Software Systems Verification and Validation

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2024-2025

Software Systems Verification and Validation

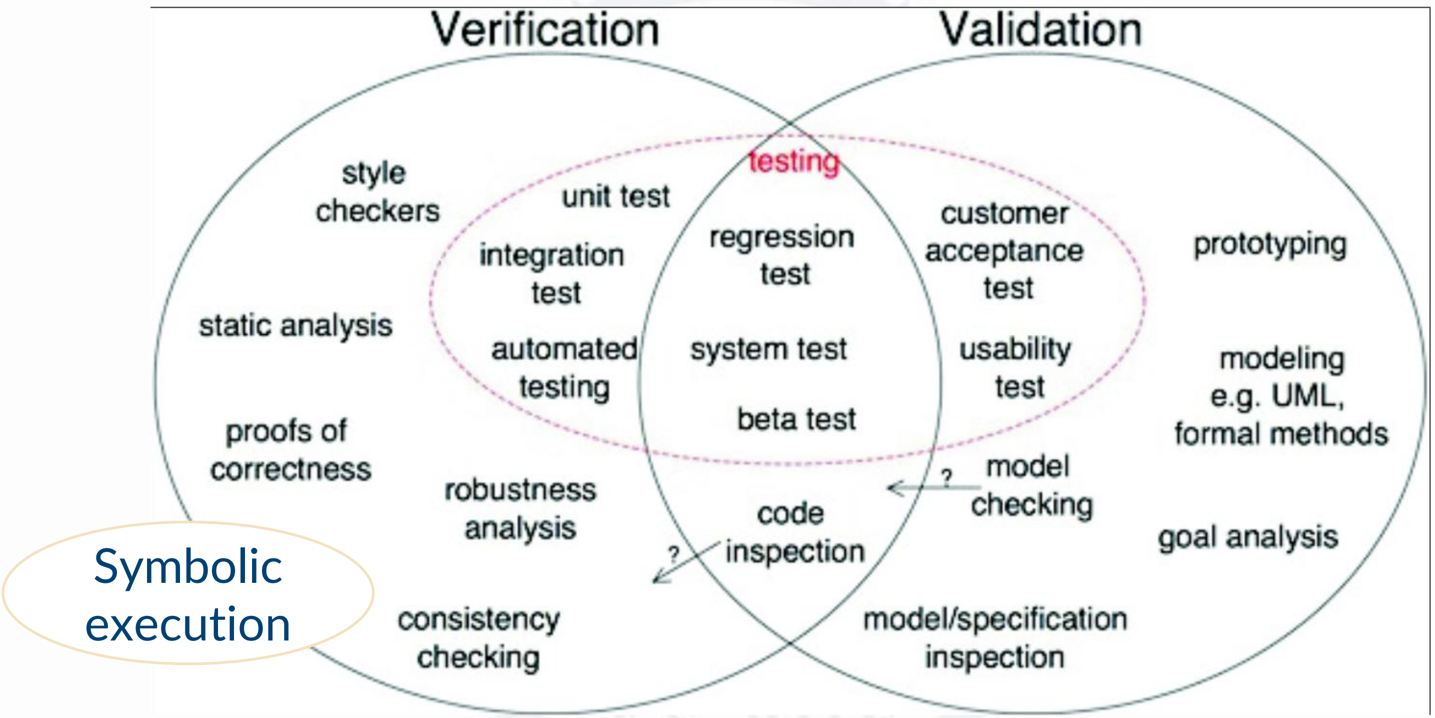
“Tell me and I forget, teach me and I may remember, involve me and I learn.”

(Benjamin Franklin)

(Next)/Today Lecture

- Correctness

What we will learn!



- http://www.easterbrook.ca/steve/2010/11/the-difference-between-verification-and-validation/

Outline

- Correctness

- Floyd's Method -Inductive assertions, Partial correctness, Termination
- Hoare Logic, Semantics of Hoare triples, Partial correctness, Total correctness
- Dijkstra's Language, Guarded commands, Nondeterminacy, Formal Derivation of Programs

- Developing correct programs from specification, Refinement, Rules of Refinement, Examples

- Static analysis, JML- Java Modeling Language, ESC/Java2- Extended Static Checker for Java

- Questions

Program verification methods - Correctness

- Lecture 1 - Verification and Validation
  - Verification/Validation
    - reviews products to ensure their quality → correctness
    - static and dynamic analysis techniques
  - A **correct program** is one that does exactly what it is intended to do, no more and no less.
  - A formally correct program is one whose correctness can be proved mathematically.
    - This requires a language for specifying precisely what the program is intended to do.
    - Specification languages are based in mathematical logic.
  - Until recently, correctness has been an academic exercise. – Now it is a key element of critical software systems.
- **Program verification - correctness**
  1. proof-based, computer-assisted, program-verification approach, mainly used for programs which we expect to terminate and produce a result
  2. model-based, automatic, property-verification approach, mainly used for concurrent, reactive systems (originally used in a post-development stage) - model checking
  3. Developing correct algorithms from specification (Carroll Morgan, "Programming from Specification)

     Correctness-by-Construction.

