MIDDLE EAST TECHNICAL UNIVERSITY

ELECTRICAL AND ELECTRONICS ENGINEERING DEPARTMENT

EE 568 Selected Topics in Electrical Machines

Final Examination 18 June 2020

Suggested Time: 300min

Exam Rules:

You are allowed to	You are NOT allowed to
• Use any additional materials, books, etc. • Use your laptops to access internet, download extra materials, run simulations • Use scientific calculators for your calculations • Have a short break, have a coffee etc.	• Use your mobile phones/tablets (not even as a calculator) • Communicate (talk, instant messaging, email etc) within or outside the class. • Sharing photos of the exam paper • Getting closer than 1.5 m to anyone

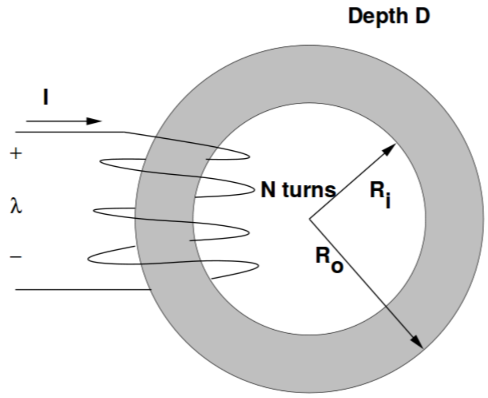
Student Name:______________________

Student ID:____________

Q1	
Q2	
Q3	
Q4	
Total	

Q1 – (20 pts) Inductor Design
Suppose that you have a toroidal core as shown in the figure below. You want to make an inductor of 10mH. Inner radius of the core is 9 cm and the outer radius is 11 cm and the depth(D) is 2 cm.

Assume the relative permeability of the core is 1400 and the core has a maximum flux density value of 1.6 T. Neglect the leakage flux throughout this problem.



Part-I: Assume the flux is homogeneously distributed along the core

a) How many turns are needed?

b) What is the maximum current that the inductor can carry without saturating the core?

c) Considering the fill factor and thermal issues, is that design feasible?

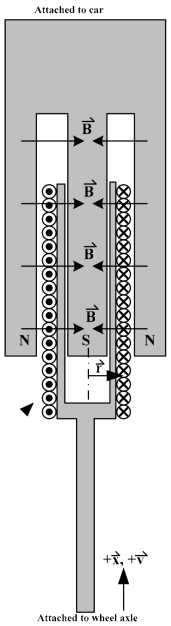
Part-II:

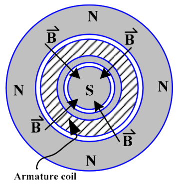
d) Repeat the part(a) and part(b), but now the flux is NOT homogeneously distributed.

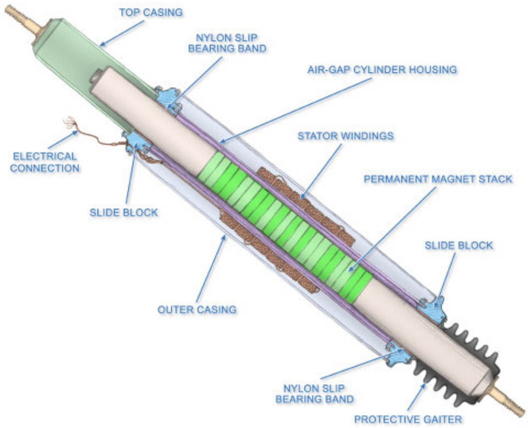
e) Plot the H_{tan} and B_{tan} versus radius under this operating condition.

Q2- (20pts) Linear Actuators

There are novel regenerative shock absorbers that use linear PM motors instead of conventional mechanical shock absorbers (Mercedes Magic Body Control uses a hydraulic version). Assume, a PM linear actuator as shown in the figure with PMs magnetized inwards and flux density is uniform in the coils.







Find the Lorentz force on the coil and vertical displacement of the absorber using the following parameters:

- r = 10 cm, B = 1.5 T, N_{coil} = 30, R= 0.5 Ω, Applied voltage= 12 V
- Total height of the stator = 20 cm
- Spring stiffness: 100 N/cm (The spring is connected in parallel to the PM actuator. Neutral position of the spring is the mid-point of the stator.

