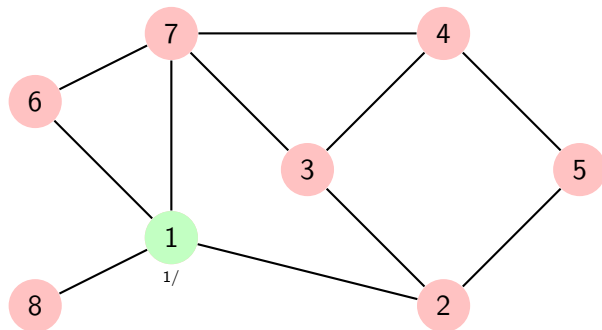


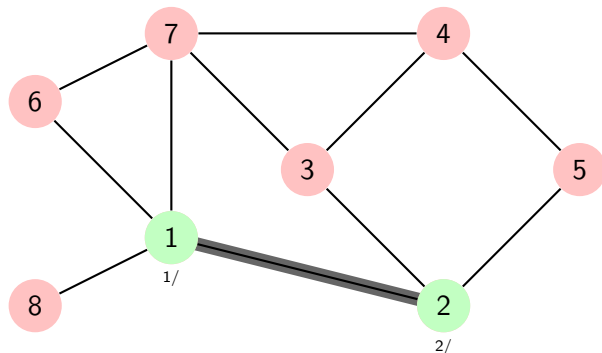
# Parcurgerea grafurilor - DFS



dfs(1)

Stiva de apeluri

# Parcurgerea grafurilor - DFS

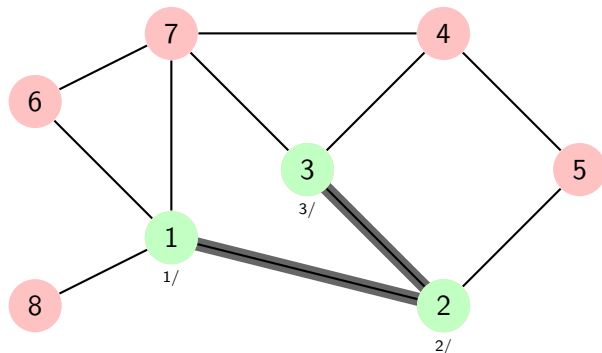


dfs(2)

dfs(1)

Stiva de apeluri

# Parcurgerea grafurilor - DFS



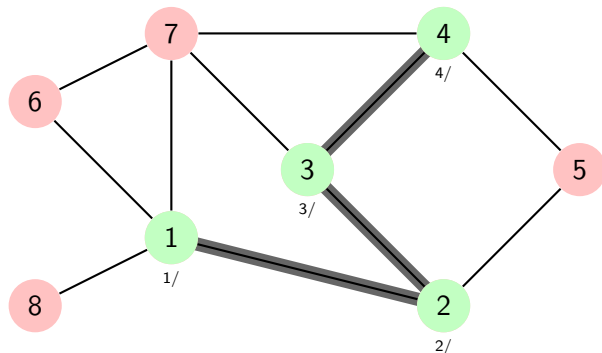
dfs(3)

dfs(2)

dfs(1)

Stiva de apeluri

# Parcurgerea grafurilor - DFS



dfs(4)

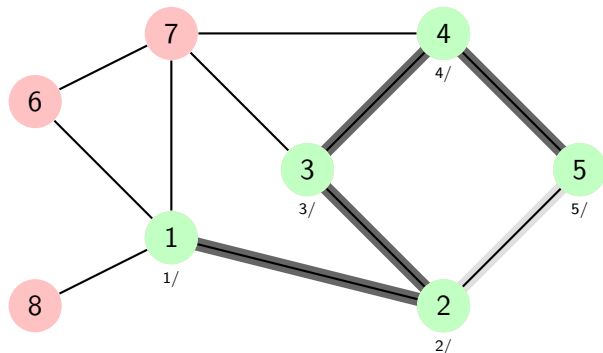
dfs(3)

dfs(2)

dfs(1)

Stiva de apeluri

# Parcurgerea grafurilor - DFS



dfs(5)

dfs(4)

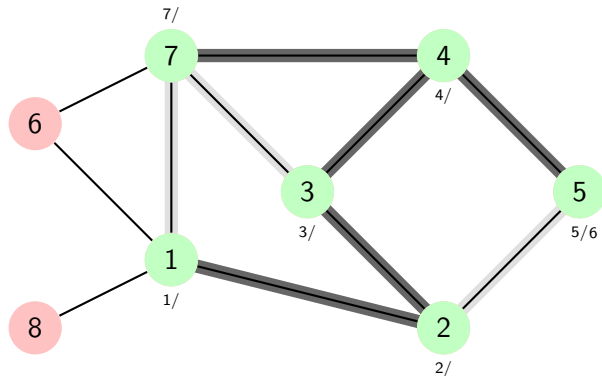
dfs(3)

dfs(2)

dfs(1)

Stiva de apeluri

# Parcurgerea grafurilor - DFS



dfs(7)

~~dfs(5)~~

dfs(4)

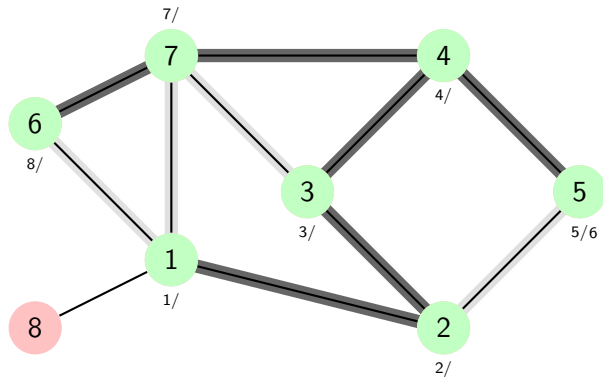
dfs(3)

dfs(2)

dfs(1)

Stiva de apeluri

# Parcurgerea grafurilor - DFS



dfs(6)

dfs(7)

~~dfs(5)~~

dfs(4)

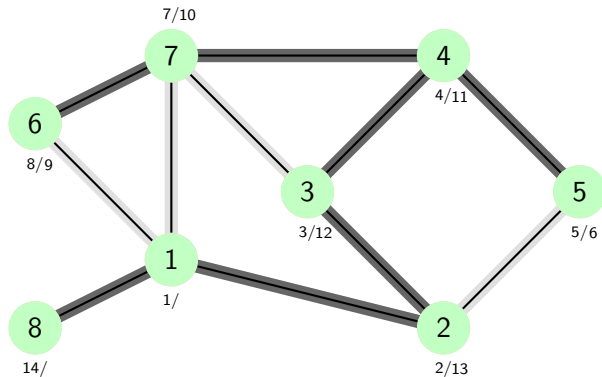
dfs(3)

dfs(2)

dfs(1)

Stiva de apeluri

# Parcurgerea grafurilor - DFS



dfs(8)

~~dfs(6)~~

~~dfs(7)~~

~~dfs(5)~~

~~dfs(4)~~

~~dfs(3)~~

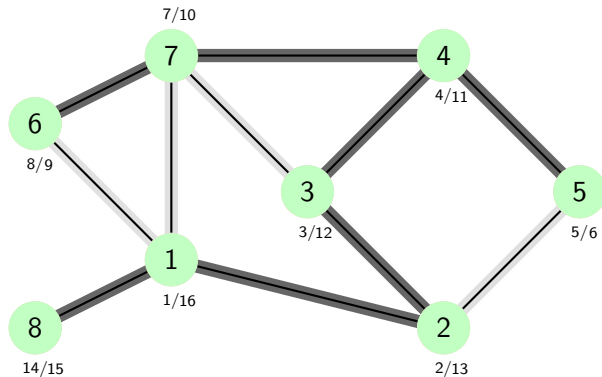
~~dfs(2)~~

dfs(1)

Stiva de apeluri



# Parcurgerea grafurilor - DFS



~~dfs(8)~~

~~dfs(6)~~

~~dfs(7)~~

~~dfs(5)~~

~~dfs(4)~~

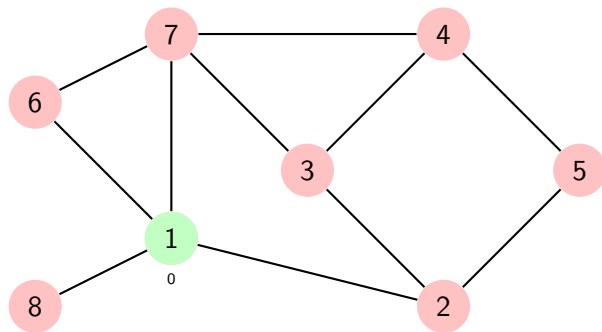
~~dfs(3)~~

~~dfs(2)~~

~~dfs(1)~~

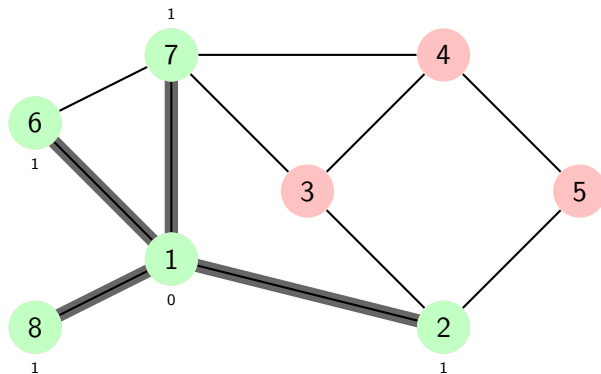
Stiva de apeluri

# Parcurgerea grafurilor - BFS



Coadă de noduri  
1

# Parcurgerea grafurilor - BFS



Coadă de noduri

1

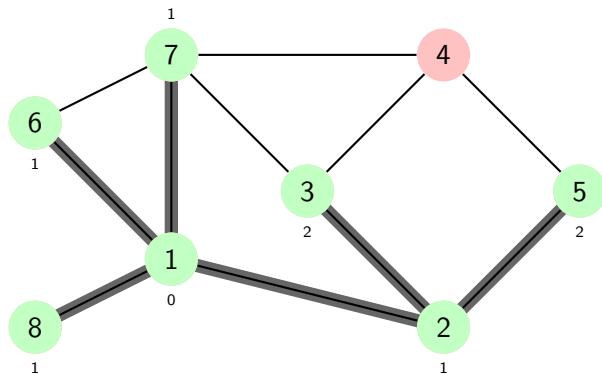
2

6

7

8

# Parcurgerea grafurilor - BFS



Coadă de noduri

1

2

6

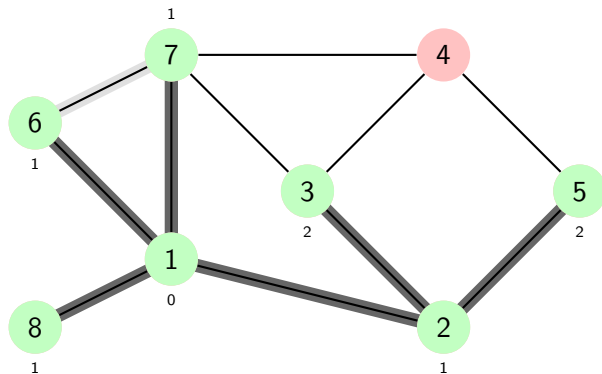
7

8

3

5

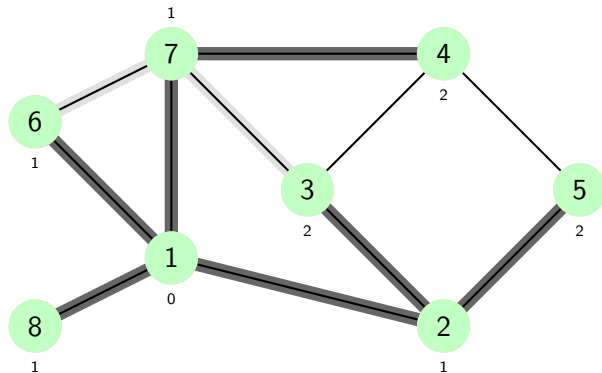
# Parcurgerea grafurilor - BFS



Coadă de noduri

1  
2  
6  
7  
8  
3  
5

# Parcurgerea grafurilor - BFS



Coadă de noduri

1

2

6

7

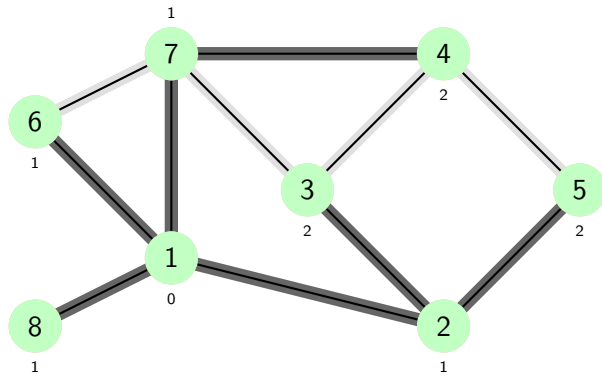
8

3

5

4

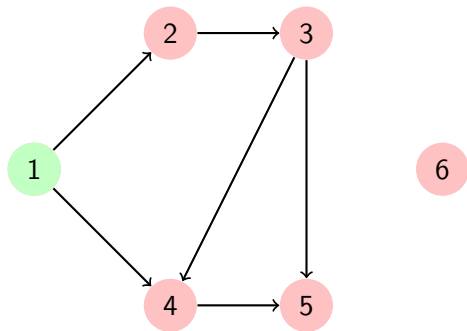
# Parcurgerea grafurilor - BFS



Coadă de noduri

1  
2  
6  
7  
8  
3  
5  
4

# Sortare topologică - Kahn

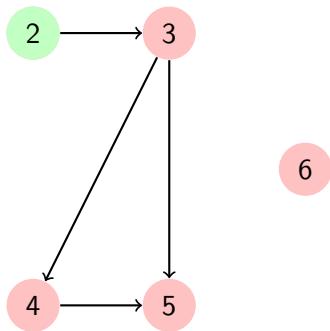


Sortare topologică

1



# Sortare topologică - Kahn

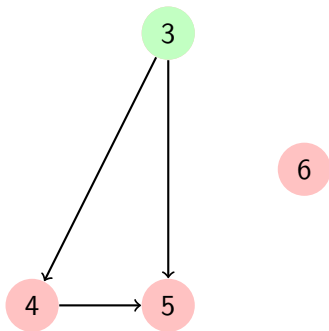


Sortare topologică

1

2

# Sortare topologică - Kahn



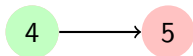
Sortare topologică

1

2

3

# Sortare topologică - Kahn



Sortare topologică

1

2

3

4

# Sortare topologică - Kahn

5

6

Sortare topologică

1

2

3

4

5

# Sortare topologică - Kahn

6

Sortare topologică

1

2

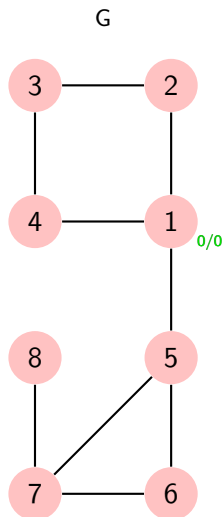
3

4

5

6

# Punți - Tarjan



DFS

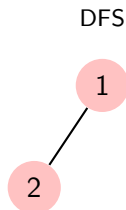
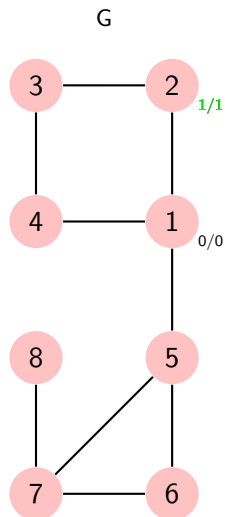


dfs(1)

Stiva de apeluri

Punte?

# Punți - Tarjan



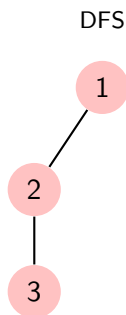
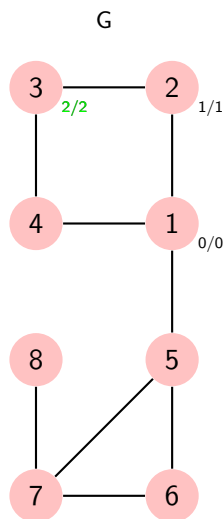
dfs(2)

dfs(1)

Stiva de apeluri

Punte?

# Punți - Tarjan



dfs(3)

dfs(2)

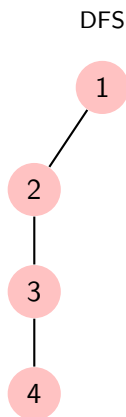
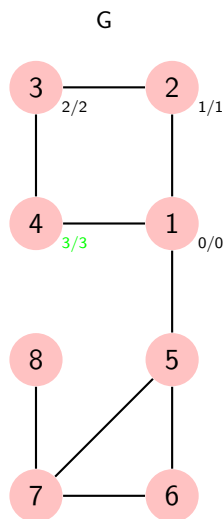
dfs(1)

Stiva de apeluri

Punte?



# Punți - Tarjan



dfs(4)

dfs(3)

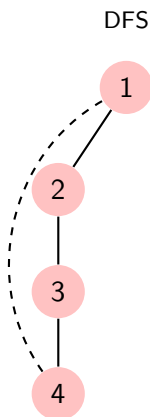
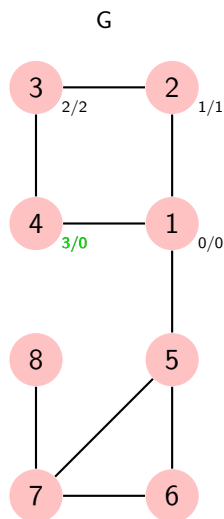
dfs(2)

dfs(1)

Stiva de apeluri

Punte?

# Punți - Tarjan



dfs(4)

dfs(3)

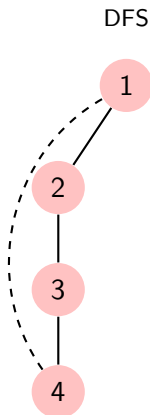
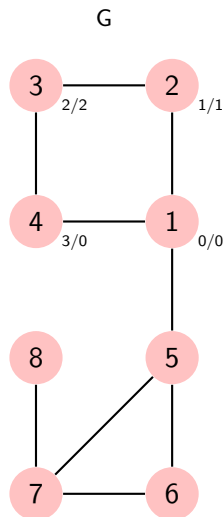
dfs(2)

dfs(1)

Stiva de apeluri

Punte?

# Punți - Tarjan



~~dfs(4)~~

$\text{low}(4) > \text{idx}(3)?$

dfs(3)

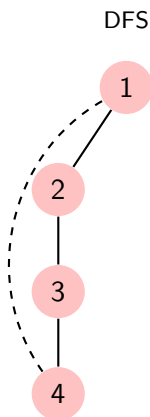
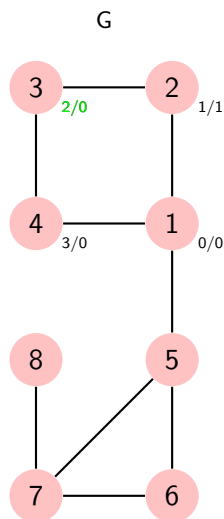
dfs(2)

dfs(1)

Stiva de apeluri

Punte?

# Punți - Tarjan



~~dfs(4)~~

dfs(3)

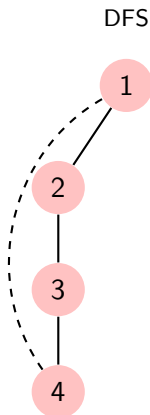
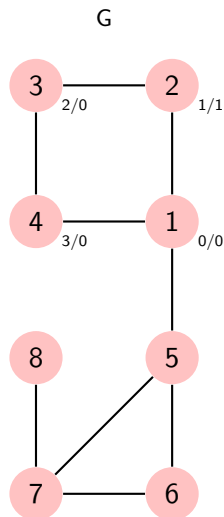
dfs(2)

dfs(1)

Stiva de apeluri

Punte?

# Punți - Tarjan



~~dfs(4)~~

~~dfs(3)~~

$\text{low}(3) > \text{idx}(2)?$

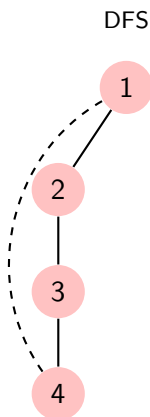
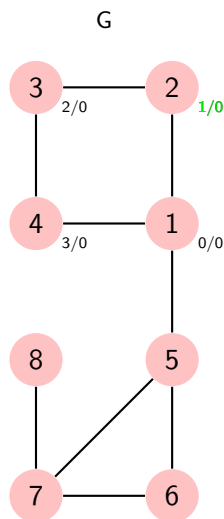
dfs(2)

dfs(1)

Stiva de apeluri

Punte?

# Punți - Tarjan



~~dfs(4)~~

~~dfs(3)~~

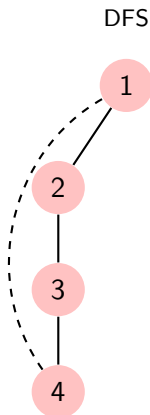
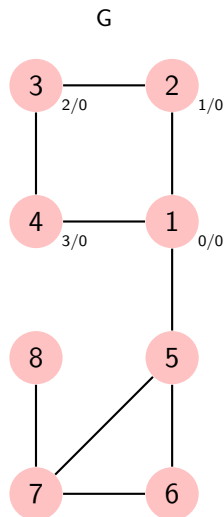
dfs(2)

dfs(1)

Stiva de apeluri

Punte?

# Punți - Tarjan



~~dfs(4)~~

~~dfs(3)~~

~~dfs(2)~~

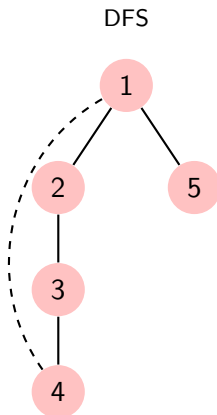
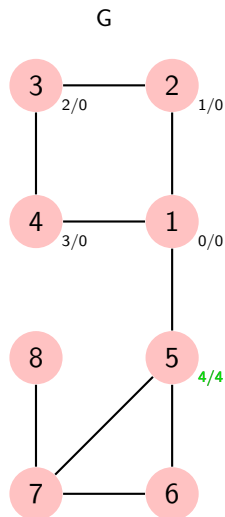
dfs(1)

$\text{low}(2) > \text{idx}(1)?$

Stiva de apeluri

Punte?

# Punți - Tarjan



dfs(5)

~~dfs(4)~~

~~dfs(3)~~

~~dfs(2)~~

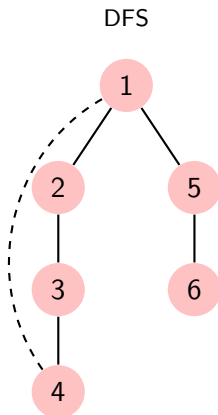
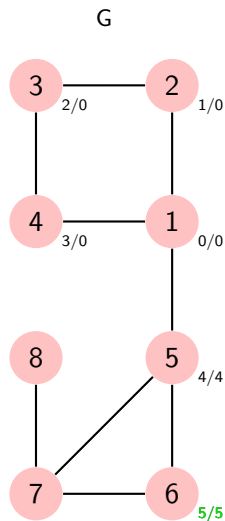
dfs(1)

Stiva de apeluri

Punte?



