Sergiu Iliev | Bayesian ML Course | Final | Problem 2 - continued

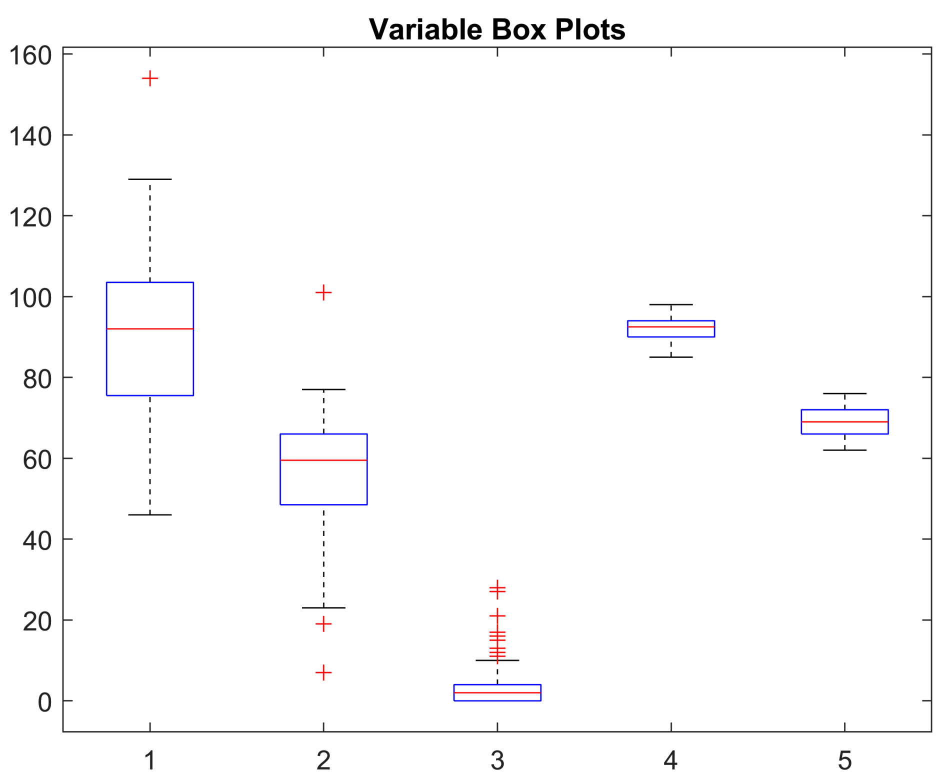
```
clc, clear
```

Load dataset to continue from Python

```
T = readtable('DF.csv');            % Create a temporary table to import the data
```

Define a forecast range and dataset select dataset subsections for traning and testing (forecasting) solar power levels

```
fh = 10; % 10 day forecast horizon
y = T.Energy(1:(end - fh));          % Define Train Energy vector
yF = T.Energy((end - fh + 1):end);   % Define Forecast Energy vector

T = removevars(T,{'Date','Energy'}); % Keep only the predictor variables in the table
X = table2array(T);
X = X(1:(end - fh),:);                 % Create the predictor array for training
XF = X((end - fh + 1):end,:);          % Create the predictor array for testing
predictornames = T.Properties.VariableNames(); % Create a vector of the predictor names

figure;
boxplot(X);
title('Variable Box Plots');
```




```
T = readtable('DF.csv');            % Regenerate the temporary table
% DF = transpose(load('DF.mat'));   % load dataset
% X = transpose(load('X.mat'));     % load training dataset
```

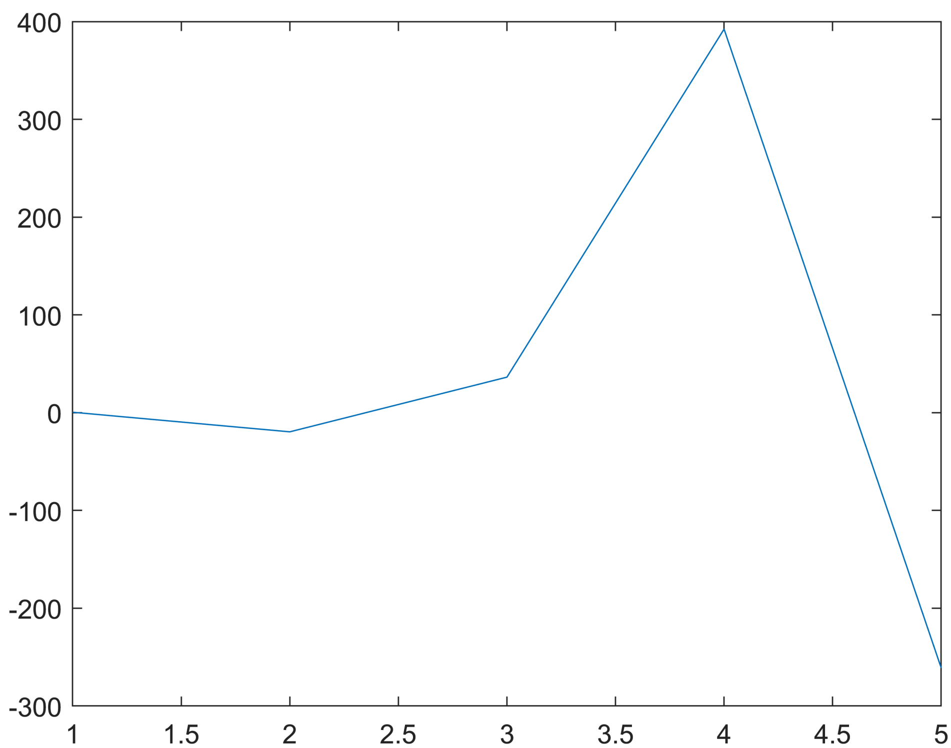
```
% y = transpose(load('y.mat'));    % load testing dataset
% fnames = fieldnames(Y);
% T = table;
% for i = 1:length(fnames)
%     x_T = table(num2cell(getfield(Y, fnames{i})));
%     x_T.Properties.VariableNames = {fnames{i}};
%     T = [T, x_T];
% end
```

