Influence of Magnet Shape on Cogging Torque and Back-emf Waveform in Permanent Magnet Machines

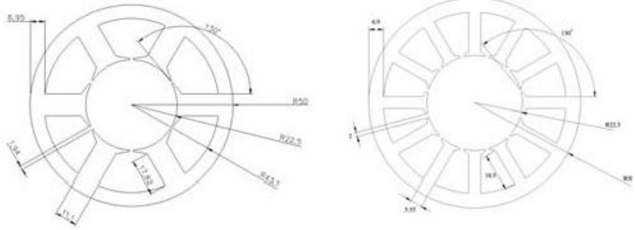
S. Ruangsinchaiwanich, Z.Q. Zhu, D. Howe
Department of Electronic and Electrical Engineering, University of Sheffield, Mappin Street, Sheffield, S1 3JD, UK

Abstract— **This paper reports the influence of the magnet shape on the cogging torque and the back-emf waveform of permanent magnet machines. Two prototype stators, together with several rotors, are employed. The first stator has 6 slots and a concentrated coil wound on every tooth, while the second stator has 12 slots and a full-pitched overlapping winding. The 4-pole rotors are equipped with surface-mounted magnets. The influence of magnetization and sinusoidal outer surface is investigated. In particular, the sinusoidal magnet shaping of arc magnets (i.e. magnet with circular bottom) and bread-loaf shaped magnets (i.e. magnet with flat bottom) are compared. It shows that the sinusoidally shaped magnets result in not only low cogging torque, but also more sinusoidal back-emf waveform. However, shaping the magnets with circular bottom, the peak of back-emf may be reduced, which can be avoided by employing either an overlapping winding stator or a bread-loaf shaped magnets.**

I. Introduction

Although various cogging torque minimization techniques are available, some have disadvantages and are difficult to apply. For example, skew, either by skewing the stator laminations or the rotor magnets, is quite difficult to realize in mass production, and increases the leakage inductance and reduces the average output torque [1]-[7]. Since the cogging torque is caused by the interaction of the permanent magnets and the stator slots, the magnet shape has a significant influence on the cogging torque, and may be more cost-effective. It is well known that the ratio of magnet pole-arc to pole-pitch has a significant influence on the cogging torque and optimum ratios exist for minimum cogging torque [3][8]. The edges of magnets affect the cogging torque as well. In general, chamfered magnets results in low cogging torque. However, both a small cogging torque and good motor performance is generally required, while an inappropriate magnet shape can result in an unacceptable high cogging torque, thus, must be avoided at the design stage.

The main focus of this paper is to study the influence of the magnet shape on the cogging torque. Two prototype stators, together with several rotors are employed. The first stator has 6 slots and a concentrated coil wound on every tooth, while the second stator has 12 slots and a full-pitched overlapping winding, as shown in Fig. 1. The main parameters which are related to the cogging torque, for example, the inner stator diameter (45mm), the outer stator diameter (110mm), the axial length (50mm), the airgap length (1mm), and the width of the slot openings (2mm), are identical for the two motors. Moreover, several prototype rotors are investigated, some parameters, such as the outer rotor diameter (43mm) and the permanent magnet material properties (NdFeB, remanence = 1.2T) being the same. In this paper, the influence of magnetization and sinusoidal outer surface in 6-slot and 12-slot, 4-pole motors is investigated. Since the shape of surface-mounted magnets may be arc-shaped or loaf-shaped [5], Fig. 2, their influence on the cogging torque are also investigated, together with the back-emf waveform.



(a) 6-slot, non-overlapping winding (b) 12-slot, overlapping winding
Fig. 1. Schematic of stator laminations.

(b) arc-shaped (c) loaf-shaped
Fig. 2. Alternative rotor magnets.

II. Sinusoidally Shaped Arc Magnet

Shaping the rotor magnets is an alternative approach to reduce the cogging torque and to achieve a sinusoidal back-emf waveform, which is desirable for brushless ac motors operation in order to reduce excitation torque ripple [1][3]. In this section, the influence of the shape of the magnets on the cogging torque and back-emf waveforms in the surface-mounted magnet motor having arc magnets is investigated. The magnet edges are assumed to be radial and the original thickness of the arc magnet is 3 mm. The outer surface of each magnet is shaped sinusoidally [9], and the edges of the magnet are varied from 3 mm to 0 mm, as shown in Fig. 3, as can be expressed by:

$$\Delta h = \Delta m \cos(p\theta) \quad (1)$$

where θ is the mechanical angular position, p is the number of pole-pairs, Δh is the magnet thickness and Δm is the difference between the original magnet thickness (i.e. 3 mm) and the edge thickness (i.e. 0 – 3 mm), as shown in Fig. 3.