     Originally intended as a mere means of programming algorithms that are correct by construction - -Dijkstra (1968), Hoare (1971),

     the approach found its way into commercial development processes of complex systems - Hall (2002), Hall and Chapman (2002)

     2012, The Correctness-by-Construction Approach to Programming, Authors: **Kourie**, Derrick G., **Watson**, Bruce W.

     2015, Experience with correctness-by-construction, B.W. Watson a, D.G. Kourie b, L. Cleophas b,∗

     2016, Correctness-by-Construction and Post-hoc Verification: Friends or Foes?, Maurice H. ter Beek1(B) , Reiner H¨ahnle2, and Ina Schaefer3

     2023, Automated Software Engineering Conference, The 5th International Workshop on Automated and verifiable Software sYstem DEvelopment (ASYDE)

     Topic: Correct-by-construction software development

     (https://conf.researchr.org/track/ase-2023/ase-2023--workshop--asyde#the-5th-international-workshop-on-automated-and-verifiable-software-system-development-aside)
- **Correctness Tools**
  - Theorem provers (PVS), Modeling languages (UML and OCL), Specification languages (JML), Programming language support (Eiffel, Java, Spark/Ada), Specification Methodology (Design by contract)
- **Methods for prooving program correctness**
  - Floyd's Method - Inductive assertions
  - Hoare - Semantics of Hoare triples
  - Dijkstra's Language- Guarded commands, Nondeterminacy and Formal Derivation of Programs

Floyd's Method - Inductive assertions [Flo67]

- **Aplicability**
  - Partial correctness of the program
  - Termination of the program
  - Total correctness = Partial correctness + Termination of the program
- **Uses**
  - The condition satisfied by the initial values of the program.
  - The condition to be satisfied by the output of the program.
  - Source code of the program.
- **Method**:
  - Cut the loops
  - Find an appropriate set of inductive assertions.
  - Construct the verification/termination conditions.
- **Theorem**: If all verification conditions are true, then the program is partially correct, i.e., whenever it terminates the result is correct.
- **Remark**. The method is useful when it is combined with termination.

Robert W Floyd
(June 8, 1936 -
September 25, 2001)

Floyd's Method - Inductive assertions [Flo67]

Partial correctness - steps

- Cutting points are chosen inside the algorithm
  1. 1 point at the beginning of the algorithm, 1 point at the end;
  2. At least 1 point for each *loop* statement
- For each cutting point an assertion (invariant predicate) is chosen.
  1. Entry point - $\varphi(X)$;
  2. Ending point - $\psi(X, Z)$.
- Construction of the verification conditions
  1. Path from i to j - α;
  2. P_i and P_j are assertions in i and j;
  3. $R_\alpha(X, Y)$ - predicate that gives the condition for path α;
  4. $r_\alpha(X, Y)$ - function that gives the transformations of the variables Y from path α;
  5. $\forall X \forall Y (P_i(X, Y) \wedge R_\alpha(X, Y) \rightarrow P_j(X, r_\alpha(X, Y)))$.
- Theorem: If all the verification conditions are true then P is partial correct.

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Floyd's Method - Inductive assertions [Flo67]

Partial correctness - example

- Algorithm for $z = x^y$

$z := 1;\ u := x;\ v := y;$ A: $\varphi(X) ::= (v > 0 \wedge (y \geq 0))$
While $(v > 0)$ execute B: $\eta(X, Y) ::= z * u^v = x^y$
If (v is even)
then $u := u * u;\ v := v/2;$
else $v := v - 1;\ z := z * u;$
endIf
endWhile
endAlg; C: $\psi(X, Z) ::= z = x^y$

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Floyd's Method - Inductive assertions [Flo67]

Termination- steps

- Cut the loops and find "good" inductive assertions.
- Choose a well-formed set M (i.e., an ordered set without infinite strictly decreasing sequences)
- To demonstrate that some termination conditions hold: passing from one cutting point to another the values of some functions in the well-ordered set decrease.
- In point i a function is chosen $u_i : D_X \times D_Y \rightarrow M$ and the termination condition on α is:
  $\forall X \forall Y(\varphi(X) \wedge R_\alpha(X, Y) \rightarrow (u_i(X, Y) > u_j(X, r_\alpha(X, Y))))$.
- **Remark**. If partial correctness was demonstrated then the termination condition can be:
  $\forall X \forall Y(P_i(X) \wedge R_\alpha(X, Y) \rightarrow (u_i(X, Y) > u_j(X, r_\alpha(X, Y))))$.
- Theorem: If all the termination conditions hold then the program P terminates.

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Floyd's Method - Inductive assertions [Flo67]

Termination- example

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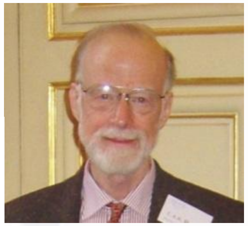
Hoare triples [Hoa69]



- The Grand Verification Challenge Hoare 2003
- Develop a compiler which verifies that the program is correct
- https://vimeo.com/39256698

Charles Antony Richard Hoare
(11 January 1934, Colombo, Sri Lanka)

- The meaning of a statement is described by a triple
  - $\{\varphi\}\ P\ \{\psi\}$, where φ is called the precondition and ψ is called the postcondition.

