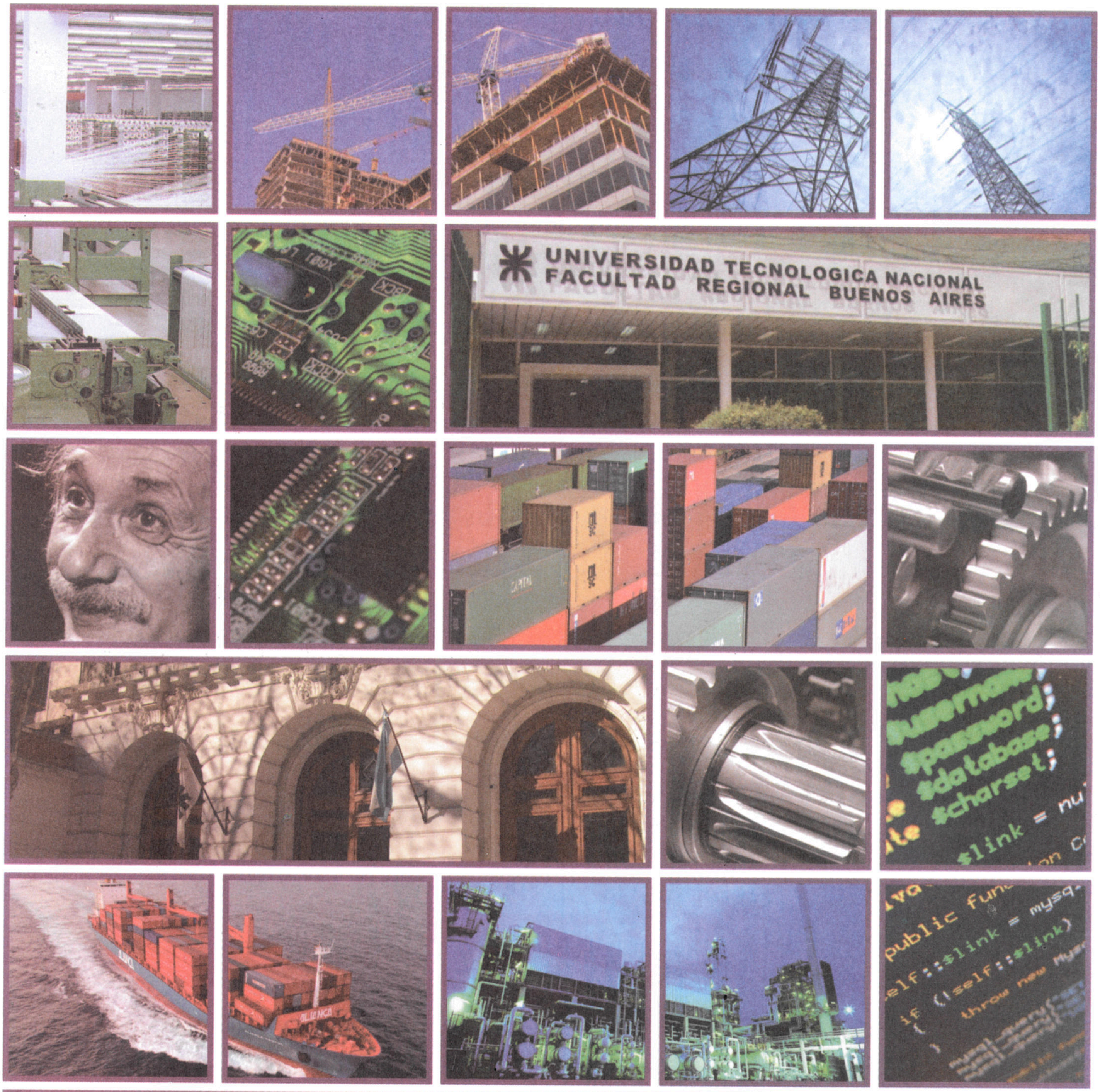

TÍTULO	# TABLA DE DERIVADAS E INTEGRALES
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NUMEROS NOTABLES

$\pi = 3{,}1415\ldots$ $e = 2{,}718281\ldots$

$\sqrt{2} = 1{,}414213\ldots$ $\sqrt{3} = 1{,}73205\ldots$

$e^{\pi} = 23{,}14069\ldots$ $\pi^{e} = 22{,}45915\ldots$

$\sqrt{\pi} = 1{,}77245\ldots = \Gamma(\frac{1}{2})$ $(\Gamma : \textit{función Gama})$ $\sqrt{e} = 1{,}64872\ldots$

$\Gamma(\frac{1}{3}) = 2{,}67893\ldots$

$\gamma = 0{,}57721566\ldots$ $(\textit{constante de Euler})$

$1\,radián = \frac{180°}{\pi} = 57{,}29577\ldots°$

$1° = \frac{\pi}{180}\,radianes = 0{,}01745\ldots radianes$

$$números\,de\,Euler\,(E_x) = \frac{\pi^{2k+1}}{2^{2k+2}(2k)t}E_x$$

$$números\,de\,Bernoulli\,(B_x) = \frac{\pi^{2k}(2^{2k}-1)}{2(2k)t}B_x$$

FUNCIONES TRIGONOMETRICAS CIRCULARES

RELACIONES ENTRE FUNCIONES TRIGONOMETRICAS

1) $\operatorname{sen}^2 x + \cos^2 x = 1$
2) $\operatorname{tg} x = \frac{\operatorname{sen} x}{\cos x}$
3) $\operatorname{cotg} x = \frac{\cos x}{\operatorname{sen} x}$
4) $\sec x = \frac{1}{\cos x}$
5) $\operatorname{cosec} x = \frac{1}{\operatorname{sen} x}$
6) $1 + \operatorname{tg}^2 x = \sec^2 x$
7) $1 + \operatorname{cotg}^2 x = \operatorname{cosec}^2 x$

FUNCIONES DE LA SUMA O DIFERENCIA DE ANGULOS

1) $\operatorname{sen}(x \pm y) = \operatorname{sen} x \cos y \pm \operatorname{sen} y \cos x$
2) $\cos(x \pm y) = \cos x \cos y \mp \operatorname{sen} x \operatorname{sen} y$
3) $\operatorname{tg}(x \pm y) = \frac{\operatorname{tg} x \pm \operatorname{tg} y}{1 \mp \operatorname{tg} x \operatorname{tg} y}$

FUNCIONES DEL DUPLO DEL ANGULO

1) $\operatorname{sen} 2x = 2 \operatorname{sen} x \cos x$
2) $\cos 2x = \cos^2 x - \operatorname{sen}^2 x = 1 - 2\operatorname{sen}^2 x = 2\cos^2 x - 1$
3) $\operatorname{tg} 2x = \frac{2 \operatorname{tg} x}{1 - \operatorname{tg}^2 x}$

FUNCIONES DEL ANGULO MITAD

1) $\operatorname{sen}(\frac{x}{2}) = \pm\sqrt{\frac{1-\cos x}{2}}$
2) $\cos(\frac{x}{2}) = \pm\sqrt{\frac{1+\cos x}{2}}$
3) $\operatorname{tg}(\frac{x}{2}) = \pm\sqrt{\frac{1-\cos x}{1+\cos x}} = \frac{\operatorname{sen} x}{1-\cos x} = \frac{1-\cos x}{\operatorname{sen} x} = \operatorname{cosec} x - \operatorname{cotg} x$

FUNCIONES POTENCIA

1) $\operatorname{sen}^2 x = \frac{1}{2}(1 - \cos 2x)$
2) $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

SUMA, DIFERENCIA Y PRODUCTO DE FUNCIONES

1) $\operatorname{sen} x + \operatorname{sen} y = 2\operatorname{sen}\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

2) $\operatorname{sen}\ x - \operatorname{sen}\ y = 2\cos\left(\frac{x+y}{2}\right)\operatorname{sen}\left(\frac{x-y}{2}\right)$

3) $\cos\ x + \cos\ y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

4) $\cos\ x - \cos\ y = -2\operatorname{sen}\left(\frac{x+y}{2}\right)\operatorname{sen}\left(\frac{x-y}{2}\right)$

5) $2\operatorname{sen} x \operatorname{sen} y = \cos(x-y) - \cos(x+y)$

6) $2\cos x \cos y = \cos(x-y) + \cos(x+y)$

7) $2\operatorname{sen} x \operatorname{sen} y = \operatorname{sen}(x-y) + \operatorname{sen}(x+y)$

FUNCIONES TRIGONOMETRICAS HIPERBOLICAS

1) $\operatorname{senh}\ x = \frac{e^{x}-e^{-x}}{2}$

2) $\cosh\ x = \frac{e^{x}+e^{-x}}{2}$

3) $\operatorname{tgh}\ x = \frac{\operatorname{senh}}{\cosh} = \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$

4) $\cot gh\ x = \frac{\cosh\ x}{\operatorname{senh}\ x} = \frac{e^{x}+e^{-x}}{e^{x}-e^{-x}}$

5) $\sec h\ x = \frac{1}{\cosh\ x}$

6) $\cos ec\ x = \frac{1}{\operatorname{senh}\ x}$

RELACIONES FUNDAMENTALES

1) $\cosh^2 x - \operatorname{senh}^2 x = 1$

2) $\operatorname{tgh}^2 x + \sec h^2 x = 1$

3) $\cot gh^2 x - \cos ech^2 x = 1$

FUNCIONES DE LA SUMA Y DIFERENCIA DE ANGULOS

1) $\operatorname{senh}(x \pm y) = \operatorname{senh} x \cosh y \pm \operatorname{senh} y \cosh x$

2) $\cosh(x \pm y) = \cosh x \cosh y \pm \operatorname{senh} x \operatorname{senh} y$

3) $\operatorname{tgh}(\ x \pm y\ ) = \frac{\operatorname{tgh}\ \pm \operatorname{tgh}\ y}{1 \pm \operatorname{tgh}\ x\ \operatorname{tgh}\ y}$

FUNCIONES DEL DUPLO DEL ANGULO

1) $\operatorname{senh} 2x = 2\operatorname{senh} x \cosh x$

2) $\cosh 2x = \cosh^2 x + \operatorname{senh}^2 x = 1 + 2\operatorname{senh}^2 x = 2\cosh^2 x - 1$

3) $\operatorname{tgh}\ 2x = \frac{2\operatorname{tgh}\ x}{1-\operatorname{tgh}^2 x}$

FUNCIONES DEL ANGULO MITAD

1) $\operatorname{senh}(\ \frac{x}{2}) = \pm\sqrt{\frac{\cosh\ x-1}{2}} \begin{cases} si\ x > 0, vale\ signo\ + \\ si\ x < 0, vale\ signo\ - \end{cases}$

2) $\cosh(\ \frac{x}{2}) = +\sqrt{\frac{1+\cosh\ x}{2}}$

3) $\operatorname{tgh}\left(\frac{x}{2}\right) = \pm\sqrt{\frac{\cosh\ x-1}{\cosh\ +1}} = \frac{\operatorname{senh}\ x}{1+\cosh\ x} = \frac{\cosh\ x-1}{\operatorname{senh}\ x}$

RELACION ENTRE LOS ARGUMENTOS HIPERBOLICOS Y LOGARITMICOS

1) $\operatorname{senh}^{-1} x = \ln\left(x + \sqrt{1+x^2}\right) \quad \forall x$

2) $\operatorname{tgh}^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right); \quad |x| < 1$

3) $\sec h^{-1} x = \ln\left(\frac{1}{x} + \sqrt{\frac{1}{x^2}-1}\right); \quad 0 < x \leq 1$

4) $\cos h^{-1} x = \ln\left(x + \sqrt{x^2-1}\right) \quad x \geq 1$

5) $\cot gh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{x-1}\right); \quad |x| > 1$

6) $\operatorname{cosech}^{-1} x = \ln\left(\frac{1}{x} + \sqrt{\frac{1}{x^2}+1}\right); \quad x \neq 0$

TABLA DE DERIVADAS

REGLAS DE DERIVACION

- Las funciones u, v y w son derivables en x.
- k, r, a y n son constantes reales.
- x es variable independiente.

a) Regla de la cadena $\frac{d}{dx} y = \frac{d}{du} y \cdot \frac{d}{dx} u$

b) $\frac{d}{dx} y = \dfrac{1}{\frac{d}{dy} x}$ c) $\dfrac{d}{dx} y = \dfrac{\frac{d}{du} y}{\frac{d}{du} x}$

FUNCION	DERIVADA
k	0
x	1
k x	k
ku	$k \frac{d}{dx} u$
u^r	$r\, u^{r-1} \frac{d}{dx} u$
u+v-w	$\frac{d}{dx} u + \frac{d}{dx} v - \frac{d}{dx} w$
uv	$\frac{d}{dx} u \cdot v + u \cdot \frac{d}{dx} v$
uvw	$\frac{d}{dx} u \cdot v \cdot w + u \cdot \frac{d}{dx} v \cdot w + u \cdot v \cdot \frac{d}{dx} w$
u/v	$\dfrac{\frac{d}{dx} u \cdot v - u \cdot \frac{d}{dx} v}{v^2}; \quad v \neq 0$
ln x	$1/x$
ln u	$\frac{1}{u} \cdot \frac{d}{dx} u$
$\log_a$ u	$\frac{1}{u \ln a} \frac{d}{dx} u \quad u > 0, a > 0, a \neq 1$
e^x	e^x
a^u	$a^u \ln a \frac{d}{dx} u$
e^u	$e^u \frac{d}{dx} u$
u^v	$u^v\left(\frac{d}{dx} v \ln u + \frac{v}{u} \frac{d}{dx} u\right); \quad u > 0$
sen u	$\cos u \frac{d}{dx} u$ u
cos u	$-\operatorname{sen} u \frac{d}{dx} u$
tg u	$\sec^2 u \frac{d}{dx} u$
cotg u	$-\operatorname{cosec}^2 u \frac{d}{dx} u$
sec u	$\sec u \operatorname{tg} u \frac{d}{dx} u$
cosec u	$-\operatorname{cosec} u \operatorname{cotg} u \frac{d}{dx} u$
senh u	$\cosh u \frac{d}{dx} u$
cosh u	$\operatorname{senh} u \frac{d}{dx} u$

tgh u	$sech^2 u \frac{d}{dx} u$
cotgh u	$-\operatorname{cosec}^2 u \frac{d}{dx} u$
sech u	$-\operatorname{sech} u \operatorname{tgh} u \frac{d}{dx} u$
cosech u	$-\operatorname{cosech} u \operatorname{cotg} u \frac{d}{dx} u$
$\operatorname{sen}^{-1} u (\operatorname{arcsen} u)$	$\frac{1}{\sqrt{1-u^2}} \cdot \frac{d}{dx} u$
$\cos^{-1} u (\arccos u)$	$-\frac{1}{\sqrt{1-u^2}} \cdot \frac{d}{dx} u$
$\operatorname{tg}^{-1} u (\operatorname{arctg} u)$	$\frac{1}{1+u^2} \cdot \frac{d}{dx} u$
$\operatorname{cotg}^{-1} u (\operatorname{arccotg} u)$	$-\frac{1}{1+u^2} \cdot \frac{d}{dx} u$
$\sec^{-1} u (\operatorname{arcsec} u)$	$\frac{1}{u\sqrt{u^2-1}} \cdot \frac{d}{dx} u$
$\operatorname{cosec}^{-1} u (\operatorname{arccos} ec\ u)$	$-\frac{1}{u\sqrt{u^2-1}} \cdot \frac{d}{dx} u$
$\operatorname{senh}^{1} u (\operatorname{arcsenh} u)$	$\frac{1}{\sqrt{u^2+1}} \frac{d}{dx} u$
$\cosh^{-1} u (\operatorname{arccosh} u)$	$\frac{1}{\sqrt{u^2-1}} \frac{d}{dx} u$
$\operatorname{tgh}^{-1} u (\operatorname{arctgh} u)$	$\frac{1}{1-u^2} \frac{d}{dx} u$
$\operatorname{cotgh}^{-1} u (\operatorname{arccotgh} u)$	$\frac{1}{1-u^2} \frac{d}{dx} u$
$\operatorname{sech}^{-1} u (\operatorname{arcsech} u)$	$-\frac{1}{u\sqrt{1-u^2}} \frac{d}{dx} u$
$\operatorname{cosech}^{-1} u (\operatorname{arcosech}^{-1} u)$	$-\frac{1}{u\sqrt{1+u^2}} \frac{d}{dx} u$

TABLA DE INTEGRALES

INTEGRALES INDEFINIDAS

REGLAS PARA UNA INTEGRACION

* Las f, u, v y w son funciones de x.

* a, b, q, r y n son constantes, r es real y n es natural.

1. $\int a\,dx = ax$
2. $\int a\,f(x)dx = a\int f(x)dx$
3. $\int (u \pm v \pm w \pm \cdots)\,dx = \int u\,dx = \pm \int v\,dx \pm \int w\,dx \pm \cdots$
4. $\int u\,dv = u\,v - \int v\,du$ *Integración por partes*
5. $\int f(ax)dx = \frac{1}{a}\int f(u)du$ *Cambio de variable* $u = ax$
6. $\int F\{f(x)\}\,dx = \int F(u)\frac{dx}{du} = \int \frac{F(u)}{f(x)}du$
7. $\int x^r dx = \frac{x^{r+1}}{r+1}$; *Con* $r \neq -1$. *Para* $r = -1$ *ver* 8
8. $\int \frac{1}{x}dx = \ln|x| = \begin{cases} \ln x \; si \; x > 0 \\ \ln(-x) \; si \; x < 0 \end{cases};\ x \neq 0$
9. $\int e^x dx = e^x$

10. $\int a^x dx = \frac{a^x}{\ln a} = a^x \log_a e \quad Para\, a > 0\, y\, a \neq 1$

11. $\int \operatorname{sen} x\, dx = -\cos x$

12. $\int \cos x\, dx = \operatorname{sen} x$

13. $\int \operatorname{tg} x\, dx = \ln \sec x = -\ln \cos x$

14. $\int \operatorname{cotg} dx = \ln \operatorname{sen} x$

15. $\int \sec x\, dx = \ln(\sec x + \operatorname{tg} x) = \ln \operatorname{tg}(\frac{x}{2} + \frac{\pi}{2})$

16. $\int cosec\, dx = \ln(cosec x - \operatorname{cotg} x) = \ln \operatorname{tg} \frac{x}{2}$

17. $\int \sec^2 dx = \operatorname{tg} x$

18. $\int cosec^2 dx = -\operatorname{cotg} x$

19. $\int \operatorname{tg}^2 x\, dx = \operatorname{tg} x - x$

20. $\int \operatorname{cotg}^2 x\, dx = -\operatorname{cotg} x - x$

21. $\int \operatorname{sen}^2 x\, dx = \frac{x}{2} - \frac{\operatorname{sen} 2x}{4} = \frac{1}{2}(x - \operatorname{sen} x \cos x)$

22. $\int \cos^2 x\, dx = \frac{x}{2} + \frac{\operatorname{sen} 2x}{4} = \frac{1}{2}(x + \operatorname{sen} x \cos x)$

23. $\int \sec x \operatorname{tg} x\, dx = \sec x$

24. $\int cosec\, x \operatorname{cotg} x\, dx = -cosec\, x$.

25. $\int \operatorname{senh} x\, dx = \cosh x$

26. $\int \cosh x\, dx = \operatorname{senh} x$

27. $\int \operatorname{tgh} x\, dx = \ln \cosh x$

28. $\int \operatorname{cotg}h\, x\, dx = \ln \operatorname{senh} x$

29. $\int \operatorname{sec}h\, x\, dx = \operatorname{sen}^{-1} x (\operatorname{tgh} x) \quad ó \quad 2 \operatorname{tg}^{-1} e^x$

30. $\int cosech\, x\, dx = \ln \operatorname{tgh} \frac{x}{2} \quad ó \quad -\cot gh^{-1} e^x$

31. $\int \sec h^2 x\, dx = \operatorname{tgh} x$

32. $\int cosech^2 x\, dx = -\operatorname{cot}gh\, x$

33. $\int \operatorname{tgh}^2 x\, dx = x - \operatorname{tgh} x$

34. $\int \operatorname{cot}gh^2 x\, dx = x - \operatorname{cot}gh\, x$

35. $\int \operatorname{senh}^2 x\, dx = \frac{\operatorname{senh} 2x}{4} - \frac{x}{2} = \frac{1}{2}(\operatorname{senh} x \cosh x - x)$

36. $\int \cosh^2 x dx = \frac{\operatorname{senh} 2x}{4} + \frac{x}{2} = \frac{1}{2}(\operatorname{senh} x \cosh x + x)$

37. $\int \sec h\, x \operatorname{tgh} x\, dx = -\sec hx$

38. $\int cosech\, x \operatorname{cot}ghx\, dx = -cosech\, x$

39. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\operatorname{tg}^{-1}\frac{x}{a}$

40. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left(\frac{x-a}{x+a}\right) = -\frac{1}{a}\operatorname{cotgh}^{-1}\frac{x}{a}; \quad x^2 > a^2$

41. $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left(\frac{x+a}{a-x}\right) = \frac{1}{a}\operatorname{tgh}^{-1}\frac{x}{a}; \quad x^2 < a^2$

42. $\int \frac{dx}{\sqrt{a^2-x^2}} = \operatorname{sen}^{-1}\frac{x}{a}$

43. $\int \frac{dx}{\sqrt{a^2+x^2}} = \ln(x+\sqrt{a^2+x^2})$ ó $\operatorname{senh}^{-1}\frac{x}{a}$

44. $\int \frac{dx}{\sqrt{x^2-a^2}} = \ln(x+\sqrt{x^2-a^2})$ ó $\cosh^{-1}\frac{x}{a}$

45. $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right|$

46. $\int \frac{dx}{x\sqrt{x^2+a^2}} = -\frac{1}{a}\ln\left[\frac{a+\sqrt{x^2+a^2}}{x}\right]$

47. $\int \frac{dx}{x\sqrt{a^2-x^2}} = -\frac{1}{a}\ln\left[\frac{a+\sqrt{a^2-x^2}}{x}\right]$

48. $\int f^{(n)}g\,dx = f^{(n-1)}g - f^{(n-2)}g' + f^{(n-3)}g'' \ldots + (-1)^n\int f.g^{(n)}dx$

METODO DE SUSTITUCION

49. $\int F(ax+b)dx = \frac{1}{a}\int F(u)du$ $\quad u = ax+b$

50. $\int F\left(\sqrt{ax+b}\right)dx = \frac{2}{a}\int uF(u)du$ $\quad u = \sqrt{ax+b}$

51. $\int F\left(\sqrt[n]{ax+b}\right)dx = \frac{n}{a}\int u^{n-1}F(u)du$ $\quad u = \sqrt[n]{ax+b}$

52. $\int F\left(\sqrt{a^2-x^2}\right)dx = a\int F(a.\cos u)\cos u\,du$ $\quad x = a\operatorname{sen}u$

53. $\int F\left(\sqrt{x^2+a2}\right)dx = a\int F(a\sec u)\sec^2 u\,du$ $\quad x = a\operatorname{tg}u$

54. $\int F\left(\sqrt{x^2-a^2}\right)dx = a\int F(a\sec u)\sec u\operatorname{tg}u\,du$ $\quad x = a\sec u$

55. $\int F(e^{ax})dx = \frac{1}{a}\int \frac{F(u)}{u}du$ $\quad u = e^{ax}$

56. $\int F(\ln u)dx = \int F(u)e^u du$ $\quad u = \ln u$

57. $\int F\left(\operatorname{sen}^{-1}\frac{x}{a}\right)dx = a\int F(u)\cos u\,du$ $\quad u = \operatorname{sen}^{-1}\frac{x}{a}$

Para otras funciones trigonometricas reciprocas se obtienen similares resultados

58. $\int F(\operatorname{sen}x.\cos x)dx = 2\int F\left(\frac{2u}{1+u^2}\frac{1-u^2}{1+u^2}\right)\frac{du}{1+u^2}$ $\quad u = \operatorname{tg}\frac{x}{2}$

Integrales indefinidas clasificadas por la forma

INTEGRALES CON ax + b

59. $\int \frac{dx}{ax+b} = \frac{1}{a}\ln(ax+b)$

60. $\int \frac{x\,dx}{ax+b} = \frac{x}{a} - \frac{b}{a^2}\ln(ax+b)$

61. $\int \frac{x^2dx}{ax+b} = \frac{(ax+b)^2}{2a^3} - \frac{2b(ax+b)}{a^3} + \frac{b^2}{a^3}\ln(ax+b)$

62. $\int \frac{x^3dx}{ax+b} = \frac{(ax+b)^2}{3a^4} - \frac{3b(ax+b)^2}{2a^4} + \frac{3b^2(ax+b)}{a^4} - \frac{b^3}{a^4}\ln(ax+b)$

63. $\int \frac{dx}{x(ax+b)} = \frac{1}{b}\ln\left(\frac{x}{ax+b}\right)$

64. $\int \frac{dx}{x^2(ax+b)} = -\frac{1}{bx} + \frac{a}{b^2}\ln\left(\frac{ax+b}{x}\right)$

65. $\int \frac{dx}{x^3(ax+b)} = \frac{2ax-b}{2b^2x^2} + \frac{a^2}{b^3}\ln\left(\frac{x}{ax+b}\right)$

66. $\int \frac{dx}{(ax+b)^2} = \frac{-1}{a(ax+b)}$

67. $\int \frac{xdx}{(ax+b)^2} = \frac{b}{a^2(ax+b)} + \frac{1}{a^2}\ln(ax+b)$

68. $\int \frac{x^2dx}{(ax+b)^2} = \frac{ax+b}{a^3} - \frac{b^2}{a^3(ax+b)} - \frac{2b}{a^3}\ln(ax+b))$

69. $\int \frac{x^3dx}{(ax+b)^2} = \frac{(ax+b)^2}{2a^4} - \frac{3b(ax+b)}{a^4} + \frac{b^3}{a^4(ax+b)} + \frac{3b^2}{a^4}\ln(ax+b)$

70. $\int \frac{dx}{x(ax+b)^2} = \frac{1}{b(ax+b)} + \frac{1}{b^2}\ln\left(\frac{x}{ax+b}\right)$

71. $\int \frac{dx}{x^2(ax+b)^2} = \frac{-a}{b^2(ax+b)} - \frac{1}{b^2x} + \frac{2a}{b^3}\ln\left(\frac{ax+b}{x}\right)$

72. $\int \frac{dx}{x^3(ax+b)^2} = -\frac{(ax+b)^2}{2b^4x^2} - \frac{a^3x}{b^4(ax+b)} + \frac{3a(ax+b)}{b^4x} - \frac{3a^2}{b^4}\ln\left(\frac{ax+b}{x}\right)$

73. $\int \frac{dx}{(ax+b)^3} = \frac{-1}{2(ax+b)^2}$

74. $\int \frac{xdx}{(ax+b)^3} = \frac{-1}{a^2(ax+b)} + \frac{b}{2a^2(ax+b)^2}$

75. $\int \frac{x^2dx}{(ax+b)^3} = \frac{2b}{a^3(ax+b)} - \frac{b2}{2a^3(ax+b)^2} + \frac{1}{a^3}\ln(ax+b)$

76. $\int \frac{x^3dx}{(ax+b)^3} = \frac{x}{a^3} - \frac{3b^2}{a^4(ax+b)} + \frac{b^3}{2a^4(ax+b)} - \frac{3b}{a4}\ \ln(ax+b)$

77. $\int \frac{dx}{x(ax+b)^3} = \frac{a^2x^2}{2b^3(ax+b)^2} - \frac{2ax}{b^3(ax+b)} - \frac{1}{b^3}\ln\left(\frac{ax+b}{x}\right)$

78. $\int \frac{dx}{x^2(ax+b)^3} = \frac{-a}{2b^2(ax+b)^2} - \frac{2a}{b^3(ax+b)} - \frac{1}{b^3x} + \frac{3a}{b^4}\ln\left(\frac{ax+b}{x}\right)$

79. $\int \frac{dx}{x^3(ax+b)^3} = \frac{a^4x^2}{2b^5(ax+b)^2} - \frac{4a^3x}{b^5(ax+b)} - \frac{(ax+b)^2}{2b^5x^2} - \frac{6a^2}{b^5}\ln\left(\frac{ax+b}{x}\right)$

80. $\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{(n+1)a}$ Si n = -1 véase 59

81. $\int x(ax+b)^n\,dx = \frac{(ax+b)^{n+2}}{(n+2)a^2} - \frac{b(ax+b)^{n+1}}{(n+1)a^2}$; $n \neq -1,-2.$

Si n=-1 ó –2 véase 62 ó 67, respectivamente.