(d)-continued: Constructing a Bayesian Linear Regression Model with a LASSO prior

```
rng('default')                          % For reproducibility

lambda = 1;                             % Specify a regularization value of 1

B = lasso(X,y,'Lambda',lambda, 'PredictorNames',predictornames) % Construct the posterior linea
```

```
B = 5×1
    0.4437
  -19.5910
   36.3150
  392.2673
 -261.1158
```

```
% [LassoBetaEstimates,FitInfo] = lasso(X,y,'PredictorNames',predictornames);

plot(B)                                 % Plot the posterior probability distribution
```



(d) Better implementation of the part above

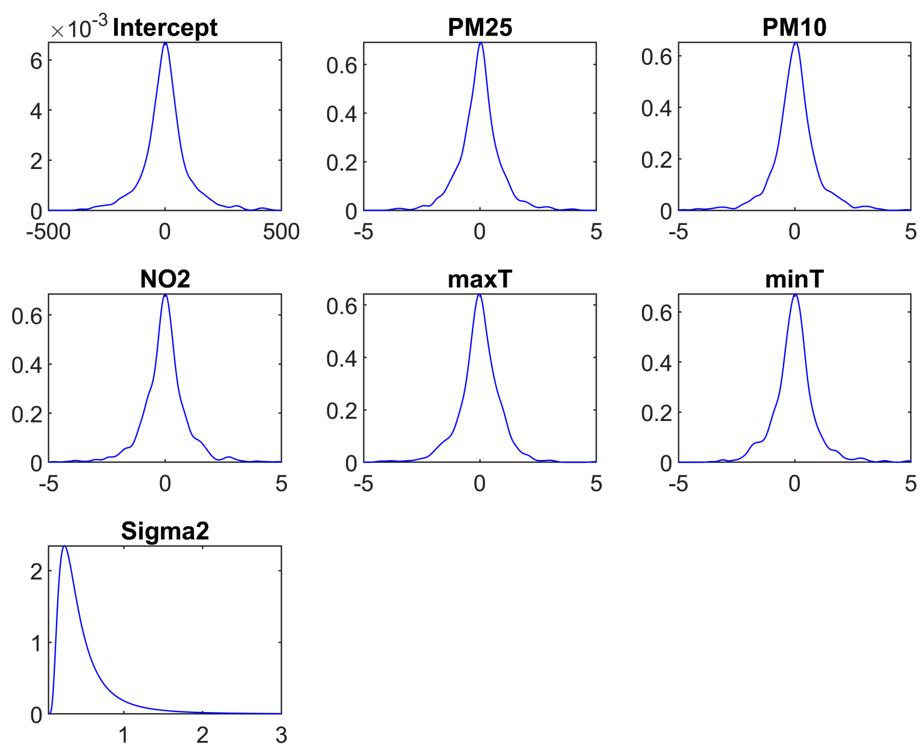
```
p = size(X,2); % Number of predictors

PriorMdl = bayeslm(p,'ModelType','lasso','VarNames',predictornames);
table(PriorMdl.Lambda,'RowNames',PriorMdl.VarNames)
```

ans = 6×1 table

	Var1
1 Intercept	0.0100
2 PM25	1.0000
3 PM10	1.0000
4 NO2	1.0000
5 maxT	1.0000
6 minT	1.0000

`PriorMdl` is a `lassoblm` model object. `lassoblm` attributes a shrinkage of `1` for each coefficient except the intercept, which has a shrinkage of `0.01`

```
plot(PriorMdl)
```



The plot above is the Prior Distribution, now we will determine the Posterior

(e) Constructing and Plotting the Posterior

```
[EstMdl,Summary] = estimate(PriorMdl,X,y,'Display',true)
```

```
Method: lasso MCMC sampling with 10000 draws
Number of observations: 76
Number of predictors:   6

           |     Mean         Std              CI95           Positive  Distribution
-------------------------------------------------------------------------------------------
 Intercept | -2579.5929    4596.5734    [-11695.474, 6339.483]     0.287     Empirical
 PM25      |    -0.0011       0.1605        [-0.341,  0.340]     0.491     Empirical
 PM10      |   -27.7022      10.2465       [-47.636, -7.502]     0.004     Empirical
 NO2       |    35.5817      27.4746       [-18.926, 89.730]     0.908     Empirical
 maxT      |   323.1037      54.2471       [217.678, 430.160]    1.000     Empirical
 minT      |  -222.3267      51.7202     [-324.413, -120.520]    0.000     Empirical
 Sigma2    |     1.3505e+06     2.1813e+05  [989107.013, 1841377.775]     1.000     Empirical

EstMdl = 
  empiricalblm with properties:

    NumPredictors: 5
        Intercept: 1
         VarNames: {6×1 cell}
        BetaDraws: [6×10000 double]
      Sigma2Draws: [1×10000 double]
```

