Mechanics Formulae

General equations

$$\vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{a} = \frac{d\vec{v}}{dt} \qquad \vec{F} = m\vec{a} \qquad W_{(A\to B)} = \int_A^B \vec{F}\cdot d\vec{r} \qquad P = \frac{dW}{dt}$$

$$\vec{p} = m\vec{v} \qquad \vec{L} = \vec{r}\times\vec{p} \qquad \vec{\tau} = \vec{r}\times\vec{F}$$

Oscillations

Simple harmonic oscillator $\quad x(t) = A\cos(\omega_0 t + \varphi) \quad$ with $\quad \omega_0 = \sqrt{k/m}$

Damped harmonic oscillator $\quad x(t) = e^{-\gamma t}\left(Ae^{\sqrt{\gamma^2-\omega_0^2}t} + Be^{-\sqrt{\gamma^2-\omega_0^2}t}\right)$

- Underdamped $\quad x(t) = Ae^{-\gamma t}\cos(\tilde{\omega}t + \varphi) \quad$ with $\quad \tilde{\omega} = \sqrt{\omega_0^2 - \gamma^2} \qquad Q = \frac{2\pi E}{\Delta E} \qquad Q \approx \omega_0\tau$
- Overdamped $\quad x(t) = e^{-\gamma t}\left(Ae^{\sqrt{\gamma^2-\omega_0^2}t} + Be^{-\sqrt{\gamma^2-\omega_0^2}t}\right)$
- Critically damped $\quad x(t) = (A + Bt)e^{-\gamma t}$

Non-homogeneous differential equations $\quad x(t) = x_h(t) + x_p(t)$

- If $F_{ext} = F_0$, $\quad x_p(t) = \frac{F_0}{m\omega_0^2}$
- If $F_{ext} = F_0\cos\omega t$, $\quad x_p(t) = A\cos(\omega t + \varphi) \quad$ with $\quad A = \frac{F_0/m}{\sqrt{\left(\omega^2-\omega_0^2\right)^2 + 4\gamma^2\omega^2}} \quad$ and $\quad \tan\varphi = \frac{2\gamma\omega}{\omega^2 - \omega_0^2}$
  - $\langle P\rangle_T = \frac{F_0^2\gamma\omega^2/m}{\left(\omega^2-\omega_0^2\right)^2 + 4\gamma^2\omega^2} \qquad \Delta\omega = 2\gamma$

Central forces

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta} \qquad \vec{F} = F_r(r)\hat{r} \qquad E = \frac{1}{2}m\dot{r}^2 + V_{ef} \quad \text{with} \quad V_{ef} = \frac{L^2}{2mr^2} + V(r)$$

$$u = \frac{1}{r} = \frac{1+\epsilon\cos(\theta-\theta_0)}{p} \quad \text{with} \quad p = \frac{L^2}{mk}, \quad \epsilon = \sqrt{1 + \frac{2L^2E}{mk^2}} \quad \text{and} \quad k = GMm$$

- If $E > 0$, $\quad \epsilon > 1 \quad r = \frac{p}{\epsilon\cos\theta - 1} \quad$ with $\quad p = a(\epsilon^2 - 1)$, $\quad$ and $\quad a = \frac{k}{2E}$
- If $E < 0$, $\quad 0 < \epsilon < 1 \quad r = \frac{p}{\epsilon\cos\theta + 1} \quad$ with $\quad p = a(1 - \epsilon^2)$, $\quad$ and $\quad a = \frac{-k}{2E} \qquad T^2 = \frac{4\pi^2a^3}{GM} \qquad E = -\frac{GMm}{2a}$
- If $E = 0$, $\quad \epsilon = 1 \quad r = \frac{a}{\cos\theta + 1} \quad$ with $\quad a = \frac{L^2}{mk}$

$$\Delta v_p = \sqrt{\frac{GM}{r_p}}\left(\sqrt{\frac{2r_a}{r_p + r_a}} - 1\right) \qquad \Delta v_a = \sqrt{\frac{GM}{r_a}}\left(1 - \sqrt{\frac{2r_p}{r_p + r_a}}\right)$$

Systems of particles

$$\vec{r}_{CM} = \frac{1}{M}\sum_{i=1}^{N} m_i\vec{r}_i \quad \text{with} \quad M = \sum_{i=1}^{N} m_i \qquad \text{or} \qquad \vec{r}_{CM} = \frac{1}{M}\int \vec{r}dm \quad \text{with} \quad M = \int dm = \int \rho dV$$

$$\mu = \frac{m_1m_2}{m_1 + m_2} \qquad \Delta v = -gt + u\ln\frac{m_0}{m} \qquad I = \int Fdt$$

Fictitious forces

$$\frac{d}{dt} = \frac{d}{dt'} + \vec{\omega}\times \qquad \vec{a}' = \vec{a} - \vec{\omega}\times(\vec{\omega}\times\vec{r}) + 2\vec{v}'\times\omega - \vec{\alpha}\times\vec{r} \qquad \vec{r}' \approx \vec{r}_0' + \vec{v}_0't + \frac{1}{2}\vec{g}t^2 + \vec{v}_0'\times\vec{\omega}t^2 + \frac{1}{3}\vec{g}\times\vec{\omega}t^3$$

Rotations

$$I = \sum m_ir_i^2 \quad \text{or} \quad I = \int r^2dm \qquad I = I_{CM} + Md^2 \qquad I_z = I_x + I_y \qquad \vec{L} = \{I\}\vec{\omega} \qquad I_{ij} = \int (r^2\delta_{ij} - x_ix_j)dm$$

$$\vec{\tau} = \frac{d\vec{L}}{dt} = \{I\}\dot{\vec{\omega}} + \vec{\omega}\times(\{I\}\vec{\omega}) \rightarrow \begin{cases} \tau_1 = I_1\dot{\omega}_1 + (I_3 - I_2)\omega_2\omega_3 \\ \tau_2 = I_2\dot{\omega}_2 + (I_1 - I_3)\omega_1\omega_3 \\ \tau_3 = I_3\dot{\omega}_3 + (I_2 - I_1)\omega_1\omega_2 \end{cases} \qquad T = \frac{1}{2}\vec{\omega}^T\{I\}\vec{\omega} \qquad \frac{dT}{dt} = \vec{\omega}\cdot\vec{\tau}$$

$$\omega_1 = sin\theta sin\psi\dot{\phi} + cos\psi\dot{\theta} \qquad \omega_2 = sin\theta cos\psi\dot{\phi} - sin\psi\dot{\theta} \qquad \omega_3 = cos\theta\dot{\phi} + \dot{\psi}$$

Lagrangian mech.

$$\delta S = 0 \quad \text{with} \quad S = \int L(q,\dot{q},t)dt \quad \text{and} \quad L(q,\dot{q},t) = T - V \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial\dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$$