82. $\int x^2(ax+b)^n\,dx = \frac{(ax+b)^{n+3}}{(n+3)a^3} - \frac{b(ax+b)^{n+2}}{(n+2)a^3} + \frac{b^2(ax+b)^{n+1}}{(n+1)a^3}; n \neq -1,-2,-3$

Si n=-1,-2 ó –3 véase 61, 68 ó 75, respectivamente.

83. $$\int x^n(ax+b)^n\,dx = \begin{cases} \frac{x^{n-1}(ax+b)^n}{m+n+1} + \frac{nb}{m+n+1}\int x^n(ax+b)^{n-1}dx \\ \frac{(x^m(ax+b)^{n+1}}{m+n+1)a} - \frac{mn}{(m+n+1)a}\int x^{m-1}(ax+b)^n\,dx \\ \frac{-x^{m+1}(ax+b)^{n+1}}{(n+1)b} + \frac{m+n+2}{(n+1)b}\int x^m(ax+b)^{n+1}dx \end{cases}$$

INTEGRALES CON $\sqrt{ax+b}$

84. $\int \frac{dx}{\sqrt{ax+b}} = \frac{2\sqrt{ax+b}}{a}$

85. $\int \frac{xdx}{\sqrt{ax+b}} = \frac{2(ax-2b)}{3a^2}\sqrt{ax+b}$

86. $\int \frac{x^2dx}{\sqrt{ax+b}} = \frac{2(3a^2x^2-4abx+8b^2)}{15a^2}\sqrt{ax+b}$

87. $\int \frac{dx}{x\sqrt{ax+b}} = \begin{cases} \frac{1}{b}\ln\left(\frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}\right) \\ \frac{2}{\sqrt{-b}}\,\mathrm{tg}^{-1}\sqrt{\frac{ax+b}{-b}} \end{cases}$ b ≠ 0

88. $\int \frac{dx}{x^2\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{bx} - \frac{a}{2b}\int \frac{dx}{x\sqrt{ax+b}}$; Véase 87 $b \neq 0$

89. $\int \sqrt{ax+b}\,.dx = \frac{2\sqrt{(ax+b)^3}}{3a}$

90. $\int x\sqrt{ax+b}\,.dx = \frac{2(3ax-2b)}{15a^2}\sqrt{(ax+b)^3}$

91. $\int x^2\sqrt{ax+b}\,.dx = \frac{2(15a^2x^2-12abx+8b^2)}{105a^3}\sqrt{(ax+b)^3}$

92. $\int \frac{\sqrt{ax+b}}{x}dx = 2\sqrt{ax+b} + b\int \frac{dx}{x\sqrt{ax+b}}$ Véase 87

93. $\int \frac{\sqrt{ax+b}}{x^2}dx = -\frac{\sqrt{ax+b}}{x} + \frac{a}{2}\int \frac{dx}{x\sqrt{ax+b}}$ Véase 87

94. $\int \frac{x^m\,dx}{\sqrt{ax+b}} = \frac{2x^m\sqrt{ax+b}}{(2m+1)a} - \frac{2mb}{(2m+1)a}\int \frac{x^{m-1}dx}{\sqrt{ax+b}}$

95. $\int \frac{dx}{x^m\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{(m-1)bx^{m-1}} - \frac{(2m-3)a}{(2m-2)b}\int \frac{dx}{x^{m-1}\sqrt{ax+b}}$; m≠ 1

96. $\int x^m\sqrt{ax+b}\,dx = \frac{2x^m}{(2m+3)a}\sqrt{(ax+b)^3} - \frac{2mb}{(2m+3)a}\int x^{m-1}\sqrt{ax+b}\,dx$

97. $\int \frac{\sqrt{ax+b}}{x^m}dx = -\frac{\sqrt{ax+b}}{(m+1)x^{m-1}} + \frac{a}{2(m-1)}\int \frac{dx}{x^{m-1}\sqrt{ax+b}}$

98. $\int \frac{\sqrt{ax+b}}{x^m}dx = -\frac{\sqrt{(ax+b)^3}}{(m-1)bx^{m-1}} - \frac{(2m-5)a}{(2m-2)b}\int \frac{\sqrt{ax+b}}{x^{m-1}}dx$

99. $\int \sqrt{(ax+b)^m}\,dx = \frac{2\sqrt{(ax+b)^{m+2}}}{a(m+2)}$

100. $\int x\sqrt{(ax+b)^m}\,dx = \frac{2\sqrt{(ax+b)^{m+4}}}{a^2(m+4)} - \frac{2b\sqrt{(ax+b)^{m+2}}}{a^2(m+2)}$

101. $\int x^2\sqrt{(ax+b)^m}\,dx = \frac{2\sqrt{(ax+b)^{m+6}}}{a^3(m+6)} - \frac{4b\sqrt{(ax+b)^{m+4}}}{a^3(m+4)} + \frac{2b^2\sqrt{(ax+b)^{m+2}}}{a^3(m+2)}$

102. $\int \frac{\sqrt{(ax+b)^m}}{x}dx = 2\frac{\sqrt{(ax+b)^m}}{m} + b\int \frac{\sqrt{(ax+b)^{m-2}}}{x}dx$

103. $\int \frac{\sqrt{(ax+b)^m}}{x^2}dx = -\frac{\sqrt{(ax+b)^{m+2}}}{bx} + \frac{ma}{2b}\int \frac{\sqrt{(ax+b)^m}}{x}dx$

104. $\int \frac{dx}{x\sqrt{(ax+b)^m}} = \frac{2}{(m-2)b\sqrt{(ax+b)^{m-2}}} + \frac{1}{b}\int \frac{dx}{x^m\sqrt{(ax+b)^{m-2}}}$

INTEGRALES CON ax + b y px + q , donde bp-aq ≠ 0

105. $\int \frac{1}{(ax+b)(px+q)}dx = \frac{1}{bp-aq}\ln\left(\frac{px+q}{ax+b}\right)$

106. $\int \frac{x}{(ax+b)(px+q)}dx = \frac{1}{bp-aq}\left(\frac{b}{a}\ln(ax+b) - \frac{q}{p}\ln(px+q)\right)$

107. $\int \frac{1}{(ax+b)^2(px+q)}dx = \frac{1}{bp-aq}\left(\frac{1}{ax+b} + \frac{p}{bp-aq}\ln\left[\frac{px+q}{ax+b}\right]\right)$

108. $\int \frac{x}{(ax+b)^2(px+q)}dx = \frac{1}{bp-aq}\left(\frac{q}{bp-aq}\ln\left[\frac{ax+b}{px+q}\right] - \frac{b}{a(ax+b)}\right)$

109. $\int \frac{x^2dx}{(ax+b)^2(px+q)} = \frac{b^2}{(bp-aq)a^2(ax+b)} + \frac{1}{(bp-aq)^2}\left(\frac{q^2}{p}\ln(px+q) + \frac{b(bp-aq)}{a^2}\ln(ax+b)\right)$

110. $\int \frac{dx}{(ax+b)^m(px+q)^n}dx = \frac{-1}{(n-1)(bp-aq)}\left(\frac{1}{(ax+b)^{m-1}(px+q)^{n-1}} + a(m+n-2)\int \frac{dx}{(ax+b)^m(px+q)^{n-1}}\right)$

111. $\int \frac{ax+b}{px+q}dx = \frac{ax}{p} + \frac{bp-aq}{p^2}\ln(px+q)$

$$112.\quad \int \frac{(ax+b)^m}{(px+q)^n}\,dx = \begin{cases} \dfrac{-1}{(n-1)(bp-aq)}\left(\dfrac{(ax+b)^{m+1}}{(px+q)^{n-1}} + a(n-m-2)\displaystyle\int \dfrac{(ax+b)^m}{(px+q)^{n-1}}\,dx\right) \\ \dfrac{-1}{(n-m-1)p}\left(\dfrac{(ax+b)^m}{(px+q)^{n-1}} + m(bp-aq)\displaystyle\int \dfrac{(ax+b)^{m-1}}{(px+q)^n}\,dx\right) \\ \dfrac{-1}{(n-1)p}\left(\dfrac{(ax+b)^m}{(px+q)^{n-1}} - ma\displaystyle\int \dfrac{(ax+b)^{m-1}}{(px+q)^{n-1}}\,dx\right) \end{cases}$$

INTEGRALES CON $\sqrt{ax+b}$ y px+q

$$113.\quad \int \frac{px+q}{\sqrt{ax+b}}\,dx = \frac{2(apx+3aq-2bp)}{3a^2}\sqrt{ax+b}$$

$$114.\quad \int \frac{dx}{(px+q)\sqrt{ax+b}} = \begin{cases} \dfrac{1}{\sqrt{bp-aq}\,\sqrt{p}}\ln\left(\dfrac{\sqrt{p(ax+b)}-\sqrt{bp-aq}}{\sqrt{p(ax+b)}+\sqrt{bp-aq}}\right) \\ \dfrac{2}{\sqrt{aq-bp}\,\sqrt{p}}\operatorname{tg}^{-1}\sqrt{\dfrac{p(ax+b)}{aq-bp}} \end{cases}$$

$$115.\quad \int \frac{\sqrt{ax+b}}{px+q}\,dx = \begin{cases} \dfrac{2\sqrt{ax+b}}{p} + \dfrac{\sqrt{bp-aq}}{p\sqrt{q}}\ln\left(\dfrac{\sqrt{p(ax+b)}-\sqrt{bp-aq}}{\sqrt{p(ax+b)}+\sqrt{bp-aq}}\right) \\ \dfrac{2\sqrt{ax+b}}{p} - \dfrac{2\sqrt{aq-bp}}{p\sqrt{q}}\operatorname{tg}^{-1}\sqrt{\dfrac{p(ax+b)}{aq-bp}} \end{cases}$$

$$116.\quad \int (px+q)^n\sqrt{ax+b}\,dx = \frac{2(px+q)^{n+1}\sqrt{ax+b}}{(2n+3)p} + \frac{bp-aq}{(2n+3)p}\int \frac{(px+q)^n}{\sqrt{ax+b}}\,dx$$

$$117.\quad \int \frac{dx}{(px+q)^n\sqrt{ax+b}} = \frac{\sqrt{ax+b}}{(n-1)(aq-bp)(px+q)^{n-1}} + \frac{(2n-3)a}{2(n-1)(aq-bp)}\int \frac{dx}{(px+q)^{n-1}\sqrt{ax+b}}$$

$$118.\quad \int \frac{(px+q)^n}{\sqrt{ax+b}} = \frac{2(px+q)^n\sqrt{ax+b}}{(2n+1)a} + \frac{2n(aq-bp)}{(2n-1)p}\int \frac{(px+q)^{n-1}}{\sqrt{ax+b}}\,dx$$

$$119.\quad \int \frac{\sqrt{ax+b}}{(px+q)^n}\,dx = \frac{-\sqrt{ax+b}}{(n-1)p(px+q)^{n-1}} + \frac{a}{2(n-1)p}\int \frac{dx}{(px+q)^{n-1}\sqrt{ax+b}}$$

INTEGRALES CON $\sqrt{ax+b}$ y $\sqrt{px+q}$

$$120.\quad \int \frac{dx}{(ax+b)(px+q)}dx = \begin{cases} \dfrac{2}{\sqrt{ap}}\ln\left(\sqrt{a(px+q)}+\sqrt{p(ax+b)}\right) \\ \dfrac{2}{\sqrt{-ap}}\operatorname{tg}^{-1}\sqrt{\dfrac{-p(ax+b)}{a(px+q)}} \end{cases}$$

$$121.\quad \int \frac{x\,dx}{\sqrt{(ax+b)(px+q)}} = \frac{\sqrt{(ax+b)(px+q)}}{ap} - \frac{bp+aq}{2ap}\int \frac{dx}{\sqrt{(ax+b)(px+q)}}$$

$$122.\quad \int \sqrt{(ax+b)(px+q)}\,dx = \frac{2apx+bp+aq}{4ap}\sqrt{(ax+b)(px+q)} - \frac{(bp-aq)^2}{8ap}\int \frac{dx}{\sqrt{(ax+b)(px+q)}}$$

$$123.\quad \int \sqrt{\frac{px+q}{ax+b}}\,dx = \frac{\sqrt{(ax+b)(px+q)}}{a} + \frac{(aq-bp)}{2a}\int \frac{dx}{\sqrt{(ax+b)(px+q)}}$$

$$124.\quad \int \frac{dx}{(px+q)\sqrt{(ax+b)(px+q)}} = \frac{2\sqrt{ax+b}}{(aq-bp)\sqrt{px+q}}$$

INTEGRALES CON x^2+a^2

$$125.\quad \int \frac{dx}{x^2+a^2} = \frac{1}{a}\operatorname{tg}^{-1}\frac{x}{a}$$

$$126.\quad \int \frac{x\,dx}{x^2+a^2} = \frac{1}{2}\ln(x^2+a^2)$$

$$127.\quad \int \frac{x^2\,dx}{x^2+a^2} = x - a\operatorname{tg}^{-1}\frac{x}{a}$$

128. $\int \frac{x^3 dx}{x^2+a^2} = \frac{x^2}{2} - \frac{a^2}{2}\ln(x^2 + a^2)$

129. $\int \frac{dx}{x(x^2+a\ )} = \frac{1}{2a^2}\ln(x^2 + a^2)$

130. $\int \frac{dx}{x^2(x^2+a\ )} = -\frac{1}{a^2 x} - \frac{1}{a^3}\operatorname{tg}^{-1}\frac{x}{a}$

131. $\int \frac{dx}{x^3(x^2+a^2)} = -\frac{1}{2a^2x^2} - \frac{1}{2a^4}\ln\left(\frac{x^2}{x^2+a^2}\right)$

132. $\int \frac{dx}{(x^2+a^2)^2} = \frac{x}{2a^2(x^2+a^2)} + \frac{1}{2a^3}\operatorname{tg}^{-1}\frac{x}{a}$

133. $\int \frac{xdx}{(x^2+a^2)} = \frac{-1}{2(x^2+a^2)}$

134. $\int \frac{x^2 dx}{(x^2+a^2)^2} = \frac{-x}{2(x^2+a^2)} + \frac{1}{2a}\operatorname{tg}^{-1}\frac{x}{a}$

135. $\int \frac{x^3 dx}{(x^2+a^2)^2} = \frac{a^2}{2(x^2+a^2)} + \frac{1}{2}\ln(x^2+a^2)$

136. $\int \frac{dx}{x(x^2+a^2)^2} = \frac{1}{2a^2(x^2+a^2)} + \frac{1}{2a^4}\ln\left(\frac{x^2}{x^2+a^2}\right)$

137. $\int \frac{dx}{x^2(x^2+a^2)^2} = -\frac{1}{a^4 x} - \frac{x}{2a^4(x^2+a^2)} - \frac{3}{2a^5}\operatorname{tg}^{-1}\frac{x}{a}$

138. $\int \frac{dx}{x^3(x^2+a^2)^2} = -\frac{1}{2a^4x^2} - \frac{1}{2a^4(x^2+a^2)} - \frac{1}{a^6}\ln\left(\frac{x^2}{x^2+a^2}\right)$

139. $\int \frac{dx}{(x^2+a\ )^n} = \frac{x}{2(n-1)a^2(x+a\ )^{n-1}} + \frac{2n-3}{(2n-2)a^2}\int \frac{dx}{(x^2+a^2)^{n-1}}$; Si n=1 Ver 125

140. $\int \frac{xdx}{(x^2+a^2)^n} = \frac{-1}{2(n-1)(x^2+a^2)^{n-1}}$ Si n=1 Ver 126

141. $\int \frac{dx}{x(x^2+a^2)^n} = \frac{1}{2(n-1)a^2(x^2+a^2)^{n-1}} + \frac{1}{a^2}\int \frac{dx}{x(x^2+a^2)^{n-1}}$; Si n=1 Ver 129

142. $\int \frac{x^m dx}{(x^2+a^2)^n} = \int \frac{x^{m-2}dx}{(x^2+a^2)^{n-1}} - a^2 \int \frac{x^{m-2}}{(x^2+a^2)^n}$

143. $\int \frac{dx}{x^m(x^2+a^2)^n} = \frac{1}{a^2}\int \frac{dx}{x^m(x^2+a^2)^{n-1}} - \frac{1}{a^2}\int \frac{dx}{x^{m-2}(x^2+a^2)^n}$

INTEGRALES CON $x^2 - a^2$; $x^2 > a^2$

144. $\int \frac{dx}{x^2-a^2)} = \frac{1}{2a}\ln\left(\frac{x-a}{x+a}\right)$ ó $\frac{-1}{a}\cot\ gh^{-1}\frac{x}{a}$

145. $\int \frac{xdx}{x^2-a^2} = \frac{1}{2}\ln(x^2 - a^2 =)$

146. $\int \frac{x^2 dx}{x^2-a^2} = x + \frac{a}{2}\ln\left(\frac{x-a}{x+a}\right)$

147. $\int \frac{x^3 dx}{x^2-a^2} = \frac{x^2}{2} + \frac{a^2}{2}\ln(x^2 - a^2)$

148. $\int \frac{dx}{x(x^2-a^2)} = \frac{1}{2a^2}\ln\left(\frac{x^2-a^2}{x^2}\right)$

149. $\int \frac{dx}{x^2(x^2-a^2)} = \frac{1}{a^2 x} + \frac{1}{2a^3}\ln\left(\frac{x-a}{x+a}\right)$

150. $\int \frac{dx}{x^3(x^2-a^2)} = \frac{1}{2a^2x^2} - \frac{1}{2a^4}\ln\left(\frac{x^2}{x^2-a^2}\right)$

151. $\int \frac{dx}{(x^2-a^2)} = \frac{-x}{2a^2(x^2-a^2)} - \frac{1}{2a^4}\ln\left(\frac{x-a}{x+a}\right)$

152. $\int \frac{xdx}{(x^2-a^2)^2} = \frac{-1}{2(x^2-a^2)}$

153. $\int \frac{x^2 dx}{(x^2-a^2)^2} = \frac{-x}{2(x^2-a^2)} + \frac{1}{4a}\ln\left(\frac{x-a}{x+a}\right)$

154. $\int \frac{x^3 dx}{(x^2-a^2)} = \frac{-a^2}{2(x^2-a^2)} + \frac{1}{2}\ln(x^2 - a^2)$

155. $\int \frac{dx}{x(x^2-a^2)} = \frac{-1}{2a^2(x^2-a^2)} + \frac{1}{2a^4}\ln\left(\frac{x^2}{x^2-a^2}\right)$

156. $\int \frac{dx}{x^2(x^2-a^2)^2} = -\frac{1}{a^4 x} - \frac{x}{2a^4(x^2-a^2)} - \frac{3}{4a^6}\ln\left(\frac{x-a}{x+a}\right)$

157. $\int \frac{dx}{x^3(x^2-a^2)^2} = -\frac{1}{2a^4x^2} - \frac{1}{2a^4(x^2-a^2)} + \frac{1}{a^6}\ln\left(\frac{x^2}{x^2-a^2}\right)$

158. $\int \frac{dx}{(x^2-a^2)^n} = \frac{-x}{2(n-1)a^2(x^2-a^2)^{n-1}} - \frac{2n-3}{(2n-2)a^2}\int \frac{dx}{(x^2-a^2)^{n-1}}$

159. $\int \frac{x\,dx}{(x^2-a^2)^n} = \frac{-1}{2(n-1)(x^2-a^2)^{n-1}}$

160. $\int \frac{dx}{x(x^2-a^2)^n} = \frac{-1}{2(n-1)a^2(x^2-a^2)^{n-1}} - \frac{1}{a^2}\int \frac{dx}{x(x^2-a^2)^{n-1}}$

161. $\int \frac{x^m\,dx}{(x^2-a^2)^n} = \int \frac{x^{m-2}\,dx}{(x^2-a^2)^{n-1}} + a^2\int \frac{x^{m-2}\,dx}{(x^2-a^2)^n}$

162. $\int \frac{dx}{x^m(x^2-a^2)^n} = \frac{1}{a^2}\int \frac{dx}{x^{m-2}(x^2-a^2)^n} - \frac{1}{a^2}\int \frac{dx}{x^m(x^2-a^2)^{n-1}}$

INTEGRALES CON a^2-x^2, $x^2<a^2$

163. $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left(\frac{a+x}{a-x}\right)$ ó $\frac{1}{a}\,tgh^{-1}\frac{x}{a}$

164. $\int \frac{x\,dx}{a^2-x^2} = -\frac{1}{2}\ln(a^2-x^2)$

165. $\int \frac{x^2\,dx}{a^2-x^2} = -x + \frac{a}{2}\ln\left(\frac{a+x}{a-x}\right)$

166. $\int \frac{x^3\,dx}{a^2-x^2} = -\frac{x^2}{2} - \frac{a^2}{2}\ln(a^2-x^2)$

167. $\int \frac{dx}{x(a^2-x^2)} = \frac{1}{2a^2}\ln\left(\frac{x^2}{a^2-x^2}\right)$

168. $\int \frac{dx}{x^2(a^2-x^2)} = -\frac{1}{a^2x} + \frac{1}{2a^3}\ln\left(\frac{a+x}{a-x}\right)$

169. $\int \frac{dx}{x^3(a^2-x^2)} = -\frac{1}{2a^2x^2} + \frac{1}{2a^4}\ln\left(\frac{x^2}{a^2-x^2}\right)$

170. $\int \frac{dx}{(a^2-x^2)^2} = \frac{x}{2a^2(a^2-x^2)} + \frac{1}{4a^3}\ln\left(\frac{a+x}{a-x}\right)$

171. $\int \frac{x\,dx}{(a^2-x^2)^2} = \frac{1}{2(a^2-x^2)}$

172. $\int \frac{x^2\,dx}{(a^2-x^2)^2} = \frac{x}{2(a^2-x^2)} - \frac{1}{4a}\ln\left(\frac{a+x}{a-x}\right)$

173. $\int \frac{x^3\,dx}{(a^2-x^2)^2} = \frac{a^2}{2(a^2-x^2)} + \frac{1}{2}\ln(a^2-x^2)$

174. $\int \frac{dx}{x(a^2-x^2)^2} = \frac{1}{2a^2(a^2-x^2)} + \frac{1}{2a^4}\ln\left(\frac{x^2}{a^2-x^2}\right)$.