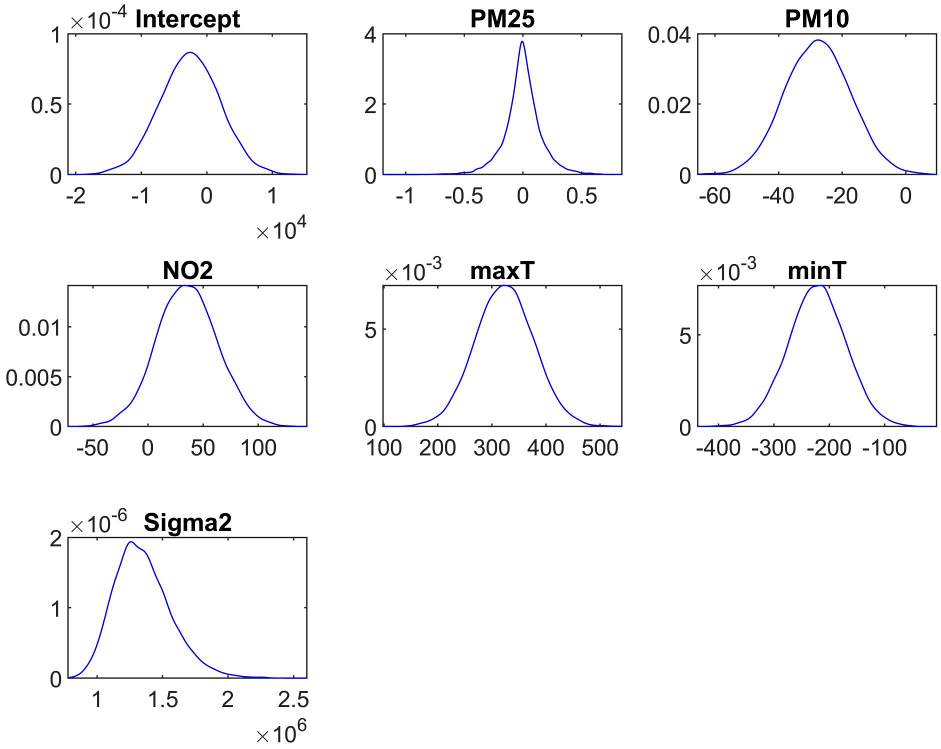

```
           |     Mean         Std              CI95           Positive  Distribution
-------------------------------------------------------------------------------------------
 Intercept | -2579.5929    4596.5734    [-11695.474, 6339.483]     0.287     Empirical
 PM25      |    -0.0011       0.1605        [-0.341,  0.340]     0.491     Empirical
 PM10      |   -27.7022      10.2465       [-47.636, -7.502]     0.004     Empirical
 NO2       |    35.5817      27.4746       [-18.926, 89.730]     0.908     Empirical
 maxT      |   323.1037      54.2471       [217.678, 430.160]    1.000     Empirical
 minT      |  -222.3267      51.7202     [-324.413, -120.520]    0.000     Empirical
 Sigma2    |     1.3505e+06     2.1813e+05  [989107.013, 1841377.775]     1.000     Empirical
```


Summary = 7×6 table

...

	Mean	Std	CI95		Positive	Distribution
1 Intercept	-2.5796e+03	4.5966e+03	-1.1695e+04	6.3395e+03	0.2875	'Empirical'
2 PM25	-0.0011	0.1605	-0.3411	0.3400	0.4909	'Empirical'
3 PM10	-27.7022	10.2465	-47.6363	-7.5023	0.0040	'Empirical'
4 NO2	35.5817	27.4746	-18.9264	89.7304	0.9085	'Empirical'
5 maxT	323.1037	54.2471	217.6779	430.1599	1.0000	'Empirical'
6 minT	-222.3267	51.7202	-324.4127	-120.5203	0	'Empirical'
7 Sigma2	1.3505e+06	2.1813e+05	9.8911e+05	1.8414e+06	1.0000	'Empirical'

```
plot(EstMdl)
```



The plot above is the Posterior distribution for Lambda = 1

Evaluating the model FMSE

```
% Fit a lasso regression model to the data. Use the default regularization path. lasso standarc
[LassoBetaEstimates,FitInfo] = lasso(X,y,'Standardize',false,...
    'PredictorNames',predictornames);
% Compute the FMSE of each model returned by lasso.
yFLasso = FitInfo.Intercept + XF*LassoBetaEstimates;
fmseLasso = sqrt(mean((yF - yFLasso).^2,1)); fmseLasso(2)
```

```
ans = 2.7914e+03
```

```
yFBayesLasso = forecast(EstMdl,XF);
fmseBayesLasso = sqrt(mean((yF - yFBayesLasso).^2));
mean(fmseBayesLasso)
```

```
ans = 2.7867e+03
```

f) PM25 Prior Alteration and effects on Posterior and RMSE

Changing the Lambda for PM25 from 1 to 100 and recalculating posterior

```
PriorMdl.Lambda(2) = 100
```

```
PriorMdl = 
  lassoblm with properties:
```

```
    NumPredictors: 5
        Intercept: 1
         VarNames: {6×1 cell}
           Lambda: [6×1 double]
                A: 3
                B: 1

           |  Mean     Std           CI95         Positive   Distribution
-----------------------------------------------------------------------------
 Intercept |  0       100    [-200.000, 200.000]    0.500   Scale mixture
 PM25      |  0      0.0100    [-0.020,  0.020]     0.500   Scale mixture
 PM10      |  0        1       [-2.000,  2.000]     0.500   Scale mixture
 NO2       |  0        1       [-2.000,  2.000]     0.500   Scale mixture
 maxT      |  0        1       [-2.000,  2.000]     0.500   Scale mixture
 minT      |  0        1       [-2.000,  2.000]     0.500   Scale mixture
 Sigma2    | 0.5000  0.5000    [ 0.138,  1.616]     1.000   IG(3.00,    1)
```