# Punți - Tarjan



dfs(6)

dfs(5)

~~dfs(4)~~

~~dfs(3)~~

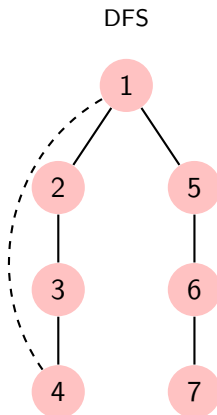
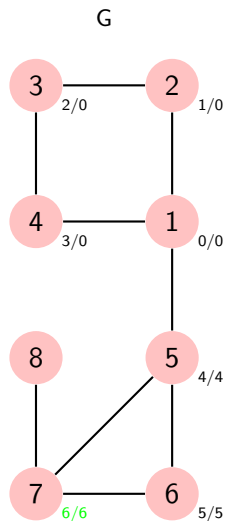
~~dfs(2)~~

dfs(1)

Stiva de apeluri

Punte?

# Punți - Tarjan



dfs(7)

dfs(6)

dfs(5)

~~dfs(4)~~

~~dfs(3)~~

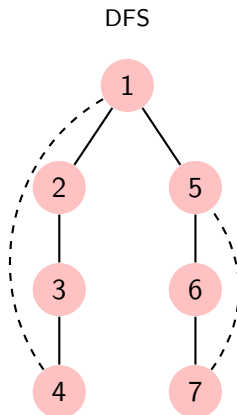
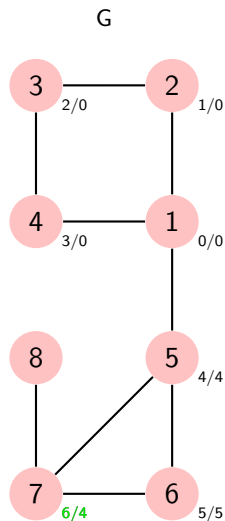
~~dfs(2)~~

dfs(1)

Stiva de apeluri

Punte?

# Punți - Tarjan



dfs(7)

dfs(6)

dfs(5)

~~dfs(4)~~

~~dfs(3)~~

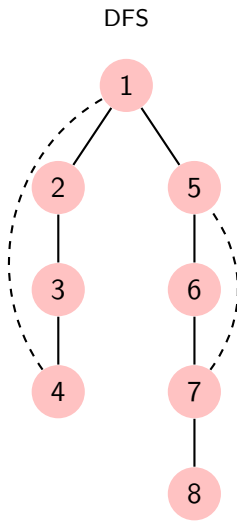
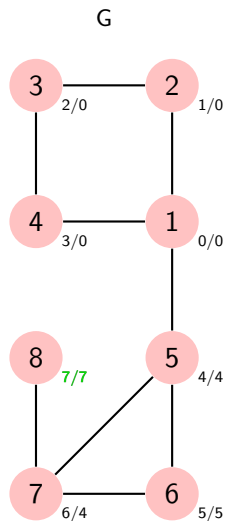
~~dfs(2)~~

dfs(1)

Stiva de apeluri

Punte?

# Punți - Tarjan



dfs(8)

dfs(7)

dfs(6)

dfs(5)

~~dfs(4)~~

~~dfs(3)~~

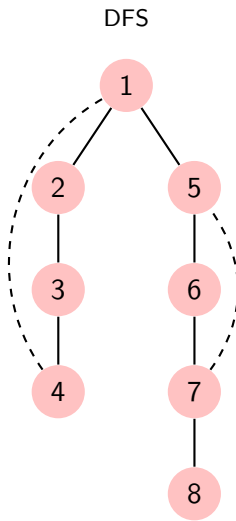
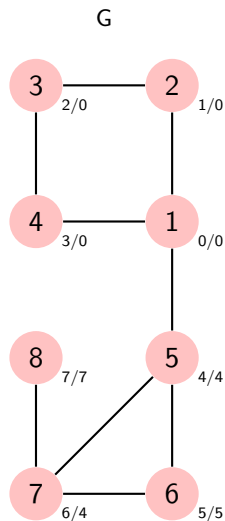
~~dfs(2)~~

dfs(1)

Stiva de apeluri

Punte?

# Punți - Tarjan



~~dfs(8)~~

$\text{low}(8) > \text{idx}(7)?$

dfs(7)

dfs(6)

dfs(5)

~~dfs(4)~~

~~dfs(3)~~

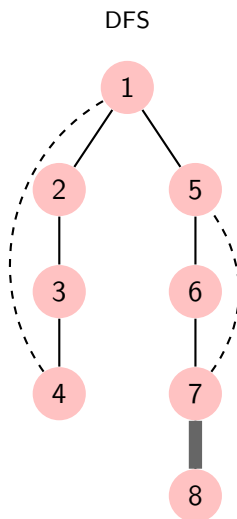
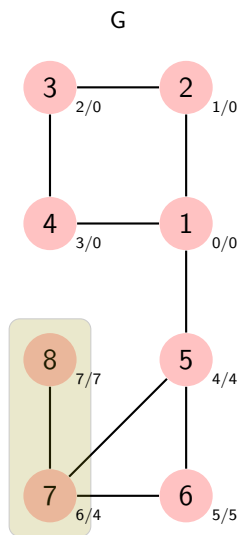
~~dfs(2)~~

dfs(1)

Stiva de apeluri

Punte?

# Punți - Tarjan



~~dfs(8)~~

dfs(7)

dfs(6)

dfs(5)

~~dfs(4)~~

~~dfs(3)~~

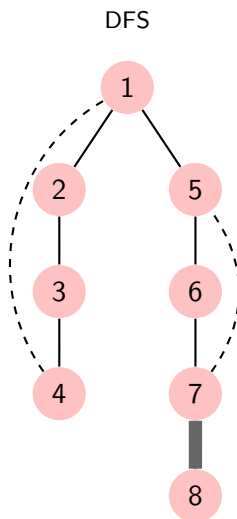
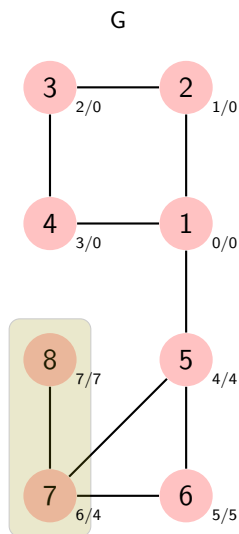
~~dfs(2)~~

dfs(1)

Stiva de apeluri

Punte?

# Punți - Tarjan



~~dfs(8)~~

~~dfs(7)~~

dfs(6)

dfs(5)

~~dfs(4)~~

~~dfs(3)~~

~~dfs(2)~~

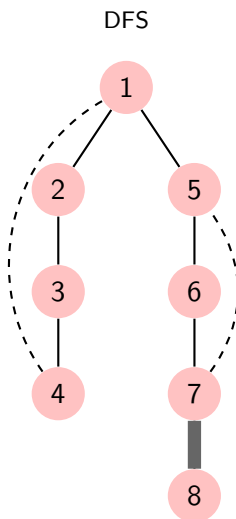
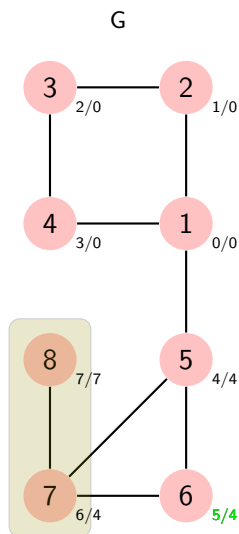
dfs(1)

low(7) > idx(6)?

Stiva de apeluri

Punte?

# Punți - Tarjan



~~dfs(8)~~

~~dfs(7)~~

dfs(6)

dfs(5)

~~dfs(4)~~

~~dfs(3)~~

~~dfs(2)~~

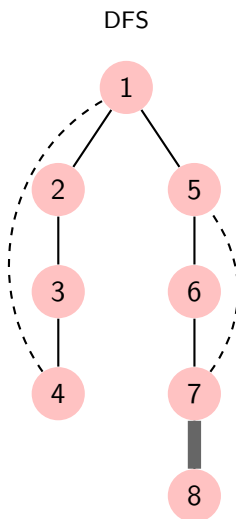
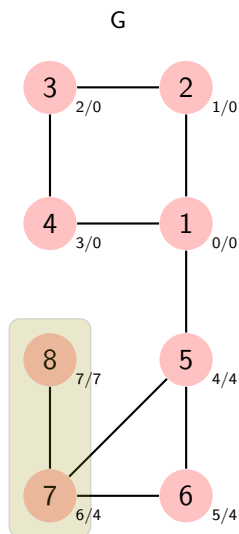
dfs(1)

Stiva de apeluri

Punte?



# Punți - Tarjan



~~dfs(8)~~

~~dfs(7)~~

~~dfs(6)~~

dfs(5)

~~dfs(4)~~

~~dfs(3)~~

~~dfs(2)~~

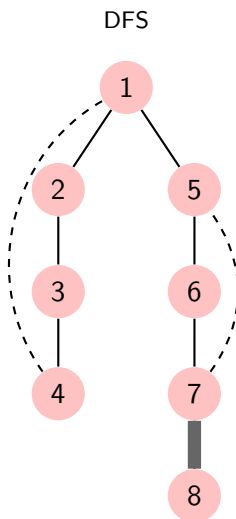
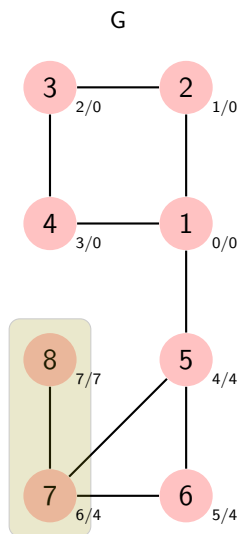
dfs(1)

$\text{low}(6) > \text{idx}(5)?$

Stiva de apeluri

Punte?

# Punți - Tarjan



~~dfs(8)~~

~~dfs(7)~~

~~dfs(6)~~

~~dfs(5)~~

~~dfs(4)~~

~~dfs(3)~~

~~dfs(2)~~

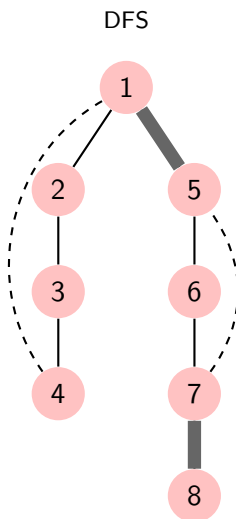
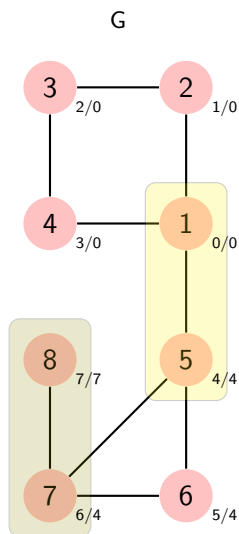
dfs(1)

low(5) > idx(1)?

Stiva de apeluri

Punte?

# Punți - Tarjan



~~dfs(8)~~

~~dfs(7)~~

~~dfs(6)~~

~~dfs(5)~~

~~dfs(4)~~

~~dfs(3)~~

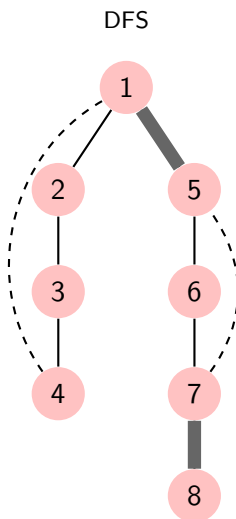
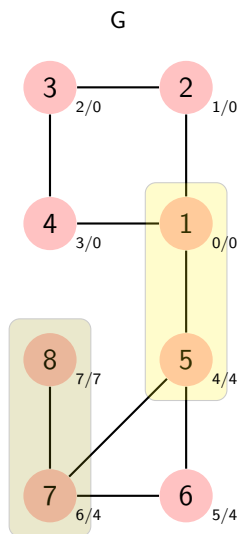
~~dfs(2)~~

dfs(1)

Stiva de apeluri

Punte?

# Punți - Tarjan



~~dfs(8)~~

~~dfs(7)~~

~~dfs(6)~~

~~dfs(5)~~

~~dfs(4)~~

~~dfs(3)~~

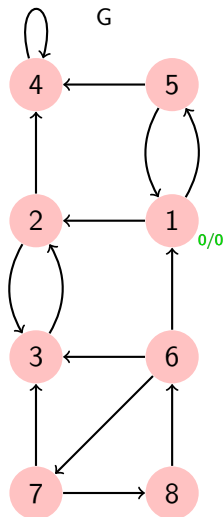
~~dfs(2)~~

~~dfs(1)~~

Stiva de apeluri

Punte?

# Tare conexitate - Tarjan



CTC

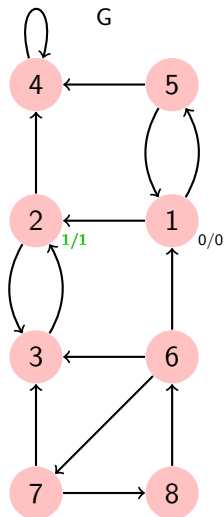
tarjan(1)

1

Stiva de apeluri

Stiva de noduri

# Tare conexitate - Tarjan



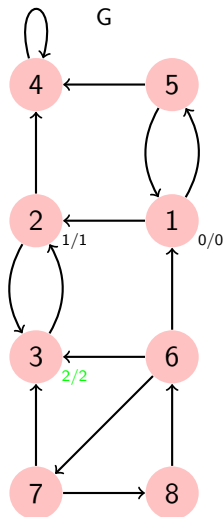
CTC

tarjan(2)                      2

tarjan(1)                      1

Stiva de apeluri      Stiva de noduri

# Tare conexitate - Tarjan



CTC

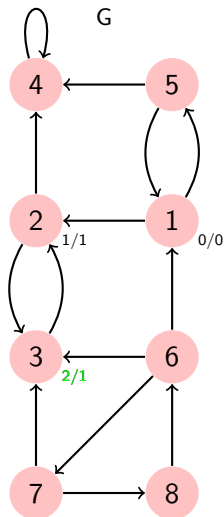
tarjan(3) 3

tarjan(2) 2

tarjan(1) 1

Stiva de apeluri Stiva de noduri

# Tare conexitate - Tarjan



tarjan(3) 3

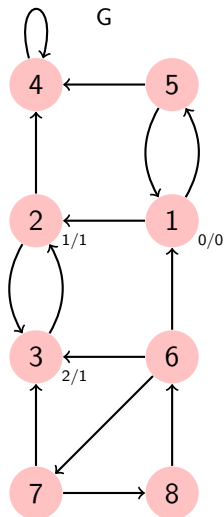
tarjan(2) 2

tarjan(1) 1

Stiva de apeluri Stiva de noduri



# Tare conexitate - Tarjan



CTC

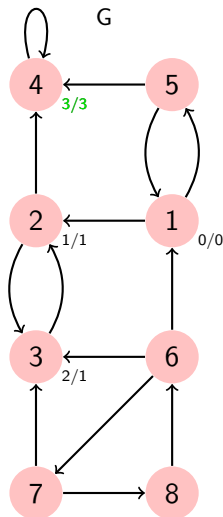
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tarjan(2) 2

tarjan(1) 1

Stiva de apeluri    Stiva de noduri

# Tare conexitate - Tarjan



CTC

tarjan(4) 4

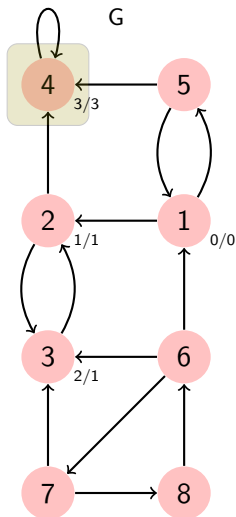
~~tarjan(3)~~ 3

tarjan(2) 2

tarjan(1) 1

Stiva de apeluri Stiva de noduri

# Tare conexitate - Tarjan



CTC

(4)

~~tarjan(4)~~ 4

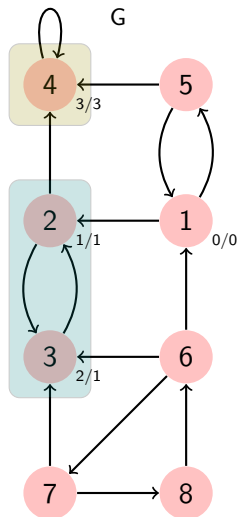
~~tarjan(3)~~ 3

tarjan(2) 2

tarjan(1) 1

Stiva de apeluri Stiva de noduri

# Tare conexitate - Tarjan



CTC

(4)

(3,2)

~~tarjan(4)~~ 4

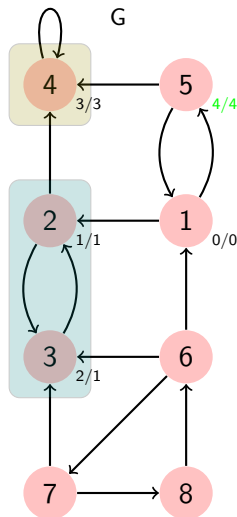
~~tarjan(3)~~ 3

~~tarjan(2)~~ 2

tarjan(1) 1

Stiva de apeluri Stiva de noduri

# Tare conexitate - Tarjan



CTC

(4)

(3,2)

tarjan(5) 5

~~tarjan(4)~~ 4

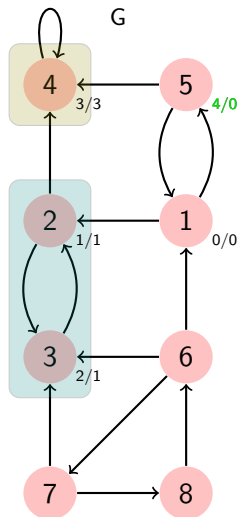
~~tarjan(3)~~ 3

~~tarjan(2)~~ 2

tarjan(1) 1

Stiva de apeluri Stiva de noduri

# Tare conexitate - Tarjan



CTC

(4)

(3,2)

tarjan(5) 5

~~tarjan(4)~~ 4

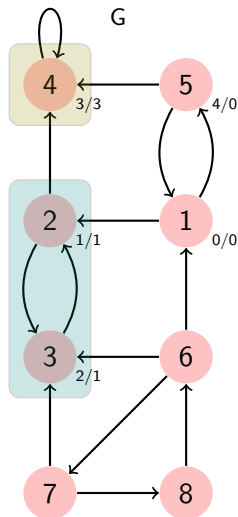
~~tarjan(3)~~ 3

~~tarjan(2)~~ 2

tarjan(1) 1

Stiva de apeluri Stiva de noduri

# Tare conexitate - Tarjan



CTC

(4)

(3,2)

~~tarjan(5)~~ 5

~~tarjan(4)~~ 4

~~tarjan(3)~~ 3

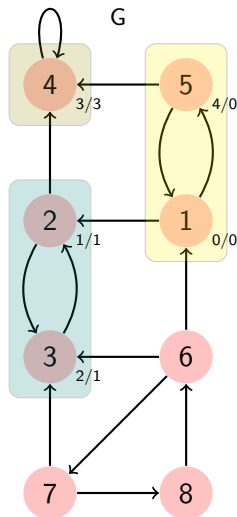
~~tarjan(2)~~ 2

tarjan(1) 1

Stiva de apeluri

Stiva de noduri

# Tare conexitate - Tarjan



CTC

(4)

(3,2)

(5,1)

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~~tarjan(4)~~

4

~~tarjan(3)~~

3

~~tarjan(2)~~

2

~~tarjan(1)~~

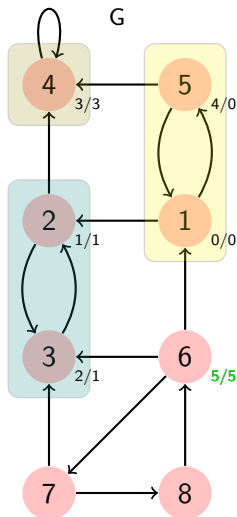
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Stiva de apeluri

Stiva de noduri



# Tare conexitate - Tarjan



CTC

(4)

(3,2)

(5,1)

tarjan(6)

6

~~tarjan(5)~~

~~5~~

~~tarjan(4)~~

~~4~~

~~tarjan(3)~~

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~~tarjan(2)~~

~~2~~

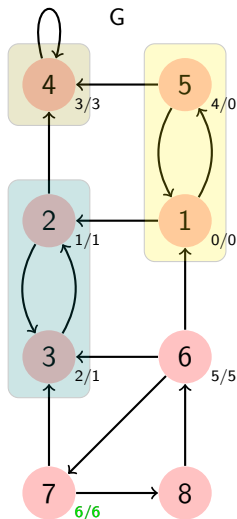
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Stiva de apeluri

Stiva de noduri

# Tare conexitate - Tarjan



CTC

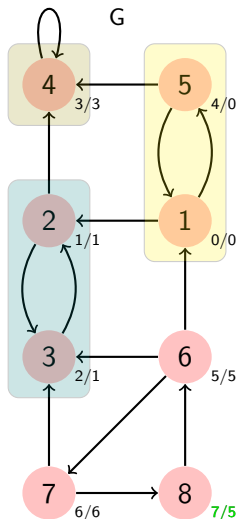
(4)	tarjan(7)	7
(3,2)	tarjan(6)	6
(5,1)	<del>tarjan(5)</del>	<del>5</del>
	<del>tarjan(4)</del>	<del>4</del>
	<del>tarjan(3)</del>	<del>3</del>
	<del>tarjan(2)</del>	<del>2</del>
	<del>tarjan(1)</del>	<del>1</del>

Stiva de apeluri

Stiva de noduri



# Tare conexitate - Tarjan



CTC

(4)

(3,2)

(5,1)

tarjan(8)

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tarjan(7)

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tarjan(6)

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~~tarjan(5)~~

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~~tarjan(4)~~

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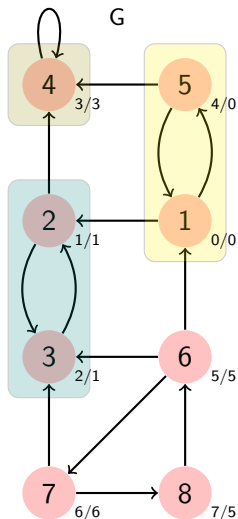
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Stiva de apeluri

Stiva de noduri

# Tare conexitate - Tarjan



CTC

(4)

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(5,1)

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~~tarjan(4)~~

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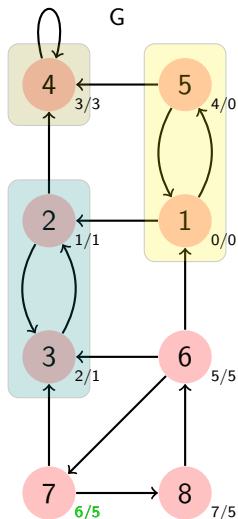
~~2~~

