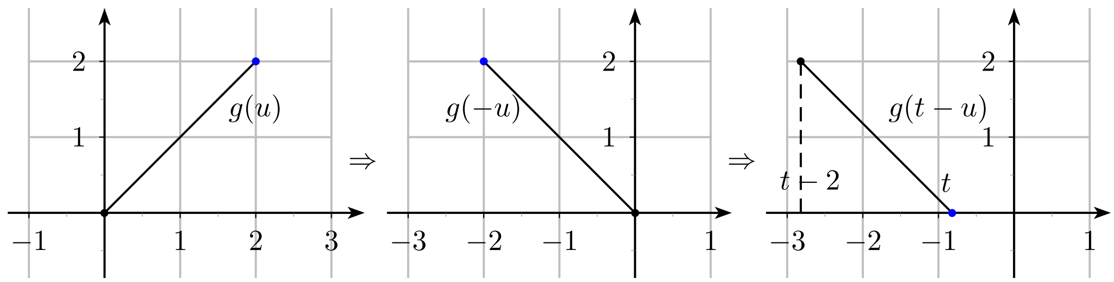
Ejemplo de Convolución

Dadas las funciones $f(t) = p_{1/2}(t - \frac{1}{2})$ y $g(t) = tp_1(t-1)$, calcular gráficamente $f * g$.

$$(f * g)(t) = \int_{-\infty}^{\infty} f(u)g(t-u)du$$

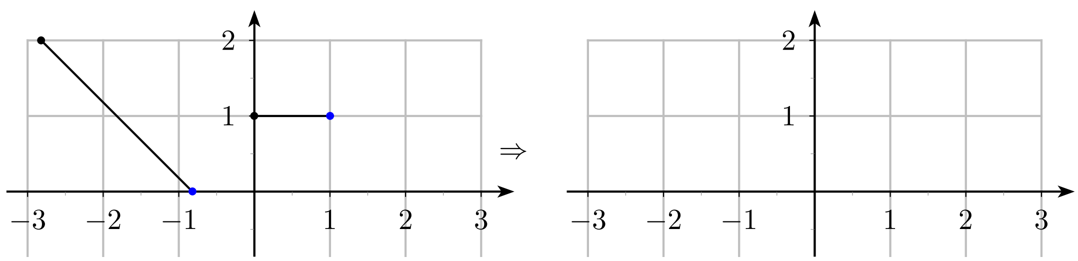
La segunda función hay que girarla y trasladarla



La función $g(t-u)$ es un trozo de recta de pendiente -1 y que pasa por el punto $(t, 0)$

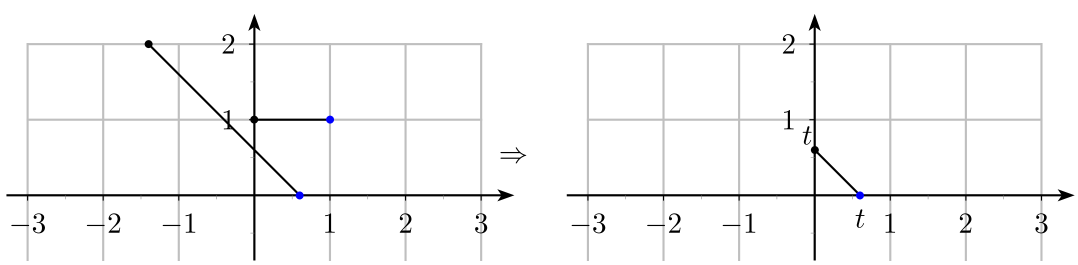
$$y = mu + n = -u + n \Rightarrow 0 = -t + n \Rightarrow n = t \Rightarrow y = -u + t$$