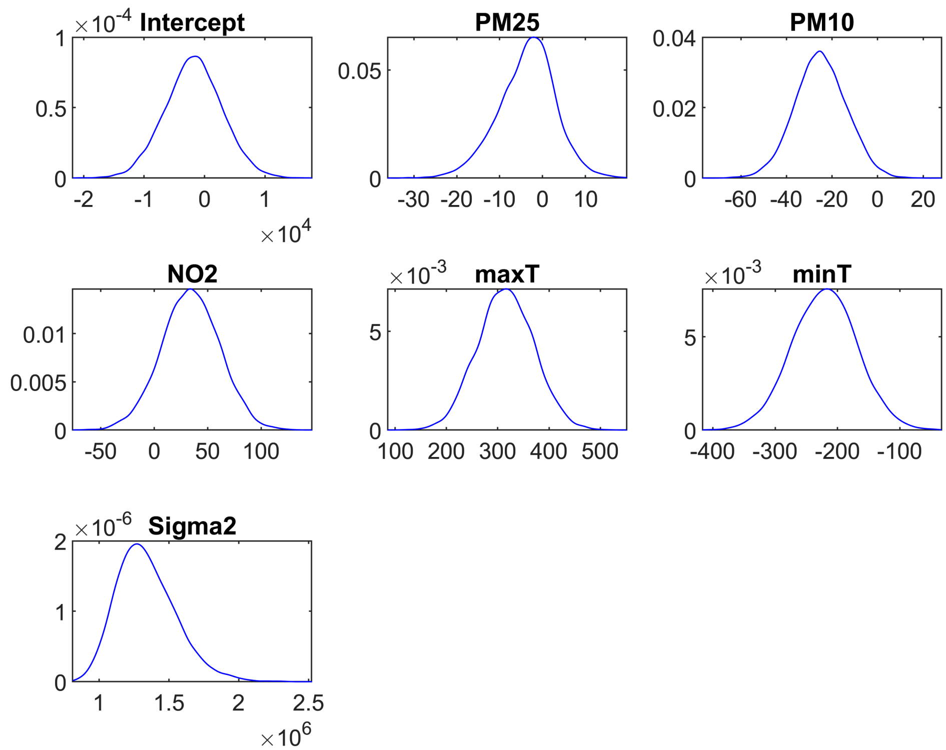
```
[EstMdl,Summary] = estimate(PriorMdl,X,y,'Display',true)
```

```
Method: lasso MCMC sampling with 10000 draws
Number of observations: 76
Number of predictors:   6

           |      Mean            Std                CI95            Positive  Distribution
---------------------------------------------------------------------------------------------
 Intercept | -1793.1094      4751.3242      [-11063.464, 7565.865]       0.344     Empirical
 PM25      |    -3.6371         6.6841         [-17.798,  8.953]        0.297     Empirical
 PM10      |   -25.0113        11.3359         [-47.363, -2.808]        0.014     Empirical
 NO2       |    33.9925        27.2863         [-19.555, 86.685]        0.895     Empirical
 maxT      |   315.4981        54.9623         [209.924, 423.502]       1.000     Empirical
 minT      |  -220.8353        51.9689        [-323.503, -119.161]      0.000     Empirical
 Sigma2    |     1.3400e+06     2.1341e+05  [986048.351, 1819575.345]   1.000     Empirical

EstMdl = 
  empiricalblm with properties:

    NumPredictors: 5
        Intercept: 1
         VarNames: {6×1 cell}
        BetaDraws: [6×10000 double]
      Sigma2Draws: [1×10000 double]

           |      Mean            Std                CI95            Positive  Distribution
---------------------------------------------------------------------------------------------
 Intercept | -1793.1094      4751.3242      [-11063.464, 7565.865]       0.344     Empirical
 PM25      |    -3.6371         6.6841         [-17.798,  8.953]        0.297     Empirical
 PM10      |   -25.0113        11.3359         [-47.363, -2.808]        0.014     Empirical
 NO2       |    33.9925        27.2863         [-19.555, 86.685]        0.895     Empirical
 maxT      |   315.4981        54.9623         [209.924, 423.502]       1.000     Empirical
 minT      |  -220.8353        51.9689        [-323.503, -119.161]      0.000     Empirical
 Sigma2    |     1.3400e+06     2.1341e+05  [986048.351, 1819575.345]   1.000     Empirical

Summary = 7×6 table
```

	Mean	Std	CI95		Positive	Distribution
1 Intercept	-1.7931e+03	4.7513e+03	-1.1063e+04	7.5659e+03	0.3439	'Empirical'
2 PM25	-3.6371	6.6841	-17.7981	8.9531	0.2966	'Empirical'
3 PM10	-25.0113	11.3359	-47.3629	-2.8084	0.0139	'Empirical'

	Mean	Std	CI95		Positive	Distribution
4 NO2	33.9925	27.2863	-19.5547	86.6846	0.8955	'Empirical'
5 maxT	315.4981	54.9623	209.9237	423.5022	1.0000	'Empirical'
6 minT	-220.8353	51.9689	-323.5032	-119.1612	0	'Empirical'
7 Sigma2	1.3400e+06	2.1341e+05	9.8605e+05	1.8196e+06	1.0000	'Empirical'

```
plot(EstMdl)
```



The plot above is the Posterior distribution for PM25 Lambda = 100

Evaluating the model FMSE

```
yFBayesLasso = forecast(EstMdl,XF);
fmseBayesLasso = sqrt(mean((yF - yFBayesLasso).^2));
mean(fmseBayesLasso)
```

```
ans = 2.7896e+03
```

Changing the Lambda for PM25 from 1 to 10000 and recalculating posterior

```
PriorMdl.Lambda(2) = 100
```

```
PriorMdl =
  lassoblm with properties:
```

```
    NumPredictors: 5
        Intercept: 1
         VarNames: {6×1 cell}
           Lambda: [6×1 double]
                A: 3
                B: 1


           |  Mean     Std          CI95         Positive   Distribution
-----------------------------------------------------------------------------
 Intercept |  0       100    [-200.000, 200.000]    0.500   Scale mixture
 PM25      |  0      0.0100    [-0.020,  0.020]     0.500   Scale mixture
 PM10      |  0        1       [-2.000,  2.000]     0.500   Scale mixture
 NO2       |  0        1       [-2.000,  2.000]     0.500   Scale mixture
 maxT      |  0        1       [-2.000,  2.000]     0.500   Scale mixture
 minT      |  0        1       [-2.000,  2.000]     0.500   Scale mixture
 Sigma2    | 0.5000  0.5000    [ 0.138,  1.616]     1.000   IG(3.00,    1)
```

```
[EstMdl,Summary] = estimate(PriorMdl,X,y,'Display',true)
```

```
Method: lasso MCMC sampling with 10000 draws
Number of observations: 76
Number of predictors:   6

           |     Mean         Std             CI95           Positive  Distribution
-----------------------------------------------------------------------------------------------
 Intercept | -1795.7640     4772.0365    [-11321.199, 7611.227]     0.357     Empirical
 PM25      |    -3.6639        6.7034       [-17.508,  9.243]      0.294     Empirical
 PM10      |   -24.8351       11.3127       [-47.103, -2.439]      0.015     Empirical
 NO2       |    33.6236       27.6746       [-20.369, 87.994]      0.890     Empirical
 maxT      |   315.8333       55.6313       [206.498, 425.390]     1.000     Empirical
 minT      |  -221.3632       51.8956      [-322.985, -120.247]    0.000     Empirical
 Sigma2    |     1.3427e+06     2.1519e+05 [989487.917, 1834540.023]   1.000     Empirical

EstMdl = 
  empiricalblm with properties:

    NumPredictors: 5
        Intercept: 1
         VarNames: {6×1 cell}
        BetaDraws: [6×10000 double]
      Sigma2Draws: [1×10000 double]


           |     Mean         Std             CI95           Positive  Distribution
-----------------------------------------------------------------------------------------------
 Intercept | -1795.7640     4772.0365    [-11321.199, 7611.227]     0.357     Empirical
 PM25      |    -3.6639        6.7034       [-17.508,  9.243]      0.294     Empirical
 PM10      |   -24.8351       11.3127       [-47.103, -2.439]      0.015     Empirical
 NO2       |    33.6236       27.6746       [-20.369, 87.994]      0.890     Empirical
 maxT      |   315.8333       55.6313       [206.498, 425.390]     1.000     Empirical
 minT      |  -221.3632       51.8956      [-322.985, -120.247]    0.000     Empirical
 Sigma2    |     1.3427e+06     2.1519e+05 [989487.917, 1834540.023]   1.000     Empirical

Summary = 7×6 table
```

