Dynamical Modelling of Tooth-Belt-Driven Industrial Servo System

Alper Senem

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1 Introduction

Tooth belt for power transmission is a mechanical system, which is widely used in industrial applications of servo control. Designing a high performance regulator for servo system significantly depends on how accurately control engineer describes the mechanical system's behaviour and dynamics.

Linear movement can be achieved by using linear motor drives or by converting rotational movement to linear one with tooth belt, lead-screw or rack and pinions. Different transmission components have accuracies and stiffnesses of their own. A higher stiffness ensures a higher resonance frequency, and as a result, a higher gain of the controller can be used. Also higher accelerations can be used, because the resonance frequency of the system will not limit the acceleration and deceleration rates of the system at such low frequencies. For example, a tooth belt axis provides the lowest stiffness, while a lead-screw and a linear drive the highest stiffness. The maximum velocity of the system is, however, limited to be quite low when a lead- screw is used.

Linear screw drives or linear motor systems offer a good solution for applications that needs very accurate movements. However, the operating speeds of screw drive systems are typically slow. Tooth belt drives on the other hand have positive features, such as a capability for high-speed use, a high efficiency, a wide operating range, and a low cost.

For precision servo applications compliance of the belt may induce mechanical vibration or high-order resonance that will reduce system stability margin in the servo loop. The flexibility of the belt is also a known source of non-linearity in the closed loop dynamics. In addition, the dynamics of the belt may change due to belt ageing and environmental factors such as temperature and humidity. Differences in the loading condition will also give variations in its dynamic behaviour. Therefore, belt-driven servo systems may only give limited repeatability and accuracy.

High performance of linear tooth belt drives requires that the system disturbances, parameter variations, and uncertain dynamics must be considered in the robust control design or compensated in order to guarantee accurate positioning.

2 Accuracy and Resolution of Machinery Systems

As the manufacturing processes are evolving, also the performance requirements set for machinery systems increase. Furthermore, production rates are rising and accuracy requirements are tightening up. The most common sources of position error in multi-axis machine tools are;

- Kinematic errors
- Resolution and accuracy of the linear measuring system
- Elastic deformation of drive components
- Inertia forces when braking or accelerating

- Friction and stick slip motion
- Control system
- Cutting force
- Vibration

Kinematic errors may result from a change in the component geometry of the structural loop of the machine, axis misalignment, and errors in the measuring systems of the machinery. The structural loop is defined as an assembly of mechanical components that maintains a relative position between specified objects. In linear tooth belt drives, the structural loop consists of gear reducer, bearings, belts, pulleys, the housing, guideways and the frame, drives and the tool, and its holders. Because of the inaccuracy of the structural loop of the linear drives, the actual end position may differ from the nominal end position, which causes a relative positioning error between the machine structures of the same kind. If the position of the axis affects the location of another axis, the error will be a function of positions similarly as is the case in multi-axis robots, which makes the machining errors more complex and challenging to compensate.

It should be pointed out that resolution, accuracy, and repeatability are not the same thing. These terms can be defined as follows: Resolution is the smallest resolvable increment of measurement or motion. This could be a software limit or a mechanical limit. Repeatability describes the ability to repeat the same motion or measurement within a certain definable limit. Accuracy can be represented as a bias between the nominal value (reference set-point) and the actual value (actual set-point).

3 Friction and Backlash

Friction is present in all mechanical systems, where the surfaces of two pieces contact each other, and widely studied especially in classical mechanical engineering. The control engineer must understand the friction phenomena, especially if high-precision control is required. Friction can be divided into three different components: Coulomb friction, static friction (stiction), and viscous damping.

Coulomb friction, also called sliding friction, has a constant value, which changes only when the direction of the motion is changed. Stiction, also known as break-away friction, has a value other than zero only when the motor speed is zero, and elsewhere the value of stiction is zero. The force required to overcome the static friction is called the break-away force. If the sliding friction is smaller than the static stiction, a Stribeck effect will occur. The viscous damping is changing as a function of motor speed. Rolling friction is less significant than the sliding friction. Detailed information about friction phenomena and models to describe its behaviour are given in previous report.

In mechanics, backlash is also quite common especially when a gearbox is used. Backlash means that the motor turns briefly before the load moves. Together with the fricton forces, backlash makes the system behaviour non-linear.

