

STATISTICAL COMPUTATIONAL METHODS

Seminar Nr. 5, Counting Processes

1. On the average, 6 airplanes per minute land at a certain international airport. Assume the number of landings is modeled by a Binomial counting process.

a) What frame length should be used to guarantee that the probability of a landing does not exceed 0.1?

b) Using the chosen frames, compute the probability of no landings during the next half a minute;

c) Using the chosen frames, compute the probability of more than 170 landed airplanes during the next 30 minutes.

2. Messages arrive at a communications center according to a Binomial counting process with 30 frames per minute. The average arrival rate is 40 messages per hour. How many messages can be expected to arrive between 10 a.m. and 10:30 a.m.? What is the standard deviation of that number of messages?

3. An internet service provider offers special discounts to every third connecting customer. Its customers connect to the internet according to a Poisson process with the rate of 5 customers per minute. Compute

a) the probability that no offer is made during the first 2 minutes;

b) the probability that no customers connect for 20 seconds;

c) expectation and standard deviation of the time of first offer.

4. Simulation and illustration of Binomial and Poisson counting processes.

a) Given sample path size N_B and probability of arrival p , simulate a Binomial counting process $X(t)$.

Application: For a frame size of 1 second, simulate the number of airplane landings from Problem 1., for 1 minute.

b) Given frequency λ and a time frame $[0, T_{max}]$, simulate a Poisson counting process $X(t)$.

Application: Simulate the number of internet connections from Problem 3., for a period of half an hour.