

STATISTICAL COMPUTATIONAL METHODS

Seminar Nr. 6, Queuing Systems

- 1.** A metered parking lot with two parking spaces is modeled by a Bernoulli two-server queuing system with capacity limited by two cars and 30-second frames. Cars arrive at the rate of one car every 4 minutes and each car is parked for 5 minutes, on the average.

 - a)** find the transition probability matrix for the number of parked cars;
 - b)** find the steady-state distribution for the number of parked cars;
 - c)** what fraction of the time are both parking spaces vacant?
 - d)** what fraction of arriving cars will not be able to park?
 - e)** every 2 minutes of parking costs 25 cents; assuming all drivers use all the parking time they pay for, how much money is the parking lot going to raise every 24 hours?
- 2.** Trucks arrive at a weigh station according to a Poisson process with average rate of 1 truck every 10 minutes. Inspection times are Exponential with the average of 3 minutes. When a truck is on the scale, the other arrived trucks stay in line waiting for their turn. Compute

 - a)** the expected number of trucks at the weigh station at any time;
 - b)** the proportion of time when the weigh station is empty;
 - c)** the expected time each truck spends at the station, from arrival to departure;
 - d)** the fraction of time there are fewer than 2 trucks in the weigh station.
- 3.** A toll area on a highway has three toll booths and works as an M/M/3 queuing system. On the average, cars arrive at the rate of one car every 5 seconds, and it takes 12 seconds to pay the toll, not including the waiting time. Compute the fraction of time when there are ten or more cars waiting in the line.
- 4.** Sports fans tune to a local sports radio station according to a Poisson process with the rate of three fans every two minutes and listen to it for an Exponential amount of time with the average of 20 minutes.

 - a)** what queuing system is the most appropriate for this situation?
 - b)** compute the expected number of concurrent listeners at any time;
 - c)** find the fraction of time when 40 or more fans are tuned to this station.

Simulations

- 5.** Messages arrive at an electronic mail server according to a Poisson process with the average frequency of 5 messages per minute. The server can process only one message at a time and messages are processed on a “first come – first serve” basis. It takes an Exponential amount of time M_1 to process any text message, plus an Exponential amount of time M_2 , independent of M_1 , to process attachments (if there are any), with $E(M_1) = 2$ seconds and $E(M_2) = 7$ seconds. Forty percent of messages contain attachments. Use Monte Carlo methods to estimate
- a)** the expected response time of this server;
 - b)** the expected waiting time of a message before it is processed.

- 6.** A small clinic has several doctors on duty, but only one patient is seen at a time. Patients are scheduled to arrive at equal 15-minute intervals, are then served in the order of their arrivals and each of them needs a Gamma time with the doctor, that has parameters $\alpha = 4$ and $\lambda = 10/3 \text{ min}^{-1}$. Use Monte Carlo simulations to estimate
- a)** the probability that a patient has to wait before seeing the doctor;
 - b)** the expected waiting time for a patient;