Q3-(20 pts) Short Answer Questions
Explain your reasoning as detailed as possible for full credit. Mathematical calculations are not required, but can be used to prove your point.

a) Assume you have two permanent magnets motors with the exact same dimensions and windings. The only difference is one of the motor has magnets of N42H, but the other one is N42UH. Compare these two machines in terms of:

i) Magnetic loading, electrical loading

ii) Insulation class, operating temperature

b) What are real world correspondence of direct and quadrature axis inductances (Ld, Lq) of electric machines? Propose a experimental method to measure Ld and Lq.

c) Assume you have an interior permanent magnet (IPM) machine. Can you generate any torque if all the magnets are demagnetized completely?

d) What are the advantages and disadvantages of fractional pitched windings over integral slot windings?-

Q4- (40 pts) Motor Analysis from Lamination Design

Assume you have the lamination design given as an attachment. The legend for the dimensions are also available. All dimensions are in mm.

- Assume M400-50A laminations are used in the machine (Datasheet is avaiable)
- NdFeB magnets of N40UH are used in the slots.

a) How many poles and slots does the machine have? (Don’t count, just read the legend)

b) Sketch the magnetic equivalent circuit for one pole and rouhgly calculate the peak airgap flux density (please clearly state any assumptions you make). Estimate the peak flux density at the stator teeth and at the back-core.

c) The slot for the magnets are slightly larger than the actual magnets. What is the purpose of the extra air-region next to the magnets?

d) Design a winding diagram for this machine. Draw the winding diagram or show the phasors for the induced voltages to clearly describe the winding diagram. You can simplify your diagrams by using any symmetry in your winding designs. Calculate the winding factor for the fundamental component. Estimate

e) Choose an axial length and rated speed for this machine. Using the standard values (electircal loading, magnetic loading etc.) estimate the power output for this machine.

f) Choose a rated voltage (you can use any standard values), then choose the number of turns per slot and standard wire size for this machine. Clearly state any assumptions such as fill factor, current density etc. Verify the electrical loading value you had chosen in part (c).

Serhat Özküçük -
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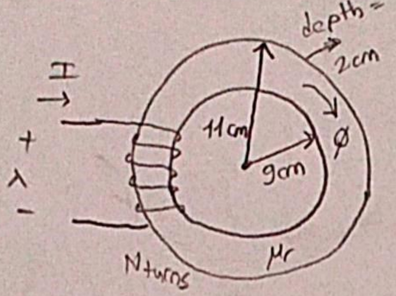
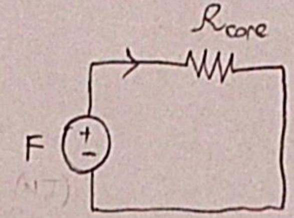
METU EE568 - Final Exam - 18.06.2020

- Solutions -

Q1) $L = 10\,mH$, $\mu_r = 1400$, $B_{max,core} = 1{,}6\,T$, neglect leakage flux.



→ magnetic model



$F = \int H.dl = NI$

$\mu = \mu_r \mu_0 = 1{,}75 \times 10^{-3}$

$\mu_0 \to 4\pi \times 10^{-7}$

Part - 1 :

a) $L = \dfrac{d\lambda}{di}$ → flux linkage change, → current change

→ $\lambda = N \phi =$ (N: turn)

$\phi = B.A$ (ϕ: flux, B: flux density (T), A: crossectional area of core (m^2))

$B = \mu H = \dfrac{\mu N I}{\ell}$ (ℓ: mean flux path length)

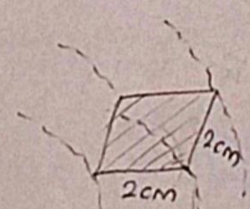
$= 2\pi \left(\dfrac{0{,}11 + 0{,}09}{2}\right)$

$= 0{,}62\ m.$

$\lambda = \dfrac{N^2 \mu I A}{\ell}$

$$\boxed{L = \frac{\lambda}{I} = \frac{N^2 \mu A}{\ell}}$$

Core Crossectional area :