{P} S {Q}

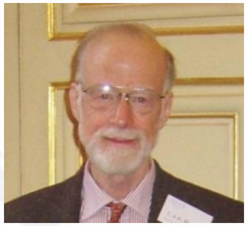
"when started in a state satisfying P, any terminating execution of S ends in a state satisfying Q "

- If P does not terminate, we make no guarantees.

- Partial correctness
  - $\models_{par} \{\varphi\}P\{\psi\}$
  - only if P actually terminates.
- Total correctness
  - $\models_{tot} \{\varphi\}P\{\psi\}$
  - the program P is guaranteed to terminate.

Hoare triples [Hoa69]

Partial correctness



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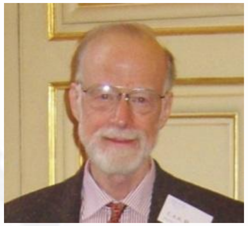
Rules

- Assignment
- Sequencing
- Conditional
- Loop

Hoare triples [Hoa69]

Partial correctness



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Assignment

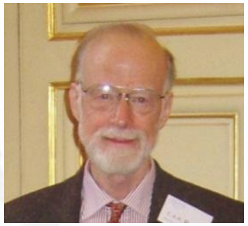
General Form: for any expression E

- $\{P\}\ X := E\{Q\}\ \textit{provided}[P \Rightarrow \langle X \leftarrow E\rangle\,(Q)]$

- Consider the triple $\{P\}X := Y + 2\{Q\}$
  - Given predicate Q, for what predicate P does this hold?
  - for any P such that $[P \Rightarrow \langle X \leftarrow Y + 2\rangle\,(Q)]$
- Examples
  - $\{P_0\}\ X := Y + 2\ \{X \le Y + 2\}$
    $P_0 \equiv \textit{true}$
  - $\{P_1\}\ X := Y + 2\ \{X < 0\}$
    $P_1 \equiv (Y + 2 < 0)$
  - $\{P_2\}\ X := Y + 2\ \{Y < 0\}$
    $P_2 \equiv (Y < 0)$
  - $\{P_3\}\ X := X + 2\ \{X\ \textit{is even}\}$
    $P_3 \equiv (X\ \textit{is even})$

Hoare triples [Hoa69]

Partial correctness



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Sequencing

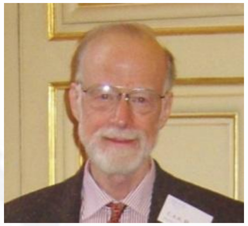
- We can conclude
  $\{P\}\ S;\ T\{Q\}$
  if we can find a predicate R such that
  $\{P\}\ S\{R\}$ *and* $\{R\}T\{Q\}$

Examples

- $\{P_0\}\ X := 2 * X;\ \ X := X + 1\{X > 0\}$
  $P_0 \equiv (2 * X + 1 > 0)]$
- $\{P_1\}\ X := Y;\ \ Y := 3\ \{X + Y < 5\}$
  $\{P_1 \equiv (Y + 3 < 5)]$

Hoare triples [Hoa69]

Partial correctness



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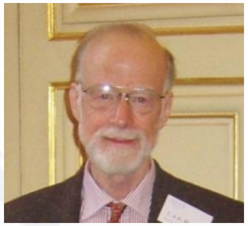
Conditional

- We can conclude
  $\{P\}\ IF\ (C)\ THEN\ S\ ELSE\ T\ END\{Q\}$
  provided we can show
  $\{P \wedge C\}\ S\{Q\}$ *and* $\{P \wedge \neg C\}T\{Q\}$

- Examples
  - $\{?\}\ \{((x > y) \Rightarrow Q_0) \wedge ((x \leq y) \Rightarrow Q_1)\}$
    $IF\ (x > y)\ THEN \quad Q_0 : \{(m|x - y) \wedge (m|y)\}$
    $x := x - y$
    $ELSE\ \ Q_1 : \{(m|x) \wedge (m|y - x)\}$
    $y := y - x$
    END
    $Q : \{(m|x) \wedge (m|y)\}$
  - So our final proof obligations are
    $[(x > y) \Rightarrow (m|x - y) \wedge (m|y)$ *and*
    $[(x \leq y) \Rightarrow (m|x) \wedge (m|y - x)]$

Hoare triples [Hoa69]

Partial correctness



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Loop

- How can we conclude
  $\{P\}$ $WHILE\ (G)\ DO\ S\ END$ $\{Q\}$
  At the end of the loop (assuming it terminates), we know $\neg G$
  But in general we dont know how often S is executed...
- Suppose we have a predicate J that is preserved by S
  $\{J\}S\{J\}$ such a J is called a loop invariant
  Then, at the end of the loop, we can conclude
  $J \wedge \neg G$
  To establish the postcondition, we need J such that
  $[J \wedge \neg G \Rightarrow Q]$

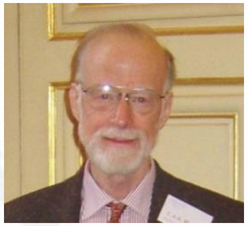
- We can conclude
  $\{P\}$ $WHILE\ (G)\ DO\ S\ END$ $\{Q\}$
  provided we can find a loop invariant J such that

