Lab report 5 (Variant 2)

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Task 1

1.1 Condition

Спростити вираз та обчислити його значення при $a = 5$:

$$\sqrt[4]{(4-4a+a^2)(a^2-3)(a+1)} : \frac{a^2+5a+4}{\sqrt[4]{a-2}}$$

1.2 Solution code

```
import sympy as sp

a = sp.symbols('a')

expr = sp.root((1 - 2*a + a**2) * (a**2 - 1) * (a - 1), 4) / ((a**2 + 2*a - 3) /
sp.root(a + 1, 4))

simplified_expr = sp.simplify(expr)

result = simplified_expr.subs(a, 5)

print("Упрощённое выражение:", simplified_expr)
print("Значение выражения при a = 5:", result)
```

1.3 Output

```
Упрощённое выражение: ((a - 1)*(a**2 - 1)*(a**2 - 2*a + 1))**(1/4)*(a + 1)**(1/4)/(a**2
+ 2*a - 3)
Значение выражения при a = 5: sqrt(6)/8
```

Task 2

2.1 Condition

1. Знайти всі розв'язки рівняння $sin^4 x + cos^4 x = 0.8$.

2.2 Solution code

```
import sympy as sp

x = sp.symbols('x')

equation = sp.sin(x)**4 - sp.cos(x)**4 - 0.8

solutions = sp.solve(equation, x)

print("sin^4(x) - cos^4(x) - 0.8 = 0:")
for sol in solutions:
    print(sol)
```

2.3 Output

```
sin^4(x) - cos^4(x) - 0.8 = 0:
-1.89254688119154
-1.24904577239825
1.24904577239825
1.89254688119154
```

Task 3

3.1 Condition

Знайти всі розв'язки системи рівнянь

$$\begin{cases} x + y + z = 6 \\ x^2 + y^2 + z^2 = 14 \\ x^3 + y^3 + z^3 = 36 \end{cases}$$

3.2 Solution code

```
import sympy as sp

x, y, z = sp.symbols('x y z')

eq1 = x + y + z - 6
eq2 = x**2 + y**2 + z**2 - 14
eq3 = x**3 + y**3 + z**3 - 36

solutions = sp.solve([eq1, eq2, eq3], (x, y, z))

print("Решения системы уравнений:")
for sol in solutions:
    print(sol)
```

3.3 Output

```
Решения системы уравнений:
(1, 2, 3)
(1, 3, 2)
(2, 1, 3)
(2, 3, 1)
(3, 1, 2)
(3, 2, 1)
```

Task 4

4.1 Condition

1. Знайти границю $\lim_{x\to 1} \frac{x^m-1}{x^n-1}$, де m, n - натуральні числа.

4.2 Solution code

```
import sympy as sp

x = sp.symbols('x')
m, n = sp.symbols('m n', positive = True, integer = True)

expr = (x**m - 1) / (x**n - 1)

limit_expr = sp.limit(expr, x, 1)

print(limit_expr)
```

4.3Output

```
m/n
```

Task 5

5.1 Condition

1. Обчислити лівосторонню границю $\lim\limits_{x \to -0} \frac{\sin|x|}{x}$.

5.2 Solution code

```
import sympy as sp

x = sp.symbols('x')

expr = sp.sin(sp.Abs(x)) / x

left_limit = sp.limit(expr, x, 0, dir='-')

print(left_limit)
```

5.3 Output

```
-1
```

Task 6

6.1 Condition

Дани функція $f(x, y) = x^3 - y^3$ і точка $M(1;1)$.

1. Обчислити градієнт функції f в точці M.
2. Обчислити похідну функції f у точці M за напрямом вектору $\bar{a} = (-3;2)$.
3. Зростає чи спадає функція f у точці M по заданому напряму?

6.2 Solution code

```
import sympy as sp

x, y = sp.symbols('x y')

f = x**3 - y**2

grad_f = [sp.diff(f, var) for var in (x, y)]
grad_at_M = [grad.subs({x: 1, y: 1}) for grad in grad_f]
a = sp.Matrix([-3, 2])

a_norm = a.norm()
a_unit = a / a_norm

grad_at_M_matrix = sp.Matrix(grad_at_M)
directional_derivative = grad_at_M_matrix.dot(a_unit)

if directional_derivative > 0:
    direction = 'inc'
elif directional_derivative < 0:
    direction = 'dec'
else:
    direction = 'constant'

print(f"grad(f) in M(1, 1): {grad_at_M}")
print(f"derivative of f in M in a direction: {directional_derivative}")
print(f"Function f in M in a direction {'inc' if direction == 'inc' else 'dec' if 
direction == 'dec' else 'constant'}")
```

6.3 Output

```
grad(f) in M(1, 1): [3, -2]
derivative of f in M in a direction: -sqrt(13)
Function f in M in a direction dec
```

Task 7

7.1 Condition

Знайти всі часткові похідні функції $f(x,y) = \cos(x+y)\,e^{xy}$ до другого порядку включно. Спростити відповіді.