$A = 2\times10^{-2} \times 2\times10^{-2}$
$= 4\times10^{-4}\ m^2$

$$L = 10\times10^{-3}\,H = \frac{N^2 \times 1{,}75\times10^{-3} \times 4\times10^{-4}}{0{,}62}$$

$N^2 = 8857{,}14$

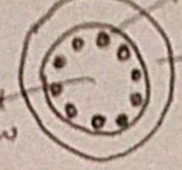
$N = 94{,}11$ turns.

b) Max flux density of the core is $B_{max} = 1{,}6\,T$

$B = \mu H = \dfrac{\mu N i}{\ell} \Rightarrow 1{,}6\,T = \dfrac{1{,}75\times10^{-3} \times 94{,}1 \times i}{0{,}62} \Rightarrow i_{max} = 6{,}02\,A$

$B \to 1{,}6\,T$ (max)

Q1, c)



Area of single conductor can be selected with respect to max current level. from AWG table,

#19 or #20 can cary 5A @ 60°C
11A @ 75°C

Area of this conductors

Area #19 : 0,653 mm² → total area : N x Area #19 = 61,44
Area #20 : 0,518 mm² → " ; N x Area #20 = 48,74
(mm²)

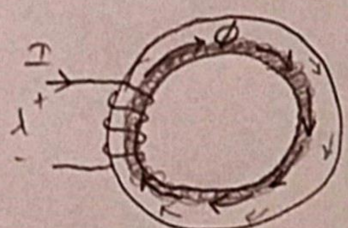
Area of core window = $\pi r^2 = \pi (90mm)^2 = 25446 \ mm^2$

$$\text{fill factor} = \frac{N \times \text{Area of cond.}}{\text{Area of window}} \Rightarrow \begin{array}{l} \text{for AWG \#19} \rightarrow k_{fill} = 2,41 \times 10^{-3} \ll 0,5 \\ \text{AWG \#20} \rightarrow k_{fill} = 1,91 \times 10^{-3} \ll 0,5 \end{array}$$

In Gererall max. fill factor is limited with 0,5 in designs. So, this inductor is feasible in thermal and mechanical considerations.

Part II :

d) In this case flux prefers the shortest path on the core which is near of inner radius 9+δ cm where δ ≪ 1.



So, mean flux path length is shorter. If we assume distribution of the flux concentrate around of inner radius;

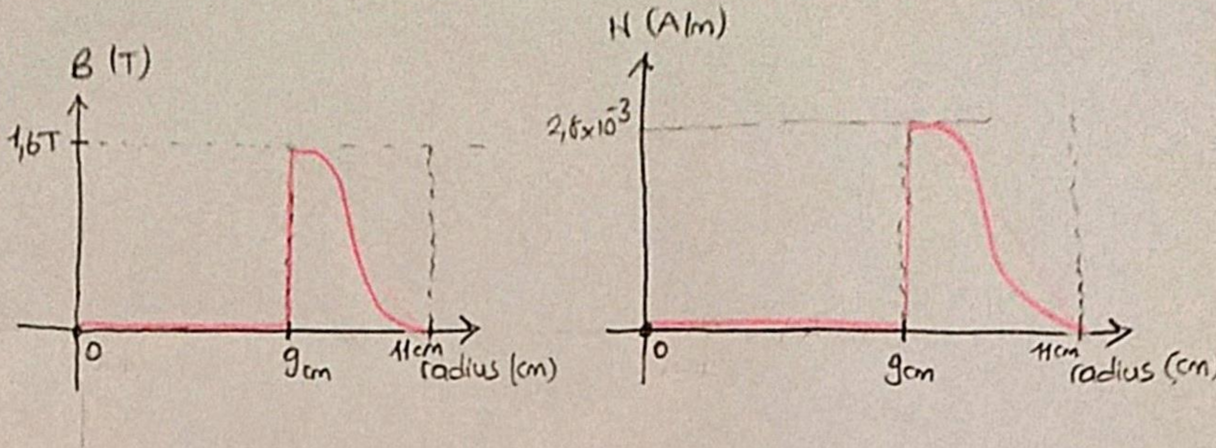
$$\text{mean flux path} = \ell = 2\pi(0,09) = 0,56 \ m.$$

$$L = 10 \times 10^{-3} H = \frac{N^2 \times 1,75 \times 10^{-3} \times 4 \times 10^{-4}}{0,56} \Rightarrow N^2 = 8000 \ , \ N = 89,44 \text{ turns}$$

$$\underset{\substack{\downarrow \\ 1,6T(max)}}{B} = \mu H = \frac{\mu N i}{\ell} \Rightarrow 1,6T = \frac{1,75 \times 10^{-3} \times 89,44 \times i}{0,56} \Rightarrow i_{max} = 5,72 \ A$$