175. $\int \frac{dx}{x^2(a^2-x^2)^2} = -\frac{1}{a^4x} + \frac{x}{2a^4(a^2-x^2)} + \frac{3}{4a^5}\ln\left(\frac{a+x}{a-x}\right)$

176. $\int \frac{dx}{x^3(a^2-x^2)^2} = -\frac{1}{2a^4x^2} + \frac{1}{2a^4(a^2-x^2)} + \frac{1}{a^6}\ln\left(\frac{x^2}{a^2-x^2}\right)$

177. $\int \frac{dx}{(a^2-x^2)} = \frac{x}{2(n-1)a^2(a^2-x^2)^{n-1}} + \frac{2n-3}{(2n-2)a^2}\int \frac{dx}{(a^2-x^2)^{n-1}}$

178. $\int \frac{x\,dx}{(a^2-x^2)^n} = \frac{1}{2(n-1)(a^2-x^2)^{n-1}}$

179. $\int \frac{dx}{x(a^2-x^2)^n} = \frac{1}{2(n-1)a^2(a^2-x^2)^{n-1}}$

180. $\int \frac{x^m\,dx}{(a^2-x^2)^n} = a^2\int \frac{x^{m-2}\,dx}{(a^2-x^2)^n} - \int \frac{x^{m-2}\,dx}{(a^2-x^2)^{n-1}}$

181. $\int \frac{dx}{x^m(a^2-x^2)} = \frac{1}{a^2}\int \frac{dx}{x^{m-2}(a^2-x^2)^n} + \frac{1}{a^2}\int \frac{dx}{x^m(a^2-x^2)^{n-1}}$

INTEGRALES CON $\sqrt{x^2+a^2}$

182. $$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left(x+\sqrt{x^2+a^2}\right) \text{ ó } \operatorname{senh}^{-1}\frac{x}{a}$$

183. $$\int \frac{x\,dx}{\sqrt{x^2+a^2}} = \sqrt{x^2+a^2}$$

184. $$\int \frac{x^2\,dx}{\sqrt{x^2+a^2}} = \frac{x\sqrt{x^2+a^2}}{2} - \frac{a^2}{2}\ln\left(x+\sqrt{x^2+a^2}\right)$$

185. $$\int \frac{x^3\,dx}{\sqrt{x^2+a^2}} = \frac{\sqrt{(x^2+a^2)^3}}{3} - a^2\sqrt{x^2+a^2}$$

186. $$\int \frac{dx}{x\sqrt{x^2+a^2}} = -\frac{1}{a}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

187. $$\int \frac{dx}{x^2\sqrt{x^2+a^2}} = -\frac{\sqrt{x^2+a^2}}{a^2x}$$

188. $$\int \frac{dx}{x^3\sqrt{x^2+a^2}} = -\frac{\sqrt{x^2+a^2}}{2a^2x^2} + \frac{1}{2a^3}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

189. $$\int \sqrt{x^2+a^2}\,dx = \frac{x\sqrt{x^2+a^2}}{2} + \frac{a^2}{2}\ln\left(x+\sqrt{x^2+a^2}\right)$$

190. $$\int x\sqrt{x^2+a^2}\,dx = \frac{\sqrt{(x^2+a^2)^3}}{3}$$

191. $$\int x^2\sqrt{x^2+a^2}\,dx = \frac{x\sqrt{(x^2+a^2)^3}}{4} - \frac{a^2x\sqrt{x^2+a^2}}{8} - \frac{a^4}{8}\ln\left(x+\sqrt{x^2+a^2}\right)$$

192. $$\int x^3\sqrt{x^2+a^2}\,dx = \frac{\sqrt{(x^2+a^2)^5}}{5} - \frac{a^2\sqrt{(x^2+a^2)^3}}{3}$$

193. $$\int \frac{\sqrt{x^2+a^2}}{x}\,dx = \sqrt{x^2+a^2} - a\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

194. $$\int \frac{\sqrt{x^2+a^2}}{x^2}\,dx = -\frac{\sqrt{x^2+a^2}}{x} + \ln\left(x+\sqrt{x^2+a^2}\right)$$

195. $$\int \frac{\sqrt{x^2+a^2}}{x^3}\,dx = -\frac{\sqrt{x^2+a^2}}{2x^2} - \frac{1}{2a}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

196. $$\int \frac{dx}{\sqrt{(x^2+a^2)^3}} = \frac{x}{a^2\sqrt{x^2+a^2}}$$

197. $$\int \frac{x\,dx}{\sqrt{(x^2+a^2)^3}} = \frac{-1}{\sqrt{x^2+a^2}}$$

198. $$\int \frac{x^2\,dx}{\sqrt{(x^2+a^2)^3}} = \frac{-x}{\sqrt{x^2+a^2}} + \ln\left(x+\sqrt{x^2+a^2}\right)$$

199. $$\int \frac{x^3\,dx}{\sqrt{(x^2+a^2)^3}} = \sqrt{x^2+a^2} + \frac{a^2}{\sqrt{x^2+a^2}}$$

200. $$\int \frac{dx}{x\sqrt{(x^2+a^2)^3}} = \frac{1}{a^2\sqrt{x^2+a^2}} - \frac{1}{a^3}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

201. $$\int \frac{dx}{x^2\sqrt{(x^2+a^2)^3}} = -\frac{\sqrt{x^2+a^2}}{a^4x} - \frac{x}{a^4\sqrt{x^2+a^2}}$$

202. $$\int \frac{dx}{x^3\sqrt{(x^2+a^2)^3}} = \frac{-1}{2a^2x^2\sqrt{x^2+a^2}} - \frac{3}{2a^4\sqrt{x^2+a^2}} + \frac{3}{2a^5}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

203. $$\int \sqrt{(x^2+a^2)^3}\,dx = \frac{x\sqrt{(x^2+a^2)^3}}{4} + \frac{3a^2x\sqrt{x^2+a^2}}{8} + \frac{3a^4}{8}\ln\left(x+\sqrt{x^2+a^2}\right)$$

204. $$\int x\sqrt{(x^2+a^2)^3}\,dx = \frac{\sqrt{(x^2+a^2)^5}}{5}$$

205. $$\int x^2\sqrt{(x^2+a^2)^3}\,dx = \frac{x\sqrt{(x^2+a^2)^5}}{6} - \frac{a^2x\sqrt{(x^2+a^2)^3}}{24} - \frac{a^4x\sqrt{x^2+a^2}}{16} - \frac{a^6}{16}\ln\left(x+\sqrt{x^2+a^2}\right)$$

206. $$\int x^3\sqrt{(x^2+a^2)^3}\,dx = \frac{\sqrt{(x^2+a^2)^7}}{7} - \frac{a^2\sqrt{(x^2+a^2)^5}}{5}$$

207. $$\int \frac{\sqrt{(x^2+a^2)^3}}{x}\,dx = \frac{\sqrt{(x^2+a^2)^3}}{3} + a^2\sqrt{x^2+a^2} - a^3\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

208. $$\int \frac{\sqrt{(x^2+a^2)^3}}{x^2}\,dx = -\frac{\sqrt{(x^2+a^2)^3}}{x} + \frac{3x\sqrt{x^2+a^2}}{2} + \frac{3}{2}a^2\ln\left(x+\sqrt{x^2+a^2}\right)$$

209. $\int \frac{\sqrt{(x^2+a^2)^3}}{x^3}dx = -\frac{\sqrt{(x^2+a^2)^3}}{2x} + \frac{3\sqrt{x^2+a^2}}{2} - \frac{3}{2}a\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$

INTEGRALES CON $\sqrt{x^2-a^2}$

210. $\int \frac{dx}{\sqrt{x^2-a^2}} = \ln(x+\sqrt{x^2-a^2})$

211. $\int \frac{x\,dx}{\sqrt{x^2-a^2}} = \sqrt{x^2-a^2}$

212. $\int \frac{x^2dx}{\sqrt{x^2-a^2}} = \frac{x\sqrt{x^2-a^2}}{2} + \frac{a^2}{2}\ln(x+\sqrt{x^2-a^2})$

213. $\int \frac{x^3dx}{\sqrt{x^2-a^2}} = \frac{\sqrt{(x^2-a^2)^3}}{3} + a^2\sqrt{x^2-a^2}$

214. $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right|$

215. $\int \frac{dx}{x^2\sqrt{x^2-a^2}} = \frac{\sqrt{x^2-a^2}}{a^2x}$

216. $\int \frac{dx}{x^3\sqrt{x^2-a^2}} = \frac{\sqrt{x^2-a^2}}{2a^2x^2} + \frac{1}{2a^3}\sec^{-1}\left|\frac{x}{a}\right|$

217. $\int \sqrt{x^2-a^2}\,dx = \frac{x\sqrt{x^2-a^2}}{2} - \frac{a^2}{2}\ln(x+\sqrt{x^2-a^2})$

218. $\int x\sqrt{x^2-a^2}\,dx = \frac{\sqrt{(x^2-a^2)^3}}{3}$

219. $\int x^2\sqrt{x^2-a^2}\,dx = \frac{x\sqrt{x^2-a^2}}{4} + \frac{a^2x\sqrt{x^2-a^2}}{8} - \frac{a^4}{8}\ln(x+\sqrt{x^2-a^2})$

220. $\int x^3\sqrt{x^2-a^2}\,dx = \frac{\sqrt{(x^2-a^2)^5}}{5} + \frac{a^2\sqrt{(x^2-a^2)^3}}{3}$

221. $\int \frac{\sqrt{x^2-a^2}}{x}dx = \sqrt{x^2-a^2} - a\sec^{-1}\left|\frac{x}{a}\right|$

222. $\int \frac{\sqrt{x^2-a^2}}{x^2}dx = -\frac{\sqrt{x^2-a^2}}{x} + \ln(x+\sqrt{x^2-a^2})$

223. $\int \frac{\sqrt{x^2-a^2}}{x^3}dx = -\frac{\sqrt{x^2-a^2}}{2x^2} + \frac{1}{2a}\sec^{-1}\left|\frac{x}{a}\right|$

224. $\int \frac{dx}{\sqrt{(x^2-a^2)^3}} = -\frac{x}{a^2\sqrt{x^2-a^2}}$

225. $\int \frac{x\,dx}{\sqrt{(x^2-a^2)^3}} = \frac{-1}{\sqrt{x^2-a^2}}$

226. $\int \frac{x^2dx}{\sqrt{(x^2-a^2)^3}} = \frac{-x}{\sqrt{x^2-a^2}} + \ln(x+\sqrt{x^2-a^2})$

227. $\int \frac{x^3dx}{\sqrt{(x^2-a^2)^3}} = \sqrt{x^2-a^2} - \frac{a^2}{\sqrt{x^2-a^2}}$

228. $\int \frac{dx}{x\sqrt{(x^2-a^2)^3}} = \frac{-1}{a^2\sqrt{x^2-a^2}} - \frac{1}{a^3}\sec^{-1}\left|\frac{x}{a}\right|$

229. $\int \frac{dx}{x^2\sqrt{(x^2-a^2)^3}} = -\frac{\sqrt{x^2-a^2}}{a^4x} - \frac{x}{a^4\sqrt{x^2-a^2}}$

230. $\int \frac{dx}{x^3\sqrt{(x^2-a^2)^3}} = \frac{1}{2a^2x^2\sqrt{x^2-a^2}} - \frac{3}{2a^4\sqrt{x^2-a^2}} - \frac{3}{2a^5}\sec^{-1}\left|\frac{x}{a}\right|$

231. $\int \sqrt{(x^2-a^2)^3}\,dx = \frac{x\sqrt{(x^2-a^2)^3}}{4} - \frac{3a^2x\sqrt{x^2-a^2}}{8} + \frac{3a^4}{8}\ln(x+\sqrt{x^2-a^2})$

232. $\int x\sqrt{(x^2-a^2)^3}\,dx = \frac{\sqrt{(x^2-a^2)^5}}{5}$

233. $\int x^2\sqrt{(x^2-a^2)^3}\,dx = \frac{x\sqrt{(x^2-a^2)^5}}{6} + \frac{a^2x\sqrt{(x^2-a^2)^3}}{24} - \frac{a^4x\sqrt{x^2-a^2}}{16} + \frac{a^6}{16}\ln(x+\sqrt{x^2-a^2})$

234. $\int x^3\sqrt{(x^2-a^2)}\,dx = \frac{\sqrt{(x^2-a^2)^7}}{7} + \frac{a^2\sqrt{(x^2-a^2)^5}}{5}$

235. $\int \frac{\sqrt{(x^2-a^2)^3}}{x}dx = \frac{\sqrt{(x^2-a^2)^3}}{3} - a^2\sqrt{x^2-a^2} + a^3 \sec^{-1}\left|\frac{x}{a}\right|$

236. $\int \frac{\sqrt{(x^2-a^2)^3}}{x^2}dx = -\frac{\sqrt{(x^2-a^2)^3}}{x} + \frac{3x\sqrt{x^2-a^2}}{2} - \frac{3}{2}a^2 \ln(x+\sqrt{x^2-a^2})$

237. $\int \frac{\sqrt{(x^2-a^2)^3}}{x^3}dx = -\frac{\sqrt{(x^2-a^2)^3}}{2x^2} + \frac{3\sqrt{x^2-a^2}}{2} - \frac{3}{2}a \sec^{-1}\left|\frac{x}{a}\right|$

INTEGRALES CON $\sqrt{a^2-x^2}$

238. $\int \frac{dx}{\sqrt{a^2-x^2}} = \operatorname{sen}^{-1}\frac{x}{a}$

239. $\int \frac{x\,dx}{\sqrt{a^2-x^2}} = -\sqrt{a^2-x^2}$

240. $\int \frac{x^2\,dx}{\sqrt{a^2-x^2}} = -\frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2}\operatorname{sen}^{-1}\frac{x}{a}$

241. $\int \frac{x^3\,dx}{\sqrt{a^2-x^2}} = \frac{\sqrt{(a^2-x^2)^3}}{3} - a^2\sqrt{a^2-x^2}$

242. $\int \frac{dx}{x\sqrt{a^2-x^2}} = -\frac{1}{a}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$

243. $\int \frac{dx}{x^2\sqrt{a^2-x^2}} = -\frac{\sqrt{a^2-x^2}}{a^2x}$

244. $\int \frac{dx}{x^3\sqrt{a^2-x^2}} = -\frac{\sqrt{a^2-x^2}}{2a^2x^2} - \frac{1}{2a^3}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$

245. $\int \sqrt{a^2-x^2}\,dx = \frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2}\operatorname{sen}^{-1}\frac{x}{a}$

246. $\int x\sqrt{a^2-x^2}\,dx = -\frac{\sqrt{(a^2-x^2)^3}}{3}$

247. $\int x^2\sqrt{a^2-x^2}\,dx = -\frac{x\sqrt{(a^2-x^2)^3}}{4} + \frac{a^2x\sqrt{a^2-x^2}}{8} - \frac{a^4}{8}\operatorname{sen}^{-1}\frac{x}{a}$

248. $\int x^3\sqrt{a^2-x^2}\,dx = \frac{\sqrt{(a^2-x^2)^5}}{5} - \frac{a^2\sqrt{(a^2-x^2)^3}}{3}$

249. $\int \frac{\sqrt{a^2-x^2}}{x}dx = \sqrt{a^2-x^2} - a\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$

250. $\int \frac{\sqrt{a^2-x^2}}{x^2}dx = -\frac{\sqrt{a^2-x^2}}{x} - \operatorname{sen}^{-1}\frac{x}{a}$

251. $\int \frac{\sqrt{a^2-x^2}}{x^3}dx = -\frac{\sqrt{a^2-x^2}}{2x^2} + \frac{1}{2a}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$

252. $\int \frac{dx}{\sqrt{(a^2-x^2)^3}} = \frac{x}{a^2\sqrt{a^2-x^2}}$

253. $\int \frac{x\,dx}{\sqrt{(a^2-x^2)^3}} = \frac{1}{\sqrt{a^2-x^2}}$

254. $\int \frac{x^2\,dx}{\sqrt{(a^2-x^2)^3}} = \frac{x}{\sqrt{a^2-x^2}} - \operatorname{sen}^{-1}\frac{x}{a}$

255. $\int \frac{x^3\,dx}{\sqrt{(a^2-x^2)^3}} = \sqrt{a^2-x^2} + \frac{a^2}{\sqrt{a^2-x^2}}$

256. $\int \frac{dx}{x\sqrt{(a^2-x^2)^3}} = \frac{1}{a^2\sqrt{a^2-x^2}} - \frac{1}{a^3}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$

257. $\int \frac{dx}{x^2\sqrt{(a^2-x^2)}} = -\frac{\sqrt{a^2-x^2}}{a^4x} + \frac{x}{a^4\sqrt{a^2-x^2}}$

258. $\int \frac{dx}{x^3\sqrt{(a^2-x^2)^3}} = \frac{-1}{2a^2x^2\sqrt{a^2-x^2}} + \frac{3}{2a^4\sqrt{a^2-x^2}} - \frac{3}{2a^6}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$

259. $\int \sqrt{(a^2-x^2)^3}\,dx = \frac{x\sqrt{(a^2-x^2)^3}}{4} + \frac{3a^2x\sqrt{a^2-x^2}}{8} + \frac{3a^4}{8}\operatorname{sen}^{-1}\frac{x}{a}$

$$260.\quad \int x\sqrt{(a^2-x^2)^3}\,dx = -\frac{\sqrt{(a^2-x^2)^5}}{5}$$

$$261.\quad \int x^2\sqrt{(a^2-x^2)^3}\,dx = -\frac{x\sqrt{(a^2-x^2)^5}}{6} + \frac{a^2x\sqrt{(a^2-x^2)^3}}{24} + \frac{a^4x\sqrt{a^2-x^2}}{16} + \frac{a^6}{16}\operatorname{sen}^{-1}\frac{x}{a}$$

$$262.\quad \int x^3\sqrt{(a^2-x^2)^3}\,dx = \frac{\sqrt{(a^2-x^2)^7}}{7} - \frac{a^2\sqrt{(a^2-x^2)^5}}{5}$$

$$263.\quad \int \frac{\sqrt{(a^2-x^2)^3}}{x}\,dx = \frac{\sqrt{(a^2-x^2)^3}}{3} + a^2\sqrt{a^2-x^2} - a^3\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$$

$$264.\quad \int \frac{\sqrt{(a^2-x^2)^3}}{x^2}\,dx = -\frac{\sqrt{(a^2-x^2)^3}}{x} - \frac{3x\sqrt{a^2-x^2}}{2} - \frac{3}{2}a^2\operatorname{sen}^{-1}\frac{x}{a}$$

$$265.\quad \int \frac{\sqrt{(a^2-x^2)^3}}{x^3}\,dx = -\frac{\sqrt{(a^2-x^2)^3}}{2x^2} - \frac{3\sqrt{a^2-x^2}}{2} + \frac{3}{2}a\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$$

INTEGRALES CON $ax^2 + bx + c$

Si $b^2 = 4ac$, se puede escribir $ax^2 + bx + c = a\,(x + b/2a)^2$ y se emplean los resultados de las páginas 11 y 12.

$$266.\quad \int \frac{dx}{ax^2+bx+c} = \begin{cases} \dfrac{2}{\sqrt{4ac-b^2}}\operatorname{tg}^{-1}\dfrac{2ax+b}{\sqrt{4ac-b^2}} \\[2ex] \dfrac{1}{\sqrt{b^2-4ac}}\ln\left(\dfrac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}}\right) \end{cases}$$

$$267.\quad \int \frac{x\,dx}{ax^2+bx+c} = \frac{1}{2a}\ln(ax^2+bx+c) - \frac{b}{2a}\int\frac{dx}{ax^2+bx+c}$$

$$268.\quad \int \frac{x^2\,dx}{ax^2+bx+c} = \frac{x}{a} - \frac{b}{2a^2}\ln(ax^2+bx+c) + \frac{b^2-2ac}{2a^2}\int\frac{dx}{ax^2+bx+c}$$

$$269.\quad \int \frac{x^m\,dx}{ax^2+bx+c} = \frac{x^{m-1}}{(m-1)a} - \frac{c}{a}\int\frac{x^{m-2}\,dx}{ax^2+bx+c} - \frac{b}{a}\int\frac{x^{m-1}\,dx}{ax^2+bx+c}$$

$$270.\quad \int \frac{dx}{x(ax^2+bx+c)} = \frac{1}{2c}\ln\left(\frac{x^2}{ax^2+bx+c}\right) - \frac{b}{2c}\int\frac{dx}{ax^2+bx+c}$$

$$271.\quad \int \frac{dx}{x^2(ax^2+bx+c)} = \frac{b}{2c^2}\ln\left(\frac{ax^2+bx+c}{x^2}\right) - \frac{1}{xc} + \frac{b^2-2ac}{2c^2}\int\frac{dx}{ax^2+bx+c}$$

$$272.\quad \int \frac{dx}{x^n(ax^2+bx+c)} = -\frac{1}{(n-1)cx^{n-1}} - \frac{b}{c}\int\frac{dx}{x^{n-1}(ax^2+bx+c)} - \frac{a}{c}\int\frac{dx}{x^{n-2}(ax^2+bx+c)}$$

273. $\int$ *NO.SE.ENTIENDE.NADA.DE.LO.QUE.DICE*

$$274.\quad \int \frac{x\,dx}{(ax^2+bx+c)^2} = -\frac{bx+2c}{(4ac-b^2)(ax^2+bx+c)} - \frac{b}{4ac-b^2}\int\frac{dx}{ax^2+bx+c}$$

$$275.\quad \int \frac{x^2\,dx}{(ax^2+bx+c)^2} = \frac{(b^2-2ac)x+bc}{a(4ac-b^2)(ax^2+bx+c)} + \frac{2c}{4ac-b^2}\int\frac{dx}{ax^2+bx+c}$$

$$276.\quad \int \frac{x^m\,dx}{(ax^2+bx+c)^n} = \frac{-x^{m-1}}{(2n-m-1)a(ax^2+bx+c)^{n-1}} + \frac{1}{(2n-m-1)a}\left\{ c\int\frac{(m-1)x^{m-2}\,dx}{(ax^2+bx+c)^n} - b\int\frac{(n-m)x^{m-1}\,dx}{(ax^2+bx+c)^n} \right\}$$

$$277.\quad \int \frac{x^{2n-1}\,dx}{(ax^2+bx+c)^n} = \frac{1}{a}\int\frac{x^{2n-3}\,dx}{(ax^2+bx+c)^{n-1}} - \frac{c}{a}\int\frac{x^{2n-3}\,dx}{(ax^2+bx+c)^n} - \frac{b}{a}\int\frac{x^{2n-2}\,dx}{(ax^2+bx+c)^n}$$

$$278.\quad \int \frac{dx}{x(ax^2+bx+c)^2} = \frac{1}{2c(ax^2+bx+c)} - \frac{b}{2c}\int\frac{dx}{(ax^2+bx+c)^2} + \frac{1}{c}\int\frac{dx}{x(ax^2+bx+c)}$$

$$279.\quad \int \frac{dx}{x^2(ax^2+bx+c)^2} = \frac{-1}{cx(ax^2+bx+c)} - \frac{3a}{c}\int\frac{dx}{(ax^2+bx+c)^2} - \frac{2b}{c}\int\frac{dx}{x(ax^2+bx+c)^2}$$

$$280.\quad \int \frac{dx}{x^m(ax^2+bx+c)^n} = \frac{-1}{(m-1)cx^{m-1}(ax^2+bx+c)^{n-1}} - \frac{(m+2n-3)a}{(m-1)c}\int\frac{dx}{x^{m-2}(ax^2+bx+c)^n} - \frac{(m+n-2)b}{(m-1)c}\int\frac{dx}{x^{m-1}(ax^2+bx+c)^n}$$

INTEGRALES CON $\sqrt{ax^2+bx+c}$

Si en las fórmulas siguientes $b^2 = 4ac$, se puede escribir $\sqrt{ax^2+bx+c} = \sqrt{a}\left(x+\frac{b}{2a}\right)$ y se emplean los resultados de las páginas 11 y 12.

