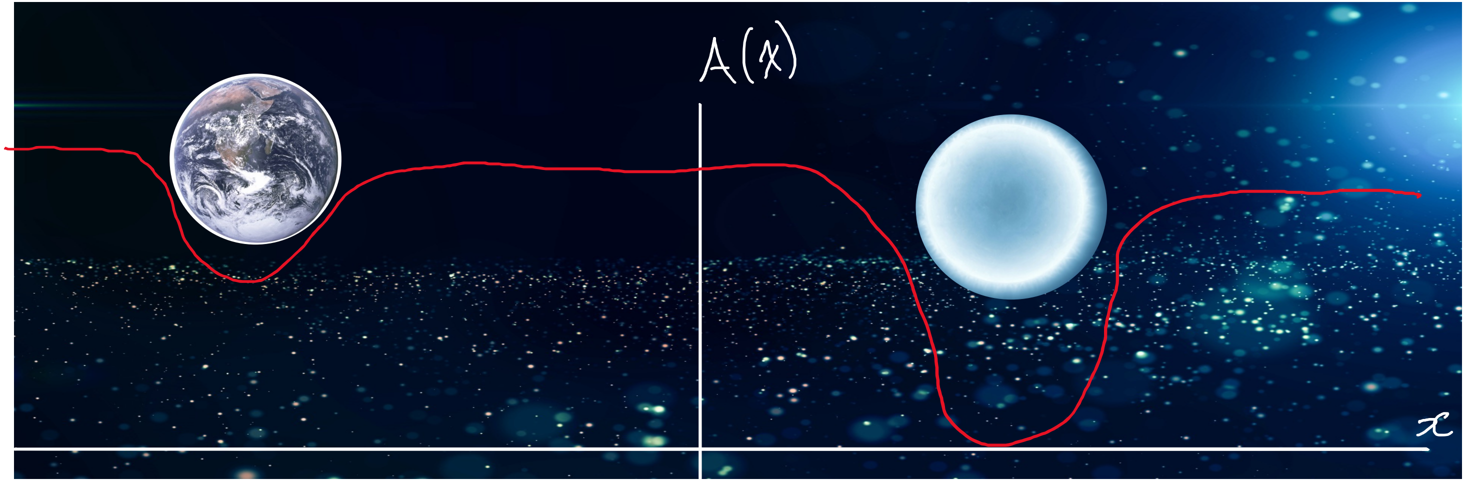


Symmetron



This effect remains independently on the field's mass or coupling strength to matter

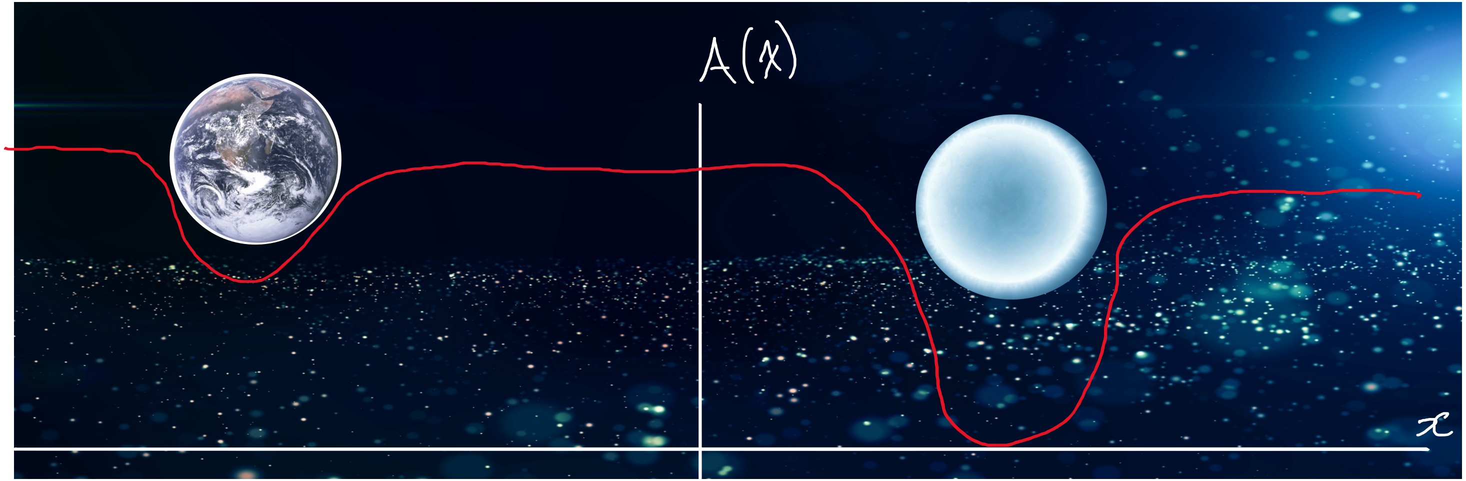
Future plans:



Calculate the real profile for the new scalar field around astrophysical objects and find observables (emitted spectroscopy, stability of other objects…)

This effect does not need to be Planck suppressed (as chameleon case). Still, densities in the universe are very inhomogeneous!