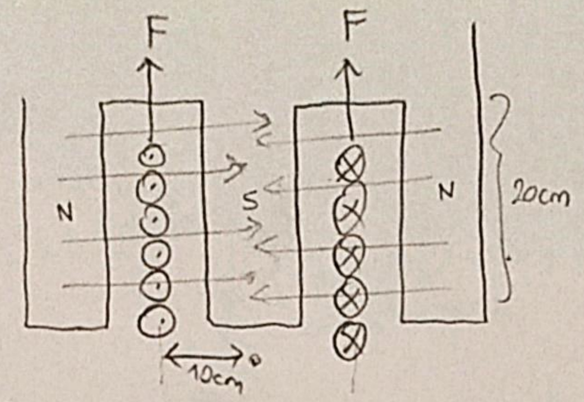
Because of the concentrated flux around of inner radius (shortest path), the core saturation can be reachable with lower current level than previous calculation. It sense logic because, homogeneous distribution condition, flux can travel via all part of core material but second (real) case flux prefers the shortest path so it saturates core with a lower current.

Q1, e)



flux is not homogeneously distributed condition.

Q2)



Lorenz force $= B \times i \times l$

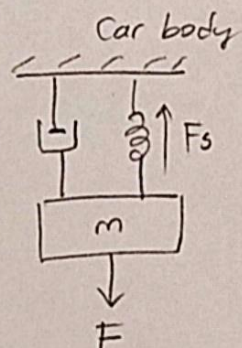
$B = 1{,}5\,T$, $R = 0{,}5\,\Omega$, $V = 12V$, $k_s = 100\,N/cm$

the current passing through the coil is $= 12/0{,}5 = 24\,A$.

length of one turn $= 2\pi \underset{10cm}{r} = 62{,}83$ cm

$F_{single} \Rightarrow$ the force from single turn (coil) $= 1{,}5 \times 24 \times 0{,}62 = 22{,}32$ N

total force $= N \times F_{single} = \boxed{669{,}6\ N}$ → this is the case that all coils are in stator.



Spring force $= F_s = k_s \times \Delta x$

Lorenz force $= 1{,}5 \times 24 \times 22{,}32 \times$ coils in Stator.

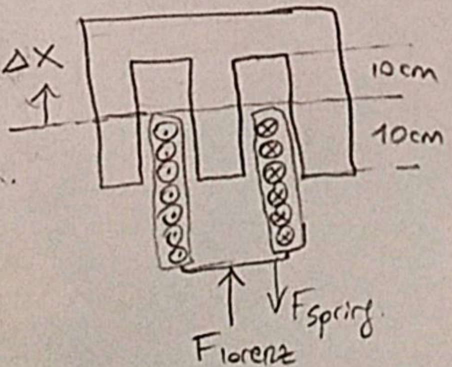
Coils in stator $\Rightarrow$ 30 turn = 20 cm

1 turn = 2/3 cm

$\Rightarrow \Delta x / 2/3$ turn

if there is no force come from wheels
$-F_s = F_{lorenz}$

Neutral point of the spring is the mid point of stator



$$-k_s \times \Delta x = B \times i \times F_{single} \times \left(\frac{10cm}{2/3} + \frac{\Delta x}{2/3}\right)$$

$$-100\,N/cm \cdot \Delta x = \underbrace{1{,}5 \times 24 \times 22{,}32}_{803{,}502\,(N/cm)} \times \left(15 + \frac{\Delta x}{2/3}\right)$$

$$0 = 12052{,}8\,N + 1305{,}25\,\Delta x$$

$$\Delta x = -9{,}23\ cm$$

if there is no any force from wheel, and neglect the mass of linear rotor the linear rotor moves from mid point to Δx direction as 9,23 cm.