281. $\int \frac{dx}{\sqrt{ax^2+bx+c}} = \begin{cases} \frac{1}{\sqrt{a}}\ln\left(2\sqrt{a}\sqrt{(ax^2+bx+c)}+2ax+b\right) \\ -\frac{1}{\sqrt{-a}}\operatorname{sen}^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right) \text{ ó } \frac{1}{\sqrt{a}}\operatorname{senh}^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right) \end{cases}$

282. $\frac{x\,dx}{\sqrt{ax^2+bx+c}} = \frac{\sqrt{ax^2+bx+c}}{a} - \frac{b}{2a}\int \frac{dx}{\sqrt{ax^2+bx+c}}$

283. $\int \frac{x^2\,dx}{\sqrt{ax^2+bx+c}} = \frac{2ax-3b}{4a^2}\sqrt{ax^2+bx+c} + \frac{3b^2-4ac}{8a^2}\int \frac{dx}{\sqrt{ax^2+bx+c}}$

284. $\int \frac{dx}{x\sqrt{ax^2+bx+c}} = \begin{cases} -\frac{1}{\sqrt{c}}\ln\left(\frac{2\sqrt{c}\sqrt{ax^2+bx+c}+bx+2c}{x}\right) \\ \frac{1}{\sqrt{-c}}\operatorname{sen}^{-1}\left(\frac{bc+2c}{|x|\sqrt{b^2-4ac}}\right) \text{ ó } -\frac{1}{\sqrt{c}}\operatorname{senh}^{-1}\left(\frac{bx+2c}{|x|\sqrt{4ac-b^2}}\right) \end{cases}$

285. $\int \frac{dx}{x^2\sqrt{ax^2+bx+c}} = -\frac{\sqrt{ax^2+bx+c}}{cx} - \frac{b}{2c}\int \frac{dx}{x\sqrt{ax^2+bx+c}}$

286. $\int \sqrt{ax^2+bx+c}\,dx = \frac{2ax+b}{4a}\sqrt{ax^2+bx+c} + \frac{4ac-b^2}{8a}\int \frac{dx}{\sqrt{ax^2+bx+c}}$

287. $\int x\sqrt{ax^2+bx+c}\,dx = \frac{\sqrt{(ax^2+bx+c)^3}}{3a} - \frac{b(2ax+b)}{8a^2}\sqrt{ax^2+bx+c} - \frac{b(4ac-b^2)}{16a^2}\int \frac{dx}{\sqrt{ax^2+bx+c}}$

288. $\int x^2\sqrt{ax^2+bx+c}\,dx = \frac{6ax-5b}{24a^2}\sqrt{(ax^2+bx+c)^3} + \frac{5b^2-4ac}{16a^2}\int \sqrt{ax^2+bx+c}\,dx$

289. $\int \frac{\sqrt{ax^2+bx+c}}{x}\,dx = \sqrt{ax^2+bx+c} + \frac{b}{2}\int \frac{dx}{\sqrt{ax^2+bx+c}} + c\int \frac{dx}{x\sqrt{ax^2+bx+c}}$

290. $\int \frac{\sqrt{ax^2+bx+c}}{x^2}\,dx = -\frac{\sqrt{ax^2+bx+c}}{x} + a\int \frac{dx}{\sqrt{ax^2+bx+c}} + \frac{b}{2}\int \frac{dx}{x\sqrt{ax^2+bx+c}}$

291. $\int \frac{dx}{\sqrt{(ax^2+bx+c)^3}} = \frac{2(2ax+b)}{(4ac-b^2)\sqrt{ax^2+bx+c}}$

292. $\int \frac{x\,dx}{\sqrt{(ax^2+bx+c)^3}} = \frac{2(bx+2c)}{(b^2-4ac)\sqrt{ax^2+bx+c}}$

293. $\int \frac{x^2\,dx}{\sqrt{(ax^2+bx+c)^3}} = \frac{(2b^2-4ac)x+2bc}{a(4ac)\sqrt{ax^2+bx+c}} + \frac{1}{a}\int \frac{dx}{\sqrt{ax^2+bx+c}}$

294. $\int \frac{dx}{x\sqrt{(ax^2+bx+c)^3}} = \frac{1}{c\sqrt{ax^2+bx+c}} + \frac{1}{c}\int \frac{dx}{x\sqrt{ax^2+bx+c}} - \frac{b}{2c}\int \frac{dx}{\sqrt{(ax^2+bx+c)^3}}$

295. $\int \frac{dx}{x^2\sqrt{(ax^2+bx+c)^3}} = \frac{-(ax^2+bx+c)}{c^2x\sqrt{ax^2+bx+c}} - \frac{3b}{2c^2}\int \frac{dx}{x\sqrt{ax^2+bx+c}} + \frac{b^2-4ac}{2c^2}\int \frac{dx}{\sqrt{(ax^2+bx+c)^3}}$

296. $\int \sqrt{(ax^2+bx+c)^{n+1}}\,dx = \frac{(2ax+b)\sqrt{(ax^2+bx+c)^{n+1}}}{4a(n+1)} + \frac{(2n+1)(4ac-b^2)}{8a(n+1)}\int \sqrt{(ax^2+bx+c)^{n-1}}\,dx$

297. $\int x\sqrt{(ax^2+bx+c)^{n+1}}\,dx = \frac{\sqrt{(ax^2+bx+c)^{n+3}}}{a(2n+3)} - \frac{b}{2a}\int \sqrt{(ax^2+bx+c)^{n+1}}\,dx$

298. $\int \frac{dx}{\sqrt{(ax^2+bx+c)^{n+1}}}\,dx = \frac{1}{(2n-1)(4ac-b^2)}\left(\frac{2(2ax+b)}{\sqrt{(ax^2+bx+c)^{n-1}}} + 8a(n-1)\int \frac{dx}{\sqrt{(ax^2+bx+c)^{n-1}}}\right)$

299.

INTEGRALES CON x^3+a^3

Para integrales con x^3-a^3, se remplaza a por -a

300. $\int \frac{dx}{x^3+a^3} = \frac{1}{6a^2}\ln\frac{(x+a)^2}{x^2-ax+a^2} + \frac{1}{a^2\sqrt{3}}\operatorname{tg}^{-1}\left(\frac{2x-a}{a\sqrt{3}}\right)$

301. $\int \frac{x\,dx}{x^3+a^3} = \frac{1}{6a}\ln\left(\frac{x^2-ax+a^2}{(x+a)^2}\right) + \frac{1}{a\sqrt{3}}\operatorname{tg}^{-1}\left(\frac{2x-a}{a\sqrt{3}}\right)$

302. $\int \frac{x^2\,dx}{x^3+a^3} = \frac{1}{3}\ln(x^3+a^3)$

303. $\int \frac{dx}{x(x^3+a^3)} = \frac{1}{3a^3}\ln\left(\frac{x^3}{x^3+a^3}\right)$

304. $\int \frac{dx}{x^2(x^3+a^3)} = -\frac{1}{a^3x} - \frac{1}{6a^4}\ln\frac{x^2-ax+a^2}{(x+a)^2} - \frac{1}{a^4\sqrt{3}}\,\mathrm{tg}^{-1}\left(\frac{2x-a}{a\sqrt{3}}\right)$

305. $\int \frac{dx}{(x^3+a^3)^2} = \frac{x}{3a^3(x^3+a^3)} + \frac{1}{9a^5}\ln\left(\frac{[x+a]^2}{x^2-ax+a^2}\right) + \frac{2}{3a^5\sqrt{3}}\,\mathrm{tg}^{-1}\left(\frac{2x-a}{a\sqrt{3}}\right)$

306. $\int \frac{xdx}{(x^3+a^3)^2} = \frac{x^2}{3a^3(x^3+a^3)} + \frac{1}{18a^4}\ln\left(\frac{x^2-ax+a^2}{[x+a]^2}\right) + \frac{1}{3a^4\sqrt{3}}\,\mathrm{tg}^{-1}\left(\frac{2x-a}{a\sqrt{3}}\right)$

307. $\int \frac{x^2dx}{(x^3+a^3)^2} = -\frac{1}{3(x^3+a^3)}$

308. $\int \frac{dx}{x(x^3+a^3)^2} = \frac{1}{3a^3(x^3+a^3)} + \frac{1}{3a^6}\ln\left(\frac{x^3}{x^3+a^3}\right)$

309. $\int \frac{dx}{x^2(x^3+a^3)^2} = -\frac{1}{a^6x} - \frac{x^2}{3a^6(x^3+a^3)} - \frac{4}{3a^6}\int\frac{xdx}{x^3+a^3}$ *Véase* 301

310. $\int \frac{x^m dx}{x^3+a^3} = \frac{x^{m-2}}{(m-2)} - a^3\int\frac{x^{m-3}dx}{x^3+a^3}$

311. $\int \frac{dx}{x^n(x^3+a^3)} = \frac{-1}{a^3(n-1)x^{n-1}} - \frac{1}{a^3}\int\frac{dx}{x^{n-3}(x^3+a^3)}$

INTEGRALES CON $x^4 \pm a^4$

312. $\int \frac{dx}{x^4+a^4} = \frac{1}{4a^3\sqrt{2}}\ln\left(\frac{x^2+ax\sqrt{2}+a^2}{x^2-ax\sqrt{2}+a^2}\right) - \frac{1}{2a^3\sqrt{2}}\,\mathrm{tg}^{-1}\left(\frac{ax\sqrt{2}}{x^2-a^2}\right)$

313. $\int \frac{xdx}{x^4+a^4} = \frac{1}{2a^2}\,\mathrm{tg}^{-1}\left(\frac{x^2}{a^2}\right)$

314. $\int \frac{x^2dx}{x^4+a^4} = \frac{1}{4a\sqrt{2}}\ln\left(\frac{x^2-ax\sqrt{2}+a^2}{x^2+ax\sqrt{2}+a^2}\right) - \frac{1}{2a\sqrt{2}}\,\mathrm{tg}^{-1}\left(\frac{ax\sqrt{2}}{x^2-a^2}\right)$

315. $\int \frac{x^3dx}{x^4+a^4=\frac{1}{4}}\ln(x^4+a^4)$

316. $\int \frac{dx}{x(x^4+a^4)} = \frac{1}{4a^4}\ln\left(\frac{x^4}{x^4+a^4}\right)$

317. $\int \frac{dx}{x^2(x^4+a^4)} = -\frac{1}{a^4x} - \frac{1}{4a^5\sqrt{2}}\ln\left(\frac{x^2-ax\sqrt{2}+a^2}{x^2+ax\sqrt{2}+a^2}\right) + \frac{1}{2a^5\sqrt{2}}\,\mathrm{tg}^{-1}\left(\frac{ax\sqrt{2}}{x^2-a^2}\right)$

318. $\int \frac{dx}{x^3(x^4+a^4)} = -\frac{1}{2a^4x^2} - \frac{1}{2a^5}\,\mathrm{tg}^{-1}\left(\frac{x^2}{a^2}\right)$

319. $\int \frac{dx}{x^4-a^4} = \frac{1}{4a^3}\ln\left(\frac{x-a}{x+a}\right) - \frac{1}{2a^3}\,\mathrm{tg}^{-1}\left(\frac{x}{a}\right)$

320. $\int \frac{xdx}{x^4-a^4} = \frac{1}{4a^2}\ln\left(\frac{x^2-a^2}{x^2+a^2}\right)$

321. $\int \frac{x^2dx}{x^4-a^4} = \frac{1}{4a}\ln\left(\frac{x-a}{x+a}\right) + \frac{1}{2a}\,\mathrm{tg}^{-1}\left(\frac{x}{a}\right)$

322. $\int \frac{x^3dx}{x^4-a^4} = \frac{1}{4}\ln(x^4-a^4)$

323. $\int \frac{dx}{x(x^4-a^4)} = \frac{1}{4a^4}\ln\left(\frac{x^4-a^4}{x^4}\right)$

324. $\int \frac{dx}{x^2(x^4-a^4)} = \frac{1}{a^4x} + \frac{1}{4a^5}\ln\left(\frac{x-a}{x+a}\right) + \frac{1}{2a^5}\,\mathrm{tg}^{-1}\left(\frac{x}{a}\right)$

325. $\int \frac{dx}{x^3(x^4-a^4)} = \frac{1}{2a^4x^2} + \frac{1}{4a^6}\ln\left(\frac{x^2-a^2}{x^2+a^2}\right)$

INTEGRALES CON $x^n \pm a^n$

326. $\int \frac{dx}{x(x^n+a^n)} = \frac{1}{na^n}\ln\left(\frac{x^n}{x^n+a^n}\right)$

327. $\int \frac{x^{n-1}dx}{x^n+a^n} = \frac{1}{n}\ln(x^n+a^n)$

328. $\int \frac{x^m dx}{(x^r+a^r)^n} = \int \frac{x^{m-r} dx}{(x^r+a^r)^{n-1}} - \frac{1}{a^r} \int \frac{dx}{x^{m-r}(x^r+a^r)^n}$

329. $\int \frac{dx}{x^m(x^r+a^r)^n} = \frac{1}{a^r} \int \frac{dx}{x^m(x^r+a^r)^{n-1}} - \frac{1}{a^r} \int \frac{dx}{x^{m-r}(x^r+a^r)^n}$

330. $\int \frac{dx}{x\sqrt{x^n+a^n}} = \frac{1}{n\sqrt{a^n}} \ln\left(\frac{\sqrt{x^n+a^n}-\sqrt{a^n}}{\sqrt{x^n+a^n}+\sqrt{a^n}}\right)$

331. $\int \frac{dx}{x(x^n-a^n)} = \frac{1}{na^n} \ln\left(\frac{x^n-a^n}{x^n}\right)$

332. $\int \frac{x^{n-1}dx}{x^n-a^n} = \frac{1}{n} \ln(x^n - a^n)$

333. $\int \frac{x^m dx}{(x^r-a^r)^n} = a^r \int \frac{x^{m-r} dx}{(x^r-a^r)^n} + \int \frac{x^{m-r} dx}{(x^r-a^r)^{n-1}}$

334. $\int \frac{dx}{x^m(\tau^r-a^r)^n} = \frac{1}{a^r} \int \frac{dx}{x^{m-r}(x^r-a^r)^n} - \frac{1}{a^r} \int \frac{dx}{x^m(x^r-a^r)^{n-1}}$

335. $\int \frac{dx}{x\sqrt{x^n-a^n}} = \frac{2}{n\sqrt{a^n}} \cos^{-1}\sqrt{\frac{a^n}{x^n}}$

336.
$$\int \frac{x^{p-1}dx}{(x^{2m}+a^{2m})} = \frac{1}{ma^{2m-p}} \sum_{k=1}^{m} \operatorname{sen}\left(\frac{[2k-1]p\pi}{2m}\right) \cdot \operatorname{tg}^{-1}\left(\frac{x+a\cos\left[\frac{(2k-1)\pi}{2m}\right]}{a\operatorname{sen}\left[\frac{(2k-1)\pi}{2m}\right]}\right) -$$
$$- \frac{1}{2ma^{2m-p}} \sum_{k=1}^{m} \cos\left(\frac{[2k-1]p\pi}{2m}\right) \cdot \ln\left\{x^2 + 2ax\cos\left(\frac{[2k-1]\pi}{2m}\right) + a^2\right\}$$

337.
$$\int \frac{x^{p-1}dx}{(x^{2m}-a^{2m}} = \frac{1}{2ma^{2m-p}} \sum_{k=1}^{m-1} \cos\frac{kp\pi}{m} \cdot \ln\left\{x^2 - 2ax\cos\left(\frac{k\pi}{m}\right) + a^2\right\} -$$
$$- \frac{1}{ma^{2m-p}} \sum_{k=1}^{m-1} \operatorname{sen}\frac{kp\lambda}{m} \cdot \operatorname{tg}^{-1}\left(\frac{x-a\cos\frac{k\pi}{m}}{a\operatorname{sen}\frac{k\lambda}{m}}\right) +$$
$$+ \frac{1}{2m\,a^{2m-p}}\{\ln(x-a) + (-1)^p \ln(x+a)\}$$

tanto en 336 como 337 es $0 < p \le 2m$

338.
$$\int \frac{x^{p-1}dx}{x^{2m+1}-a^{2m+1}} = \frac{2(-1)^{p-1}}{(2m+1)a^{2m-p+1}} \sum_{k=1}^{m} \operatorname{sen}\frac{2kp\pi}{2m+1} \cdot \operatorname{tg}^{-1}\left(\frac{x+a\cos\frac{2k\pi}{2m+1}}{a\operatorname{sen}\frac{2k\pi}{2m+1}}\right) -$$
$$- \frac{(-1)^{p-1}}{(2m+1)a^{2m-p+1}} \sum_{k=1}^{m} \cos\frac{2kp\pi}{2m+1} \cdot \ln\{x^2 - 2ax\cos\frac{2k\pi}{2m+1} + a^2\} +$$
$$+ \frac{(-1)^{p-1}\ln(x+a)}{(2m+1)a^{2m-p+1}}$$

339.
$$\int \frac{x^{p-1}dx}{x^{2m+1}-a^{2m-1}} = \frac{-2}{(2m+1)a^{2m-p+1}} \sum_{k=1}^{m} \operatorname{sen}\frac{2kp\pi}{2m+1} \cdot \operatorname{tg}^{-1}\left(\frac{x-a\cos\frac{2k\pi}{2m+1}}{a\operatorname{sen}\frac{2k\pi}{2m+1}}\right) +$$
$$+ \frac{1}{(2m+1)a^{2m-p+1}} \sum_{k=1}^{m} \cos\frac{2kp\pi}{2m+1} \cdot \ln\{x^2 - 2ax\cos\frac{2k\pi}{2m+1} + a^2\} +$$
$$+ \frac{\ln(x-a)}{(2m+1)a^{2m-p+1}}$$

tanto en 338 como en 339 es $0 < p \le 2m+1$

INTEGRALES CON sen ax

340. $\int \operatorname{sen} ax\, dx = -\frac{\cos ax}{a}$

341. $\int x \operatorname{sen} ax\, dx = \frac{\operatorname{sen} ax}{a^2} - \frac{x \cos ax}{a}$

342. $\int x^2 \operatorname{sen} ax\, dx = \frac{2x}{a^2}\operatorname{sen} ax + \left(\frac{2}{a^3} - \frac{x^2}{a}\right)\cos ax$

343. $\int x^3 \operatorname{sen} ax\, dx = \left(\frac{3x^2}{a^2} - \frac{6}{a^4}\right)\operatorname{sen} ax + \left(\frac{6x}{a^3} - \frac{x^3}{a}\right)\cos ax$

344. $\int \frac{\operatorname{sen} ax}{x}\, dx = ax - \frac{(ax)^3}{3\cdot 3!} + \frac{(ax)^5}{5\cdot 5!} - \cdots = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}(ax)^{2n-1}}{(2n-1)(2n-1)!}$

345. $\int \frac{\operatorname{sen} ax}{x^2}\, dx = -\frac{\operatorname{sen} ax}{x} + a\int \frac{\cos ax}{x}\, dx$ *Véase* 374

346. $\int \frac{dx}{\operatorname{sen} ax} = \frac{1}{a}\ln(\operatorname{cosec} ax - \operatorname{cotg} ax) = \frac{1}{a}\ln \operatorname{tg}\frac{ax}{2}$

347. $\int \frac{x\,dx}{\operatorname{sen} ax} = \frac{1}{a^2}\left\{ax + \frac{(ax)^3}{18} + \frac{7(ax)^5}{1800} + \cdots + \frac{2(2^{n-1}-1)B_n(ax)^{2n-1}}{(2n+1)!} + \cdots\right\}$ B_{bn} *es n° de Bernoulli*

348. $\int \operatorname{sen}^2 ax\, dx = \frac{x}{2} - \frac{\operatorname{sen} ax}{4a}$

349. $\int x \operatorname{sen}^2 dx = \frac{x^2}{4} - \frac{x \operatorname{sen} 2ax}{4a} - \frac{\cos 2ax}{8a^2}$

350. $\int \operatorname{sen}^3 ax\, dx = -\frac{\cos ax}{a} + \frac{\cos^3 ax}{3a}$

351. $\int \operatorname{sen}^4 ax\, dx = \frac{3x}{8} - \frac{\operatorname{sen} 2ax}{4a} + \frac{\operatorname{sen} 4ax}{32a}$

352. $\int \frac{dx}{\operatorname{sen}^2 ax} = -\frac{1}{a}\operatorname{cotg} ax$

353. $\int \frac{dx}{\operatorname{sen}^3 ax} = -\frac{\cos ax}{2a \operatorname{sen}^2 ax} + \frac{1}{2a}\ln \operatorname{tg}\frac{ax}{2}$

354. $\int \operatorname{sen} px \operatorname{sen} qx\, dx = \frac{\operatorname{sen}(p-q)x}{2(p-q)} - \frac{\operatorname{sen}(p+q)}{2(p+q)}$ *Si* $p = \pm q$, *véase* 348

355. $\int \frac{dx}{1-\operatorname{sen} ax} = \frac{1}{a}\operatorname{tg}\left[\frac{\pi}{4} + \frac{ax}{2}\right]$

356. $\int \frac{x\,dx}{1-\operatorname{sen} ax} = \frac{x}{a}\operatorname{tg}\left(\frac{\pi}{4} + \frac{ax}{2}\right) + \frac{2}{a^2}\ln \operatorname{sen}\left(\frac{\pi}{4} - \frac{ax}{2}\right)$

357. $\int \frac{dx}{1+\operatorname{sen} ax} = -\frac{1}{a}\operatorname{tg}\left(\frac{\pi}{4} - \frac{ax}{2}\right)$

358. $\int \frac{x\,dx}{1+\operatorname{sen} ax} = -\frac{x}{a}\operatorname{tg}\left(\frac{\pi}{4} - \frac{ax}{2}\right) + \frac{2}{a^2}\ln \operatorname{sen}\left(\frac{\pi}{4} + \frac{ax}{2}\right)$

359. $\int \frac{dx}{(1-\operatorname{sen} ax)^2} = \frac{1}{2a}\operatorname{tg}\left(\frac{\pi}{4} + \frac{ax}{2}\right) + \frac{1}{6a}\operatorname{tg}^3\left(\frac{\pi}{4} + \frac{ax}{2}\right)$

360. $\int \frac{dx}{(1+\operatorname{sen} ax)^2} = -\frac{1}{2a}\operatorname{tg}\left(\frac{\pi}{4} - \frac{ax}{2}\right) - \frac{1}{6a}\operatorname{tg}^3\left(\frac{\pi}{4} - \frac{ax}{2}\right)$

361. $\int \frac{dx}{p+q \operatorname{sen} ax} = \begin{cases} \frac{2}{a\sqrt{p^2-q^2}}\operatorname{tg}^{-1}\left(\frac{p \operatorname{tg}\frac{ax}{2}+p}{\sqrt{p^2-q^2}}\right) \\ \frac{1}{a\sqrt{q^2-p^2}}\ln\left(\frac{p \operatorname{tg}\frac{ax}{2}+q-\sqrt{q^2-p^2}}{p \operatorname{tg}\frac{ax}{2}+q+\sqrt{q^2-p^2}}\right) \end{cases}$ *Si* $p = \pm q$. *Véase* 355 *y* 357

362. $\int \frac{dx}{(p+q \operatorname{sen} ax)^2} = \frac{q \cos ax}{a(p^2-q^2)(p+q \operatorname{sen} ax)} + \frac{p}{(p^2-q^2)}\int \frac{dx}{p+q \operatorname{sen} ax}$ *Si* $p = \pm q$, *véase* 359 *y* 360

363. $\int \frac{dx}{p^2+q^2 \operatorname{sen}^2 ax} = \frac{1}{ap\sqrt{p^2+q^2}} \operatorname{tg}^{-1}\left(\frac{\sqrt{p^2+q^2}\cdot \operatorname{tg} ax}{p}\right)$

364. $\int \frac{dx}{p^2-q^2 \operatorname{sen}^2 ax} = \begin{cases} \frac{1}{ap\sqrt{p^2-q^2}} \operatorname{tg}^{-1}\left(\frac{\sqrt{p-q}\cdot \operatorname{tg} ax}{p}\right) \\ \frac{1}{2ap\sqrt{q^2-p^2}} \ln\left(\frac{\sqrt{q^2-p^2}\cdot \operatorname{tg} ax + p}{\sqrt{q^2-p^2}\cdot \operatorname{tg} ax - p}\right) \end{cases}$

365. $\int x^m \operatorname{sen} ax\, dx = -\frac{x^m \cos ax}{a} + \frac{mx^{m-1} \operatorname{sen} ax}{a^2} - \frac{m(m-1)}{a^2} \int x^{m-2} \operatorname{sen} ax\, dx$

366. $\int \frac{\operatorname{sen} ax}{x^n} dx = -\frac{\operatorname{sen} ax}{(n-1)x^{n-1}} + \frac{a}{n-1} \int \frac{\cos ax}{x^{n-1}} dx$ *Véase* 396

367. $\int \operatorname{sen}^n ax\, dx = -\frac{\operatorname{sen}^{n-1} ax \cos ax}{an} + \frac{n-1}{n} \int \operatorname{sen}^{n-2} ax\, dx$

368. $\int \frac{dx}{\operatorname{sen}^n ax} = \frac{-\cos ax}{a(n-1)\operatorname{sen}^{n-1} ax} + \frac{(n-2)}{(n-1)} \int \frac{dx}{\operatorname{sen}^{n-2} ax}$

369. $\int \frac{x dx}{\operatorname{sen}^n ax} = \frac{-x \cos ax}{a(n-1)\operatorname{sen}^{n-1} ax} - \frac{1}{a^2(n-1)(n-2)\operatorname{sen}^{n-2} ax} + \frac{(n-2)}{(n-1)} \int \frac{x dx}{\operatorname{sen}^{n-2} ax}$

INTEGRALES CON cos ax

370. $\int \cos ax\, dx = \frac{\operatorname{sen} ax}{a}$

371. $\int x \cos ax\, dx = \frac{\cos ax}{a^2} + \frac{x \operatorname{sen} ax}{a}$

372. $\int x^2 \cos ax\, dx = \frac{2x}{a^2} \cos ax + \left(\frac{x^2}{a} - \frac{2}{a^3}\right) \operatorname{sen} ax$

373. $\int x^3 \cos ax\, dx = \left(\frac{3x^2}{a^2} - \frac{6}{a^4}\right)\cos ax + \left(\frac{x^3}{a} - \frac{6x}{a^3}\right)\operatorname{sen} ax$

374. $\int \frac{\cos ax}{x} dx = \ln x - \frac{(ax)^2}{2\cdot 2!} + \frac{(ax)^4}{4\cdot 4!} - \frac{(ax)^6}{6\cdot 6!} + \cdots = \ln x + \sum_{n=1}^{\infty} \frac{(-1)^n (ax)^{2n}}{(2n)\cdot(2n)!}$

375. $\int \frac{\cos ax}{x^2} dx = -\frac{\cos ax}{x} - a \int \frac{\operatorname{sen} ax}{x} dx$ *Véase* 374