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Stiva de apeluri

Stiva de noduri

# Tare conexitate - Tarjan



CTC

(4)

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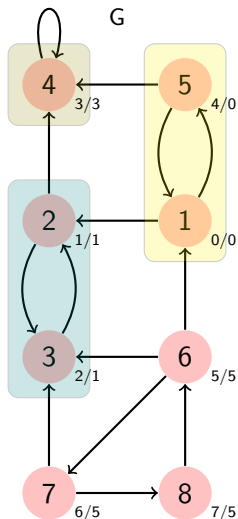
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Stiva de apeluri

Stiva de noduri

# Tare conexitate - Tarjan



CTC

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(3,2)

(5,1)

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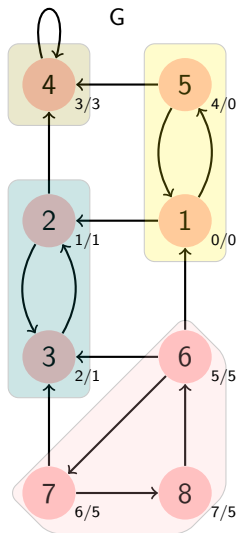
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Stiva de apeluri

Stiva de noduri

# Tare conexitate - Tarjan



CTC

(4)  
(3,2)  
(5,1)  
(8,7,6)

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~~tarjan(7)~~

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~~tarjan(2)~~

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~~tarjan(1)~~

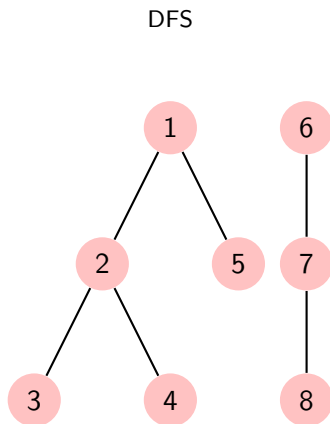
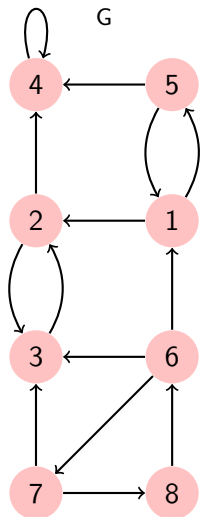
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Stiva de apeluri

Stiva de noduri



# Tare conexitate - Kosaraju



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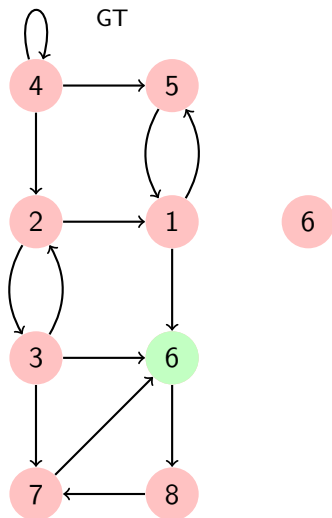
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Stiva de  
noduri finalizate

# Tare conexitate - Kosaraju



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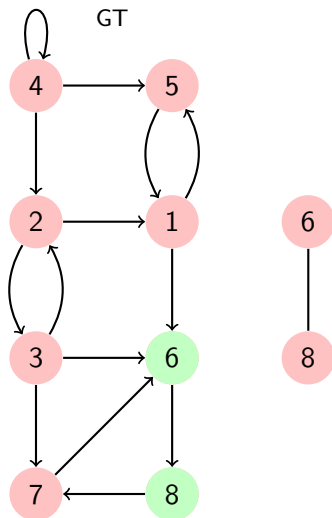
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Stiva de  
noduri finalizate

# Tare conexitate - Kosaraju



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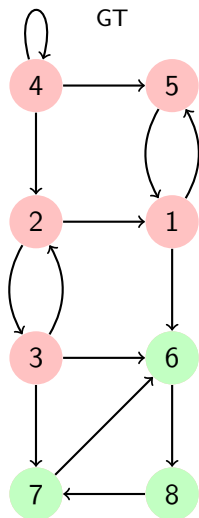
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Stiva de  
noduri finalizate

# Tare conexitate - Kosaraju



DFST



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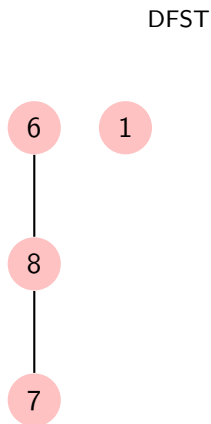
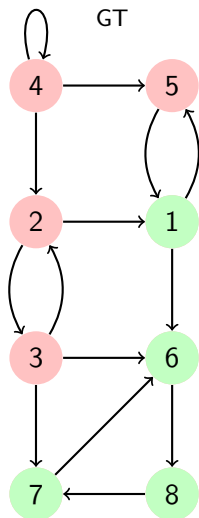
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Stiva de  
noduri finalizate

# Tare conexitate - Kosaraju



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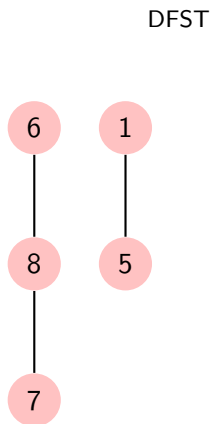
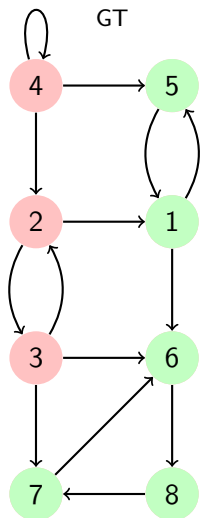
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Stiva de  
noduri finalizate

# Tare conexitate - Kosaraju



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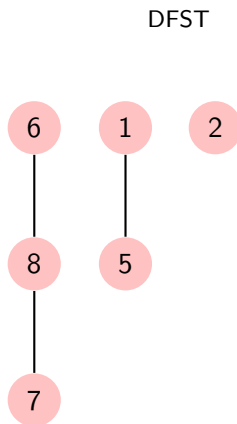
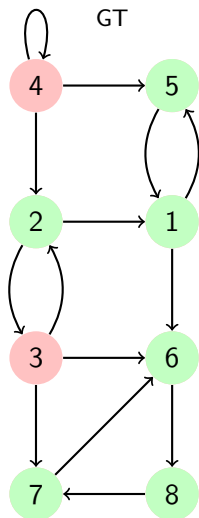
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Stiva de  
noduri finalizate

# Tare conexitate - Kosaraju



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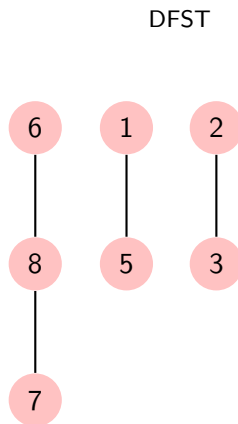
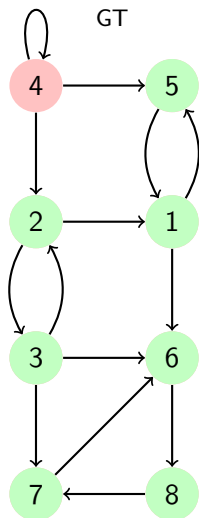
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Stiva de  
noduri finalizate

# Tare conexitate - Kosaraju



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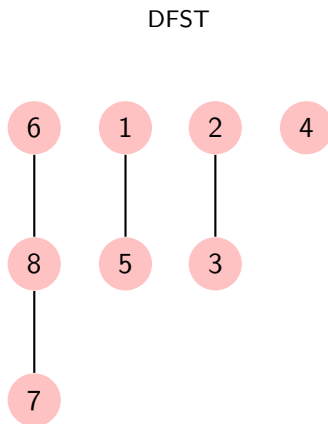
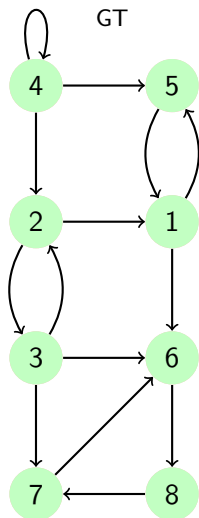
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Stiva de  
noduri finalizate



# Tare conexitate - Kosaraju



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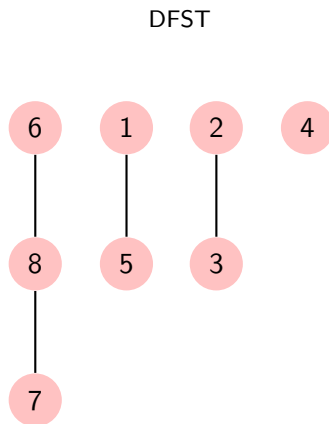
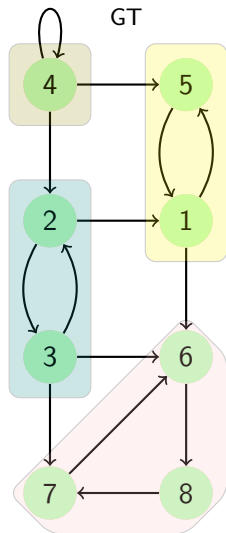
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Stiva de  
noduri finalizate

# Tare conexitate - Kosaraju



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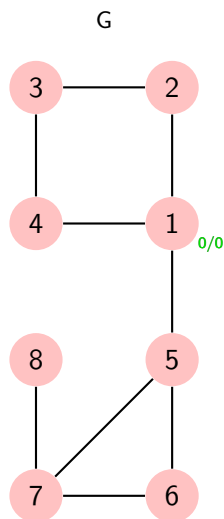
2

4

3

Stiva de  
noduri finalizate

# Puncte de articulație - Tarjan



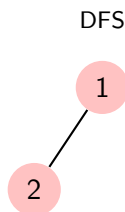
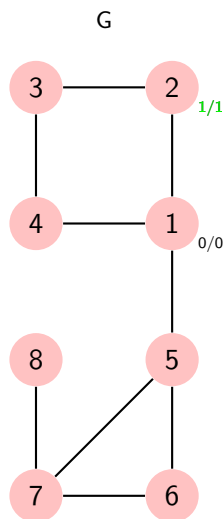
DFS



dfs(1)

Stiva de apeluri    Punct critic?

# Puncte de articulație - Tarjan

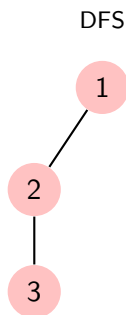
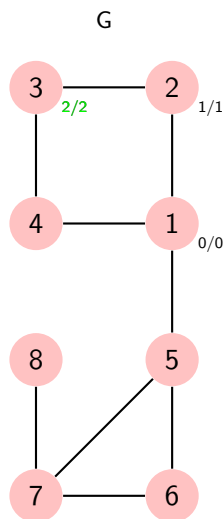


dfs(2)

dfs(1)

Stiva de apeluri    Punct critic?

# Puncte de articulație - Tarjan



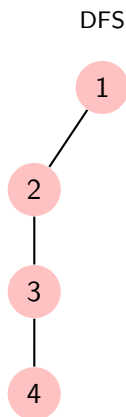
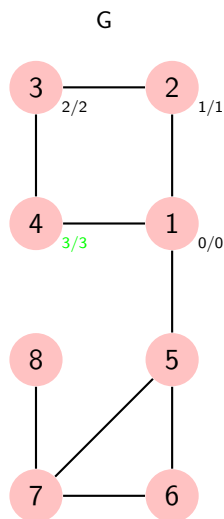
dfs(3)

dfs(2)

dfs(1)

Stiva de apeluri    Punct critic?

# Puncte de articulație - Tarjan



dfs(4)

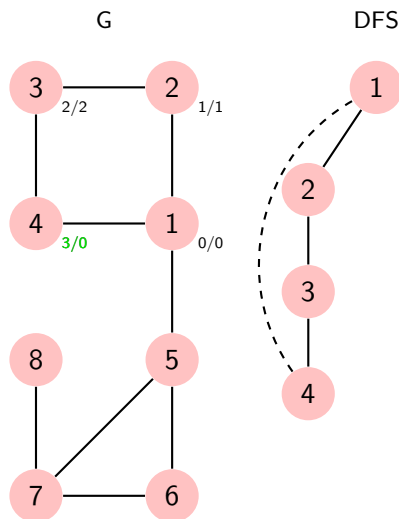
dfs(3)

dfs(2)

dfs(1)

Stiva de apeluri    Punct critic?

# Puncte de articulație - Tarjan



dfs(4)

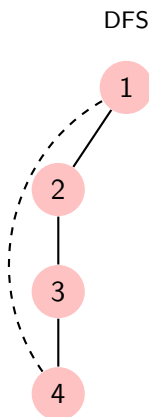
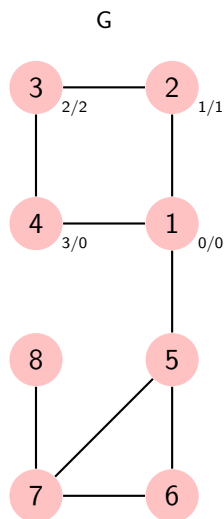
dfs(3)

dfs(2)

dfs(1)

Stiva de apeluri    Punct critic?

# Puncte de articulație - Tarjan



~~dfs(4)~~

$\text{low}(4) \geq \text{idx}(3)?$

dfs(3)

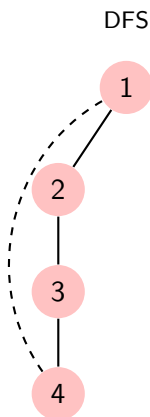
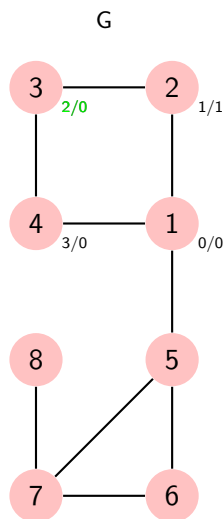
dfs(2)

dfs(1)

Stiva de apeluri    Punct critic?



# Puncte de articulație - Tarjan



~~dfs(4)~~

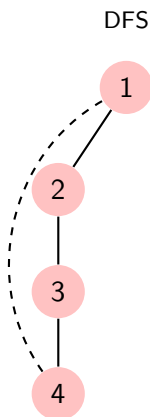
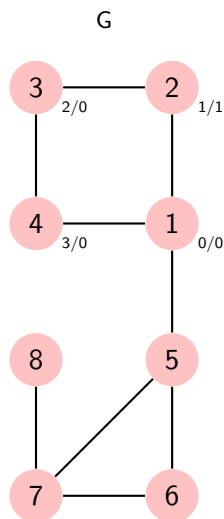
dfs(3)

dfs(2)

dfs(1)

Stiva de apeluri    Punct critic?

# Puncte de articulație - Tarjan



~~dfs(4)~~

~~dfs(3)~~

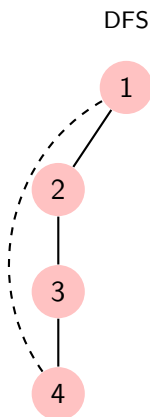
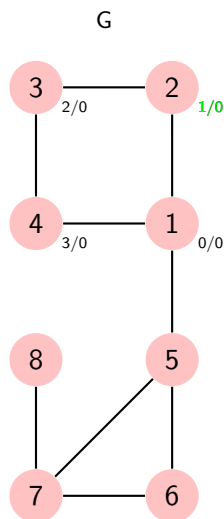
$\text{low}(3) \geq \text{idx}(2)?$

dfs(2)

dfs(1)

Stiva de apeluri    Punct critic?

# Puncte de articulație - Tarjan



~~dfs(4)~~

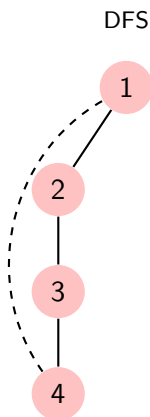
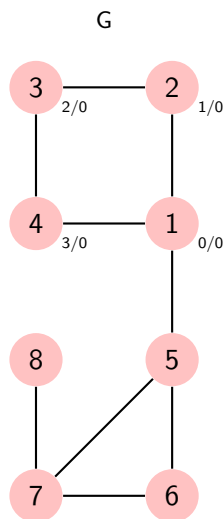
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dfs(2)

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Stiva de apeluri    Punct critic?

# Puncte de articulație - Tarjan



~~dfs(4)~~

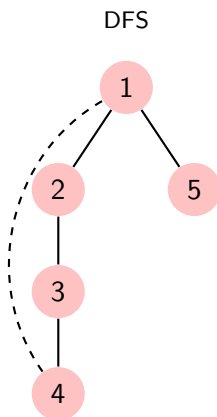
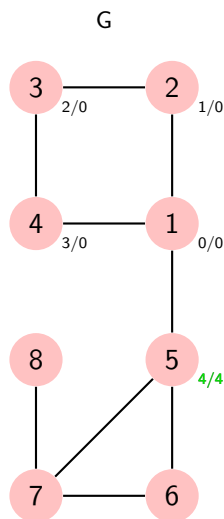
~~dfs(3)~~

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Stiva de apeluri    Punct critic?

# Puncte de articulație - Tarjan



dfs(5)

~~dfs(4)~~

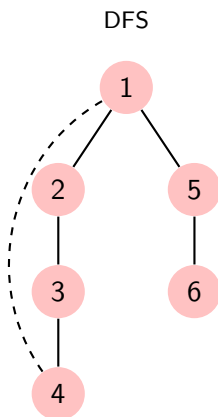
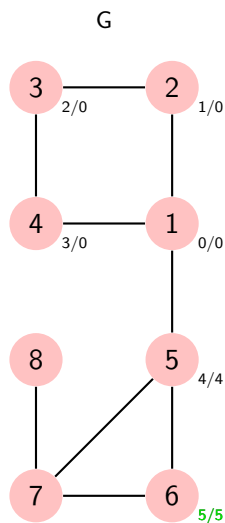
~~dfs(3)~~

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dfs(1)

Stiva de apeluri    Punct critic?

# Puncte de articulație - Tarjan



dfs(6)

dfs(5)

~~dfs(4)~~

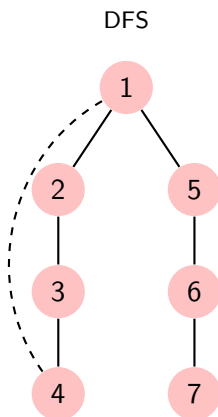
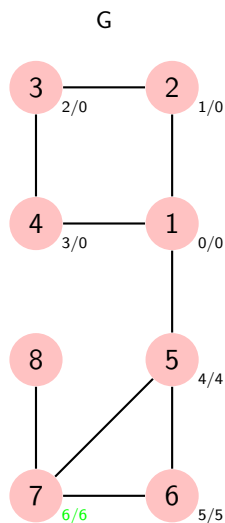
~~dfs(3)~~

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dfs(1)

Stiva de apeluri    Punct critic?

# Puncte de articulație - Tarjan



dfs(7)

dfs(6)

dfs(5)

~~dfs(4)~~

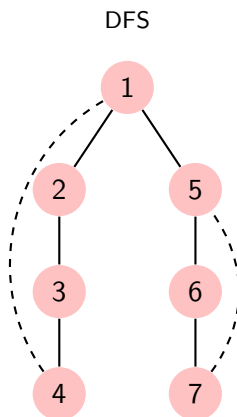
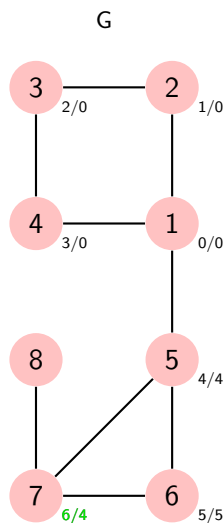
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dfs(1)

Stiva de apeluri    Punct critic?

# Puncte de articulație - Tarjan



dfs(7)

dfs(6)

dfs(5)

~~dfs(4)~~

~~dfs(3)~~

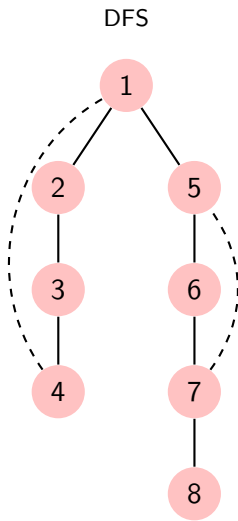
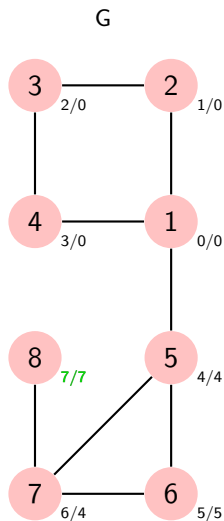
~~dfs(2)~~

dfs(1)

Stiva de apeluri    Punct critic?



# Puncte de articulație - Tarjan



dfs(8)

dfs(7)

dfs(6)

dfs(5)

~~dfs(4)~~

~~dfs(3)~~

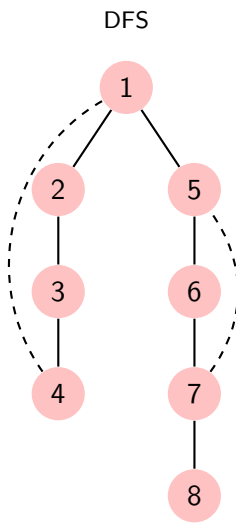
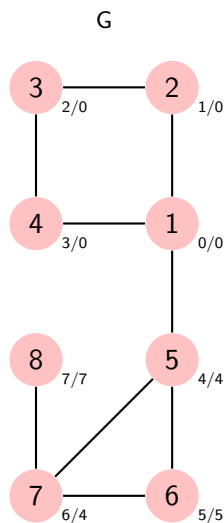
~~dfs(2)~~

dfs(1)

Stiva de apeluri

Punct critic?

# Puncte de articulație - Tarjan



~~dfs(8)~~

$\text{low}(8) \geq \text{idx}(7)?$

dfs(7)

dfs(6)

dfs(5)

~~dfs(4)~~

~~dfs(3)~~

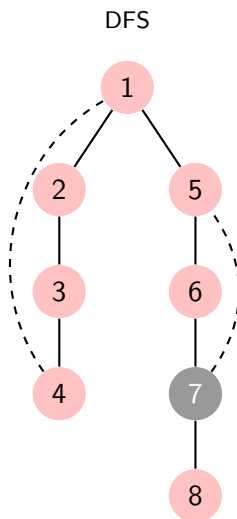
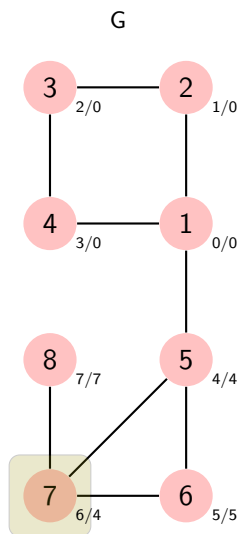
~~dfs(2)~~

dfs(1)

Stiva de apeluri

Punct critic?

