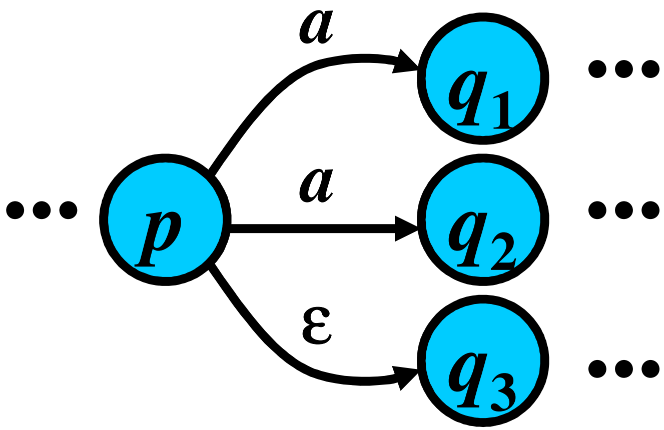
Part IV. Variants of Finite Automata

Theory vs. Practice

a) Configuration: pax



Next Configuration:

q_1x or q_2x or q_3ax ?

Theory: ☺ × Practice: ☹

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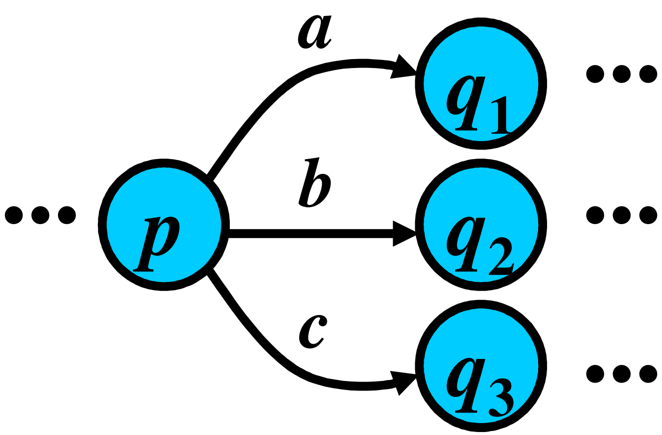
b) Configuration: pax



Next Configuration:

only q_1x

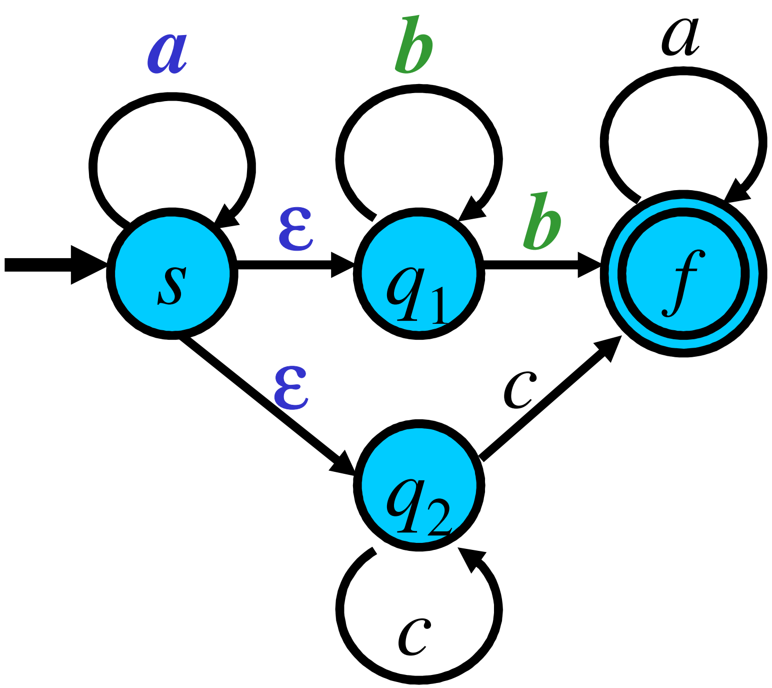
Theory: ☹ × Practice: ☺

Use of FA in General

Simulation of all possible moves from every configuration.

Example:

FA M is defined as:



Question: $ab \in L(M)$?

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Simulation of all possible moves from every configuration.

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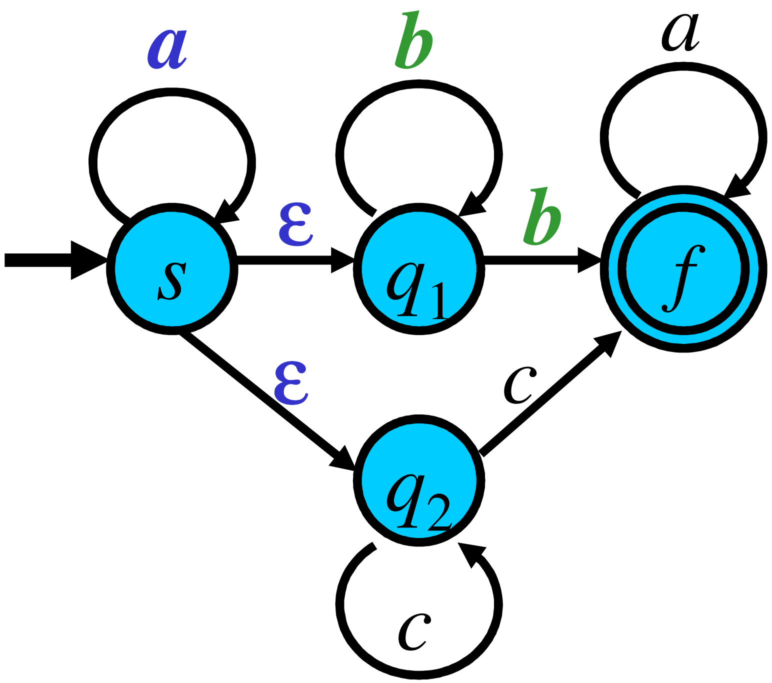

sab

Question: $ab \in L(M)$ **?**

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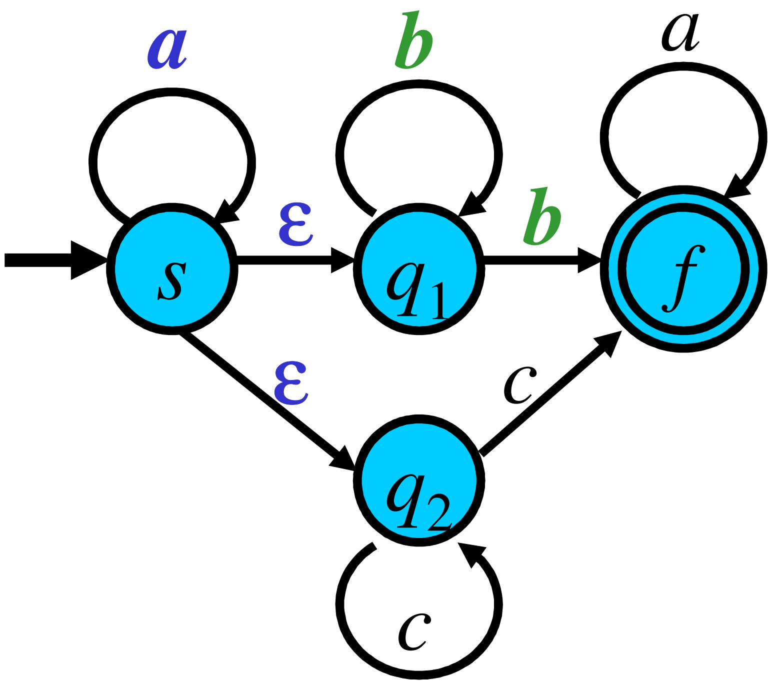

$$sab \begin{cases} \vdash q_1ab \\ \vdash q_2ab \\ \vdash sb \end{cases}$$

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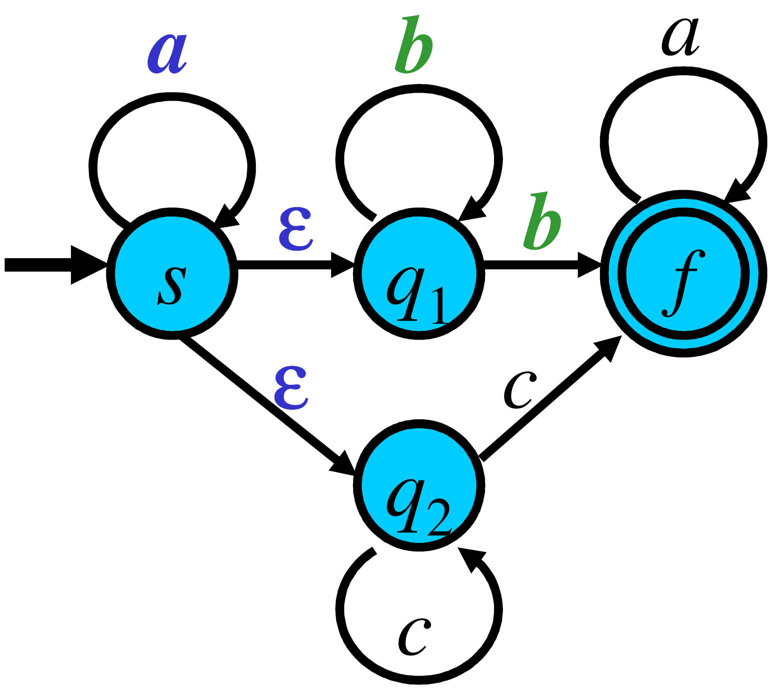

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No next configuration

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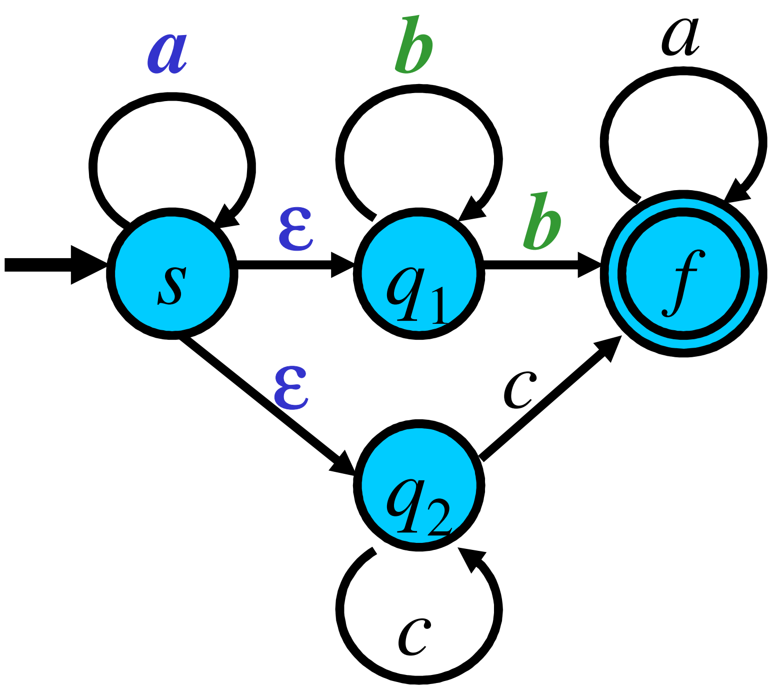

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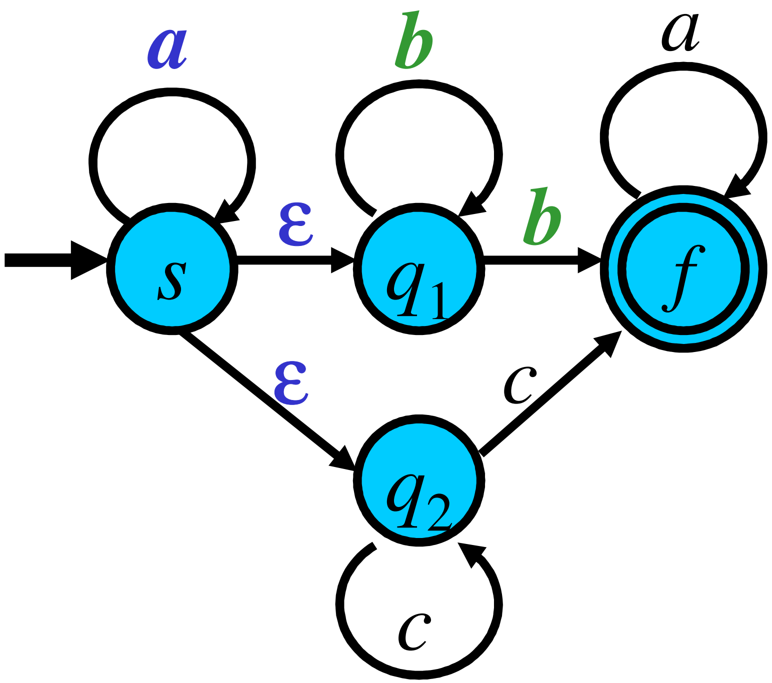

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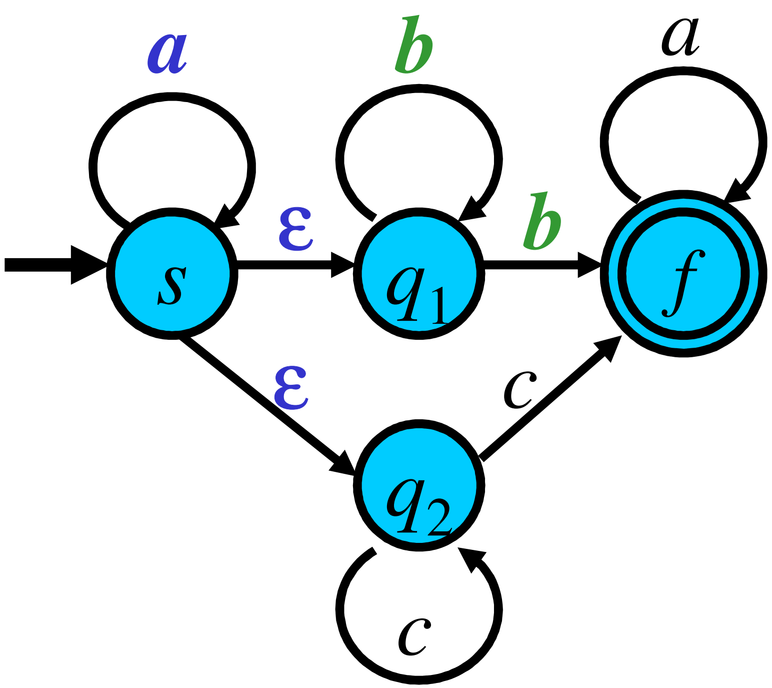

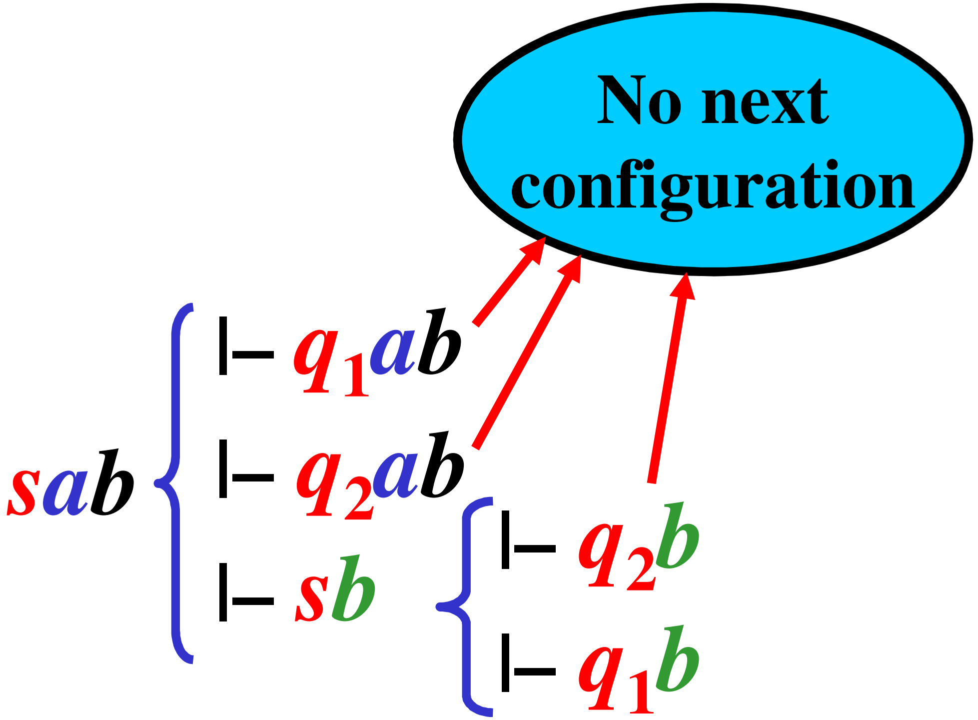

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Use of FA in General

Simulation of all possible moves from every configuration.

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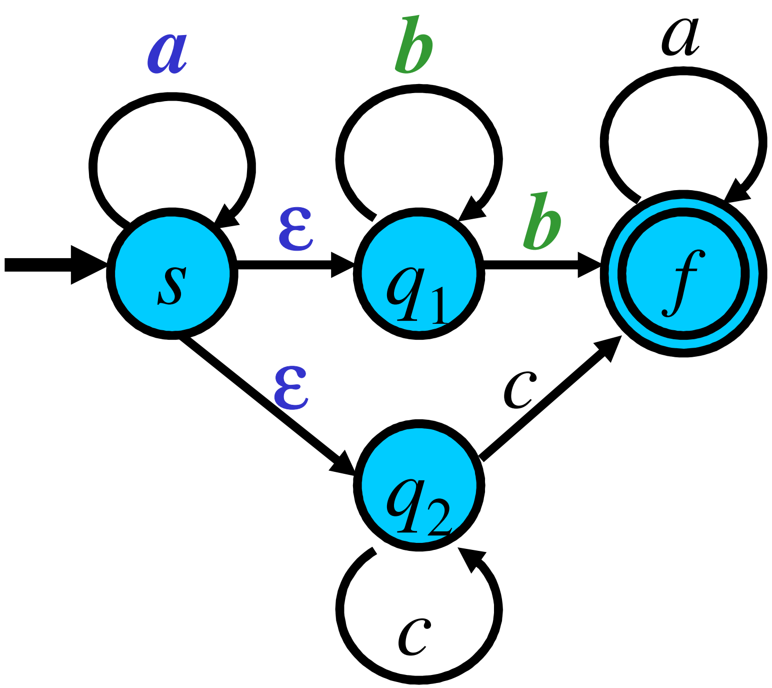

No next configuration

$$sab \begin{cases} \vdash q_1ab \\ \vdash q_2ab \\ \vdash sb \begin{cases} \vdash q_2b \\ \vdash q_1b \begin{cases} \vdash q_1 \\ \vdash f \end{cases} \end{cases} \end{cases}$$

Question: $ab \in L(M)$?

Use of FA in General

Simulation of all possible moves from every configuration.

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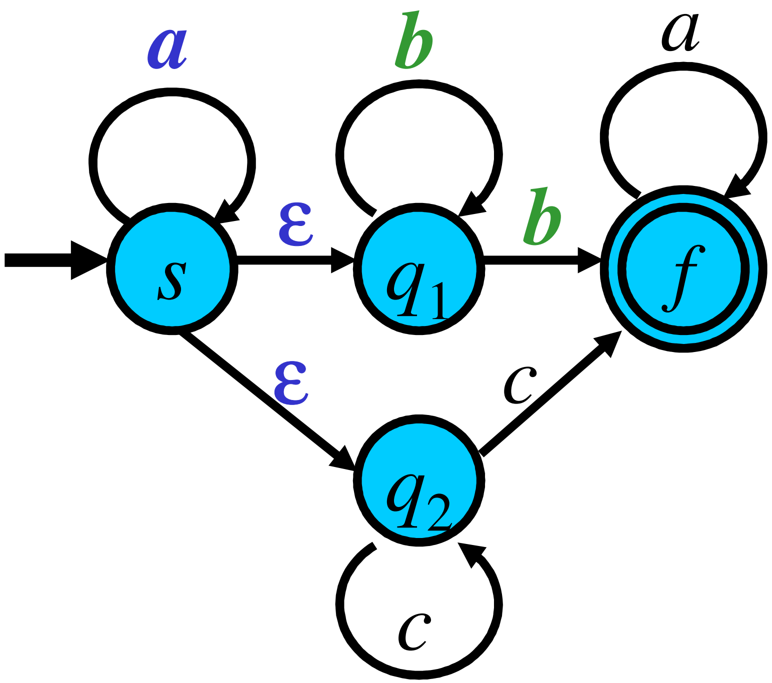

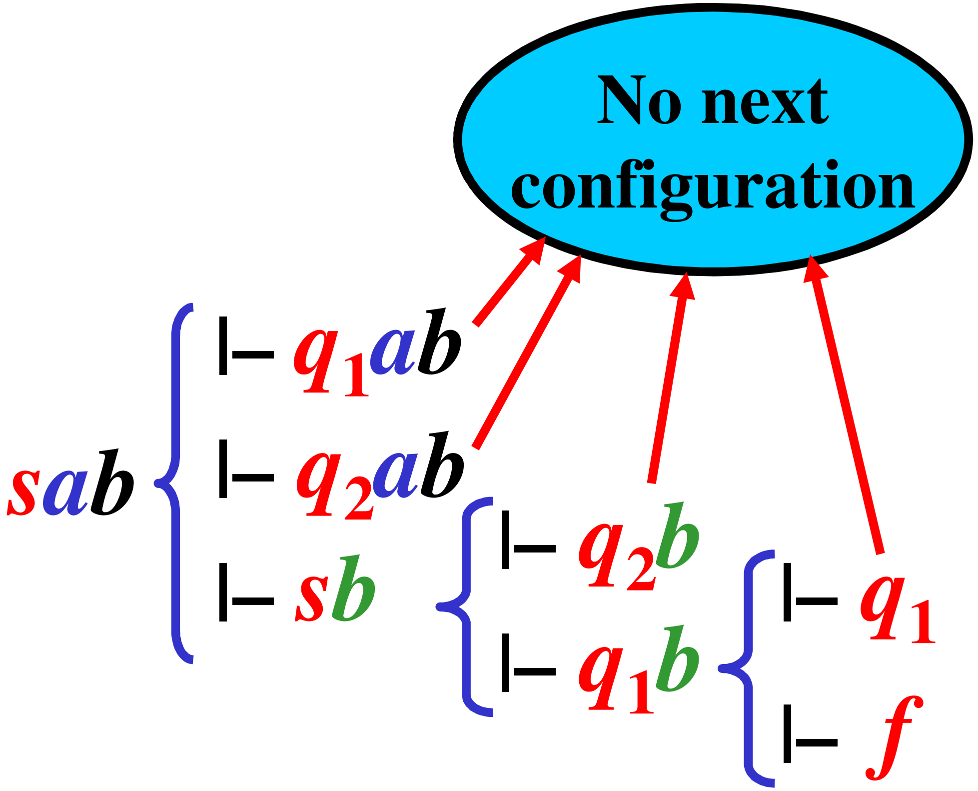

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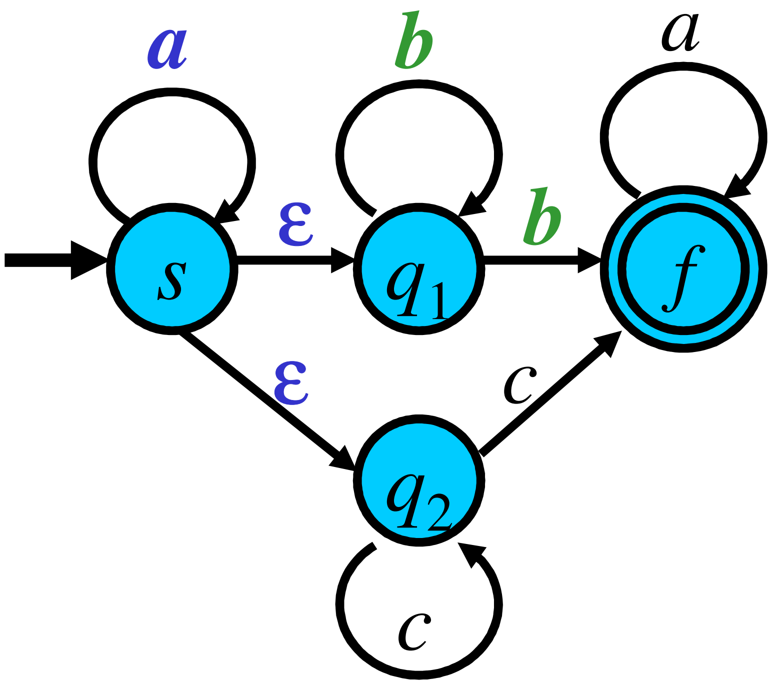
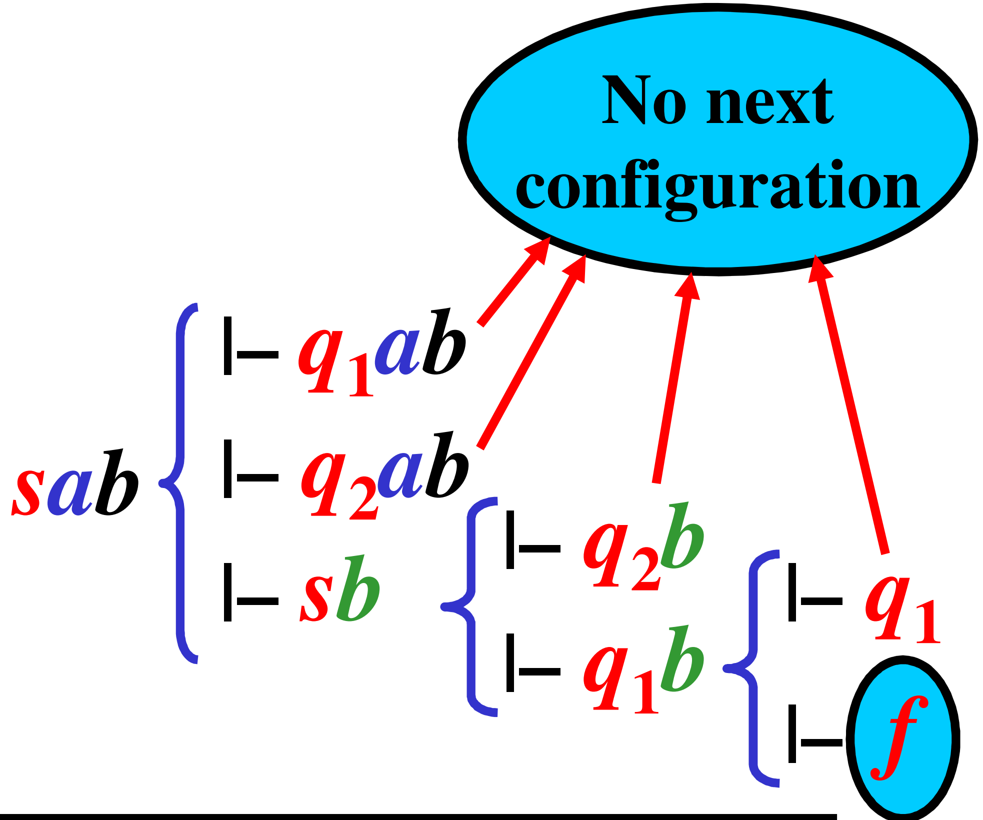




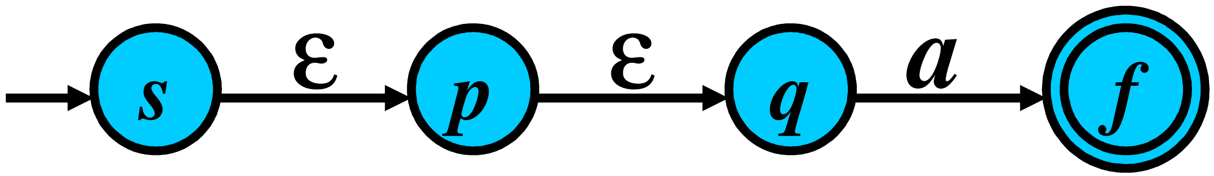
Question: $ab \in L(M)$?

Answer: YES, $ab \in L(M)$ because $f \in F$.

From FA to DFA in Essence 1/2

Preference in practice: *Determinictic FA* (DFA) that makes no more than one move from every configuration.

