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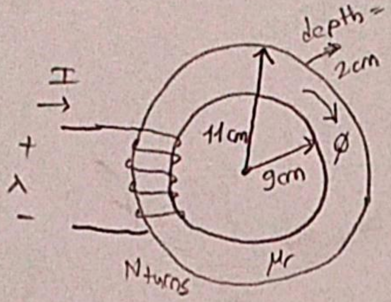
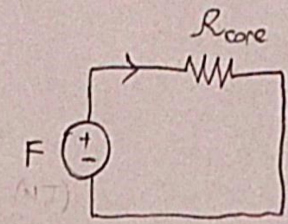
METU EE568 - Final Exam - 18.06.2020

- Solutions -

Q1) $L = 10\,mH$, $\mu_r = 1400$, $B_{max,core} = 1{,}6\,T$, neglect leakage flux.



→ magnetic model



$F = \int H.dl = NI$

$\mu = \mu_r \mu_0 = 1{,}75 \times 10^{-3}$

$\mu_0 = 4\pi \times 10^{-7}$

Part - 1 :

a) $L = \dfrac{d\lambda}{di}$ → flux linkage change, → current change → $\lambda = N \phi =$ (turn)

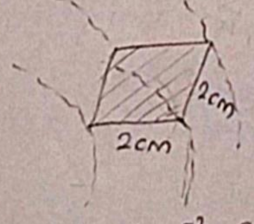
$\phi = B.A$ — ϕ: flux; B: flux density (T); A: crossectional area of core (m^2)

$B = \mu H = \dfrac{\mu N I}{l}$ — l: mean flux path length $= 2\pi \left(\dfrac{0{,}11 + 0{,}09}{2}\right) = 0{,}62\ m.$

$\lambda = \dfrac{N^2 \mu I A}{l}$

$$\boxed{L = \frac{\lambda}{I} = \frac{N^2 \mu A}{l}}$$

Core Crossectionel area :



$A = 2 \times 10^{-2} \times 2 \times 10^{-2} = 4 \times 10^{-4}\ m^2$

$L = 10 \times 10^{-3}\ H = \dfrac{N^2 \times 1{,}75 \times 10^{-3} \times 4 \times 10^{-4}}{0{,}62}$

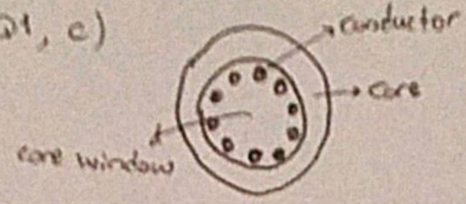
$N^2 = 8857{,}14$

$N = 94{,}11$ turns.

b) Max flux dusity of the core is $B_{max} = 1{,}6\,T$

$B = \mu H = \dfrac{\mu N i}{l}$ (B → $1{,}6\,T$ (max)) $\Rightarrow 1{,}6\,T = \dfrac{1{,}75 \times 10^{-3} \times 94{,}1 \times i}{0{,}62} \Rightarrow i_{max} = 6{,}02\ A$

Q1, c)



Area of single conductor can be selected with respect to max current level. from AWG table,

#19 or #20 can cary 5A @ 60°C
11A @ 75°C

Area of this conductors

Area #19 : 0,653 mm² → total area : N x Area #19 = 61,44
Area #20 : 0,518 mm² → " ; N x Area #20 = 48,74
(mm²)

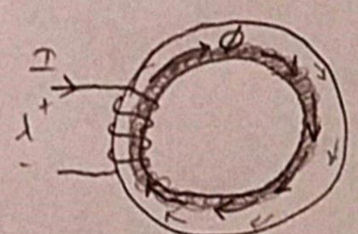
Area of core window $= \pi r^2 = \pi (90mm)^2 = 25446\ mm^2$

$$\text{fill factor} = \frac{N \times \text{Area of cond.}}{\text{Area of window}} \Rightarrow \begin{array}{l} \text{for AWG \#19} \rightarrow k_{fill} = 2,41 \times 10^{-3} \ll 0,5 \\ \text{AWG \#20} \rightarrow k_{fill} = 1,91 \times 10^{-3} \ll 0,5 \end{array}$$

In Gererell max. fill factor is limited with 0,5 in designs. So, this inductor is feasible in thermal and mechanical considerations.