# Puncte de articulație - Tarjan



~~dfs(8)~~

dfs(7)

dfs(6)

dfs(5)

~~dfs(4)~~

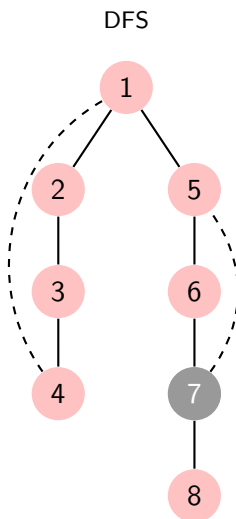
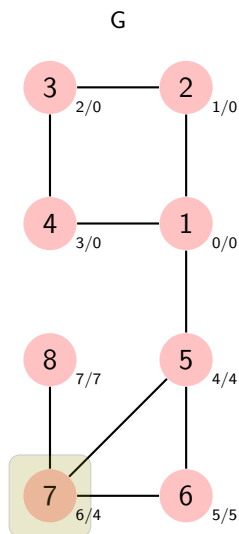
~~dfs(3)~~

~~dfs(2)~~

dfs(1)

Stiva de apeluri      Punct critic?

# Puncte de articulație - Tarjan



~~dfs(8)~~

~~dfs(7)~~

$low(7) \geq idx(6)?$

dfs(6)

dfs(5)

~~dfs(4)~~

~~dfs(3)~~

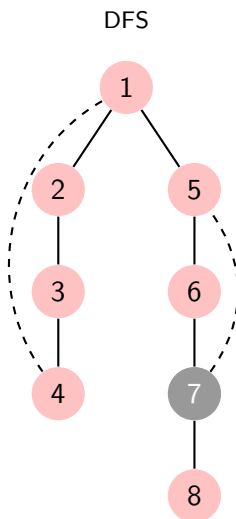
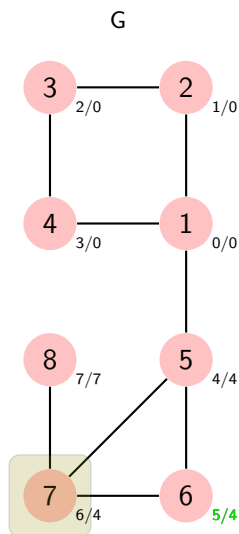
~~dfs(2)~~

dfs(1)

Stiva de apeluri

Punct critic?

# Puncte de articulație - Tarjan



~~dfs(8)~~

~~dfs(7)~~

dfs(6)

dfs(5)

~~dfs(4)~~

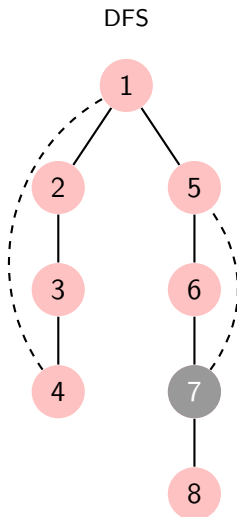
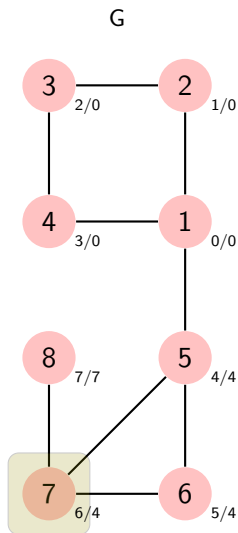
~~dfs(3)~~

~~dfs(2)~~

dfs(1)

Stiva de apeluri      Punct critic?

# Puncte de articulație - Tarjan



~~dfs(8)~~

~~dfs(7)~~

~~dfs(6)~~

dfs(5)

~~dfs(4)~~

~~dfs(3)~~

~~dfs(2)~~

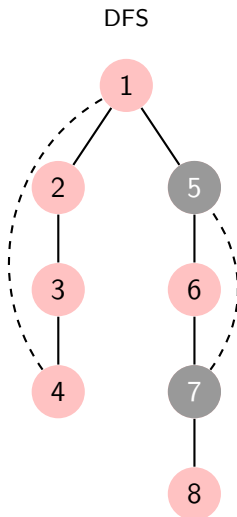
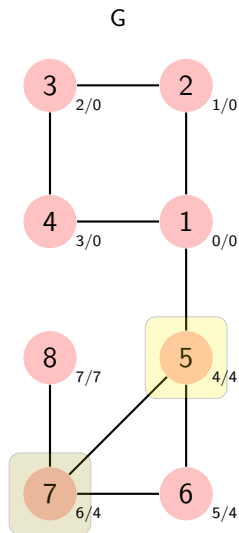
dfs(1)

$\text{low}(6) \geq \text{idx}(5)?$

Stiva de apeluri

Punct critic?

# Puncte de articulație - Tarjan



~~dfs(8)~~

~~dfs(7)~~

~~dfs(6)~~

dfs(5)

~~dfs(4)~~

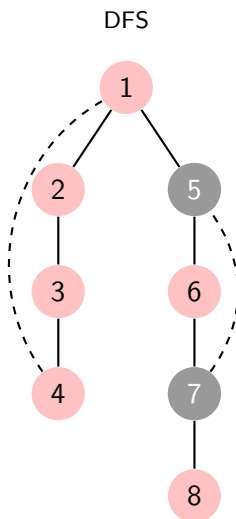
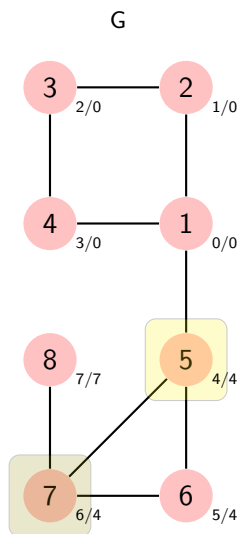
~~dfs(3)~~

~~dfs(2)~~

dfs(1)

Stiva de apeluri      Punct critic?

# Puncte de articulație - Tarjan



~~dfs(8)~~

~~dfs(7)~~

~~dfs(6)~~

~~dfs(5)~~

~~dfs(4)~~

~~dfs(3)~~

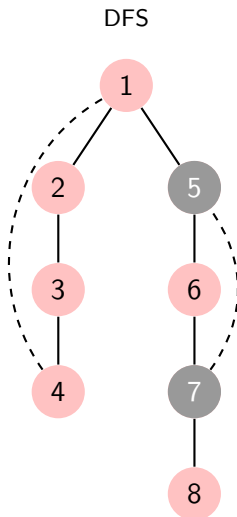
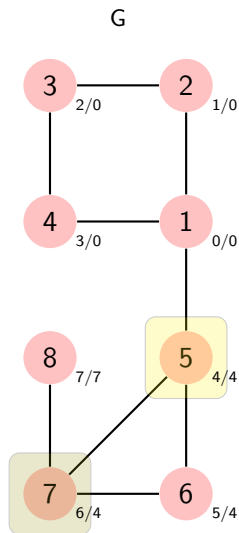
~~dfs(2)~~

dfs(1)

Stiva de apeluri      Punct critic?



# Puncte de articulație - Tarjan



~~dfs(8)~~

~~dfs(7)~~

~~dfs(6)~~

~~dfs(5)~~

~~dfs(4)~~

~~dfs(3)~~

~~dfs(2)~~

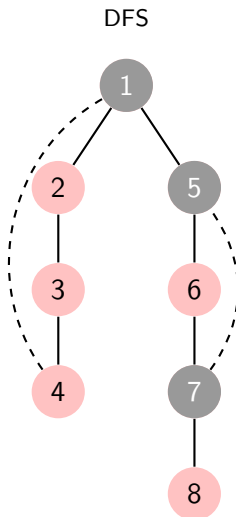
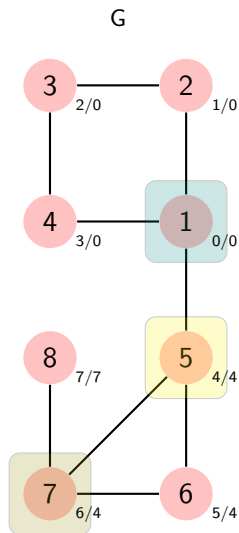
dfs(1)

$\text{copii}(1) \geq 2?$

Stiva de apeluri

Punct critic?

# Puncte de articulație - Tarjan



~~dfs(8)~~

~~dfs(7)~~

~~dfs(6)~~

~~dfs(5)~~

~~dfs(4)~~

~~dfs(3)~~

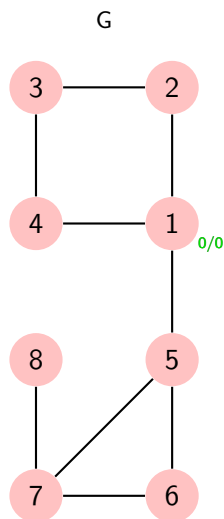
~~dfs(2)~~

~~dfs(1)~~

Stiva de apeluri

Punct critic?

# Componente biconexe - Tarjan



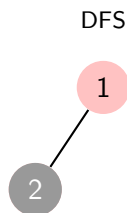
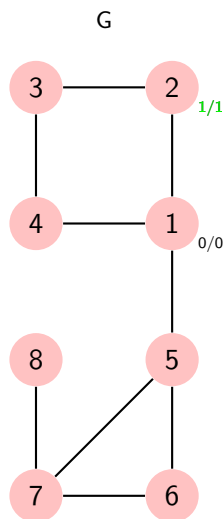
DFS

1

1

Stiva de noduri    Punct critic?

# Componente biconexe - Tarjan

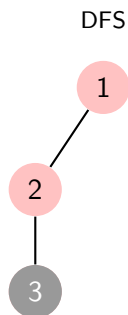
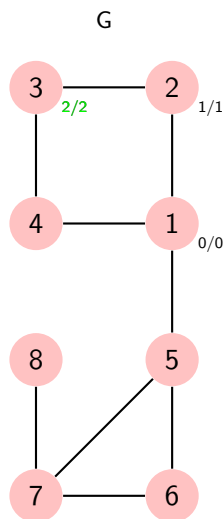


2

1

Stiva de noduri    Punct critic?

# Componente biconexe - Tarjan



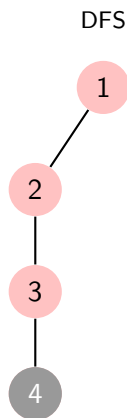
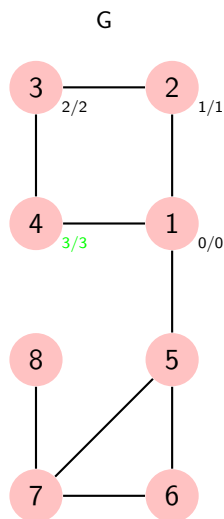
3

2

1

Stiva de noduri    Punct critic?

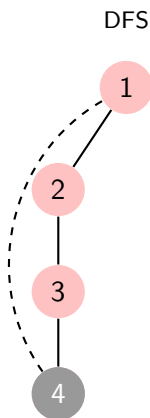
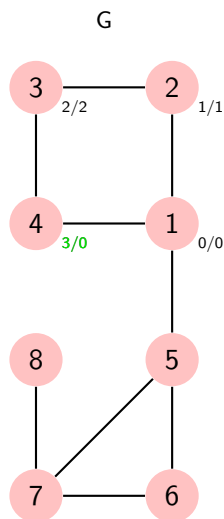
# Componente biconexe - Tarjan



4  
3  
2  
1

Stiva de noduri    Punct critic?

# Componente biconexe - Tarjan



4

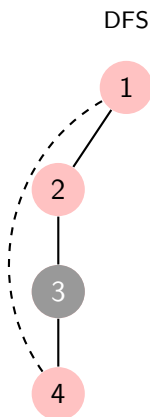
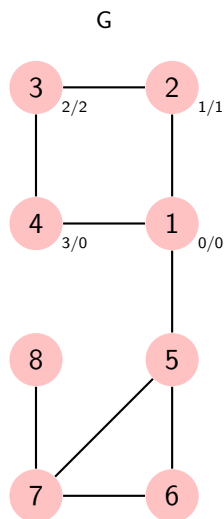
3

2

1

Stiva de noduri    Punct critic?

# Componente biconexce - Tarjan



4

$\text{low}(4) \geq \text{idx}(3)?$

3

2

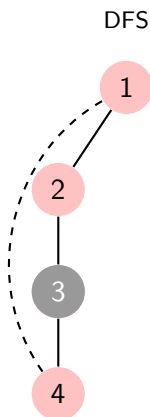
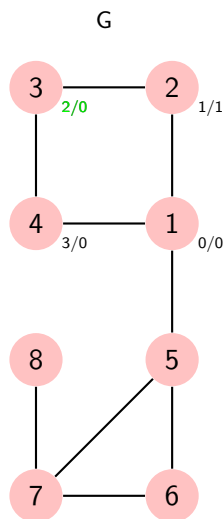
1

Stiva de noduri

Punct critic?



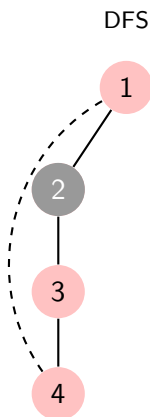
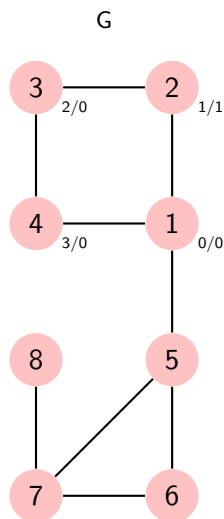
# Componente biconexce - Tarjan



4  
3  
2  
1

Stiva de noduri    Punct critic?

# Componente biconexce - Tarjan



4

3

2

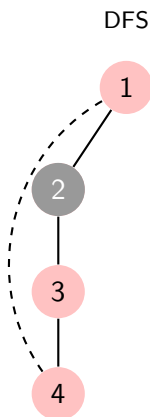
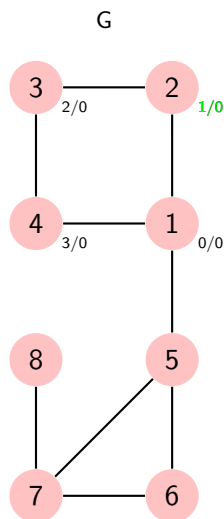
1

$\text{low}(3) \geq \text{idx}(2)?$

Stiva de noduri

Punct critic?

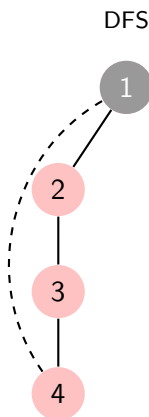
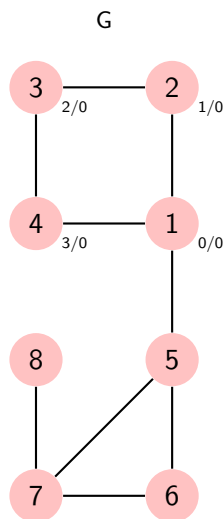
# Componente biconexce - Tarjan



4  
3  
2  
1

Stiva de noduri    Punct critic?

# Componente biconexce - Tarjan



4

3

2

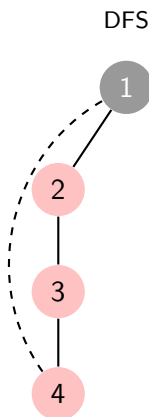
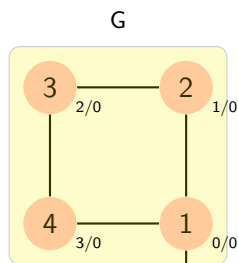
1

$\text{low}(2) \geq \text{idx}(1)?$

Stiva de noduri

Punct critic?

# Componente biconexce - Tarjan



4

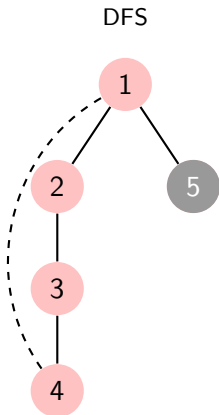
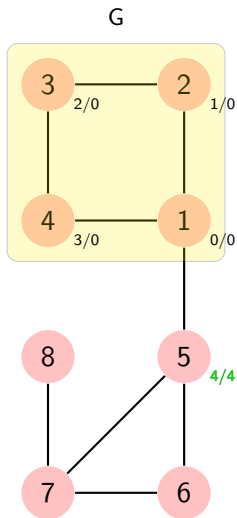
3

2

1

Stiva de noduri    Punct critic?

## Componente biconexe - Tarjan



5

4

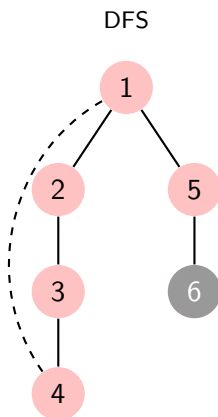
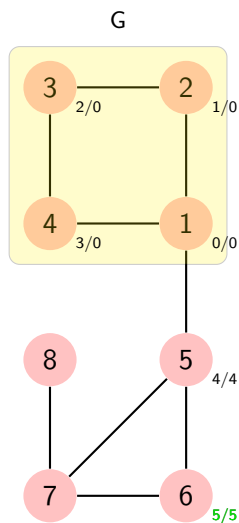
3

2

1

Stiva de noduri      Punct critic?

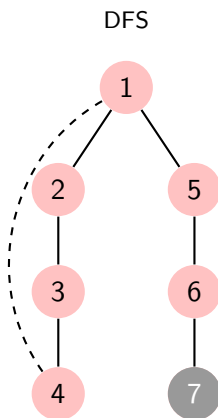
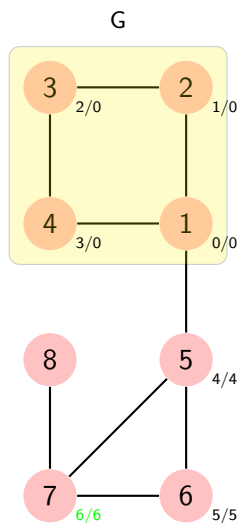
# Componente biconexe - Tarjan



6  
5  
4  
3  
2  
1

Stiva de noduri    Punct critic?

# Componente biconexce - Tarjan

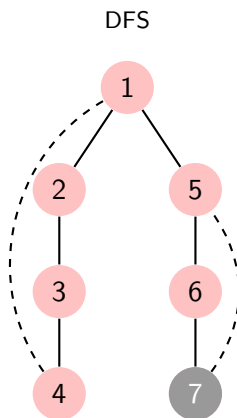
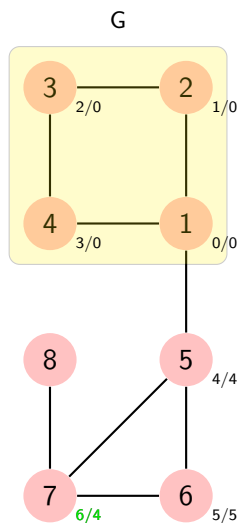


7  
6  
5  
4  
3  
2  
1

Stiva de noduri    Punct critic?



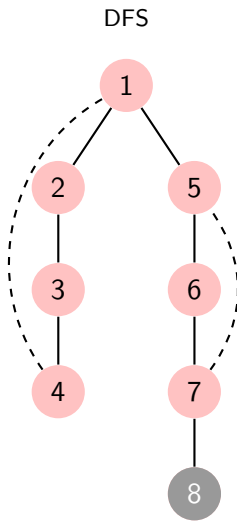
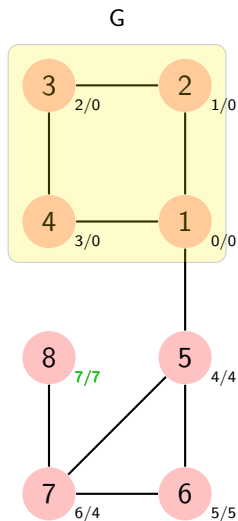
# Componente biconexce - Tarjan



7  
6  
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Stiva de noduri    Punct critic?

# Componente biconexce - Tarjan



8

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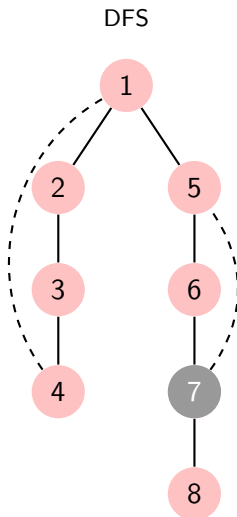
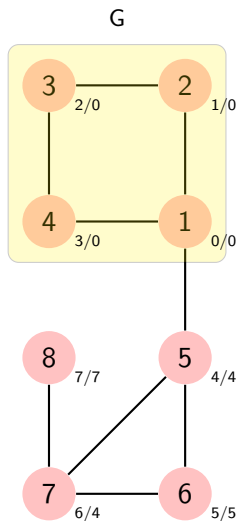
2

1

Stiva de noduri

Punct critic?

# Componente biconexce - Tarjan



8

$\text{low}(8) \geq \text{idx}(7)?$

7

6

5

4

3

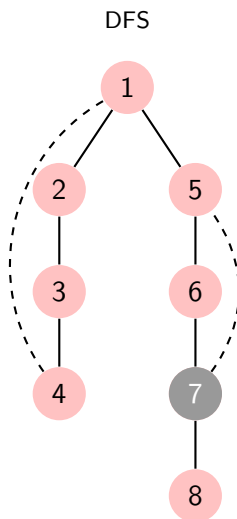
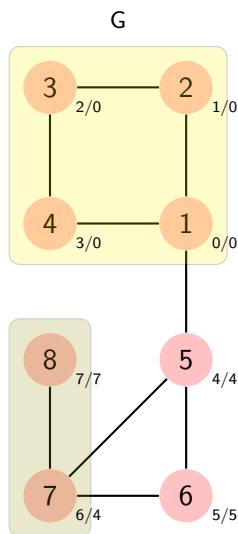
2

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Stiva de noduri

Punct critic?

# Componente biconexce - Tarjan



8

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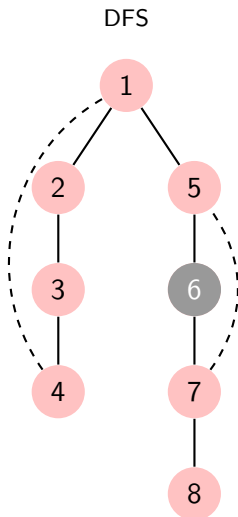
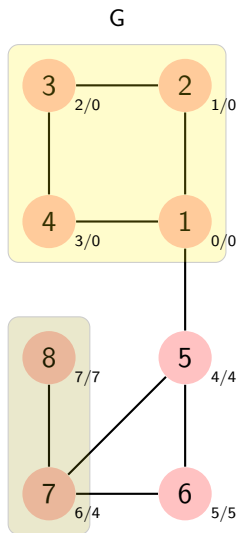
2

1

Stiva de noduri

Punct critic?