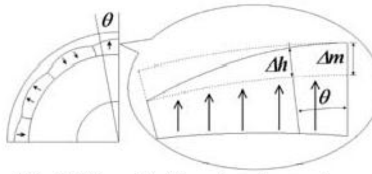
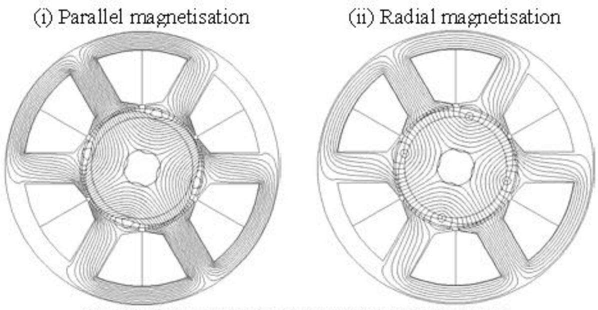
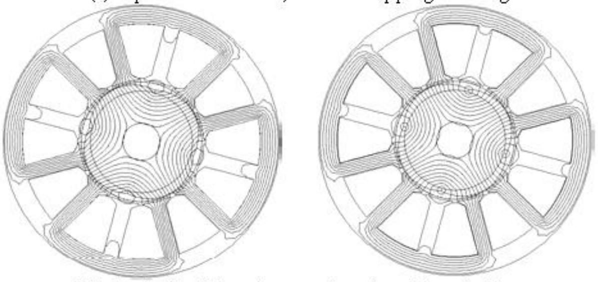


Fig. 3. Sinusoidal shaping of magnet arc.



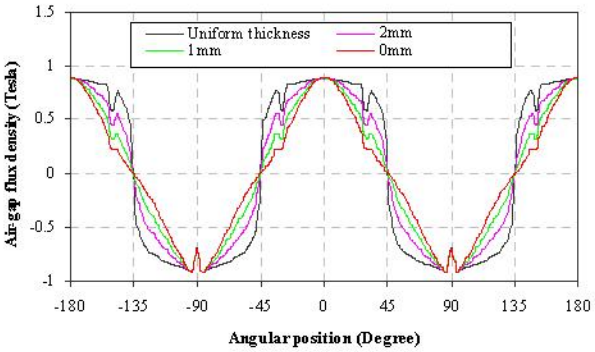
(a) 4-pole 6-slot motor, non-overlapping winding



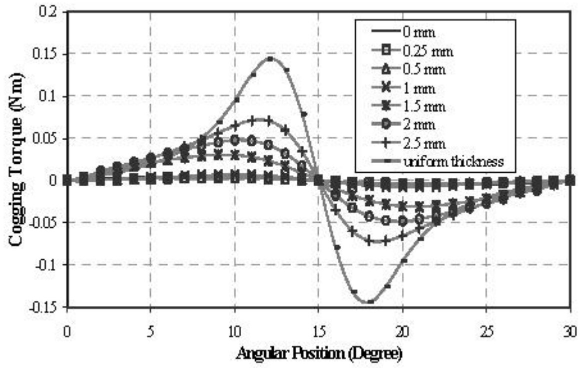
(b) 4-pole 12-slot motor, overlapping stator winding

Fig. 4. Open-circuit field distributions of sinusoidally shaped arc magnet rotor motors, edge thickness=1 mm.

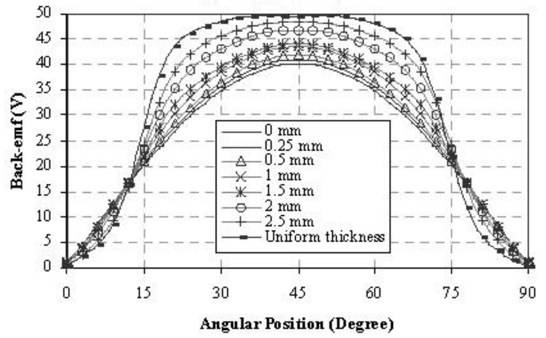
Fig. 4(a) shows the magnetic field distributions, while Figs. 5 and 6 show the resultant cogging torque and back-emf waveforms for parallel-magnetized and radial-magnetized 4-pole 6-slot non-overlapping winding permanent magnet motors, respectively, when the magnets are sinusoidally shaped with different edge thickness. As the magnet thickness rotor is varied from being uniform to being sinusoidal, the resultant air-gap field distribution becomes more sinusoidal, and the amplitude of the cogging torque is significantly reduced. By the way of example, although, in general, the parallel magnetization produces relatively lower cogging torque than the radial magnetization, the cogging torque can be reduced by 95.2 % for parallel magnetization and by 91.5% for radial magnetization, when the magnet edge thickness is reduced, and the resultant cogging torque being 3.7mNm for parallel magnetization and 1.2mNm for radial magnetization when the magnet edge thickness is reduced to zero.