...

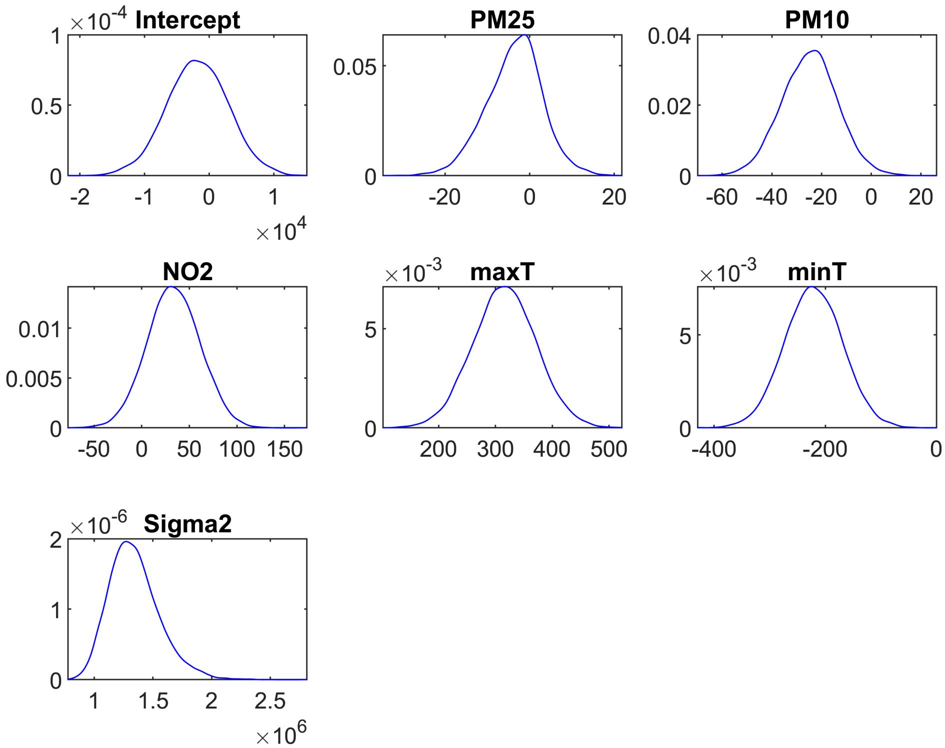
	Mean	Std	CI95		Positive	Distribution
1 Intercept	-1.7958e+03	4.7720e+03	-1.1321e+04	7.6112e+03	0.3567	'Empirical'
2 PM25	-3.6639	6.7034	-17.5078	9.2431	0.2945	'Empirical'
3 PM10	-24.8351	11.3127	-47.1034	-2.4391	0.0146	'Empirical'

	Mean	Std	CI95		Positive	Distribution
4 NO2	33.6236	27.6746	-20.3695	87.9940	0.8896	'Empirical'
5 maxT	315.8333	55.6313	206.4975	425.3900	1.0000	'Empirical'
6 minT	-221.3632	51.8956	-322.9846	-120.2473	0.0001	'Empirical'
7 Sigma2	1.3427e+06	2.1519e+05	9.8949e+05	1.8345e+06	1.0000	'Empirical'

```
SLRMdlFull = fitlm(X,y,'VarNames',T.Properties.VariableNames(2:end))
```

```
SLRMdlFull = 
Linear regression model:
    minT ~ 1 + Energy + PM25 + PM10 + NO2 + maxT

Estimated Coefficients:
                   Estimate      SE       tStat       pValue  
                   ________    ______    ________    __________

    (Intercept)    -1556.8     5290.7    -0.29425       0.76944
    Energy         -5.9438     8.9961    -0.66071       0.51097
    PM25           -23.458     12.639      -1.856      0.067664
    PM10            33.406     29.454      1.1342       0.26059
    NO2             315.44      60.54      5.2104    1.8132e-06
    maxT           -222.42     55.435     -4.0123     0.0001487


Number of observations: 76, Error degrees of freedom: 70
Root Mean Squared Error: 1.23e+03
R-squared: 0.39,  Adjusted R-Squared: 0.347
F-statistic vs. constant model: 8.96, p-value = 1.29e-06
```

```
plot(EstMdl)
```



The plot above is the Posterior distribution for PM25 Lambda = 10000

Evaluating the model FMSE

```
yFBayesLasso = forecast(EstMdl,XF);
fmseBayesLasso = sqrt(mean((yF - yFBayesLasso).^2));
mean(fmseBayesLasso)
```

```
ans = 2.7880e+03
```

