

**MIDDLE EAST TECHNICAL UNIVERSITY**  
**ELECTRICAL AND ELECTRONICS ENGINEERING DEPARTMENT**

**EE 568 Selected Topics in Electrical Machines**

**Final Examination**

18 June 2020

**Suggested Time: 300min**

**Exam Rules:**

<b>You are allowed to</b>	<b>You are NOT allowed to</b>
<ul style="list-style-type: none"><li>• Use any additional materials, books, etc.</li><li>• Use your laptops to access internet, download extra materials, run simulations</li><li>• Use scientific calculators for your calculations</li><li>• Have a short break, have a coffee etc.</li></ul>	<ul style="list-style-type: none"><li>• Use your mobile phones/tablets (not even as a calculator)</li><li>• Communicate (talk, instant messaging, email etc) within or outside the class.</li><li>• Sharing photos of the exam paper</li><li>• Getting closer than 1.5 m to anyone</li></ul>

**Student Name:** \_\_\_\_\_

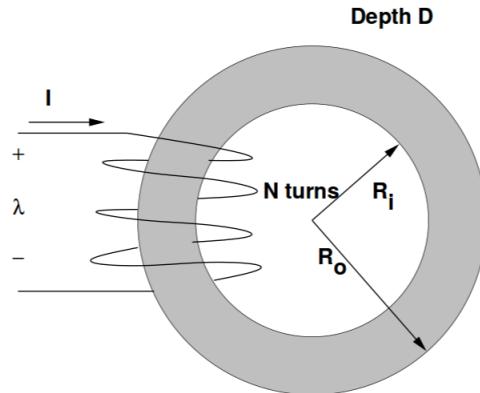
**Student ID:** \_\_\_\_\_

Q1	
Q2	
Q3	
Q4	
Total	

### **Q1 – (20 pts) Inductor Design**

Suppose that you have a toroidal core as shown in the figure below. You want to make an inductor of 10mH. Inner radius of the core is 9 cm and the outer radius is 11 cm and the depth(D) is 2 cm.

Assume the relative permeability of the core is 1400 and the core has a maximum flux density value of 1.6 T. Neglect the leakage flux throughout this problem.



**Part-I:** Assume the flux is homogeneously distributed along the core

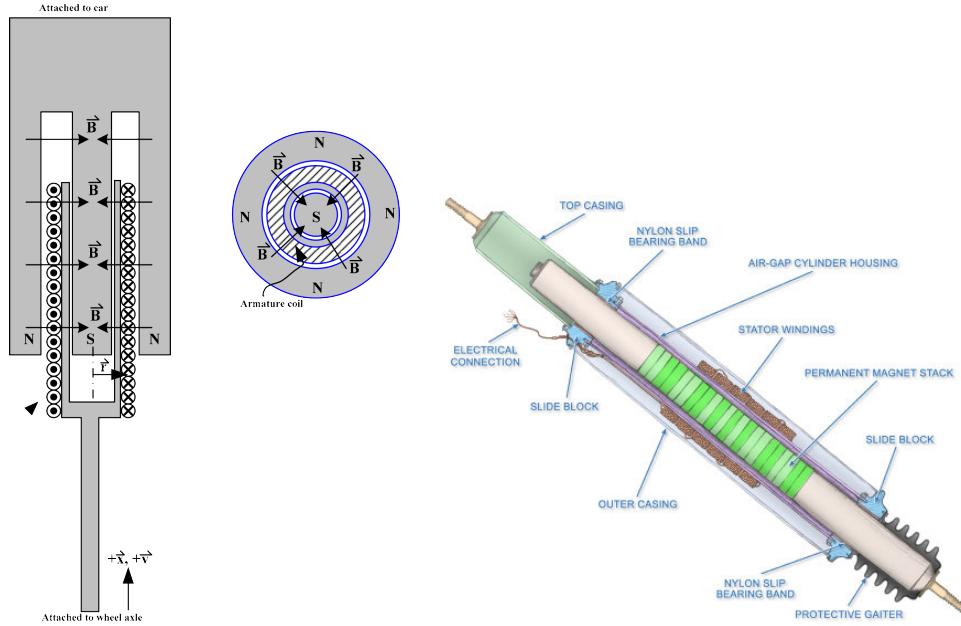
- How many turns are needed?
- What is the maximum current that the inductor can carry without saturating the core?
- Considering the fill factor and thermal issues, is that design feasible?