$[P \Rightarrow J]$	J holds at loop entry
$[J \wedge \neg G \Rightarrow Q]$	J establishes Q at loop exit
$\{G \wedge J\}S\{J\}$	J is preserved by each iteration

- Exponentiation using multiplication
  - $\{(A > 0) \wedge (B \geq 0)\}$ S $\{R = A^B\}$

$\{(A > 0) \wedge (B \geq 0)\}$
$R :=?; b := 0$ R:=1
WHILE $(b \neq B)$ DO $J : R = A^b$
$R :=?;\ R := R * A;$
$b := b + 1$
END
$\{R = A^B\}$

Hoare triples [Hoa69]



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"when started in a state satisfying P, any terminating execution of S ends in a state satisfying Q "

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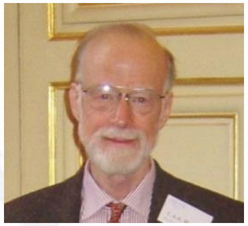
- Partial correctness
  - $\models_{par} \{\varphi\}P\{\psi\}$
  - only if P actually terminates.
- Total correctness
  - $\models_{tot} \{\varphi\}P\{\psi\}$
  - the program P is guaranteed to terminate.

- The "total correctness" interpretation also requires termination
"when started in a state satisfying P, any execution of S must terminate in a state satisfying Q "

Hoare triples [Hoa69]

Termination



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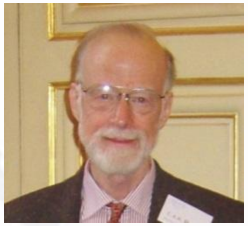
Rules

- Assignment
- Sequencing
- Conditional
- Loop

- Assignment
  $\{P\}\ X := E\ \{Q\} provided [P \Rightarrow \langle X \leftarrow E \rangle (Q)]$
- Sequencing
  $\{P\}\ S; T\{Q\}$ provided
  $\{P\}\ S\ \{R\}$ *and* $\{R\} T\ \{Q\}$ for some R
- Conditional
  $\{P\}\ IF\ (G)\ THEN\ S\ ELSE\ T\ END\ \{Q\}$ provided
  $\{P \wedge G\}\ S\ \{Q\}$ *and* $\{P \wedge \neg G\}\ T\{Q\}$
- Note: Same as the rules for partial correctness!

- Total correctness rule for loops
- Consider
  $\{P\}\ WHILE\ (G)\ DO\ S\ END\ \{Q\}$
- How do we show that the loop terminates?
- One method
  find an integer expression V such that
  the value of V is nonnegative (that is $V \geq 0$), and
  the value of V (strictly) decreases in every iteration that is,
  $\{V = K\}\ S\ \{V < K\}$
- Such an expression is called a "loop variant"

Hoare triples [Hoa69]



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Exponentiation using multiplication

- $\{(A > 0) \wedge (B \geq 0)\}$ S $\{R = A^B\}$
- Recall loop invariant $J : R = A^b \ \wedge (B \geq b);$

$\{(A > 0) \wedge (B \geq 0)\}$
$R := 1; b := 0$
WHILE $(b \neq B)$ DO $\quad J: \ R = A^b \wedge (B \geq b);$
$R := R * A;$
$b := b + 1$
END
$\{R = A^B\}$

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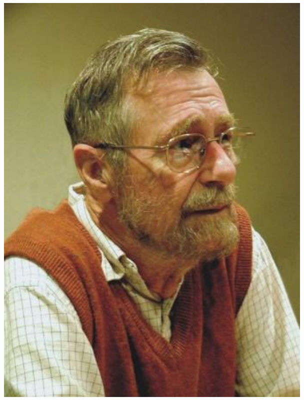
Edsger Wybe Dijkstra [Dij75]

Guarded command

- "guarded command" - a statement list prefixed by a boolean expression: only when this boolean expression is initially true, is the statement list eligible for execution
- $<$ *guarded command* $>::=<$ *guard* $>\rightarrow<$ *guarded list* $>$
- $<$ *guard* $>::=<$ *boolean expression* $>$
- $<$ *guarded list* $>::=<$ *statement* $>\{;<$ *statement* $>\}$
- $<$ *guarded command set* $>::=$
  $<$ *guarded command* $>\{\Box<$ *guarded command* $>\}$
- $<$ *alternative construct* $>::=$ **if** $<$ *guarded command set* $>$ **fi**
- $<$ *repetitive construct* $>::=$ **do** $<$ *guarded command set* $>$ **do**
- $<$ *statement* $>::=<$ *alternative construct* $>$ |
  $<$ *repetitive construct* $>$ | "*other statements*"



Edsger Wybe Dijkstra
(May 11, 1930 - August 6, 2002)

Edsger Wybe Dijkstra [Dij75]

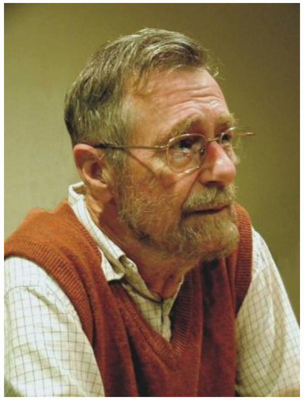
Nondeterminacy

- Example 1

  if $x \geq y \rightarrow m := x$
  $\Box y \geq x \rightarrow m := y$
  fi

- Example 2 - compute k s.t. for fixed value n and fixed function *f(i) (defined for* $0 \leq i < n$*), k will eventually satisfy* $0 \leq k < n$ *and* $(\forall i : 0 \leq i < n : \; f(k) \geq f(i))$.