# Componente biconexce - Tarjan



8

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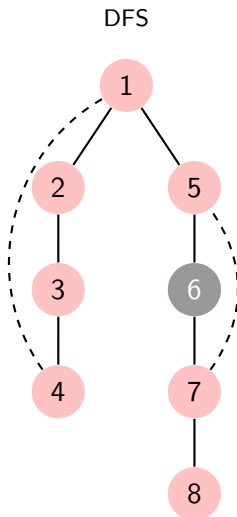
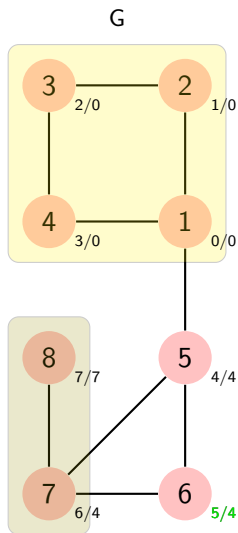
1

$\text{low}(7) \geq \text{idx}(6)?$

Stiva de noduri

Punct critic?

# Componente biconexe - Tarjan



8

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6

5

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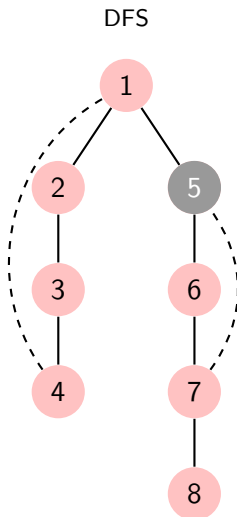
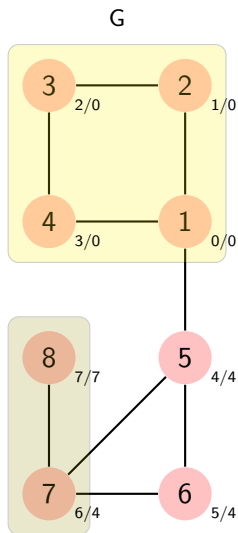
2

1

Stiva de noduri

Punct critic?

# Componente biconexce - Tarjan



8

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4

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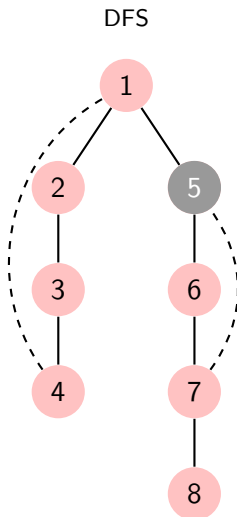
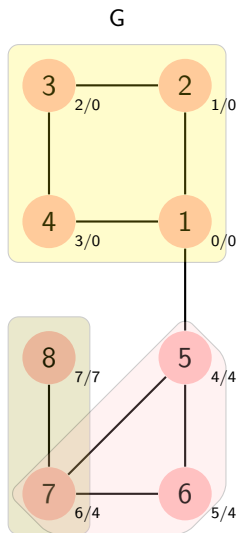
1

$\text{low}(6) \geq \text{idx}(5)?$

Stiva de noduri

Punct critic?

# Componente biconexe - Tarjan



8

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6

5

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3

2

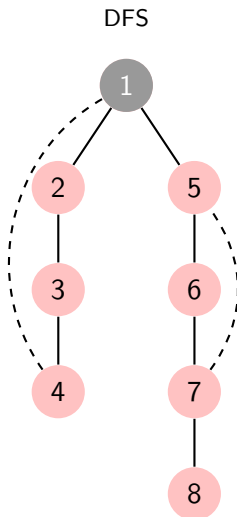
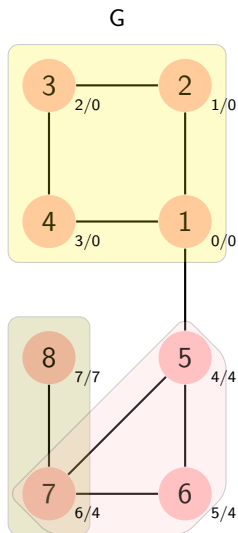
1

Stiva de noduri

Punct critic?



# Componente biconexce - Tarjan



8

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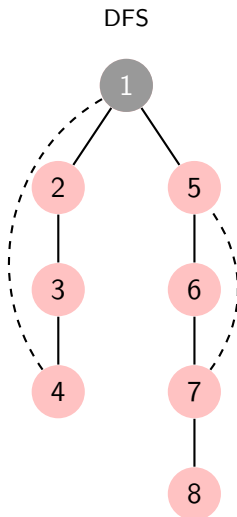
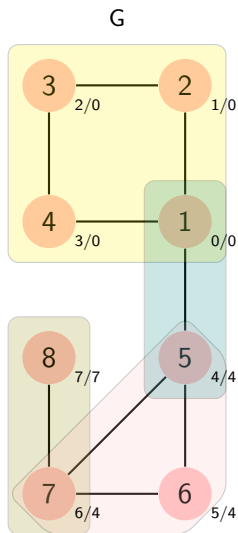
1

$\text{low}(5) \geq \text{idx}(1)?$

Stiva de noduri

Punct critic?

# Componente biconexce - Tarjan



8

7

6

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4

3

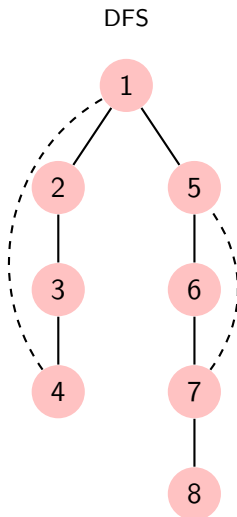
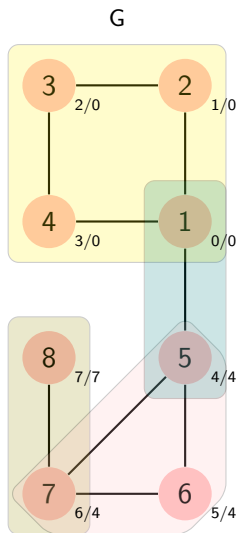
2

1

Stiva de noduri

Punct critic?

# Componente biconexe - Tarjan



8

7

6

5

4

3

2

1

Stiva de noduri

Punct critic?

## Arbori minimi de acoperire – Kruskal

**Kruskal**(V, E)

E = sort(E) după cost

AMA = {}

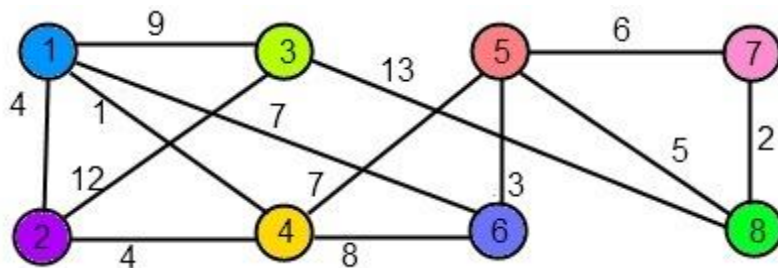
for (e : E)

if AMA U {e} n-are cicluri

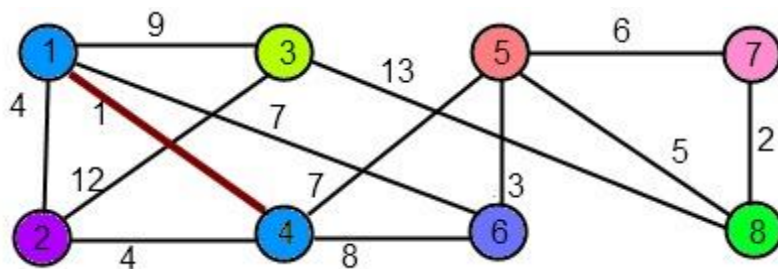
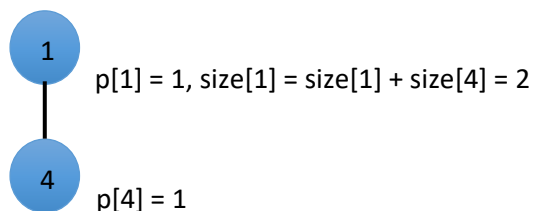
AMA = AMA U {e}

Pentru a determina că AMA nu are cicluri, folosim o structură de tip **union-find**:

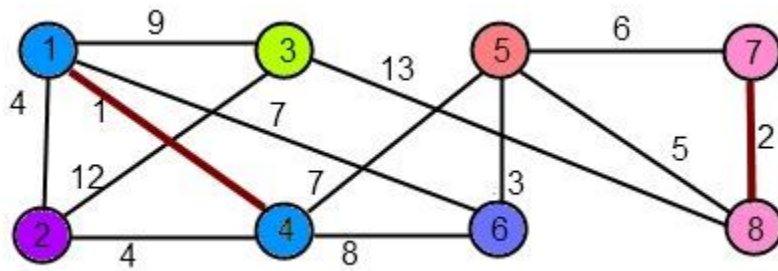
- Inițial fiecare nod face parte dintr-un arbore distinct cu rădăcina (root) în nodul respectiv
- Când adăugăm o muchie (u,v) la AMA
  - ea trebuie să unească arbori distincti, adică: **findroot(u) ≠ findroot(v)**
  - cei 2 arbori trebuie uniți (union), adică de acum încolo, pentru orice nod x din arbore cu u și orice nod y din arbore cu v: **findroot(x) = findroot(y)**
- Pentru a optimiza operațiile de union/find
  - vom folosi un vector de părinți pentru fiecare nod (doar root-ul unui arbore va fi propriul său părinte), restul nodurilor trebuie să circule din părinte în părinte spre root
  - la find: nodul care circulă din părinte în părinte spre root va actualiza, pe parcursul acestei operații, părintele tuturor nodurilor din cale la root, astfel încât data viitoare să indice direct la acesta: **p[nod] = findroot(p[nod])**
  - la union: pentru a micșora lungimea căilor spre root, vom adăuga mereu arborele mai mic ca subarbore al arborelui mai mare (deci e nevoie de un vector size): **p[root\_mic] = root\_mare**



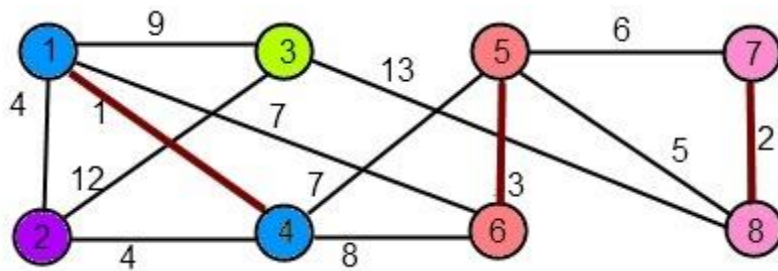
E = {(1,4), (7,8), (5,6), (1,2), (2,4), (5,8), (5,7), (1,6), (4,5), (4,6), (1,3), (2,3), (3,8)}



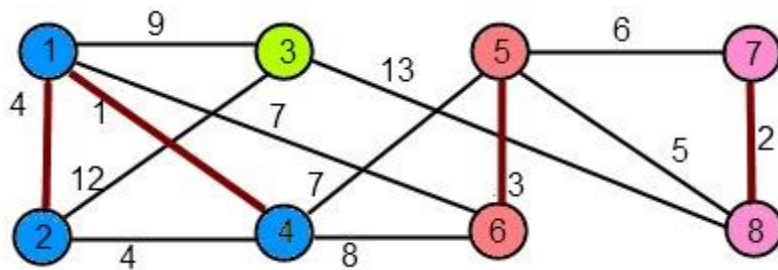
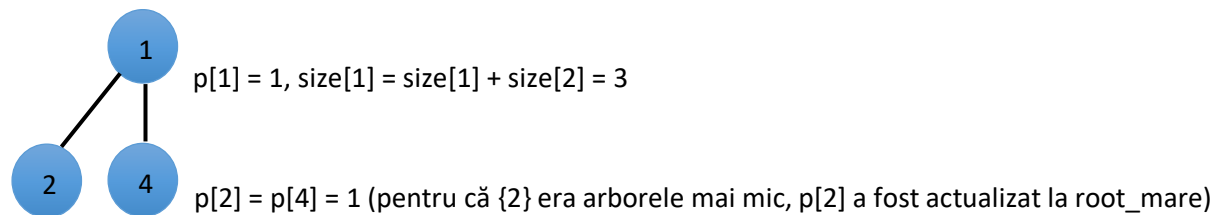
$E = \{(1,4), (7,8), (5,6), (1,2), (2,4), (5,8), (5,7), (1,6), (4,5), (4,6), (1,3), (2,3), (3,8)\}$



$E = \{(1,4), (7,8), (5,6), (1,2), (2,4), (5,8), (5,7), (1,6), (4,5), (4,6), (1,3), (2,3), (3,8)\}$

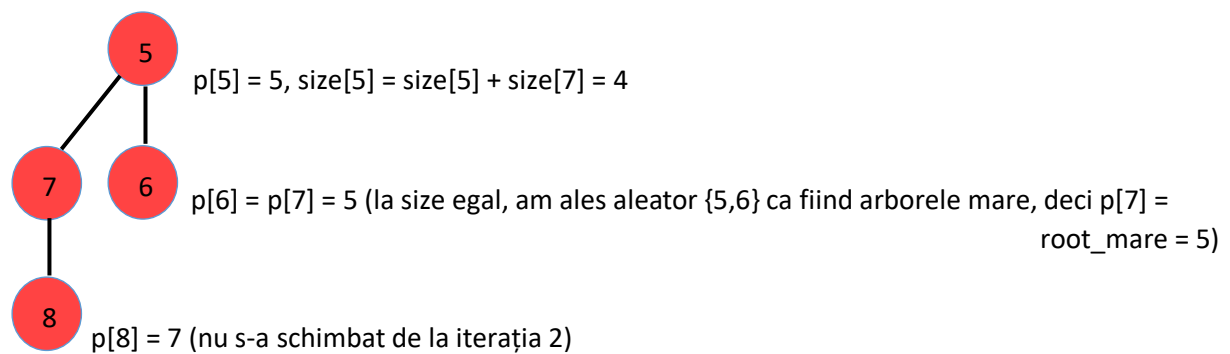


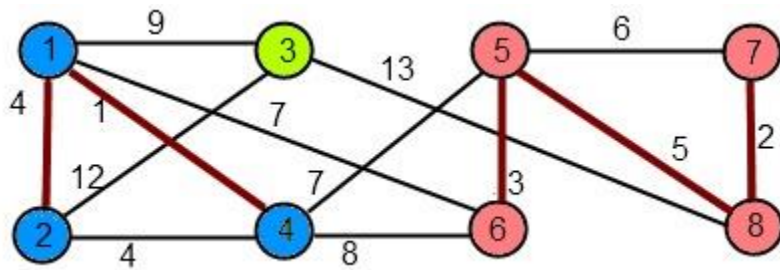
$E = \{(1,4), (7,8), (5,6), (1,2), (2,4), (5,8), (5,7), (1,6), (4,5), (4,6), (1,3), (2,3), (3,8)\}$



$E = \{(1,4), (7,8), (5,6), (1,2), (2,4), (5,8), (5,7), (1,6), (4,5), (4,6), (1,3), (2,3), (3,8)\}$

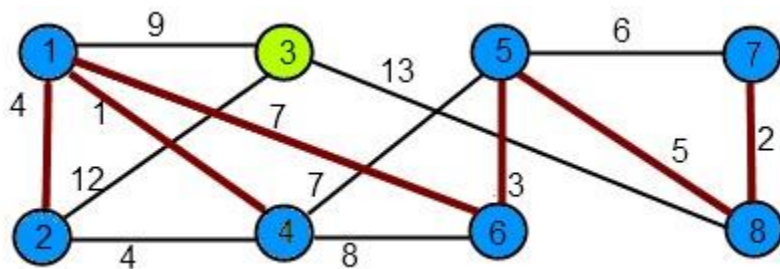
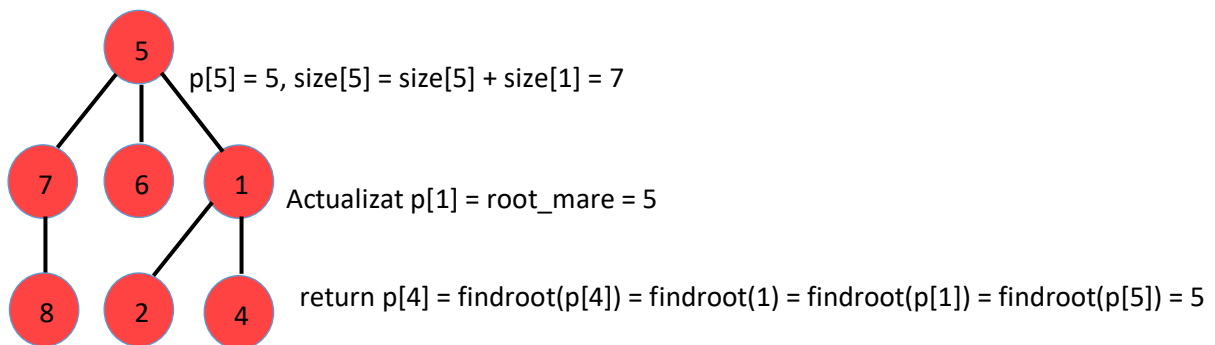
$\text{findroot}(2) = \text{findroot}(4) = 1$  (deci ignorăm muchia  $(2,4)$ )





$E = \{(1,4), (7,8), (5,6), (1,2), (2,4), (5,8), (5,7), (1,6), (4,5), (4,6), (1,3), (2,3), (3,8)\}$

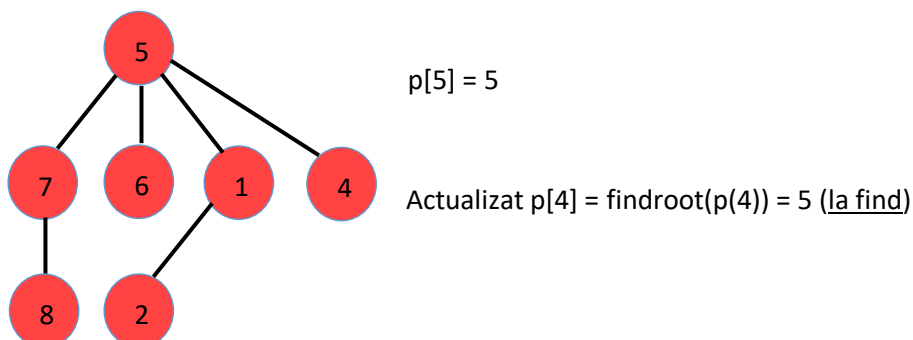
$\text{findroot}(5) = \text{findroot}(7) = 5$  (deci ignorăm muchia  $(5,7)$ )



$E = \{(1,4), (7,8), (5,6), (1,2), (2,4), (5,8), (5,7), (1,6), (4,5), (4,6), (1,3), (2,3), (3,8)\}$

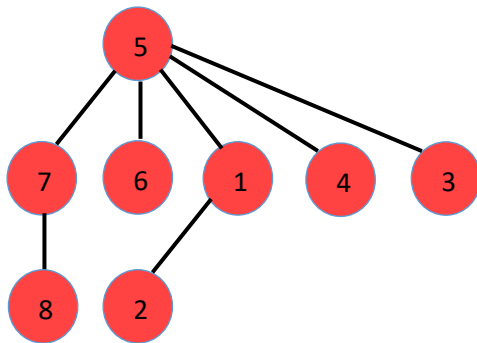
$\text{findroot}(4) = \text{findroot}(5) = 5$  (deci ignorăm muchia  $(5,7)$ )

!!! În plus, în timpul  $\text{findroot}(4)$ :  $p(4) = \text{findroot}(p(4)) = 5$



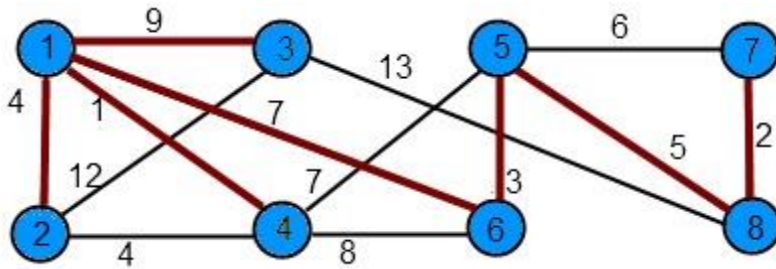
$\text{findroot}(4) = \text{findroot}(6) = 5$  (deci ignorăm muchia  $(4,6)$ )

!!! Am profitat de calea scurtată de la 4 la 5



$p[5] = 5, \text{size}[5] = \text{size}[5] + \text{size}[3]$

Actualizat  $p[3] = \text{root\_mare} = 5$



$E = \{(1,4), (7,8), (5,6), (1,2), (2,4), (5,8), (5,7), (1,6), (4,5), (4,6), (1,3), (2,3), (3,8)\}$

Putem să continuăm să respingem muchii sau, dacă am reținut numărul de noduri/muchii deja adăugate, să ne oprim aici pentru că știm că avem deja un AAM.