(a) Air-gap field distribution

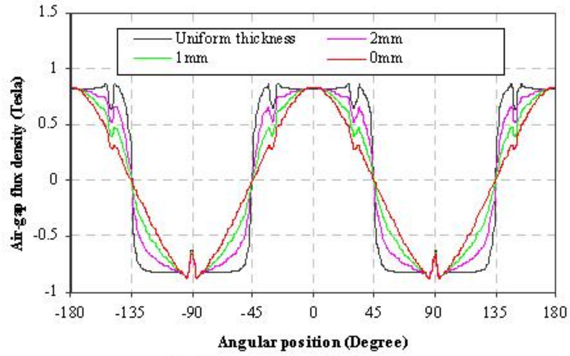


(b) Cogging torque

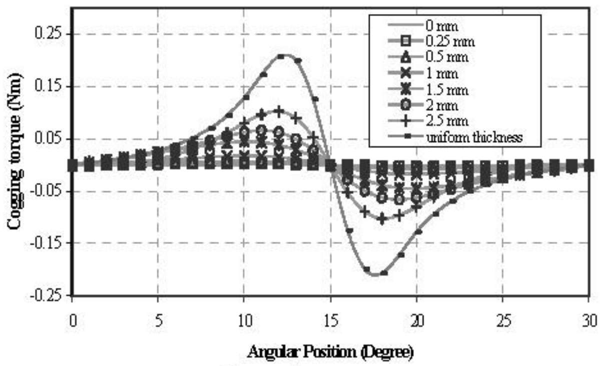


(c) Back-emf (500 rpm)

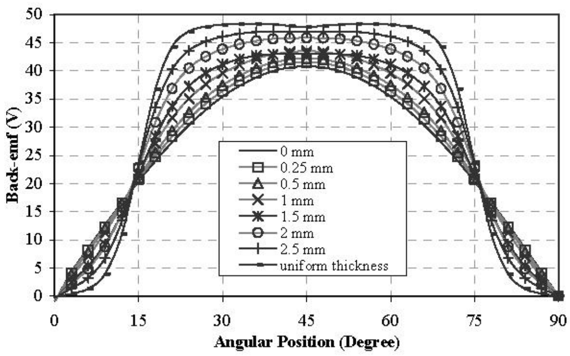
Fig. 5. Influence of arc magnet shape on air-gap field, cogging torque, and back-emf, 4-pole 6-slot motor, parallel magnetization.



(a) Air-gap field distributions



(b) Cogging torque

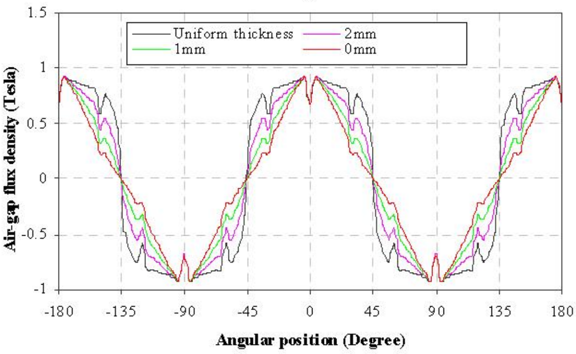


(c) Back-emf (500 rpm)

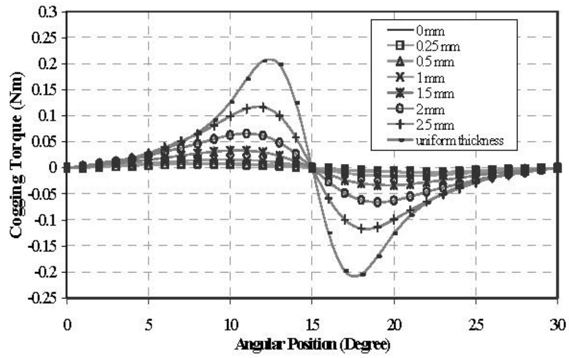
Fig. 6. Influence of arc magnet shape on air-gap field, cogging torque, and back-emf, 4-pole 6-slot motor, radial magnetization.