**Part-II:**

- Repeat the part(a) and part(b), but now the flux is NOT homogeneously distributed.
- Plot the  $H_{tan}$  and  $B_{tan}$  versus radius under this operating condition.

## Q2- (20pts) Linear Actuators

There are novel regenerative shock absorbers that use linear PM motors instead of conventional mechanical shock absorbers (Mercedes Magic Body Control uses a hydraulic version). Assume, a PM linear actuator as shown in the figure with PMs magnetized inwards and flux density is uniform in the coils.



Find the Lorentz force on the coil and vertical displacement of the absorber using the following parameters:

- $r = 10 \text{ cm}$ ,  $B = 1.5 \text{ T}$ ,  $N_{\text{coil}} = 30$ ,  $R = 0.5 \Omega$ , Applied voltage = 12 V
- Total height of the stator = 20 cm
- Spring stiffness: 100 N/cm (The spring is connected in parallel to the PM actuator. Neutral position of the spring is the mid-point of the stator.)

**Q3-(20 pts) Short Answer Questions**

Explain your reasoning as detailed as possible for full credit. Mathematical calculations are not required, but can be used to prove your point.

- a) Assume you have two permanent magnets motors with the exact same dimensions and windings. The only difference is one of the motor has magnets of N42H, but the other one is N42UH. Compare these two machines in terms of:
- i) Magnetic loading, electrical loading
  - ii) Insulation class, operating temperature
- b) What are real world correspondence of direct and quadrature axis inductances ( $L_d$ ,  $L_q$ ) of electric machines? Propose a experimental method to measure  $L_d$  and  $L_q$ .
- c) Assume you have an interior permanent magnet (IPM) machine. Can you generate any torque if all the magnets are demagnetized completely?
- d) What are the advantages and disadvantages of fractional pitched windings over integral slot windings?-

#### **Q4- (40 pts) Motor Analysis from Lamination Design**

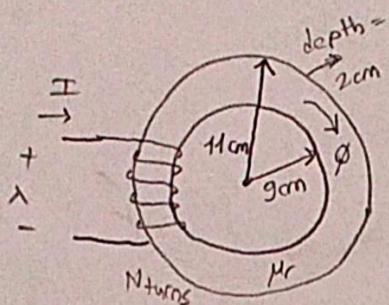
Assume you have the lamination design given as an attachment. The legend for the dimensions are also available. All dimensions are in mm.

- Assume M400-50A laminations are used in the machine (Datasheet is available)
- NdFeB magnets of N40UH are used in the slots.

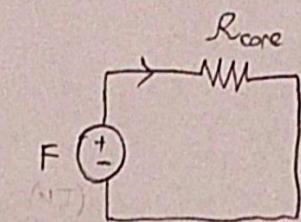
- How many poles and slots does the machine have? (Don't count, just read the legend)
- Sketch the magnetic equivalent circuit for one pole and roughly calculate the peak airgap flux density (please clearly state any assumptions you make). Estimate the peak flux density at the stator teeth and at the back-core.
- The slot for the magnets are slightly larger than the actual magnets. What is the purpose of the extra air-region next to the magnets?
- Design a winding diagram for this machine. Draw the winding diagram or show the phasors for the induced voltages to clearly describe the winding diagram. You can simplify your diagrams by using any symmetry in your winding designs. Calculate the winding factor for the fundamental component. Estimate
- Choose an axial length and rated speed for this machine. Using the standard values (electrical loading, magnetic loading etc.) estimate the power output for this machine.
- Choose a rated voltage (you can use any standard values), then choose the number of turns per slot and standard wire size for this machine. Clearly state any assumptions such as fill factor, current density etc. Verify the electrical loading value you had chosen in part (c).