1) Gist: Removal of ε-moves



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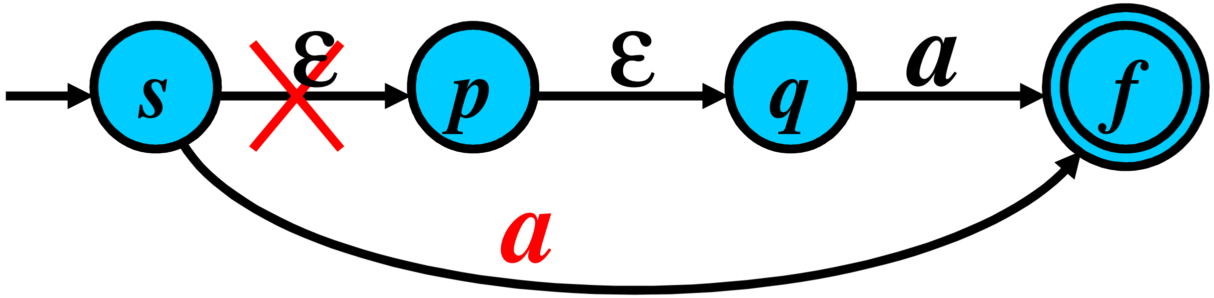

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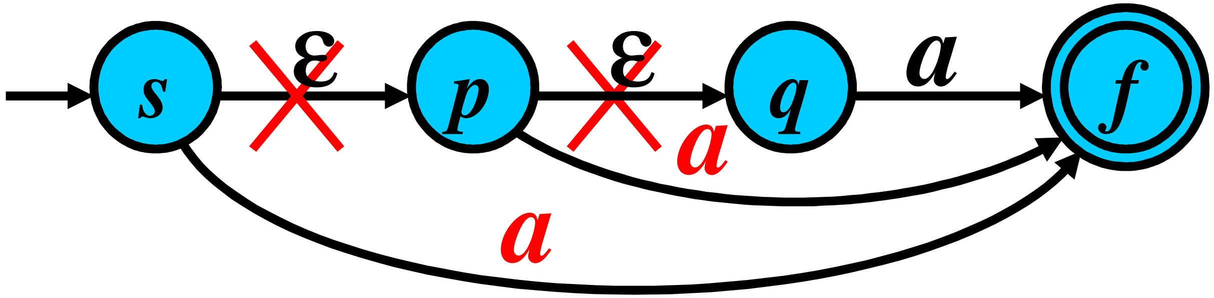

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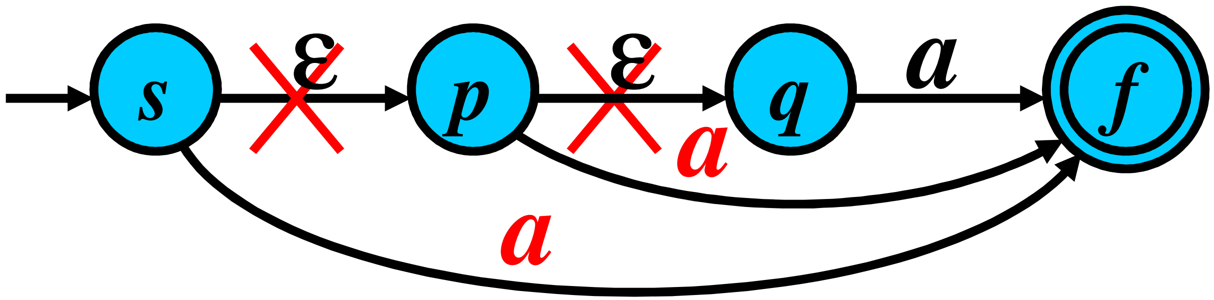
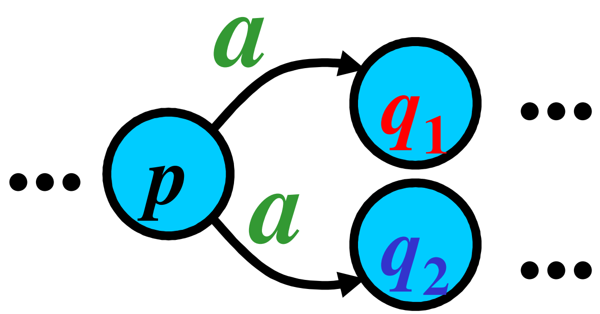


Definition: Let $M = (Q, \Sigma, R, s, F)$ be a FA. M is an ***ε-free finite automaton*** if for all rules $pa \rightarrow q \in R$, where $p, q \in Q$, holds

$$a \in \Sigma \; (a \neq \varepsilon)$$

From FA to DFA in Essence 2/2

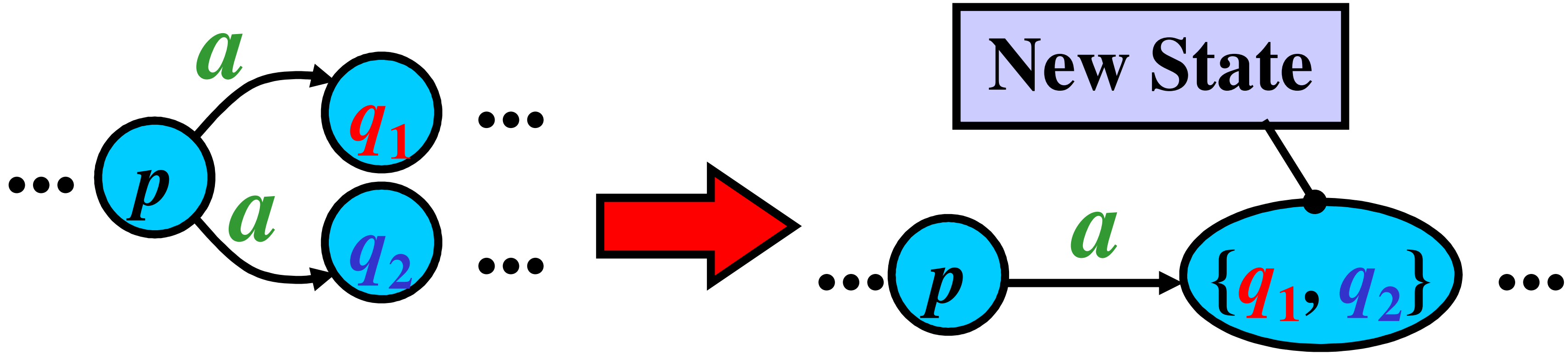
2) Gist: Removal of nodeterminism



Definition: Let $M = (Q, \Sigma, R, s, F)$ be an **ε-free FA**. M is a ***deterministic finite automaton*** (DFA) if for each rule $pa \rightarrow q \in R$ it holds that $R - \{pa \rightarrow q\}$ contains no rule with the left-hand side equal to pa.

From FA to DFA in Essence 2/2

2) Gist: Removal of nodeterminism



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Theorem

• For every FA M, there is an equivalent DFA M_d.

Proof is based on these conversions:

Finite Automaton M

↓

ε-free Finite Automaton M' — $L(M) = L(M')$

↓

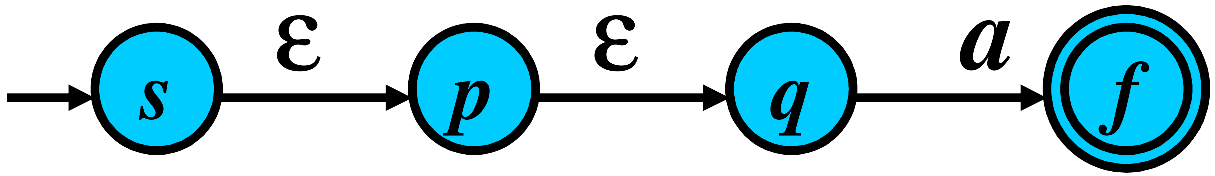
DFA M_d — $L(M') = L(M_d)$

ε-closure

Gist: *q* is in ε-closure(*p*) if FA can reach *q* from *p* without reading.

Definition: For every states $p \in Q$, we define a set $\varepsilon\text{-}closure(p)$ as ε-closure(p) = $\{q: q \in Q, p \vdash^* q\}$

Example:



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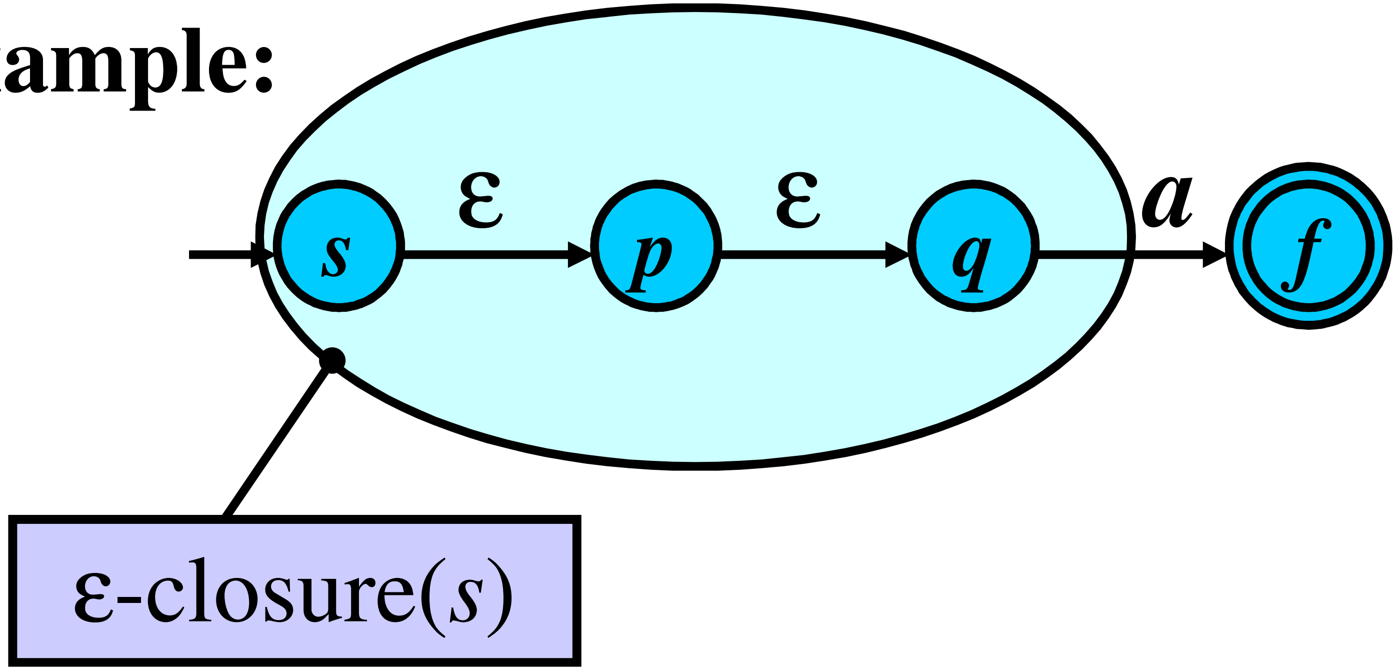

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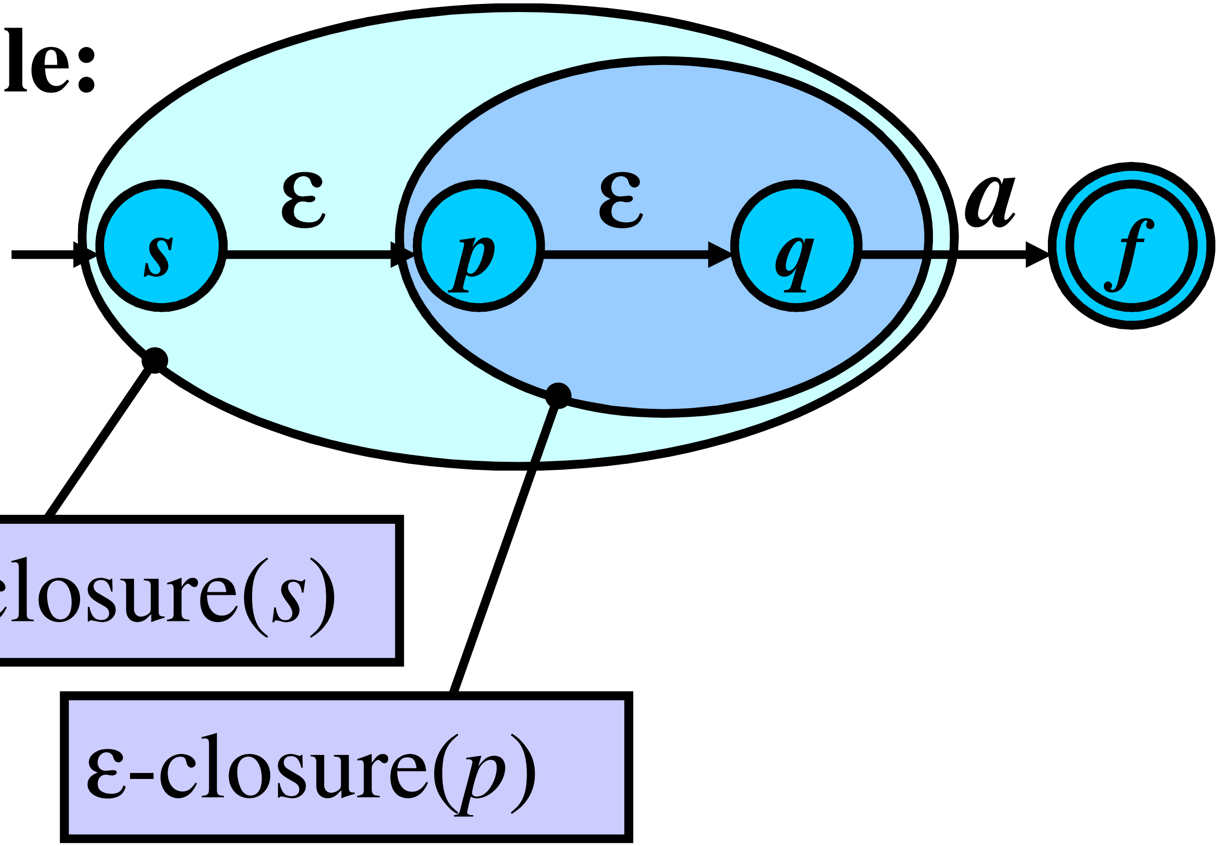

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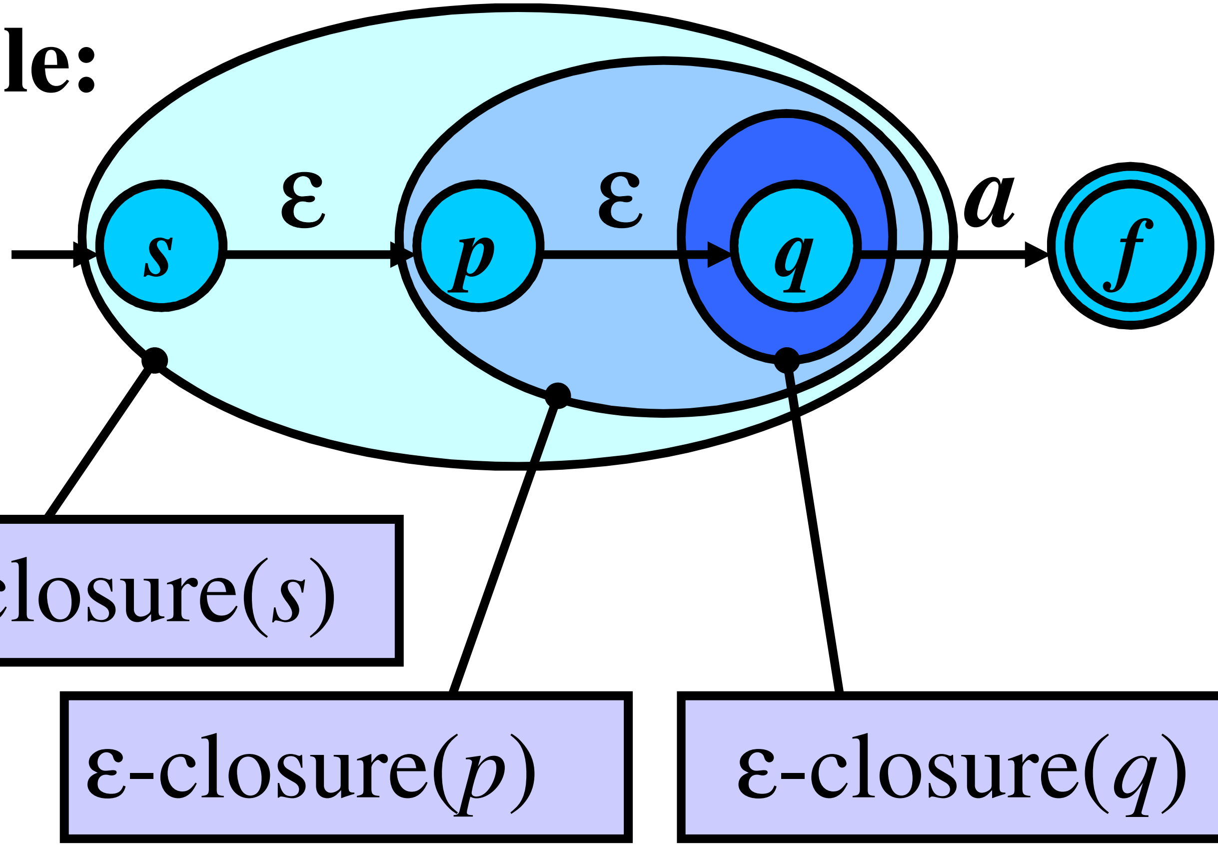

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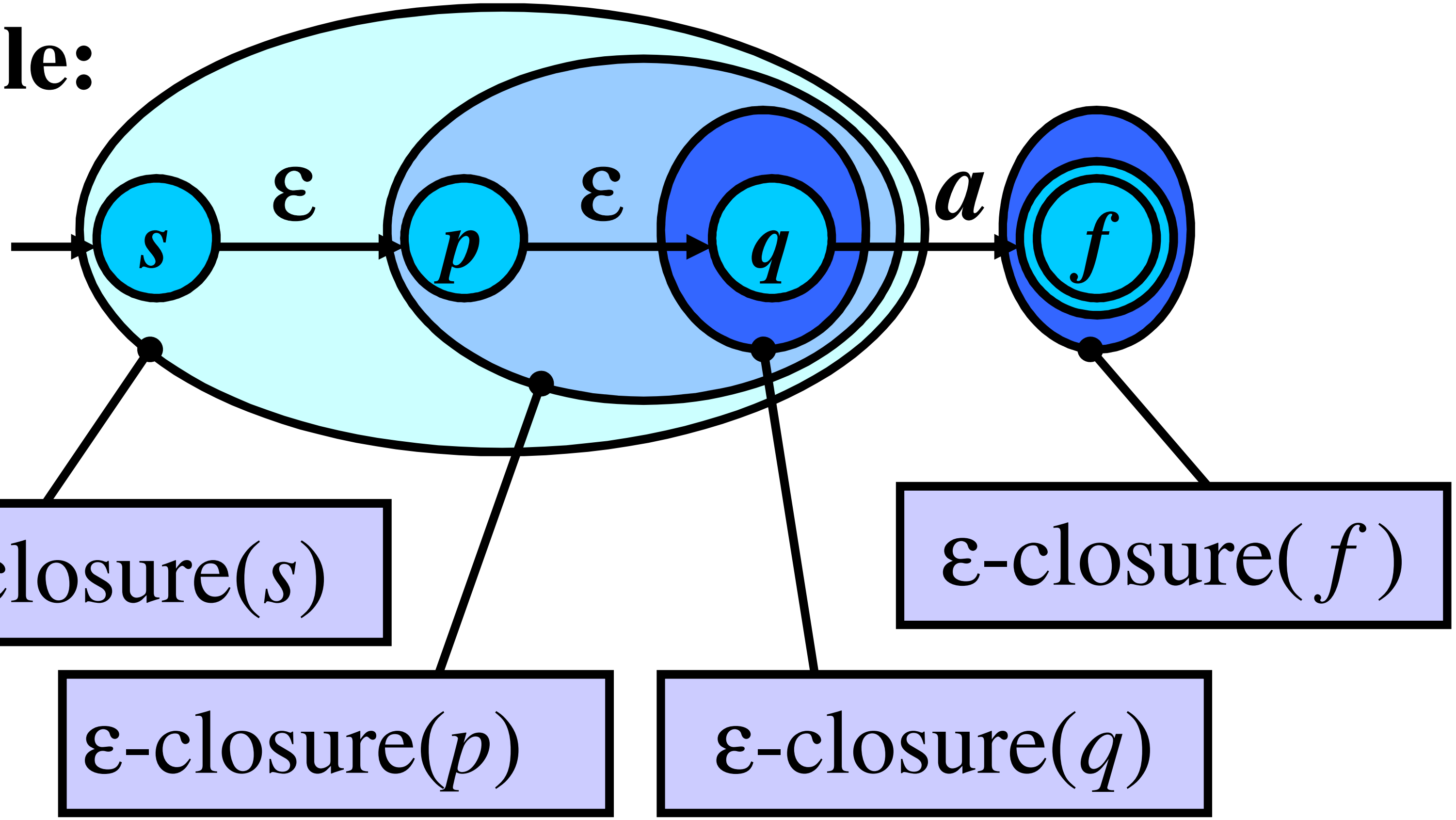

Algorithm: ε-closure

- **Input:** $M = (Q, \Sigma, R, s, F)$; $p \in Q$
- **Output:** ε-closure(p)

- **Method:**
- $i := 0$; $Q_0 := \{p\}$;
- **repeat**

  $i := i + 1$;

  $Q_i := Q_{i-1} \cup \{\, p' : p' \in Q,\ q \rightarrow p' \in R,$
  $q \in Q_{i-1} \,\}$;

until $Q_i = Q_{i-1}$;

- ε-closure(p) $:= Q_i$.

ε-closure: Example

$M = (Q, \Sigma, R, s, F)$, where: $Q = \{s, p, q, f\}$, $\Sigma = \{a\}$,
$R = \{s \rightarrow p, p \rightarrow q, qa \rightarrow f\}$, $F = \{f\}$

Task: ε-closure(s)

ε-closure: Example

$M = (Q, \Sigma, R, s, F)$, where: $Q = \{s, p, q, f\}$, $\Sigma = \{a\}$,
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Task: ε-closure(s)

$Q_0 = \{s\}$

ε-closure: Example

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Task: ε-closure(s)

$Q_0 = \{\boldsymbol{s}\}$

---

1) $\boldsymbol{s} \rightarrow p'; p' \in Q$: $\boldsymbol{s} \rightarrow \boldsymbol{p}$

$Q_1 = \{\boldsymbol{s}\} \cup \{\boldsymbol{p}\} = \{\boldsymbol{s}, \boldsymbol{p}\}$

ε-closure: Example

$M = (Q, \Sigma, R, s, F)$, where: $Q = \{s, p, q, f\}$, $\Sigma = \{a\}$,
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$\boldsymbol{p} \to p'; p' \in Q$: $\boldsymbol{p} \to \boldsymbol{q}$

$Q_2 = \{\boldsymbol{s}, \boldsymbol{p}\} \cup \{\boldsymbol{p}, \boldsymbol{q}\} = \{\boldsymbol{s}, \boldsymbol{p}, \boldsymbol{q}\}$

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---

3) $\mathbf{s} \rightarrow p'; p' \in Q$: $\mathbf{s} \rightarrow \mathbf{p}$

$\mathbf{p} \rightarrow p'; p' \in Q$: $\mathbf{p} \rightarrow \mathbf{q}$

$\mathbf{q} \rightarrow p'; p' \in Q$: **none**

$Q_3 = \{\mathbf{s}, \mathbf{p}, \mathbf{q}\} \cup \{\mathbf{p}, \mathbf{q}\} = \{\mathbf{s}, \mathbf{p}, \mathbf{q}\} = Q_2 =$ **ε-closure(s)**

Algorithm: FA to ε-free FA

Gist: Skip all ε-moves

- **Input:** FA $M = (Q, \Sigma, R, s, F)$
- **Output:** ε-free FA $M' = (Q, \Sigma, R', s, F')$

- **Method:**
- $R' := \emptyset$;
- **for all** $p \in Q$ **do**

$$R' := R' \cup \{ pa \rightarrow q: p'a \rightarrow q \in R, a \in \Sigma, p' \in \varepsilon\text{-closure}(p), q \in Q \};$$

- $F' := \{ p: p \in Q, \varepsilon\text{-closure}(p) \cap F \neq \emptyset \}$.

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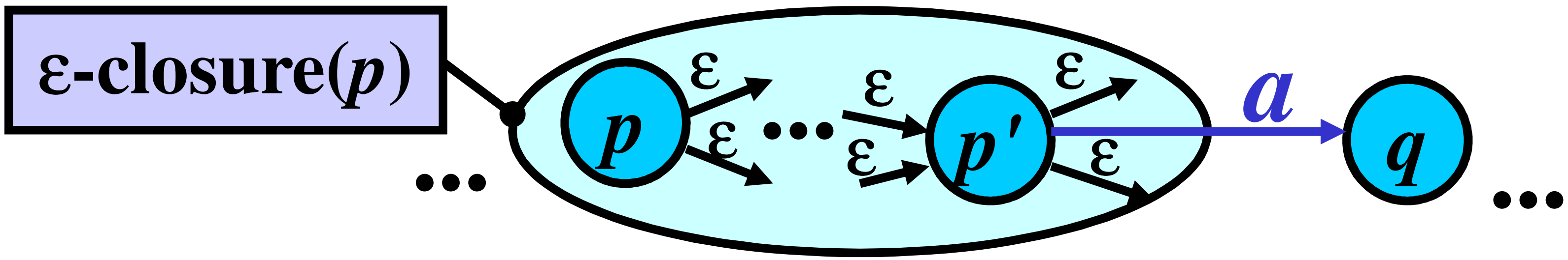

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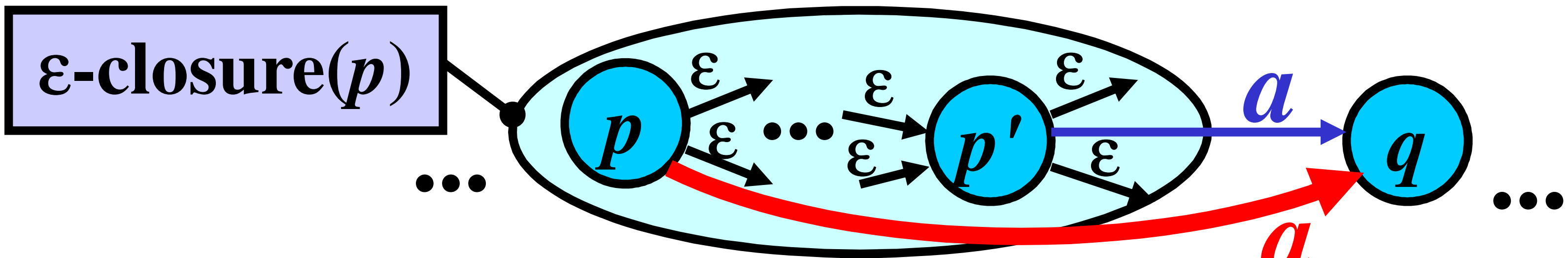

FA to ε-free FA: Example 1/3

$M = (Q, \Sigma, R, s, F)$, where:

$Q = \{s, q_1, q_2, f\}$; $\Sigma = \{a, b, c\}$;

$R = \{sa \rightarrow s, s \rightarrow q_1, q_1b \rightarrow q_1, q_1b \rightarrow f, s \rightarrow q_2,$
$q_2c \rightarrow q_2, q_2c \rightarrow f, fa \rightarrow f\}$; $F = \{f\}$

FA to ε-free FA: Example 1/3

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---

1) for $p = s$: ε-closure(s) = $\{s, q_1, q_2\}$

A. $sd \rightarrow q',\ d \in \Sigma,\ q' \in Q$: $sa \rightarrow s$

B. $q_1d \rightarrow q',\ d \in \Sigma,\ q' \in Q$: $q_1b \rightarrow q_1,\ q_1b \rightarrow f$

C. $q_2d \rightarrow q',\ d \in \Sigma,\ q' \in Q$: $q_2c \rightarrow q_2,\ q_2c \rightarrow f$

$R' = \emptyset \cup \{sa \rightarrow s,\ sb \rightarrow q_1,\ sb \rightarrow f,\ sc \rightarrow q_2,\ sc \rightarrow f\}$

FA to ε-free FA: Example 2/3

FA to ε-free FA: Example 2/3

2) for $p = q_1$: ε-closure(q_1) = $\{q_1\}$

A. $q_1d \rightarrow q'; d \in \Sigma; q' \in Q$: $q_1b \rightarrow q_1, q_1b \rightarrow f$

$R' = R' \cup \{q_1b \rightarrow q_1, q_1b \rightarrow f\}$

FA to ε-free FA: Example 2/3

2) for $p = q_1$: ε-closure(q_1) = $\{q_1\}$

A. $q_1 d \rightarrow q'; d \in \Sigma; q' \in Q$: $q_1 b \rightarrow q_1, q_1 b \rightarrow f$

$R' = R' \cup \{q_1 b \rightarrow q_1, q_1 b \rightarrow f\}$

---

3) for $p = q_2$: ε-closure(q_2) = $\{q_2\}$

A. $q_2 d \rightarrow q'; d \in \Sigma; q' \in Q$: $q_2 c \rightarrow q_2, q_2 c \rightarrow f$

$R' = R' \cup \{q_2 c \rightarrow q_2, q_2 c \rightarrow f\}$

FA to ε-free FA: Example 2/3

2) for $p = q_1$: ε-closure(q_1) = $\{q_1\}$

A. $q_1 d \rightarrow q'; d \in \Sigma; q' \in Q$: $q_1 b \rightarrow q_1$, $q_1 b \rightarrow f$

$R' = R' \cup \{q_1 b \rightarrow q_1, q_1 b \rightarrow f\}$

---

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A. $q_2 d \rightarrow q'; d \in \Sigma; q' \in Q$: $q_2 c \rightarrow q_2$, $q_2 c \rightarrow f$