Part II :

d) In this case flux prefers the shortest path on the core which is near of inner radius 9+δ cm where δ ≪ 1.



So, mean flux path length is shorter. If we assume distribution of the flux concentrate around of inner radius ;

$$\text{mean flux path} = l = 2\pi\,(0,09) = 0,56\ m.$$

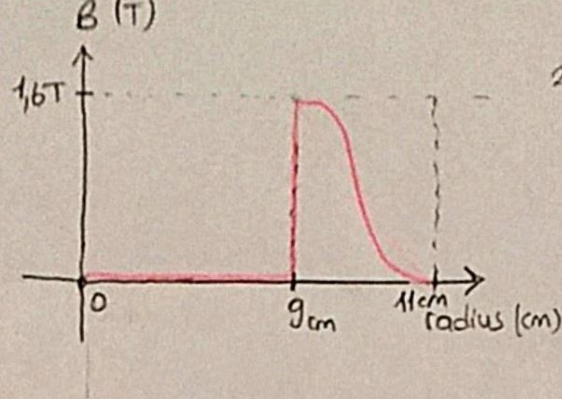
$$L = 10 \times 10^{-3} H = \frac{N^2 \times 1,75 \times 10^{-3} \times 4 \times 10^{-4}}{0,56} \Rightarrow N^2 = 8000\ ,\ N = 89,44 \text{ turns}$$

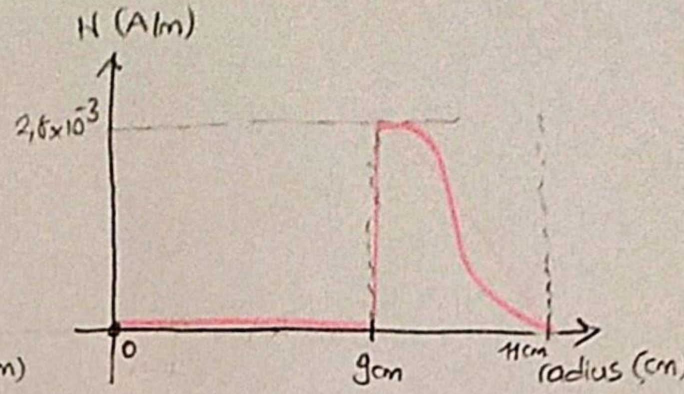
$$B = \mu H = \frac{\mu N i}{l} \Rightarrow 1,6T = \frac{1,75 \times 10^{-3} \times 89,44 \times i}{0,56} \Rightarrow i_{max} = 5,72\ A$$

1,6T(max)

Because of the concentrated flux around of inner radius (shortest path), the core saturation can be reachable with lower current level than previous calculation. It sense logic because, homogeneous distribution condition, flux can travel via all part of core material but second (real) case flux prefers the shortest path so it saturates core with a lower current.

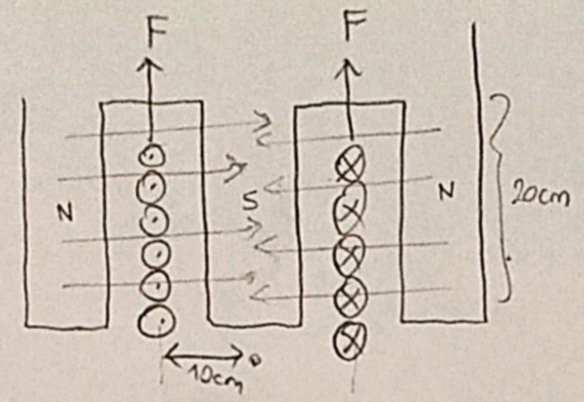
Q1, e)





flux is not homogeneously distributed condition.

Q2)



Lorenz force $= B \times i \times l$

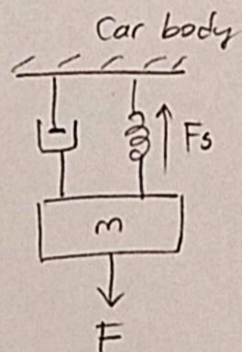
$B = 1{,}5\,T$, $R = 0{,}5\,\Omega$, $V = 12V$, $k_s = 100 N/cm$

the current passing through the coil is $= 12/0{,}5 = 24\,A$.

length of one turn $= 2\pi \underset{10cm}{\underset{\downarrow}{r}} = 62{,}83$ cm

F_{single} ⇒ the force from single turn (coil) $= 1{,}5 \times 24 \times 0{,}62 = 22{,}32\,N$

total force $= N \times F_{single} = \boxed{669{,}6\,N}$ → this is the case that all coils are in stator.