## - Solutions -

Q1)  $L = 10 \text{ mH}$ ,  $\mu_r = 1400$ ,  $B_{\max, \text{core}} = 1,6 \text{ T}$ , neglect leakage flux.



→ magnetic model



$$F = \int H \cdot dL = NI$$

$$\mu = \mu_r \mu_0 = 1,75 \times 10^{-3}$$

$\downarrow$

$$4\pi \times 10^{-7}$$

Part - 1 :

a)  $L = \frac{d\lambda}{di} \rightarrow$  flux linkage change  
 $\lambda = \frac{d\lambda}{di} \rightarrow$  current change

$$\lambda = \underbrace{N \phi}_{\text{turn}} =$$

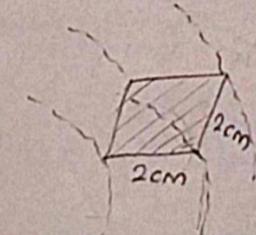
$\downarrow$

$$\lambda = \frac{N^2 \mu I A}{l}$$

$\phi = B \cdot A$   
 flux  
 flux density  
 (T)  
 ↓  
 cross sectional  
 area of core  
 (m²)

$$L = \frac{\lambda}{I} = \frac{N^2 \mu A}{l}$$

Core Cross sectional area :



$$A = 2 \times 10^{-2} \times 2 \times 10^{-2} = 4 \times 10^{-4} \text{ m}^2$$

$$L = 10 \times 10^{-3} \text{ H} = \frac{N^2 \times 1,75 \times 10^{-3} \times 4 \times 10^{-4}}{0,62}$$

$$N^2 = 8857,14$$

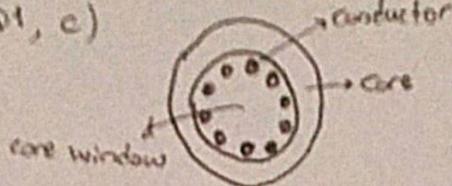
$$N = 94,11 \text{ turns.}$$

$$\begin{aligned} B &= \mu H = \frac{\mu N i}{l} \\ &\downarrow \\ &= 2\pi (0,11 + 0,09) \\ &= 0,62 \text{ m.} \end{aligned}$$

b) Max flux density of the core is  $B_{\max} = 1,6 \text{ T}$

$$\begin{aligned} B &= \mu H = \frac{\mu N i}{l} \Rightarrow 1,6 \text{ T} = \frac{1,75 \times 10^{-3} \times 94,1 \times i}{0,62} \Rightarrow i_{\max} = 6,02 \text{ A} \\ &\downarrow \\ &1,6 \text{ T (max)} \end{aligned}$$

Q1, c)



Area of single conductor can be selected with respect to max current level. From AWG table,

#19 or #20 can carry 5A @ 60°C  
11A @ 75°C

Area of this conductors

$$\text{Area } \#19 : 0,653 \text{ mm}^2 \rightarrow \text{total area} : N \times \text{Area } \#19 = 67,64$$

$$\text{Area } \#20 : 0,518 \text{ mm}^2 \rightarrow \quad ; N \times \text{Area } \#20 = 48,74$$

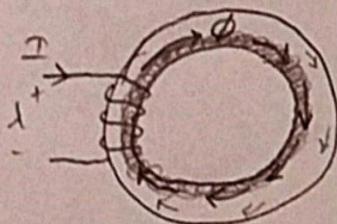
$$\text{Area of core window} = \pi r^2 = \pi (90 \text{ mm})^2 = 25446 \text{ mm}^2 \quad (\text{mm}^2)$$

$$\text{fill factor} = \frac{N \times \text{Area of cond.}}{\text{Area of window}} \Rightarrow \begin{array}{l} \text{for AWG } \#19 \rightarrow k_{\text{fill}} = 2,41 \times 10^{-3} \ll 0,5 \\ \text{AWG } \#20 \rightarrow k_{\text{fill}} = 1,91 \times 10^{-3} \ll 0,5 \end{array}$$

In general max. fill factor is limited with 0,5 in designs. So, this inductor is feasible in thermal and mechanical considerations.