Forecasting

Fit a Linear Model for Comparison

```
SLRMdlFull = fitlm(X,y,'VarNames',T.Properties.VariableNames(2:end))
```

```
SLRMdlFull = 
Linear regression model:
    minT ~ 1 + Energy + PM25 + PM10 + NO2 + maxT

Estimated Coefficients:
```

	Estimate	SE	tStat	pValue
(Intercept)	-1556.8	5290.7	-0.29425	0.76944
Energy	-5.9438	8.9961	-0.66071	0.51097
PM25	-23.458	12.639	-1.856	0.067664
PM10	33.406	29.454	1.1342	0.26059
NO2	315.44	60.54	5.2104	1.8132e-06
maxT	-222.42	55.435	-4.0123	0.0001487

```
Number of observations: 76, Error degrees of freedom: 70
Root Mean Squared Error: 1.23e+03
R-squared: 0.39,  Adjusted R-Squared: 0.347
F-statistic vs. constant model: 8.96, p-value = 1.29e-06
```

Results Interpretation:

Increasing the value of the Lasso shrinkage parameter Lambda, reduces the influence of the predictor variable. In the case of PM25:

- when Lambda = 1, FRMSE = 2787
- when Lambda = 100, FRMSE = 2790
- when Lambda = 10000, FRMSE = 2788

These results suggest that PM25 has a minor contribution to the prediction capacity of the model. As we approach turning off this parameter by increasing Lambda to 10000 the RMSE does not change.

Therefore PM25 is an unimportant predictor variable.

g) PM10 Prior Alterationa and effects on Posterior and RMSE

Changing the Lambda for PM25 from 1 to 100 and recalculating posterior

```
PriorMdl.Lambda(3) = 100
```

```
PriorMdl =
  lassoblm with properties:

    NumPredictors: 5
        Intercept: 1
         VarNames: {6×1 cell}
           Lambda: [6×1 double]
                A: 3
                B: 1


           |  Mean     Std           CI95          Positive   Distribution
---------------------------------------------------------------------------
 Intercept |  0       100    [-200.000, 200.000]    0.500   Scale mixture
 PM25      |  0      0.0100    [-0.020,  0.020]     0.500   Scale mixture
 PM10      |  0      0.0100    [-0.020,  0.020]     0.500   Scale mixture
 NO2       |  0        1       [-2.000,  2.000]     0.500   Scale mixture
 maxT      |  0        1       [-2.000,  2.000]     0.500   Scale mixture
 minT      |  0        1       [-2.000,  2.000]     0.500   Scale mixture
 Sigma2    | 0.5000  0.5000    [ 0.138,  1.616]     1.000   IG(3.00,    1)
```

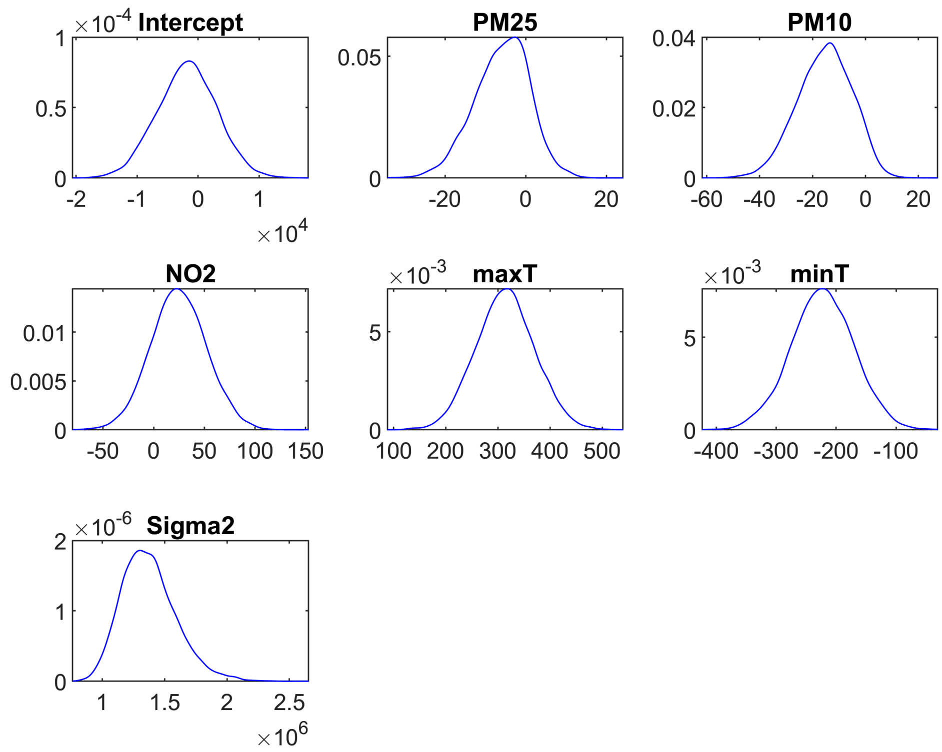
```
[EstMdl,Summary] = estimate(PriorMdl,X,y,'Display',true)
```

```
Method: lasso MCMC sampling with 10000 draws
Number of observations: 76
Number of predictors:   6

           |      Mean              Std                   CI95              Positive  Distribution
--------------------------------------------------------------------------------------------------
 Intercept | -1819.1573      4880.1771       [-11314.392, 7638.331]       0.352     Empirical
 PM25      |     -5.9489        6.8705          [-20.311,   6.502]       0.188     Empirical
 PM10      |    -15.4871       10.3390          [-36.715,   2.806]       0.058     Empirical
```

```
 NO2       |    24.7916        27.4699          [-28.991, 78.964]         0.817     Empirical
 maxT      |   314.2101        56.1702          [205.218, 424.424]        1.000     Empirical
 minT      |  -223.0117        52.4490         [-328.343, -120.366]       0.000     Empirical
 Sigma2    |     1.3752e+06     2.1991e+05  [1007242.185, 1872316.897]    1.000     Empirical

EstMdl = 
  empiricalblm with properties:

    NumPredictors: 5
        Intercept: 1
         VarNames: {6×1 cell}
        BetaDraws: [6×10000 double]
      Sigma2Draws: [1×10000 double]


           |      Mean            Std                 CI95             Positive  Distribution
-----------------------------------------------------------------------------------------------
 Intercept | -1819.1573      4880.1771       [-11314.392, 7638.331]       0.352     Empirical
 PM25      |    -5.9489         6.8705          [-20.311,  6.502]         0.188     Empirical
 PM10      |   -15.4871        10.3390          [-36.715,  2.806]         0.058     Empirical
 NO2       |    24.7916        27.4699          [-28.991, 78.964]         0.817     Empirical
 maxT      |   314.2101        56.1702          [205.218, 424.424]        1.000     Empirical
 minT      |  -223.0117        52.4490         [-328.343, -120.366]       0.000     Empirical
 Sigma2    |     1.3752e+06     2.1991e+05  [1007242.185, 1872316.897]    1.000     Empirical

Summary = 7×6 table
```