In a linear belt driven servo system, rotational friction appears both in the motor bearings and in the belt pulleys. Linear sliding friction is generated when the cart is moving.Instability can occur in a servo system because of mechanical resonances, which are typically caused by the compliance of power transmission components, such as gearboxes, drive trains, couplings, and belts. These resonances are not usually taken into account when analysing and designing servo control laws. The servo control laws are formulated for rigidly-coupled loads, although in practical machines, resonances are always present. These resonances decrease the stability margins of the system; usually this is compensated by reducing the controller gains, which further reduces the performance of the servo system. In tooth belt systems, the resonant frequency varies in a wide range as a function of cart position.

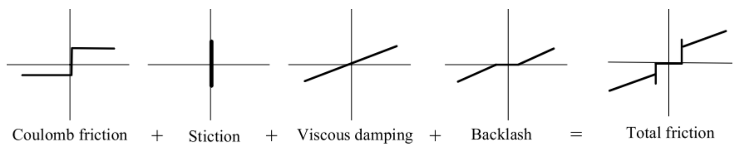

Figure 1: Behaviour of different frictions and backlash.

4 Elasticity of Belt

The belt has elasticity properties; therefore it is possible to change the length through the application external forces, caused by motor torque and cart mass. This quantity can be described by generalized Hooke's law in terms of the concepts of stress and strain. Stress is a quantity that is proportional to the force causing a deformation; strain is the measure of the degree of the deformation.

Tooth belt drives are flexible systems, and the resonances of the system occur at low frequencies. Flexibility can be the major challenge for the control system, and cause difficulties to the desired trajectory tracking. The vibration caused by the resonance of the system can degrade the position accuracy during motion and also in the setpoint position. Solutions to the problem of mechanical vibration include stiffening of the mechanical structure, adding damping to the system, using additional vibration sensors and vibration controllers, making the closed- loop controller more sophisticated and complex, or using the references that do not excite resonances.

5 Mathematical Model

Developing a high-performance position control for linear tooth belt drives requires a mathematical model that sufficiently represents the essential system dynamics. The main drawbacks related to linear tooth belt drives come from the elastic structure of the tooth belt itself, the position-dependent resonance frequency, and the large variation of the system parameters. Although a properly designed controller can cope with large parameter variation, certain knowledge regarding the system behaviour is crucial for a robust control design.

A linear belt drive guide is considered which consists of an actuator (electric motor),a gearbox, two belt pulleys, a moving cart, and a toothed belt, which connects the cart and the belt pulleys together. In the model the driven end and the moving cart dynamics dominate in the system. Therefore, the free end dynamics are neglected which simplifies the system model to a two-mass system. Rubber tooth belt has also damping feature that is considered in the modelling.

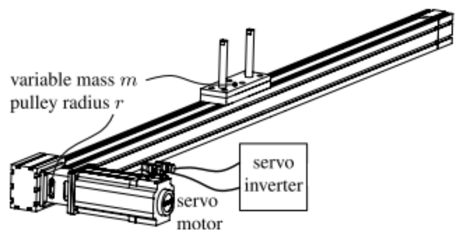

Figure 2: Structure of Belt Driven system.

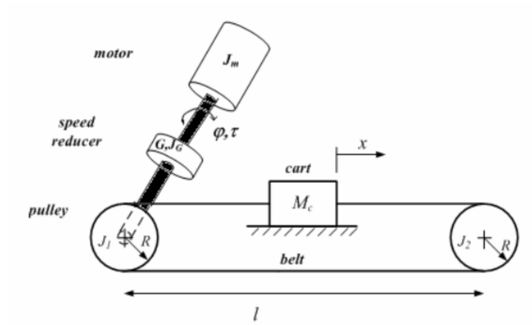

Figure 3: Linear Belt Drive system schematic.

The derivation of model described in Fig.3 is carried out under the following basic assumptions:

- The belt operates within the linear elastic range
- The belt's stiffness is inversely proportional to belt's length
- The belt can be modelled by linear massless spring
- Friction that is concentrated in the motor and pulley bearings, the speed reducer (gearbox), and in the carriage guidance are considered as an unknown exogenous disturbance
- Link between the motor shaft and the driving pulley is rigid
- The motor can ensure a high- dynamic-torque response with a negligible time delay