376. $\int \frac{dx}{\cos ax} = \frac{1}{a} \ln(\sec ax + \operatorname{tg} ax) = \frac{1}{a} \ln \operatorname{tg}\left(\frac{\pi}{4} + \frac{ax}{2}\right)$

377. $\int \frac{x dx}{\cos ax} = \frac{1}{a^2}\left\{\frac{(ax)^2}{2} + \frac{(ax)^4}{8} + \frac{5(ax)^6}{144} + \cdots + \frac{E_n (ax)^{2n+2}}{(2n+2)(2n)!} + \cdots\right\}$ E_n *es* n^0 *de Euler*

378. $\int \cos^2 ax\, dx = \frac{x}{2} + \frac{\operatorname{sen} 2ax}{4a}$

379. $\int x \cos^2 ax\, dx = \frac{x^2}{4} + \frac{x \operatorname{sen} 2ax}{4a} + \frac{\cos 2ax}{8a^2}$

380. $\int \cos^3 ax\, dx = \frac{\operatorname{sen} ax}{a} - \frac{\operatorname{sen}^3 ax}{3a}$

381. $\int \cos^4 ax\, dx = \frac{3x}{8} + \frac{\operatorname{sen} 2ax}{4a} + \frac{\operatorname{sen} 4ax}{32a}$

382. $\int \frac{dx}{\cos^2 ax} = \frac{1}{a} \operatorname{tg} ax$

383. $\int \frac{dx}{\cos^3 ax} = \frac{\operatorname{sen} ax}{2a\cos^3 ax} + \frac{1}{2a} \ln \operatorname{tg}\left(\frac{\pi}{4} + \frac{ax}{2}\right)$

384. $\int \cos px \cos qx \, dx = \frac{\operatorname{sen}(p-q)x}{2(p-q)} + \frac{\operatorname{sen}(p+q)x}{2(p+q)}$

385. $\int \frac{dx}{1-\cos ax} = \frac{1}{a} \operatorname{cotg} \frac{ax}{2}$

386. $\int \frac{x\,dx}{1-\cos ax} = -\frac{x}{a} \cot g \frac{ax}{2} + \frac{2}{a^2} \ln \operatorname{sen} \frac{ax}{2}$

387. $\int \frac{dx}{1+\cos ax} = \frac{1}{a} \operatorname{tg} \frac{ax}{2}$

388. $\int \frac{x\,dx}{1+\cos ax} = \frac{x}{a} \operatorname{tg} \frac{ax}{2} + \frac{2}{a^2} \ln \cos \frac{ax}{2}$

389. $\int \frac{dx}{(1-\cos ax)^2} = -\frac{1}{2a} \operatorname{cotg} \frac{ax}{2} - \frac{1}{6a} \operatorname{cotg}^3 \frac{ax}{2}$

390. $\int \frac{dx}{(1+\cos ax)^2} = \frac{1}{2a} \operatorname{tg} \frac{ax}{2} + \frac{1}{6a} \operatorname{tg}^3 \frac{ax}{2}$

391. $\int \frac{dx}{p+q\cos ax} = \begin{cases} \frac{2}{a\sqrt{p^2-q^2}} \operatorname{tg}^{-1} \sqrt{\frac{p-q}{p+q}} \cdot \operatorname{tg} \frac{ax}{2} \\ \frac{1}{a\sqrt{q^2-p^2}} \ln\left(\frac{\operatorname{tg}\frac{ax}{2} + \sqrt{\frac{q+p}{q-p}}}{\operatorname{tg}\frac{ax}{2} - \sqrt{\frac{q+p}{q-p}}} \right) \end{cases}$ *Si* $p = \pm q$, *Véanse* 385 *y* 387

392. $\int \frac{dx}{(p+q\cos ax)^2} = \frac{\operatorname{sen} ax}{a(q^2-p^2)(p+q\cos ax)} - \frac{p}{(q^2-p^2)} \int \frac{dx}{p+q\cos ax}$ *Si* $p = \pm q$, *véanse* 389 *y* 390

393. $\int \frac{dx}{p^2+q^2\cos^2 ax} = \frac{1}{ap\sqrt{p^2+q^2}} \operatorname{tg}^{-1} \frac{p \operatorname{tg} ax}{\sqrt{p^2+q^2}}$

394. $\int \frac{dx}{p^2-q^2\cos^2 ax} = \begin{cases} \frac{1}{ap\sqrt{p^2+q^2}} \operatorname{tg}^{-1} \frac{p \operatorname{tg} ax}{\sqrt{p^2+q^2}} \\ \frac{1}{2ap\sqrt{q^2-p^2}} \ln\left(\frac{p \operatorname{tg} ax - \sqrt{q^2-p^2}}{p \operatorname{tg} ax + \sqrt{q^2+p^2}} \right) \end{cases}$

395. $\int x^m \cos ax \, dx = \frac{x^m \operatorname{sen} ax}{a} + \frac{m x^{m-1} \cos ax}{a^2} - \frac{m(m-1)}{a^2} \int x^{m-2} \cos ax \, dx$

396. $\int \frac{\cos ax}{x^n} dx = -\frac{\cos ax}{(n-1)x^{n-1}} - \frac{a}{n-1} \int \frac{\operatorname{sen} ax}{x^{n-1}} dx$ *Véase* 366

397. $\int \cos^n ax \, dx = \frac{\cos^{n-1} ax \operatorname{sen} ax}{an} + \frac{n-1}{n} \int \cos^{n-2} ax \, dx$

398. $\int \frac{dx}{\cos^n ax} = \frac{\operatorname{sen} ax}{a(n-1)\cos^{n-1} ax} + \frac{(n-2)}{(n-1)} \int \frac{dx}{\cos^{n-2} ax}$

399. $\int \frac{x\,dx}{\cos^n ax} = \frac{x \operatorname{sen} ax}{a(n-1)\cos^{n-1} ax} - \frac{1}{a^2(n-1)(n-2)\cos^{n-2} ax} + \frac{(n-2)}{(n-1)} \int \frac{x\,dx}{\cos^{n-2} ax}$

INTEGRALES CON sen ax y cos ax

400. $\int \operatorname{sen} ax \cos ax \, dx = \frac{\operatorname{sen}^2 ax}{2a}$

401. $\int \operatorname{sen} px \cos qx \, dx = -\frac{\cos(p-q)x}{2(p-q)} - \frac{\cos(p+q)}{2(p+q)}$

402. $\int \operatorname{sen}^n ax \cos ax \, dx = \frac{\operatorname{sen}^{n+1} ax}{(n+1)a}$ *Si* $n = -1$, *véase* 441

403. $\int \operatorname{sen} ax \cos^n ax \, dx = -\frac{\cos^{n-1} ax}{(n+1)a}$ *Si* $n = -1$, *véase* 430

404. $\int \operatorname{sen}^2 ax \cos^2 ax \, dx = \frac{x}{8} - \frac{\operatorname{sen} 4ax}{32a}$

405. $\int \frac{dx}{\operatorname{sen} ax \cos ax} = \frac{1}{a} \ln \operatorname{tg} ax$

406. $\int \frac{dx}{\operatorname{sen}^2 ax \cos ax} = \frac{1}{a} \ln \operatorname{tg}\left[\frac{\pi}{4} + \frac{ax}{2}\right] - \frac{1}{a \operatorname{sen} ax}$

407. $\int \frac{dx}{\operatorname{sen} ax \cos^2 ax} = \frac{1}{a} \ln \operatorname{tg} \frac{ax}{2} + \frac{1}{a \cos ax}$

408. $\int \frac{dx}{\operatorname{sen}^2 ax \cos^2 ax} = -\frac{2 \cot g\, 2ax}{a}$

409. $\int \frac{\operatorname{sen}^2 ax}{\cos ax} dx = \frac{1}{a} \ln \operatorname{tg}\left(\frac{\pi}{4} + \frac{ax}{2}\right) - \frac{\operatorname{sen} ax}{a}$

410. $\int \frac{\cos^2 ax}{\operatorname{sen} ax} dx = \frac{1}{a} \ln \operatorname{tg} \frac{ax}{2} + \frac{\cos ax}{a}$

411. $\int \frac{dx}{(1 \pm \operatorname{sen} ax) \cos ax} = \pm \frac{1}{2a(1 \pm \operatorname{sen} ax)} + \frac{1}{2a} \ln \operatorname{tg}\left(\frac{\pi}{4} + \frac{ax}{2}\right)$

412. $\int \frac{dx}{(1 \pm \cos ax) \operatorname{sen} ax} = \pm \frac{1}{2a(1 \pm \cos ax)} + \frac{1}{2a} \ln \operatorname{tg} \frac{ax}{2}$

413. $\int \frac{dx}{\operatorname{sen} ax \pm \cos ax} = -\frac{1}{a\sqrt{2}} \ln \operatorname{tg}\left(\pm \frac{\pi}{8} + \frac{ax}{2}\right)$

414. $\int \frac{\operatorname{sen} ax\, dx}{\operatorname{sen} ax \pm \cos ax} = \frac{x}{2} \mp \frac{1}{2a} \ln(\operatorname{sen} ax \pm \cos ax)$

415. $\int \frac{\cos ax\, dx}{\operatorname{sen} ax \pm \cos ax} = \pm \frac{x}{2} + \frac{1}{2a} \ln(\operatorname{sen} ax \pm \cos ax)$

416. $\int \frac{\operatorname{sen} ax\, dx}{p + q \cos ax} = -\frac{1}{aq} \ln(p + q \cos ax)$

417. $\int \frac{\cos ax\, dx}{p + q \operatorname{sen} ax} = \frac{1}{aq} \ln(p + q \operatorname{sen} ax)$

418. $\int \frac{\operatorname{sen} ax\, dx}{(p + q \cos ax)^n} = \frac{1}{aq(n-1)(p + q \cos ax)^{n-1}}$

419. $\int \frac{\cos ax\, dx}{(p + q \operatorname{sen} ax)^n} = \frac{-1}{aq(n-1)(p + q \operatorname{sen} x)^{n-1}}$

420. $\int \frac{dx}{p \operatorname{sen} ax + q \cos ax} = \frac{1}{a\sqrt{p^2 + q^2}} \ln \operatorname{tg}\left(\frac{ax + \operatorname{tg}}{2}\right)$

421. $$\int \frac{dx}{p \operatorname{sen} ax + q \cos ax + r} = \begin{cases} \frac{2}{a\sqrt{r^2 - p^2 - q^2}} \operatorname{tg}^{-1}\left(\frac{p + (r-q) \operatorname{tg}(ax/2)}{\sqrt{r^2 - p^2 - q^2}}\right) \\ \frac{1}{a\sqrt{p^2 + q^2 - r^2}} \ln\left(\frac{p - \sqrt{p^2 + q^2 - r^2} + (r-q) \operatorname{tg}(ax/2)}{p + \sqrt{p^2 + q^2 - r^2} + (r-q) \operatorname{tg}(ax/2)}\right) \end{cases}$$

Si $r = q$ véase 422. Si $r^2 = p^2$ véase 423

422. $\int \frac{dx}{p \operatorname{sen} ax + q(1 + \cos ax)} = \frac{1}{ap} \ln\left(q + p \operatorname{tg} \frac{ax}{2}\right)$

423. $\int \frac{dx}{p \operatorname{sen} ax + q \cos ax \pm \sqrt{p^2 + q^2}} = \frac{-1}{a\sqrt{p^2 + q^2}} \operatorname{tg}\left(\frac{\pi}{4} \mp \frac{ax + \operatorname{tg}^{-1} q/p)}{2}\right)$

424. $\int \frac{dx}{p^2 \operatorname{sen}^2 ax + q^2 \cos^2 ax} = \frac{1}{apq} \operatorname{tg}^{-1}\left(\frac{p \operatorname{tg} ax}{q}\right)$

425. $\int \frac{dx}{p^2 \operatorname{sen}^2 ax - q^2 \cos^2 ax} = \frac{1}{2apq} \ln\left(\frac{p \operatorname{tg} ax - q}{p \operatorname{tg} ax + q}\right)$

426. $$\int \operatorname{sen}^m ax \cos^n ax\, dx = \begin{cases} -\frac{\operatorname{sen}^{m-1} ax \cos^{n-1} ax}{a(m+n)} + \frac{(m-1)}{(m+n)} \int \operatorname{sen}^{m-2} ax \cos^n ax\, dx \\ \frac{\operatorname{sen}^{m+1} ax \cos^{n-1} ax}{a(m+n)} + \frac{(n-1)}{(m+n)} \int \operatorname{sen}^m ax \cos^{n-2} ax\, dx \end{cases}$$

427. $$\int \frac{\operatorname{sen}^m ax}{\cos^n ax} dx = \begin{cases} \frac{\operatorname{sen}^{m-1} ax}{a(n-1)\cos^{n-1} ax} + \frac{m-1}{n-1} \int \frac{\operatorname{sen}^{m-2} ax}{\cos^{n-2} ax} dx \\ \frac{\operatorname{sen}^{m+1} ax}{a(n-1)\cos^{n-1} ax} - \frac{m-1}{n-1} \int \frac{\operatorname{sen}^m ax}{\cos^{n-2} ax} dx \\ \frac{-\operatorname{sen}^{m-1} ax}{a(m-n)\cos^{n-1} ax} + \frac{m-1}{n-1} \int \frac{\operatorname{sen}^{m-2} ax}{\cos^n ax} dx \end{cases}$$

428. $$\int \frac{\cos^m ax}{\operatorname{sen}^n ax}\,dx = \begin{cases} \dfrac{-\cos^{m-1} ax}{a(n-1)\operatorname{sen}^{n-1} ax} - \dfrac{m-1}{n-1}\displaystyle\int \frac{\cos^{m-2} ax}{\operatorname{sen}^{n-2} ax}\,dx \\ \dfrac{-\cos^{m+1} ax}{a(n-1)\operatorname{sen}^{n-1} ax} - \dfrac{m-n+2}{n-1}\displaystyle\int \frac{\cos^{m} ax}{\operatorname{sen}^{n-2} ax}\,dx \\ \dfrac{\cos^{m-1} ax}{a(m-n)\operatorname{sen}^{n-1} ax} + \dfrac{m-1}{n-1}\displaystyle\int \frac{\cos^{m-2} ax}{\operatorname{sen}^{n} ax}\,dx \end{cases}$$

429. $$\int \operatorname{sen}^m ax \cos^n ax\,dx = \begin{cases} \dfrac{1}{a(n-1)\operatorname{sen}^{m-1} ax \cos^{n-1} ax} + \dfrac{m+n-2}{n-1}\displaystyle\int \frac{dx}{\operatorname{sen}^{m} ax \cos^{n-2} ax} \\ \dfrac{-1}{a(m-1)\operatorname{sen}^{m-1} ax \cos^{n-1} ax} + \dfrac{m+n-2}{m-1}\displaystyle\int \frac{dx}{\operatorname{sen}^{m-2} ax \cos^{n} ax} \end{cases}$$

INTEGRALES CON tg ax

430. $\int \operatorname{tg} ax\,dx = -\frac{1}{a}\ln\cos ax = \frac{1}{a}\ln\sec ax$

431. $\int \operatorname{tg}^2 ax\,dx = \frac{\operatorname{tg} ax}{a} - x$

432. $\int \operatorname{tg}^3 ax\,dx = \frac{\operatorname{tg}^2 ax}{2a} + \frac{1}{a}\ln\cos ax$

433. $\int \operatorname{tg}^n ax \sec^2 ax\,dx = \frac{\operatorname{tg}^{n+1} ax}{(n+1)a}$

434. $\int \frac{\sec^2 ax}{\operatorname{tg} ax}\,dx = \frac{1}{a}\ln\operatorname{tg} ax$

435. $\int \frac{dx}{\operatorname{tg} ax} = \frac{1}{a}\ln\operatorname{sen} ax$

436. $\int x\operatorname{tg} ax\,dx = \frac{1}{a^2}\left\{\frac{(ax)^3}{3} + \frac{(ax)^5}{5} + \frac{2(ax)^7}{105} + \cdots + \frac{2^{2n}(2^{2n}-1)B_n(ax)^{2n+1}}{(2n+1)!} + \cdots\right\}$

B_n es n° de Bernoulli tanto en 436 como en 437.

437. $\int \frac{\operatorname{tg} ax}{x}\,dx = ax + \frac{(ax)^3}{9} + \frac{2(ax)^5}{75} + \cdots + \frac{2^{2n}(2^{2n}-1)B_n(ax)^{2n-1}}{(2n-1)(2n)!} + \cdots$

438. $\int x\operatorname{tg}^2 ax\,dx = \frac{x\operatorname{tg} ax}{a} + \frac{1}{a^2}\ln\cos ax - \frac{x^2}{2}$

439. $\int \frac{dx}{p+q\operatorname{tg} ax} = \frac{px}{p^2+q^2} + \frac{q}{a(p^2+q^2)}\ln(q\operatorname{sen} ax + p\cos ax)$

440. $\int \operatorname{tg}^n ax\,dx = \frac{\operatorname{tg}^{n-1} ax}{(n-1)a} - \int \operatorname{tg}^{n-2} ax\,dx$

INTEGRALES CON cotg ax

441. $\int \operatorname{cotg} ax\,dx = \frac{1}{a}\ln\operatorname{sen} ax$

442. $\int \operatorname{cotg}^2 ax\,dx = -\frac{\operatorname{cotg} ax}{a} - x$

443. $\int \operatorname{cotg}^3 ax\,dx = -\frac{\operatorname{cotg}^2 ax}{2a} - \frac{1}{a}\ln\operatorname{sen} ax$

444. $\int \operatorname{cotg}^n ax \operatorname{cosec}^2 ax\,dx = -\frac{\operatorname{cotg}^{n+1} ax}{(n+1)a}$

445. $\int \frac{\operatorname{cosec}^2 ax}{\operatorname{cotg} ax}\,dx = -\frac{1}{a}\ln\operatorname{cotg} ax$

446. $\int \frac{dx}{\operatorname{cotg} ax} = -\frac{1}{a}\ln\cos ax$

447. $\int x\operatorname{cotg} ax\,dx = \frac{1}{a^2}\left(ax - \frac{(ax)^3}{9} - \frac{(ax)^5}{225} - \cdots - \frac{2^{2n}B_n(ax)^{2n+1}}{(2n+1)!}\right)$

B_n es n° de Bernoulli tanto en 447 como en 448

448. $\int \frac{\operatorname{cotg} ax}{x}\,dx = -\frac{1}{ax} - \frac{ax}{3} - \frac{(ax)^3}{135} - \cdots - \frac{2^{2n}B_n(ax)^{2n-1}}{(2n-1)(2n)!}$

449. $\int x \cot g^2 ax\, dx = \frac{x \cot g\, ax}{a} + \frac{1}{a^2} \ln \operatorname{sen} ax - \frac{x^2}{2}$

450. $\int \frac{dx}{p+q \cot g\, ax} dx = \frac{px}{p^2+q^2} - \frac{q}{a(p^2+q^2)} \ln(q \operatorname{sen} ax + p \cos ax)$

451. $\int \cot g^n ax\, dx = -\frac{\cot g^{n-1} ax}{(n-1)a} - \int \cot g^{n-2} ax\, dx$

INTEGRALES CON sec ax

452. $\int \sec ax\, dx = \frac{1}{a} \ln(\sec ax + \operatorname{tg} ax) = \frac{1}{a} \ln \operatorname{tg}(\frac{\pi}{4} + \frac{ax}{2})$

453. $\int \sec^2 ax\, dx = \frac{\operatorname{tg} ax}{a}$

454. $\int \sec^3 ax\, dx = \frac{\sec ax \operatorname{tg} ax}{2a} + \frac{1}{2a} \ln(\sec ax + \operatorname{tg} ax)$

455. $\int \sec^n ax \operatorname{tg} ax\, dx = \frac{\sec^n ax}{na}$

456. $\int \frac{dx}{\sec ax} = \frac{\operatorname{sen} ax}{a}$

457. $\int x \sec ax\, dx = \frac{1}{a^2} \{ \frac{(ax)^2}{2} + \frac{(ax)^4}{8} + \frac{5(ax)^6}{144} + \cdots + \frac{E_n (ax)^{2n+2}}{(2n+2)(2n)!}$

E_n *es n° de Euler*

458. $\int \frac{\sec ax}{x} dx = \ln x + \frac{(ax)^2}{4} + \frac{5(ax)^4}{96} + \frac{61(ax)^6}{4320} + \cdots + \frac{E_n (ax)^{2n}}{2n(2n)!}$

459. $\int x \sec^2 ax\, dx = \frac{x}{a} \operatorname{tg} ax + \frac{1}{a^2} \ln \cos ax$

460. $\int \frac{dx}{q+p \sec ax} = \frac{x}{q} - \frac{p}{q} \int \frac{dx}{q+p \cos ax}$ *Véase* 391

461. $\int \sec^n ax\, dx = \frac{\sec^{n-2} ax \operatorname{tg} ax}{a(n-1)} + \frac{n-2}{n-1} \int \sec^{n-2} ax\, dx$

INTEGRALES CON cosec ax

462. $\int \operatorname{cosec} ax\, dx = \frac{1}{a} \ln(\operatorname{cosec} ax - \cot g\, ax) = \frac{1}{a} \ln \operatorname{tg} \frac{ax}{2}$

463. $\int \operatorname{cosec}^2 ax\, dx = -\frac{\cot g\, ax}{a}$

464. $\int \operatorname{cosec}^3 ax\, dx = -\frac{\operatorname{cosec} ax \cot g\, ax}{2a} + \frac{1}{2a} \ln \operatorname{tg} \frac{ax}{2}$

465. $\int \operatorname{cosec}^n ax \cot g\, ax\, dx = -\frac{\operatorname{cosec}^n ax}{na}$

466. $\int \frac{dx}{\operatorname{cosec}} = -\frac{\cos ax}{a}$

467. $\int x \operatorname{cosec} ax\, dx = \frac{1}{a^2} \{ ax + \frac{(ax)^3}{18} + \frac{7(ax)^5}{1800} + \cdots + \frac{2(2^{2n-1}-1) B_n (ax)^{2n+1}}{(2n+1)!} + \cdots \}$

B_n *es n° de Bernoulli*

468. $\int \frac{\operatorname{cosec} ax}{x} dx = -\frac{1}{ax} + \frac{ax}{6} + \frac{7(ax)^3}{1080} + \cdots + \frac{2(2^{n-1}-1) B_n (ax)^{2n-1}}{(2n-1)(2n)!} + \cdots \}$

469. $\int x \operatorname{cosec}^2 ax\, dx = -\frac{x}{a} \cot g\, ax + \frac{1}{a^2} \ln \operatorname{sen} ax$

470. $\int \frac{dx}{q+p \operatorname{cosec} ax} = \frac{x}{p} - \frac{p}{q} \int \frac{dx}{q+p \operatorname{sen} ax}$ *Véase* 361

471. $\int \operatorname{cosec} ax\, dx = -\frac{\operatorname{cosec}^{n-2} ax \cot g\, ax}{a(n-1)} + \frac{n-2}{n-1} \int \operatorname{cosec}^{n-2} ax\, dx$

INTEGRALES DE FUNCIONES TRIGONOMETRICAS INVERSAS

 472. $\int \operatorname{sen}^{-1} \frac{x}{a} dx = x \operatorname{sen}^{-1} \frac{x}{a} + \sqrt{a^2 - x^2}$

473. $\int x \operatorname{sen}^{-1}\frac{x}{a}\,dx = \left(\frac{x^2}{2}-\frac{a^2}{4}\right)\operatorname{sen}^{-1}\frac{x}{a} + \frac{x\sqrt{a^2-x^2}}{4}$

474. $\int x^2 \operatorname{sen}^{-1}\frac{x}{a}\,dx = \frac{x^3}{3}\operatorname{sen}^{-1}\frac{x}{a} + \frac{(x^2+2a^2)\sqrt{a^2-x^2}}{9}$

475. $\int x^m \operatorname{sen}^{-1}\frac{x}{a}\,dx = \frac{x^{m+1}}{m+1}\operatorname{sen}^{-1}\frac{x}{a} - \frac{1}{m+1}\int \frac{x^{m+1}}{\sqrt{a^2-x^2}}\,dx$

476. $\int \frac{\operatorname{sen}^{-1}(x/a)}{x}\,dx = \frac{x}{a} + \frac{(x/a)^3}{2\cdot 3\cdot 3} + \frac{1\cdot 3(x/a)^5}{2\cdot 4\cdot 5\cdot 5} + \frac{1\cdot 3\cdot 5(x/a)^7}{2\cdot 4\cdot 6\cdot 7\cdot 7} + \cdots$

477. $\int \frac{\operatorname{sen}^{-1}(x/a)}{x^2}\,dx = -\frac{\operatorname{sen}^{-1}(x/a)}{x} - \frac{1}{a}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$

478. $\int \left(\operatorname{sen}^{-1}\frac{x}{a}\right)^2 dx = x\left(\operatorname{sen}^{-1}\frac{x}{a}\right)^2 - 2x + 2\sqrt{a^2-x^2}\operatorname{sen}^{-1}\frac{x}{a}$

479. $\int \cos^{-1}\frac{x}{a}\,dx = x\cos^{-1}\frac{x}{a} - \sqrt{a^2-x^2}$

480. $\int x\cos^{-1}\frac{x}{a}\,dx = \left(\frac{x^2}{2}-\frac{a^2}{4}\right)\cos^{-1}\frac{x}{a} - \frac{\sqrt{a^2-x^2}}{4}$