  $k := 0;\; j := 1;$
  do $j \neq n \rightarrow$ **if** $f(j) \leq f(k) \rightarrow j := j + 1$
  $\Box f(j) \geq f(k) \rightarrow k := j;\; j := j + 1$
  fi
  od



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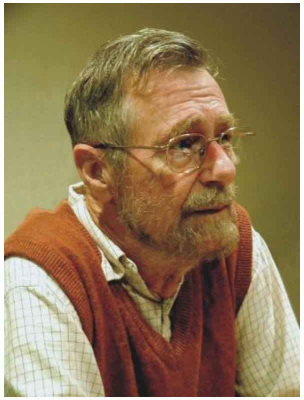
Edsger Wybe Dijkstra [Dij75]

Weakest pre-conditions



Edsger Wybe Dijkstra
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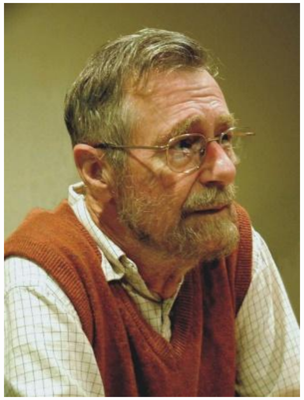
- Hoare - introduced sufficient pre-conditions such that the mechanism will not produce the wrong result but may fail to terminate.
- Dijkstra - introduced necessary and sufficient pre-conditions such that the mechanism are guaranteed to produce the right result.

  = weakest pre-conditions
- wp(S, R), where S denotes a statement list, R some condition on the state of the system.
- wp - called a “ predicate transformer” - because it associates a pre-condition to any post-condition R.

Edsger Wybe Dijkstra [Dij75]

Properties of wp

1. Law of the Excluded Miracle
   For any S, for all states: $wp(S, F) = F$
2. For any S and any two post-conditions, such that for all states $P \Rightarrow Q$, for all states:
   $$wp(S, P) \Rightarrow wp(S, Q)$$
3. For any S and any two post-conditions P and Q, for all states:
   $$wp(S, P) \textbf{ and } wp(S, Q) = wp(S, P \textbf{ and } Q)$$
4. For any deterministic S and any post-conditions P and Q, for all states:
   $$(wp(S,\ P) \textbf{ or } wp(S,\ Q)) \Rightarrow wp(S, P \textbf{ or } Q)$$



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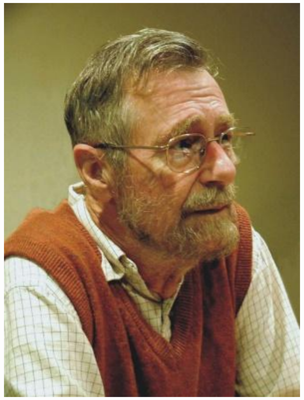
Edsger Wybe Dijkstra [Dij75]

Assignment and concatenation operator



Edsger Wybe Dijkstra
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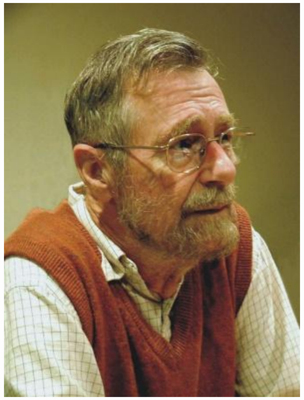
- Assignment
  The semantics of $x := E$ are given by:
  $wp(\text{“}x := E\text{''}, R) = R^x_E$, R^x_E -denotes a copy of the predicate defining R in which each occurrence of the variable x is replaced by E.
- Concatenation operator ;
  The semantics of the ; operator are given by:
  $wp(\text{“}S1\ ; S2\text{''}, R) = wp(S1, wp(S2, R))$.

Edsger Wybe Dijkstra [Dij75]

The Alternative Construct

- Let IF denote: $\textbf{if} B_1 \rightarrow SL_1 \Box ... \Box B_n \rightarrow SL_n \textbf{fi}$
  Let BB denote: $(\exists i: \ 1 \leq i \leq n: \ B_i)$, then, by definition
  $wp(IF, R) = (BB \ and \ (\forall i: \ 1 \leq i \leq n: \ B_i \Rightarrow wp(SL_i, R)))$.
- Theorem 1
  From $(\forall i: 1 \leq i \leq n: \ (Q \textbf{ and } B_i) \Rightarrow wp(SL_i, R)$ for all states we can conclude that $(Q \textbf{ and } BB) \Rightarrow wp(IF, R)$ holds for all states.)
- Let t denote some integer function, and $wdec(S, t)$
- Theorem 2
  From $(\forall i: 1 \leq i \leq n: (Q \textbf{ and } B_i) \Rightarrow wdec(SL_i, t))$ for all states we can conclude that $(Q \textbf{ and } BB) \Rightarrow wdec(IF, t)$ hold for all states.
- By definition,
  $wdec(S, t) = (tmin(X) \leq t(X) - 1) = (tmin(X) < t(X))$.