Spring force $= F_s = k_s \times \Delta x$

Lorenz force $= 1{,}5 \times 24 \times 22{,}32 \times$ coils in Stator.

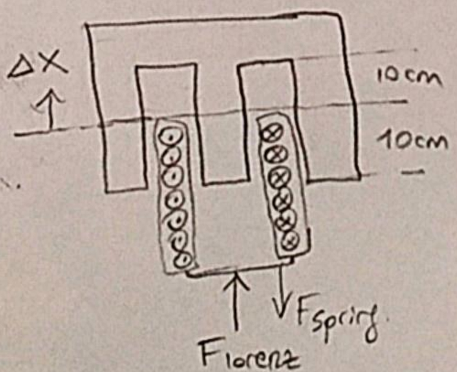
Coils in stator ⇒ 30 turn = 20 cm

1 turn = 2/3 cm

⇒ $\Delta x / 2/3$ turn

if there is no force come from wheels
$-F_s = F_{lorenz}$

Neutral point of the spring is the mid point of stator



$$-k_s \times \Delta x = B \times i \times F_{single} \times \left(\frac{10cm}{2/3} + \frac{\Delta x}{2/3}\right)$$

$$-100\,N/cm \cdot \Delta x = \underbrace{1{,}5 \times 24 \times 22{,}32}_{803{,}502\,(N/cm)} \times \left(15 + \frac{\Delta x}{2/3}\right)$$

$$0 = 12052{,}8\,N + 1305{,}25\,\Delta x$$

$$\Delta x = -9{,}23\ cm$$

if there is no any force from wheel, and neglect the mass of linear rotor the linear rotor moves from mid point to Δx direction as 9,23 cm.

	1st machine	2nd machine
Q3) a)	N42H up to 128C	N42UH up to 180°C
	$B_r = 1{,}3\,T$	$B_r = 1{,}3\,T$
	intrinsic Coer. (kA/m) = 1353	intrinsic coercivity (kA/m) = 1989

Magnetic Loading = $\bar{B} = \dfrac{\text{total flux}}{\text{total area of airgap}} = \dfrac{P\,\phi_p}{\pi D_i L}$

ϕ_p → pole flux, L → machine length, D_i → inner diameter

Electrical Loading = $\bar{A} = \dfrac{\text{total current}}{\text{rotor circumference}} = \dfrac{N_{turn_{slot}} \times Q \times i}{\pi D_i}$

Q → slot number, i → current

i) Up to 128°C operation temperature, they have same magnetic and electrical loading performance. After 120°C, 1st machine can not handle required magnetic and electrical loading so it needs cooling operation because even if its windings have 128c durable insulation, the magnets will be demagnetize.

ii) 1st machine insulation class must be Class A : 105°c, because after 120°c operating condition, the PMs will be demagnetize and loss its performance. So, even if the current increases, magnets can not provides required flux for required performance (under no cooling condition). If we select the class B : 130°c, at this time machine works but for all 120°c - 130°c operating times, magnet lifes will be shorter. If you want to reliable, safe machine, insulation class must be 130°c but machine operating condition must be define under 120°c. If you consider cost, insulation class must be class A : 105°c and operation conditions must be defined under 105°c.

Q3, continue) for 2^{nd} machine, insulation class must be Class H: 180°C.

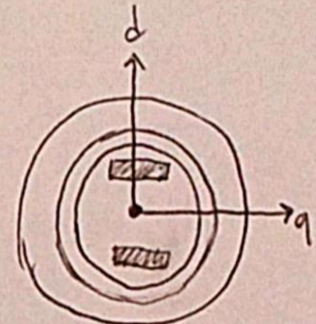
If we summarize.

1^{st} machine, Class A: 105°C, max operating temp = 105°C
with safety and reliability tolerance. But low performance

2^{nd} machine, Class H: 180°C, max op. temp. = 180°C

Alternatively.