481. $\int x^2\cos^{-1}\frac{x}{a}\,dx = \frac{x^3}{3}\cos^{-1}\frac{x}{a} - \frac{(x^2+2a^2)\sqrt{a^2-x^2}}{9}$

482. $\int x^m\cos^{-1}\frac{x}{a}\,dx = \frac{x^{m+1}}{m+1}\cos^{-1}\frac{x}{a} + \frac{1}{m+1}\int \frac{x^{m+1}}{\sqrt{a^2-x^2}}\,dx$

483. $\int \frac{\cos^{-1}(x/a)}{x}\,dx = \frac{\pi}{2}\ln x - \int \frac{\operatorname{sen}^{-1}(x/a)}{x}\,dx$ *Véase 476*

484. $\int \frac{\cos^{-1}(x/a)}{x^2}\,dx = -\frac{\cos^{-1}(x/a)}{x} + \frac{1}{a}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$

485. $\int \left(\cos^{-1}\frac{x}{a}\right)^2 dx = x\left(\cos^{-1}\frac{x}{a}\right)^2 - 2x - 2\sqrt{a^2-x^2}\cos^{-1}\frac{x}{a}$

486. $\int \operatorname{tg}^{-1}\frac{x}{a}\,dx = x\operatorname{tg}^{-1}\frac{x}{a} - \frac{a}{2}\ln(x^2+a^2)$

487. $\int x\operatorname{tg}^{-1}\frac{x}{a}\,dx = \frac{1}{2}(x^2+a^2)\operatorname{tg}^{-1}\frac{x}{a} - \frac{ax}{2}$

488. $\int x^2\operatorname{tg}^{-1}\frac{x}{a}\,dx = \frac{x^3}{3}\operatorname{tg}^{-1}\frac{x}{a} - \frac{ax^2}{6} + \frac{a^3}{6}\ln(x^2+a^2)$

489. $\int x^m\operatorname{tg}^{-1}\frac{x}{a}\,dx = \frac{x^{m+1}}{m+1}\operatorname{tg}^{-1}\frac{x}{a} - \frac{a}{m+1}\int \frac{x^{m+1}}{x^2+a^2}\,dx$

490. $\int \frac{\operatorname{tg}^{-1}(x/a)}{x}\,dx = \frac{x}{a} - \frac{(x/a)^3}{3^2} + \frac{(x/a)^5}{5^2} - \frac{(x/a)^7}{7^2} + \cdots$

491. $\int \frac{\operatorname{tg}^{-1}(x/a)}{x^2}\,dx = -\frac{1}{x}\operatorname{tg}^{-1}\frac{x}{a} - \frac{1}{2a}\ln\left(\frac{x^2+a^2}{x^2}\right)$

492. $\int \operatorname{cotg}^{-1}\frac{x}{a}\,dx = x\operatorname{cotg}^{-1}\frac{x}{a} + \frac{a}{2}\ln(x^2+a^2)$

493. $\int x\operatorname{cotg}^{-1}\frac{x}{a}\,dx = \frac{1}{2}(x^2+a^2)\operatorname{cotg}^{-1}\frac{x}{a} + \frac{ax}{2}$

494. $\int x^2\operatorname{cotg}^{-1}\frac{x}{a}\,dx = \frac{x^3}{3}\operatorname{cotg}^{-1}\frac{x}{a} + \frac{ax^2}{6} - \frac{a^3}{6}\ln(x^2+a^2)$

495. $\int x^m\operatorname{cotg}^{-1}\frac{x}{a}\,dx = \frac{x^{m+1}}{m+1}\operatorname{cotg}^{-1}\frac{x}{a} + \frac{a}{m+1}\int \frac{x^{m+1}}{x^2+a^2}\,dx$

496. $\int \frac{\operatorname{cotg}^{-1}(x/a)}{x}\,dx = \frac{\pi}{2}\ln x - \int \frac{\operatorname{tg}^{-1}(x/a)}{x}\,dx$ *Véase 490*

497. $\int \frac{\operatorname{cotg}^{-1}(x/a)}{x^2}\,dx = -\frac{1}{x}\operatorname{cotg}^{-1}\frac{x}{a} + \frac{1}{2a}\ln\left(\frac{x^2+a^2}{x^2}\right)$

498. $\int \sec^{-1}\frac{x}{a}\,dx = \begin{cases} x\sec^{-1}\frac{x}{a} - a\ln(x+\sqrt{x^2-a^2}); & 0<\sec^{-1}\frac{x}{a}<\frac{\pi}{2} \\ x\sec^{-1}\frac{x}{a} + a\ln(x+\sqrt{x^2-a^2}); & \frac{\pi}{2}<\sec^{-1}\frac{x}{a}<\pi \end{cases}$

499. $\int x\sec^{-1}\frac{x}{a}\,dx = \begin{cases} \frac{x^2}{2}\sec^{-1}\frac{x}{a} - \frac{a\sqrt{x^2-a^2}}{2}; & 0<\sec^{-1}\frac{x}{a}<\frac{\pi}{2} \\ \frac{x^2}{2}\sec^{-1}\frac{x}{a} + \frac{a\sqrt{x^2-a^2}}{2}; & \frac{\pi}{2}<\sec^{-1}\frac{x}{a}<\pi \end{cases}$

500. $$\int x^2 \sec^{-1}\frac{x}{a}\,dx = \begin{cases} \frac{x^3}{3}\sec^{-1}\frac{x}{a} - \frac{ax\sqrt{x^2-a^2}}{2} - \frac{a^3}{6}\ln(x+\sqrt{x^2-a^2}); & 0<\sec^{-1}\frac{x}{a}<\frac{\pi}{2} \\ \frac{x^3}{3}\sec^{-1}\frac{x}{a} + \frac{ax\sqrt{x^2-a^2}}{2} + \frac{a^3}{6}\ln(x+\sqrt{x^2-a^2}); & \frac{\pi}{2}<\sec^{-1}\frac{x}{a}<\pi \end{cases}$$

501. $$\int x^m \sec^{-1}\frac{x}{a}\,dx = \begin{cases} \frac{x^{m+1}}{m+1}\sec^{-1}\frac{x}{a} - \frac{a}{m+1}\int\frac{x^m\,dx}{\sqrt{x^2-a^2}}; & 0<\sec^{-1}\frac{x}{a}<\frac{\pi}{2} \\ \frac{x^{m+1}}{m+1}\sec^{-1}\frac{x}{a} + \frac{a}{m+1}\int\frac{x^m\,dx}{\sqrt{x^2-a^2}}; & \frac{\pi}{2}<\sec^{-1}<\pi \end{cases}$$

502. $$\int\frac{\sec^{-1}(x/a)}{x}\,dx = \frac{\pi}{2}\ln x + \frac{a}{x} + \frac{(x/a)^3}{2\cdot3\cdot3} + \frac{1\cdot3\,(x/a)^5}{2\cdot4\cdot5\cdot5} + \frac{1\cdot3\cdot5\,(x/a)^7}{2\cdot4\cdot6\cdot7\cdot7} + \cdots$$

503. $$\int\frac{\sec^{-1}(x/a)}{x^2}\,dx = \begin{cases} -\frac{\sec^{-1}(x/a)}{x} + \frac{\sqrt{x^2-a^2}}{ax}; & 0<\sec^{-1}\frac{x}{a}<\frac{\pi}{2} \\ -\frac{\sec^{-1}(x/a)}{x} - \frac{\sqrt{x^2-a^2}}{ax}; & \frac{\pi}{2}<\sec^{-1}\frac{x}{a}<\pi \end{cases}$$

504. $$\int\operatorname{cosec}^{-1}\frac{x}{a}\,dx = \begin{cases} x\operatorname{cosec}^{-1}\frac{x}{a} + a\ln(x+\sqrt{x^2-a^2}); & 0<\operatorname{cosec}^{-1}\frac{x}{a}<\frac{\pi}{2} \\ x\operatorname{cosec}^{-1}\frac{x}{a} - a\ln(x+\sqrt{x^2-a^2}); & -\frac{\pi}{2}<\operatorname{cosec}^{-1}\frac{x}{a}<0 \end{cases}$$

505. $$\int x\operatorname{cosec}^{-1}\frac{x}{a}\,dx = \begin{cases} \frac{x^2}{2}\operatorname{cosec}^{-1}\frac{x}{a} + \frac{a\sqrt{x^2-a^2}}{2}; & 0<\operatorname{cosec}^{-1}\frac{x}{a}<\frac{\pi}{2} \\ \frac{x^2}{2}\operatorname{cosec}^{-1}\frac{x}{a} - \frac{a\sqrt{x^2-a^2}}{2}; & -\frac{\pi}{2}<\operatorname{cosec}^{-1}\frac{x}{a}<0 \end{cases}$$

506. $$\int x^2\operatorname{cosec}^{-1}\frac{x}{a}\,dx = \begin{cases} \frac{x^3}{3}\operatorname{cosec}^{-1}\frac{x}{a} + \frac{ax\sqrt{x^2-a^2}}{2} + \frac{a^3}{6}\ln(x+\sqrt{x^2-a^2}); & 0<\operatorname{cosec}^{-1}\frac{x}{a}<\frac{\pi}{2} \\ \frac{x^3}{3}\operatorname{cosec}^{-1}\frac{x}{a} - \frac{ax\sqrt{x^2-a^2}}{2} - \frac{a^3}{6}\ln(x+\sqrt{x^2-a^2}); & -\frac{\pi}{2}<\operatorname{cosec}^{-1}\frac{x}{a}<0 \end{cases}$$

507. $$\int x^m\operatorname{cosec}^{-1}\frac{x}{a}\,dx = \begin{cases} \frac{x^{m+1}}{m+1}\operatorname{cosec}^{-1}\frac{x}{a} + \frac{a}{m+1}\int\frac{x^m\,dx}{\sqrt{x^2-a^2}}; & 0<\operatorname{cosec}^{-1}\frac{x}{a}<\frac{\pi}{2} \\ \frac{x^{m+1}}{m+1}\operatorname{cosec}^{-1}\frac{x}{a} - \frac{a}{m+1}\int\frac{x^m\,dx}{\sqrt{x^2-a^2}}; & \frac{\pi}{2}<\operatorname{cosec}^{-1}\frac{x}{a}<\pi \end{cases}$$

508. $$\int\frac{\operatorname{cosec}^{-1}(x/a)}{x}\,dx = -\left(\frac{a}{x} + \frac{(x/a)^3}{2\cdot3\cdot3} + \frac{1\cdot3\cdot(x/a)^5}{2\cdot4\cdot5\cdot5} + \frac{1\cdot3\cdot5\,(x/a)^7}{2\cdot4\cdot6\cdot7\cdot7} + \cdots\right.$$

509. $$\int\frac{\operatorname{cosec}^{-1}(x/a)}{x^2}\,dx = \begin{cases} -\frac{\operatorname{cosec}^{-1}(x/a)}{x} - \frac{\sqrt{x^2-a^2}}{ax}; & 0<\operatorname{cosec}^{-1}\frac{x}{a}<\frac{\pi}{2} \\ -\frac{\operatorname{cosec}^{-1}(x/a)}{x} + \frac{\sqrt{x^2-a^2}}{ax}; & -\frac{\pi}{2}<\sec^{-1}\frac{x}{a}<0 \end{cases}$$

INTEGRALES CON e^{ax}

510. $$\int e^{ax}\,dx = \frac{e^{ax}}{a}$$

511. $$\int x\,e^{ax}\,dx = \frac{e^{ax}}{a}\left(x - \frac{1}{a}\right)$$

512. $$\int x^2 e^{ax}\,dx = \frac{e^{ax}}{a}\left(x^2 - \frac{2x}{a} + \frac{2}{a^2}\right)$$

513. $$\int x^n e^{ax}\,dx = \frac{x^n e^{ax}}{a} - \frac{n}{a}\int x^{n-1}e^{ax}\,dx \quad \left(= \sum_{k=1}^{n}\frac{(-1)^k k!}{a^k}\right.$$ *Si n es natural*)

514. $$\int\frac{e^{ax}}{x}\,dx = \ln x + \frac{ax}{1\cdot1!} + \frac{(ax)^2}{2\cdot2!} + \frac{(ax)^3}{3\cdot3!} + \cdots$$

515. $$\int\frac{e^{ax}}{x^n}\,dx = \frac{-e^{ax}}{(n-1)x^{n-1}} + \frac{a}{n-1}\int\frac{e^{ax}}{x^{n-1}}\,dx$$

516. $$\int\frac{dx}{p+q\,e^{ax}} = \frac{x}{p} - \frac{1}{ap}\ln(p+q\,e^{ax})$$

517. $$\int\frac{dx}{(p+q\,e^{ax})^2} = \frac{x}{p^2} + \frac{1}{ap\,(p+q\,e^{ax})} - \frac{1}{ap^2}\ln(p+q\,e^{ax})$$

518. $\int \frac{dx}{p\, e^{ax} + q\, e^{ax}} = \begin{cases} \frac{1}{a\sqrt{pq}} \operatorname{tg}^{-1}\left(\sqrt{\frac{p}{q}}\, e^{ax}\right) \\ \frac{1}{2a\sqrt{-pq}} \ln\left(\frac{e^{ax} - \sqrt{-q/p}}{e^{ax} + \sqrt{-q/p}}\right) \end{cases}$

519. $\int e^{ax} \cdot \operatorname{sen} bx dx = \frac{e^{ax}\,(a \operatorname{sen} bx - b \cos bx)}{a^2 + b^2}$

520. $\int e^{ax} \cos bx\, dx = \frac{e^{ax}\,(a \cos bx + b \operatorname{sen} bx)}{a^2 + b^2}$

521. $\int x e^{ax} \operatorname{sen} bx dx = \frac{x e^{ax}\,(a \operatorname{sen} bx - b \cos bx)}{a^2+b^2} - \frac{e^{ax}\{(a^2-b^2)\operatorname{sen} bx - 2ab \cos bx\}}{(a^2+b^2)^2}$

522. $\int x\, e^{ax} \cos bx dx = \frac{x e^{ax}\,(a \cos bx - b \operatorname{sen} bx)}{a^2+b^2} - \frac{e^{ax}\{(a^2-b^2)\cos bx + 2ab \operatorname{sen} bx}{(a^2+b^2)^2}$

523. $\int e^{ax} \ln x\, dx = \frac{e^{ax} \ln x}{a} - \frac{1}{a} \int \frac{e^{ax}}{x} dx$

524. $\int e^{ax} \operatorname{sen}^n bx dx = \frac{e^{ax} \operatorname{sen}^{-1} bx\,(a \operatorname{sen} bx - nb \cos bx)}{a^2+n^2b^2} + \frac{n(n+1)b^2}{a^2+n^2b^2} \int e^{ax} \operatorname{sen}^{n-2} bx dx$

525. $\int e^{ax} \cos^n bx dx = \frac{e^{ax} \cos^{-1} bx\,(a \cos bx + nb \operatorname{sen} bx)}{a^2+n^2b^2} + \frac{n(n-1)b^2}{a^2+n^2b^2} \int e^{ax} \cos^{n-2} bx dx$

INTEGRALES CON lnx

526. $\int \ln x\, dx = x \ln x - x$

527. $\int x \ln x\, dx = \frac{x^2}{2}(\ln x - \frac{1}{2})$

528. $\int x^m \ln x\, dx = \frac{x^{m+1}}{m+1}(\ln x - \frac{1}{m+1})$ *Si* $m = -1$, *véase* 529

529. $\int \frac{\ln x}{x} dx = \frac{1}{2} \ln^2 x$

530. $\int \frac{\ln x}{x^2} dx = -\frac{\ln x}{x} - \frac{1}{x}$

531. $\int \ln^2 x\, dx = x \ln^2 x - 2x \ln x + 2x$

532. $\int \frac{\ln^n x}{x} dx = \frac{\ln^{n+1} x}{n+1}$ *Si* $n = -1$, *véase* 533

533. $\int \frac{dx}{x \ln x} = \ln(\ln x)$

534. $\int \frac{dx}{\ln x} = \ln(\ln x) + \ln x + \frac{\ln^2 x}{2 \cdot 2!} + \frac{\ln^3 x}{3 \cdot 3!} + \cdots$

535. $\int \frac{x^m dx}{\ln x} = \ln(\ln x) + (m+1) \ln x + \frac{(m+1)^2 \ln^2 x}{2 \cdot 2!} + \frac{(m+1)^3 \ln^3 x}{3 \cdot 3!} + \cdots$

536. $\int \ln^n x\, dx = x \ln^n x - n \int \ln^{n-1} x\, dx$

537. $\int x^m \ln^n x\, dx = \frac{\ln^n x}{m+1} - \frac{n}{m+1} \int x^m \ln^{n-1} x\, dx$ *Si* $m = -1$, *véase* 532

538. $\int \ln(x^2 + a^2) dx = x \ln(x^2 + a^2) - 2x + 2a \operatorname{tg}^{-1} \frac{x}{a}$

539. $\int \ln(x^2 - a^2) dx = x \ln(x^2 - a^2) - 2x + a \ln\left(\frac{x+a}{x-a}\right)$

540. $\int x^m \ln(x^2 \pm a^2) dx = \frac{x^{m+1} \ln(x^2 \pm a^2)}{m+1} - \frac{2}{m+1} \int \frac{x^{m+2}}{(x^2 \pm a^2)} dx$

INTEGRALES CON senh ax

541. $\int \operatorname{senh} ax\, dx = \frac{\cosh ax}{a}$

542. $\int x \operatorname{senh} ax\, dx = -\frac{\operatorname{senh} ax}{a^2} + \frac{x \cosh ax}{a}$

543. $\int x^2 \operatorname{senh} ax\, dx = -\frac{2x}{a^2}\operatorname{senh} ax + \left(\frac{2}{a^3} + \frac{x^2}{a}\right)\cosh ax$

544. $\int \frac{\operatorname{senh} ax}{x} dx = ax + \frac{(ax)^3}{3\cdot 3!} + \frac{(ax)^5}{5\cdot 5!} - \cdots = \sum^{\infty} \frac{(ax)^{2n-1}}{(2n-1)!}$

545. $\int \frac{\operatorname{senh} ax}{x^2} dx = -\frac{\operatorname{senh} ax}{x} + a\int \frac{\cosh ax}{x} dx$ *Véase 566*

546. $\int \frac{dx}{\operatorname{senh} ax} = \frac{1}{a}\ln \operatorname{tgh}\frac{ax}{2}$

547. $\int \frac{x\,dx}{\operatorname{senh} ax} = \frac{1}{a^2}\left\{ax - \frac{(ax)^3}{18} + \frac{7(ax)^5}{1800} + \cdots + \frac{2(-1)^n(2^{2n-1}-1)B_n(ax)^{2n+1}}{(2n+1)!} + \cdots\right\}$

B_n es n° de Bernoulli

548. $\int \operatorname{senh}^2 ax\, dx = -\frac{x}{2} + \frac{\operatorname{senh} ax \cdot \cosh ax}{2a}$

549. $\int x \operatorname{senh}^2 ax\, dx = -\frac{x^2}{4} + \frac{x \operatorname{senh} 2ax}{4a} - \frac{\cosh 2ax}{8a^2}$

550. $\int \frac{dx}{\operatorname{senh}^2 ax} = -\frac{1}{a}\operatorname{cotgh} ax$

551. $\int \operatorname{senh} px \operatorname{senh} qx\, dx = -\frac{\operatorname{senh}(p-q)x}{2(p-q)} + \frac{\operatorname{senh}(p+q)x}{2(p+q)}$ *Si $p = \pm q$, véase 548*

552. $\int \operatorname{senh} px \operatorname{sen} qx\, dx = \frac{p\cosh px \operatorname{sen} qx - q\operatorname{senh} px \cos qx}{p^2+q^2}$

553. $\int \operatorname{senh} px \cos qx\, dx = \frac{p\cosh px \cos qx + q\operatorname{senh} px \operatorname{sen} qx}{p^2+q^2}$

554. $\int \frac{dx}{p+q\operatorname{senh} ax} = \frac{1}{a\sqrt{p^2+q^2}}\ln\left(\frac{qe^{ax}+p-\sqrt{p^2+q^2}}{qe^{ax}+p+\sqrt{p^2+q^2}}\right)$

555. $\int \frac{dx}{(p+q\operatorname{senh} ax)^2} = \frac{-q\cosh ax}{a(p^2+q^2)(p+q\operatorname{senh} ax)} + \frac{p}{(p^2+q^2)}\int \frac{dx}{p+q\operatorname{senh} ax}$

556. $\int \frac{dx}{p^2+q^2\operatorname{senh}^2 ax} = \begin{cases} \frac{1}{ap\sqrt{q^2-p^2}}\operatorname{tg}^{-1}\frac{\sqrt{q^2-p^2}\cdot\operatorname{tgh} ax}{p} \\ \frac{1}{2ap\sqrt{p^2-q^2}}\ln\left(\frac{p+\sqrt{p^2-q^2}\cdot\operatorname{tgh} ax}{p-\sqrt{p^2-q^2}\cdot\operatorname{tgh} ax}\right)\end{cases}$

557. $\int \frac{dx}{p^2-q^2\operatorname{senh}^2 ax} = \frac{1}{2ap\sqrt{p^2+q^2}}\ln\left(\frac{p+\sqrt{p^2+q^2}\cdot\operatorname{tgh} ax}{p-\sqrt{p^2+q^2}\cdot\operatorname{tgh} ax}\right)$

558. $\int x^m \operatorname{senh} ax\, dx = \frac{x^m\cosh ax}{a} - \frac{m}{a}\int x^{m-1}\cosh ax\, dx$ *Véase 586*

559. $\int \frac{\operatorname{senh} ax}{x^n} dx = -\frac{\operatorname{senh} ax}{(n-1)x^{n-1}} + \frac{a}{n-1}\int \frac{\cosh ax}{x^{n-1}} dx$ *Véase 587*

560. $\int \operatorname{senh}^n ax\, dx = \frac{\operatorname{senh}^{n-1} ax \cosh ax}{an} - \frac{n-1}{n}\int \operatorname{senh}^{n-2} ax\, dx$

561. $\int \frac{dx}{\operatorname{senh}^n ax} = \frac{-\cosh ax}{a(n-1)\operatorname{senh}^{n-1} ax} - \frac{n-2}{n-1}\int \frac{dx}{\operatorname{senh}^{n-2} ax}$

562. $\int \frac{x\,dx}{\operatorname{senh}^n ax} = \frac{-x\cosh ax}{a(n-1)\operatorname{senh}^{n-1} ax} - \frac{1}{a^2(n-1)(n-2)\operatorname{senh}^{n-2} ax} - \frac{n-2}{n-1}\int \frac{x\,dx}{\operatorname{senh}^{n-2} ax}$

INTEGRALES CON cosh ax

563. $\int \cosh ax\, dx = \frac{\operatorname{senh} ax}{a}$

564. $\int x\cosh ax\, dx = -\frac{\cosh ax}{a^2} + \frac{x\operatorname{senh} ax}{a}$

565. $\int x^2\cosh ax\, dx = -\frac{2x}{a^2}\cosh ax + \left(\frac{x^2}{a} + \frac{2}{a^3}\right)\operatorname{senh} ax$

566. $\int \frac{\cosh ax}{x} dx = \ln x + \frac{(ax)^2}{2\cdot 2!} + \frac{(ax)^4}{4\cdot 4!} + \frac{(ax)^6}{6\cdot 6!} + \cdots = \ln x + \sum_{n=1}^{\infty}\frac{(ax)^{2n}}{(2n)\cdot(2n)!}$

567. $\int \frac{\cosh ax}{x^2}dx = -\frac{\cosh ax}{x} + a\int \frac{\operatorname{senh} ax}{x}\,dx$ *Véase 544*

568. $\int \frac{dx}{\cosh ax} = \frac{2}{a}\operatorname{tg}^{-1} e^{ax}$

569. $\int \frac{x\,dx}{\cosh ax} = \frac{1}{a^2}\left\{\frac{(ax)^2}{2} - \frac{(ax)^4}{8} + \frac{5(ax)^6}{144} - \cdots + \frac{(-1)^n E_n (ax)^{2n+2}}{(2n+2)(2n)!} + \cdots\right\}$ E_n *es n° de Euler*

570. $\int \cosh^2 ax\,dx = \frac{x}{2} + \frac{\operatorname{senh} ax \cosh ax}{2}$

571. $\int x\cosh^2 ax\,dx = \frac{x^2}{4} + \frac{x\operatorname{senh} 2ax}{4a} - \frac{\cosh 2ax}{8a^2}$

572. $\int \frac{dx}{\cosh^2 ax} = \frac{1}{a}\operatorname{tgh} ax$

573. $\int \cosh px \cosh qx\,dx = \frac{\operatorname{senh}(p-q)x}{2(p-q)} + \frac{\operatorname{senh}(p+q)x}{2(p+q)}$ *Si* $p = \pm q$, *véase* 570

574. $\int \cosh px \operatorname{sen} qx\,dx = \frac{p\operatorname{senh} px \operatorname{sen} qx - q\cosh px \cos qx}{p^2+q^2}$

575. $\int \cosh px \cos qx\,dx = \frac{p\operatorname{senh} px \cos qx + q\cosh px \cos qx}{p^2+q^2}$

576. $\int \frac{dx}{1-\cosh ax} = \frac{1}{a}\operatorname{cot}gh\,\frac{ax}{2}$

577. $\int \frac{x\,dx}{1-\cosh ax} = \frac{x}{a}\operatorname{cot}gh\,\frac{ax}{2} - \frac{2}{a^3}\ln \operatorname{senh}\frac{ax}{2}$