Future plans:



5.51 g/cm³ 10^{-29} g/cm³ 10^{6} g/cm³

Calculate the real profile for the new scalar field around astrophysical objects and find observables (emitted spectroscopy, stability of other objects…)

This effect does not need to be Planck suppressed (as chameleon case). Still, densities in the universe are very inhomogeneous!

Thank you for your attention

Any questions?

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E. Copeland, P. Millington and SSM (2111.06357)

Multi–fermion case

Let's take a Lagrangian with multiple mass formation mechanisms

In the Jordan frame:

$$S_{\rm m}[g_{\mu\nu}] = \int {\rm d}x^4 \sqrt{-g}\Bigg[\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \\ + \left(\sum_\alpha^N i\bar{\psi}_\alpha\gamma^\mu\nabla_\mu\psi_\alpha - y_\alpha\bar{\psi}_\alpha\phi\psi_\alpha\right) \\ + \left(\sum_\beta^M i\bar{\tau}_\beta\gamma^\mu\nabla_\mu\tau_\beta - m_{\beta,0}\bar{\tau}_\beta\tau_\beta\right) \\ -U(\varphi) + \frac{1}{2}\mu^2\phi^2 - \frac{\lambda}{4!}\phi^4 - \frac{3\mu^4}{2\lambda}\Bigg],$$

Multi-fermion case

Let's take a Lagrangian with multiple mass formation mechanisms

In the Einstein frame

$$S_{\mathrm{m}}[\tilde{g}_{\mu\nu}] = \int \mathrm{d}x^4 \sqrt{-\tilde{g}} \Bigg[\frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \chi \partial_\nu \chi + A^2(\chi) \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$
$$+ \left(\sum_\alpha^N i A^3(\chi) \bar{\psi}_\alpha \tilde{e}^\mu_a \gamma^a \tilde{\nabla}_\mu \psi_\alpha - y_\alpha A^4(\chi) \bar{\psi}_\alpha \phi \psi_\alpha \right)$$
$$+ \left(\sum_\beta^M i A^3(\chi) \bar{\tau}_\beta \tilde{e}^\mu_a \gamma^a \tilde{\nabla}_\mu \tau_\beta - m_{\beta,0} A^4(\chi) \bar{\tau}_\beta \tau_\beta \right)$$
$$- V(\chi) + A^4(\chi) \left(\frac{1}{2} \mu^2 \phi^2 - \frac{\lambda}{4!} \phi^4 - \frac{3\mu^4}{2\lambda} \right) \Bigg],$$

Multi-fermion case

Rescaling the fields as before:

$$\phi \to A^{-1}(\chi)\tilde{\phi}, \qquad \psi_i \to A^{-3/2}(\chi)\tilde{\psi}_i, \qquad \tau_i \to A^{-3/2}(\chi)\tilde{\tau}_i,$$

$$\begin{aligned} S_{\mathrm{m}}[\tilde{g}_{\mu\nu}] = \int \mathrm{d}x^4 \sqrt{-\tilde{g}}\Bigg[&\frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\tilde{\chi}\partial_\nu\tilde{\chi} + \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\tilde{\phi}\partial_\nu\tilde{\phi} - \frac{\tilde{A}'(\tilde{\chi})}{\tilde{A}(\tilde{\chi})}\tilde{\phi}\tilde{g}^{\mu\nu}\partial_\mu\tilde{\chi}\partial_\nu\tilde{\phi} \\ &+ \left(\sum_\alpha^N i\bar{\tilde{\psi}}_\alpha \tilde{e}^\mu_a \gamma^a \tilde{\nabla}_\mu \tilde{\psi}_\alpha - y_\alpha \bar{\tilde{\psi}}_\alpha \phi \tilde{\psi}_\alpha\right) \\ &+ \left(\sum_\beta^M i\bar{\tilde{\tau}}_\beta \tilde{e}^\mu_a \gamma^a \tilde{\nabla}_\mu \tilde{\tau}_\beta - m_{\beta,0}\tilde{A}(\tilde{\chi})\bar{\tilde{\tau}}_\beta \tilde{\tau}_\beta\right) \\ &- \tilde{V}(\tilde{\chi}) + \left(\frac{1}{2}\mu^2\tilde{A}(\tilde{\chi})^2\tilde{\phi}^2 - \frac{\lambda}{4!}\tilde{\phi}^4 - \frac{3\mu^4}{2\lambda}\tilde{A}(\tilde{\chi})^4\right)\Bigg], \end{aligned}$$