Part II :

- d) In this case flux prefers the shortest path on the core which is near of inner radius  $9 + \delta$  cm where  $\delta \ll 1$ .



So, mean flux path length is shorter. If we assume distribution of the flux concentrate around of inner radius;

$$\text{mean flux path} = l = 2\pi (0,09) = 0,56 \text{ m.}$$

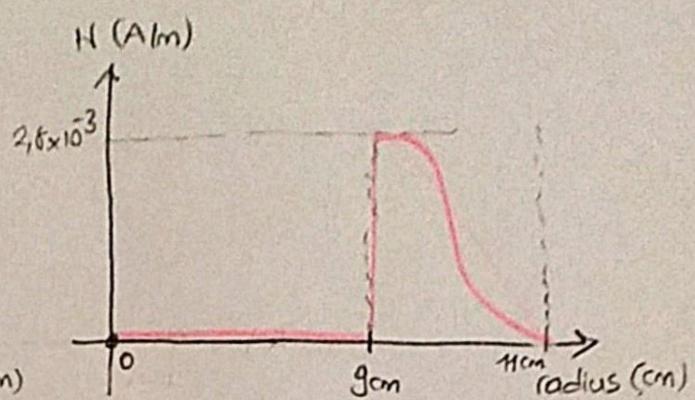
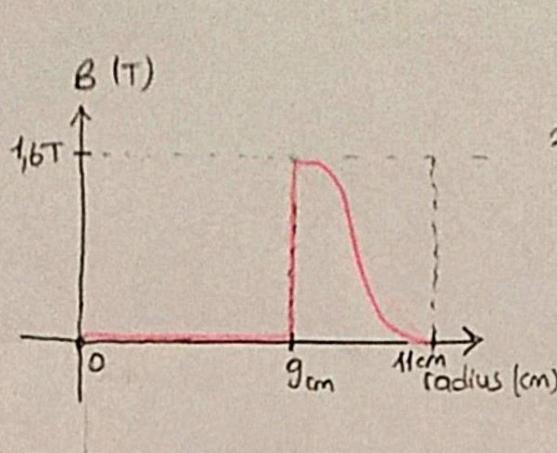
$$L = 10 \times 10^{-3} \text{ H} = \frac{N^2 \times 1,75 \times 10^{-3} \times 4 \times 10^{-4}}{0,56} \rightarrow N^2 = 8000, \quad N = 89,44 \text{ turns}$$

$$B = \mu H = \frac{\mu N i}{l} \Rightarrow 1,6T = \frac{1,75 \times 10^{-3} \times 89,44 \times i}{0,56} \Rightarrow i_{\text{max}} = 5,72 \text{ A}$$

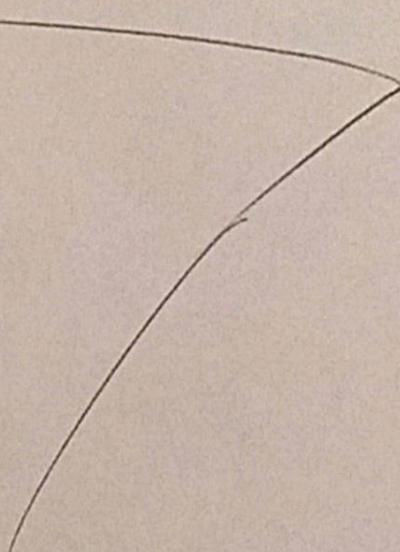
1,6T(max)

Because of the concentrated flux around of inner radius (shortest path), the core saturation can be reachable with lower current level than previous calculation. It sense logic because, homogeneous distribution condition, flux can travel via all part of core material but second (real) case flux prefers the shortest path so it saturates core with a lower current.