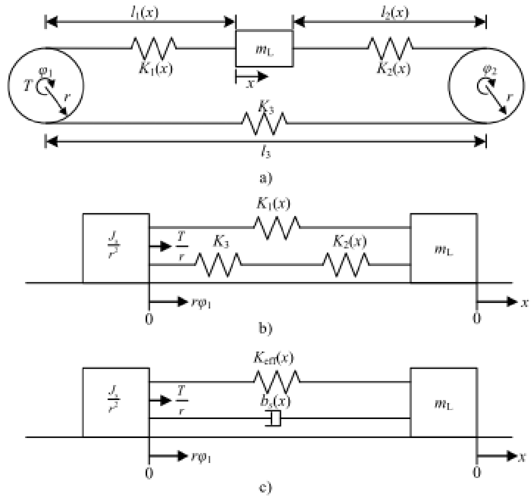

Figure 4: System models. a) Model of the linear tooth belt axis. b) Simplified model without free-end dynamics c) Simplified two-mass model with belt damping.

5.1 Three-Mass Model

The tooth belt axis can be presented as a spring-mass system as illustrated in Fig.4. It represents a complex non-linear distributed-parameter system. The mathematical model of the belt-driven servomechanism can be obtained using modal analysis. One can develop the sixth-order mathematical model that is presented by the motion;

$$\begin{gathered}
\left(J_1 + G^2\left(J_G + J_m\right)\right) \cdot \ddot{\varphi}_1 + \tau_{f1} = G\tau - R \cdot \left[K_1(x) \cdot (R \cdot \varphi_1 - x) - K_3 \cdot (R \cdot \varphi_2 - R \cdot \varphi_1)\right] \\
J_2 \cdot \ddot{\varphi}_2 + \tau_{f2} = R \cdot \left[K_2(x) \cdot (x - R \cdot \varphi_2) - K_3 \cdot (R \cdot \varphi_2 - R \cdot \varphi_1)\right] \\
M_c \cdot \ddot{x} + f_f = K_1(x) \cdot (R \cdot \varphi_1 - x) - K_2(x) \cdot (x - R \cdot \varphi_2)
\end{gathered} \tag{1}$$

where

J_1, J_2	inertia moment of the driving and the driven pulley, respectively;
J_G, J_m	inertia moment of the speed reducer and the motor, respectively;
M_c	mass of the carriage;
G	speed-reducer ratio;
R	radius of the pulleys;
K_1, K_2, K_3	position-dependent elasticity coefficients of the belt;
q_1, q_2, φ	angular position of the driving pulley, driven pulley, and the motor, respectively;
x	carriage position;
τ	torque developed by the motor;
τ_{f1}, τ_{f2}	friction torque which affects the pulleys;
f_f	friction force on the carriage.

5.2 Two-Mass Model

The model (1) is a highly coupled and non-linear system with exogenous disturbances which enter at the driving side, as well as the load side, of the system. However, the pulley inertia is small in comparison with the motor and the load-side inertia. Therefore, the model can be simplified and reduced to a two-mass system.

Reduced-order system model represents the essential system dynamics in different operating points and can be used to design high-performance robust position control for tooth belt drives. Thus, in the model, the driven end and the moving cart dynamics dominate in the system. Therefore, the free end dynamics can be neglected, also the inertias of the pulleys are small compared with the inertia of the motor and the load, which simplifies the system model to a two-mass system.

$$G^2 \cdot J_{\mathrm{M}} \cdot \ddot{\varphi}_1 + \tau_{\mathrm{f}} = G \cdot \tau - b_s \cdot R \cdot \dot{w} - K_{eff}(x) \cdot R \cdot w \quad (2)$$

$$M_{\mathrm{c}} \cdot \ddot{x} + f_{\mathrm{f}} f_{\mathrm{d}} = b_s \cdot \dot{w} + K_{eff}(x) \cdot w \quad (3)$$

where $w = (R \cdot \varphi_1 - x)$ and $\dot{w} = (R \cdot \dot{\varphi}_1 - \dot{x})$.

b_s : damping constant of the belt.

f_{d} : external disturbance on the table.