Moreover, the back-emf waveform becomes more sinusoidal, albeit with a reduction in amplitude when the edges of magnets are reduced in thickness. For example, when the magnets are shaped such that the edges are only 1.0 mm in thickness, the amplitude of the back-emf for parallel magnetization is reduced to 45.5 volts, i.e. ~ 4.5% reduction, and to 45.8 volts, i.e. ~ 4.6% reduction for radial magnetization, while when the edges are reduced in thickness to zero, the amplitude of the back-emf is reduced by ~ 20% for both magnetization distributions.

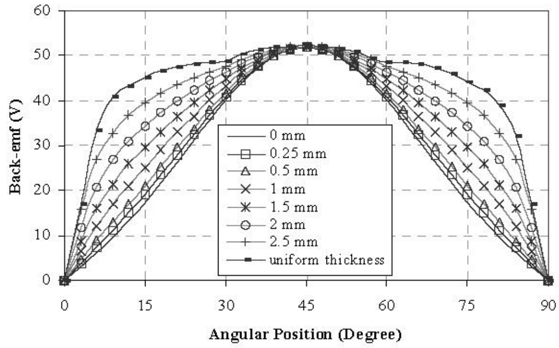
One solution for minimizing the detrimental influence of sinusoidally shaped magnets on the back-emf waveform is to employ an overlapping stator winding accommodated in 12 stator slots, the corresponding field distribution being shown in Fig. 4(b). Figs. 7 and 8 show the resultant cogging torque and back-emf waveforms when the magnets are sinusoidally shaped with different edge thickness. As will be seen, the amplitude of resultant cogging torque is almost double that of the 6-slot motor with the non-overlapping stator winding [8], due to the increase in the slot number since in the 4-pole 6-slot motor when half of the magnet edges face the slot openings and other half of the edges face the teeth, whilst in the 4-pole 12-slot motor all magnet edges will face the slot openings at the same time. However, the cogging torque waveforms are similar, in that the period is 30 degrees, since the least common multiple between the pole number and slot number for both motors is identical, i.e. 12.



(a) Air-gap field distributions

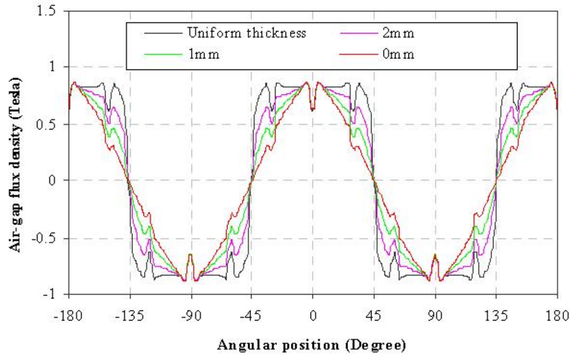


(b) Cogging torque waveforms

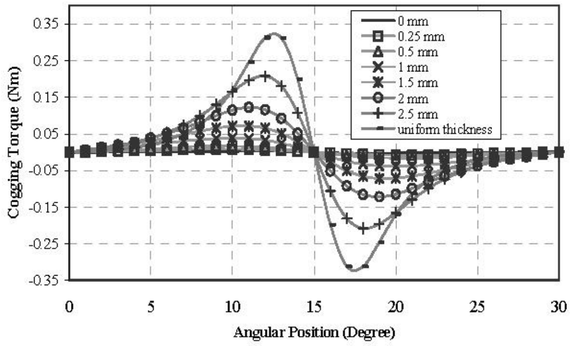


(c) Back-emf waveforms (500 rpm)

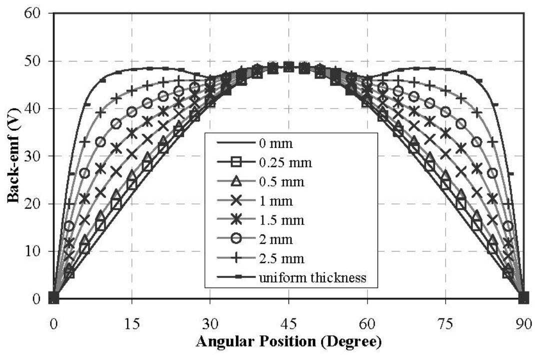
Fig. 7. Results for sinusoidally shaped arc magnet rotor motor with overlapping winding stator, 4-pole 12-slot motor, parallel magnetization.