$R' = R' \cup \{q_2 c \rightarrow q_2, q_2 c \rightarrow f\}$

---

4) for $p = f$: ε-closure(f) = $\{f\}$

A. $fd \rightarrow q'; d \in \Sigma; q' \in Q$: $fa \rightarrow f$

$R' = R' \cup \{fa \rightarrow f\}$

FA to ε-free FA: Example 2/3

2) for $p = q_1$: ε-closure(q_1) = $\{q_1\}$

A. $q_1 d \to q'$; $d \in \Sigma$; $q' \in Q$: $q_1 b \to q_1$, $q_1 b \to f$

$R' = R' \cup \{q_1 b \to q_1, q_1 b \to f\}$

---

3) for $p = q_2$: ε-closure(q_2) = $\{q_2\}$

A. $q_2 d \to q'$; $d \in \Sigma$; $q' \in Q$: $q_2 c \to q_2$, $q_2 c \to f$

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---

4) for $p = f$: ε-closure(f) = $\{f\}$

A. $fd \to q'$; $d \in \Sigma$; $q' \in Q$: $fa \to f$

$R' = R' \cup \{fa \to f\}$

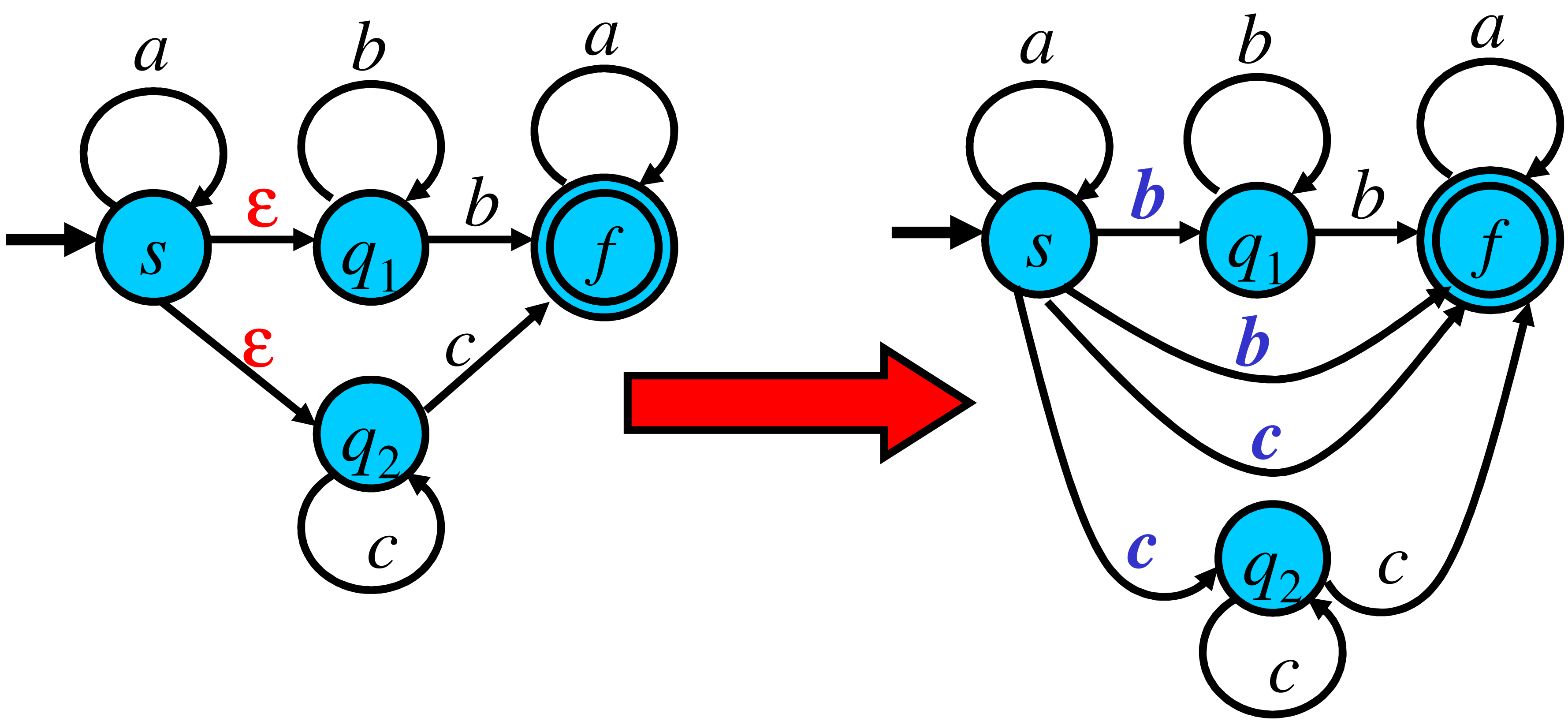
$R' = \{sa \to s, sb \to q_1, sb \to f, sc \to q_2, sc \to f,$
$q_1 b \to q_1, q_1 b \to f, q_2 c \to q_2, q_2 c \to f, fa \to f\}$

FA to ε-free FA: Example 3/3

$$\left.\begin{array}{l}
\varepsilon\text{-closure}(s) \cap F = \{s, q_1, q_2\} \cap \{f\} = \emptyset \\
\varepsilon\text{-closure}(q_1) \cap F = \{q_1\} \cap \{f\} = \emptyset \\
\varepsilon\text{-closure}(q_2) \cap F = \{q_2\} \cap \{f\} = \emptyset \\
\varepsilon\text{-closure}(f) \cap F = \{f\} \cap \{f\} = \{f\} \neq \emptyset
\end{array}\right\} \boldsymbol{F'} = \{f\}$$

FA to ε-free FA: Example 3/3

$$\left.\begin{array}{l}
\varepsilon\text{-closure}(s) \cap F = \{s, q_1, q_2\} \cap \{f\} = \emptyset \\
\varepsilon\text{-closure}(q_1) \cap F = \{q_1\} \cap \{f\} = \emptyset \\
\varepsilon\text{-closure}(q_2) \cap F = \{q_2\} \cap \{f\} = \emptyset \\
\varepsilon\text{-closure}(f) \cap F = \{f\} \cap \{f\} = \{f\} \neq \emptyset
\end{array}\right\} \boldsymbol{F'} = \{f\}$$



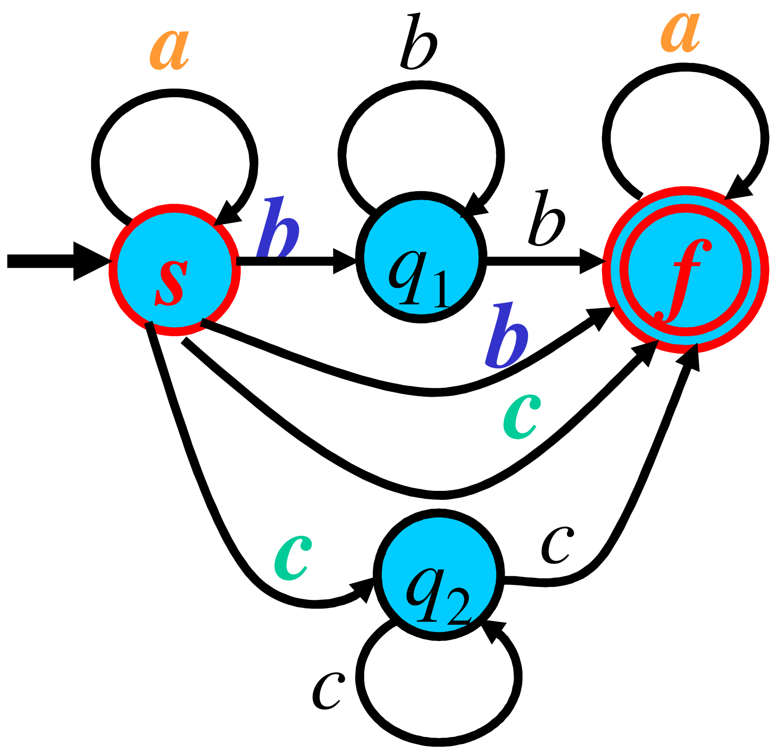
Algorithm: ε-free FA to DFA 1/2

Gist: In DFA, make states from all subsets of states in ε-free FA and move between them so that all possible states of ε-free FA are <u>simultaneously</u> simulated.

Algorithm: ε-free FA to DFA 1/2

Gist: In DFA, make states from all subsets of states in ε-free FA and move between them so that all possible states of ε-free FA are simultaneously simulated.

Illustration:



$Q_{DFA} = \{\{s\}, \{q_1\}, \{q_2\}, \{f\}, \{s,q_1\}, \{s,q_2\}, \{s, f\}, \{q_1,q_2\}, \{q_1, f\}, \{q_2, f\}, \{s,q_1,q_2\}, \{s,q_1, f\}, \{s,q_2, f\}, \{q_1,q_2, f\}, \{s,q_1,q_2, f\}\}$

Algorithm: ε-free FA to DFA 1/2

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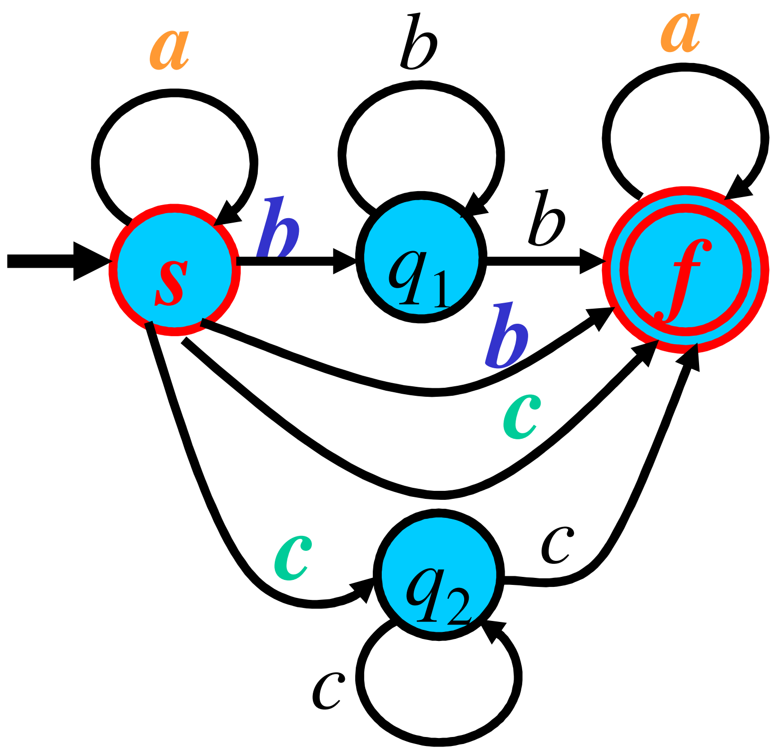
Illustration:



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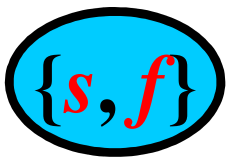
For state $\{s\}$: …

⋮

For state $\{s, f\}$: 

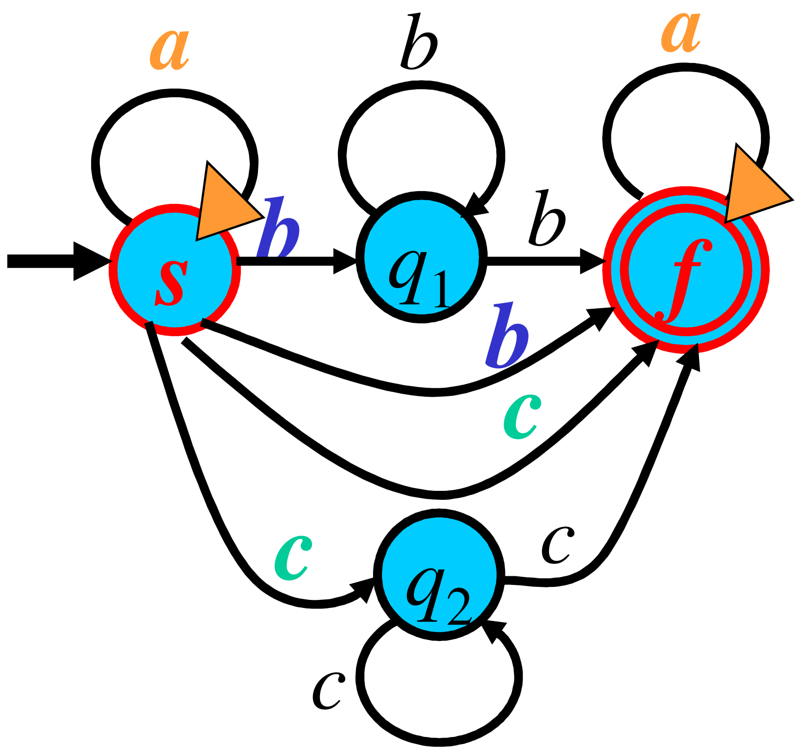
⋮

For state $\{s,q_1,q_2,f\}$: …

Algorithm: ε-free FA to DFA 1/2

Gist: In DFA, make states from all subsets of states in ε-free FA and move between them so that all possible states of ε-free FA are <u>simultaneously</u> simulated.

Illustration:

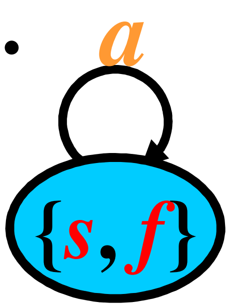


$Q_{DFA} = \{\{s\}, \{q_1\}, \{q_2\}, \{f\}, \{s,q_1\}, \{s,q_2\}, \{s,f\}, \{q_1,q_2\}, \{q_1,f\}, \{q_2,f\}, \{s,q_1,q_2\}, \{s,q_1,f\}, \{s,q_2,f\}, \{q_1,q_2,f\}, \{s,q_1,q_2,f\}\}$

For state $\{s\}$: …

⋮

For state $\{s, f\}$:



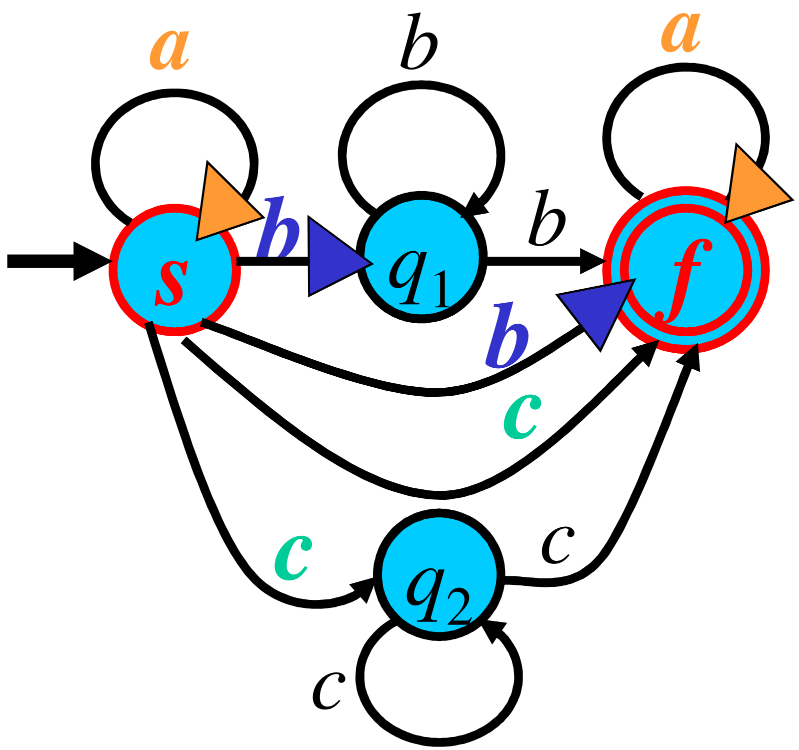
⋮

For state $\{s,q_1,q_2,f\}$: …

Algorithm: ε-free FA to DFA 1/2

Gist: **In DFA, make states from all subsets of states in ε-free FA and move between them so that all possible states of ε-free FA are <u>simultaneously</u> simulated.**

Illustration:



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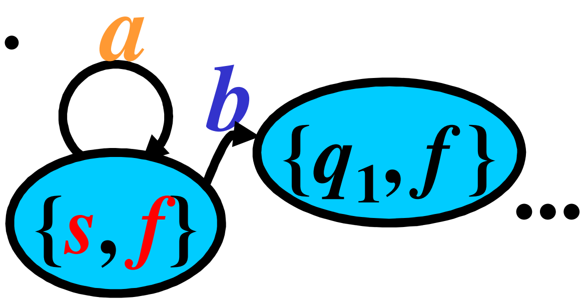
For state {*s*}: …

⋮

For state {*s*, *f*}:



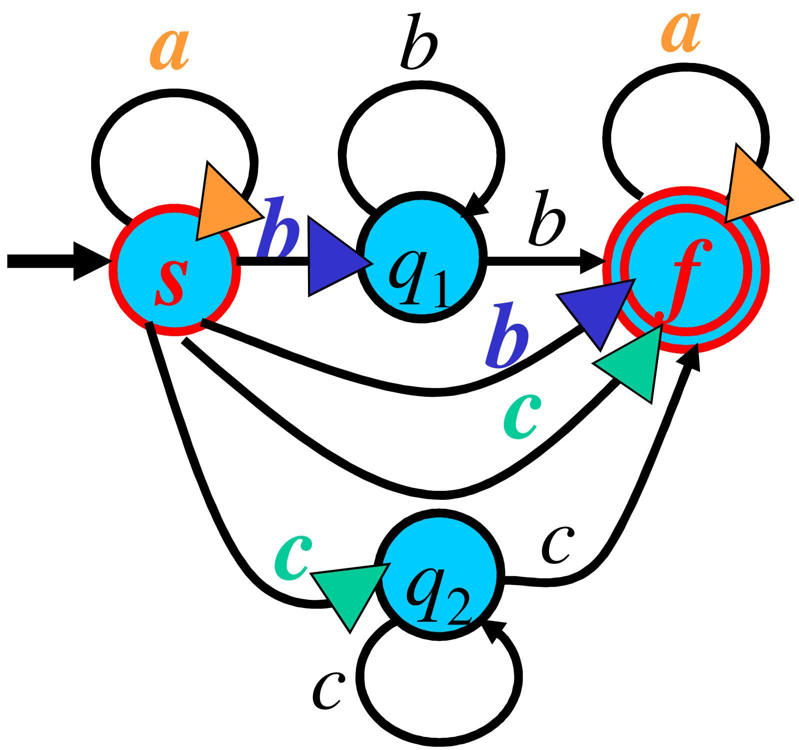
⋮

For state {*s*,*q*₁,*q*₂,*f*}: …

Algorithm: ε-free FA to DFA 1/2

Gist: In DFA, make states from all subsets of states in ε-free FA and move between them so that all possible states of ε-free FA are <u>simultaneously</u> simulated.

Illustration:



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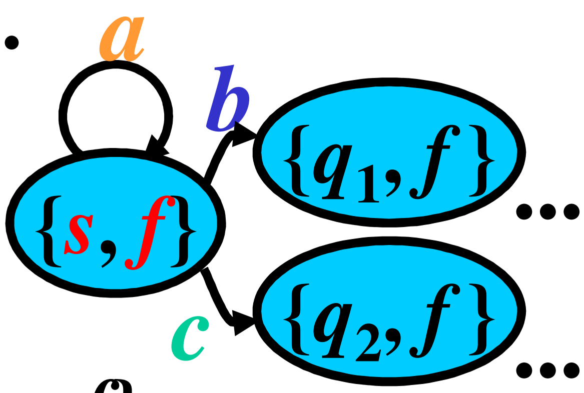
For state $\{s\}$: …

⋮

For state $\{s, f\}$:



⋮

For state $\{s,q_1,q_2,f\}$: …

Algorithm: ε-free FA to DFA 2/2

- **Input:** ε-free FA: $M = (Q, \Sigma, R, s, F)$
- **Output:** DFA: $M_d = (Q_d, \Sigma, R_d, s_d, F_d)$

- **Method:**
- $Q_d := \{Q' : Q' \subseteq Q, Q' \neq \emptyset\}$; $R_d := \emptyset$;
- **for each** $Q' \in Q_d$, **and** $a \in \Sigma$ **do begin**

  $Q'' := \{q : p \in Q', pa \rightarrow q \in R\}$;

  if $Q'' \neq \emptyset$ **then** $R_d := R_d \cup \{Q'a \rightarrow Q''\}$;

  end
- $s_d := \{s\}$;
- $F_d := \{F' : F' \in Q_d, F' \cap F \neq \emptyset\}$.

ε-free FA to DFA: Example 1/5

$M = (Q, \Sigma, R, s, F)$, where:

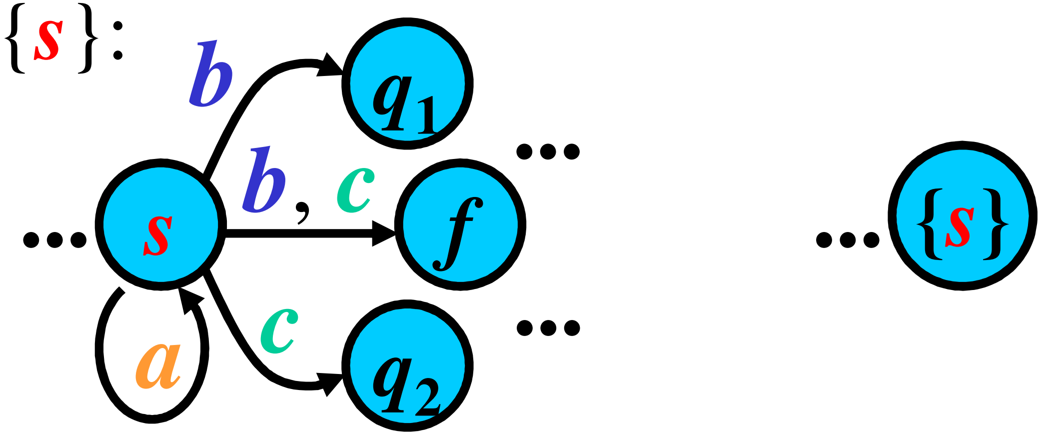
$Q = \{s, q_1, q_2, f\}$; $\Sigma = \{a, b, c\}$; $F = \{f\}$

$R = \{sa \rightarrow s, sb \rightarrow q_1, sb \rightarrow f, sc \rightarrow q_2, sc \rightarrow f,$
$q_1b \rightarrow q_1, q_1b \rightarrow f, q_2c \rightarrow q_2, q_2c \rightarrow f, fa \rightarrow f\}$;

$Q_d = \{\{s\}, \{s,q_1\}, \{s,q_1,q_2\}, \{s,q_1,f\}, \{s,q_1,q_2,f\}, \{s,q_2\}, \{s,q_2,f\},$
$\{s,f\}, \{q_1\}, \{q_1,q_2\}, \{q_1,f\}, \{q_1,q_2,f\}, \{q_2\}, \{q_2,f\}, \{f\}\}$

---

for $Q' = \{s\}$:



ε-free FA to DFA: Example 1/5

$M = (Q, \Sigma, R, s, F)$, where:

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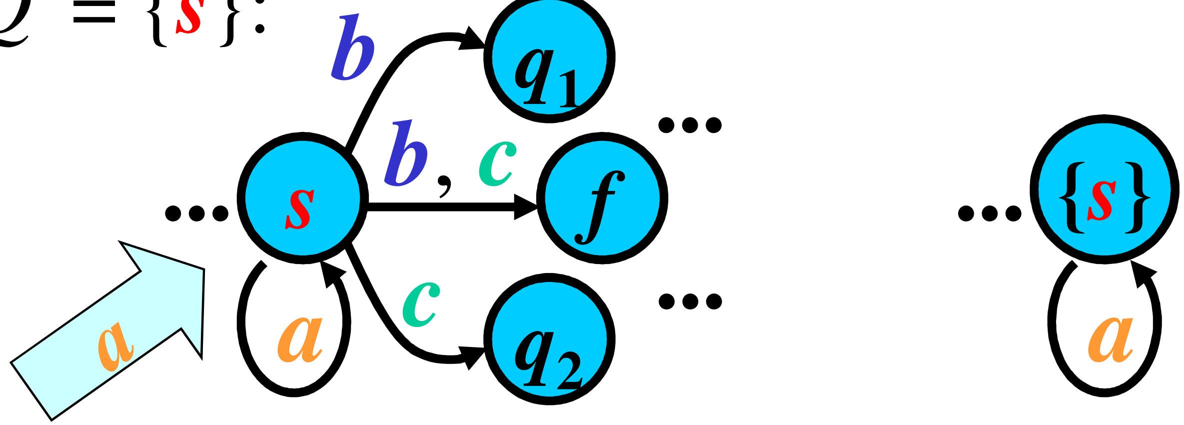
---

for $Q' = \{s\}$:



ε-free FA to DFA: Example 1/5

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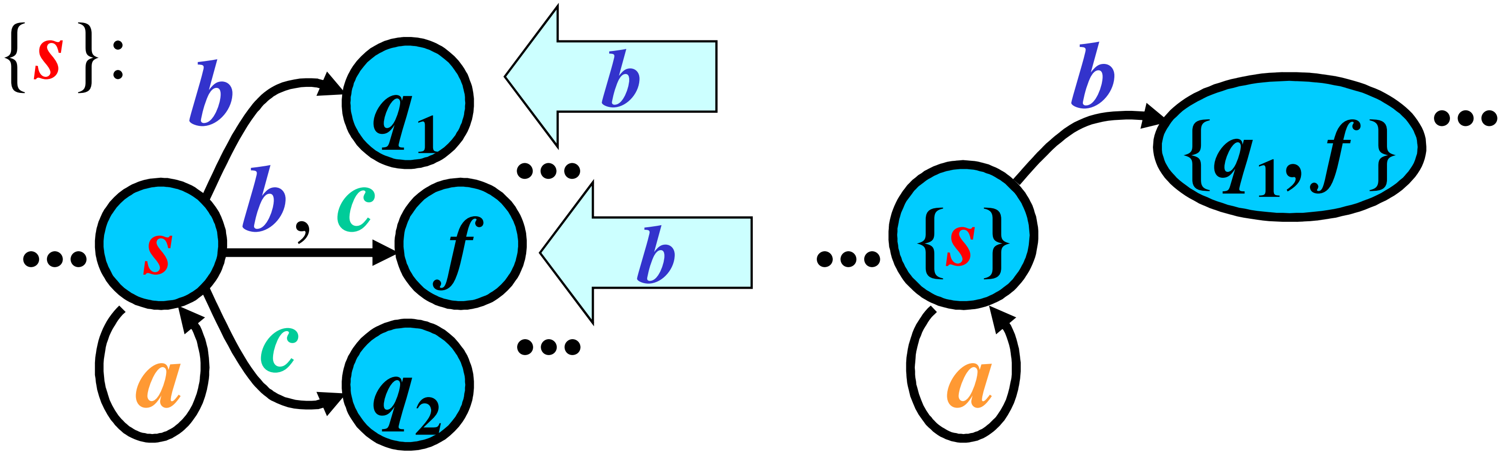
$\{s,f\}, \{q_1\}, \{q_1,q_2\}, \{q_1,f\}, \{q_1,q_2,f\}, \{q_2\}, \{q_2,f\}, \{f\}\}$

---

for $Q' = \{s\}$:



ε-free FA to DFA: Example 1/5

$M = (Q, \Sigma, R, s, F)$, where:

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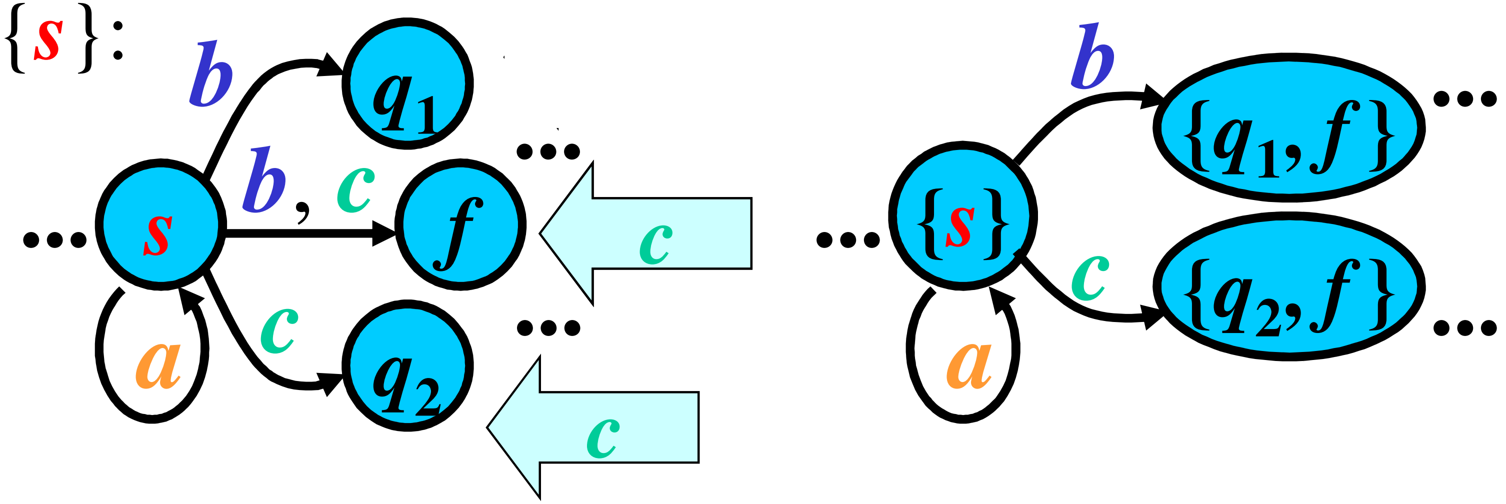

ε-free FA to DFA: Example 1/5

$M = (Q, \Sigma, R, s, F)$, where:

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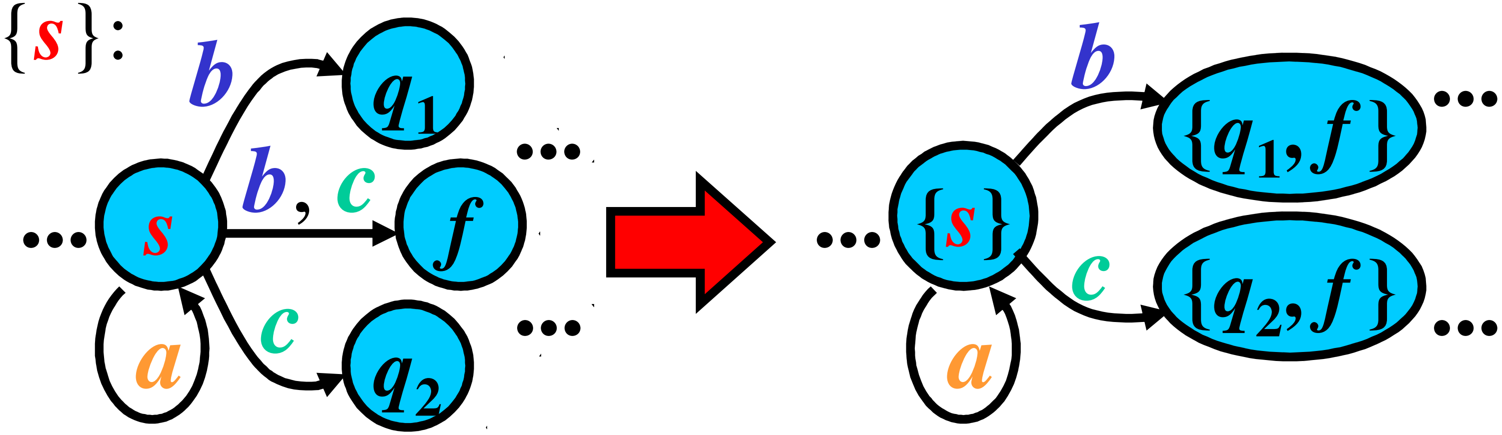
---