The equivalent spring-constant K_{eff} can be derived, when considering the system as a two-mass system with three springs connecting the masses together. The equivalent spring constant $K_{eff}(x)$ can be calculated by first connecting two serial springs ($K_2(x)$ and K_3) together, and then connecting the result parallel with the spring $K_1(x)$.

$$K_{\mathrm{eff}}(x) = K_1(x) + \frac{K_2(x) K_3}{K_2(x) + K_3} \quad (4)$$

In a real system, however, only the spring constant K_3 is stationary, while the other two spring constants are functions of cart position. Equations for spring-constants can be derived from Hooke's law when the belt stretching characteristics are known. Tensile strain can be written as follows

$$\varepsilon = \frac{\delta}{l_0} \quad (5)$$

where ε is the stretched part of the particle, when stretching with the force F, and l_0 is the initial length of the particle. An equation for the spring constants can be then derived from Hooke's law.

$$\begin{aligned} &\delta = \varepsilon \cdot l_0 \\ &F = k \cdot x = k \cdot \delta = k \cdot \varepsilon \cdot l_0 \\ &\Rightarrow k = \frac{F}{\varepsilon} \cdot \frac{1}{l_0} \end{aligned} \quad (6)$$

Functions for the position-dependent spring constants $K_1(x)$ and $K_2(x)$ can be found, and the value for the stationary spring constant K_3 can be calculated.

$$K_1(x) = \frac{F}{\varepsilon} \cdot \frac{1}{l_1 + x}$$
$$K_2(x) = \frac{F}{\varepsilon} \cdot \frac{1}{l_2 - x} \tag{7}$$
$$K_3 = \frac{F}{\varepsilon} \cdot \frac{1}{l_3}$$

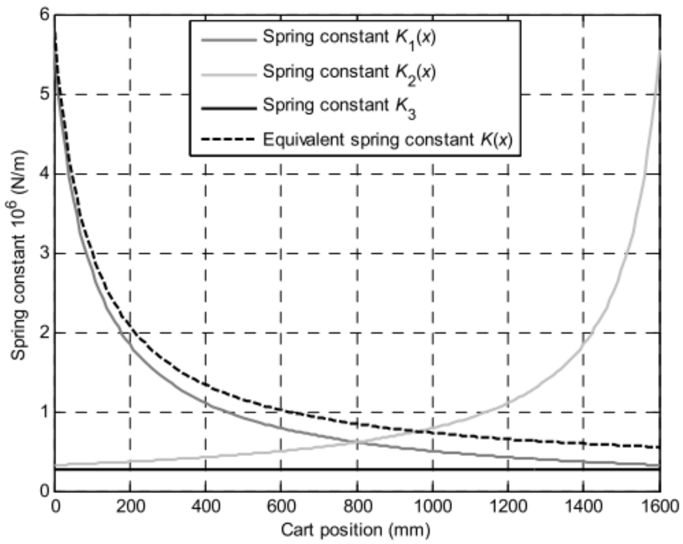

Figure 5: An example of spring constants as functions of cart position.

Spring constants K_1, K_2, K_3 and K_{eff} are shown as a function of cart position in Fig.5. It can be seen that the spring constant K_1 mostly affects the equivalent spring constant. When moving away from the motor the equivalent spring constant differs more and more from the spring constant K_1.

When considering the spring constants K_1 and K_2, the initial length l_0 varies when the cart is moving. The constant part F/ε can be found from the manufacturer data sheets for tooth belt axes. This constant part F/ε should be accurate if the target is that the simulation model matches the real system. The maximum feed force for the belt is also an important quantity, because it is the decisive limiting factor affecting the performance and the production rate of the system.

The damping constant of the belt is considered in the modelling by using the following approximation:

$$b_s(x) = \frac{K_{\text{eff}}(x)}{\omega_{\text{res}}(x) \cdot Q_k} \tag{8}$$

where $\omega_{res}(x)$ is the position-dependent resonance frequency for the system, and Q_k represents the constant for different coupling approximations.

$$Q_k = \frac{1}{2\zeta} \tag{9}$$

The spring constant of the belt is position dependent, as shown in (7), and thus, the system dynamics of the flexible part varies as a function of position, which also makes the resonant frequency of the system vary as a function of position, which leads to a problem that a simple notch band-stop filter cannot be used to block the mechanical resonances when designing an accurate and fast position control.

6 Conclusion

A linear tooth belt drive is a complicated system, the operation of which is greatly affected by the non-linear friction and the flexibility of the belt. The first resonant frequency of the system varies as a function of cart position and given belt characteristics. All these reduce the performance of the system and make the use of linear control design methods difficult. Therefore, compensation of non-linearities such as friction, flexibility and backlash are essential, and will be investigated deeply on later studies.

It is also should be noted that backlash effect is omitted and friction model is selected simpler models. Since their effects on the servo control bandwidth is critical, they have to be considered on system models and compensated accordingly.

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