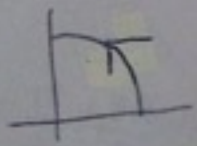
Integrals de línia amb parametritzacions diferents

- Camp escalar: indep. de la parametrització
- Camp vectorial: només depèn de l'orientació.

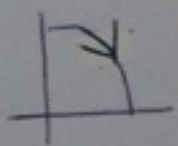
Exemples: Calcular $\int_C f\,ds$ amb $f(x,y)=x$ sobre la corba C, amb les dues parametritzacions següents:

$$\left.\begin{aligned}\sigma_1 : [0,1] &\longrightarrow \mathbb{R}^2\\ t &\longmapsto \left(\cos\tfrac{\pi}{2}t,\ \sin\tfrac{\pi}{2}t\right)\end{aligned}\right\}$$



i

$$\left.\begin{aligned}\sigma_2 : [0,\tfrac{\pi}{2}] &\longrightarrow \mathbb{R}^2\\ t &\longmapsto (\sin t,\ \cos t)\end{aligned}\right\}$$



(1) $$\int_{\sigma_1} f\,ds = \int_0^1 \cos\left(\tfrac{\pi}{2}t\right)\cdot\tfrac{\pi}{2}\,dt = \left[\sin\tfrac{\pi}{2}t\right]_0^1 = 1$$

$$\sigma_1' = \left(-\tfrac{\pi}{2}\sin\tfrac{\pi}{2}t,\ \tfrac{\pi}{2}\cos\tfrac{\pi}{2}t\right) \Rightarrow \|\sigma_1'\| = \tfrac{\pi}{2}$$

$$f(\sigma_1(t)) = \cos\tfrac{\pi}{2}t$$

(2) $$\int_{\sigma_2} f\,ds = \int_0^{\frac{\pi}{2}} \sin t\,dt = \left[-\cos t\right]_0^{\frac{\pi}{2}} = 1$$

$$\sigma_2' = (\cos t,\ -\sin t) \Rightarrow \|\sigma_2'\| = 1$$

$$f(\sigma_2(t)) = \sin t$$

Amb les mateixes parametritzacions, calcular $\int_{\sigma_1}\vec{F}\,d\vec{s}$ i $\int_{\sigma_2}\vec{F}\,d\vec{s}$ amb $\vec{F}=(x^2,y)$

$$\int_{\sigma_1} \vec{F}\, d\vec{s} = \int_0^1 \left(\cos^2\frac{\pi}{2}t, \sin\frac{\pi}{2}t\right)\left(-\frac{\pi}{2}\sin\frac{\pi}{2}t, \frac{\pi}{2}\cos\frac{\pi}{2}t\right) dt =$$

$$\vec{F}(\sigma_1(t)) = \left(\cos^2\frac{\pi}{2}t, \sin\frac{\pi}{2}t\right)$$

$$\sigma_1'(t) = \left(-\frac{\pi}{2}\sin\frac{\pi}{2}t, \frac{\pi}{2}\cos\frac{\pi}{2}t\right)$$

$$= \int_0^1 \frac{\pi}{2}\left(\sin\frac{\pi}{2}t \cdot \cos\frac{\pi}{2}t - \cos^2\frac{\pi}{2}t \sin\frac{\pi}{2}t\right) dt = \left[\left(\frac{\cos^3\left(\frac{\pi}{2}t\right)}{3} - \frac{\cos^2\left(\frac{\pi}{2}t\right)}{2}\right)\right]_0^1 =$$

$$= \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

$$\int_{\sigma_2} \vec{F}\, d\vec{s} = \int_0^{\frac{\pi}{2}} (\sin^2 t, \cos t)(\cos t, -\sin t)\, dt = \int_0^{\frac{\pi}{2}} \cos t\,(\sin^2 t - \sin t)\, dt =$$

$$\vec{F}(\sigma_2(t)) = (\sin^2 t, \cos t)$$

$$\sigma_2'(t) = (\cos t, -\sin t)$$

$$= \left[\frac{\sin^3 t}{3} - \frac{\sin^2 t}{2}\right]_0^1 = \frac{1}{3} - \frac{1}{2} = -\frac{1}{6}$$