MATRICES ASOCIADAS [1]

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ÁLGEBRA Y GEOMETRIA

[1]CURSO 20-21, Q1.

PROBLEMA 1: ENDOMORFISMO IDENTIDAD

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- $I_{\mathbb{R}^2}(\vec{v}) = \vec{v}$:

$$\begin{array}{ccc} \mathbb{R}^2 & \xrightarrow{I_{\mathbb{R}^2}} & \mathbb{R}^2 \\ (x,y) & \longmapsto & (x,y) \\ \vec{e}_1 = (1,0) & \longmapsto & \vec{e}_1 = (1,0) \\ \vec{e}_2 = (0,1) & \longmapsto & \vec{e}_2 = (0,1) \\ \vec{u}_1 = (2,3) & \longmapsto & \vec{u}_1 = (2,3) \\ \vec{u}_2 = (1,2) & \longmapsto & \vec{u}_2 = (1,2) \end{array}$$

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- $h(\vec{v}) = 3\vec{v}$:

$$\begin{array}{ccc} \mathbb{R}^2 & \xrightarrow{h} & \mathbb{R}^2 \\ (x,y) & \longmapsto & (3x,3y) \\ \vec{e}_1 = (1,0) & \longmapsto & 3\vec{e}_1 = (3,0) \\ \vec{e}_2 = (0,1) & \longmapsto & 3\vec{e}_2 = (0,3) \\ \vec{u}_1 = (2,3) & \longmapsto & 3\vec{u}_1 = (6,9) \\ \vec{u}_2 = (1,2) & \longmapsto & 3\vec{u}_2 = (3,6) \end{array}$$

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- $B = \begin{pmatrix} 6 & 3 \\ 9 & 6 \end{pmatrix} = AP$; $B' = \begin{pmatrix} 6 & -3 \\ -9 & 6 \end{pmatrix} = P^{-1}A$
- $\begin{cases} 3\vec{u}_1 = 6 \cdot \vec{e}_1 + 9 \cdot \vec{e}_2 \\ \\ 3\vec{u}_2 = 3 \cdot \vec{e}_1 + 6 \cdot \vec{e}_2 \end{cases} \Rightarrow \begin{cases} [3\vec{u}_1]_{\mathcal{C}} = \begin{pmatrix} 6 \\ 9 \end{pmatrix} \\ \\ [3\vec{u}_2]_{\mathcal{C}} = \begin{pmatrix} 3 \\ 6 \end{pmatrix} \end{cases} \Rightarrow [h]_{\mathcal{BC}} = B$

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- $g(\vec{v}) = \vec{w}$:

$$\begin{array}{ccc} \mathbb{R}^2 & \xrightarrow{g} & \mathbb{R}^2 \\ (x,y) & \longmapsto & (-y,x) \\ \vec{e}_1 = (1,0) & \longmapsto & \vec{e}_2 = (0,1) \\ \vec{e}_2 = (0,1) & \longmapsto & -\vec{e}_1 = (-1,0) \\ \vec{u}_1 = (2,3) & \longmapsto & \vec{v}_1 = (-3,2) \\ \vec{u}_2 = (1,2) & \longmapsto & \vec{v}_2 = (-2,1) \end{array}$$

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- $B = \begin{pmatrix} -8 & -5 \\ 13 & 8 \end{pmatrix} = P^{-1}AP$

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- $\left\{ \begin{array}{l} \vec{e}_2 = 0 \cdot \vec{e}_1 + 1 \cdot \vec{e}_2 \\ \\ -\vec{e}_1 = 1 \cdot \vec{e}_1 + 0 \cdot \vec{e}_2 \end{array} \right. \Rightarrow \left\{ \begin{array}{l} [\vec{e}_2]_{\mathcal{C}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ \\ [-\vec{e}_1]_{\mathcal{C}} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \end{array} \right. \Rightarrow [g]_{\mathcal{C}} = A$

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- $B' = \begin{pmatrix} -3 & -2 \\ 2 & 1 \end{pmatrix} = AP$; $B'' = \begin{pmatrix} -1 & -2 \\ 2 & 3 \end{pmatrix} = P^{-1}A$
- $\left\{ \begin{array}{l} \vec{v}_1 = -3 \cdot \vec{e}_1 + 2 \cdot \vec{e}_2 \\ \\ \vec{v}_2 = -2 \cdot \vec{e}_1 + 1 \cdot \vec{e}_2 \end{array} \right. \Rightarrow \left\{ \begin{array}{l} [\vec{v}_1]_{\mathcal{C}} = \begin{pmatrix} -3 \\ 2 \end{pmatrix} \\ \\ [\vec{v}_2]_{\mathcal{C}} = \begin{pmatrix} -2 \\ 1 \end{pmatrix} \end{array} \right. \Rightarrow [g]_{\mathcal{BC}} = B'$

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- $\left\{ \begin{array}{l} \vec{e}_2 = -1 \cdot \vec{u}_1 + 2 \cdot \vec{u}_2 \\ -\vec{e}_1 = -2 \cdot \vec{u}_1 + 3 \cdot \vec{u}_2 \end{array} \right. \Rightarrow \left\{ \begin{array}{l} [\vec{e}_2]_{\mathcal{B}} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \\ [-\vec{e}_1]_{\mathcal{B}} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} \end{array} \right. \Rightarrow [g]_{\mathcal{CB}} = B''$