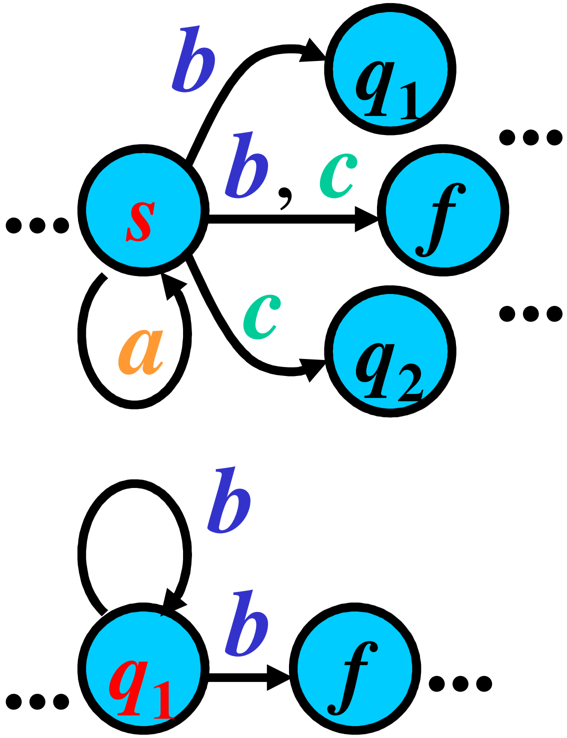
for $Q' = \{s\}$:



$R_d = \emptyset \cup \{\{s\}a \rightarrow \{s\}, \{s\}b \rightarrow \{q_1, f\}, \{s\}c \rightarrow \{q_2, f\}\}$

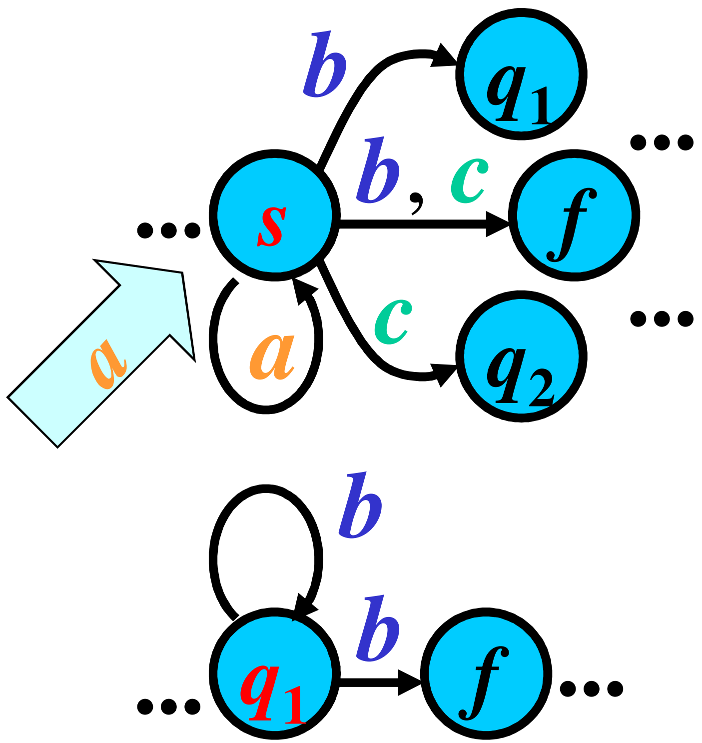
ε-free FA to DFA: Example 2/5

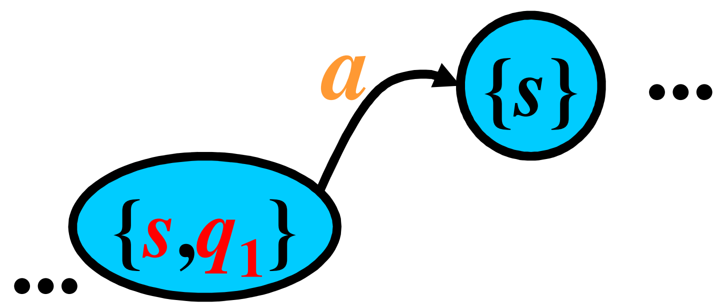
for $Q' = \{s, q_1\}$:



ε-free FA to DFA: Example 2/5

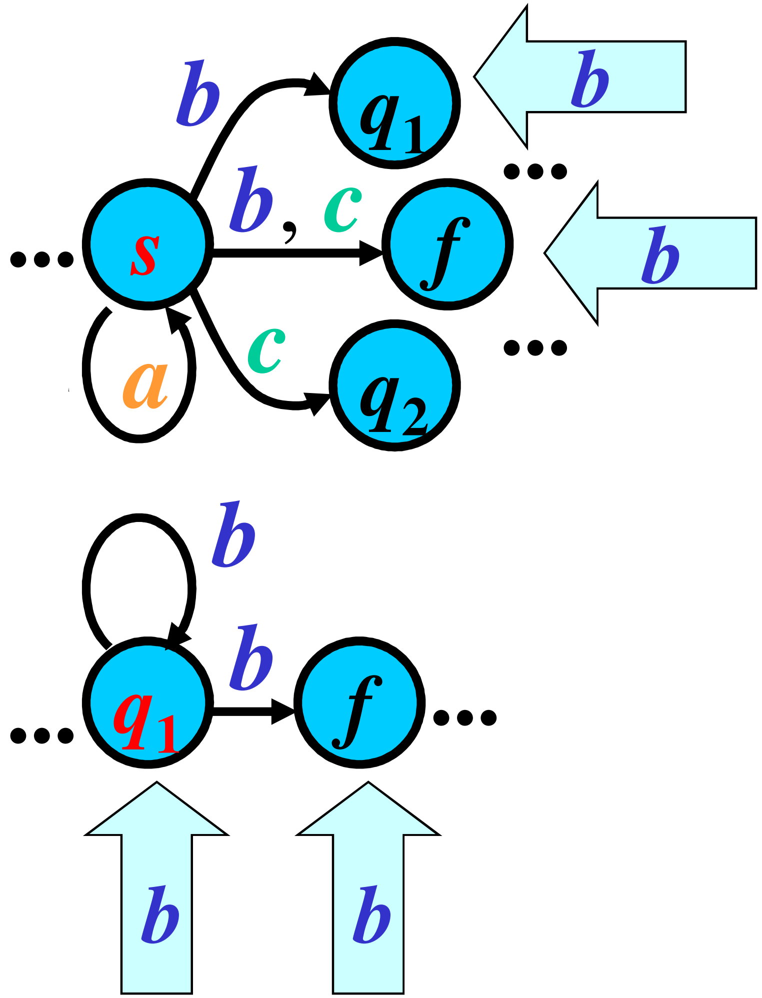
for $Q' = \{s, q_1\}$:

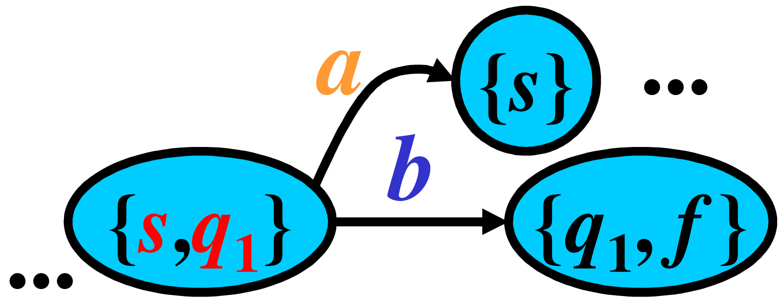




ε-free FA to DFA: Example 2/5

for $Q' = \{s, q_1\}$:





ε-free FA to DFA: Example 2/5

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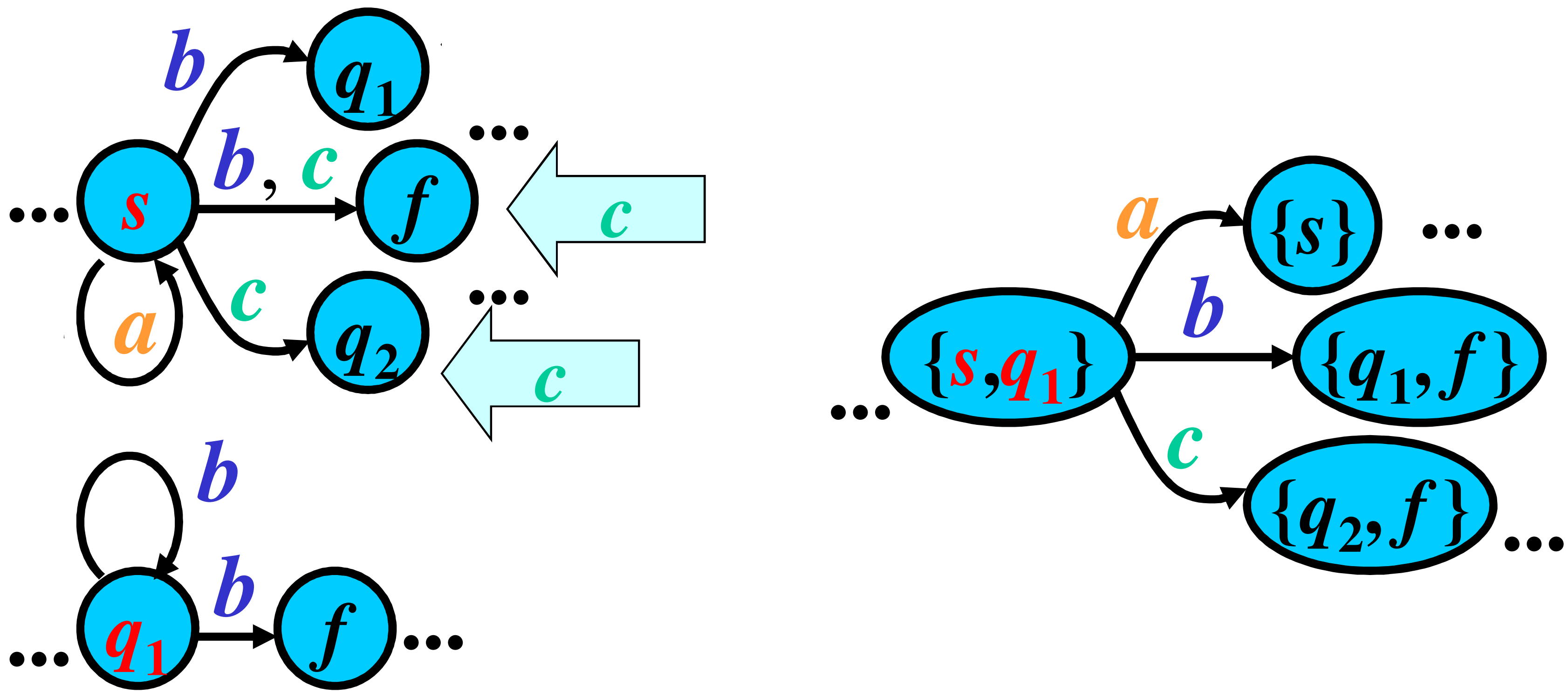

ε-free FA to DFA: Example 2/5

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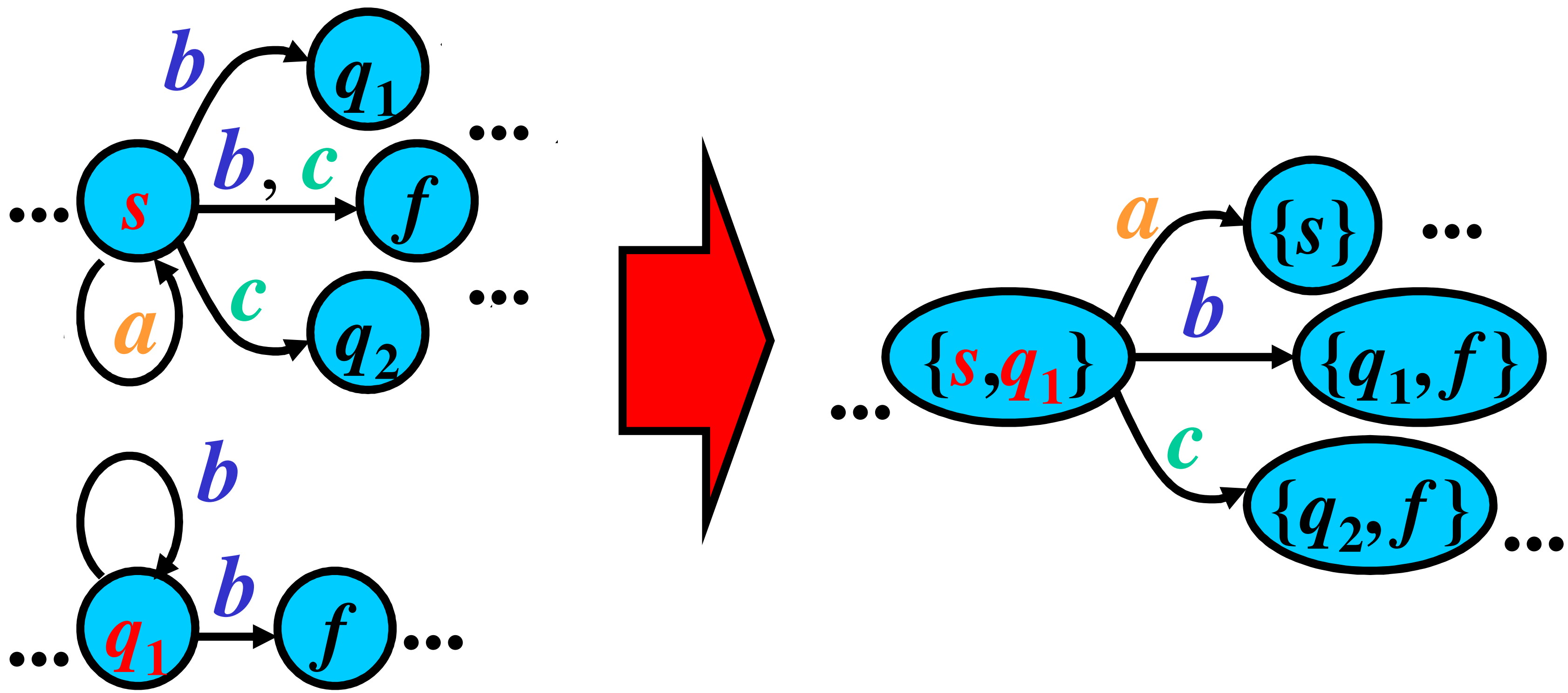
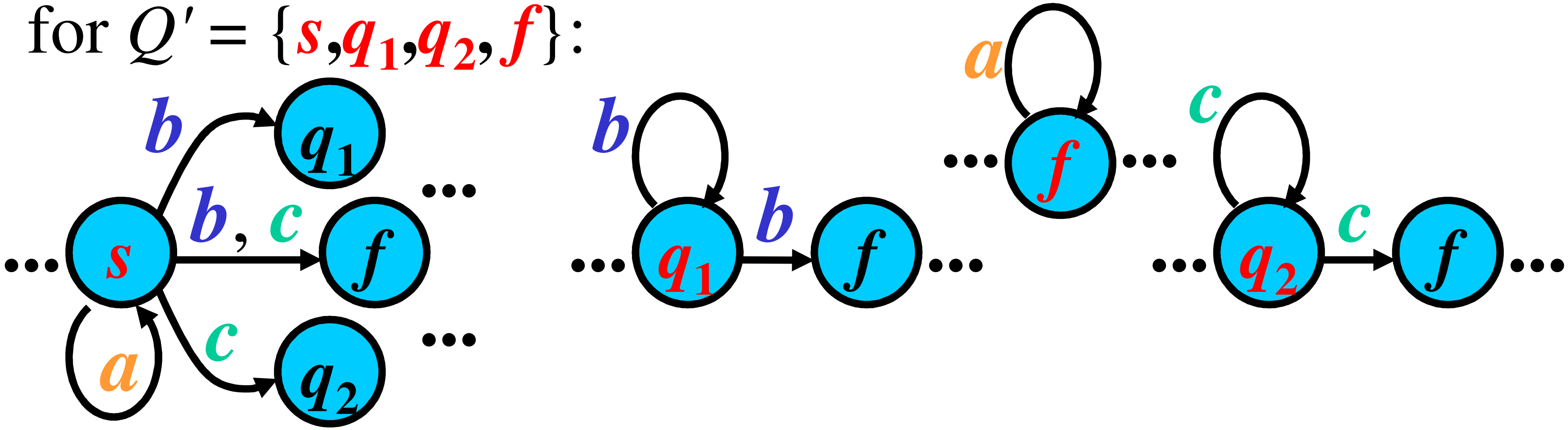


$$R_d = R_d \cup \{\{s,q_1\}a \rightarrow \{s\},\ \{s,q_1\}b \rightarrow \{q_1,f\},\ \{s,q_1\}c \rightarrow \{q_2,f\}\}$$

ε-free FA to DFA: Example 3/5

for $Q' = \{s, q_1, q_2, f\}$:



ε-free FA to DFA: Example 3/5

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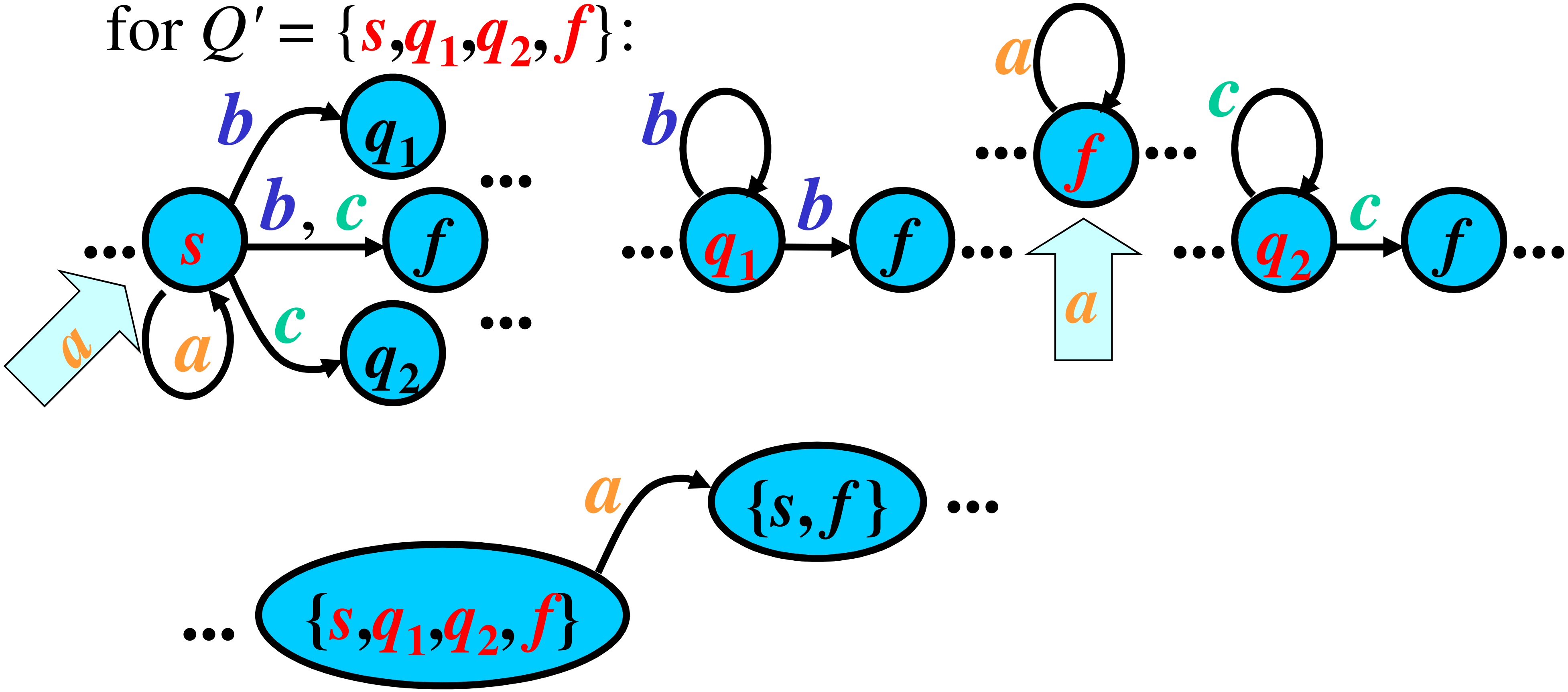
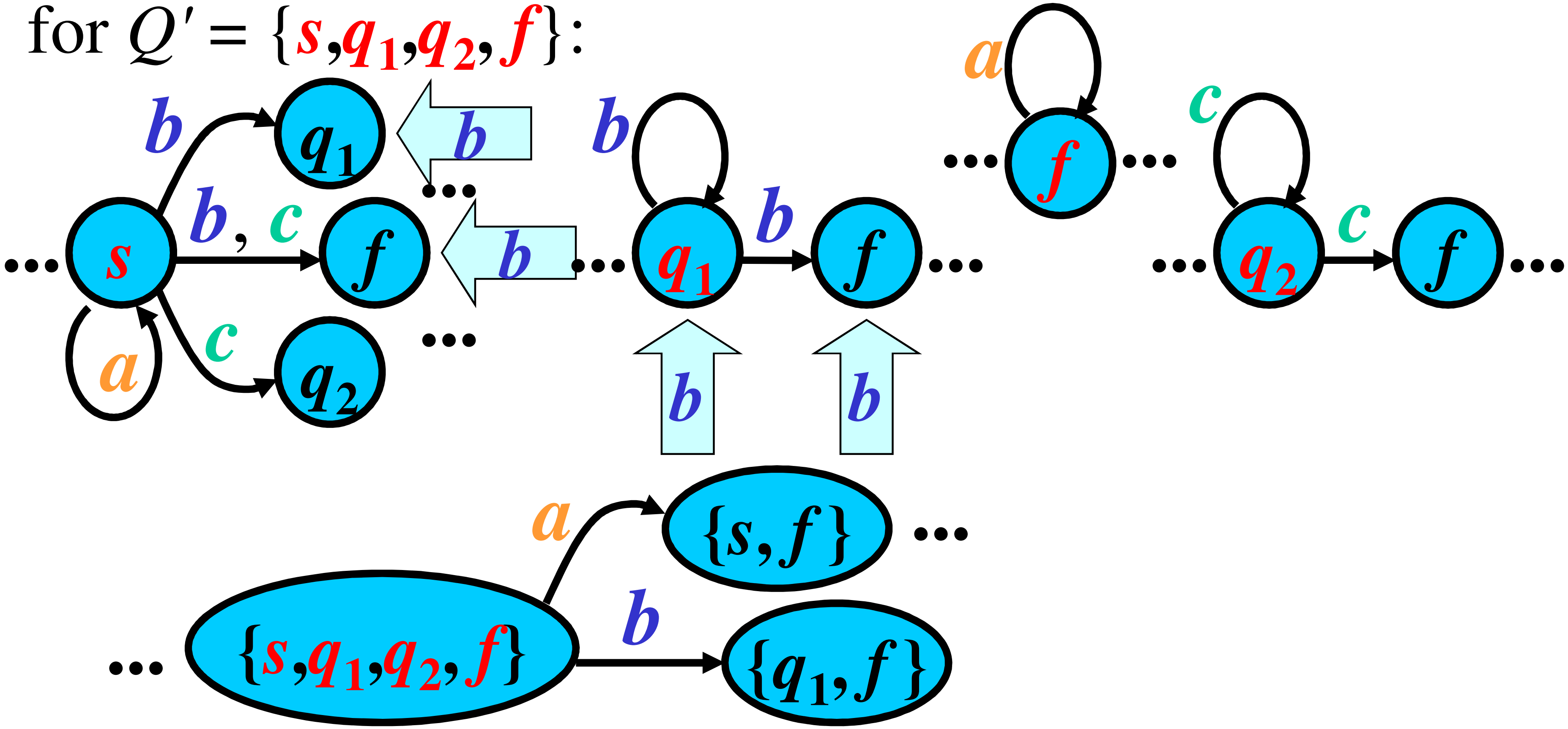


ε-free FA to DFA: Example 3/5

for $Q' = \{s, q_1, q_2, f\}$:



ε-free FA to DFA: Example 3/5

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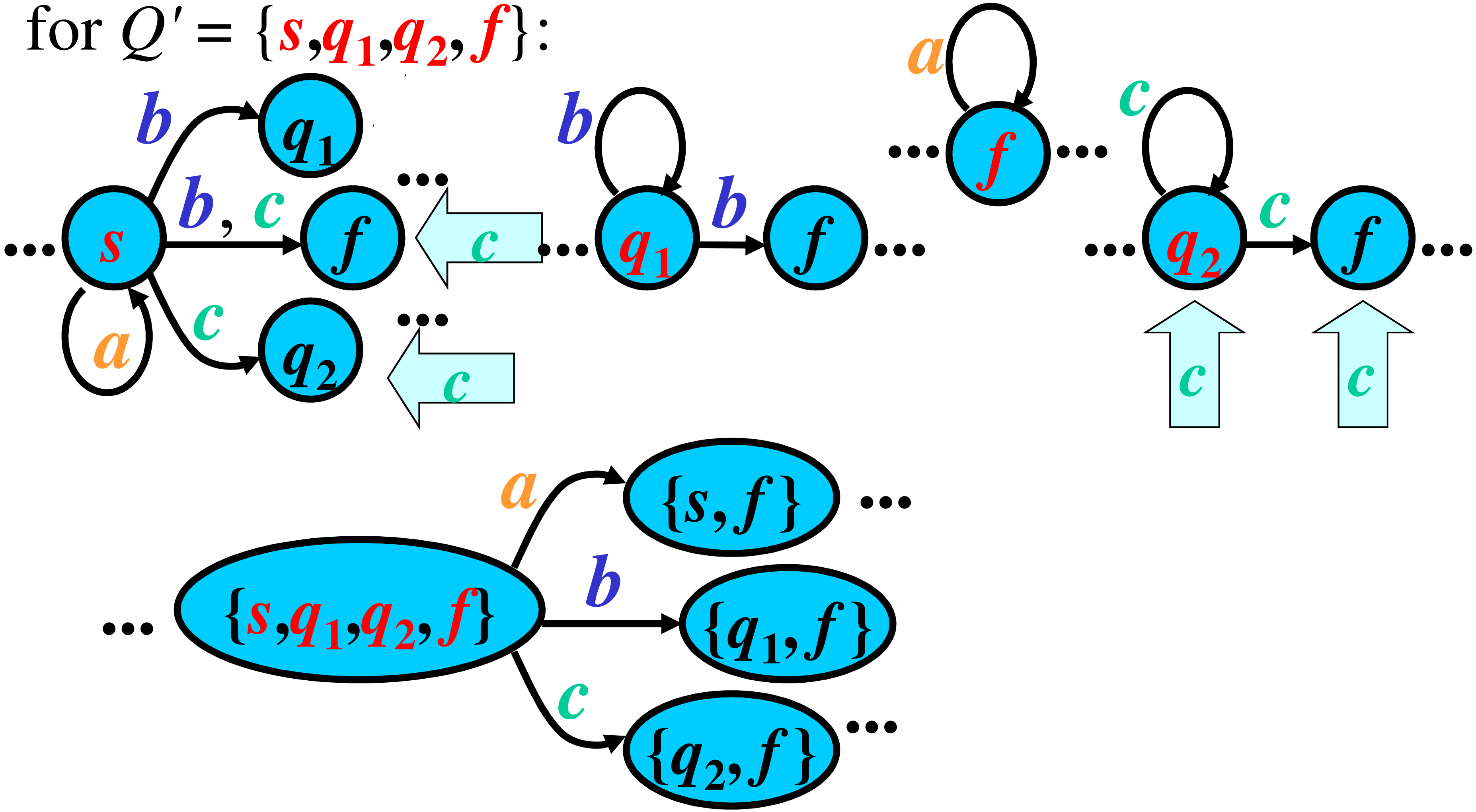
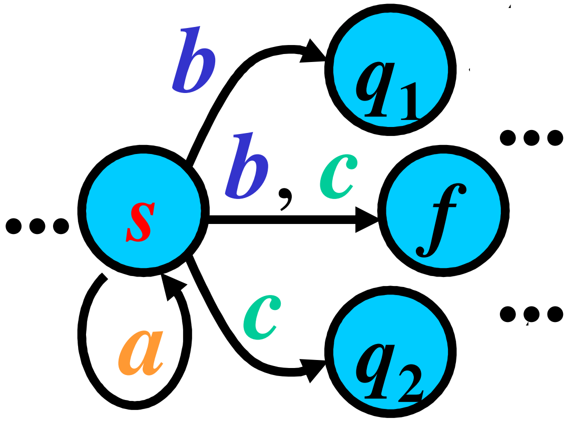
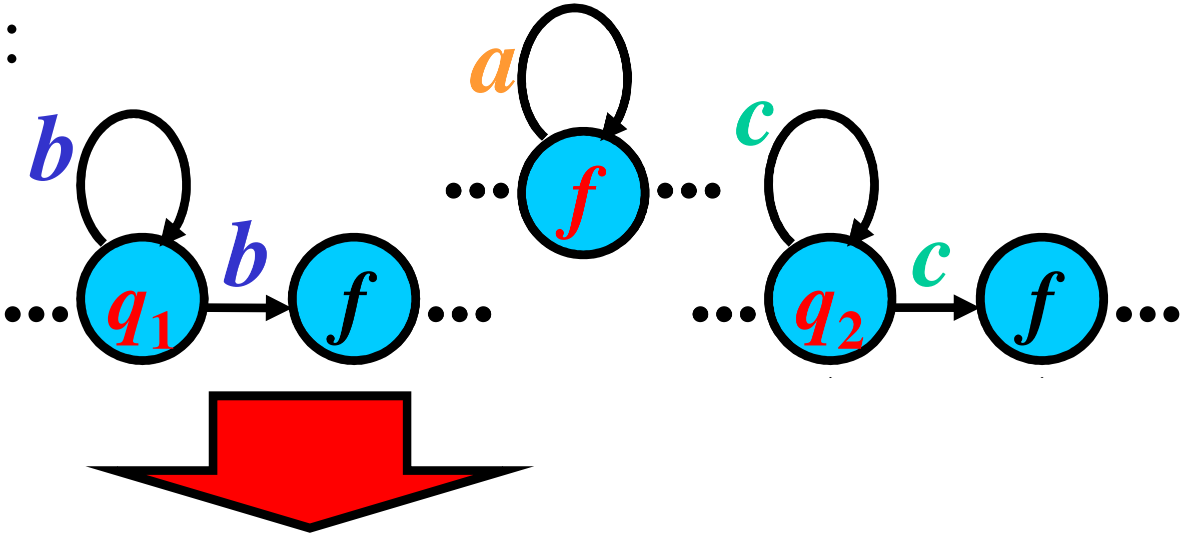
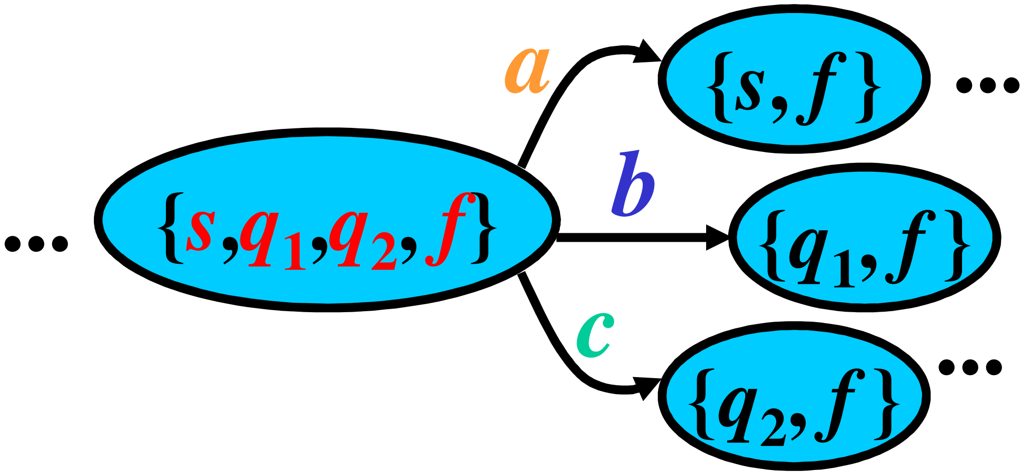