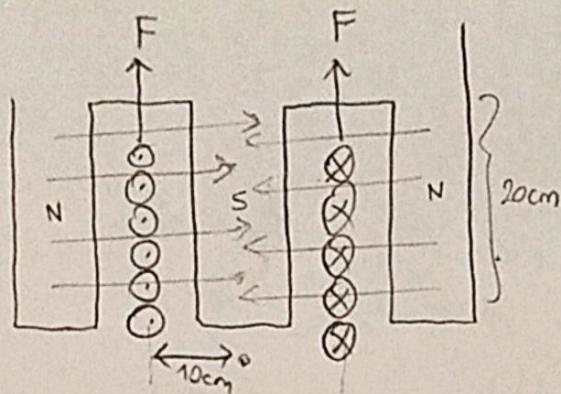
Q1, e)



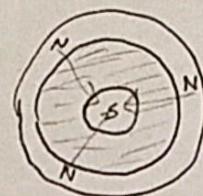
flux is not homogeneously distributed condition



Q2)



$$\text{Lorenz force} = B \times i \times l$$



$$B = 1,5 \text{ T}, R = 0,5 \Omega, V = 12 \text{ V}, k_s = 100 \text{ N/cm}$$

the current passing through the coil is  $= 12 / 0,5 = 24 \text{ A}$ .

$$\text{length of one turn} = 2\pi r = 62,83 \text{ cm}$$

$\downarrow$   
10cm

$$F_{\text{single}} \Rightarrow \text{the force from single turn (coil)} = 1,5 \times 24 \times 0,62 = 22,32 \text{ N}$$

$$\text{total force} = N \times F_{\text{single}} = 669,6 \text{ N} \rightarrow \text{this is the case that all coils are in stator.}$$

$$\text{Spring force} = F_s = k_s \times \Delta x$$

$$\text{Lorenz force} = 1,5 \times 24 \times 22,32 \times \text{coils in stator.}$$

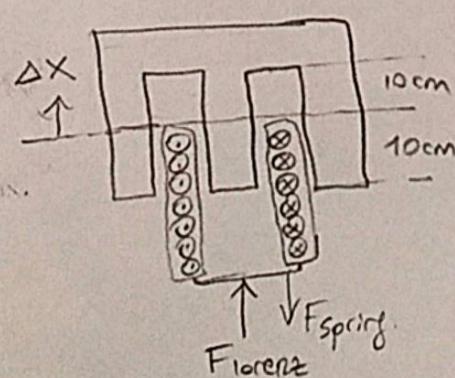
$$\text{Coils in stator} \Rightarrow 30 \text{ turn} = 20 \text{ cm}$$

$$1 \text{ turn} = \frac{2}{3} \text{ cm}$$

$$\Rightarrow \Delta x / \frac{2}{3} \text{ turn}$$

if there is no force come from wheels  
 $-F_s = F_{\text{Lorenz}}$

Neutral point of the spring is the mid point of stator



$$-k_s \times \Delta x = B \times i \times F_{\text{single}} \times \left( \frac{10 \text{ cm}}{\frac{2}{3}} + \frac{\Delta x}{\frac{2}{3}} \right)$$

$$-100 \text{ N/cm} \cdot \Delta x = 1,5 \times 24 \times 22,32 \times \left( 15 + \frac{\Delta x}{\frac{2}{3}} \right)$$

803,502 (N/cm)

$$0 = 12052,8 \text{ N} + 1305,25 \Delta x$$

$$\Delta x = -9,23 \text{ cm}$$

if there is no any force from wheel, and neglect the mass of linear rotor  
 the linear rotor moves from mid point to  $\Delta x$  direction or 9,23 cm.

