STATISTICAL COMPUTATIONAL METHODS

Seminar Nr. 1, Random Variables and Applications

1. (Memoryless Property) The Exponential $Exp(\lambda), \lambda > 0$ and Shifted Geometric $SGeo(p)$, $p \in (0,1)$ variables "lose memory"; in predicting the future, the past gets "forgotten", only the present matters, i.e. if $X \in Exp(\lambda)$ or $X \in SGeo(p)$,

$$P(X > x + y \mid X > y) = P(X > x),\ \forall x, y \geq 0, \forall x, y \in \mathbb{N}, \text{respectively.}$$

2. Messages arrive at an electronic message center at random times, with an average of 9 messages per hour. What is the probability of
a) receiving *exactly* 5 messages during the next hour (event A)?
b) receiving *at least* 5 messages during the next hour (event B)?

3. After a computer virus entered the system, a computer manager checks the condition of all important files. He knows that each file has probability 0.2 to be damaged by the virus, independently of other files. Find the probability that
a) at least 5 of the first 20 files checked, are damaged (event A);
b) the manager has to check at least 6 files in order to find 3 that are undamaged (event B).

4. Consider a satellite whose work is based on block A, independently backed up by a block B. The satellite performs its task until both blocks A and B fail. The lifetimes of A and B are Exponentially distributed with mean lifetime of 10 years. What is the probability that the satellite will work for more than 10 years (event E)?

5. Compilation of a computer program consists of 3 blocks that are processed sequentially, one after the other. Each block takes Exponential time with the mean of 5 minutes, independently of other blocks. Compute the probability that the entire program is compiled in less than 12 minutes (event A). Use the Gamma-Poisson formula to compute this probability two ways.

6. Under good weather conditions, 80% of flights arrive on time. During bad weather, only 50% of flights arrive on time. Tomorrow, the chance of good weather is 60%. What is the probability that your flight will arrive on time?