(a) Air-gap field distributions



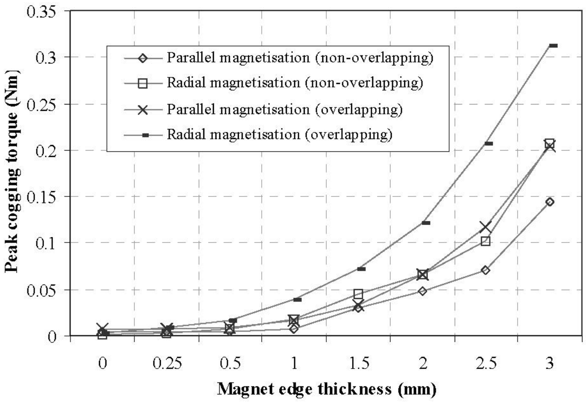
(b) Cogging torque waveforms



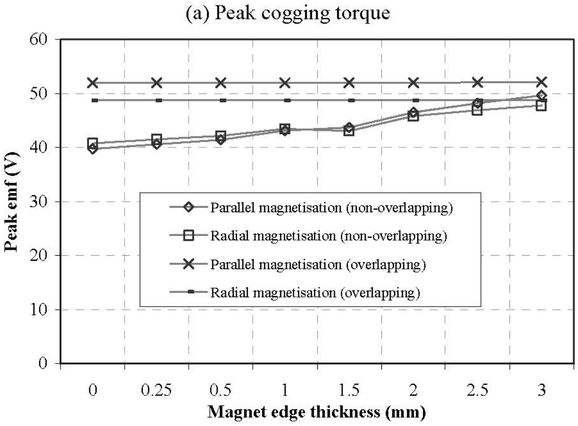
(c) Back-emf waveforms (500 rpm)

Fig. 8. Results for sinusoidally shaped arc magnet rotor motor with overlapping winding stator, 4-pole 12-slot motor, radial magnetization.

Fig. 9 compares the variation of the peak cogging torque and the peak back-emf with magnet edge thickness for both overlapping and non-overlapping winding motors and both parallel and radial magnetized magnets. Again, the cogging torque for the radially-magnetized motor is higher than that for the parallel-magnetized motor. It will also be noted that the amplitude of the back-emf when the overlapping winding is employed is more or less constant as the cogging torque is gradually reduced by reducing the edge thickness, as shown in Fig. 9.



(a) Peak cogging torque



(b) Peak back-emf (500 rpm)

Fig. 9. Comparison of peak cogging torque and back-emf in sinusoidally shaped arc magnet rotor motors.

Although the resultant cogging torque can be almost eliminated by reducing the thickness of the edges of the magnets to zero, this may results in mechanical problem and may also compromise the demagnetization withstand capability of the machine. Hence, a minimum edge thickness of around 1.0 mm may be employed, which for machine under consideration reduces cogging torque by ~ 95.2% and 91.5% for parallel and radial magnetization, respectively.

III. Loaf-Shaped Magnets

An alternative to employing profiled magnet arc is to use loaf-shaped magnets with a flat inner surface, as shown in Fig. 10. Such flat-bottomed magnets are easier to mount on the rotor rub. However, since the rotor back-iron structure is then different to that in the surface-mounted magnet rotors discussed in the previous sections, the d-axis inductance will be smaller than the q-axis inductance. In this section, the influence of the shape of the outer surface of such magnets on the cogging torque and back-emf waveforms is investigated. Fig. 11 shows typical open-circuit field distributions when motors equipped with loaf-shaped magnets, Fig. 10, for both overlapping and non-overlapping windings. Figs. 12 to 15 show air-gap field distributions, and cogging torque and back-emf waveforms, which result when the magnets are sinusoidally shaped with different edge thickness for both radial and parallel magnetizations. They show that the loaf-shaped magnets are effective in minimizing cogging torque. Fig. 16 shows that when the thickness of the magnet edges is reduced the peak cogging torque is reduced, while the peak back-emf remains more or less constant. Further, the peak back-emf is slightly larger than that which results with sinusoidally shaped arc magnets, due to increase magnet thickness and larger magnet volume.