	<u>1<sup>st</sup> machine</u>	<u>2<sup>nd</sup> machine</u>
Q3) a)	N42H up to 128°C $B_r = 1.3\text{ T}$ intrinsic Coer. (kA/m) = 1350	N42 UH up to 180°C $B_r = 1.3\text{ T}$ intrinsic coercivity (kA/m) = 1389
Magnetic Loading = $\bar{B}$ =	$\frac{\text{total flux}}{\text{total area of airgap}}$	$= \frac{P Q_p}{\pi D_1 L} \rightarrow \text{pole flux}$ machine length inner diameter slot number
Electrical Loading = $\bar{A}$ =	$\frac{\text{total current}}{\text{Rotor circumference}}$	$= \frac{N_{\text{slot}} \times Q \times i}{\pi D_1} \rightarrow \text{current}$

- i) Up to 128°C operation temperature, they have same magnetic and electrical loading performance. After 128°C, 1<sup>st</sup> machine can not handle required magnetic and electrical loading so it needs cooling operation because even if its windings have 128°C durable insulation, the magnets will be demagnetize.
- ii) 1<sup>st</sup> machine insulation class must be Class A : 105°C, because after 128°C operating condition, the PMs will be demagnetize and loss its performance. So, even if the current increases, magnets can not provides required flux for required performance (under no cooling condition). If we select the class B : 130°C, at this time machine works but for all 120°C - 130°C operating times, magnet lifes will be shorter. If you want to reliable, safe machine, insulation class must be 130°C but machine operating condition must be define under 128°C. If you consider cost, insulation class must be class A : 105°C and operation conditions must be defined under 105°C.

Q3, continue) for 2<sup>nd</sup> machine, insulation class must be Class H: 180°C.

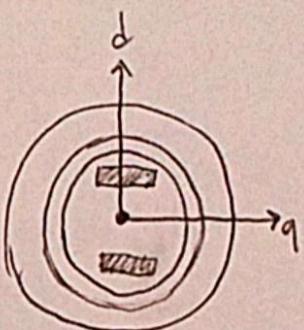
If we summarize.

1<sup>st</sup> machine, Class A: 105°C , max operating temp = 105°C  
with safety and reliability tolerance. But low performance

2<sup>nd</sup> machine, Class H: 180°C , max op. temp. = 180°C

Alternatively.

1<sup>st</sup> machine, class B: 130°C , max. op. temp = 120°C  
with safety and reliability tolerance, High performance



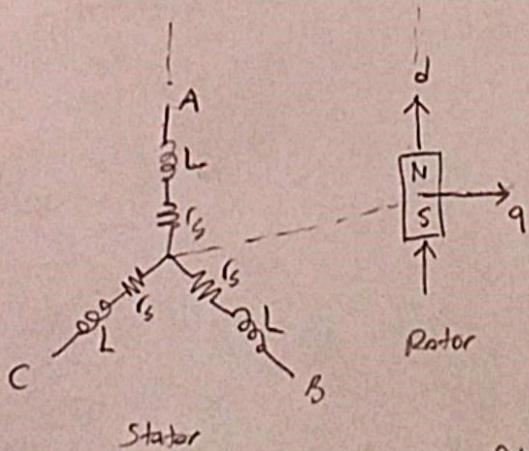
- b)  $L_d \rightarrow$  Inductance related with magnetic axis direction  
 $L_q \rightarrow "$  non magnetic axis direction.

$L_q$  is related with reluctance torque production. (Saliency)

$L_d$  is related electromagnetic torque production.

$L_d, L_q$  are representation of the machine inductances in synchronously rotating ref. frame.

$L_d$  and  $L_q$  values depend on rotor position,  $\theta_{elec}$ . If we can align the rotor magnetic axis to phase A axis of stator like in figure,



$$L_d = \frac{2}{3} L \rightarrow \theta_{elec} = 0^\circ$$

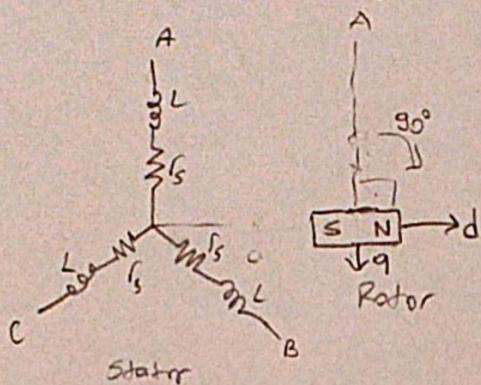
$L$  can be measured by with step excitation to phase A, phase B and C are grounded. Rotor is locked at that position.