ε-free FA to DFA: Example 3/5

for $Q' = \{s, q_1, q_2, f\}$:







Construct the other states analogically…

$R_d = R_d \cup \{\{s,q_1,q_2,f\}a \rightarrow \{s, f\}, \{s,q_1,q_2,f\}b \rightarrow \{q_1,f\},$
$\{s,q_1,q_2,f\}c \rightarrow \{q_2,f\}\}$

ε-free FA to DFA: Example 4/5

Final states: $F_d := \{F' : F' \in Q_d, F' \cap F \neq \emptyset\}$

for $F = \{f\}$:

$\{s\} \cap \{f\} = \emptyset \Rightarrow \{s\} \notin F_d$

$\{s, q_1\} \cap \{f\} = \emptyset \Rightarrow \{s, q_1\} \notin F_d$

$\{s, q_1, q_2\} \cap \{f\} = \emptyset \Rightarrow \{s, q_1, q_2\} \notin F_d$

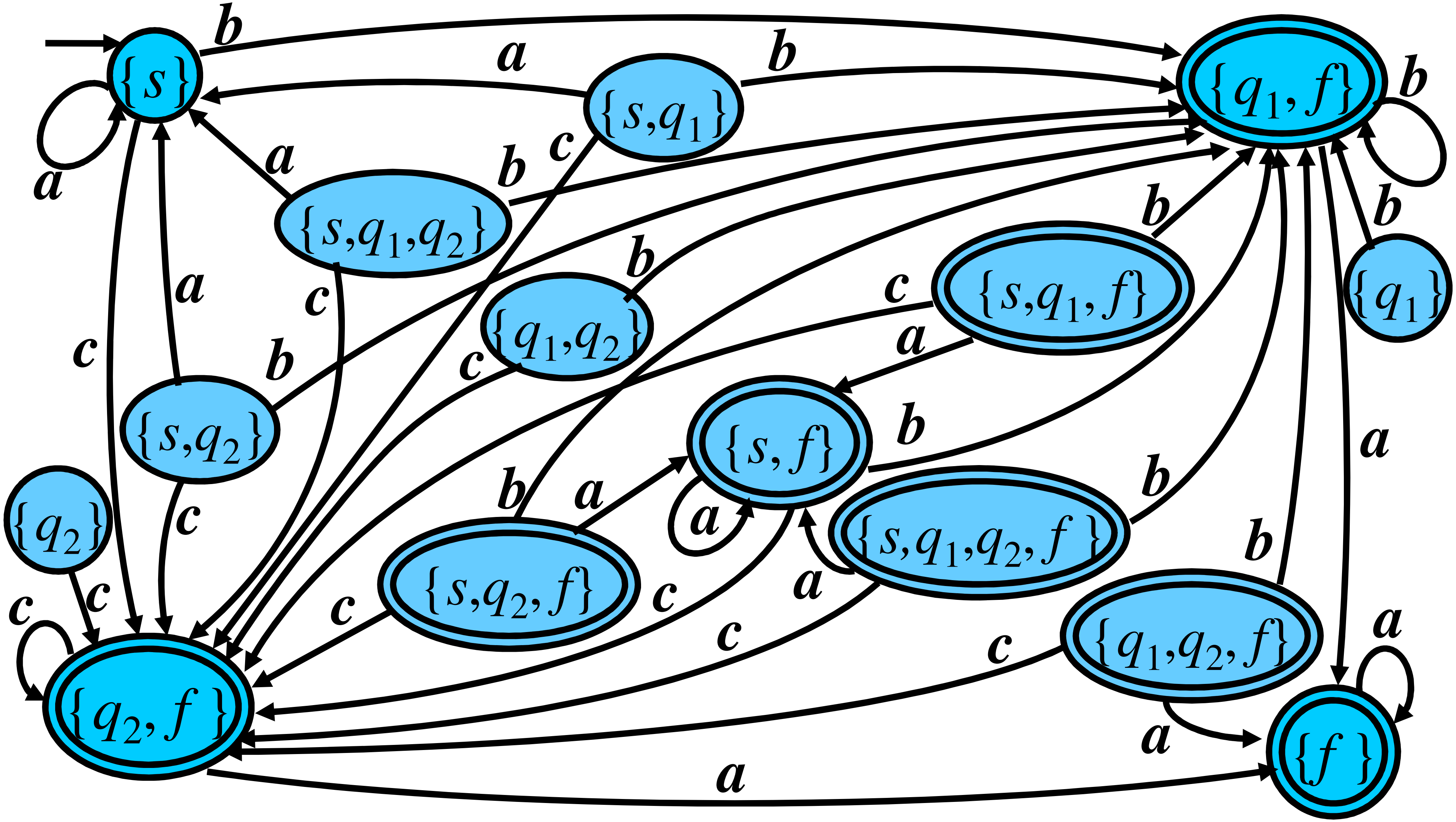
$\{s, q_1, f\} \cap \{f\} = \{f\} \neq \emptyset \Rightarrow \{s, q_1, f\} \in F_d$

$\{s, q_1, q_2, f\} \cap \{f\} = \{f\} \neq \emptyset \Rightarrow \{s, q_1, q_2, f\} \in F_d$

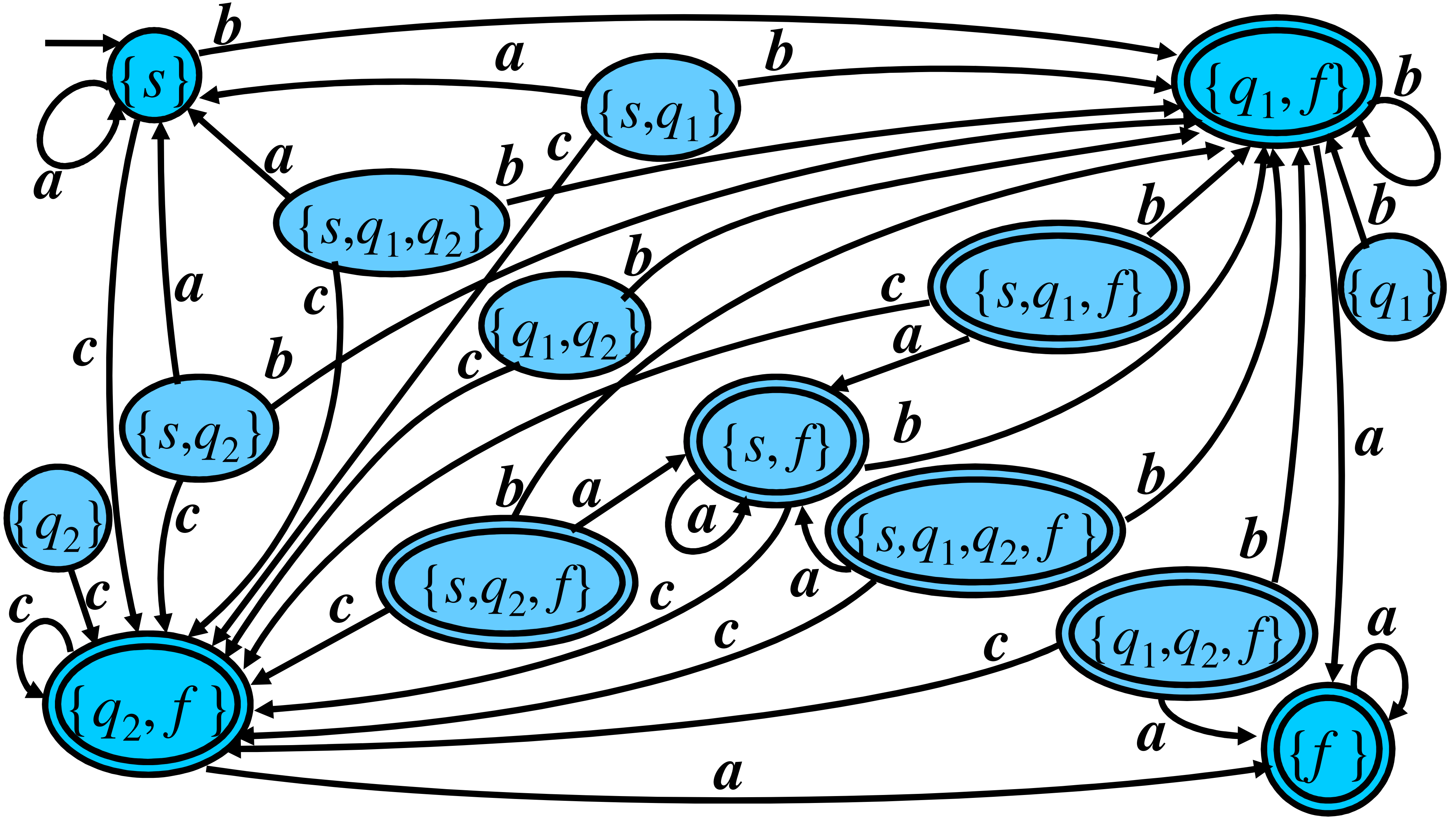
$\vdots$

$F_d = \{\{s, q_1, f\}, \{s, q_1, q_2, f\}, \{s, q_2, f\}, \{s, f\},$
$\{q_1, f\}, \{q_1, q_2, f\}, \{q_2, f\}, \{f\}\}$

ε-free FA to DFA: Example 5/5

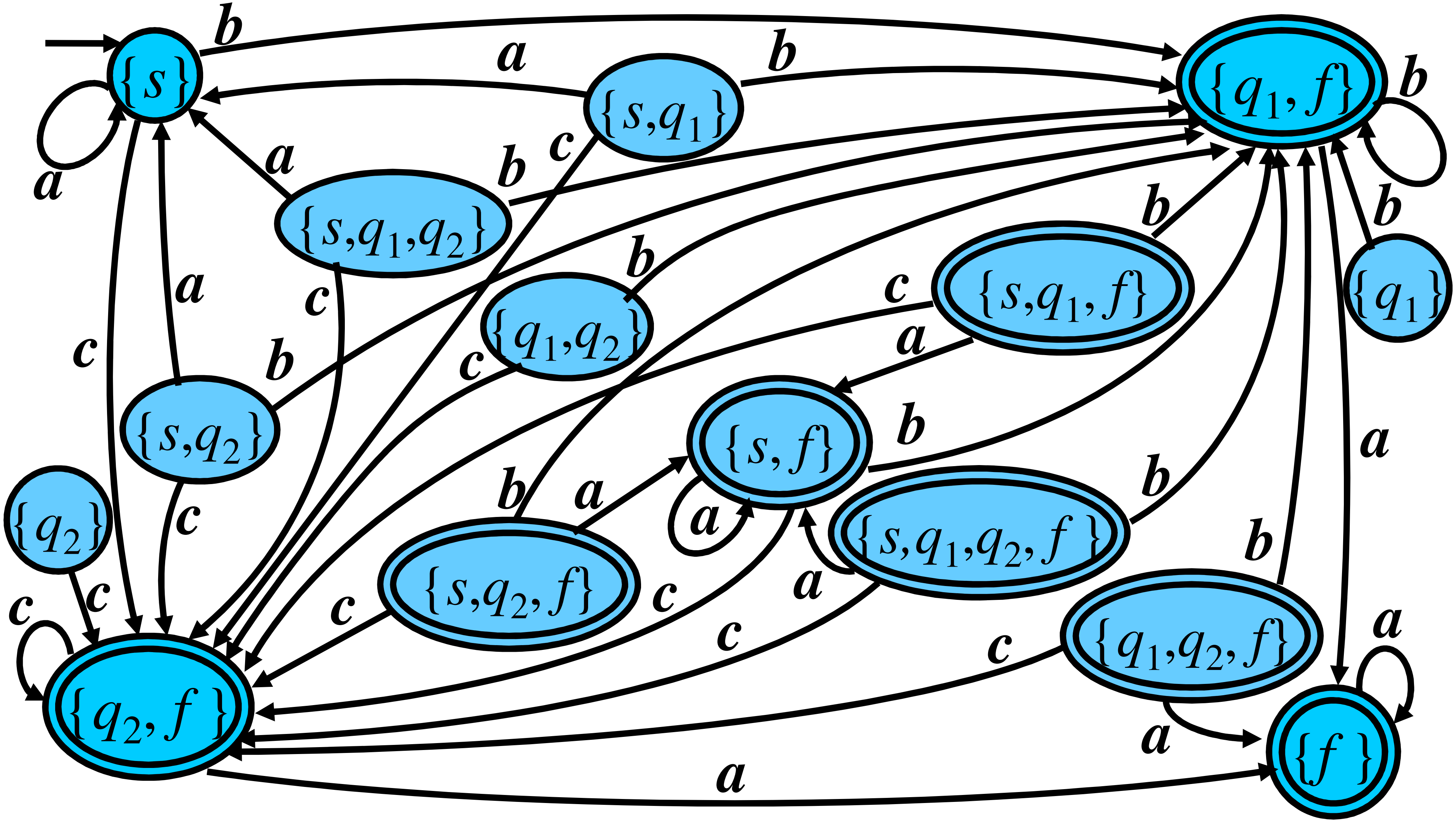


ε-free FA to DFA: Example 5/5



Question: Can we make DFA smaller?

ε-free FA to DFA: Example 5/5



Question: Can we make DFA smaller?

Answer: YES

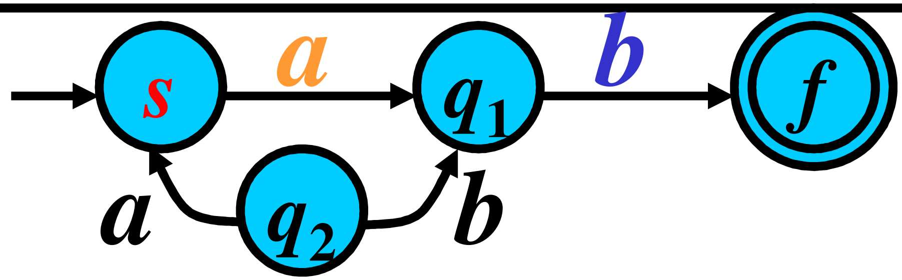
Accessible States

Gist: State q is *accessible* if a string takes DFA from s (the start state) to q.

Definition: Let $M = (Q, \Sigma, R, s, F)$ be an FA.
A state $q \in Q$ is *accessible* if there exists $w \in \Sigma^*$ such that $sw \vdash^* q$; otherwise, q is *inaccessible*.

Note: Each inaccesible state can be removed from FA

Example:



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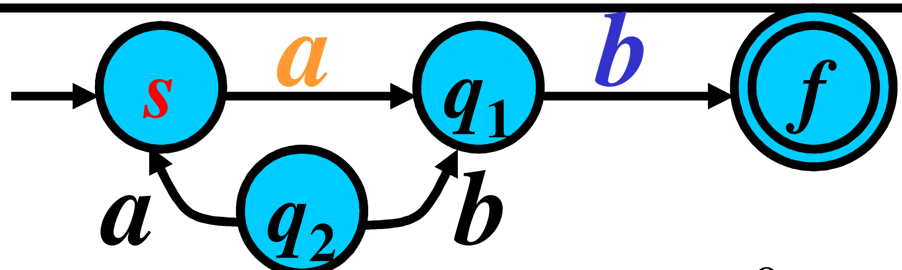
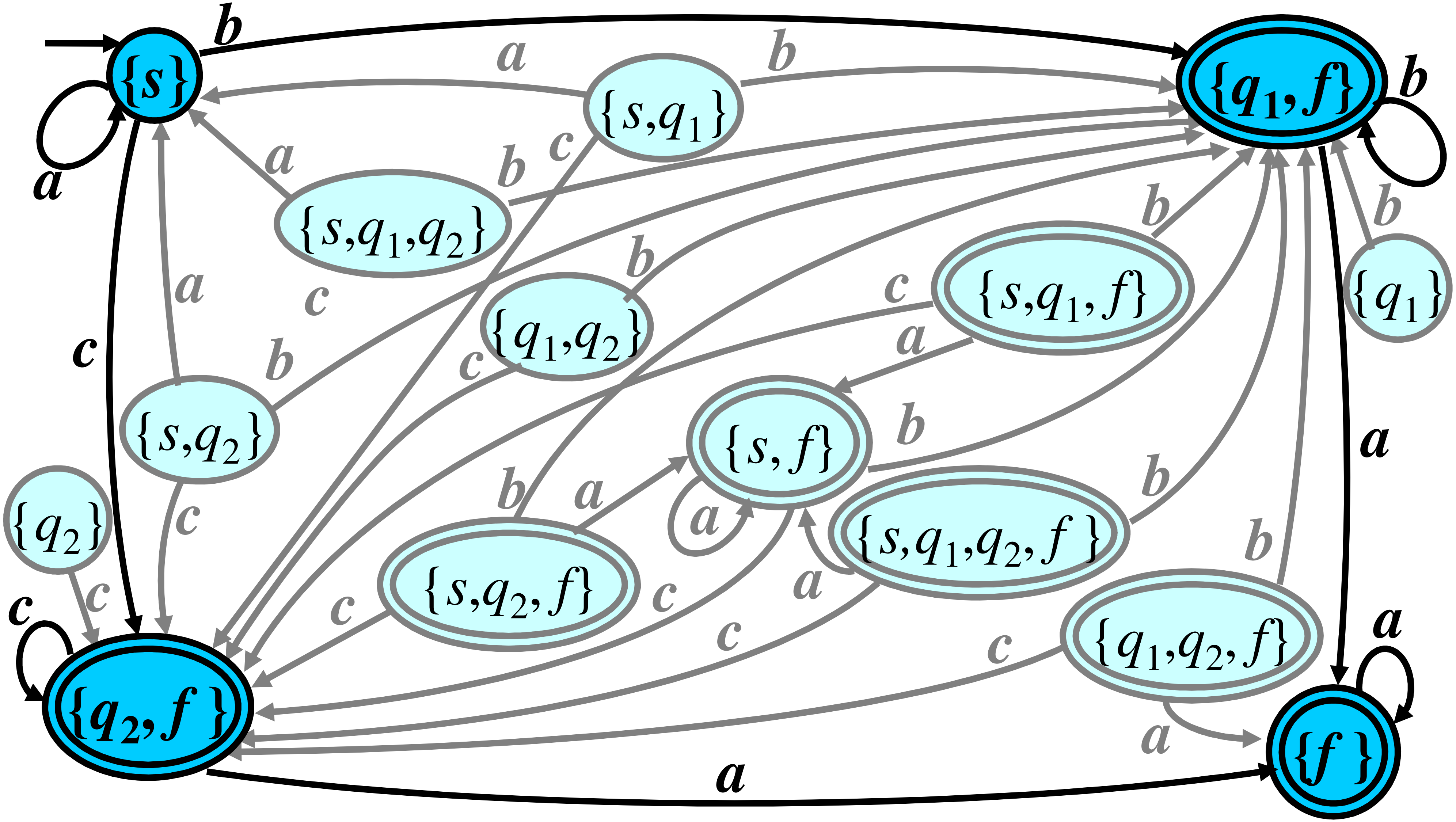
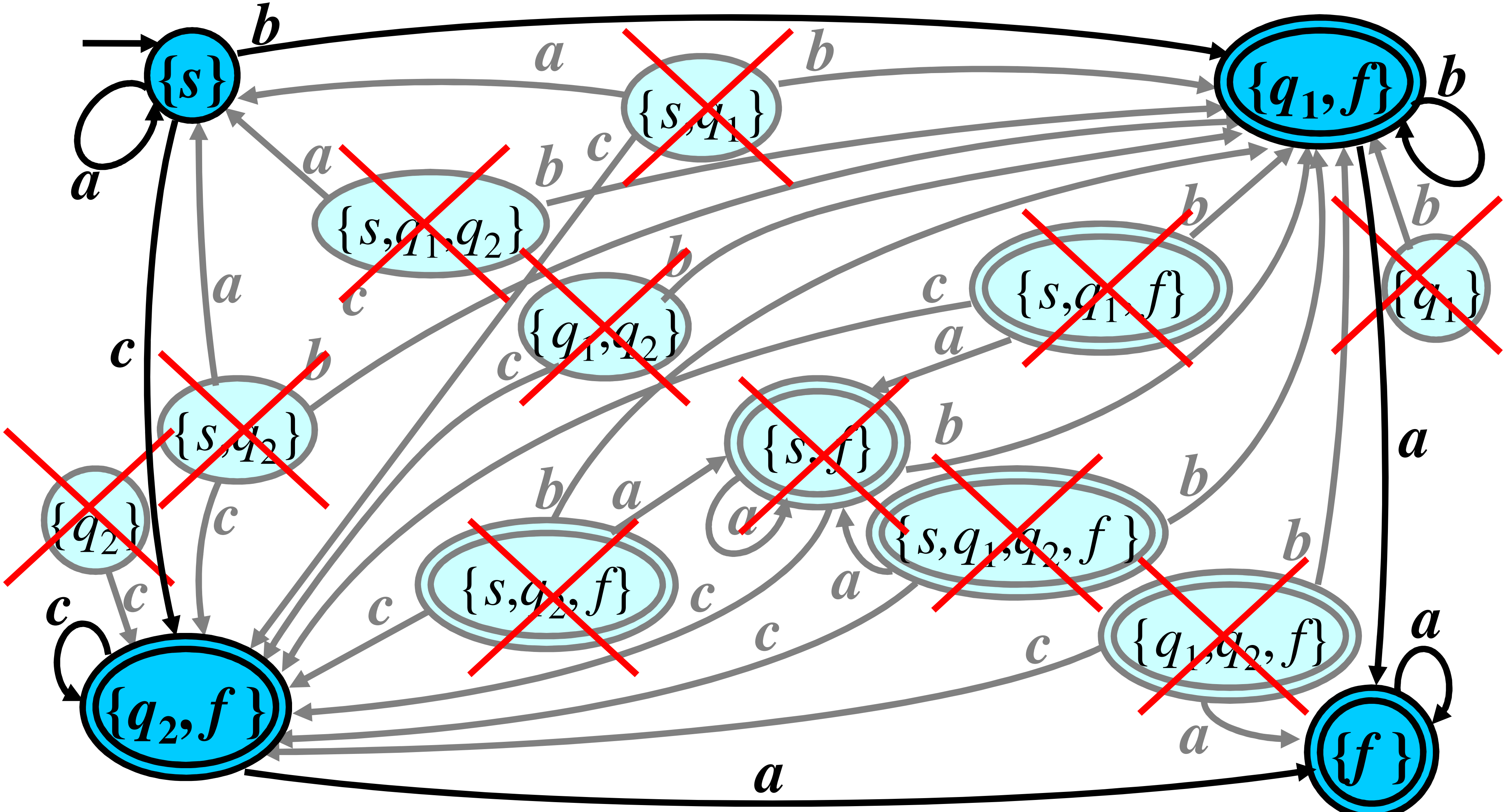


State s - accesible: $w = \varepsilon$: $s \vdash^0 s$
State q_1 - accesible: $w = a$: $sa \vdash q_1$
State f - accesible: $w = ab$: $sab \vdash q_1 b \vdash f$
State q_2 - **inaccessible** (there is no $w \in \Sigma^*$ such that $sw \vdash^* q_2$)

Previous Example



Previous Example



Many inaccessible states

Algorithm II: ε-free FA to DFA 1/2

Gist: Analogy to the previous algorithm except that only sets of accessible states are introduced.

Algorithm II: ε-free FA to DFA 1/2

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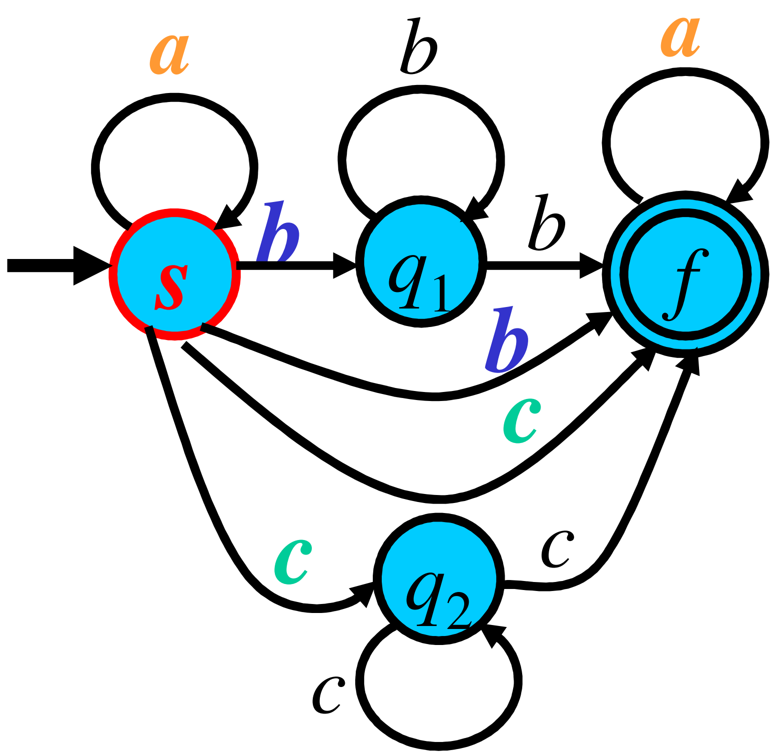
Illustration:

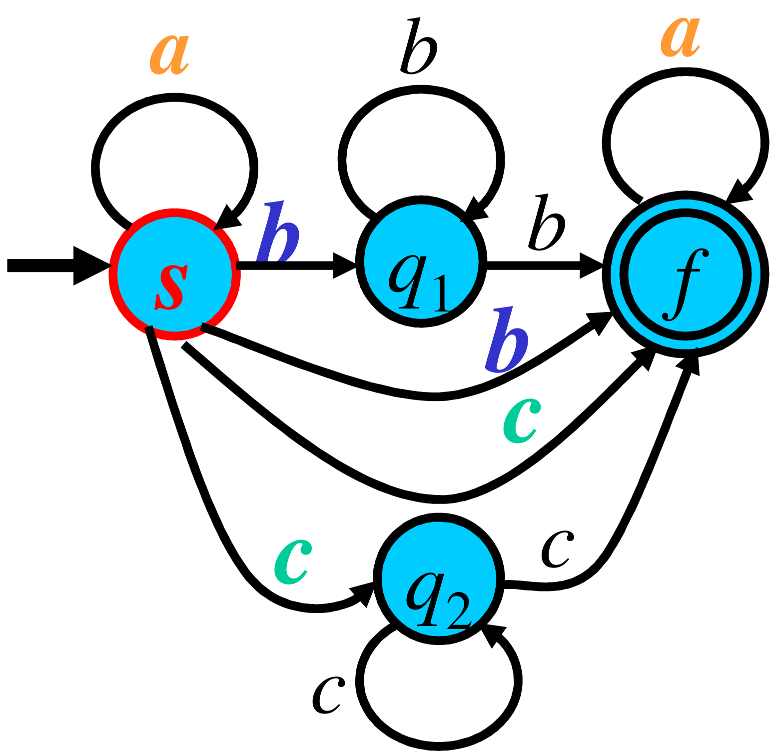


$Q_{DFA} = \{\{s\}\}$

Algorithm II: ε-free FA to DFA 1/2

Gist: Analogy to the previous algorithm except that only sets of accessible states are introduced.