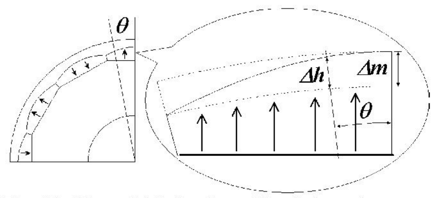
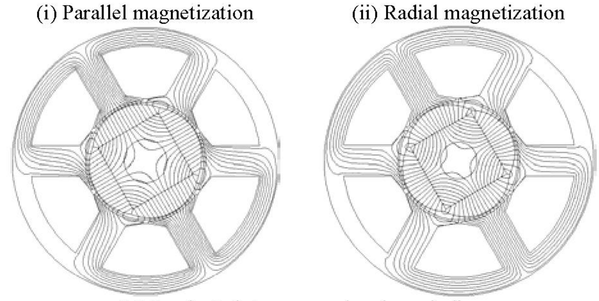
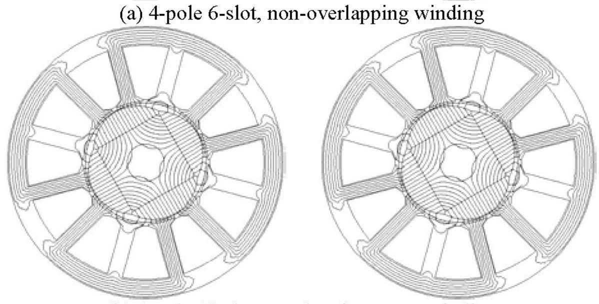


Fig. 10. Sinusoidal shaping of loaf-shaped magnet.

(i) Parallel magnetization (ii) Radial magnetization

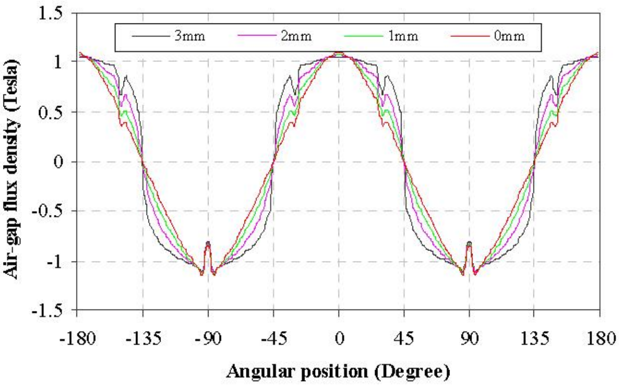


(a) 4-pole 6-slot, non-overlapping winding

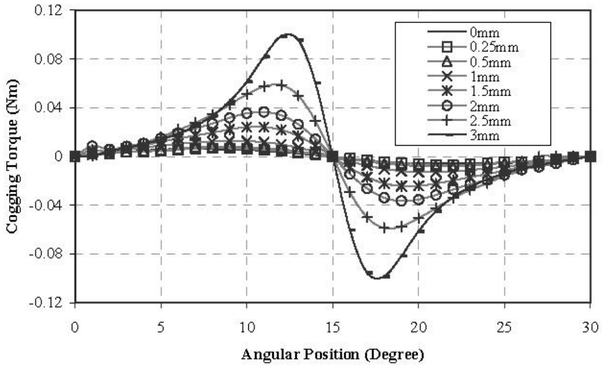


(b) 4-pole 12-slot, overlapping stator winding

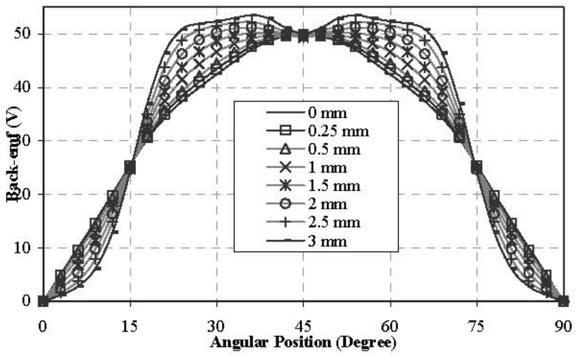
Fig. 11 Open-circuit field distributions of loaf-shaped magnet motor, edge thickness = 1 mm.