So RL circuit is obtained.  $i_d = \frac{V}{R} (1 - e^{-t \frac{R}{L}})$

$$L_d = \frac{2}{3} L$$

Q3, continue 2 )

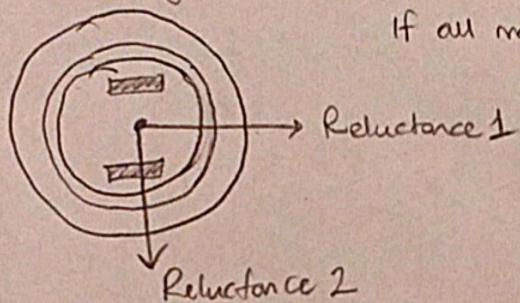
b) Similarly measuring the  $L_q$ ,  $\theta_{elec}$  is set  $90^\circ$



$$L_q = \frac{2}{3} L \rightarrow \theta_{elec} = 90^\circ$$

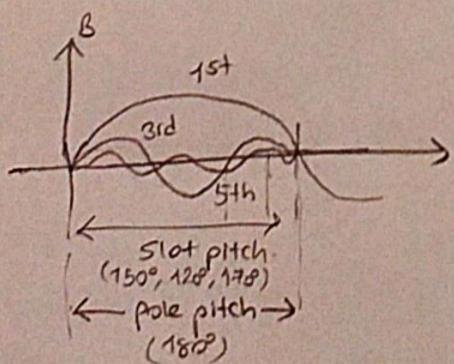
$L$  can be measured by step excitation to phase A, phase B and C are grounded. Rotor is locked at  $90^\circ$  first order RL circuit is solved.  $L$  is found.

c) Yes, we can generate a small amount torque because of saliency.



If all magnets are demagnetized, rotor will be salient. so  $R_2 > R_1$  and, small amount of reluctance torque is produced.

d) In fractional slot winding we can set the pitch angle and eliminate the effects of harmonics. Also, cogging torque can prevent or limited by using fractional slot winding. Another important advantage of fractional slot winding is, better emf waveform in case of generators (it is related with harmonic elimination)



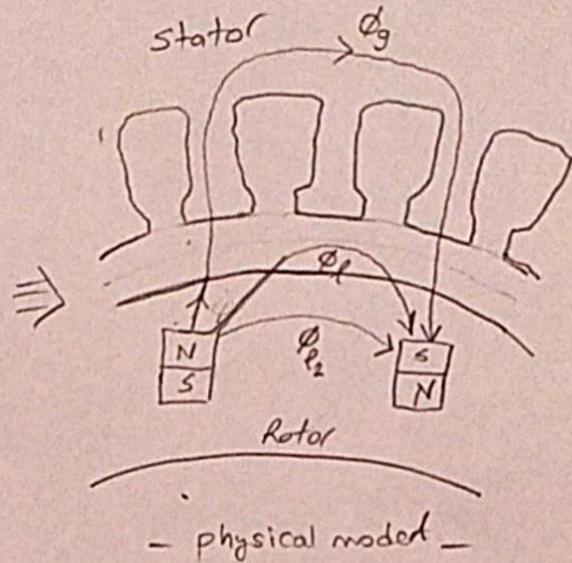
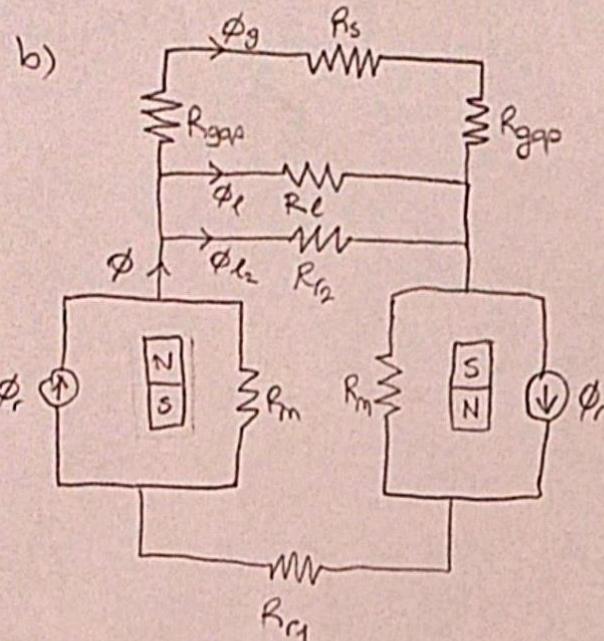
Q4)