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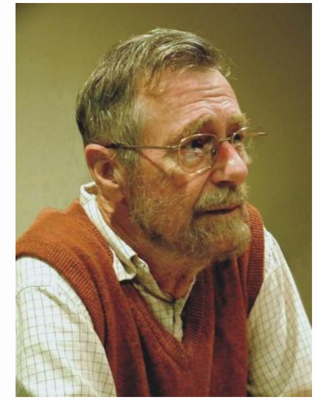
Edsger Wybe Dijkstra [Dij75]

The Alternative Construct - example

- The formal requirements for performing $m := max(x, y)$ is: $R : \; (m = x$ **or** $m = y)$ **and** $m \geq x$ **and** $m \geq y$.
- Assignment $m := x$ for $m = x$?
  $wp(``m := x", R) = (x = x$ **or** $x = y)$ **and** $x \geq x$ **and** $x \geq y = x \geq y$
- Theorem 1: **if**$x \geq y \rightarrow m := x$**fi**
- But $B \neq T$, so we weakening BB means looking for alternatives which might introduce new guards.
- Alternative: "$m := y$" that introduces the new guard
  $wp(``m" := y, R) = y \geq x$
  if$x \geq y \rightarrow m := x$
  $\square y \geq x \rightarrow m := y$
  fi

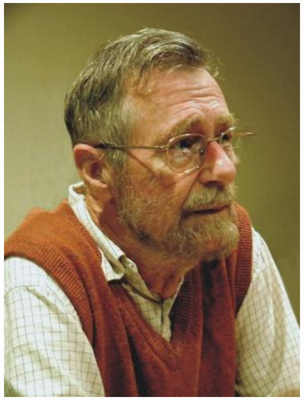


Edsger Wybe Dijkstra
(May 11, 1930 - August 6, 2002)

Edsger Wybe Dijkstra [Dij75]

The Repetitive Construct

- Let DO denote: $\mathbf{do} B_1 \rightarrow SL_1 \square ... \square B_n \rightarrow SL_n \mathbf{do}$
  Let $H_0 = (R \textbf{ and non } BB)$
  and for $k > 0$, $H_k(R) = (wp(IF, H_{k-1}(R))) \textbf{ or } H_0(R)$
  then, by definition: $wp(DO, R) = (\exists k : k \geq 0 : H_k(R))$.
- Theorem 3
  If we have all the states
  $(P \textbf{ and } BB) \Rightarrow (wp(IF, P) \textbf{ and } wdec(IF, t) \textbf{ and } t \geq 0)$ we can conclude that we have for all states $P \Rightarrow wp(DO, P \textbf{ and non } BB)$
- T is the condition satisfied by all states, and $wp(S, T)$ is the weakest pre-condition guaranteeing proper termination of S.
- Theorem 4
  From $(P \textbf{ and } BB) \Rightarrow wp(IF, P)$ for all states, we can conclude that we have for all states
  $(P \textbf{ and } wp(DO, T) \Rightarrow wp(DO, P \textbf{ and non } BB))$



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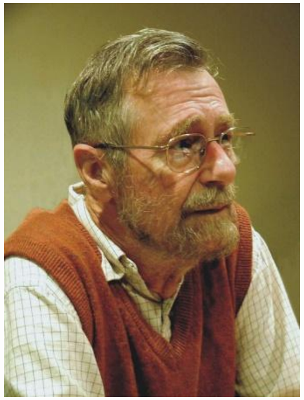
The Repetitive Construct - example

- The greatest common divisor: $x = gcd(X, Y)$
- Choose an invariant relation and variant function.
  establish the relation P to be kept invariant
  do "decrease t as long as possible under variance of P" **od**
- invariant relation (established by $x := X; y := Y$):
  $P : gcd(X, Y) = gcd(x, y)$ **and** $x > 0$ **and** $y > 0$
- $(P$ **and** $B) \Rightarrow wp(\text{“}x, y : E1, E2\text{''}, P))$
  $= (gcd(X, Y) = gcd(E1, E2)$ *and* $E1 > 0$ **and** $E2 > 0)$.
  - $gcd(X, Y) = gcd(E1, E2)$ is implied by P
  - invariant for (x, y) : $wp(\text{“}x := x - y, P) = (gcd(X, Y) = gcd(x - y, y)$ **and** $x - y > 0$ **and** $y > 0)$, and guard $x > y$
  - decrease of the variant function $t = x + y$
    $wp(\text{“}x := x - y\text{''}, t \leq t_0) = (x \leq t_0)$
    $tmin = x,\ wdec(\text{''}x := x - y\text{''}, t) = (x < x + y) = y > 0$



Edsger Wybe Dijkstra
(May 11, 1930 - August 6, 2002)

- x:=X; y:=Y
  do $x > y \rightarrow x := x - y$ **od**
- But P **and** BB - are not allowed to conclude $x = gcd(X, Y)$
  the alternative $y := y - x$ requires a guard $y > x$
- x:=X; y:=Y
  do $x > y \rightarrow$ x:=x-y
  $\Box y > x \rightarrow y := y - x$
  od