(a) Air-gap field distributions

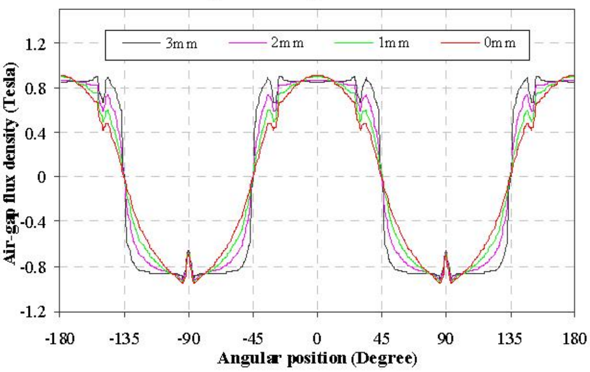


(b) Cogging torque waveforms

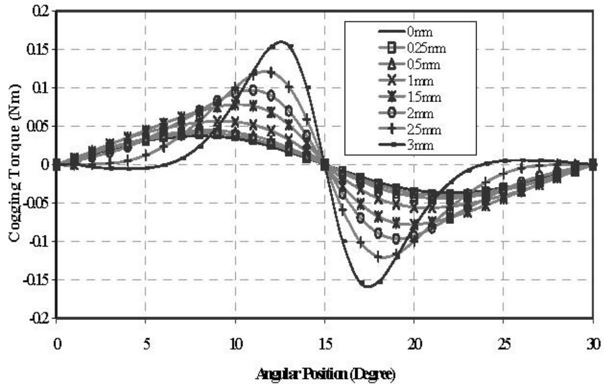


(c) Back-emf waveforms (500 rpm)

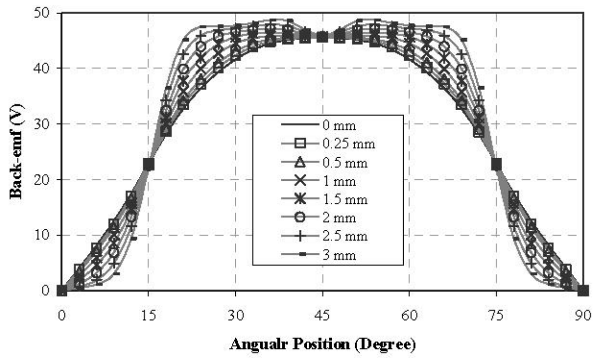
Fig. 12. Results for sinusoidally loaf-shaped magnet motor, 4-pole 6-slot, parallel magnetization.



(a) Air-gap field distributions

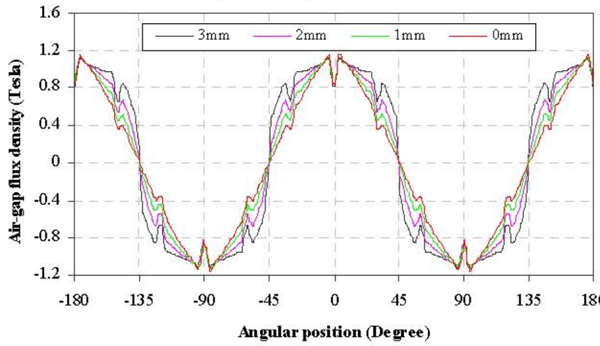


(b) Cogging torque waveforms

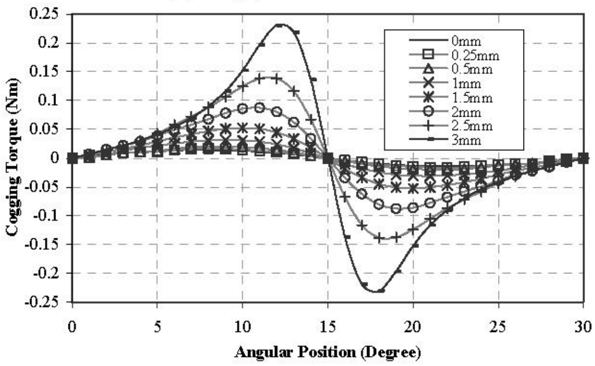


(c) Back-emf waveforms (500 rpm)

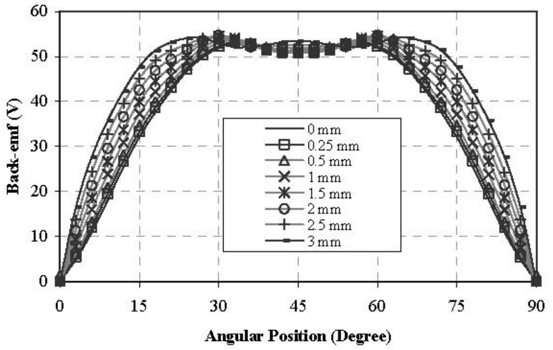
Fig. 13. Results for sinusoidally loaf-shaped magnet motor, 4-pole 6-slot, radial magnetization.