578. $\int \frac{dx}{1+\cosh ax} = \frac{1}{a}\operatorname{tgh}\frac{ax}{2}$

579. $\int \frac{x\,dx}{1+\cosh ax} = \frac{x}{a}\operatorname{tgh}\frac{ax}{2} - \frac{2}{a^2}\ln\cosh\frac{ax}{2}$

580. $\int \frac{dx}{(1-\cosh ax)^2} = \frac{1}{2a}\operatorname{cot}gh\,\frac{ax}{2} - \frac{1}{6a}\operatorname{cot}gh^3\,\frac{ax}{2}$

581. $\int \frac{dx}{(1+\cosh ax)^2} = \frac{1}{2a}\operatorname{tgh}\frac{ax}{2} - \frac{1}{6a}\operatorname{tgh}^3\frac{ax}{2}$

582. $$\int \frac{dx}{p+q\cosh ax} = \begin{cases} \dfrac{2}{a\sqrt{q^2-p^2}}\operatorname{tg}^{-1}\dfrac{p+qe^{ax}}{\sqrt{q^2-p^2}} \\[2ex] \dfrac{1}{a\sqrt{p^2-q^2}}\ln\left(\dfrac{qe^{ax}+p-\sqrt{p^2-q^2}}{qe^{ax}+p+\sqrt{p^2-q}}\right) \end{cases}$$

583. $\int \frac{dx}{(p+q\cosh ax)^2} = \frac{q\operatorname{senh} ax}{a(q^2-p^2)(p+q\cosh ax)} - \frac{p}{(q^2-p^2)}\int\frac{dx}{p+q\cosh ax}$

584. $$\int \frac{dx}{p^2+q^2\cosh^2 ax} = \begin{cases} \dfrac{1}{2ap\sqrt{p^2+q^2}}\ln\left(\dfrac{p\operatorname{tgh} ax+\sqrt{p^2+q^2}}{p\operatorname{tgh} ax-\sqrt{p^2+q^2}}\right) \\[2ex] \dfrac{1}{ap\sqrt{p^2+q^2}}\operatorname{tg}^{-1}\dfrac{p\operatorname{tgh} ax}{\sqrt{p^2+q^2}} \end{cases}$$

585. $$\int \frac{dx}{p^2-q^2\cosh^2 ax} = \begin{cases} \dfrac{-1}{ap\sqrt{q^2-p^2}}\operatorname{tg}^{-1}\dfrac{p\operatorname{tgh} ax}{\sqrt{q^2-p^2}} \\[2ex] \dfrac{1}{2ap\sqrt{p^2-q^2}}\ln\left(\dfrac{p\operatorname{tgh} ax+\sqrt{p^2-q^2}}{p\operatorname{tgh} ax-\sqrt{p^2-q^2}}\right) \end{cases}$$

586. $\int x^m\cosh ax\,dx = \frac{x^m\operatorname{senh} ax}{a} - \frac{m}{a}\int x^{m-1}\operatorname{senh} ax\,dx$ *Véase 558*

587. $\int \frac{\cosh ax}{x^n}dx = -\frac{\cosh ax}{(n-1)x^{n-1}} + \frac{a}{n-1}\int\frac{\operatorname{senh} ax}{x^{n-1}}dx$ *Véase 559*

588. $\int \cosh^n ax\,dx = \frac{\cosh^{n-1} ax \operatorname{senh} ax}{an} + \frac{n-1}{n}\int\cosh^{n-2} ax\,dx$

589. $\int \frac{dx}{\cosh^n ax} = \frac{\operatorname{senh} ax}{a(n-1)\cosh^{n-1} ax} + \frac{n-2}{n-1}\int\frac{dx}{\cosh^{n-2} ax}$

590. $\int \frac{x\,dx}{\cosh^n ax} = \frac{x\operatorname{senh} ax}{a(n-1)\cosh^{n-1} ax} + \frac{1}{a^2(n-1)(n-2)\cosh^{-2} ax} + \frac{n-2}{n-1}\int\frac{x\,dx}{\cosh^{n-2} ax}$

591. $\int \operatorname{senh} ax \cosh ax \, dx = \frac{\operatorname{senh}^2 ax}{2a}$

592. $\int \operatorname{senh} px \cosh qx \, dx = \frac{\cosh(p-q)x}{2(p-q)} + \frac{\cosh(p+q)x}{2(p+q)}$

593. $\int \operatorname{senh}^n ax \cosh ax \, dx = \frac{\operatorname{senh}^{n+1} ax}{(n+1)a}$ *Si* $n=-1$, *véase* 615

594. $\int \operatorname{senh} ax \cosh^n ax \, dx = \frac{\cosh^{n+1} ax}{(n+1)a}$ *Si* $n=-1$, *véase* 604

595. $\int \operatorname{senh}^2 ax \cosh^2 ax \, dx = -\frac{x}{8} + \frac{\operatorname{senh} 4ax}{32a}$

596. $\int \frac{dx}{\operatorname{senh} ax \cosh ax} = \frac{1}{a} \ln \operatorname{tgh} ax$

597. $\int \frac{dx}{\operatorname{senh}^2 ax \cosh ax} = -\frac{1}{a} \operatorname{tg}^{-1} \operatorname{senh} ax - \frac{\cosh ax}{a}$

598. $\int \frac{dx}{\operatorname{senh} ax \cosh^2 ax} = \frac{1}{a} \ln \operatorname{tgh} \frac{ax}{2} + \frac{\sec hax}{a}$

599. $\int \frac{dx}{\operatorname{senh}^2 ax \cosh^2 ax} = -\frac{2 \cot gh\, 2ax}{a}$

600. $\int \frac{\operatorname{senh}^2 ax}{\cosh ax} dx = -\frac{1}{a} \operatorname{tg}^{-1} \operatorname{senh} ax + \frac{\operatorname{senh} ax}{a}$

601. $\int \frac{\cosh^2 ax}{\operatorname{senh} ax} dx = \frac{1}{a} \ln \operatorname{tgh} \frac{ax}{2} + \frac{\cosh ax}{a}$

602. $\int \frac{dx}{(1+\operatorname{senh} ax)\cosh ax} = \frac{1}{2a} \ln\left(\frac{1+\operatorname{senh} ax}{\cosh ax}\right) + \frac{1}{a} \operatorname{tg}^{-1} e^{ax}$

603. $\int \frac{dx}{(\cosh ax \pm 1)\operatorname{senh} ax} = \pm \frac{1}{2a(\cosh ax \pm 1)} \pm \frac{1}{2a} \ln \operatorname{tgh} \frac{ax}{2}$

INTEGRALES CON tgh ax

604. $\int \operatorname{tgh} ax \, dx = \frac{1}{a} \ln \cosh ax$

605. $\int \operatorname{tgh}^2 ax \, dx = -\frac{\operatorname{tgh} ax}{a} + x$

606. $\int \operatorname{tgh}^3 ax \, dx = -\frac{\operatorname{tgh}^2 ax}{2a} + \frac{1}{a} \ln \cosh ax$

607. $\int \operatorname{tgh}^n ax \sec h^2 ax \, dx = \frac{\operatorname{tgh}^{n+1} ax}{(n+1)a}$

608. $\int \frac{\sec h^2 ax}{\operatorname{tgh} ax} dx = \frac{1}{a} \ln \operatorname{tgh} ax$

609. $\int \frac{dx}{\operatorname{tgh} ax} = \frac{1}{a} \ln \operatorname{senh} ax$

610. $\int x \operatorname{tgh} ax \, dx = \frac{1}{a^2}\left(\frac{(ax)^3}{3} - \frac{(ax)^5}{15} + \frac{2(ax)^7}{105} - \cdots + \frac{(-1)^{n-1} 2^{2n}(2^{2n}-1)B_n (ax)^{2n+1}}{(2n+1)!} + \cdots\right)$

B_n es un n° de Bernoulli tanto en 610 como en 611.

611. $\int \frac{\operatorname{tgh} ax}{x} dx = ax - \frac{(ax)^3}{9} + \frac{2(ax)^5}{75} - \cdots + \frac{(2^{2n}-1)B_n (ax)^{2n-1}}{(2n-1)(2n)!} + \cdots$

612. $\int x \operatorname{tgh}^2 ax \, dx = -\frac{x \operatorname{tgh} ax}{a} + \frac{1}{a^2} \ln \cosh ax + \frac{x^2}{2}$

613. $\int \frac{dx}{p+q \operatorname{tgh} ax} = \frac{px}{p^2-q^2} - \frac{q}{a(p^2-q^2)} \ln(q \operatorname{senh} ax + p \cosh ax)$

614. $\int \operatorname{tgh}^n ax \, dx = -\frac{\operatorname{tgh}^{n-1} ax}{(n-1)a} + \int \operatorname{tgh}^{n-2} ax \, dx$

INTEGRALES CON cotgh ax

615. $\int \operatorname{cotgh} ax \, dx = \frac{1}{a} \ln \operatorname{senh} ax$

 616. $\int \operatorname{cotgh}^2 ax \, dx = -\frac{\operatorname{cotgh} ax}{a} + x$

617. $\int \operatorname{cotgh}^3 ax\,dx = -\frac{\operatorname{cotgh}^2 ax}{2a} + \frac{1}{a}\ln \operatorname{senh} ax$

618. $\int \operatorname{cotgh}^n ax\,\operatorname{cosech}^2 ax\,dx = -\frac{\operatorname{cotgh}^{n+1} ax}{(n+1)a}$

619. $\int \frac{\operatorname{cosech}^2 ax}{\operatorname{cotgh} ax}dx = -\frac{1}{a}\ln \operatorname{cotgh} ax$

620. $\int \frac{dx}{\operatorname{cotgh} ax} = \frac{1}{a}\ln \cosh ax$

621. $\int x \operatorname{cotgh} ax\,dx = \frac{1}{a^2}\left(ax + \frac{(ax)^3}{9} - \frac{(ax)^5}{225} + \cdots + \frac{(-1)^{n-1}2^{2n}B_n(ax)^{2n+1}}{(2n+1)!} + \cdots\right)$

B_n *es n° de Bernoulli, tanto en 621 como en 622*

622. $\int \frac{\operatorname{cotgh} ax}{x}dx = -\frac{1}{ax} + \frac{ax}{3} - \frac{(ax)^3}{135} + \cdots + \frac{(-1)^n 2^{2n} B_n (ax)^{2n-1}}{(2n-1)!} + \cdots$

623. $\int x \operatorname{cotgh}^2 ax\,dx = -\frac{x \operatorname{cotgh} ax}{a} + \frac{1}{a^2}\ln \operatorname{senh} ax + \frac{x^2}{2}$

624. $\int \frac{dx}{p + q \operatorname{cotgh} ax} = \frac{px}{p^2 - q^2} - \frac{q}{a(p^2 - q^2)}\ln(p \operatorname{senh} ax + q\cosh ax)$

625. $\int \operatorname{cotgh}^n ax\,dx = -\frac{\operatorname{cotgh}^{n-1} ax}{(n-1)a} + \int \operatorname{cotgh}^{n-1} ax\,dx$

INTEGRALES CON sech ax

626. $\int \operatorname{sech} ax\,dx = \frac{2}{a}\operatorname{tg}^{-1} e^{ax}$

627. $\int \operatorname{sech}^2 ax\,dx = \frac{\operatorname{tgh} ax}{a}$

628. $\int \operatorname{sech}^3 ax\,dx = \frac{\operatorname{sech} ax \operatorname{tgh} ax}{2a} + \frac{1}{2a}\operatorname{tg}^{-1}\operatorname{senh} ax$

629. $\int \operatorname{sech}^n ax\,\operatorname{tgh} ax\,dx = -\frac{\operatorname{sech}^n ax}{na}$

630. $\int \frac{dx}{\operatorname{sech} ax} = \frac{\operatorname{senh} ax}{a}$

631. $\int x \operatorname{sech} ax\,dx = \frac{1}{a^2}\left\{\frac{(ax)^2}{2} - \frac{(ax)^4}{8} + \frac{5(ax)^6}{144} - \cdots + \frac{(-1)^n E_n (ax)^{2n+2}}{(2n+2)(2n)!} + \cdots\right\}$ E_n *es n°de Euler*

632. $\int \frac{\operatorname{sech} ax}{x}dx = \ln x - \frac{(ax)^2}{4} + \frac{5(ax)^4}{96} - \frac{61(ax)^6}{4320} + \cdots + \frac{(-1)^n E_n (ax)^{2n}}{2n(2n)!} + \cdots$

633. $\int x \operatorname{sech}^2 ax\,dx = \frac{x}{a}\operatorname{tgh} ax - \frac{1}{a^2}\ln\cosh ax$

634. $\int \frac{dx}{q + p \operatorname{sech} ax} = \frac{x}{q} - \frac{p}{q}\int \frac{dx}{p + q\cosh ax}$ *Véase* 582

635. $\int \operatorname{sech}^n ax\,dx = \frac{\operatorname{sech}^{n-2} ax \operatorname{tgh} ax}{a(n-1)} + \frac{n-2}{n-1}\int \operatorname{sech}^{n-2} ax\,dx$

INTEGRALES CON cosech ax

636. $\int \operatorname{cosech} ax\,dx = \frac{1}{a}\ln \operatorname{tgh}\frac{ax}{2}$

637. $\int \operatorname{cosech}^2 ax\,dx = -\frac{\operatorname{cotgh} ax}{a}$

638. $\int \operatorname{cosech}^3 ax\,dx = -\frac{\operatorname{cosech} ax \operatorname{cotgh} ax}{2a} - \frac{1}{2a}\ln \operatorname{tgh}\frac{ax}{2}$

639. $\int \operatorname{cosech}^n ax \operatorname{cotgh} ax\,dx = -\frac{\operatorname{cosech}^n ax}{na}$

640. $\int \frac{dx}{\operatorname{cosech} x} = \frac{\cosh ax}{a}$

641. $\int x \operatorname{cosech} ax\,dx = \frac{1}{a^2}\left\{ax - \frac{(ax)^3}{18} + \frac{7(ax)^5}{1800} - \cdots + \frac{(-1)^n 2(2^{2n-1}-1)B_n (ax)^{2n+1}}{(2n+1)!} + \cdots\right\}$

B_n *es n° de Bernoulli*

642. $\int \frac{\operatorname{cosech} ax}{x}\,dx = -\frac{1}{ax} - \frac{ax}{6} + \frac{7(ax)^3}{1080} \cdots + \frac{(-1)^n 2(2^{2n-1}-1)B_n(ax)^{2n-1}}{(2n-1)(2n)!} + \cdots$

643. $\int x \operatorname{cosech}^2 ax\,dx = -\frac{x}{a}\operatorname{cotgh} ax + \frac{1}{a^2}\ln \operatorname{senh} ax$

644. $\int \frac{dx}{q + p \operatorname{cosech} ax} = \frac{x}{p} - \frac{p}{q}\int \frac{dx}{p + q \operatorname{senh} ax}$ *Véase* 554

645. $\int \operatorname{cosech}^n ax\,dx = -\frac{\operatorname{cosech}^{n-2} ax \operatorname{cotgh} ax}{a(n-1)} + \frac{n-2}{n-1}\int \operatorname{cosech}^{n-2} ax\,dx$

INTEGRALES DE FUNCIONES HIPERBOLICAS INVERSAS

646. $\int \operatorname{senh}^{-1}\frac{x}{a}\,dx = x \operatorname{senh}^{-1}\frac{x}{a} - \sqrt{a^2+x^2}$

647. $\int x \operatorname{senh}^{-1}\frac{x}{a}\,dx = \left(\frac{x^2}{2} + \frac{a^2}{4}\right)\operatorname{senh}^{-1}\frac{x}{a} - \frac{x\sqrt{a^2+x^2}}{4}$

648. $\int x^2 \operatorname{senh}^{-1}\frac{x}{c}\,dx = \frac{x^3}{3}\operatorname{senh}^{-1}\frac{x}{a} + \frac{(-x^2+2a^2)\sqrt{a^2+x^2}}{9}$

649. $\int x^m \operatorname{sen}^{-1}\frac{x}{a}\,dx = \frac{x^{m+1}}{m+1}\operatorname{senh}^{-1}\frac{x}{a} - \frac{1}{m+1}\int \frac{x^{m+1}}{\sqrt{a^2+x^2}}\,dx$

650. $\int \frac{\operatorname{senh}^{-1} x/a}{x}\,dx = \begin{cases} \frac{x}{a} - \frac{(x/a)^3}{2\cdot 3\cdot 3} + \frac{1\cdot 3\,(x/a)^5}{2\cdot 4\cdot 5\cdot 5} - \frac{1\cdot 3\cdot 5\,(x/a)^7}{2\cdot 4\cdot 6\cdot 7\cdot 7} + \cdots & |x| < a \\ \frac{\ln^2(2x/a)}{2} - \frac{(a/x)^2}{2\cdot 2\cdot 2} + \frac{1\cdot 3\,(a/x)^4}{2\cdot 4\cdot 4\cdot 4} - \frac{1\cdot 3\cdot 5\,(a/x)^6}{2\cdot 4\cdot 6\cdot 6\cdot 6} + \cdots & x > a \\ \frac{-\ln^2(-2x/a)}{2} + \frac{(a/x)^2}{2\cdot 2\cdot 2} - \frac{1\cdot 3\,(a/x)^4}{2\cdot 4\cdot 4\cdot 4} + \frac{1\cdot 3\cdot 5\,(a/x)^6}{2\cdot 4\cdot 6\cdot 6\cdot 6} - \cdots & x < -a \end{cases}$

651. $\int \frac{\operatorname{senh}^{-1} x/a}{x^2}\,dx = -\frac{\operatorname{senh}^{-1} x/a}{x} - \frac{1}{a}\ln\left(\frac{a+\sqrt{a^2+x^2}}{x}\right)$

652. $\int \cosh^{-1}\frac{x}{c}\,dx = \begin{cases} x\cosh^{-1}\frac{x}{a} - \sqrt{x^2-a^2}; & \cosh^{-1}\frac{x}{a} > 0 \\ x\cosh^{-1}\frac{x}{a} + \sqrt{x^2-a^2}; & \cosh^{-1}\frac{x}{a} < 0 \end{cases}$

653. $\int x\cosh^{-1}\frac{x}{a}\,dx = \begin{cases} \left(\frac{x^2}{2} - \frac{a^2}{4}\right)\cos^{-1}\frac{x}{a} - \frac{x\sqrt{x^2-a^2}}{4}; & \cosh^{-1}\frac{x}{a} > 0 \\ \left(\frac{x^2}{2} - \frac{a^2}{4}\right)\cos^{-1}\frac{x}{a} - \frac{x\sqrt{x^2-a^2}}{4}; & \cosh^{-1}\frac{x}{a} < 0 \end{cases}$

654. $\int x^2\cosh^{-1}\frac{x}{a}\,dx = \begin{cases} \frac{x}{3}\cosh^{-1}\frac{x}{a} - \frac{x+a\sqrt{x^2-a^2}}{9}; & \cosh^{-1}\frac{x}{a} > 0 \\ \frac{x^3}{3}\cosh^{-1}\frac{x}{a} + \frac{(x^2+2a^2)\sqrt{x^2-a^2}}{9}; & \cosh^{-1}\frac{x}{a} < 0 \end{cases}$

655. $\int x^m\cosh^{-1}\frac{x}{a}\,dx = \begin{cases} \frac{x^{m+1}}{m+1}\cosh^{-1}\frac{x}{a} - \frac{1}{m+1}\int \frac{x^{m+1}}{\sqrt{x^2-a^2}}\,dx; & \cosh^{-1}\frac{x}{a} > 0 \\ \frac{x^{m+1}}{m+1}\cosh^{-1}\frac{x}{a} + \frac{1}{m+1}\int \frac{x^{m+1}}{\sqrt{x^2-a^2}}\,dx; & \cosh^{-1}\frac{x}{a} < 0 \end{cases}$

656. $\int \frac{\cosh^{-1} x/a}{x}\,dx = \pm\left(\frac{\ln^2(2a)}{2} + \frac{(a/x)^2}{2\cdot 2\cdot 2} + \frac{1\cdot 3\,(a/x)^4}{2\cdot 4\cdot 4\cdot 4} + \frac{1\cdot 3\cdot 5\,(a/x)^6}{2\cdot 4\cdot 6\cdot 6\cdot 6} + \cdots\right)$

$+\,si\ \cosh^{-1}\frac{x}{a} > 0; -\,si\ \cosh^{-1}\frac{x}{a} < 0$

657. $\int \frac{\cosh^{-1}(x/a)}{x^2}\,dx = -\frac{\cosh^{-1}(x/a)}{x} \mp \frac{1}{a}\ln\left(\frac{a+\sqrt{a^2+x^2}}{x}\right)$

$-\,si\ \cosh^{-1}\frac{x}{a} > 0; +\,si\ \cosh^{-1}\frac{x}{a} < 0$

658. $\int \operatorname{tgh}^{-1}\frac{x}{a}\,dx = x\operatorname{tgh}^{-1}\frac{x}{a} + \frac{a}{2}\ln(a^2-x^2)$

659. $\int x\operatorname{tgh}^{-1}\frac{x}{a}\,dx = \frac{1}{2}(x^2-a^2)\operatorname{tgh}^{-1}\frac{x}{a} + \frac{ax}{2}$

660. $\int x^2\operatorname{tgh}^{-1}\frac{x}{a}\,dx = \frac{x^3}{3}\operatorname{tgh}^{-1}\frac{x}{a} + \frac{ax^2}{6} + \frac{a^3}{6}\ln(a^2-x^2)$

661. $\int x^m\operatorname{tgh}^{-1}\frac{x}{a}\,dx = \frac{x^{m+1}}{m+1}\operatorname{tgh}^{-1}\frac{x}{a} - \frac{a}{m+1}\int \frac{x^{m+1}}{a^2-x^2}\,dx$

662. $\int \frac{\operatorname{tgh}^{-1}(x/a)}{x}\,dx = \frac{x}{a} + \frac{(x/a)^3}{3^2} + \frac{(x/a)^5}{5^2} + \frac{(x/a)^7}{7^2} + \cdots$

663. $\int \frac{\operatorname{tgh}^{-1}(x/a)}{x^2}\,dx = -\frac{1}{x}\operatorname{tgh}^{-1}\frac{x}{a} + \frac{1}{2a}\ln\left(\frac{x^2}{a^2-x^2}\right)$

664. $\int \operatorname{cotgh}^{-1}\frac{x}{a}\,dx = x\operatorname{cotgh}^{-1}\frac{x}{a} + \frac{a}{2}\ln(x^2-a^2)$

665. $\int x\operatorname{cotgh}^{-1}\frac{x}{a}\,dx = \frac{1}{2}(x^2-a^2)\operatorname{cotgh}^{-1}\frac{x}{a} + \frac{ax}{2}$

666. $\int x^2\operatorname{cotgh}^{-1}\frac{x}{a}\,dx = \frac{ax^2}{6} + \frac{x^3}{3}\operatorname{cotgh}^{-1}\frac{x}{a} + \frac{a^3}{6}\ln(x^2-a^2)$

667. $\int x^m\operatorname{cotgh}^{-}\frac{x}{a}\,dx = \frac{x^{m+1}}{m+1}\operatorname{cotgh}^{-1}\frac{x}{a} - \frac{a}{m+1}\int \frac{x^{m+1}}{a^2-x^2}\,dx$

668. $\int \frac{\operatorname{cotgh}^{-1}(x/a)}{x}\,dx = -\left(\frac{a}{x} + \frac{(a/x)^3}{3^2} + \frac{(a/x)^5}{5^2} + \frac{(a/x)^7}{7^2} + \cdots\right)$

669. $\int \frac{\operatorname{cotgh}^{-1}(x/a)}{x^2}\,dx = -\frac{1}{x}\operatorname{cotgh}^{-1}\frac{x}{a} + \frac{1}{2a}\ln\left(\frac{x^2}{x^2-a^2}\right)$

670. $\int \operatorname{sech}^{-1}\frac{x}{a}\,dx = \begin{cases} x\operatorname{sech}^{-1}\frac{x}{a} + a\operatorname{sen}^{-1}(x/a); & \sec^{-1}\frac{x}{a} > 0 \\ x\operatorname{sech}^{-1}\frac{x}{a} - a\operatorname{sen}^{-1}(x/a); & \sec^{-1}\frac{x}{a} < 0 \end{cases}$

671. $\int x\operatorname{sech}^{-1}\frac{x}{a}\,dx = \begin{cases} \frac{x}{2}\sec h^{-1}\frac{x}{a} - \frac{a\sqrt{a^2-x^2}}{2}; & \sec^{-1}\frac{x}{a} > 0 \\ \frac{x}{2}\sec h^{-1}\frac{x}{a} + \frac{a\sqrt{a^2-x^2}}{2}; & \sec^{-1}\frac{x}{a} < 0 \end{cases}$

672. $\int x^m\operatorname{sech}^{-1}\frac{x}{a}\,dx = \begin{cases} \frac{x^{m+1}}{m+1}\operatorname{sech}^{-1}\frac{x}{a} + \frac{a}{m+1}\int \frac{x^m dx}{\sqrt{a^2-x^2}}; & \operatorname{sech}^{-1}\frac{x}{a} > 0 \\ \frac{x^{m+1}}{m+1}\operatorname{sech}^{-1}\frac{x}{a} - \frac{a}{m+1}\int \frac{x^m dx}{\sqrt{a^2-x^2}}; & \operatorname{sech}^{-1}\frac{x}{a} < 0 \end{cases}$