Outline

- Correctness

- Floyd's Method -Inductive assertions, Partial correctness, Termination
- Hoare Logic, Semantics of Hoare triples, Partial correctness, Total correctness
- Dijkstra's Language, Guarded commands, Nondeterminacy, Formal Derivation of Programs

- Developing correct programs from specification, Refinement, Rules of Refinement, Examples

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2023, Automated Software Engineering Conference, The 5th International Workshop on Automated and verifiable Software sYstem DEvelopment (ASYDE)

Topic: Correct-by-construction software development

(https://conf.researchr.org/track/ase-2023/ase-2023--workshop--asyde#the-5th-international-workshop-on-automated-and-verifiable-software-system-development-aside)

Static analysis, JML- Java Modeling Language, ESC/Java2- Extended Static Checker for Java

- Questions

Developing correct programs from specification[Mor98]

Refinement

- Input data: X $\varphi(X)$
  Output data: Z $\psi(X, Z)$
- Abstract program
  $Z : [\varphi, \psi]$
- Refinement
  $P_1 \prec P_2 \prec ... \prec P_{n-1} \prec P_n$
- Rules of refinement
  - Assignment rule
  - Sequential composition rule
  - Alternation rule
  - Iteration rule

Carroll Morgan

https://my.cse.unsw.edu.au/staff/staff_details.php?ID=carrollm

Developing correct programs from specification[Mor98]

Rules of Refinement

- Assignment rule: $[\varphi(v/e), \psi] \prec v := e$
- Sequential composition rule (γ – *middlepredicate*)
  $[\eta_1, \eta_2] \prec [\eta_1, \gamma]$
  $\qquad\qquad\quad [\gamma, \eta_2]$
- Alternation rule, $G = g_1 \vee g_2 \vee \ldots \vee g_n$
  $[\eta_1, \eta_2] \prec$
  if $g_1 \rightarrow [\eta_1 \wedge g_1, \eta_2]$
  $\quad \square g_2 \rightarrow [\eta_1 \wedge g_2, \eta_2]$
  $\quad \vdots$
  $\quad \square g_n \rightarrow [\eta_1 \wedge g_n, \eta_2]$
  fi
- Iteration rule $G = g_1 \vee g_2 \vee \ldots \vee g_n$
  $[\eta, \eta \wedge \neg G] \prec$
  do $g_1 \rightarrow [\eta \wedge g_1, \eta \wedge TC]$
  $\quad \square g_2 \rightarrow [\eta \wedge g_2, \eta \wedge TC]$
  $\quad \vdots$
  $\quad \square g_n \rightarrow [\eta \wedge g_n, \eta \wedge TC]$
  do

Program verification methods - Correctness

- Lecture 1 - Verification and Validation
  - Verification/Validation
    - reviews products to ensure their quality → correctness
    - static and dynamic analysis techniques
  - A **correct program** is one that does exactly what it is intended to do, no more and no less.
  - A formally correct program is one whose correctness can be proved mathematically.
    - This requires a language for specifying precisely what the program is intended to do.
    - Specification languages are based in mathematical logic.
  - Until recently, correctness has been an academic exercise. – Now it is a key element of critical software systems.
- **Program verification - correctness**
  1. proof-based, computer-assisted, program-verification approach, mainly used for programs which we expect to terminate and produce a result
  2. model-based, automatic, property-verification approach, mainly used for concurrent, reactive systems (originally used in a post-development stage) - model checking (Lecture 9)
  3. Developing correct algorithms from specification (Carroll Morgan, "Programming from Specification)

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- **Correctness Tools**
  - Theorem provers (PVS), Modeling languages (UML and OCL), Specification languages (JML), Programming language support (Eiffel, Java, Spark/Ada), Specification Methodology (Design by contract)
- **Methods for prooving program correctness**
  - Floyd's Method - Inductive assertions
  - Hoare - Semantics of Hoare triples
  - Dijkstra's Language- Guarded commands, Nondeterminacy and Formal Derivation of Programs

Program verification methods - Correctness

- **Software engineering problem:** building/maintaining **correct** systems.
  - How?
    - Specification
    - Tools
- Formal Methods in Software Engineering
  - Formal languages guarantee
    - Precision (no ambiguity)
    - Certainty (modeling errors)
    - Automation (automatic verification tools).

- Things to do:
  1) make a ***formal model***
  2) ***specify properties*** for the model
  3) ***verify/check*** the properties

- Formal methods and JML (Java Modeling Language):
  1) formal model is ***Java programming language***
  2) the properties are specified in ***JML***
  3) Properties may be
     - ***Tested*** using ***jmlrac***
     - ***Checked*** using ***ESC2Java***

What is JML?