(a) Air-gap field distributions

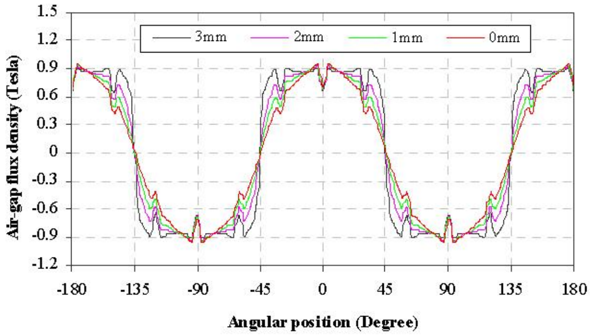


(b) Cogging torque waveforms

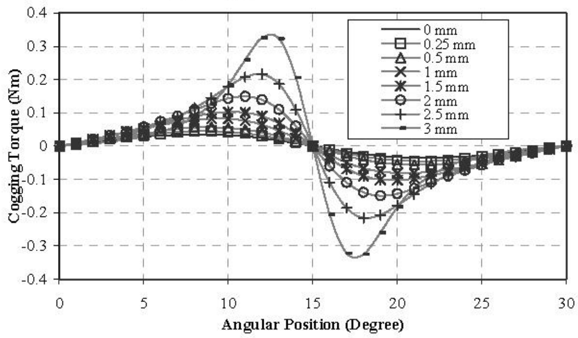


(c) Back-emf waveforms (500 rpm)

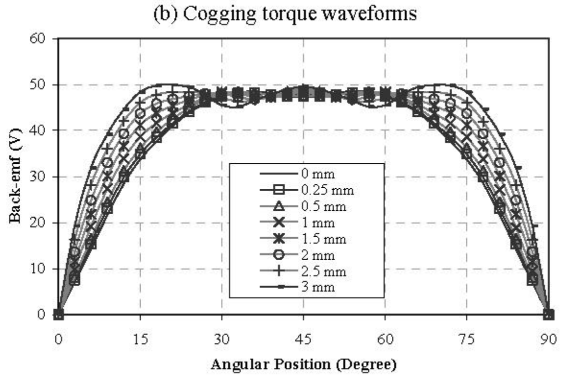
Fig. 14. Results for sinusoidally loaf-shaped magnet motor with overlapping winding stator, 4-pole 12-slot, parallel magnetization.



(a) Air-gap field distributions



(b) Cogging torque waveforms

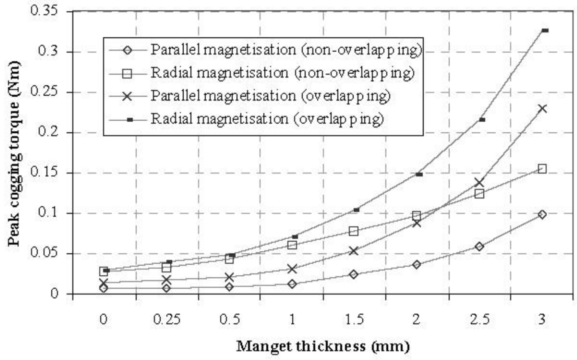


(c) Back-emf waveforms (500 rpm)

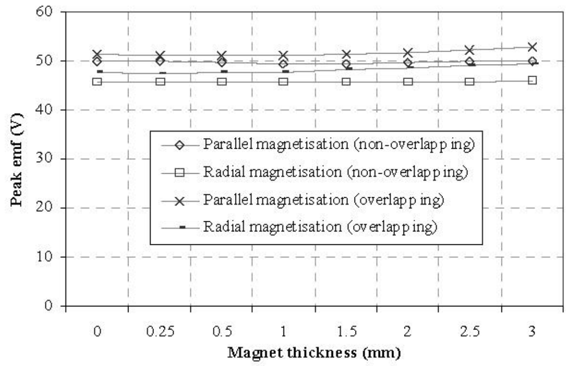
Fig. 15. Results for sinusoidally loaf-shaped magnet motor with overlapping winding stator, 4-pole 6-slot, radial magnetization.

IV. Conclusions

The cogging torque is highly dependent on the shape of the magnets. Indeed, shaping magnet can effectively reduce the cogging torque and can potentially be applied in the massive production, in particular sinusoidally shaped magnets not only result in low cogging torque, but also a more sinusoidal back-emf waveform. However, shaped magnet arcs may reduce the peak back-emf, which is also significant factor to consider due to the motor performance. This can be avoided by employing an overlapping winding or using loaf-shaped magnets. The conclusions are applicable to both surface-mounted and inset magnet motors.



(a) Peak cogging torque



(b) Peak back-emf (500 rpm)

Fig. 16. Comparison of peak cogging torque and back-emf in sinusoidally loaf-shaped magnet rotor motors.

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