a) Si $t < 0$ el producto de convolución vale 0



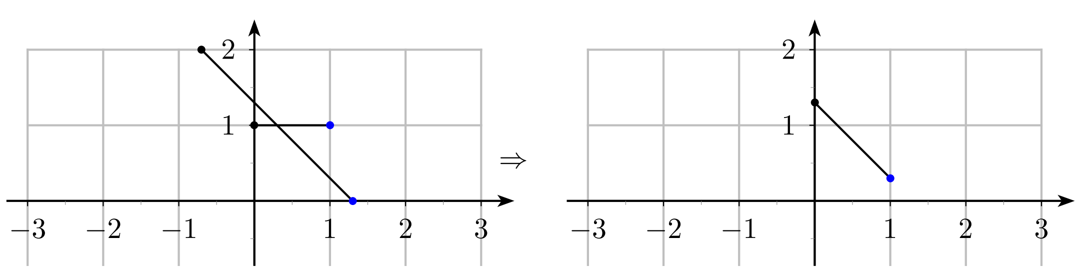
$$f(u)g(t-u) = 0 \Rightarrow \int_{-\infty}^{\infty} f(u)g(t-u)du = 0$$

b) Si $0 < t < 1$



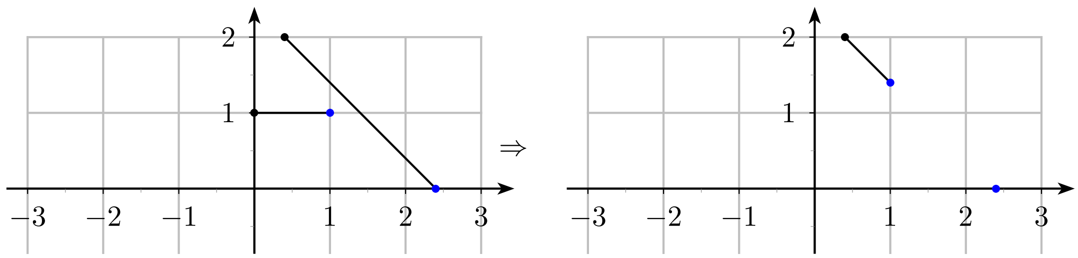
$$\int_{-\infty}^{\infty} f(u)g(t-u)du = \int_0^t -u + t\,du = \left[-\frac{u^2}{2} + tu \right]_0^t = -\frac{t^2}{2} + t^2 = \frac{t^2}{2}$$

c) Si $1 < t < 2$



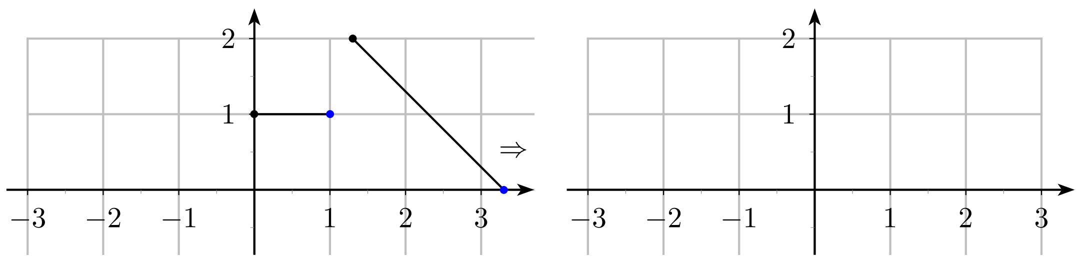
$$\int_{-\infty}^{\infty} f(u)g(t-u)du = \int_0^1 -u+t\,du = \left[-\frac{u^2}{2}+tu\right]_0^1 = -\frac{1}{2}+t$$

d) Si $2 < t < 3$



$$\int_{-\infty}^{\infty} f(u)g(t-u)du = \int_{t-2}^{1} -u+t\,du = \left[-\frac{u^2}{2}+tu\right]_{t-2}^{1} = -\frac{1}{2}+t-\left(-\frac{(t-2)^2}{2}+t(t-2)\right) = -\frac{t^2}{2}+t+\frac{3}{2}$$

e) Si $3 < t$



$$f(u)g(t-u) = 0 \Rightarrow \int_{-\infty}^{\infty} f(u)g(t-u)du = 0$$

Solución:

$$f(t)*g(t) = \begin{cases} 0 & si \quad t<0 \\ \frac{t^2}{2} & si \quad 0<t<1 \\ -\frac{1}{2}+t & si \quad 1<t<2 \\ -\frac{t^2}{2}+t+\frac{3}{2} & si \quad 2<t<3 \\ 0 & si \quad 3<t \end{cases}$$