1^{st} machine, class B: 130°C, max. op. temp = 120°C
with safety and reliability tolerance, High performance

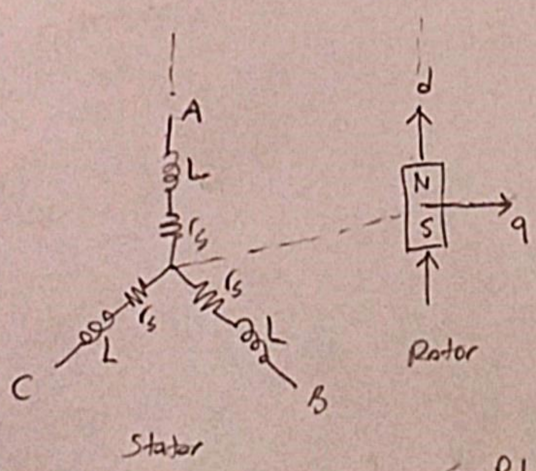


b) L_d → Inductance related with magnetic axis direction
L_q → " " non magnetic axis direction.

L_q is related with reluctance torque production. (saliency)
L_d is related electromagnetic torque production.

L_d, L_q are representation of the machine inductances in synchronously rotating ref. frame.

L_d and L_q values depend on rotor position, θ_{elec}. If we can allign the rotor magnetic axis to phase A axis of stator like in figure,



$L_d = \frac{2}{3} L \longrightarrow \theta_{elec} = 0°$

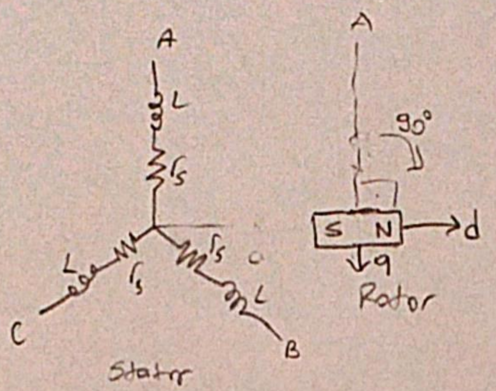
L can be measured by with step excitation to phase A, phase B and C are grounded. Rotor is locked at that position.

So RL circuit is obtained. $i_d = \frac{V}{R}\left(1 - e^{-\frac{tR}{L}}\right)$

$L_d = \frac{2}{3} L$

Q3, continue 2)

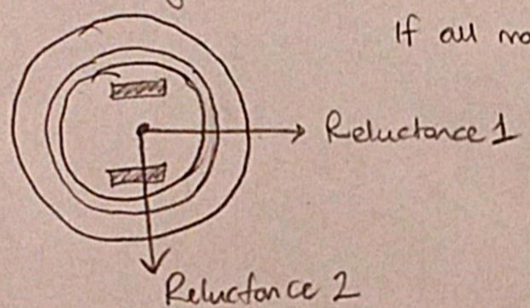
b) Similarly measuring the L_q, θ_{elec} is set $90°$



$L_q = \frac{2}{3} L \rightarrow \theta_{elec} = 90°$

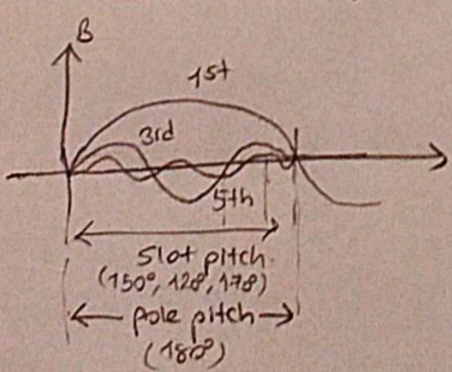
L can be measured by step excitation to phase A, phase B and C are grounded. Rotor is locked at $90°$ first order RL circuit is solved. L is found.

c) Yes, we can generate a small amount torque because of saliency.



If all magnets are demagnetized, rotor will be salient. So $\mathcal{R}_2 > \mathcal{R}_1$ and, small amount of reluctance torque is produced.

d) In fractional slot winding we can set the pitch angle and eliminate the effects of harmonics. Also, cogging torque can prevent or limited by using fractional slot winding. Another important advantage of fractional slot winding is, better emf waveform in case of generators (it is related with harmonic elimination)