$\text{costAMA} = \text{suma costurilor muchiilor verzi} = 1 + 2 + 3 + 4 + 5 + 7 + 9 = 31$

### Arbori minimi de acoperire – Prim

**Prim(V, E)**

alege un nod sursă root

pq = coadă de priorități inițializată cu vecinii lui root (distanțele minime au prioritate)

while (pq nevidă)

$u = \text{top}(pq)$  // cel mai apropiat nod de AMA

    if  $u \notin \text{AMA}$

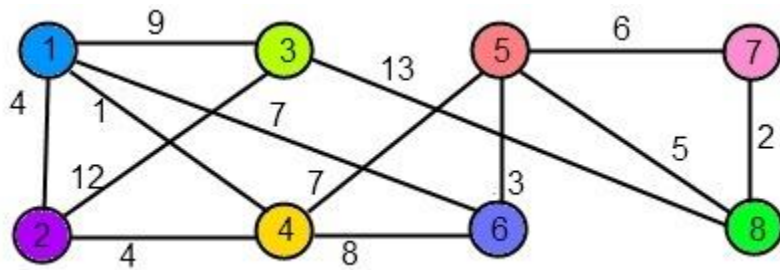
$\text{AMA} = \text{AMA} \cup \{(p(u), u)\}$  //  $p(u)$  = cel mai apropiat vecin al lui  $u$  în AMA

        for ( $v$  : vecini( $u$ ))

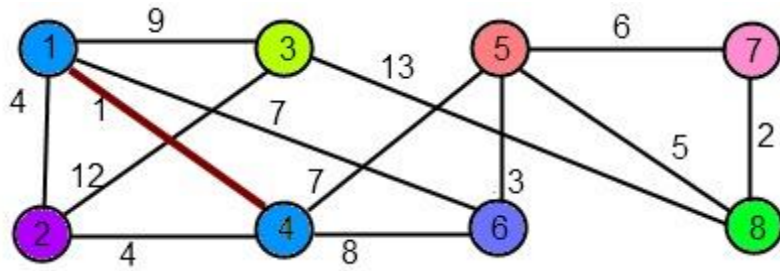
            actualizează în pq distanța lor față de AMA

Pentru a construi AMA, elementele din pq vor conține toată informația utilă:

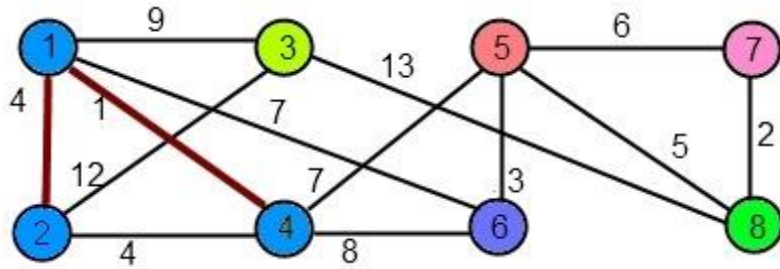
- nodul  $u$  de adăugat în AMA
- părintele  $p[u]$  (cel mai apropiat vecin al lui  $u$  care este deja în AMA), necesar dacă
  - vrem să știm ce muchie adăugăm în AMA
- distanța  $d[u] = w(p[u], u)$  (costul muchiei cu care vrem să legăm  $u$  la AMA), necesar pentru
  - a ordona elementele în pq
  - a calcula rezultatul final costAMA



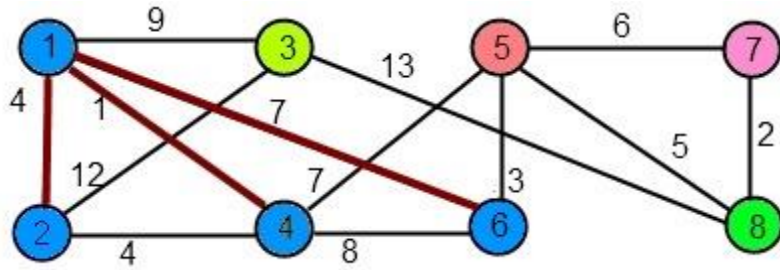
u	2	3	4	5	6	7	8
p[u]	1	1	1		1		
d[u]	4	9	1		7		



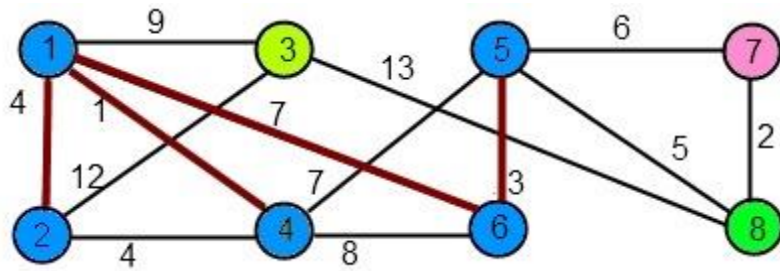
u	2	3	4	5	6	7	8
p[u]	1	1		4	1		
d[u]	4	9		7	7		



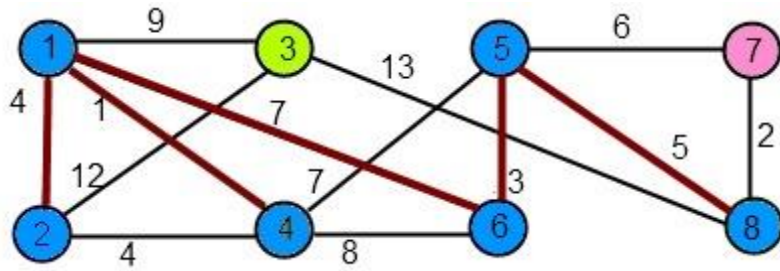
u	2	3	4	5	6	7	8
p[u]		1		4	1		
d[u]		9		7	7		



u	2	3	4	5	6	7	8
p[u]		1		6			
d[u]		9		3			

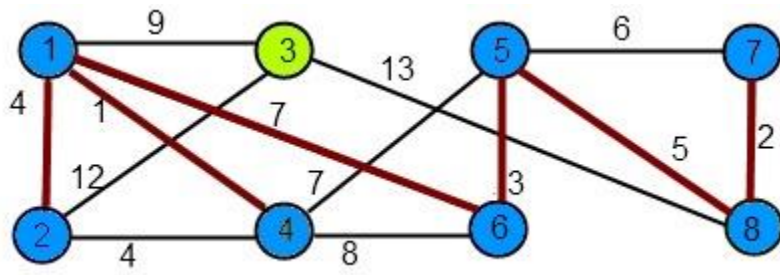


u	2	3	4	5	6	7	8
p[u]		1				5	5
d[u]		9				6	5

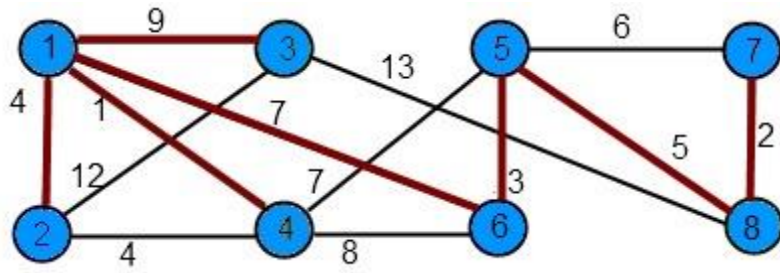


u	2	3	4	5	6	7	8
p[u]		1				8	
d[u]		9				2	



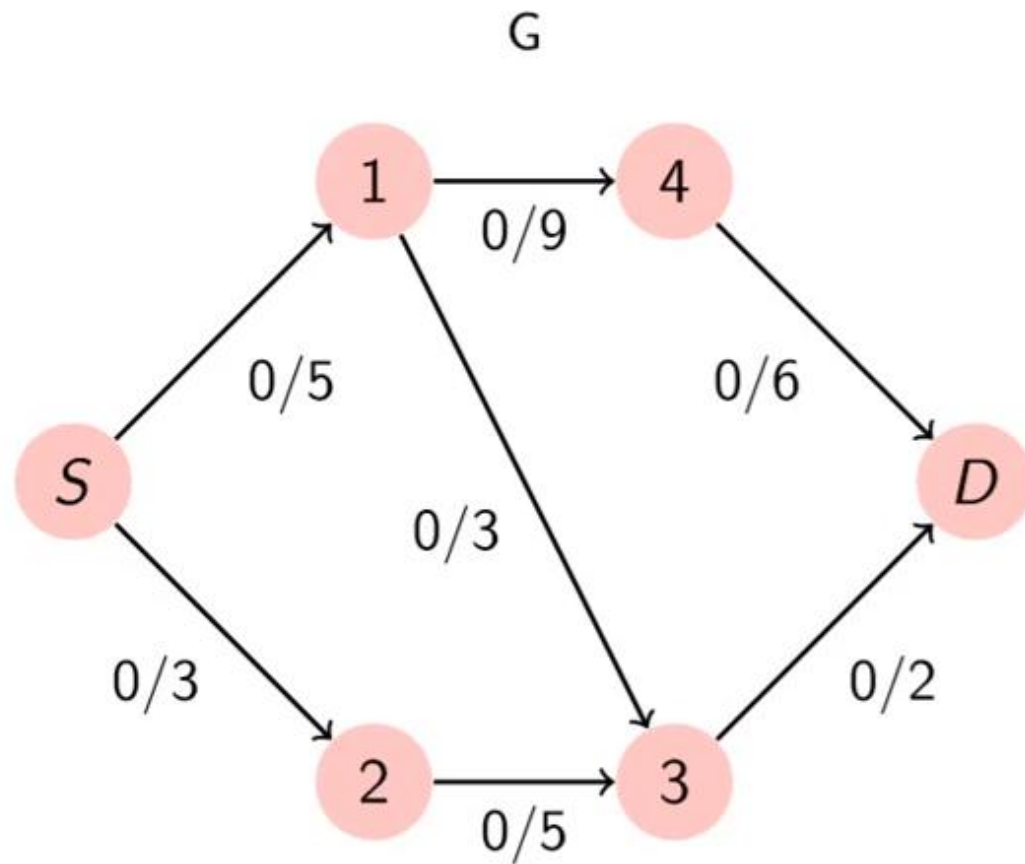


u	2	3	4	5	6	7	8
p[u]		1					
d[u]		9					



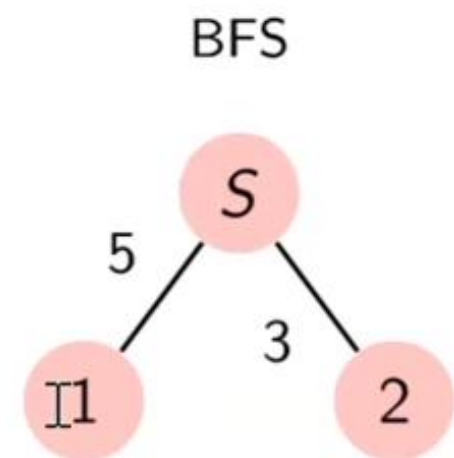
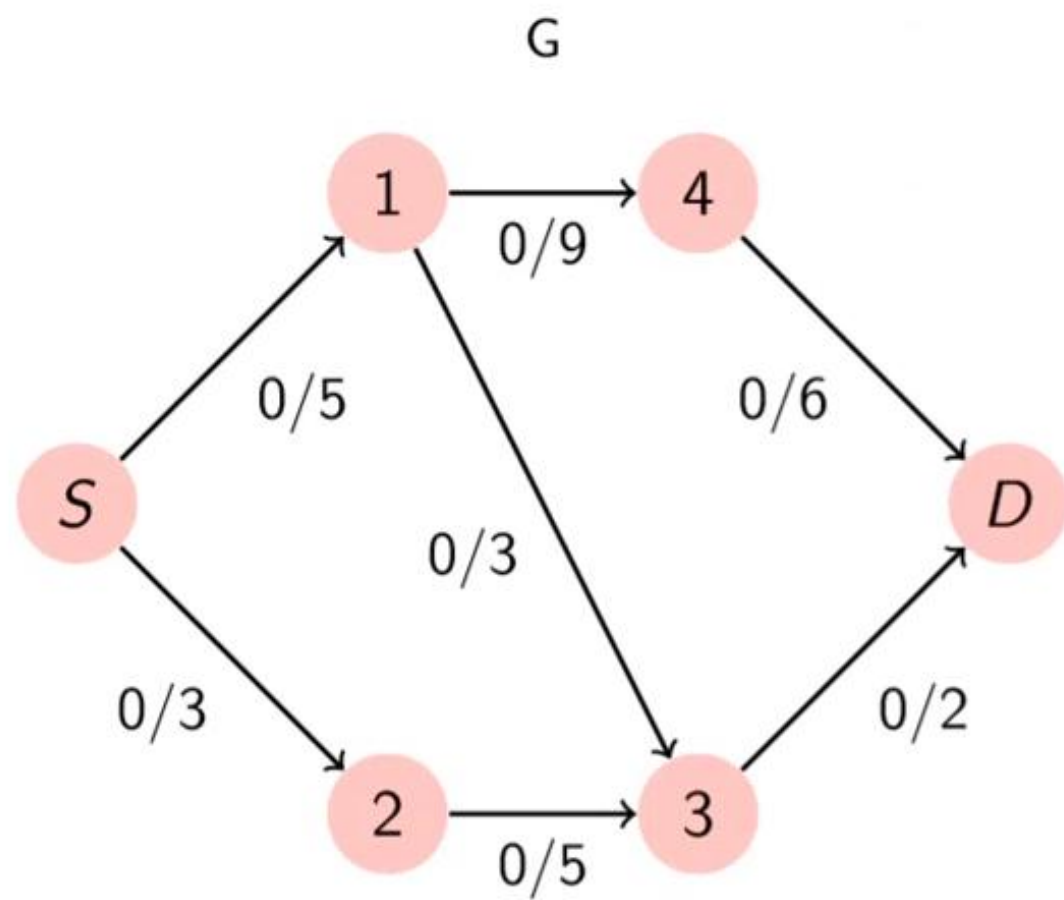
u	2	3	4	5	6	7	8
p[u]							
d[u]							

# Flux maxim - Edmonds-Karp

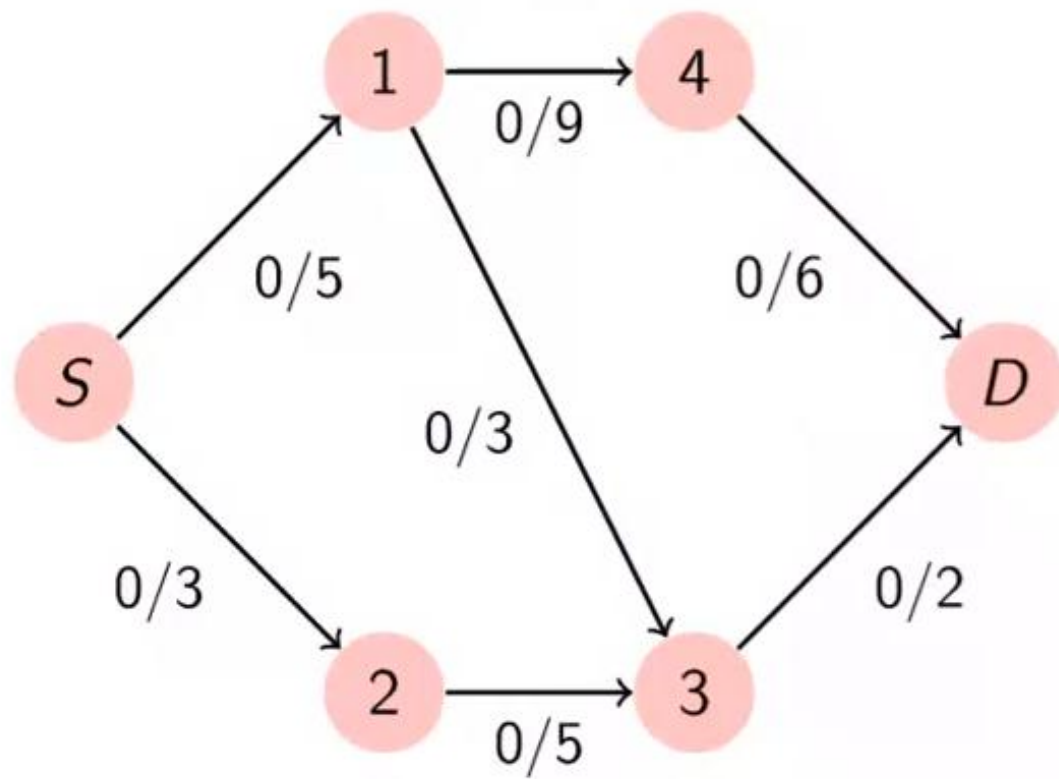


BFS

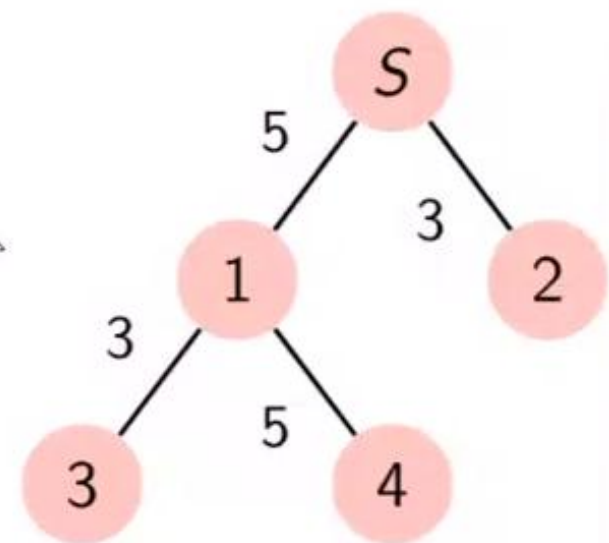




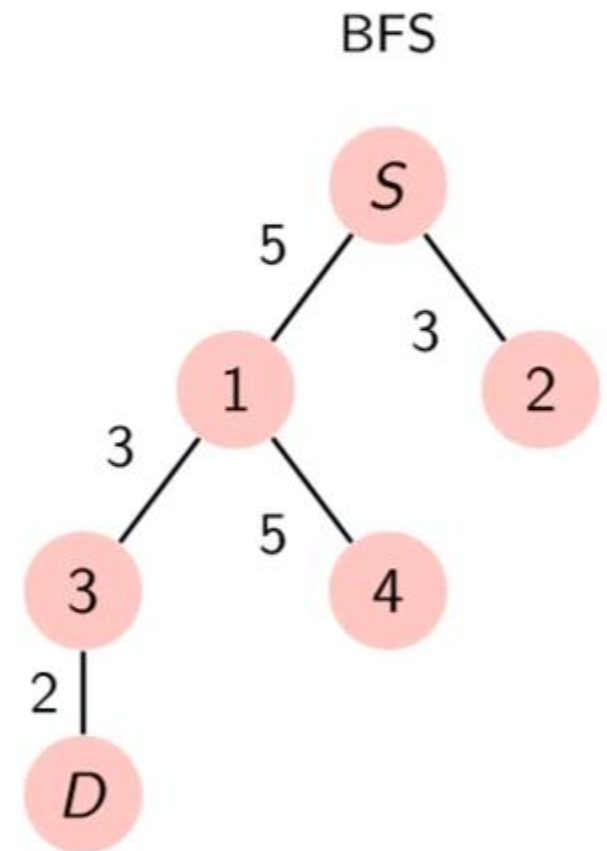
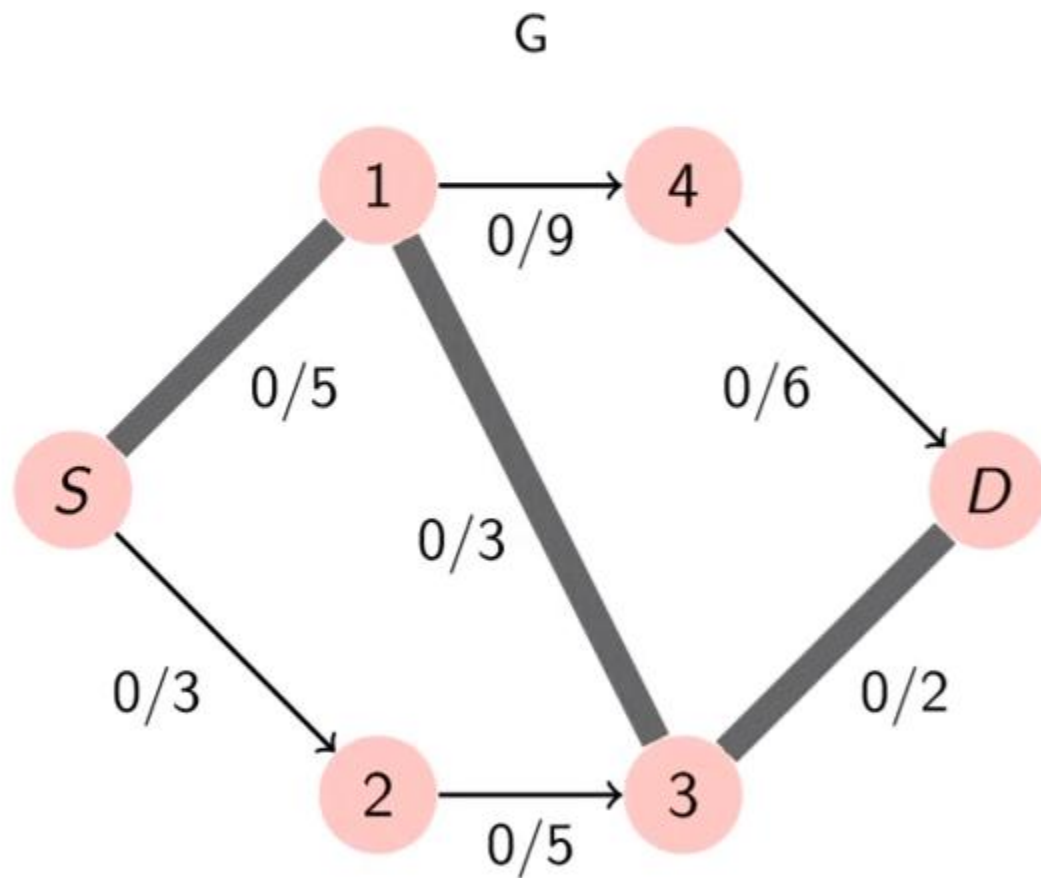
G