Illustration:



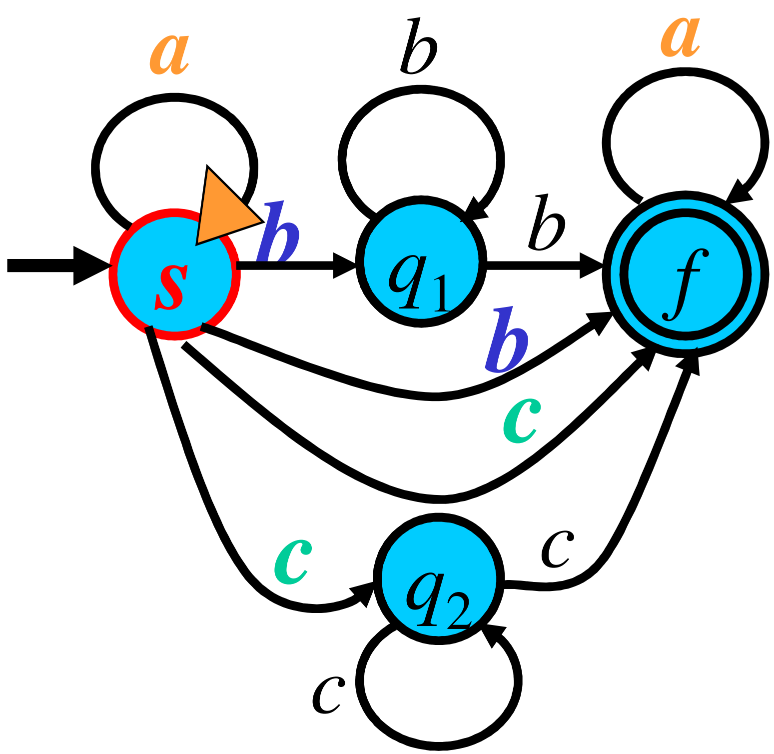
$Q_{DFA} = \{\{s\}\}$

For state {*s*}:

Algorithm II: ε-free FA to DFA 1/2

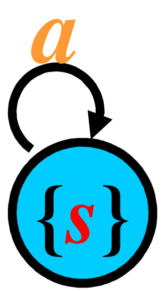
Gist: Analogy to the previous algorithm except that only sets of accessible states are introduced.

Illustration:



$Q_{DFA} = \{\{s\}\}$

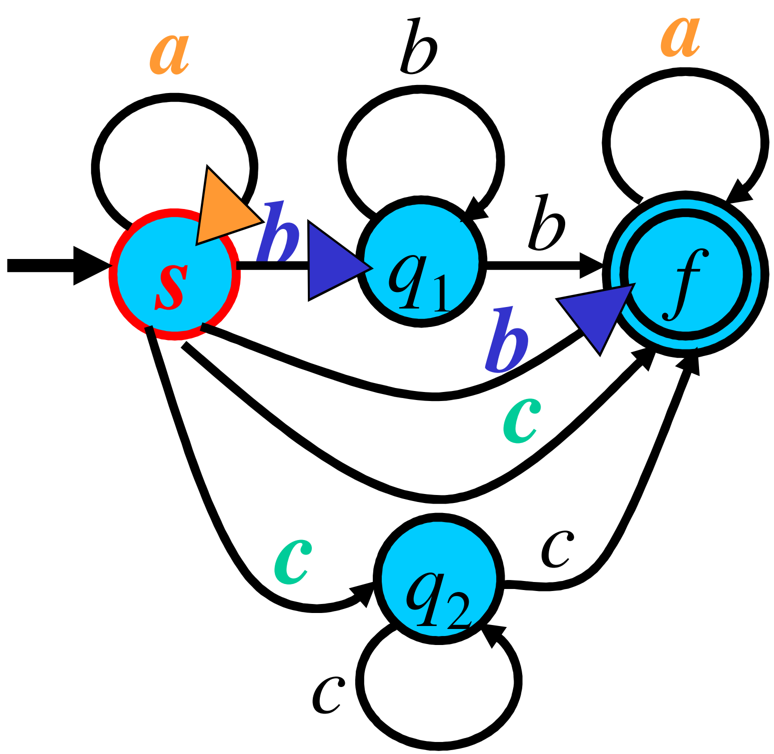
For state $\{s\}$:



Algorithm II: ε-free FA to DFA 1/2

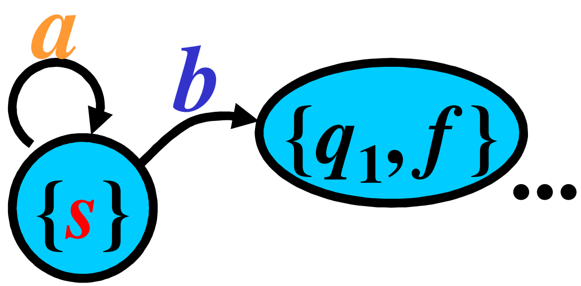
Gist: **Analogy to the previous algorithm except that only sets of accessible states are introduced.**

Illustration:



$Q_{DFA} = \{\{s\}\}$

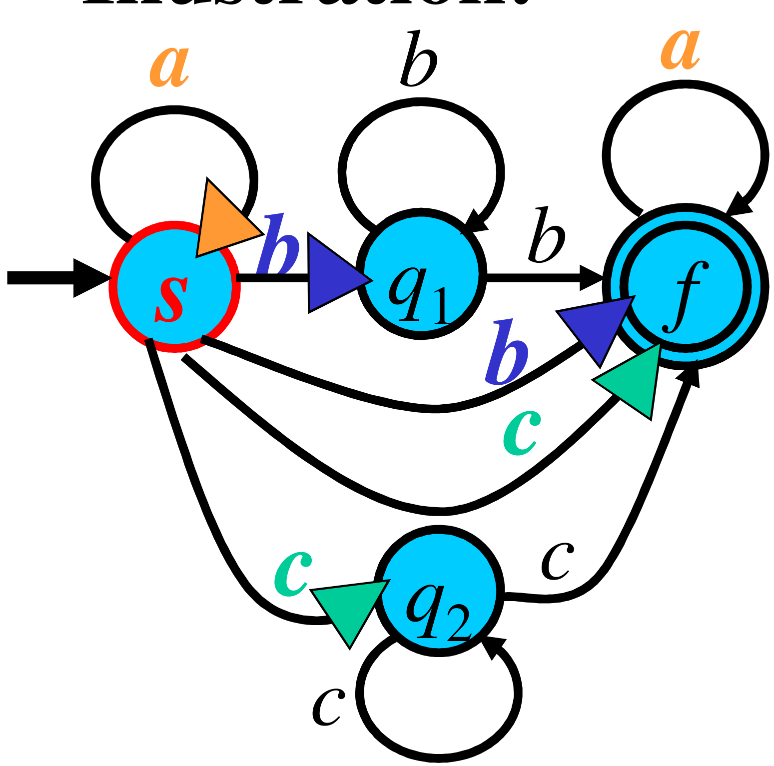
For state $\{s\}$:



Algorithm II: ε-free FA to DFA 1/2

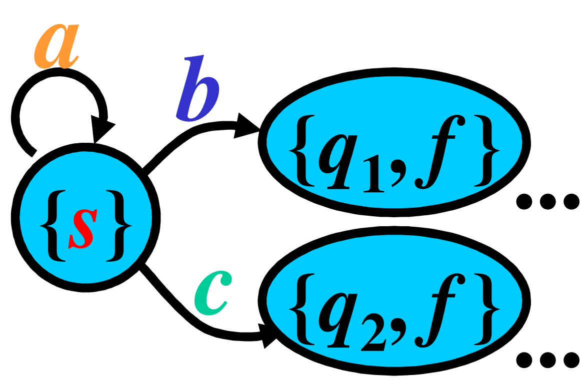
Gist: Analogy to the previous algorithm except that only sets of accessible states are introduced.

Illustration:



$Q_{DFA} = \{\{s\}\}$

For state {s}:

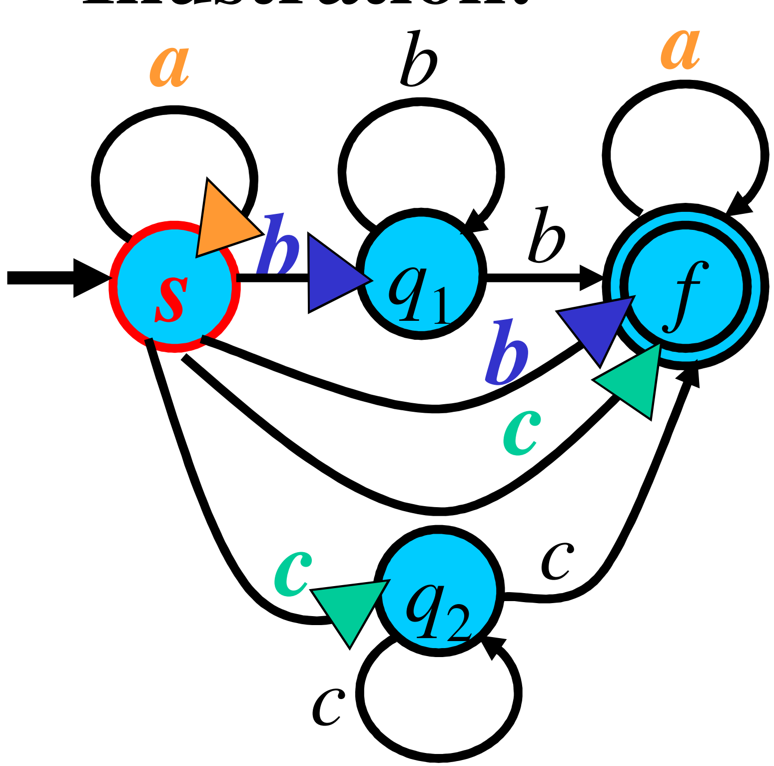


Add new states $\{q_1, f\}$, $\{q_2, f\}$ to Q_{DFA}

Algorithm II: ε-free FA to DFA 1/2

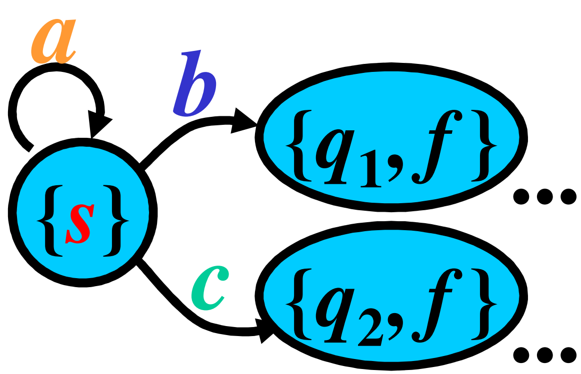
Gist: Analogy to the previous algorithm except that only sets of accessible states are introduced.

Illustration:



$Q_{DFA} = \{\{s\}\}$

For state $\{s\}$:



Add new states $\{q_1, f\}$, $\{q_2, f\}$ to Q_{DFA}

For state $\{q_1, f\}$: …

For state $\{q_2, f\}$: …

Add new states …

⋮

Algorithm II: ε-free FA to DFA 2/2

- **Input:** ε-*free* FA: $M = (Q, \Sigma, R, s, F)$
- **Output:** DFA: $M_d = (Q_d, \Sigma, R_d, s_d, F_d)$

without any inaccessible states

- **Method:**
- $s_d := \{s\}$; $Q_{new} := \{s_d\}$; $R_d := \emptyset$; $Q_d := \emptyset$; $F_d := \emptyset$;
- **repeat**

  let $Q' \in Q_{new}$; $Q_{new} := Q_{new} - \{Q'\}$; $Q_d := Q_d \cup \{Q'\}$;

  for each $a \in \Sigma$ **do begin**

  $Q'' := \{q: p \in Q', pa \rightarrow q \in R\}$;

  if $Q'' \neq \emptyset$ **then** $R_d := R_d \cup \{Q'a \rightarrow Q''\}$;

  if $Q'' \notin Q_d \cup \{\emptyset\}$ **then** $Q_{new} := Q_{new} \cup \{Q''\}$

  end;

  if $Q' \cap F \neq \emptyset$ **then** $F_d := F_d \cup \{Q'\}$

until $Q_{new} = \emptyset$.

ε-free FA to DFA: Example 1/3

$M = (Q, \Sigma, R, s, F)$, where:

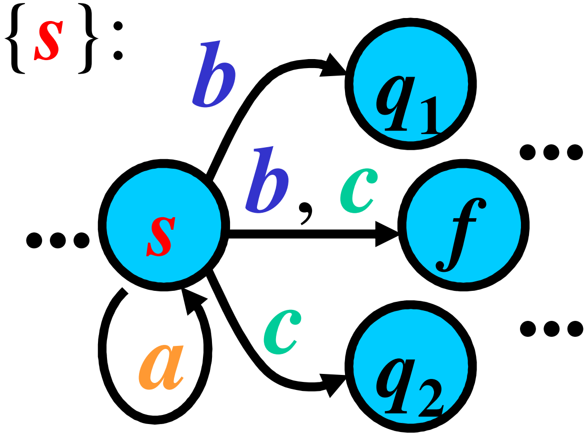
$Q = \{s, q_1, q_2, f\}$; $\Sigma = \{a, b, c\}$; $F = \{f\}$

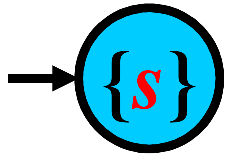
$R = \{sa \rightarrow s, sb \rightarrow q_1, sb \rightarrow f, sc \rightarrow q_2, sc \rightarrow f,$
$q_1b \rightarrow q_1, q_1b \rightarrow f, q_2c \rightarrow q_2, q_2c \rightarrow f, fa \rightarrow f\}$;

$Q_{new} = \{\{s\}\}$; $R_d = \emptyset$; $Q_d = \emptyset$; $F_d = \emptyset$

---

for $Q' = \{s\}$:





ε-free FA to DFA: Example 1/3

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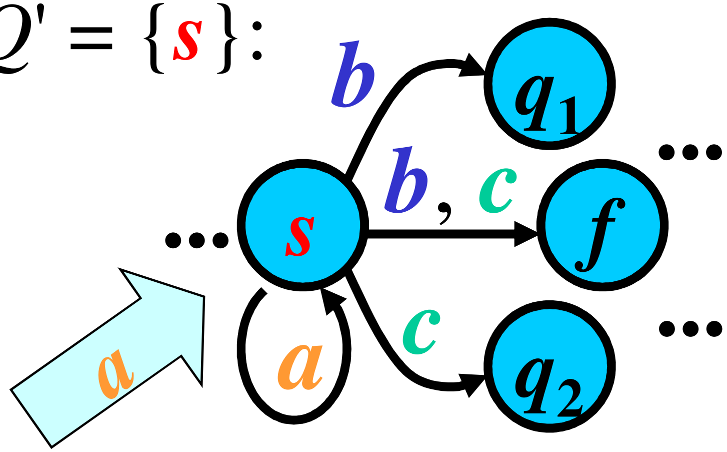
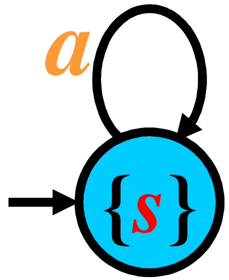
for $Q' = \{s\}$:





ε-free FA to DFA: Example 1/3

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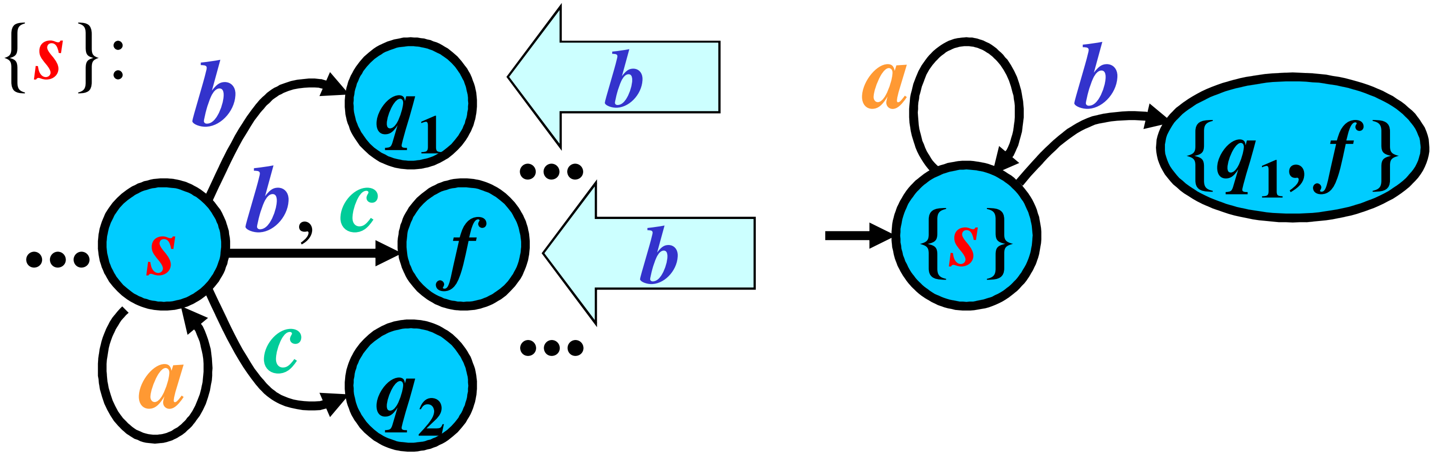
---

for $Q' = \{s\}$:



ε-free FA to DFA: Example 1/3

$M = (Q, \Sigma, R, s, F)$, where:

$Q = \{s, q_1, q_2, f\}$; $\Sigma = \{a, b, c\}$; $F = \{f\}$

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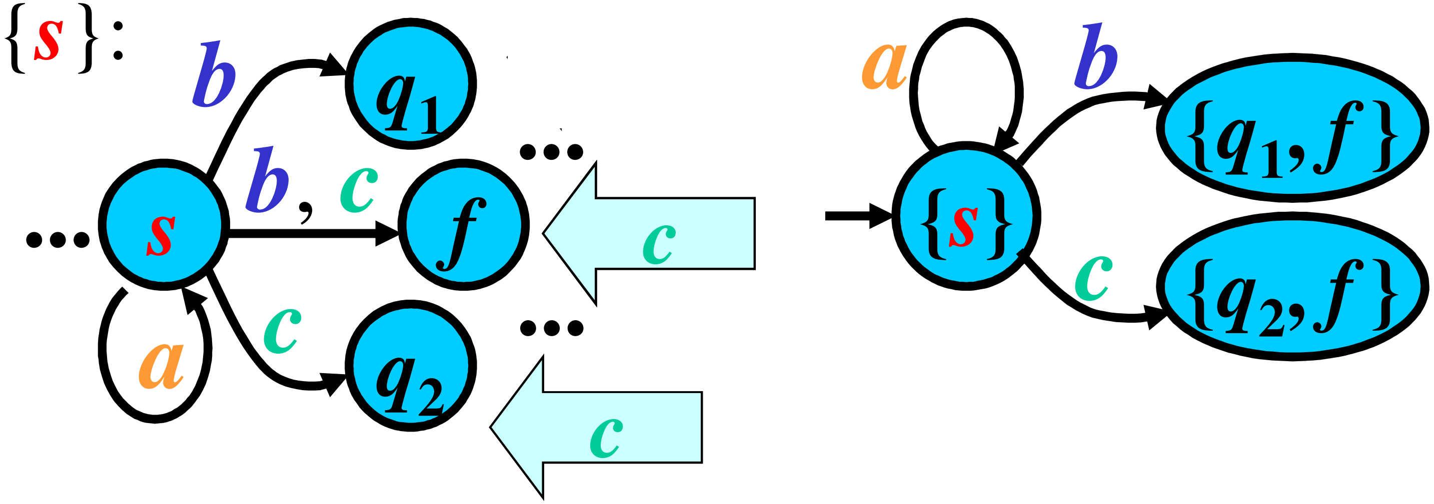
---

for $Q' = \{s\}$:



ε-free FA to DFA: Example 1/3

$M = (Q, \Sigma, R, s, F)$, where:

$Q = \{s, q_1, q_2, f\}$; $\Sigma = \{a, b, c\}$; $F = \{f\}$

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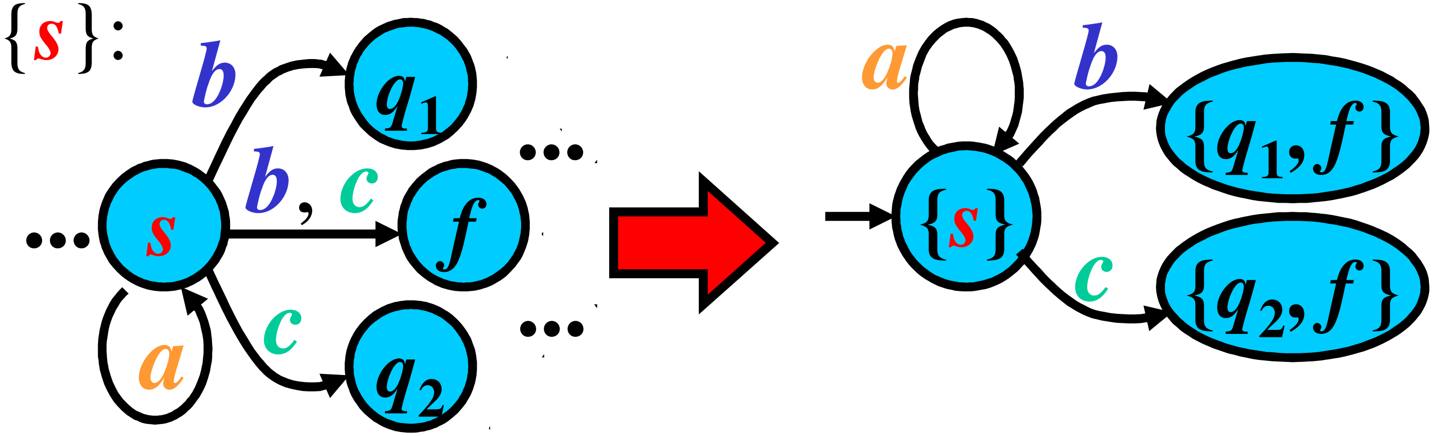
$Q_{new} = \{\{s\}\}$; $R_d = \emptyset$; $Q_d = \emptyset$; $F_d = \emptyset$

---

for $Q' = \{s\}$:

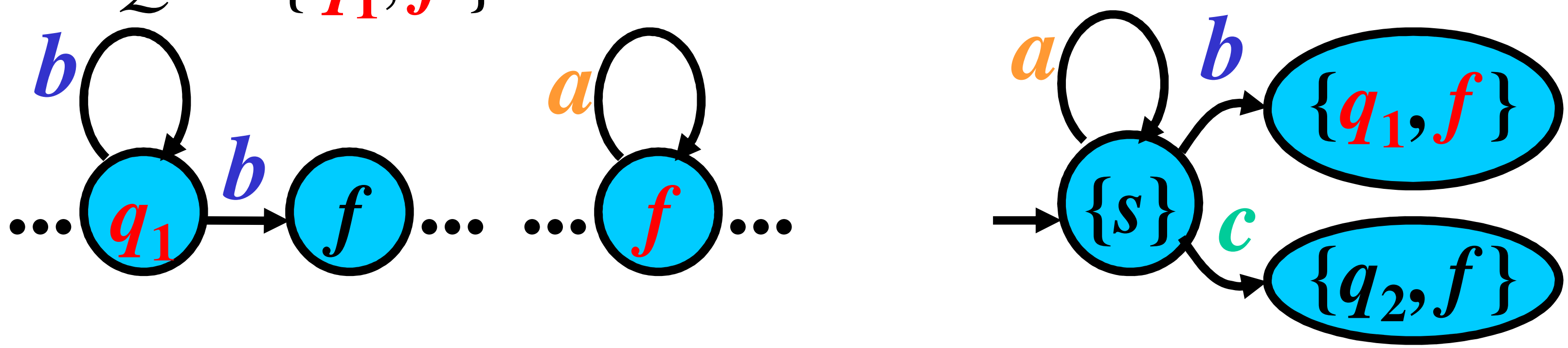


$R_d := \emptyset \cup \{\{s\}a \rightarrow \{s\}, \{s\}b \rightarrow \{q_1, f\}, \{s\}c \rightarrow \{q_2, f\}\}$

$Q_{new} = \{\{q_1, f\}, \{q_2, f\}\}$, $Q_d = \emptyset \cup \{\{s\}\}$, $F_d = \emptyset$

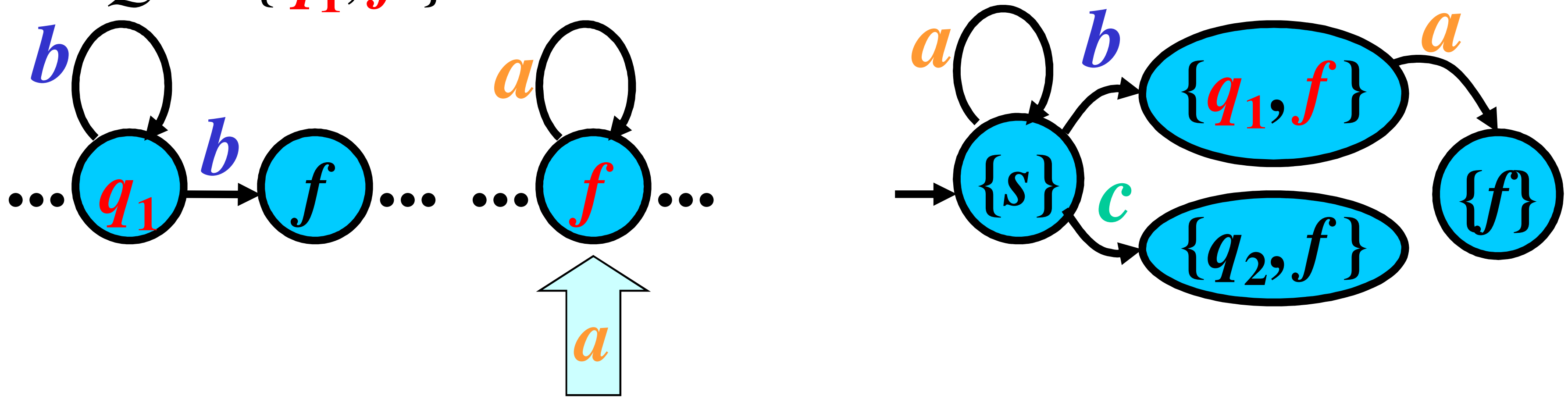
ε-free FA to DFA: Example 2/3

for $Q' = \{q_1, f\}$:



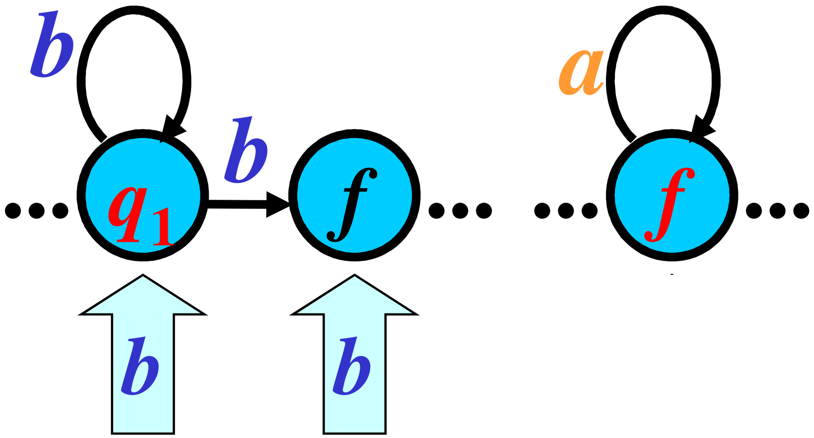
ε-free FA to DFA: Example 2/3

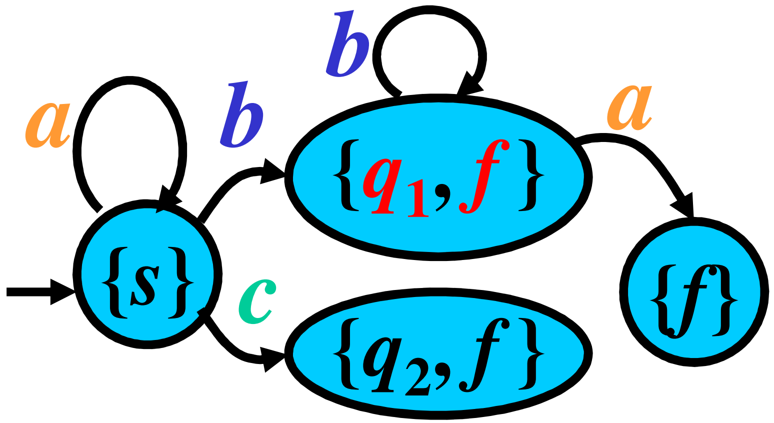
for $Q' = \{q_1, f\}$:



ε-free FA to DFA: Example 2/3

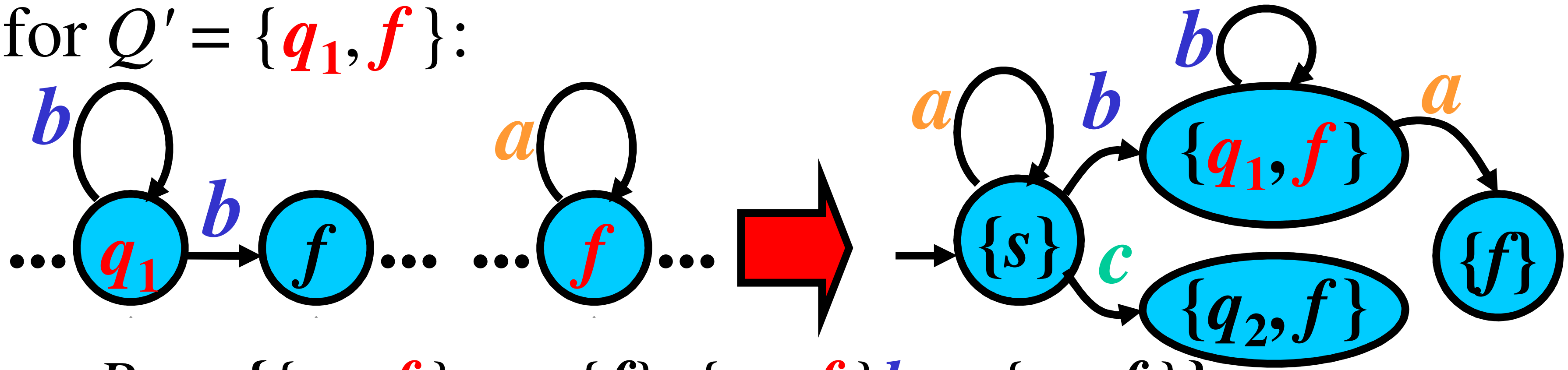
for $Q' = \{q_1, f\}$:





ε-free FA to DFA: Example 2/3

for $Q' = \{q_1, f\}$:

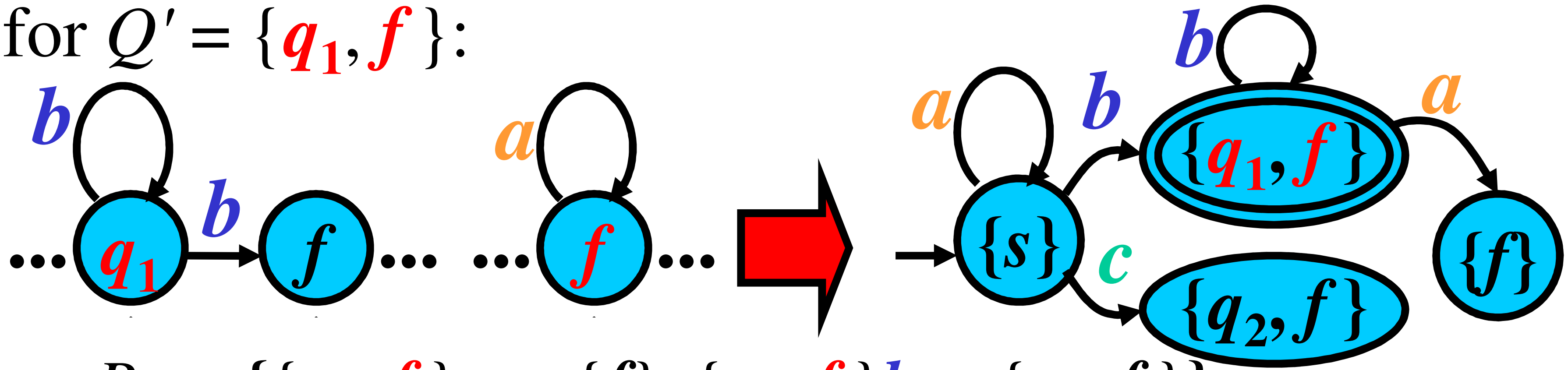


$R_d := R_d \cup \{\{q_1, f\}a \rightarrow \{f\}, \{q_1, f\}b \rightarrow \{q_1, f\}\}$

$Q_{new} = \{\{q_2, f\}, \{f\}\}$, $Q_d = Q_d \cup \{\{q_1, f\}\}$,

ε-free FA to DFA: Example 2/3

for $Q' = \{q_1, f\}$:



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$Q_{new} = \{\{q_2, f\}, \{f\}\}, Q_d = Q_d \cup \{\{q_1, f\}\}, F_d := \emptyset \cup \{\{q_1, f\}\}$

ε-free FA to DFA: Example 2/3

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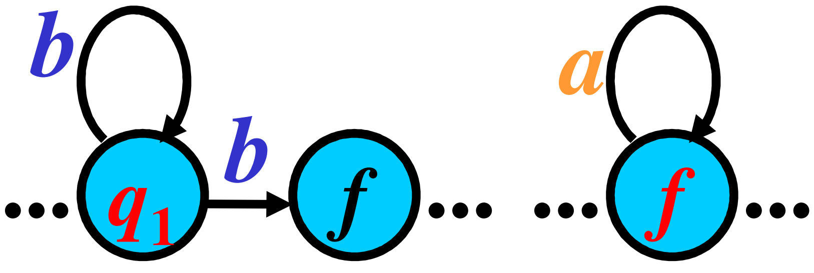

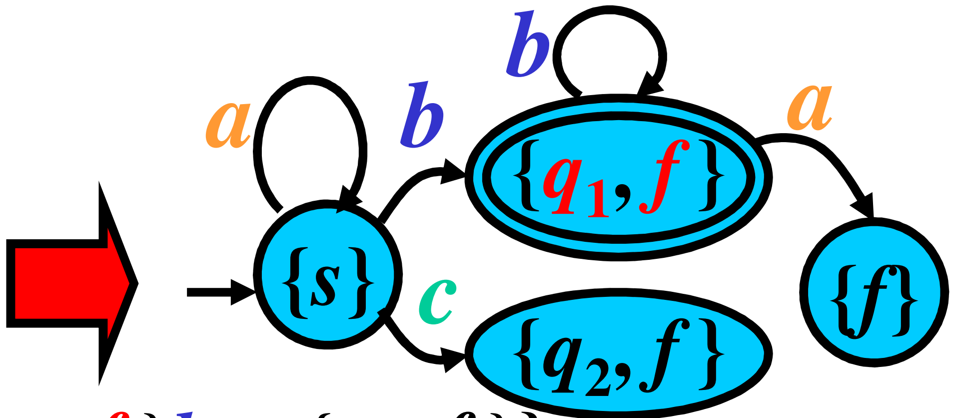

$R_d := R_d \cup \{\{q_1, f\}a \to \{f\}, \{q_1, f\}b \to \{q_1, f\}\}$

$Q_{new} = \{\{q_2, f\}, \{f\}\}$, $Q_d = Q_d \cup \{\{q_1, f\}\}$, $F_d := \emptyset \cup \{\{q_1, f\}\}$

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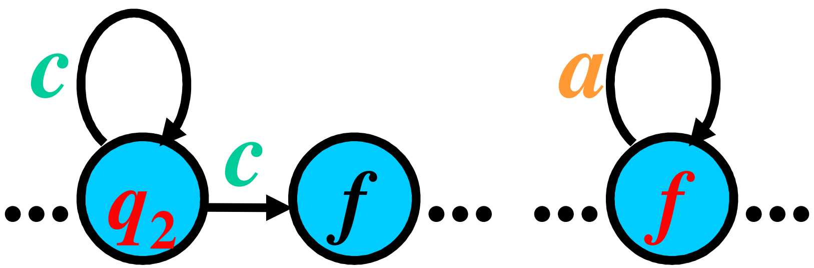
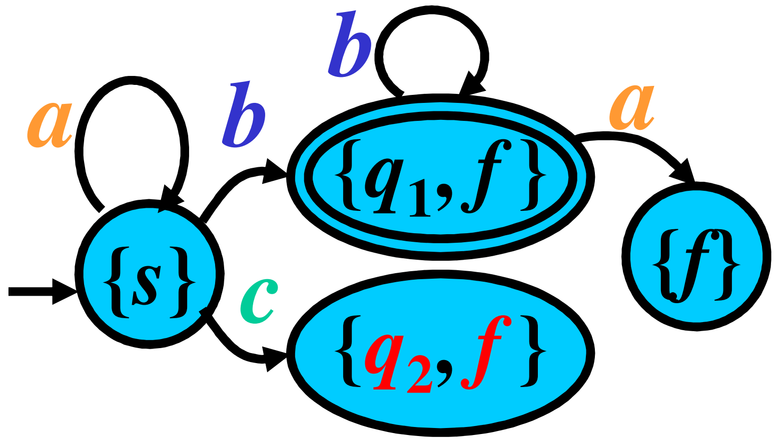
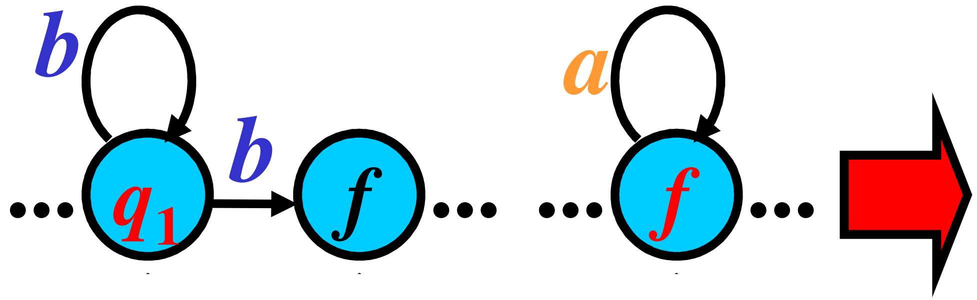
for $Q' = \{q_2, f\}$:

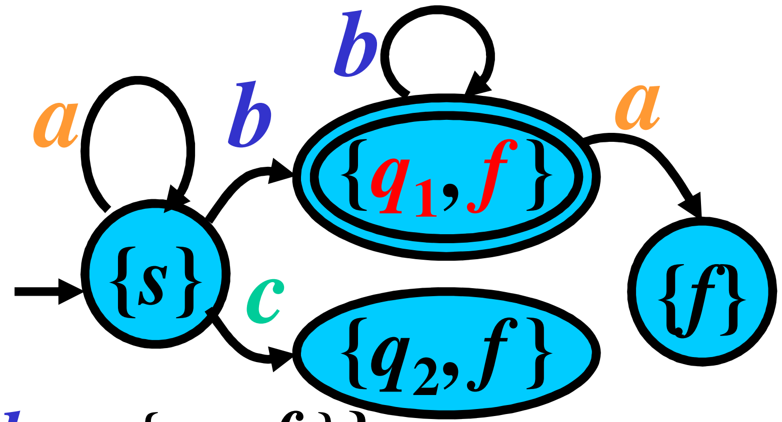




ε-free FA to DFA: Example 2/3

for $Q' = \{q_1, f\}$:





$$R_d := R_d \cup \{\{q_1, f\}a \to \{f\},\ \{q_1, f\}b \to \{q_1, f\}\}$$

$$Q_{new} = \{\{q_2, f\}, \{f\}\},\ Q_d = Q_d \cup \{\{q_1, f\}\}, F_d := \emptyset \cup \{\{q_1, f\}\}$$

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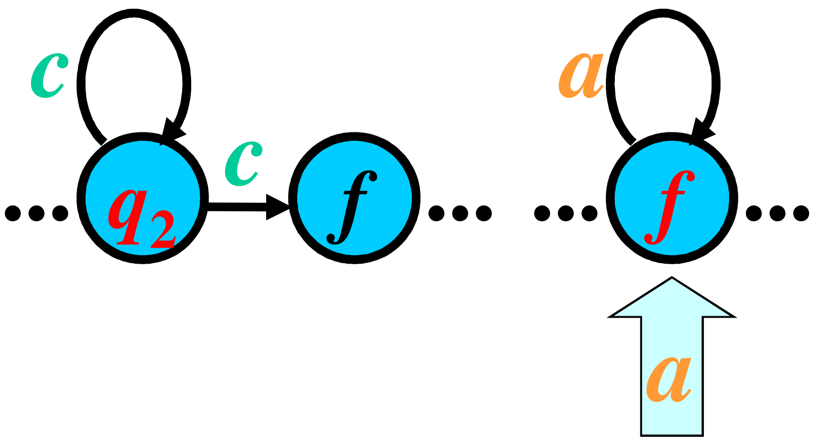
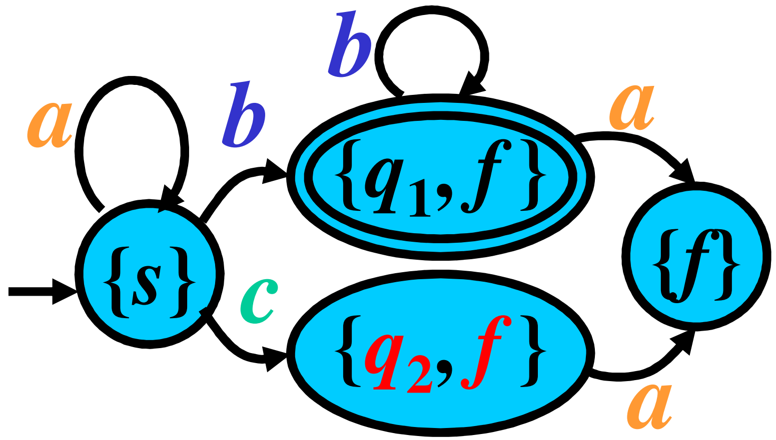
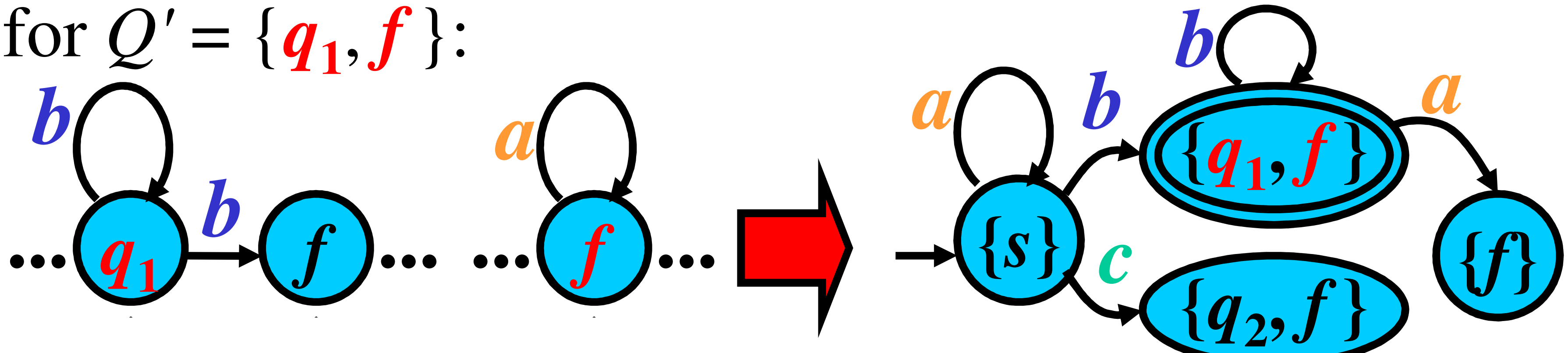
for $Q' = \{q_2, f\}$:





ε-free FA to DFA: Example 2/3

for $Q' = \{q_1, f\}$:

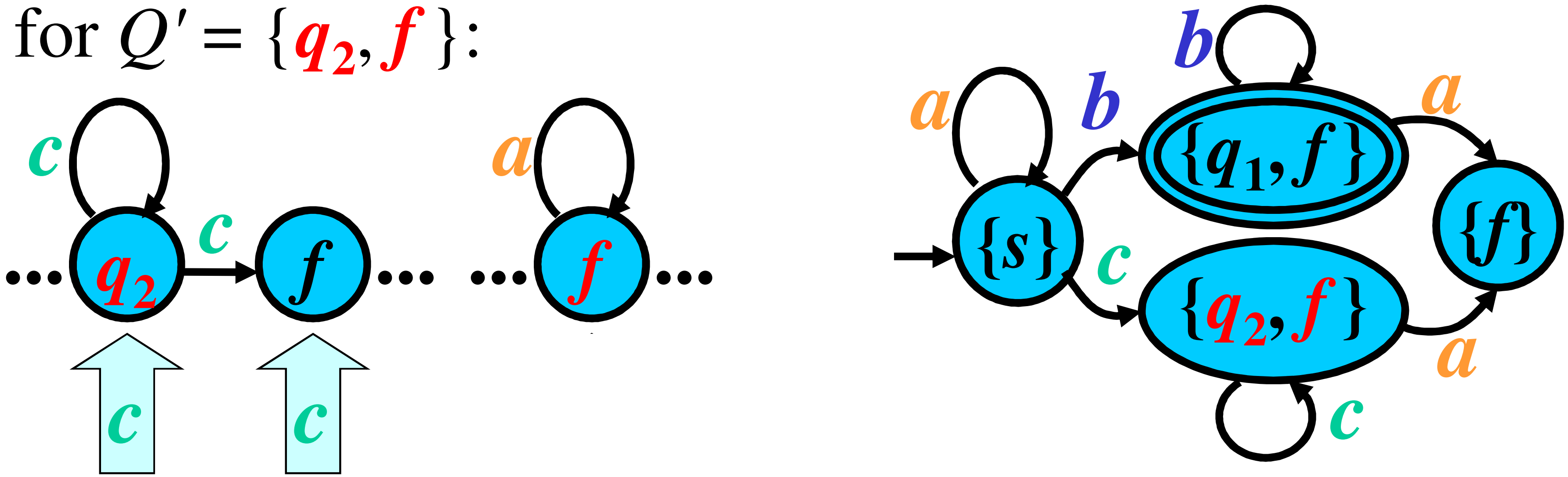


$R_d := R_d \cup \{\{q_1, f\}a \rightarrow \{f\}, \{q_1, f\}b \rightarrow \{q_1, f\}\}$

$Q_{new} = \{\{q_2, f\}, \{f\}\}$, $Q_d = Q_d \cup \{\{q_1, f\}\}$, $F_d := \emptyset \cup \{\{q_1, f\}\}$

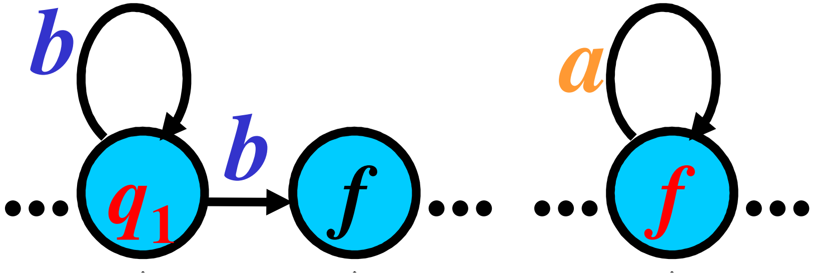
---

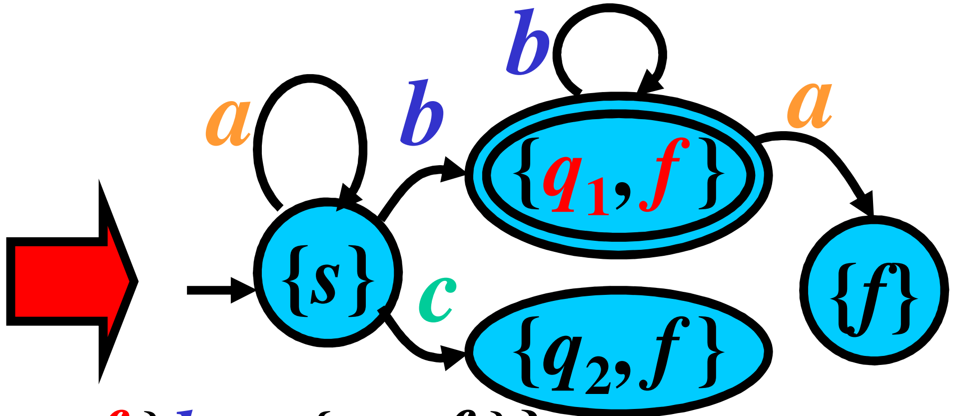
for $Q' = \{q_2, f\}$:



ε-free FA to DFA: Example 2/3

for $Q' = \{q_1, f\}$:





$R_d := R_d \cup \{\{q_1, f\}a \rightarrow \{f\}, \{q_1, f\}b \rightarrow \{q_1, f\}\}$

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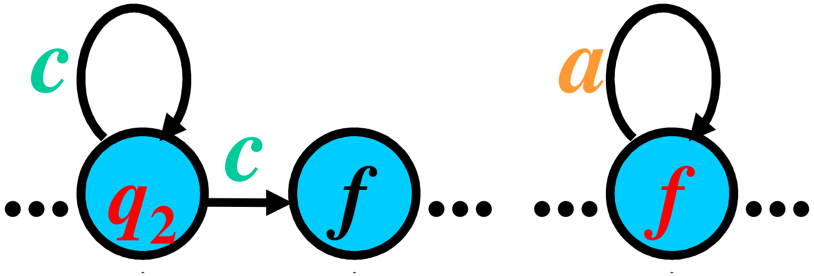
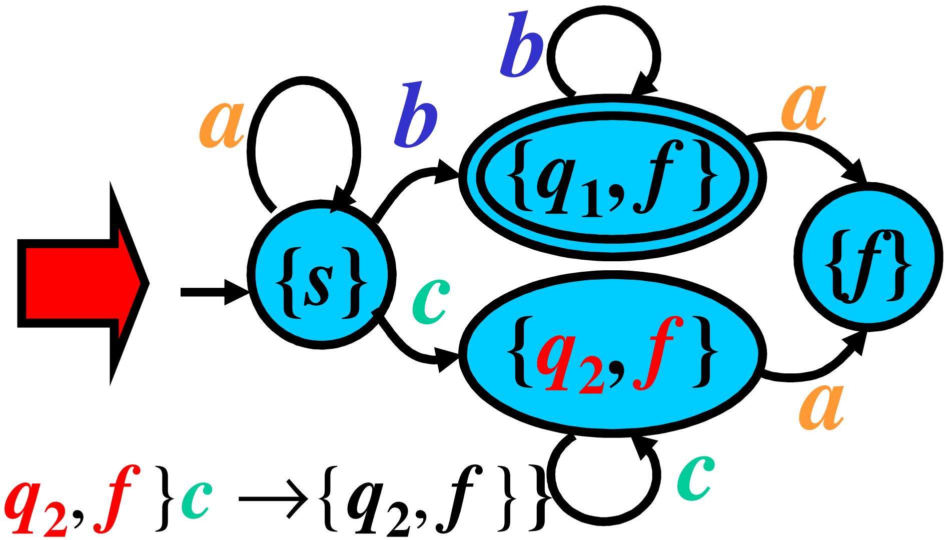
for $Q' = \{q_2, f\}$:





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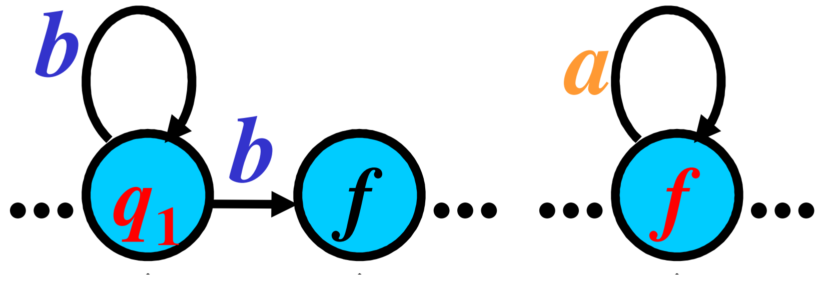
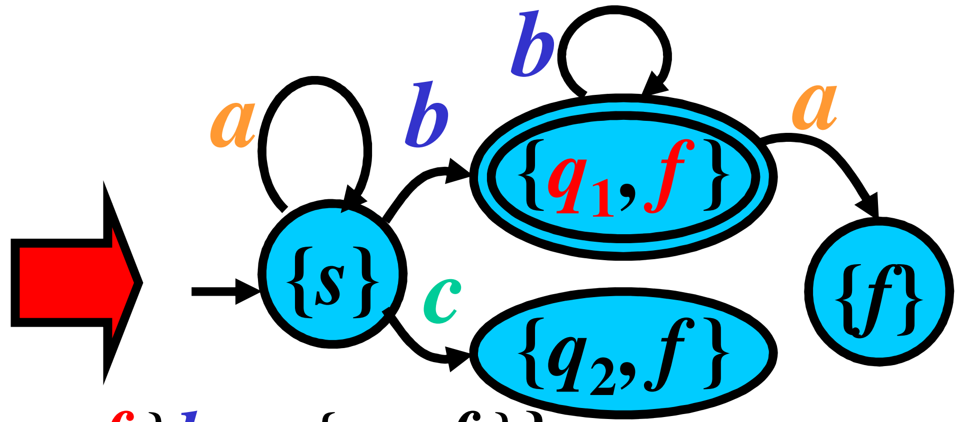
$Q_{new} = \{\{f\}\}, Q_d = Q_d \cup \{\{q_2, f\}\},$

ε-free FA to DFA: Example 2/3

for $Q' = \{q_1, f\}$:





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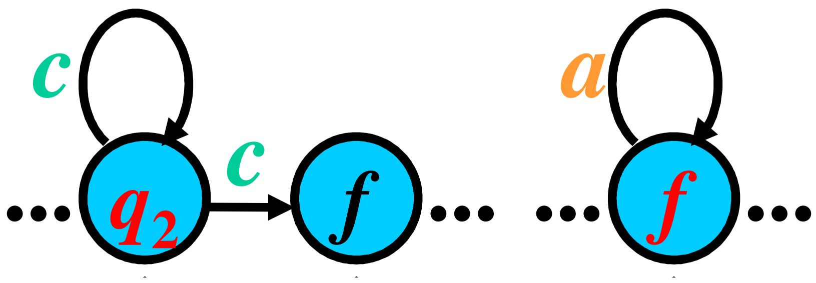
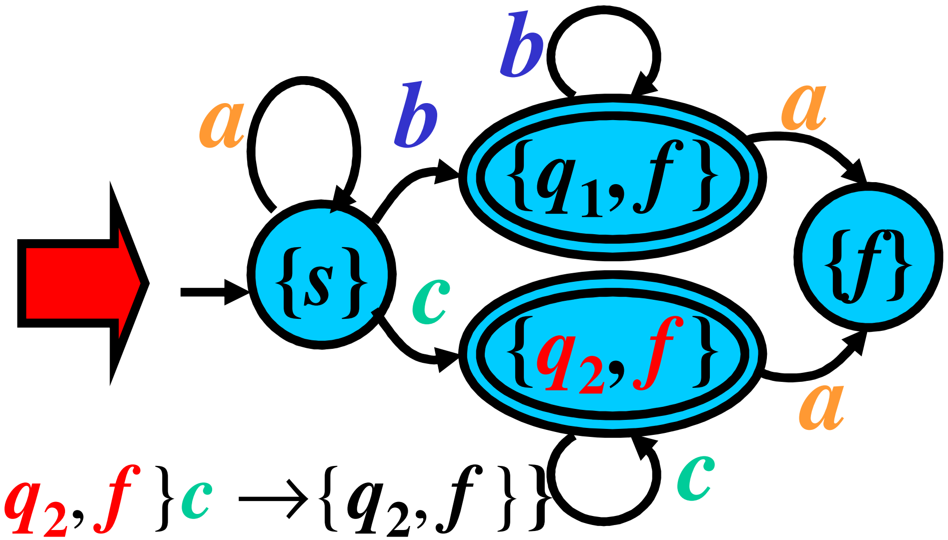
$Q_{new} = \{\{q_2, f\}, \{f\}\}, Q_d = Q_d \cup \{\{q_1, f\}\}, F_d := \emptyset \cup \{\{q_1, f\}\}$

---

for $Q' = \{q_2, f\}$:



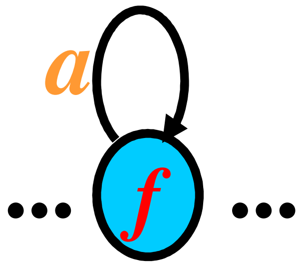


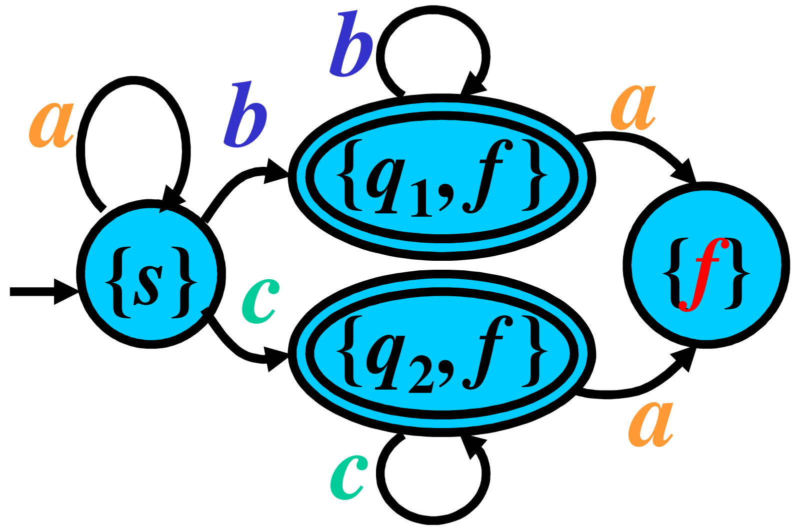
$R_d := R_d \cup \{\{q_2, f\}a \rightarrow \{f\}, \{q_2, f\}c \rightarrow \{q_2, f\}\}$

$Q_{new} = \{\{f\}\}, Q_d = Q_d \cup \{\{q_2, f\}\}, F_d := F_d \cup \{\{q_2, f\}\}$

ε-free FA to DFA: Example 3/3

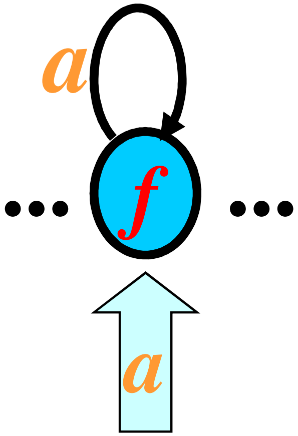
for $Q' = \{f\}$:

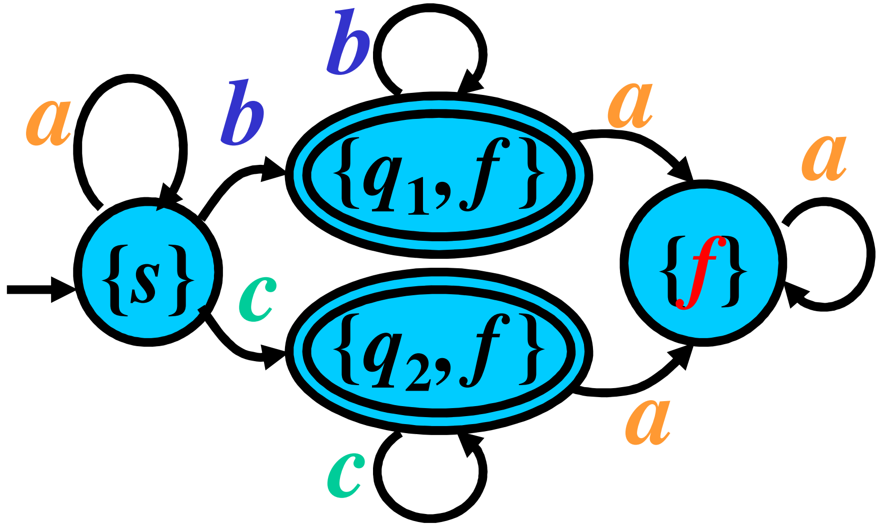




ε-free FA to DFA: Example 3/3

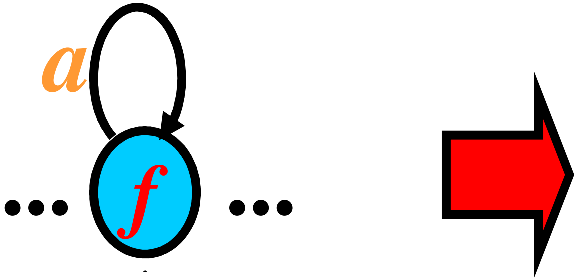
for $Q' = \{f\}$:

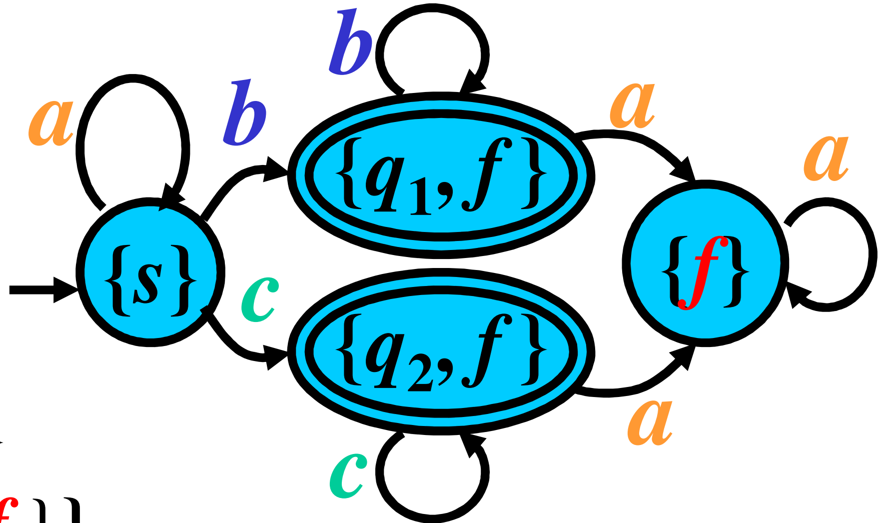




ε-free FA to DFA: Example 3/3

for $Q' = \{f\}$:





$R_d := R_d \cup \{\{f\}a \to \{f\}\}$

$Q_{new} = \emptyset$, $Q_d = Q_d \cup \{\{f\}\}$,

ε-free FA to DFA: Example 3/3

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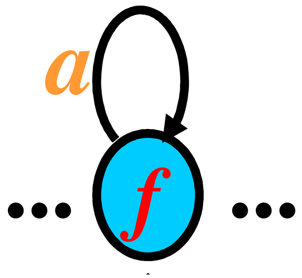

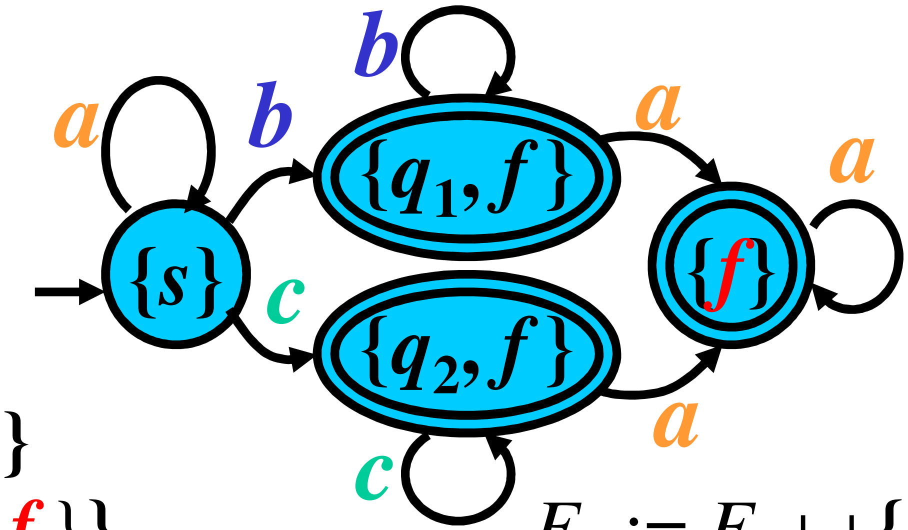

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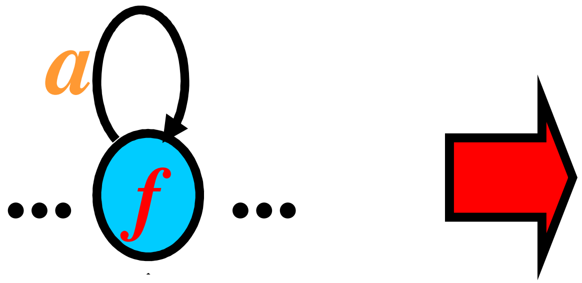

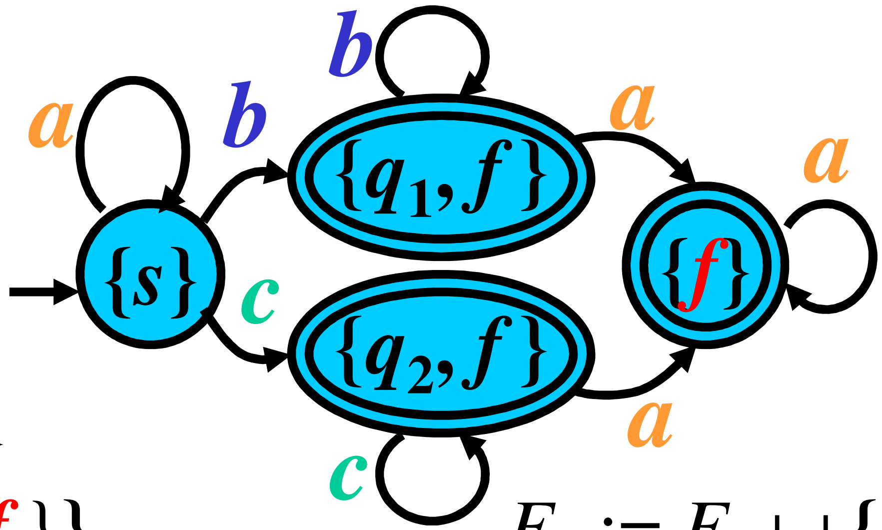

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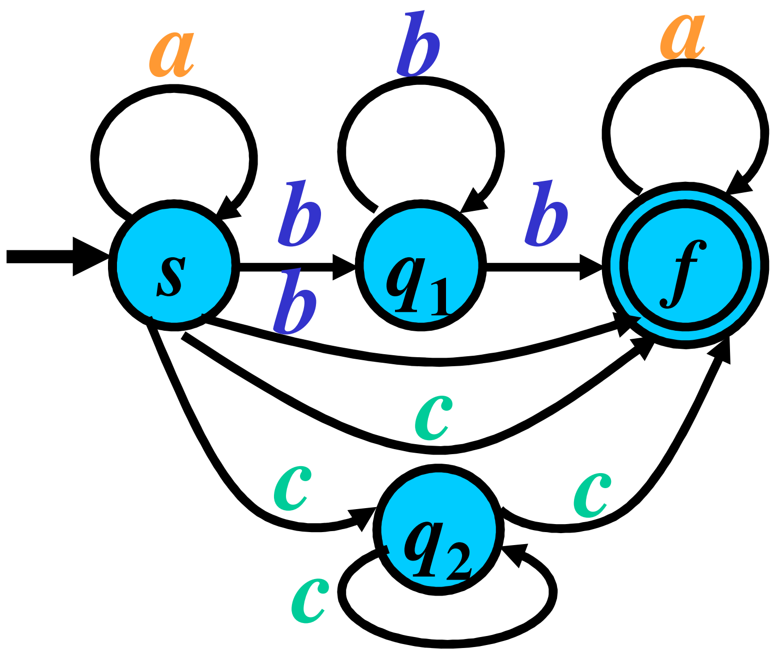
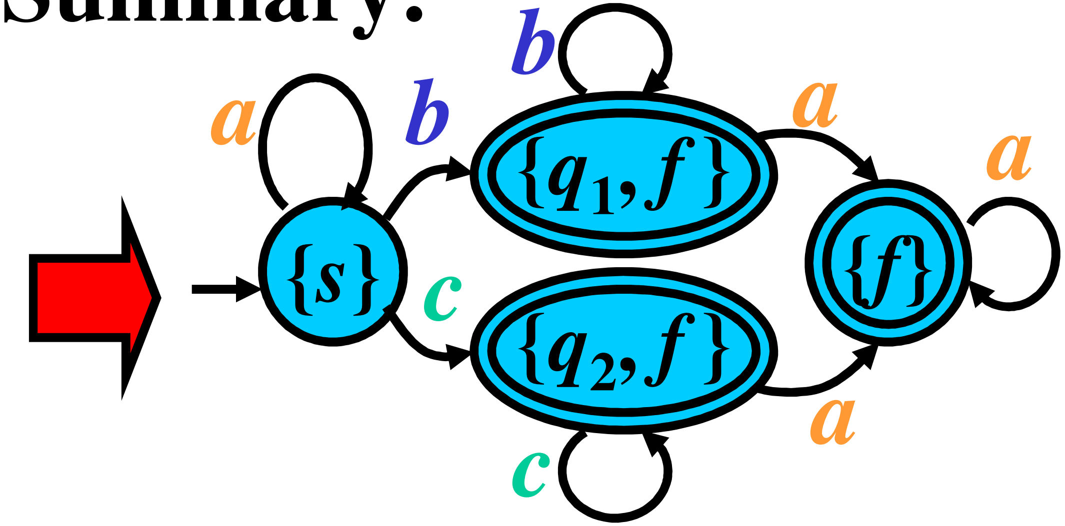




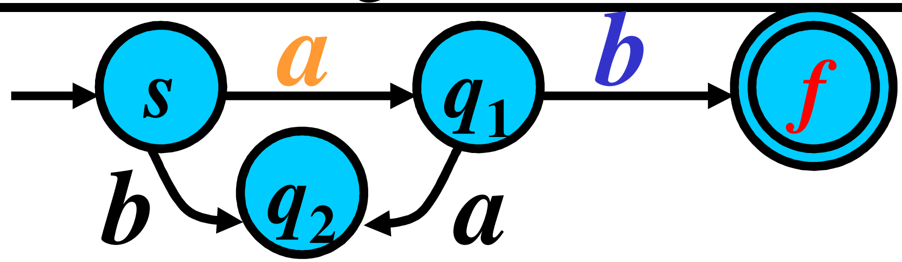
Terminating States

Gist: State q is *terminating* if a string takes DFA from q to a final state.

Definition: Let $M = (Q, \Sigma, R, s, F)$ be a DFA. A state $q \in Q$ is *terminating* if there exists $w \in \Sigma^*$ such that $qw \vdash^* f$ with $f \in F$; otherwise, q is *nonterminating*.

Note: Each nonterminating state can be removed from DFA

Example:



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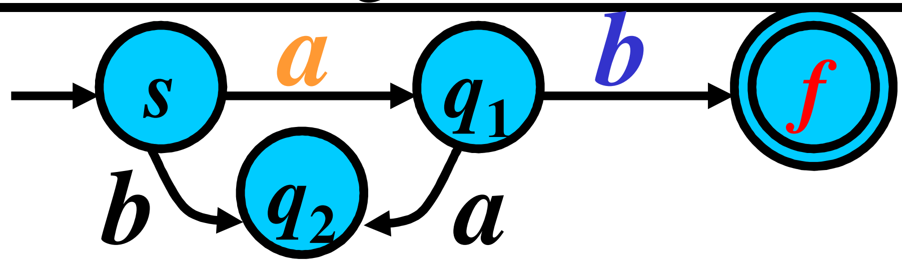

State s - terminating: $w = ab$: $sab \vdash q_1b \vdash f$

State q_1 - terminating: $w = b$: $q_1b \vdash f$

State f - terminating: $w = \varepsilon$: $f \vdash^0 f$

State q_2 - **nonterminating** (there is no $w \in \Sigma^*$ such that $q_2w \vdash^* q, q \in F$)

Algorithm: Removal of nont. states

- **Input:** DFA: $M = (Q, \Sigma, R, s, F)$
- **Output:** DFA: $M_t = (Q_t, \Sigma, R_t, s, F)$

- **Method:**
- $Q_0 := F$; $i := 0$;
- **repeat**

  $i := i + 1$;

  $Q_i := Q_{i-1} \cup \{q: qa \rightarrow p \in R, a \in \Sigma, p \in Q_{i-1}\}$;

until $Q_i = Q_{i-1}$;
- $Q_t := Q_i$;
- $R_t := \{qa \rightarrow p: qa \rightarrow p \in R,\ p, q \in Q_t, a \in \Sigma\}$.

Nonterminating States: Example

$M = (Q, \Sigma, R, s, F)$, where: $Q = \{s, q_1, q_2, f\}$, $\Sigma = \{a, b\}$, $R = \{sa \rightarrow q_1, sb \rightarrow q_2, q_1a \rightarrow q_2, q_1b \rightarrow f\}$, $F = \{f\}$

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$Q_1 = \{f\} \cup \{q_1\} = \{f, q_1\}$

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$qd \rightarrow q_1$; $q \in Q$; $d \in \Sigma$: $\quad sa \rightarrow q_1$

$Q_2 = \{f, q_1\} \cup \{q_1, s\} = \{f, q_1, s\}$

Nonterminating States: Example

$M = (Q, \Sigma, R, s, F)$, where: $Q = \{s, q_1, q_2, f\}$, $\Sigma = \{a, b\}$,
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3) $qd \rightarrow f$; $q \in Q$; $d \in \Sigma$: $\quad q_1b \rightarrow f$

$qd \rightarrow q_1$; $q \in Q$; $d \in \Sigma$: $\quad sa \rightarrow q_1$

$qd \rightarrow s$; $q \in Q$; $d \in \Sigma$: **none**

$Q_3 = \{f, q_1, s\} \cup \{q_1, s\} = \{f, q_1, s\} = Q_2 = Q_t$

Nonterminating States: Example

$M = (Q, \Sigma, R, s, F)$, where: $Q = \{s, q_1, q_2, f\}$, $\Sigma = \{a, b\}$,
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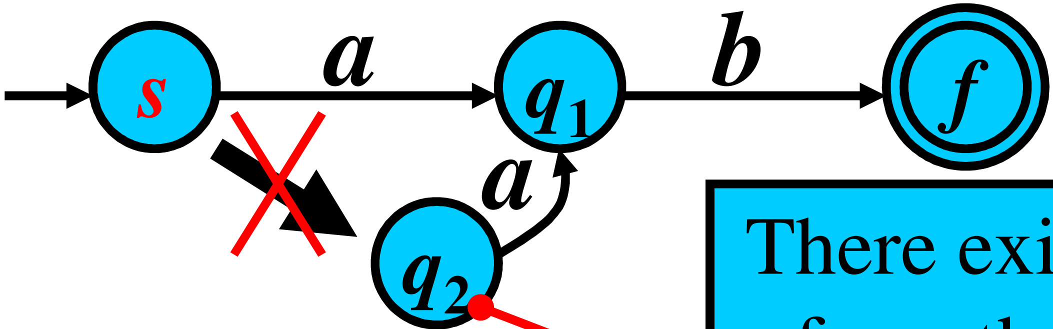
$qd \rightarrow s$; $q \in Q$; $d \in \Sigma$: **none**

$Q_3 = \{f, q_1, s\} \cup \{q_1, s\} = \{f, q_1, s\} = Q_2 = Q_t$

$R_t = \{sa \rightarrow q_1, \cancel{sb \rightarrow q_2}, \cancel{q_1a \rightarrow q_2}, q_1b \rightarrow f\}$

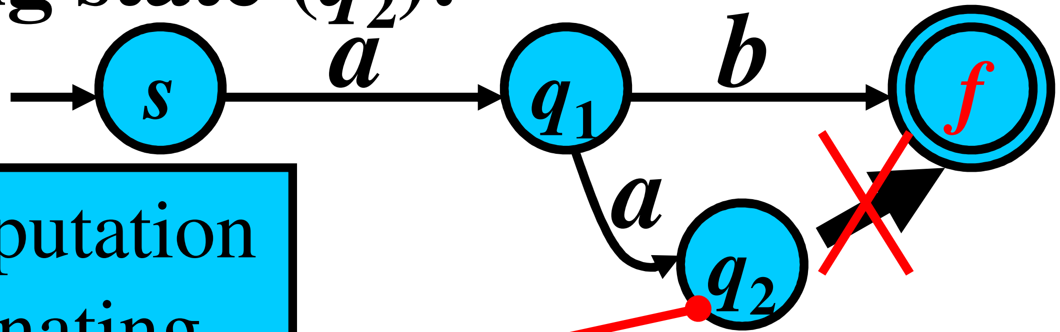
Summary: States to Remove

1) Inaccessible state (q_2):



There exists no computation from the start state to this inaccessible state.

2) Nonterminating state (q_2):



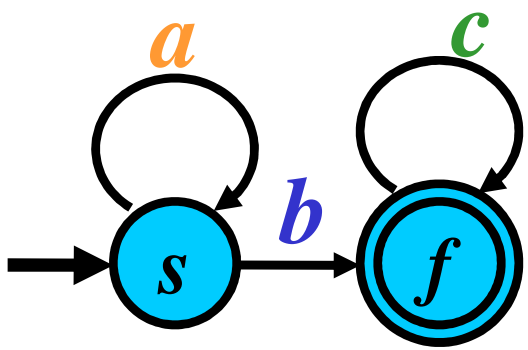
There exists no computation from this nonterminating state to a final state.

Complete DFA

Gist: Complete DFA cannot get stuck.

Definition: Let $M = (Q, \Sigma, R, s, F)$ be a **DFA**. M is ***complete***, if for any $p \in Q$, $a \in \Sigma$ there is exactly one rule of the form $pa \rightarrow q \in R$ for some $q \in Q$; otherwise, M is ***incomplete***

Conversion: **Incomplete DFA**



$\Sigma = \{a, b, c\}$

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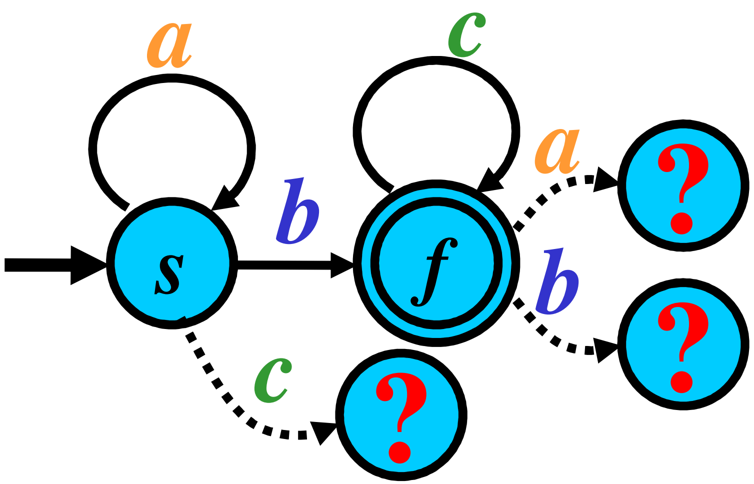

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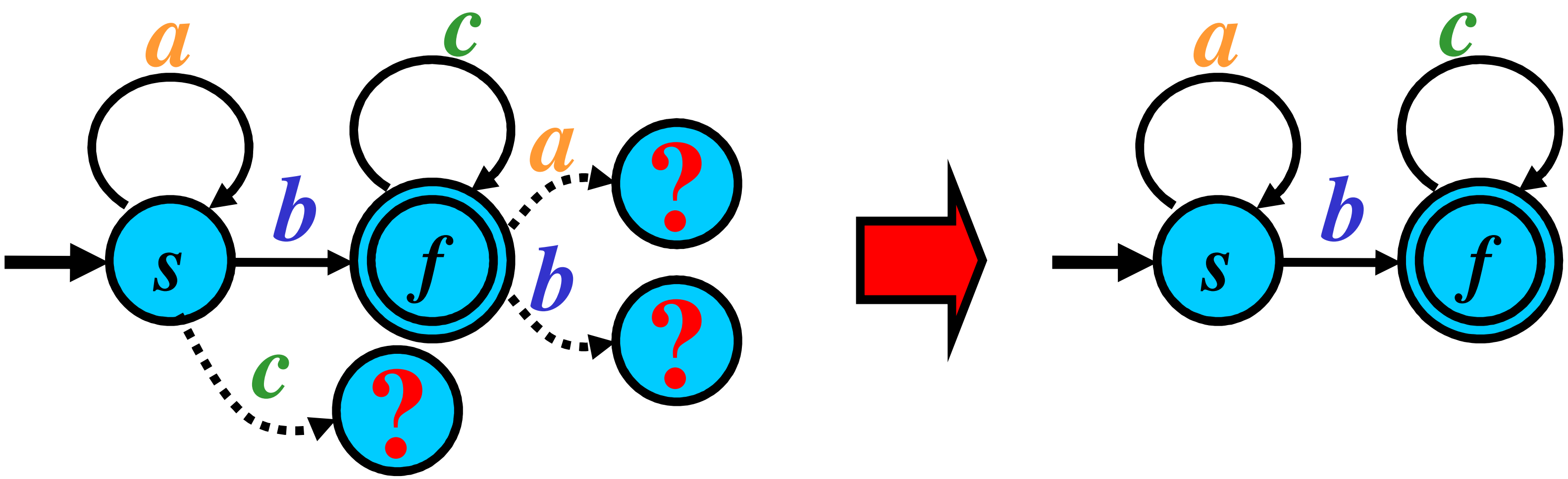
Conversion: **Incomplete DFA** **to Complete DFA**



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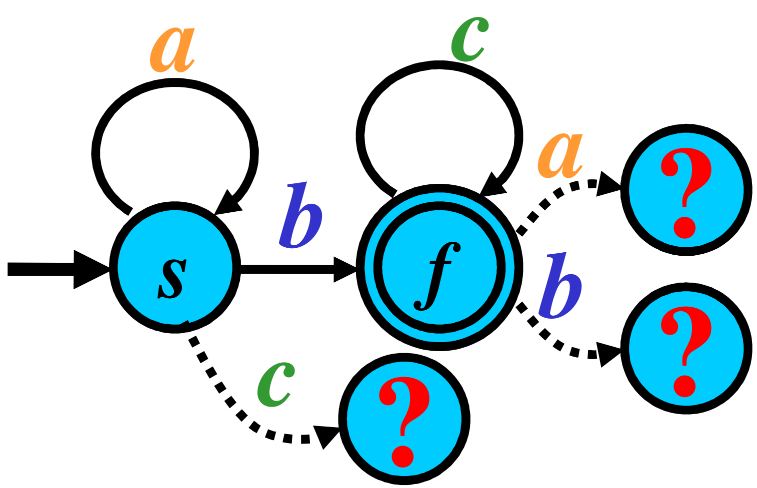

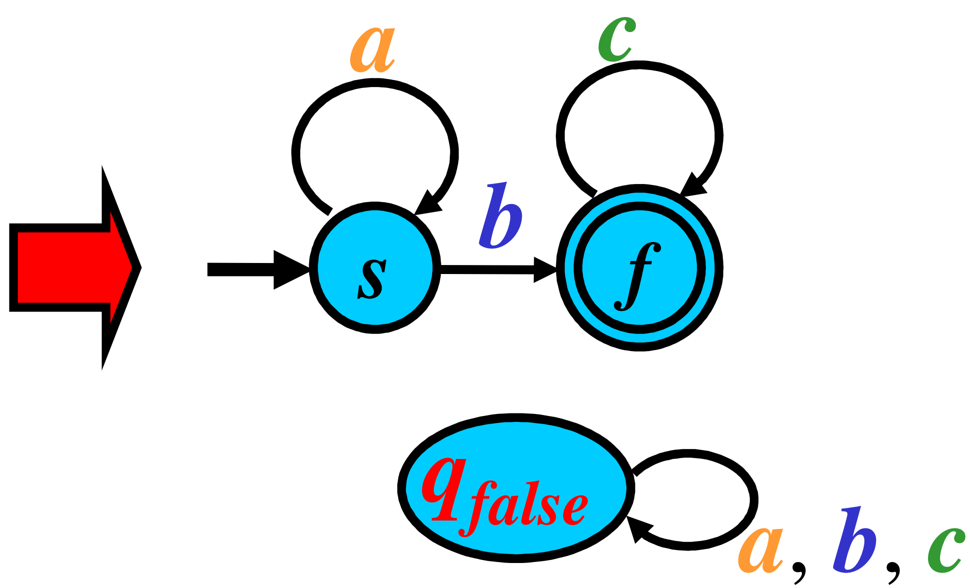

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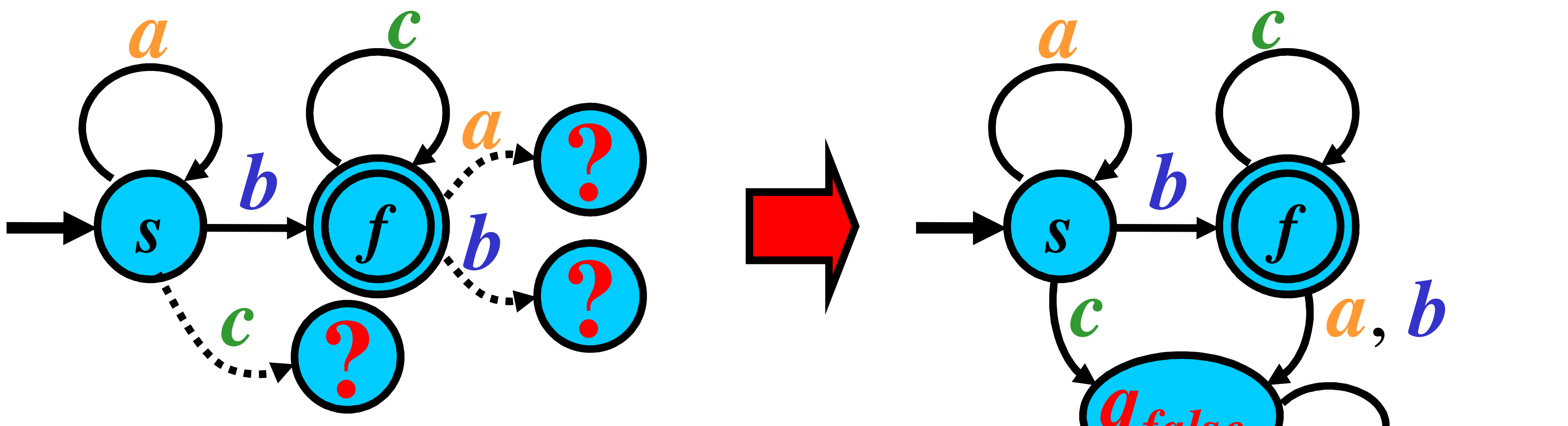

$\Sigma = \{a, b, c\}$

Algorithm: DFA to Complete DFA

Gist: Add a "trap" state

- **Input:** Incomplete DFA $M = (Q, \Sigma, R, s, F)$
- **Output:** Complete DFA $M_c = (Q_c, \Sigma, R_c, s, F)$

- **Method:**
- $Q_c := Q \cup \{q_{false}\}$;
- $R_c := R \cup \{qa \to q_{false}: a \in \Sigma, q \in Q_c,$

  $qa \to p \notin R, \; p \in Q\}$.

Well-Specified FA

Definition: Let $M = (Q, \Sigma, R, s, F)$ be a **complete DFA**. Then, M is ***well-specified FA*** (WSFA) if:

1) Q has no inaccessible state

2) Q has no more than one nonterminating state

Note: If well-specified FA has one nonterminating state, then it is q_{false} from the previous algorithm.

Theorem: For every FA M, there is an equivalent WSFA M_{ws}.

Proof: Use the next algorithm.

Algorithm: FA to WSFA

- **Input:** FA M
- **Output:** WSFA M_{ws}

- **Method:**
- convert a FA M to an equivalent ε-free FA M'
- convert a M' to an equivalent DFA M_d without any inaccessible state
- convert M_d to an equivalent DFA M_t without any nonterminating state
- convert M_t to an equivalent complete DFA M_c
- $M_{ws} := M_c$

Note: No more than one nonterminating state in M_{ws}—q_{false}

Variants of FA: Summary

	FA	ε-free FA	DFA	Complete FA	WSFA
Number of rules of the form $p \rightarrow q$, where $p, q \in Q$	**0-*n***	**0**	**0**	**0**	**0**
Number of rules of the form $pa \rightarrow q$, for any $p \in Q$, $a \in \Sigma$	**0-*n***	**0-*n***	**0-1**	**1**	**1**
Number of inaccessible states	**0-*n***	**0-*n***	**0-*n***	**0-*n***	**0**
Number of nonterminating states	**0-*n***	**0-*n***	**0-*n***	**0-*n***	**0-1**
Number of this FAs for any regular language.	∞	∞	∞	∞	∞