Q4)

from legend :

a) 250 / 12 . 270 $\rightarrow$ Core length

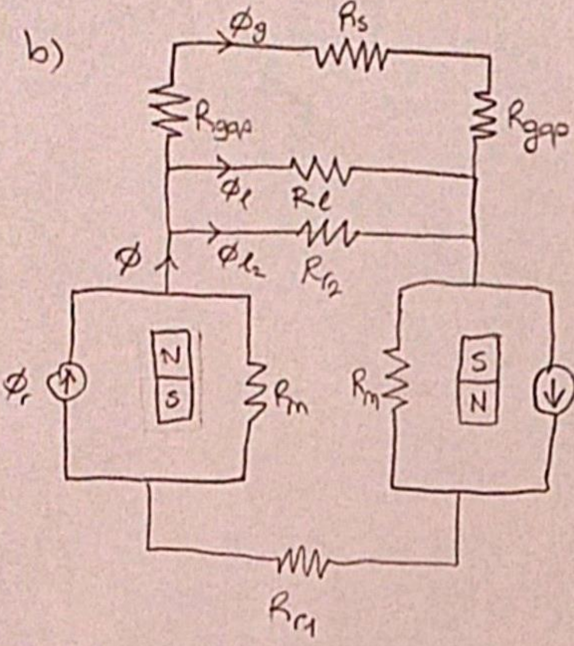
250 $\rightarrow$ frame height

12 $\rightarrow$ pole number

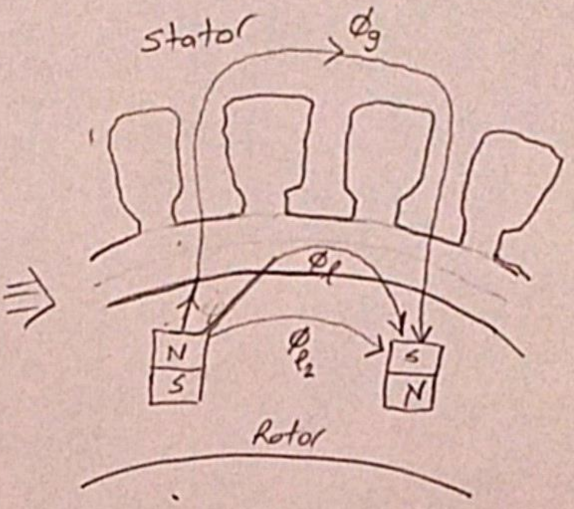
N	A	b_2
54	--	--

54 is Slot number ✓

b)

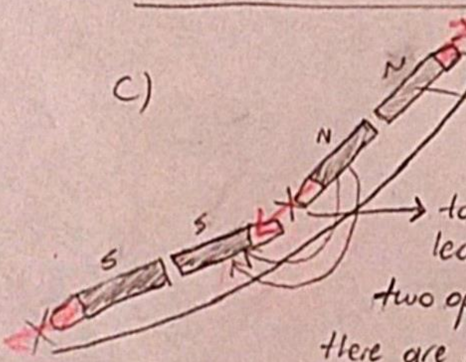


– magnetic circuit –



– physical model –

c)



slot (extra magnet region)

to prevent the leakage flux between two opposite poles magnets, there are placed air gap (creating extra high reluctance) at the nearest locations of two opposite pole magnets

In the schematic, 4 stator teeth against the one pole.

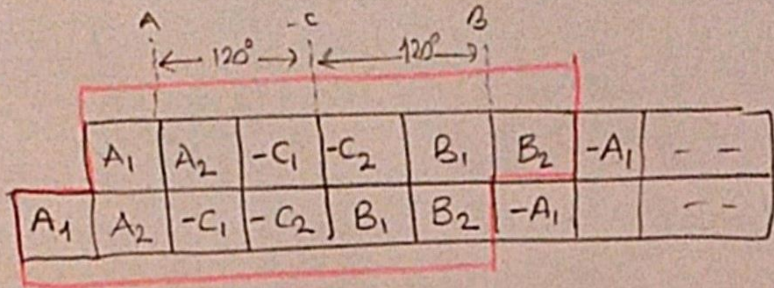
Bsat level is 1,8 T is given in datasheet.

So we can say the back core flux density is nearly 1,8T, So, this flux goes through via 4 teeths, so teeth flux density must be nearly $1{,}8T/4 = 0{,}45T$

air gap flux density will be $0{,}9T = 1{,}8T/2$

d) 54 slots, 12 pole, 3 phase

$q = 3/2$ slot/per pole/per phase, double layer. 4,5 slots/per pole



e) Axial length → 270 mm, rated speed → 1200 rpm.

f) ? time is up :(