	1^{st} machine	2^{nd} machine
Q3) a)	N42H up to 128C	N42UH up to 180°C
	$B_r = 1{,}3\,T$	$B_r = 1{,}3\,T$
	intrinsic Coer. (kA/m) = 1353	intrinsic coercivity (kA/m) = 1989

$$\text{Magnetic Loading} = \bar{B} = \frac{\text{total flux}}{\text{total area of airgap}} = \frac{p\,\underbrace{\Phi_p}_{\text{pole flux}}}{\pi\,\underbrace{D_i}_{\text{inner diameter}}\,\underbrace{L}_{\text{machine length}}}$$

$$\text{Electrical Loading} = \bar{A} = \frac{\text{total current}}{\text{rotor circumference}} = \frac{N_{turn_{slot}} \times \underbrace{Q}_{\text{slot number}} \times \underbrace{i}_{\text{current}}}{\pi D_i}$$

i) Up to 128°C operation temperature, they have same magnetic and electrical loading performance. After 120°C, 1^{st} machine can not handle required magnetic and electrical loading so it needs cooling operation because even if its windings' have 128c durable insulation, the magnets will be demagnetize.

ii) 1^{st} machine insulation class must be Class A : 105°c, because after 120°c operating condition, the PMs will be demagnetize and Loss its performance. So, even if the current increases, magnets can not provides required flux for required performance (under no cooling condition). If we select the class B : 130°c, at this time machine works but for all 120°c - 130°c operating times, magnet lifes will be shorter. If you want to reliable, safe machine, insulation class must be 130°c but machine operating condition must be define under 120°c. If you consider cost, insulation class must be class A : 105°c and operation conditions must be defined under 105°c.

Q3, continue) for 2^{nd} machine, insulation class must be Class H: 180°C.

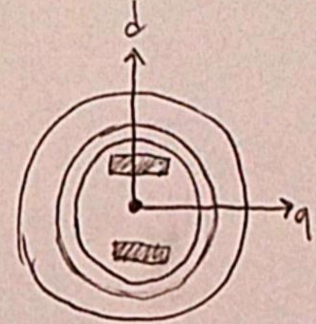
If we summarize.

1^{st} machine, Class A: 105°C, max operating temp = 105°C
with safety and reliability tolerance. But low performance

2^{nd} machine, Class H: 180°C, max op. temp. = 180°C

Alternatively.

1^{st} machine, class B: 130°C, max. op. temp = 120°C
with safety and reliability tolerance, High performance

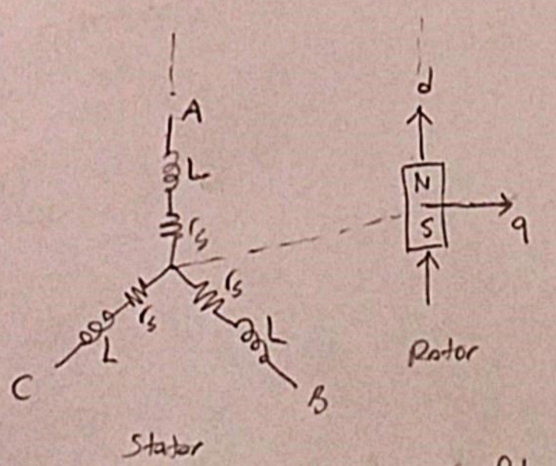


b) $L_d \rightarrow$ Inductance related with magnetic axis direction
$L_q \rightarrow$ " " non magnetic axis direction.

L_q is related with reluctance torque production. (saliency)
L_d is related electromagnetic torque production.

L_d, L_q are representation of the machine inductances in synchronously rotating ref. frame.

L_d and L_q values depend on rotor position, θ_{elec}. If we can allign the rotor magnetic axis to phase A axis of stator like in figure,



$L_d = \frac{2}{3} L \longrightarrow \theta_{elec} = 0°$

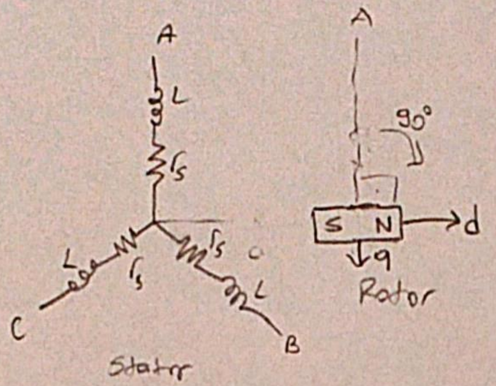
L can be measured by with step excitation to phase A, phase B and C are grounded. Rotor is locked at that position.