	Mean	Std	CI95		Positive	Distribution
1 Intercept	-1.8192e+03	4.8802e+03	-1.1314e+04	7.6383e+03	0.3520	'Empirical'
2 PM25	-5.9489	6.8705	-20.3105	6.5022	0.1878	'Empirical'
3 PM10	-15.4871	10.3390	-36.7152	2.8062	0.0581	'Empirical'
4 NO2	24.7916	27.4699	-28.9912	78.9638	0.8169	'Empirical'
5 maxT	314.2101	56.1702	205.2180	424.4239	1.0000	'Empirical'
6 minT	-223.0117	52.4490	-328.3427	-120.3662	0	'Empirical'
7 Sigma2	1.3752e+06	2.1991e+05	1.0072e+06	1.8723e+06	1.0000	'Empirical'

```
plot(EstMdl)
```



The plot above is the Posterior distribution for PM10 Lambda = 100

Evaluating the model FMSE

```
yFBayesLasso = forecast(EstMdl,XF);
fmseBayesLasso = sqrt(mean((yF - yFBayesLasso).^2));
mean(fmseBayesLasso)
```

```
ans = 2.7612e+03
```

Changing the Lambda for PM25 from 1 to 10000 and recalculating posterior

```
PriorMdl.Lambda(3) = 10000
```

```
PriorMdl =
  lassoblm with properties:

    NumPredictors: 5
        Intercept: 1
         VarNames: {6×1 cell}
           Lambda: [6×1 double]
                A: 3
                B: 1

           |  Mean      Std           CI95          Positive   Distribution
---------------------------------------------------------------------------
 Intercept |  0       100      [-200.000, 200.000]    0.500    Scale mixture
 PM25      |  0        1         [-2.000,   2.000]    0.500    Scale mixture
```

```
 PM10      |  0      0.0001   [-0.000,  0.000]     0.500   Scale mixture
 NO2       |  0       1       [-2.000,  2.000]     0.500   Scale mixture
 maxT      |  0       1       [-2.000,  2.000]     0.500   Scale mixture
 minT      |  0       1       [-2.000,  2.000]     0.500   Scale mixture
 Sigma2    | 0.5000  0.5000   [ 0.138,  1.616]     1.000   IG(3.00,    1)
```

```
[EstMdl,Summary] = estimate(PriorMdl,X,y,'Display',true)
```

```
Method: lasso MCMC sampling with 10000 draws
Number of observations: 76
Number of predictors:   6

           |     Mean          Std                CI95              Positive  Distribution
----------------------------------------------------------------------------------------------
 Intercept | -672.0733      4966.0646      [-10391.819, 9118.265]      0.447     Empirical
 PM25      |  -14.1946         7.4351         [-28.728,  0.546]      0.030     Empirical
 PM10      |   -0.0056         0.1682          [-0.364,  0.337]      0.487     Empirical
 NO2       |   12.4457        25.8196         [-37.996, 63.150]      0.687     Empirical
 maxT      |  300.2241        57.3047         [186.429, 412.692]     1.000     Empirical
 minT      | -221.9849        52.9240        [-325.395, -119.192]    0.000     Empirical
 Sigma2    |    1.4036e+06     2.2660e+05  [1034611.768, 1920640.068]  1.000     Empirical

EstMdl = 
  empiricalblm with properties:

    NumPredictors: 5
        Intercept: 1
         VarNames: {6×1 cell}
        BetaDraws: [6×10000 double]
      Sigma2Draws: [1×10000 double]


           |     Mean          Std                CI95              Positive  Distribution
----------------------------------------------------------------------------------------------
 Intercept | -672.0733      4966.0646      [-10391.819, 9118.265]      0.447     Empirical
 PM25      |  -14.1946         7.4351         [-28.728,  0.546]      0.030     Empirical
 PM10      |   -0.0056         0.1682          [-0.364,  0.337]      0.487     Empirical
 NO2       |   12.4457        25.8196         [-37.996, 63.150]      0.687     Empirical
 maxT      |  300.2241        57.3047         [186.429, 412.692]     1.000     Empirical
 minT      | -221.9849        52.9240        [-325.395, -119.192]    0.000     Empirical
 Sigma2    |    1.4036e+06     2.2660e+05  [1034611.768, 1920640.068]  1.000     Empirical

Summary = 7×6 table
```

...