673. $\int \frac{\operatorname{sech}^{-1}(x/a)}{x}\,dx = \mp\left(\frac{\ln(x/a)\ln(4x/a)}{2} + \frac{(x/a)^2}{2\cdot 2\cdot 2} + \frac{1\cdot 3(x/a)^4}{2\cdot 4\cdot 4\cdot 4} + \frac{1\cdot 3\cdot 5(x/a)^6}{2\cdot 4\cdot 6\cdot 6\cdot 6} + \cdots\right)$

674. $\int \operatorname{cosech}^{-1}\frac{x}{a}\,dx = x\operatorname{cosech}^{-1}\frac{x}{a} \pm a\operatorname{senh}^{-1}\frac{x}{a}$; $[+\ si\ x > 0,\ -\ si\ x < 0]$

675. $\int x\operatorname{cosech}^{-1}\frac{x}{a}\,dx = \frac{x^2}{2}\operatorname{cosech}^{-1}\frac{x}{a} \pm \frac{a\sqrt{x^2+a^2}}{2}$; $[+\ si\ x > 0,\ -\ si\ x < 0]$

676. $\int x^m\operatorname{cosech}^{-1}\frac{x}{a}\,dx = \frac{x^{m+1}}{m+1}\operatorname{cosech}^{-1}\frac{x}{a} \pm \frac{a}{m+1}\int \frac{x^m dx}{\sqrt{x^2+a^2}}$; $[+si\ x > 0,\ -\ si\ x < 0]$

677. $\int \frac{\operatorname{cosech}^{-1}(x/a)}{x}\,dx = \begin{cases} \frac{\ln(x/a)\ln(x/a)}{2} + \frac{(x/a)^2}{2\cdot 2\cdot 2} - \frac{1\cdot 3(x/a)^4}{2\cdot 4\cdot 4\cdot 4} + \cdots; & 0 < x < a \\ \frac{\ln(-x/a)\ln(-x/a)}{2} - \frac{(x/a)^2}{2\cdot 2\cdot 2} + \frac{1\cdot 3(x/a)^4}{2\cdot 4\cdot 4\cdot 4} - \cdots; & -a < x < 0 \\ -\frac{a}{x} + \frac{(a/x)^3}{2\cdot 3\cdot 3} - \frac{1\cdot 3(a/x)^5}{2\cdot 4\cdot 5\cdot 5} + \cdots; & |x| > a \end{cases}$

INTEGRALES DEFINIDAS

PROPIEDADES

678. $\int_a^b [k\ f(x) + r\ g(x) - n\ h(x)]dx = k\int_a^b f(x)dx + r\int_a^b g(x)dx - n\int_a^b h(x)dx;$
$a, b, k, r, n \in R$

679. $\int_a^a f(x)dx = 0$

680. $\int_a^b f(x)dx = -\int_b^a f(x)dx$

681. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

682. $\int_a^b f(x)dx = f(c)(b-a)$ *Para algún c tal que* $a < c < b$, f *continua en* $[a, b]$

683. $Si\ f'(x) = F(x) \Rightarrow \int_a^b f(x)dx = F(b) - F(a)$ (Regla de Barrow)

INTEGRALES IMPROPIAS

684. $\int_a^{+\infty} f(x)dx = \lim_{z \to +\infty} \int_a^z f(x)dx$

685. $\int_{-\infty}^{+\infty} f(x)dx = \lim_{\substack{b \to +\infty \\ a \to -\infty}} \int_a^b f(x)dx$

686. $\int_a^b f(x)dx = \lim_{\theta \to 0} \int_a^{b-\theta} f(x)dx$ Si b es punto singular de $f, \theta > 0$

687. $\int_a^b f(x)dx = \lim_{\theta \to 0} \int_{a+\theta}^b f(x)dx$ Si a es punto singular de $f, \theta > 0$

INTEGRALES DEFINIDAS O IMPROPIAS DE FUNCIONES TRIGONOMETRICAS

688. $\int_0^\pi \operatorname{sen} nx \operatorname{sen} kx\, dx = \int_0^\pi \cos nx \cos kx\, dx = \begin{cases} 0 & si\ n \neq k \\ \frac{\pi}{2} & si\ n = k \end{cases} ; n, k \in Z$

689. $\int_0^\pi \operatorname{sen} kx \cos nx\, dx = \begin{cases} 0 & si\ n+k\ es\ impar \\ \frac{2k}{k^2-n^2} & si\ n+k\ es\ par \end{cases} ; n, k \in Z$

690. $\int_0^{\pi/2} \operatorname{sen}^2 x\, dx = \int_0^{\pi/2} \cos^2 x\, dx = \frac{\pi}{4}$

691. $\int_0^{\pi/2} \operatorname{sen}^{2k} x\, dx = \int_0^{\pi/2} \cos^{2k} x\, dx = \frac{1 \cdot 3 \cdot 5 \cdots (2k-1)}{2 \cdot 4 \cdot 6 \cdots 2k} \cdot \frac{\pi}{2} \quad k \in N$

692. $\int_0^{\pi/2} \operatorname{sen}^{2k+1} x\, dx = \int_0^{\pi/2} \cos^{2k+1} x\, dx = \frac{2 \cdot 4 \cdot 6 \cdots 2k}{1 \cdot 3 \cdot 5 \cdots (2k+1)} \quad k \in N$

693. $\int \operatorname{sen}^{2p-1} x \cos^{2q-1} x\, dx = \frac{\Gamma(p)\Gamma(q)}{2 \cdot \Gamma(p+q)}$ Γ : Función Gamma (ver apéndice)

694. $\int_0^\infty \frac{\operatorname{sen} kx}{x} dx = \begin{cases} \frac{\pi}{2} & k > 0 \\ 0 & k = 0 \\ \frac{-\pi}{2} & k < 0 \end{cases}$

695. $\int_0^\infty \frac{\operatorname{sen} px \cos qx}{x} dx = \begin{cases} \pi/2 & 0 < p < q \\ 0 & 0 < q < p \\ \pi/4 & p = q > 0 \end{cases}$

696. $\int_0^\infty \frac{\operatorname{sen} px \cos qx}{x^2} dx = \begin{cases} p\pi/2 & 0 < p < q \\ q\pi/2 & 0 < q \leq p \end{cases}$

697. $\int_0^\infty \frac{\operatorname{sen}^2 px}{x^2} dx = \frac{p\pi}{2}$

698. $\int_0^\infty \frac{1 - \cos px}{x^2} dx = \frac{p\pi}{2}$

699. $\int_0^\infty \frac{\cos px - \cos qx}{x} dx = \ln \frac{q}{p}$

700. $\int_0^\infty \frac{\cos px - \cos qx}{x^2} dx = \frac{(q-p)\pi}{2}$

701. $\int_0^\infty \frac{\cos nx}{x^2 + a^2} dx = \frac{\pi}{2a} e^{-na}$

702. $\int_0^\infty \frac{x \operatorname{sen} nx}{x^2 + a^2} dx = \frac{\pi}{2} e^{-na}$

703. $\int_0^\infty \frac{\operatorname{sen} nx}{x(x^2+a^2)}dx = \frac{\pi}{2a^2}(1-e^{-na})$

704. $\int_0^{2\pi} \frac{dx}{a+b\operatorname{sen} x} = \frac{2\pi}{\sqrt{a^2-b^2}}$

705. $\int_0^{\pi/2} \frac{dx}{a+b\cos x} = \frac{\cos^{-1}(b/a)}{\sqrt{a^2-b^2}}$

706. $\int_0^{2\pi} \frac{dx}{(a+b\operatorname{sen} x)^2} = \int_0^{2\pi} \frac{dx}{(a+b\cos x)^2} = \frac{2\pi a}{\sqrt{(a^2-b^2)^3}}$

707. $\int_0^{2\pi} \frac{dx}{1-2a\cos x+a^2} = \frac{2\pi}{1-a^2} \quad 0 < a < 1$

708. $\int_0^{\pi} \frac{x\operatorname{sen} x\, dx}{1-2a\cos x+a^2} = \begin{cases} \frac{\pi}{a}\ln(1+a) & si\ |a| < 1 \\ \pi\ln(1+\frac{1}{a}) & |a| > 1 \end{cases}$

709. $\int_0^{\pi} \frac{\cos kx\, dx}{1-2a\cos x+a^2} = \frac{\pi a^k}{1-a^2} \quad si\ |a| < 1,\ k \in N$

710. $\int_0^\infty \operatorname{sen} ax^2 dx = \int_0^\infty \cos ax^2 dx = \sqrt{\frac{\pi}{8a}}$

711. $\int_0^\infty \operatorname{sen} ax^n dx = \frac{\Gamma(1/n)}{n a^{1/n}}\operatorname{sen}\frac{\pi}{2n}; \quad n > 1,\ \Gamma:$ *Función Gamma (Ver apéndice)*

712. $\int_0^\infty \cos ax^n dx = \frac{\Gamma(1/n)}{n a^{1/n}}\cos\frac{\pi}{2n}; \quad n > 1$

713. $\int_0^\infty \frac{\operatorname{sen} x}{\sqrt{x}} dx = \int_0^\infty \frac{\cos x}{\sqrt{x}} dx = \sqrt{\frac{\pi}{2}}$

714. $\int_0^\infty \frac{\operatorname{sen} x}{x^p} dx = \frac{\pi}{2\Gamma(p)\operatorname{sen}(p\pi/2)}; \quad 0 < p < 1$

715. $\int_0^\infty \frac{\cos x}{x^p} dx = \frac{\pi}{2\Gamma(p)\operatorname{sen}(p\pi/2)}; \quad 0 < p < 1$

716. $\int_0^\infty \operatorname{sen} ax^2 \cos 2bx\, dx = \sqrt{\frac{\pi}{8a}}(\cos\frac{b^2}{a} + \operatorname{sen}\frac{b^2}{a})$

717. $\int_0^\infty \cos ax^2 \operatorname{sen} 2bx\, dx = \sqrt{\frac{\pi}{8a}}(\cos\frac{b^2}{a} + \operatorname{sen}\frac{b^2}{a})$

718. $\int_0^\infty \frac{\operatorname{sen}^3 x}{x^3} dx = \frac{3\pi}{8}$

719. $\int_0^\infty \frac{\operatorname{sen}^4 x}{x^4} dx = \frac{\pi}{3}$

720. $\int_0^\infty \frac{\operatorname{tg} x}{x} dx = \frac{\pi}{2}$

721. $\int_0^{\pi/2} \frac{dx}{1+\operatorname{tg}^n x} = \frac{\pi}{4}$

722. $\int_0^{\pi/2} \frac{x\, dx}{\operatorname{sen} x} = 2\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)^2}$

723. $\int_0^1 \frac{\operatorname{tg}^{-1} x}{x} dx = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)^2}$

724. $\int_0^1 \frac{\operatorname{sen}^{-1} x}{x} dx = \frac{\pi}{2}\ln 2$

725. $\int_0^1 \frac{1-\cos x}{x} dx - \int_1^\infty \frac{\cos x}{x} dx = \gamma, \quad \gamma:$ *Constante de Euler*

726. $\int_0^\infty \left(\frac{1}{1+x^2} - \cos x\right)\frac{dx}{x} = \gamma$

727. $\int_0^\infty \frac{\operatorname{tg}^{-1} px - \operatorname{tg}^{-1} qx}{x} dx = \frac{\pi}{2}\ln\frac{p}{q}$

INTEGRALES DEFINIDAS O IMPROPIAS DE FUNCIONES RACIONALES E IRRACIONALES

728. $\int_0^\infty \frac{dx}{x^2+a^2} = \frac{\pi}{2a}$

729. $\int_0^\infty \frac{x^{p-1}dx}{1+x} = \frac{\pi}{\operatorname{sen} p\pi}$; $\quad 0 < p < 1$

730. $\int_0^\infty \frac{x^m dx}{x^n+a^n} = \frac{\pi\, a^{m-n+1}}{n \operatorname{sen}\left(\frac{m+1}{n}\pi\right)}$; $\quad 0 < m+1 < n$

731. $\int_0^\infty \frac{x^m dx}{1+2x\cos\beta+x^2} = \frac{\pi}{\operatorname{sen} m\pi} \cdot \frac{\operatorname{sen} m\beta}{\operatorname{sen}\beta}$

732. $\int_0^a \frac{dx}{\sqrt{a^2-x^2}} = \frac{\pi}{2}$

733. $\int_0^a \sqrt{a^2-x^2}\,dx = \frac{\pi a^2}{4}$

734. $\int_0^a x^m(a^n-x^n)^p dx = \frac{a^{m+np+1}}{n} \cdot \frac{\Gamma(m+1/n)\cdot\Gamma(p+1)}{\Gamma(m+1/n+p+1)}$ $\quad \Gamma$: *Función Gamma*

735. $\int_0^\infty \frac{x^m dx}{(a^n+x^n)^r} = \frac{(-1)^{r-1}\pi\, a^{m-nr+1}}{n \operatorname{sen}\left(\frac{m+1}{n}\pi\right)\cdot(r-1)!} \cdot \frac{\Gamma(m+1/n)\cdot}{\Gamma(m+1/n-r+1)}$; $\quad 0 < m+1 < nr$

INTEGRALES DEFINIDAS O IMPROPIAS DE FUNCIONES EXPONENCIALES

736. $\int_0^\infty e^{-ax}\cos bx\; dx = \frac{a}{a^2+b^2}$

737. $\int_0^\infty e^{-ax}\operatorname{sen} bx\; dx = \frac{b}{a^2+b^2}$

738. $\int_0^\infty \frac{e^{-ax}\operatorname{sen} bx}{x}\,dx = \operatorname{tg}^{-1}\frac{b}{a}$

739. $\int_0^\infty \frac{e^{-ax}-e^{-bx}}{x}\,dx = \ln\frac{b}{a}$

740. $\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$

741. $\int_0^\infty e^{-ax^2}\cos bx\,dx = \frac{1}{2}\sqrt{\frac{\pi}{2}}\,e^{-(b^2/4a)}$

742. $\int_0^\infty e^{-(ax^2+bx+c)}dx = \frac{1}{2}\sqrt{\frac{\pi}{2}}e^{(b^2-4a)/4a}\cdot f_{cer}\frac{b}{2\sqrt{a}}$; $\quad$ *Siendo* $f_{cer}(p) = \frac{2}{\sqrt{\pi}}\int_0^\infty e^{-k^2}dk$

743. $\int_0^\infty e^{-(ax^2+bx+c)}dx = \sqrt{\frac{\pi}{a}}e^{(b^2-4a)/4a}$

744. $\int_0^\infty x^n e^{-ax}dx = \frac{\Gamma(n+1)}{a^{n+1}}$; $\quad \Gamma$: *Función Gamma*

745. $\int_0^\infty x^m e^{-ax^2}dx = \frac{\Gamma(m+1/2)}{2a^{(m+1/2)}}$

746. $\int_0^\infty e^{-(ax^2+b/m^2)}dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}\,e^{-2\sqrt{ab}}$

747. $\int_0^\infty \frac{x\,dx}{e^x-1} = \sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{\pi^2}{6}$

748. $\int_0^\infty \frac{x^{n-1}dx}{e^x-1} = \Gamma(n+1)\sum_{k=1}^{\infty}\frac{1}{k^n}$; $\quad \Gamma$: *Función Gamma*

Si n es par esta serie se puede hallar con ayuda de los números de Bernoulli (ver apéndice)

749. $\int_0^{\infty} \frac{x\,dx}{e^x+1} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$

750. $\int_0^{\infty} \frac{x^{n-1}dx}{e^x+1} = \Gamma(n+1)\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^n}$ $\quad \Gamma$: *Función Gamma*

751. $\int_0^{\infty} \frac{\operatorname{sen}\ mx\ dx}{e^{2\pi x}-1} = \frac{1}{4}\cot\ gh\ \frac{m}{2} - \frac{1}{2m}$

752. $\int_0^{\infty} \left(\frac{1}{1+x} - e^{-x}\right)\frac{dx}{x} = \gamma$ $\quad \gamma$: *Constante de Euler*

753. $\int_0^{\infty} \frac{e^{-x^2}-e^{-x}}{x}\,dx = \frac{\gamma}{2}$

754. $\int_0^{\infty} \left(\frac{1}{e^{-x}-1} - \frac{e^{-x}}{x}\right)dx = \gamma$ $\quad \gamma$: *Constante de Euler*

755. $\int_0^{\infty} \frac{e^{-ax}-e^{-bx}}{x\sec\ px}\,dx = \frac{1}{2}\ln\ \frac{b^2+p^2}{a^2+p^2}$

756. $\int_0^{\infty} \frac{e^{-ax}-e^{-bx}}{x\operatorname{cosec}\ px}\,dx = \operatorname{tg}^{-1}\frac{b}{p} - \operatorname{tg}^{-1}\frac{a}{p}$

757. $\int_0^{\infty} \frac{e^{-ax}(1-\cos x)}{x^2}\,dx = \operatorname{cotg}^{-1}a - \frac{a}{2}\ln(a^2+1)$

INTEGRALES DEFINIDAS O IMPROPIAS DE FUNCIONES LOGARITMICAS

758. $\int_0^1 x^m \ln^n x\,dx = \frac{(-1)^n n!}{(m+1)^{n+1}}; \quad m > -1, n \in N_o$

759. $\int_0^1 x^m \ln^n x\,dx = \frac{(-1)^n \Gamma(n+1)}{(m+1)^{n+1}}; \quad m > -1, n \notin N_o, \Gamma$: *Función Gamma*

760. $\int_0^1 \frac{\ln x}{1+x}dx = -\frac{\pi^2}{12}$

761. $\int_0^1 \frac{\ln x}{1-x}\,dx = -\frac{\pi^2}{6}$

762. $\int_0^1 \frac{\ln(1+x)}{x}\,dx = \frac{\pi^2}{12}$

763. $\int_0^1 \frac{\ln(1-x)}{x}\,dx = -\frac{\pi^2}{6}$

764. $\int_0^1 \ln x \ln(1+x)\,dx = 2(1-\ln 2) - \frac{\pi^2}{12}$

765. $\int_0^1 \ln x \ln(1-x)dx = 2 - \frac{\pi^2}{6}$

766. $\int_0^{\infty} \frac{x^{p-1}\ln x}{1+x}\,dx = -\pi^2 \operatorname{cosec}\ p\pi \cdot \operatorname{cotg}\ p\pi; \quad 0 < p < 1$

767. $\int_0^1 \frac{x^m - x^n}{\ln x}\,dx = \ln\ \frac{m+1}{n+1}$

768. $\int_0^{\infty} e^{-x}\ln x\,dx = -\gamma$ $\quad \gamma$: *Constante de Euler*

769. $\int_0^{\infty} e^{-x^2}\ln x\,dx = -\frac{\sqrt{\pi}}{4}(\gamma + 2\ln 2)$

770. $\int_0^{\infty} \ln\ \frac{e^x+1}{e^x-1}\,dx = \frac{\pi^2}{4}$

771. $\int_0^{\pi/2} \ln \operatorname{sen} x\,dx = \int_0^{\pi/2} \ln\cos x\,dx = -\frac{\pi\ln 2}{2}$

772. $\int_0^{\pi/2} (\ln \operatorname{sen} x)^2 \, dx = \int_0^{\pi/2} (\ln \cos x)^2 \, dx = \frac{\pi^3}{4t} + \frac{\pi \ln^2 2}{2}$

773. $\int_0^{\pi} x \ln \operatorname{sen} x \, dx = -\frac{\pi^2 \ln 2}{2}$

774. $\int_0^{\pi/2} \operatorname{sen} x \ln \operatorname{sen} x \, dx = \ln \frac{2}{e}$

775. $\int_0^{2\pi} \ln(a + b \operatorname{sen} x) dx = \int_0^{2\pi} \ln(a + b \cos x) dx = 2\pi \ln(a + \sqrt{a^2 - b^2})$

776. $\int_0^{\pi} \ln(a + b \cos x) \, dx = \pi \ln \frac{a + \sqrt{a^2 - b^2}}{2}$

777. $\int_0^{\pi} \ln(a^2 + 2ab \cos x + b^2) \, dx = \begin{cases} 2\pi \ln b; & si\ 0 < a \le b \\ 2\pi \ln a; & si\ 0 < b \le a \end{cases}$

778. $\int_0^{\pi/4} \ln(1 + \operatorname{tg} x) \, dx = \frac{\pi \ln 2}{8}$

779. $\int_0^{\pi/2} \sec x \ln\left(\frac{1 + b \cos x}{1 + a \cos x}\right) dx = \frac{(\cos^{-1} a)^2 - (\cos^{-1} b)^2}{2}$

780. $\int_0^{a} \ln(2 \operatorname{sen} \frac{x}{2}) dx = -\sum_{n=1}^{\infty} \frac{\operatorname{sen} an}{n^2}$

INTEGRALES IMPROPIAS DE FUNCIONES HIPERBOLICAS

781. $\int_0^{\infty} \frac{\operatorname{sen} ax}{\operatorname{senh} bx} dx = \frac{\pi}{2b} \operatorname{tgh} \frac{a\pi}{2b}$

782. $\int_0^{\infty} \frac{\cos ax}{\cosh} dx = \frac{\pi}{2b} \sec h \frac{a\pi}{2b}$

783. $\int_0^{\infty} \frac{x dx}{\operatorname{senh} ax} = \frac{\pi^2}{4a^2}$

784. $\int_0^{\infty} \frac{x^n dx}{\operatorname{senh} ax} = \frac{2^{n+1} - 1}{2^{n+1} \cdot a^{n+1}} \Gamma(n+1) \sum_{k=1}^{\infty} \frac{1}{k^{n+1}}$; Γ : *Función Gamma*

785. $\int_0^{\infty} \frac{\operatorname{senh} ax}{e^{bx} + 1} = \frac{\pi}{2b} \operatorname{cosec} \frac{a\pi}{b} - \frac{1}{2a}$

786. $\int_0^{\infty} \frac{\operatorname{senh} ax}{e^{bx} - 1} dx = \frac{1}{2a} - \frac{\pi}{2b} \operatorname{cotg} \frac{a\pi}{b}$

APENDICE

FUNCION GAMMA

Definición: $\Gamma(n) = \int_0^{\infty} t^{n-1} e^{-t} dt; \quad n > 0$

Formula de recurrencia: $\Gamma(n+1) = n\Gamma(n)$

$Si\ n \in N \Rightarrow \Gamma(n+1) = n\,t \qquad Si\ n < 0 \Rightarrow \Gamma(n) = \frac{\Gamma(n+1)}{n}$

Propiedades: a) $\Gamma(p)\Gamma(1-p) = \frac{\pi}{\operatorname{sen} p\pi}$ b) $\frac{2^{2x-1}}{\sqrt{\pi}} = \frac{\Gamma(2x)}{\Gamma(x)\Gamma(x+\frac{1}{2})}$

FUNCION BETA

Definición: $B(m,n) = \int_0^{1} t^{m-1} (1-t)^{n-1} dt; \quad m > 0, n > 0$

Relación con la función Gamma: $B(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

Otras formas de expresar la función Beta: $B(m,n) = B(n,M0 =$

$$= 2\int_0^{\pi/2} \operatorname{sen}^{2m-1} x \cos^{2n-1} x\,dx = \int_0^{\infty} u^{m-1}(1+u)^{-m-n}\,du = r^n(r+1)^m \int_0^1 \frac{t^{m-1}(1-t)^{n-1}}{(r+t)^{m+n}}dt$$

NUMEROS DE BERNOULLI Y EULER

Definición: $B(m,n) = \int_0^1 t^{m-1}(1-t)^{n-1}dt; \quad m > 0\,; n > 0$

Relación con la función Gamma: $B(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

Otras formas de expresar la función Beta: $B(m,n) = B(n,m) =$

$$= 2\int_0^{\pi/2} \operatorname{sen}^{2m-1} x \cos^{2n-1} x\,dx = \int_0^{\infty} u^{m-1}(1+u)^{-m-n}\,du = r^n(r+1)^m \int_0^1 \frac{t^{m-1}(1-t)^{n-1}}{(r+t)^{m+n}}\,dt$$

NUMEROS DE BERNOULLI Y EULER

a) Bernoulli: los números $B_1\,; B_2\,; B_3\,; \cdots$ *se definen por las series* :

$$\frac{x}{e^x-1} = 1 - \frac{x}{2} + \frac{B_1 x^2}{2!} + \frac{B_2 x^4}{4!} + \frac{B_3 x^6}{6!} + \cdots \qquad |x| < 2\pi$$

ó tambien $1 - \frac{x}{2}\operatorname{cotg}(\frac{x}{2}) = \frac{B_1 x^2}{2!} + \frac{B_2 x^4}{4!} + \frac{B_3 x^6}{6!} + \cdots \qquad |x| < \pi$

b) Euler: los números de Euler $E_1\,; E_2\,; E_3\,; \cdots$ *se definen por las series* :

$$\operatorname{sech} x = 1 - \frac{E_1 x^2}{2!} + \frac{E_2 x^4}{4!} - \frac{E_3 x^6}{6!} + \cdots \qquad |x| < \pi/2$$

$$\sec x = 1 + \frac{E_1 x^2}{2!} + \frac{E_2 x^4}{4!} + \frac{E_3 x^6}{6!} + \cdots \qquad |x| < \pi/2$$

Tabla de algunos números B_n y E_n

n	B_n	E_n
1	1/6	1
2	1/30	5
3	1/42	61
4	1/30	1385
5	5/66	50521
6	691/2730	2702765
7	7/6	199360981
8	3617/510	19391512145
9	43867/798	2404879675441
10	174611/330	370371188237525