So RL circuit is obtained. $i_d = \frac{V}{R}\left(1 - e^{-\frac{tR}{L}}\right)$

$L_d = \frac{2}{3} L$

Q3, continue 2)

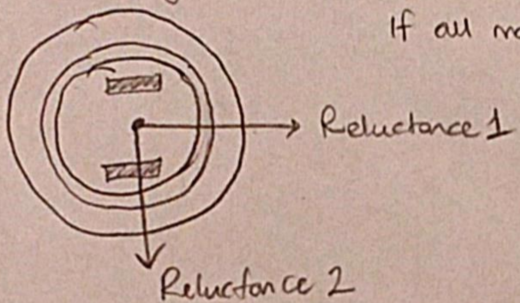
b) Similarly measuring the L_q, θ_{elec} is set 90°



$L_q = \frac{2}{3} L \rightarrow \theta_{elec} = 90^\circ$

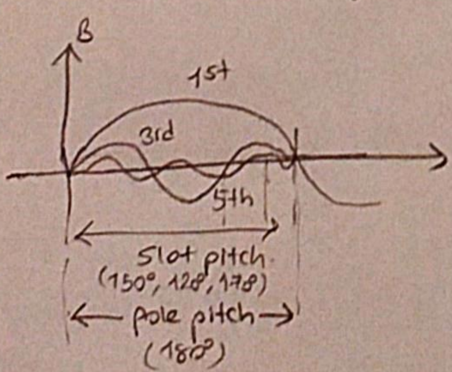
L can be measured by step excitation to phase A, phase B and C are grounded. Rotor is locked at 90° first order RL circuit is solved. L is found.

c) Yes, we can generate a small amount torque because of saliency.



If all magnets are demagnetized, rotor will be salient. So $R_2 > R_1$ and, small amount of reluctance torque is produced.

d) In fractional slot winding we can set the pitch angle and eliminate the effects of harmonics. Also, cogging torque can prevent or limited by using fractional slot winding. Another important advantage of fractional slot winding is, better emf waveform in case of generators (it is related with harmonic elimination)



Q4)

from legend :

a) 250 / 12 . 270 → Core length

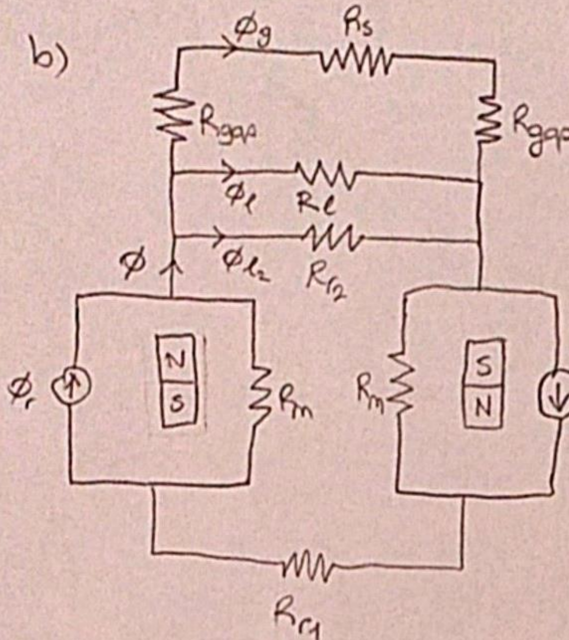
250 → frame height

12 → pole number

N	A	b_z
54	--	--

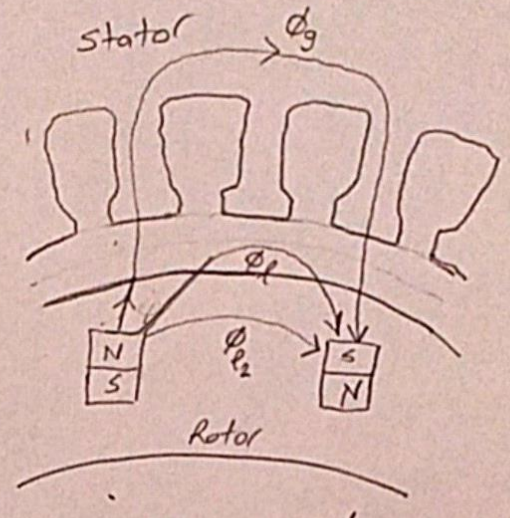
54 is Slot number ✓

b)