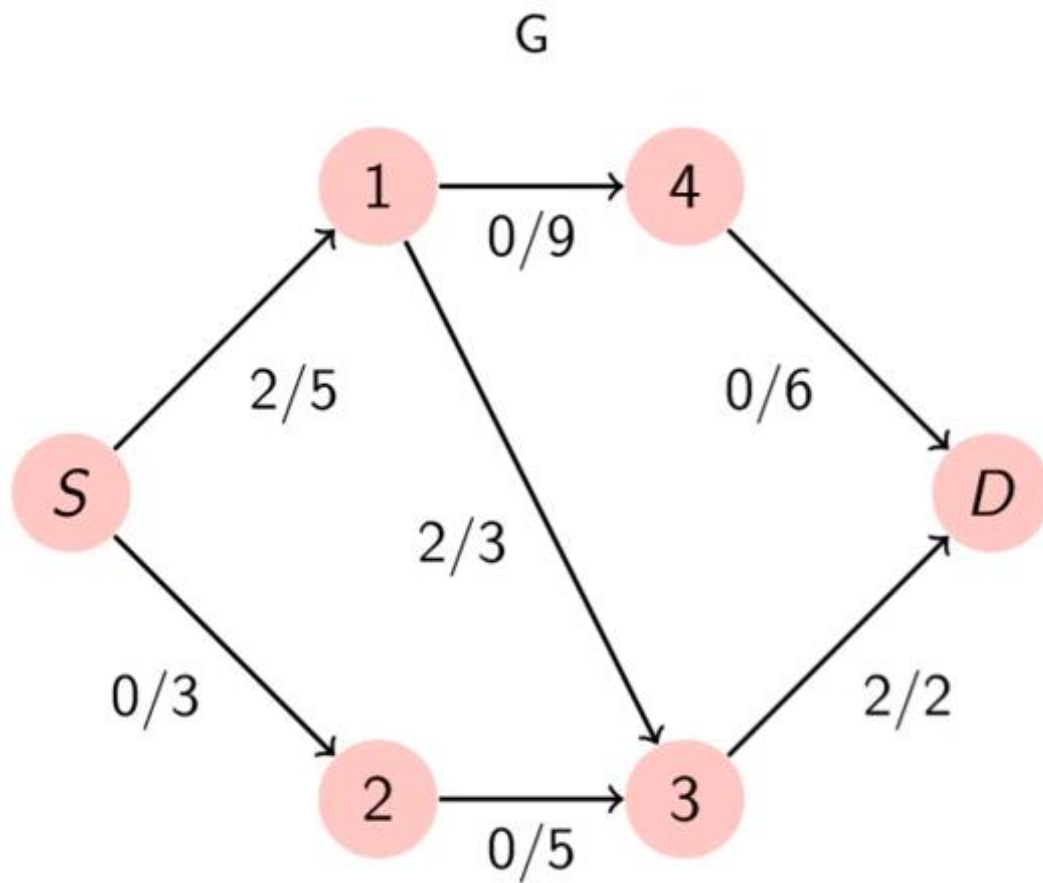
BFS



# Flux maxim - Edmonds-Karp



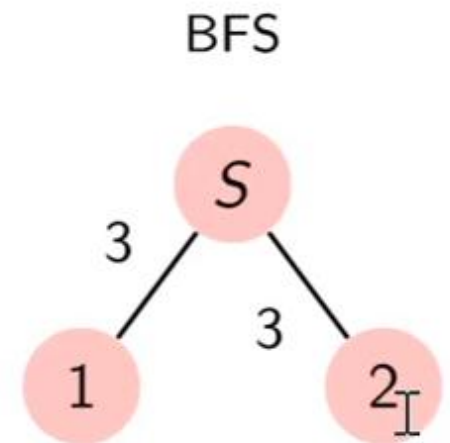
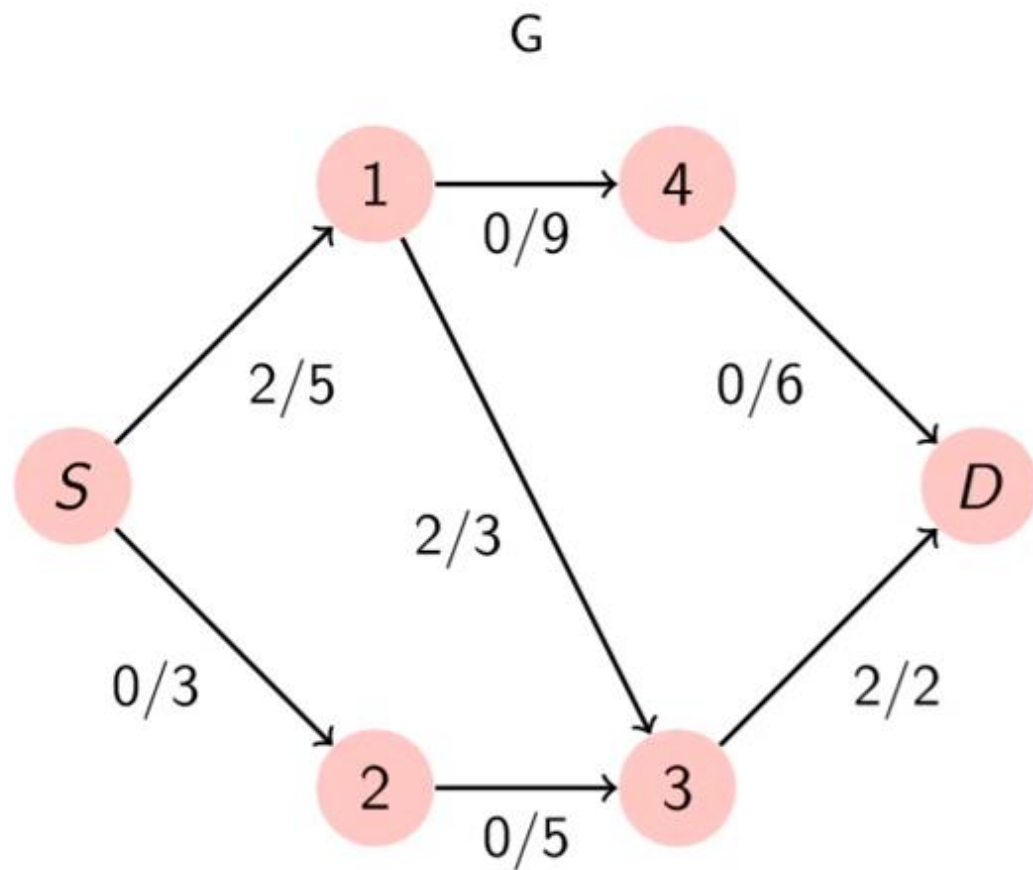
# Flux maxim - Edmonds-Karp



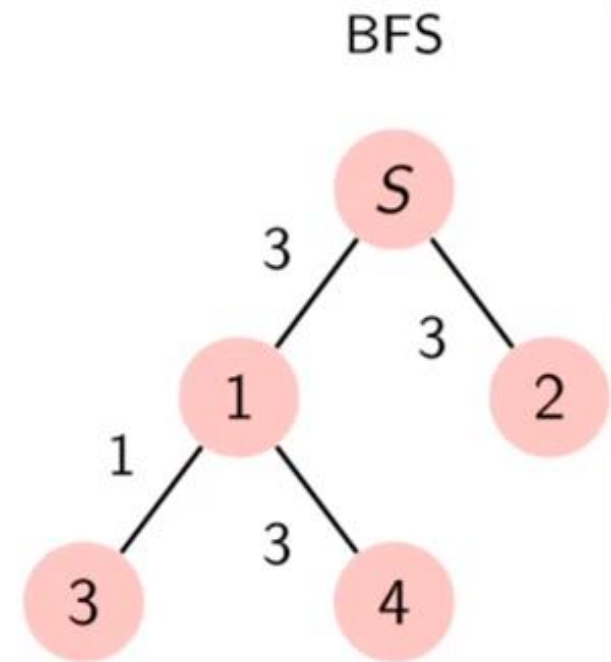
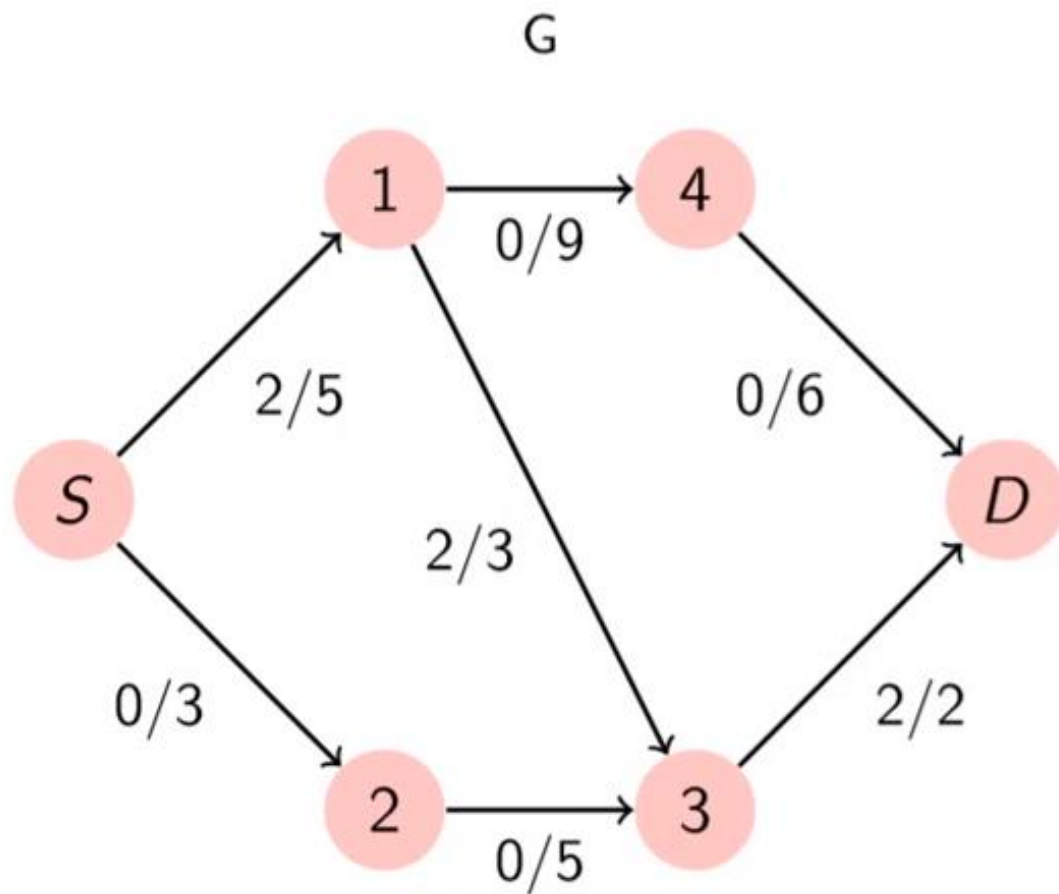
BFS



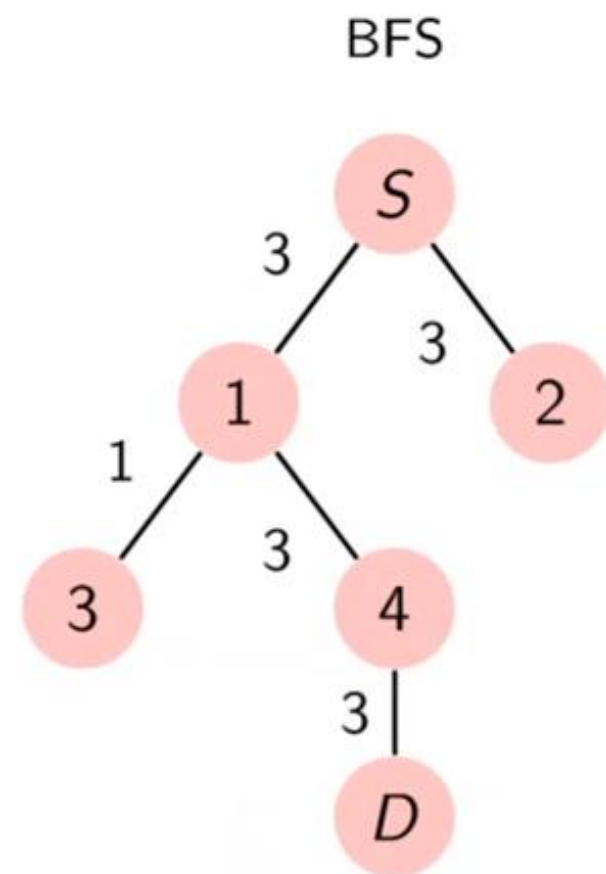
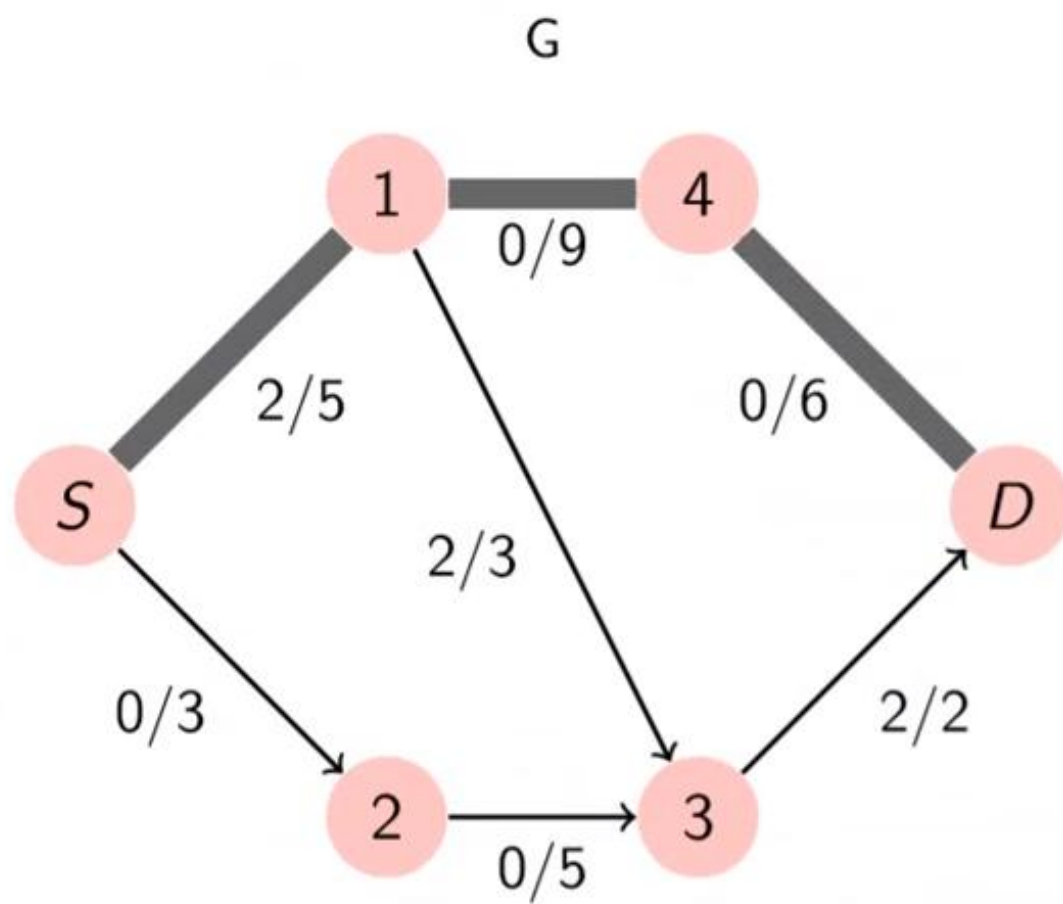
## Flux maxim - Edmonds-Karp



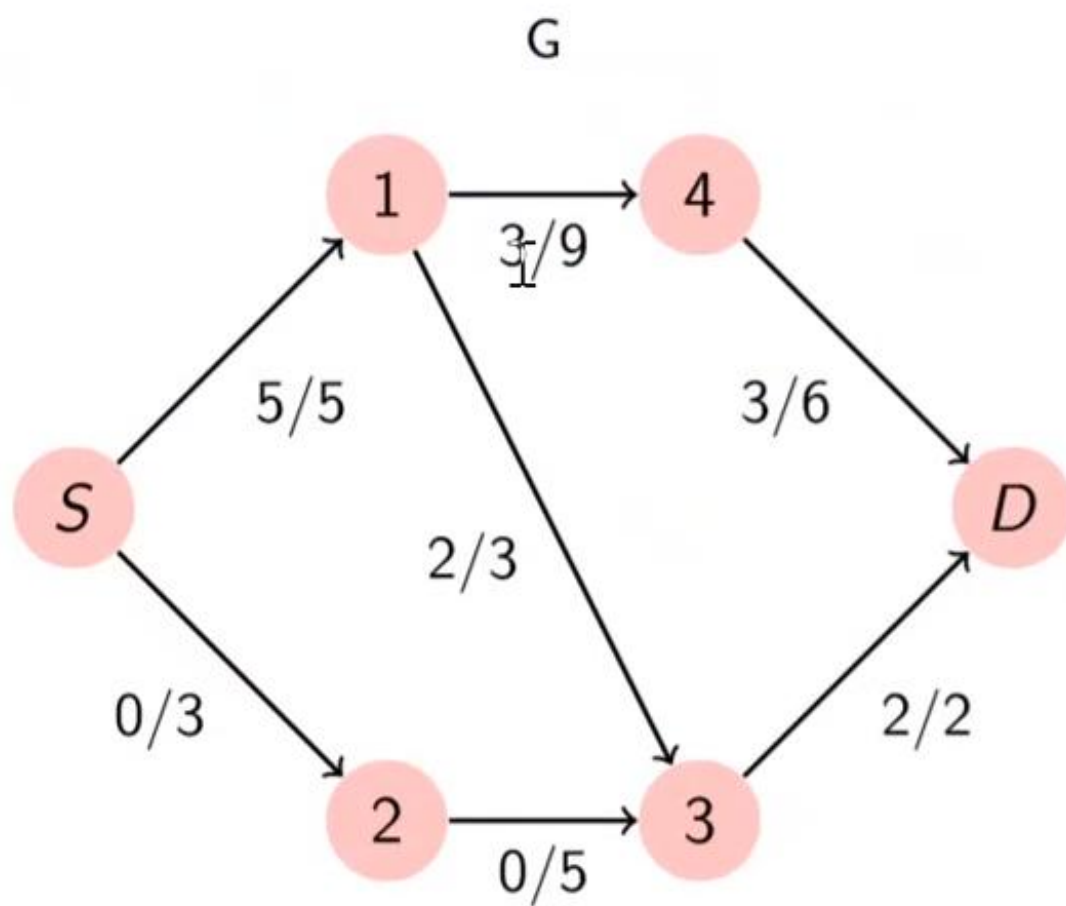
## Flux maxim - Edmonds-Karp







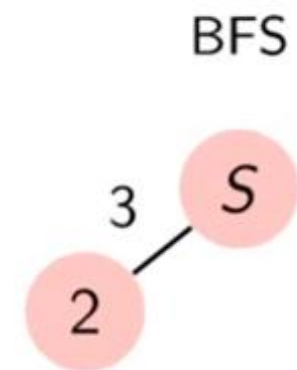
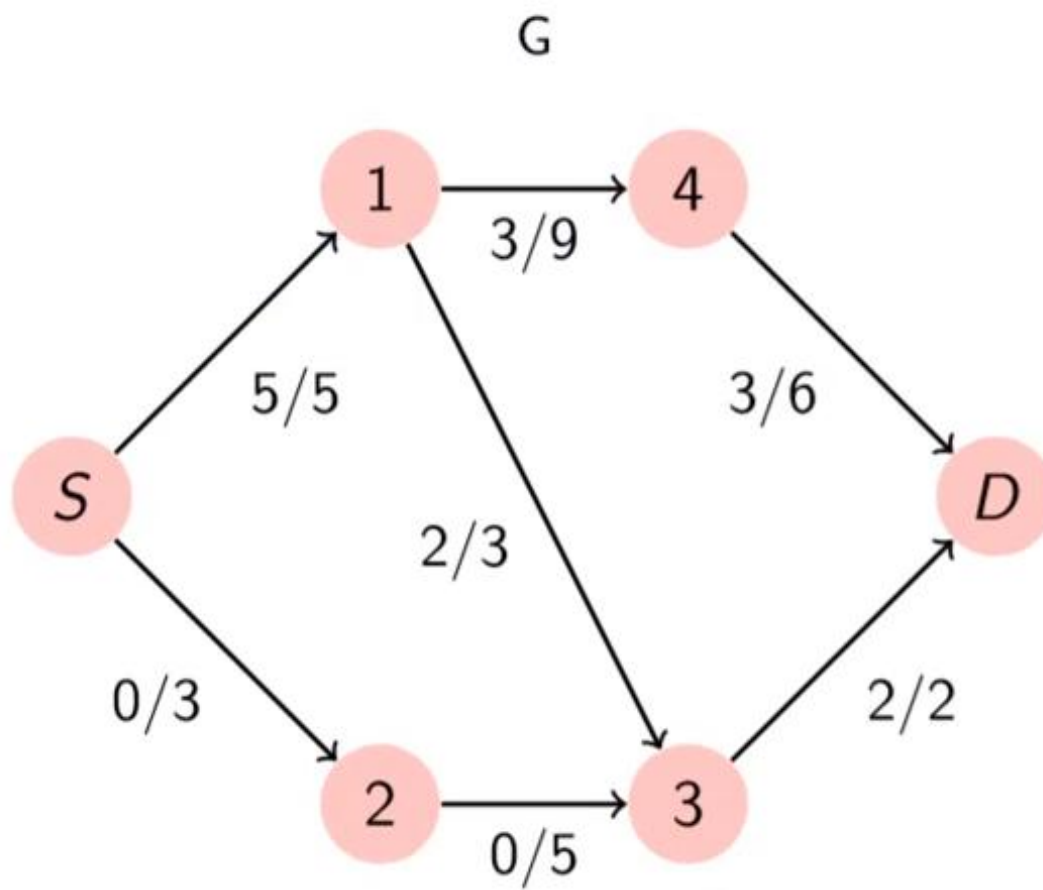
## Flux maxim - Edmonds-Karp



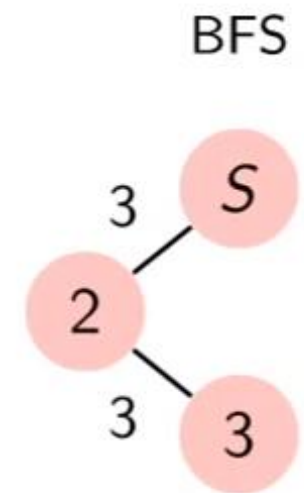
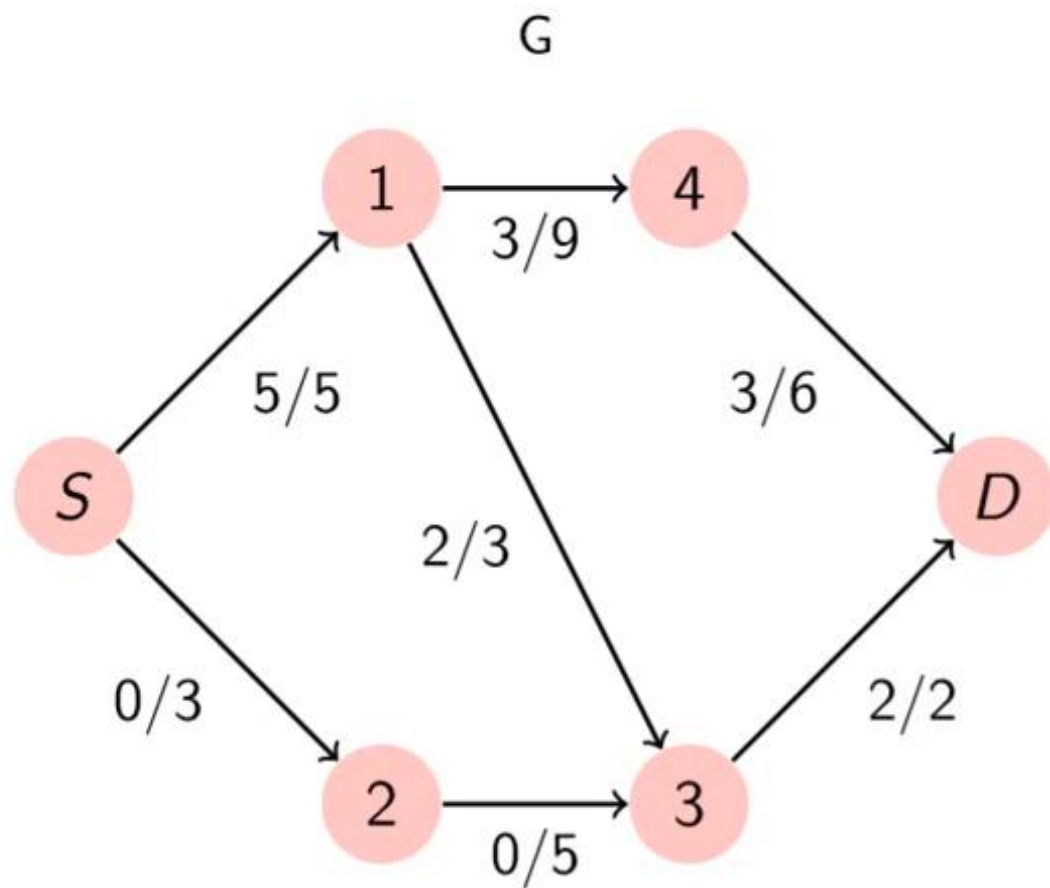
BFS