Multi–fermion case

As earlier, we just need to calculate the vev of the fields, including the background's density by

$$\rho_\psi = \sum_\alpha^N y_\alpha v_{\tilde{\phi}} \bar{\tilde{\psi}}_\alpha \tilde{\psi}_\alpha \qquad \rho_\tau = \sum_\beta^M m_{\beta,0} \bar{\tilde{\tau}}_\beta \tilde{\tau}_\beta$$

$$\frac{\lambda}{6} v_{\tilde{\phi}}^4 - \mu^2 v_{\tilde{\phi}}^2 A^2(v_{\tilde{\chi}}) + \rho_\psi = 0,$$

$$\tilde{A}'(v_{\tilde{\chi}}) \tilde{A}^3(v_{\tilde{\chi}}) \frac{6\mu^4}{\lambda} - \mu^2 v_{\tilde{\phi}}^2 \tilde{A}'(v_{\tilde{\chi}}) \tilde{A}(v_{\tilde{\chi}}) + \tilde{V}'(v_{\tilde{\chi}}) + \frac{\tilde{A}'(v_{\tilde{\chi}})}{\tilde{A}(v_{\tilde{\chi}})} \rho_\tau = 0.$$

Multi-fermion case

As earlier, we just need to calculate the vev of the fields, including the background's density by

$$\rho_\psi = \sum_\alpha^N y_\alpha v_{\tilde{\phi}} \bar{\tilde{\psi}}_\alpha \tilde{\psi}_\alpha \qquad \rho_\tau = \sum_\beta^M m_{\beta,0} \bar{\tilde{\tau}}_\beta \tilde{\tau}_\beta$$

$$\frac{\lambda}{6} v_{\tilde{\phi}}^4 - \mu^2 v_{\tilde{\phi}}^2 A^2(v_{\tilde{\chi}}) + \rho_\psi = 0,$$

$$\tilde{A}'(v_{\tilde{\chi}}) \tilde{A}^3(v_{\tilde{\chi}}) \frac{6\mu^4}{\lambda} - \mu^2 v_{\tilde{\phi}}^2 \tilde{A}'(v_{\tilde{\chi}}) \tilde{A}(v_{\tilde{\chi}}) + \tilde{V}'(v_{\tilde{\chi}}) \boxed{+ \frac{\tilde{A}'(v_{\tilde{\chi}})}{\tilde{A}(v_{\tilde{\chi}})} \rho_\tau} = 0.$$

Multi-fermion case

As earlier, we just need to calculate the vev of the fields, including the background's density by

$$v_{\tilde{\phi}}^2 = \frac{v^2}{2}\tilde{A}^2(v_{\tilde{\chi}})\left(1+\sqrt{1-\frac{4\rho_\psi}{v^2\mu^2\tilde{A}^4(v_{\tilde{\chi}})}}\right)$$

$$\tilde{V}'(v_{\tilde{\chi}}) = -\frac{\tilde{A}'(v_{\tilde{\chi}})}{\tilde{A}(v_{\tilde{\chi}})}(\rho_\psi+\rho_T),$$

All sources of scale breaking contribute equally

Multi-fermion case

Moreover, recalling the Lagrangian

$$+\left(\sum_{\alpha}^{N} i\bar{\tilde{\psi}}_\alpha \tilde{e}_a^\mu \gamma^a \tilde{\nabla}_\mu \tilde{\psi}_\alpha - y_\alpha \bar{\tilde{\psi}}_\alpha \phi \tilde{\psi}_\alpha\right)$$
$$+\left(\sum_{\beta}^{M} i\bar{\tilde{\tau}}_\beta \tilde{e}_a^\mu \gamma^a \tilde{\nabla}_\mu \tilde{\tau}_\beta - m_{\beta,0}\tilde{A}(\tilde{\chi})\bar{\tilde{\tau}}_\beta \tilde{\tau}_\beta\right)$$

We find the effective masses to be

$$m_\alpha = y_\alpha v_{\tilde{\phi}} = y_\alpha v \tilde{A}(v_{\tilde{\chi}})$$
$$m_\beta = m_{\beta,0}\tilde{A}(v_{\tilde{\chi}}),$$

All sources of scale breaking get also rescaled equally