from legend :

a)  $250 / 12 = 270 \rightarrow$  Core length  
 ↓  
 frame height pole number

N	A	b <sub>2</sub>
54	—	—

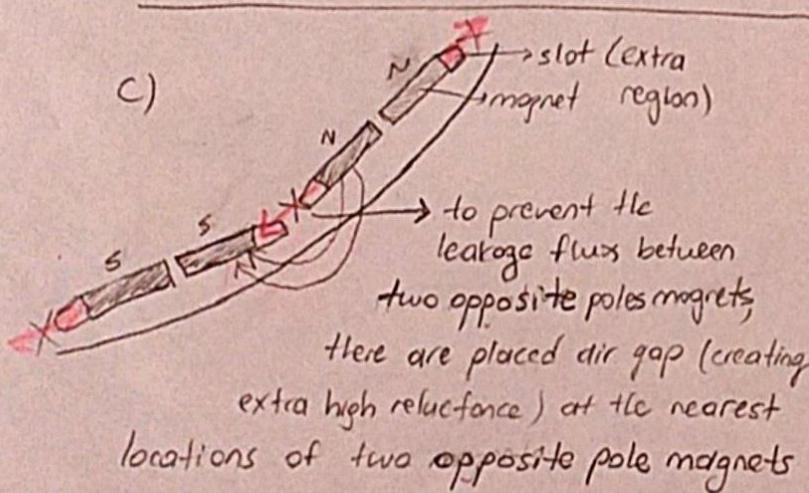
54 is Slot number ✓



- physical model -

- magnetic circuit -

c)



In the schematic, 4 stator teeth against the one pole.

B<sub>sat</sub> level is 1,8 T is given in Datasheet.

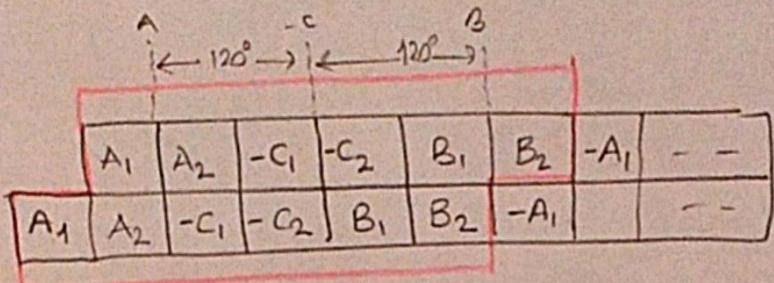
So we can say the back core flux density is nearly 1,8 T,

So, this flux goes through via 4 teeths, so teeth flux density must be nearly  $1,8 \text{ T} / 4 = 0,45 \text{ T}$

air gap flux density will be  $0,9 \text{ T} = 1,8 \text{ T} / 2$

d) 54 slots, 12 pole, 3 phase

$q = 3/2$  slot / per pole / per phase, double layer. 4.5 slots / per pole



e) Axial length  $\rightarrow$  270 mm, rated speed  $\rightarrow$  1200 rpm.

f) ? Time is up :(