- Gary T. Leavens's JML group at the University of Central Florida
- http://www.eecs.ucf.edu/~leavens/JML//index.shtml

- a behavioral interface specification language
- used to specify the behavior of Java modules
- combines
  - design by contract approach
  - the model-based specification approach
  - some elements of the refinement calculus

Tools for using JML

- Runtime assertion checkers (e.g. **jmlc/jmlrac**)
- Static checkers (**ESC2Java**)

- Test generation (e.g. jmlunit)
- Formal verification tools (e.g. KeY)
- Design tools (e.g. AutoJML)

Tools for JML

Runtime assertion checking with jmlc/jmlrac

- Special compiler inserts runtime tests for all JML assertions. Any assertion violation results in a special exception.
- checks specs at run-time
- only **tests** correctness of **specs**.
- **Find violations at runtime.**

Extended static checking with ESC/Java

- Automatically tries to prove simple JML assertions at compile time.
- checks specs at compile-time
- **proves** correctness of **specs**.
- **Warn about likely runtime exceptions and violations.**

JML web page

- http://www.eecs.ucf.edu/~leavens/JML//index.shtml

ESC/Java2 web page

http://www.kindsoftware.com/products/opensource/ESCJava2/download.html

Design by contract

Contract?

Method contract

Precondition

Specifies "caller's responsibility"

- Constraints on parameter values and target object's state.
- Valid object's states, in which a method can be called.

Intuitively

- Expression that must hold at the entry to the method.

Postcondition

Specifies "implementation's responsibility"

- Constraints on the method's return value and side effects.
- Relation between initial and final state of the method.

Intuitively

- Expression that must hold at the exit from the method.

Class contract

Invariant

- Specifies caller's responsibility at the entry to a method and implementation's responsibility at the exit from a method.
- Valid states of class instances (values of fields).

Intuitively

- Expression that must hold at the entry and exit of each method in the class.

Tools for JML

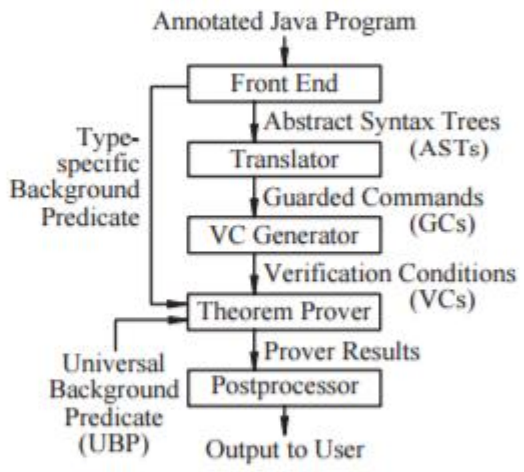
Runtime assertion checking with jmlc/jmlrac

- Special compiler inserts runtime tests for all JML assertions. Any assertion violation results in a special exception.
- checks specs at run-time
- only **tests** correctness of **specs**.
- **Find violations at runtime.**

***jmlc* and *jmlrac* – by example**

- Compile and Run
- Compile
- jmlc FileName.java
- Run
- jmlrac FileName listOfParam

- Demo 01: Factorial
- Demo02: Integer sqrt



- Unsound ?

- Incomplete ?

Tools for JML

Extended static checking with ESC/Java

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- checks specs at compile-time
- **proves** correctness of **specs**
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ESC/Java2 – by example

- Run
- escj FileName.java

- Demo 01: Fast exponentiation
- Demo 02: MyArray
- Demo 03: MySet

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Correct-by-construction

Correctness by Construction How Can We Build Better Software, Date: May 31, 2023

SPEAKER Ina Schaefer, Professor of Software Engineering, Karlsruhe Institute of Technology (KIT), Germany

https://www.youtube.com/watch?v=5Nno9lSggPo

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Floyd, Dijkstra, Hoare (25XP)

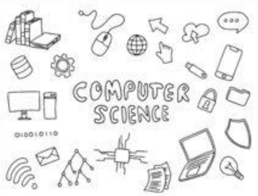
- **Robert Floyd** OR **Edsger Wybe Dijkstra** OR **Charles Antony Richard Hoare**

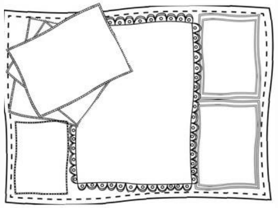
- 1 page A4 information (electronic format and printed format)
  - short bio
  - profession
  - Institution
  - known by…
  - awards
  - interesting facts

- Feel free to select a format: plain text, mindmap, other
- Delivery:
  - Today, after 10 minutes
  - 3-4 students per team

In-class

Doodle map





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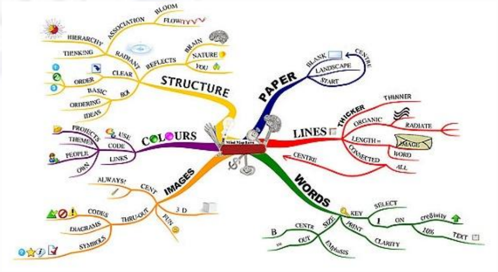

Next Lecture

Lecture attendance: 25XP

- Altom Invited Lecture

Challenges with Automation for Games

Presenters:	**Tuesday**
Rotaru Alexandru	**6 May 2025, 8-10 am**
	Room: 6/II (Main Building)

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References

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Software Systems Verification and Validation

"Tell me and I forget, teach me and I may remember, involve me and I learn."

(Benjamin Franklin)