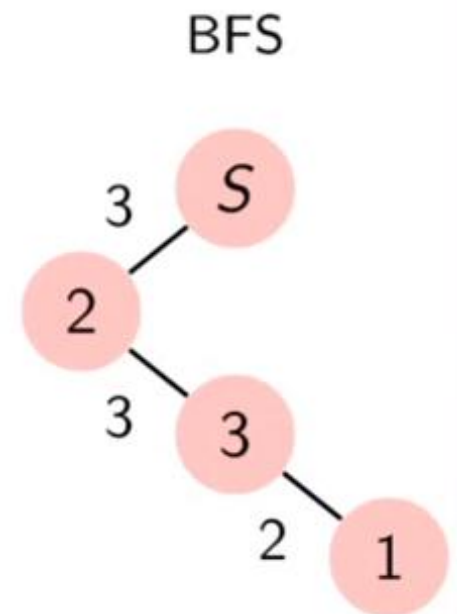
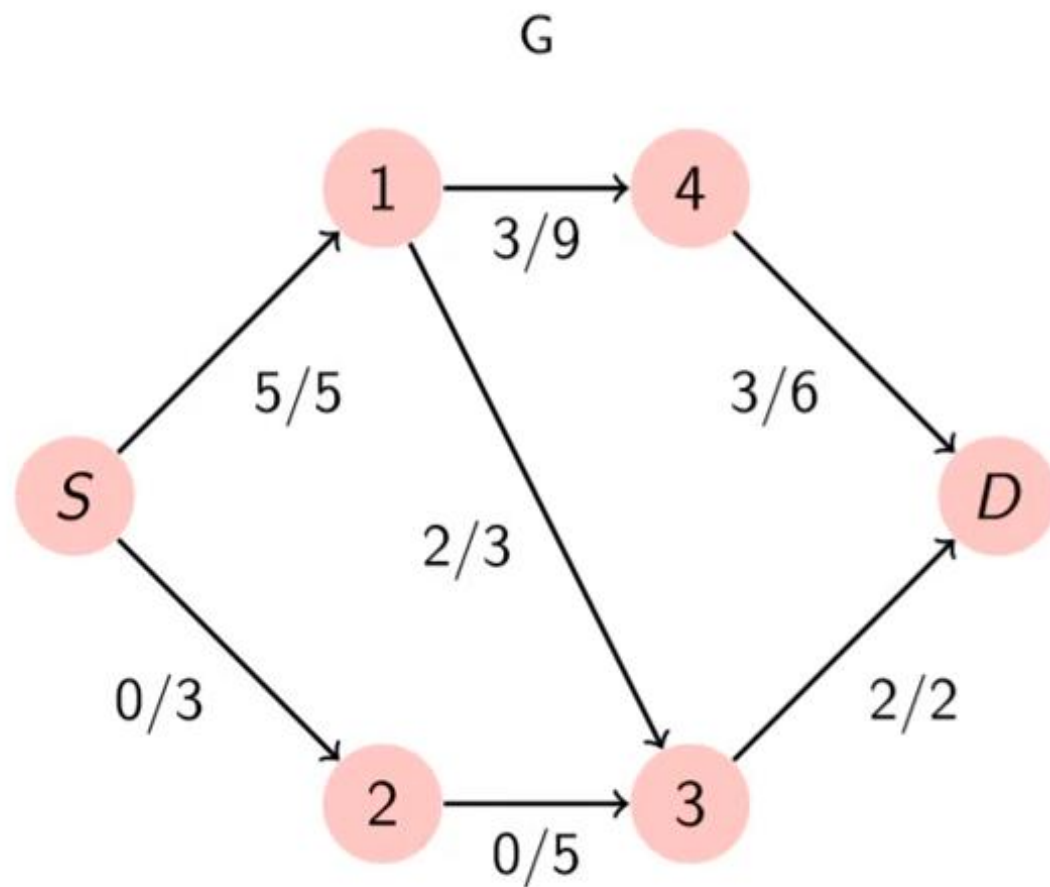
# Flux maxim - Edmonds-Karp



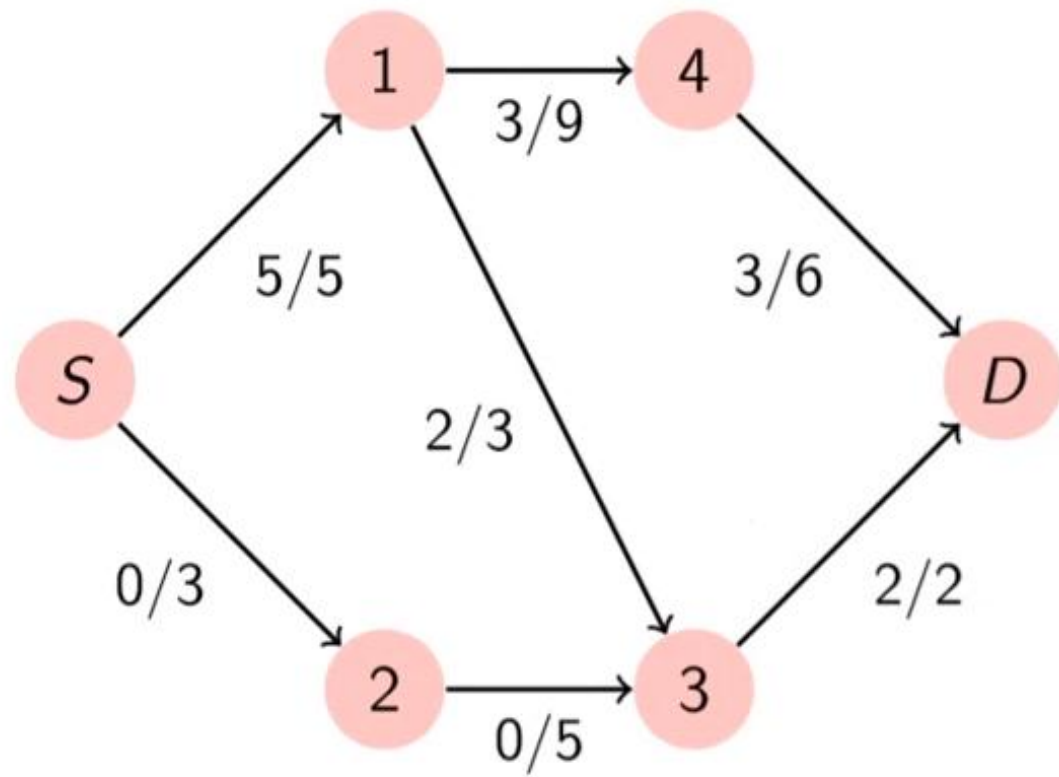
## Flux maxim - Edmonds-Karp



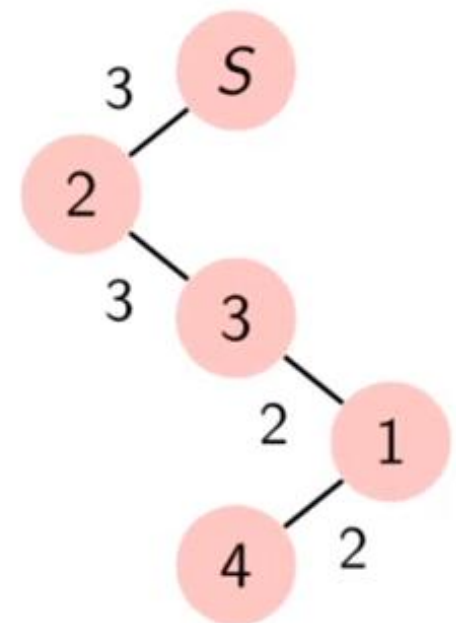
## Flux maxim - Edmonds-Karp



$\bar{G}$

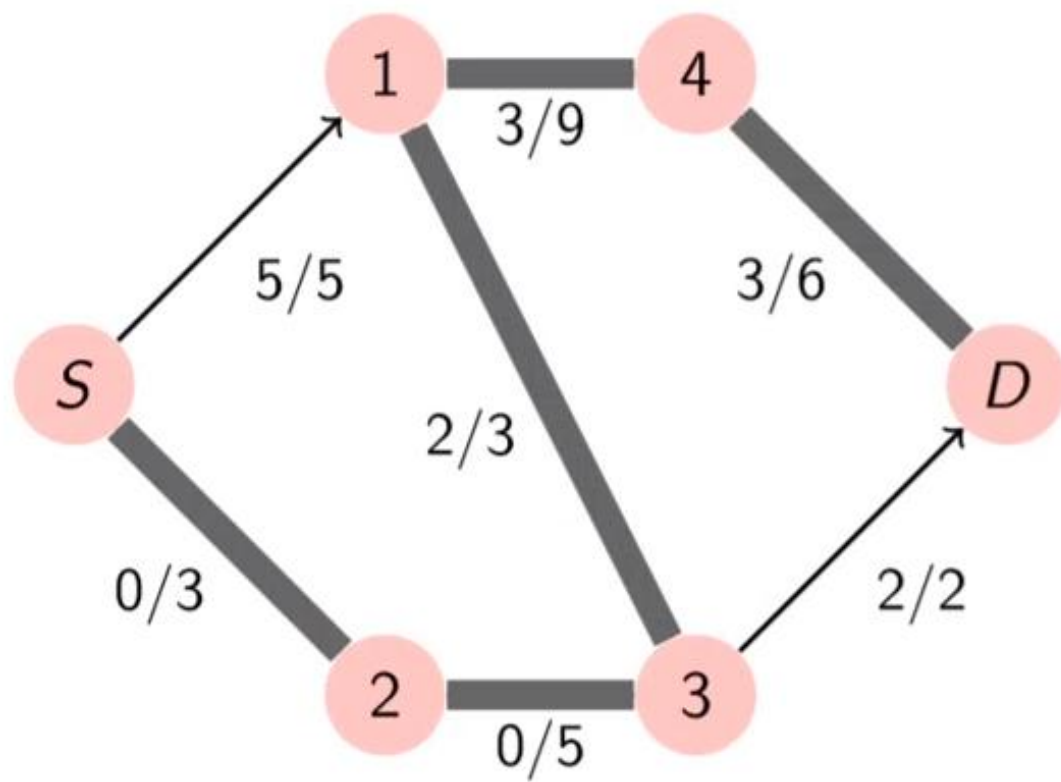


BFS

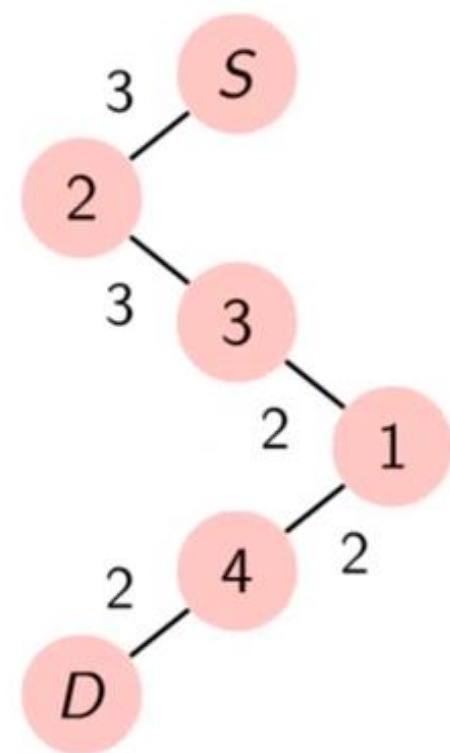




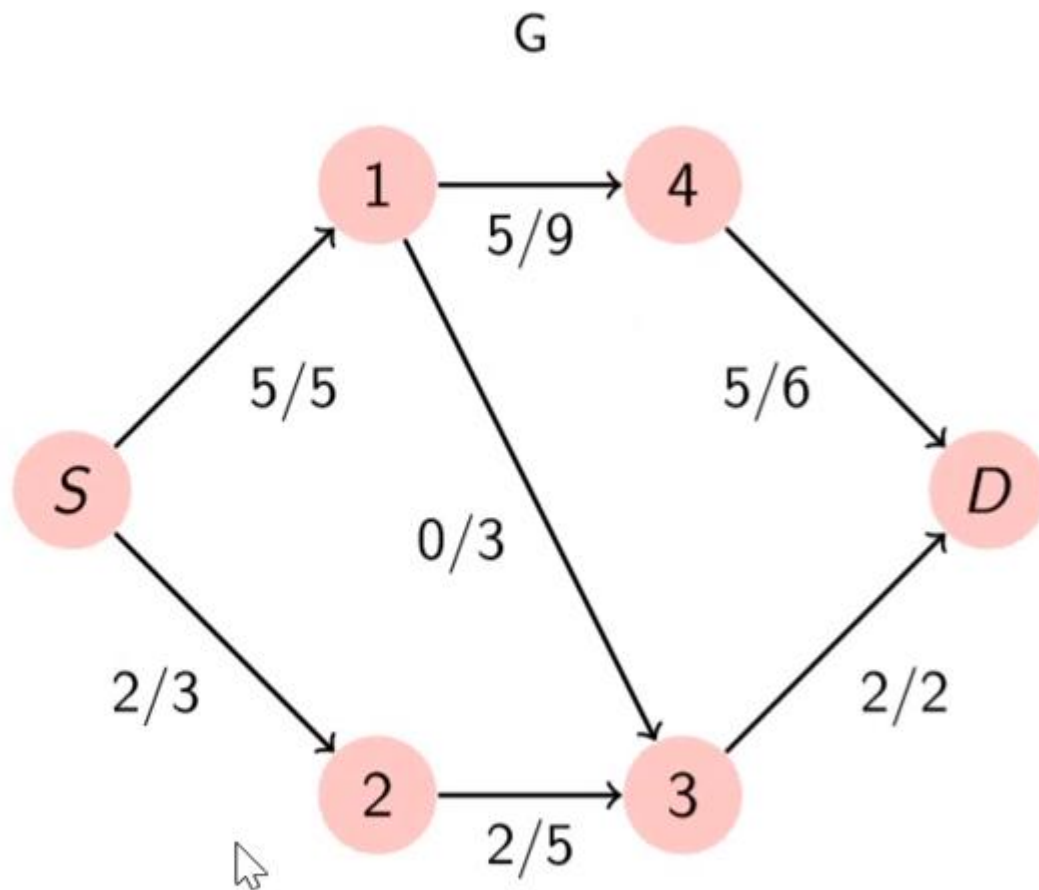
G



BFS



# Flux maxim - Edmonds-Karp

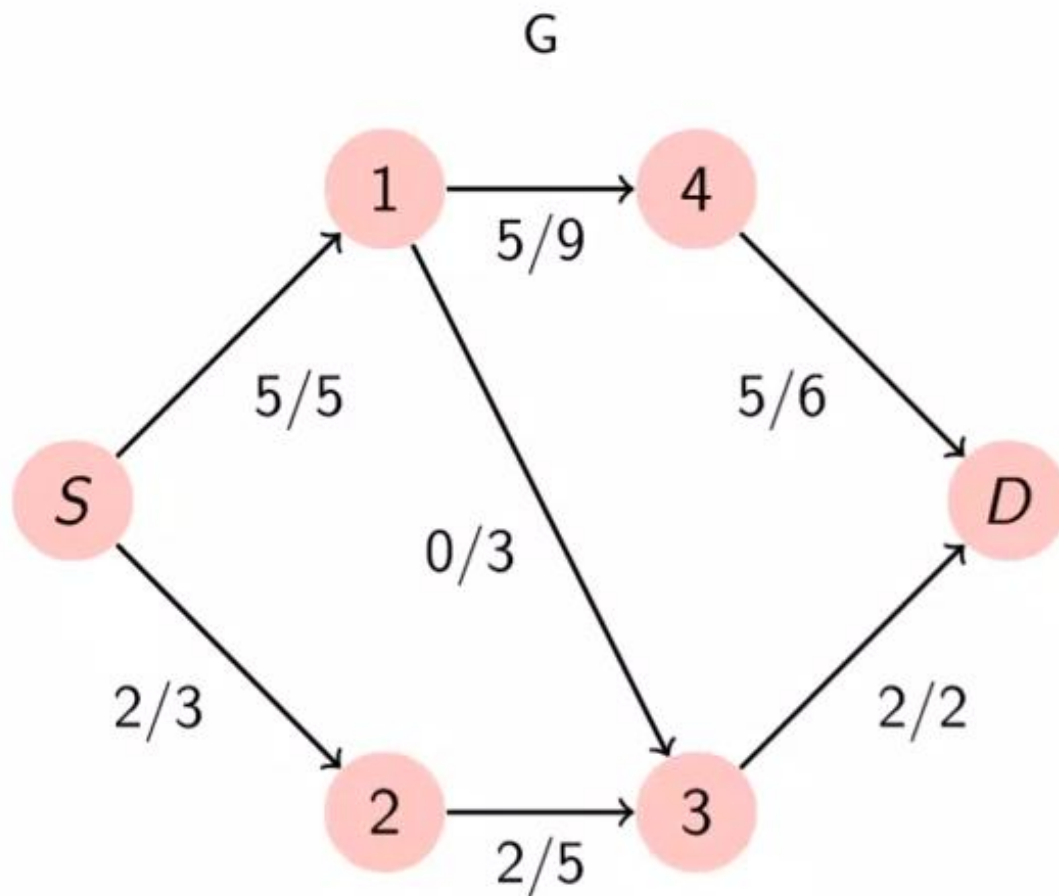


BFS

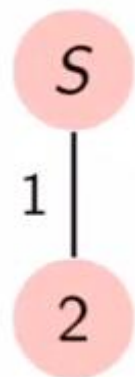




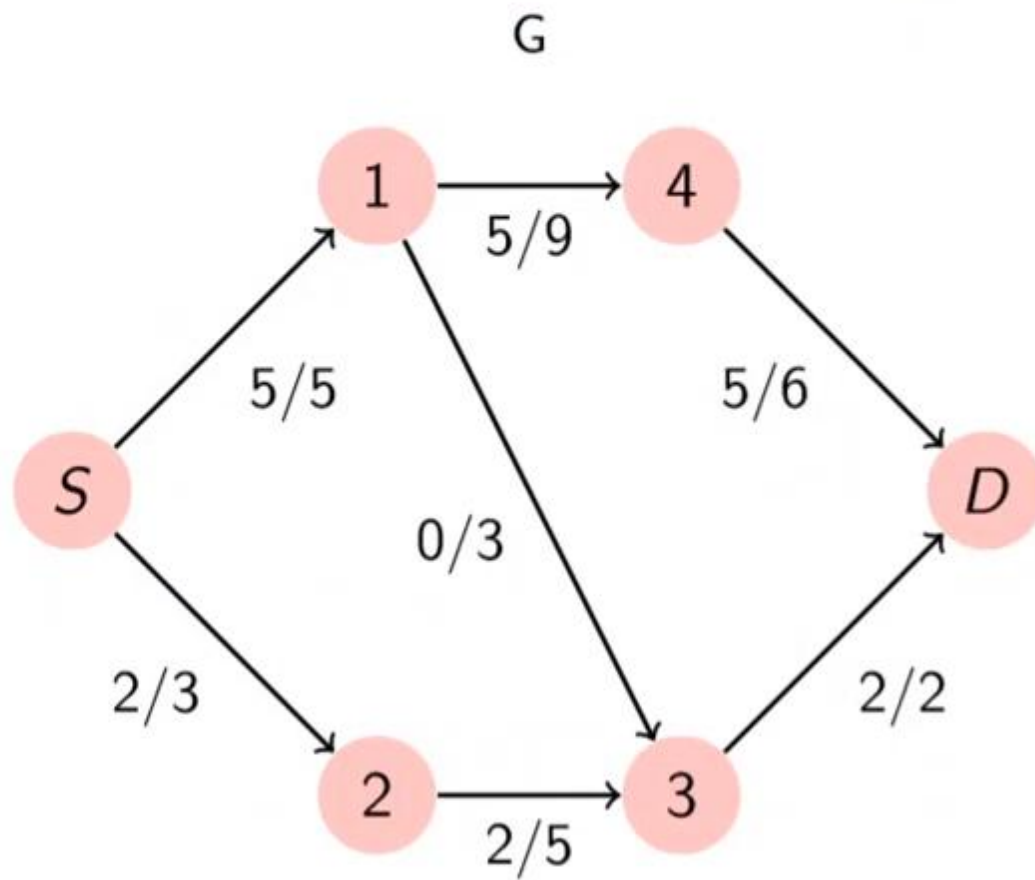
## Flux maxim - Edmonds-Karp



BFS



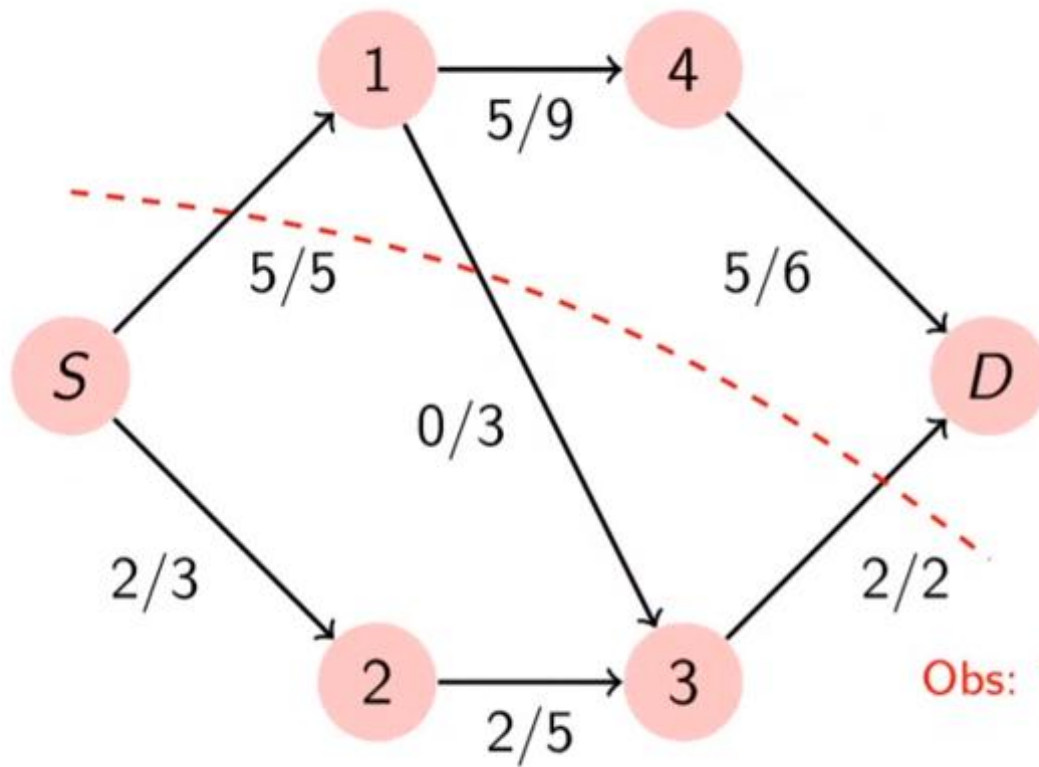
## Flux maxim - Edmonds-Karp



BFS



G



BFS



Obs: Tăietura ((S,2,3), (1,4,D)) este saturată.