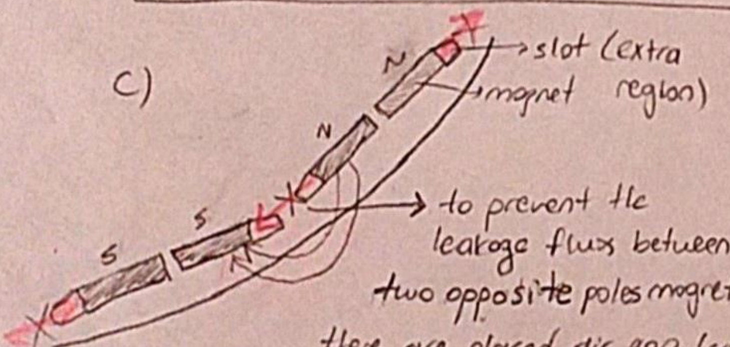
– magnetic circuit –

⇒



– physical model –

c)



to prevent the leakage flux between two opposite poles magnets, there are placed air gap (creating extra high reluctance) at the nearest locations of two opposite pole magnets

In the schematic, 4 stator teeth against the one pole.

Bsat level is 1,8 T is given in datasheet.

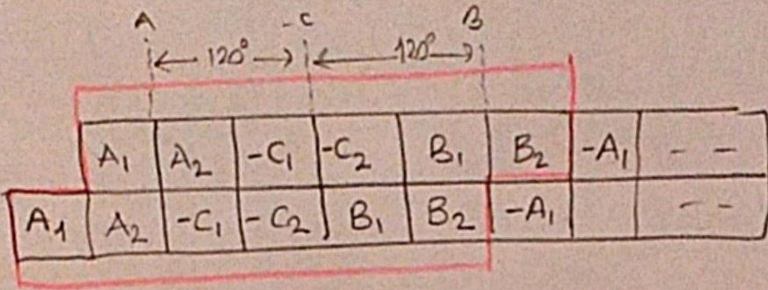
So we can say the back core flux density is nearly 1,8T,

So, this flux goes through via 4 teeths, so teeth flux density must be nearly $1,8T/4 = 0,45T$

air gap flux density will be $0,9T = 1,8T/2$

d) 54 slots, 12 pole, 3 phase

$q = 3/2$ slot/per pole/per phase, double layer. 4,5 slots/per pole

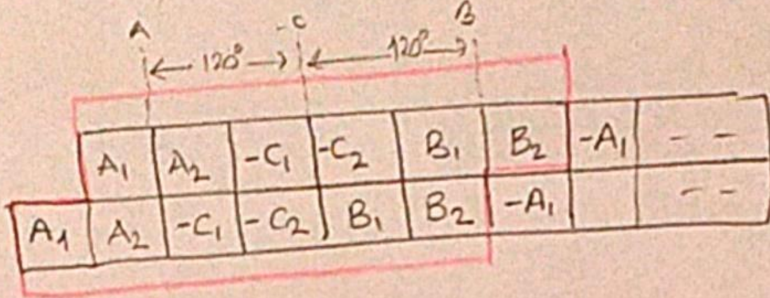


e) Axial length → 270 mm, rated speed → 1200 rpm.

f) ? time is up :(

d) 54 slots, 12 pole, 3 phase

$q = 3/2$ slot/per pole/per phase, double layer. 4,5 slots/per pole



e) Axial length → 270 mm, rated speed → $\frac{120f}{P} = n_{rated}$ (rpm)

$$\frac{120 \times 60\,Hz}{12\,pole} = 600 \text{ rpm (rated speed)}$$

Power out $= T \cdot \omega$.

f) ? time is up :(