	Mean	Std	CI95		Positive	Distribution
1 Intercept	-672.0733	4.9661e+03	-1.0392e+04	9.1183e+03	0.4469	'Empirical'
2 PM25	-14.1946	7.4351	-28.7278	0.5461	0.0298	'Empirical'
3 PM10	-0.0056	0.1682	-0.3636	0.3368	0.4872	'Empirical'
4 NO2	12.4457	25.8196	-37.9963	63.1495	0.6868	'Empirical'
5 maxT	300.2241	57.3047	186.4290	412.6916	1.0000	'Empirical'
6 minT	-221.9849	52.9240	-325.3946	-119.1923	0.0001	'Empirical'
7 Sigma2	1.4036e+06	2.2660e+05	1.0346e+06	1.9206e+06	1.0000	'Empirical'

```
SLRMdlFull = fitlm(X,y,'VarNames',T.Properties.VariableNames(2:end))
```

```
SLRMdlFull = 
```

```
Linear regression model:
    minT ~ 1 + Energy + PM25 + PM10 + NO2 + maxT

Estimated Coefficients:
                   Estimate      SE        tStat        pValue
                   ________    ______    ________    _________

    (Intercept)    -1556.8     5290.7    -0.29425       0.76944
    Energy         -5.9438     8.9961    -0.66071       0.51097
    PM25           -23.458     12.639      -1.856      0.067664
    PM10            33.406     29.454      1.1342       0.26059
    NO2             315.44      60.54      5.2104    1.8132e-06
    maxT           -222.42     55.435     -4.0123     0.0001487


Number of observations: 76, Error degrees of freedom: 70
Root Mean Squared Error: 1.23e+03
R-squared: 0.39,  Adjusted R-Squared: 0.347
F-statistic vs. constant model: 8.96, p-value = 1.29e-06
```

```
plot(EstMdl)
```

The plot above is the Posterior distribution for PM10 Lambda = 10000

Evaluating the model FMSE

```
yFBayesLasso = forecast(EstMdl,XF);
fmseBayesLasso = sqrt(mean((yF - yFBayesLasso).^2));
mean(fmseBayesLasso)
```

```
ans = 2.7361e+03
```

Forecasting & Results Interpretation:

Increasing the value of the Lasso shrinkage parameter Lambda, reduces the influence of the predictor variable. In the case of PM10:

- when Lambda = 1, FRMSE = 2787
- when Lambda = 100, FRMSE = 27612
- when Lambda = 10000, FRMSE = 2723

These results suggest that PM10 has a minor contribution to the prediction capacity of the model. As we approach turning off this parameter by increasing Lambda to 10000 the RMSE does not change.

Similarly these results sugges PM10 is an unimportant predictor variable.

e) Model Forecasting Plot

Selecting the PM10 model with lambda = 1 and PM25 model with lambda = 100

```
PriorMdl.Lambda(3) = 1;
PriorMdl.Lambda(1) = 100;
[EstMdl,Summary] = estimate(PriorMdl,X,y,'Display',true);
```

```
Method: lasso MCMC sampling with 10000 draws
```

```
Number of observations: 76
Number of predictors:   6

           |     Mean          Std                CI95           Positive  Distribution
-------------------------------------------------------------------------------------------
 Intercept |    0.0871      16.3853         [-33.088, 34.285]       0.502     Empirical
 PM25      |   -7.0128       7.8197         [-22.375,  8.084]       0.182     Empirical
 PM10      |  -22.9849      11.8176         [-46.186,  0.135]       0.026     Empirical
 NO2       |   29.8274      24.9835         [-17.871, 79.596]       0.885     Empirical
 maxT      |  299.7063      37.4389         [226.176, 374.317]      1.000     Empirical
 minT      | -222.7302      50.6031        [-321.986, -123.126]     0.000     Empirical
 Sigma2    |    1.3438e+06   2.1964e+05  [982034.146, 1843525.581]  1.000     Empirical
```

```
yFBayesLasso = forecast(EstMdl,XF);
plot(yFBayesLasso,'color','red');
hold on
plot(yF,'color','blue')
```

extra) Hyperparameter Sweep and Best Model Selection

The previous approach only considered the fluctuation of PM10 and PM25, the Lasso method can allow us to adjust the shrinkage valua automatically to turn of predictor variables that do not have an impact on the model's predictive power. This section considers a Hyprparameter sweep and selects the best model

```
% Fit a lasso regression model to the data.
[LassoBetaEstimates,FitInfo] = lasso(X,y,'PredictorNames',predictornames);

% Compute the FMSE of each model returned by lasso.
yFLasso = FitInfo.Intercept + XF*LassoBetaEstimates;
fmseLasso = sqrt(mean((yF - yFLasso).^2,1)); fmseLasso(1)
```

```
ans = 2.7914e+03
```

```
% Plot the magnitude of the regression coefficients with respect to the shrinkage value.
hax = lassoPlot(LassoBetaEstimates,FitInfo);
L1Vals = hax.Children.XData;
yyaxis right
h = plot(L1Vals,fmseLasso,'LineWidth',2,'LineStyle','--');
legend(h,'FMSE','Location','SW');
ylabel('FMSE');
title('Frequentist Lasso')
```

```
%% Determine the Best Lasso Model -- the plot above suggests the best results (FMSE around 2720
fmsebestlasso = min(fmseLasso(FitInfo.DF == 1));
idx = fmseLasso == fmsebestlasso;
bestLasso = [FitInfo.Intercept(idx); LassoBetaEstimates(:,idx)];
```

```
table(bestLasso,'RowNames',["Intercept" predictornames])
```

```
ans = 6×1 table
```

	bestLasso
1 Intercept	1.0491e+04
2 PM25	0
3 PM10	0
4 NO2	0
5 maxT	3.7000e-14
6 minT	0

The frequentist lasso analysis suggests that the variables maxT predictor has the strongest predictive power.

To see how the other variables combare let us compute the shrinkage parameter for each of the 5:

```
fmsebestlasso = min(fmseLasso(FitInfo.DF == 5));
idx = fmseLasso == fmsebestlasso;
bestLasso = [FitInfo.Intercept(idx); LassoBetaEstimates(:,idx)];
table(bestLasso,'RowNames',["Intercept" predictornames])
```

```
ans = 6×1 table
```

	bestLasso
1 Intercept	2.4510e+03
2 PM25	-6.6516
3 PM10	-10.0156
4 NO2	2.6471
5 maxT	220.4625
6 minT	-161.6840

This suggest minT has the strongest predictive power after maxT (negatively correlated). Then PM25 and PM10 are either insignificant or redundant, compared to them PM10 has a stronger predictive power.