7.2 Solution code

```
import sympy as sp

x, y = sp.symbols('x y')

f = sp.cos(x + y) * sp.exp(x * y)

f_x = sp.diff(f, x)
f_y = sp.diff(f, y)

f_xx = sp.diff(f_x, x)
f_yy = sp.diff(f_y, y)
f_xy = sp.diff(f_x, y)
f_yx = sp.diff(f_y, x)

f_x = sp.simplify(f_x)
f_y = sp.simplify(f_y)
f_xx = sp.simplify(f_xx)
f_yy = sp.simplify(f_yy)
f_xy = sp.simplify(f_xy)
f_yx = sp.simplify(f_yx)

print(f"x: {f_x}")
print(f"y: {f_y}")
print(f"xx: {f_xx}")
print(f"yy: {f_yy}")
print(f"xy: {f_xy}")
print(f"yx: {f_yx}")
```

7.3 Output

```
x: (y*cos(x + y) - sin(x + y))*exp(x*y)
y: (x*cos(x + y) - sin(x + y))*exp(x*y)
xx: (y**2*cos(x + y) - 2*y*sin(x + y) - cos(x + y))*exp(x*y)
yy: (x**2*cos(x + y) - 2*x*sin(x + y) - cos(x + y))*exp(x*y)
xy: (x*y*cos(x + y) - x*sin(x + y) - y*sin(x + y))*exp(x*y)
yx: (x*y*cos(x + y) - x*sin(x + y) - y*sin(x + y))*exp(x*y)
```

Task 8

8.1 Condition

Знайти загальний розв’язок диференціального рівняння $y'' + y = \tan x$.

8.2 Solution code

```
import sympy as sp

x = sp.symbols('x')
y = sp.Function('y')(x)

eq = sp.Derivative(y, x, x) + y - sp.tan(x)

solution = sp.dsolve(eq)

sp.pprint(solution)
```

8.3 Output

$$y(x) = C_2 \cdot \sin(x) + \left(C_1 + \frac{\log(\sin(x) - 1)}{2} - \frac{\log(\sin(x) + 1)}{2}\right) \cdot \cos(x)$$

Task 9

9.1 Condition

Показати, що функція $u = \frac{1}{\sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + (x_3 - y_3)^2}}$ задовільняє рівняння еліптичного типу $\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2} = 0$.

9.2Solution code

```
import sympy as sp

x1, x2, x3, y1, y2, y3 = sp.symbols('x1 x2 x3 y1 y2 y3')

u = 1 / sp.sqrt((x1 - y1)**2 + (x2 - y2)**2 + (x3 - y3)**2)

u_x1 = sp.diff(u, x1, 2)
u_x2 = sp.diff(u, x2, 2)
u_x3 = sp.diff(u, x3, 2)

laplacian = u_x1 + u_x2 + u_x3

laplacian_simplified = sp.simplify(laplacian)

print(laplacian_simplified)
```

9.3 Output

```
0
```

Task 10

10.1 Condition

Знайти первісну функції $f(x) = \frac{x}{3-2x^2}$. Перевірити результат диференціюванням.

10.2Solution code

```
import sympy as sp

x = sp.symbols('x')

f = x / (3 - 2 * x**2)

F = sp.integrate(f, x)

F_prime = sp.diff(F, x)

print(f"antiderivative f(x): {F}")
print(f"(d/dx(F)): {F_prime}")
```

10.3 Output

```
antiderivative f(x): -log(2*x**2 - 3)/4
(d/dx(F)): -x/(2*x**2 - 3)
```

Task 11

11.1 Condition

Обчислити визначений інтеграл

$$\int_0^{\pi} x \sin x \, dx$$

11.2Solution code

```
import sympy as sp

x = sp.symbols('x')

f = x * sp.sin(x)

integral_result = sp.integrate(f, (x, 0, sp.pi))

print(integral_result)
```

11.3 Output

```
pi
```

Task 12

12.1 Condition

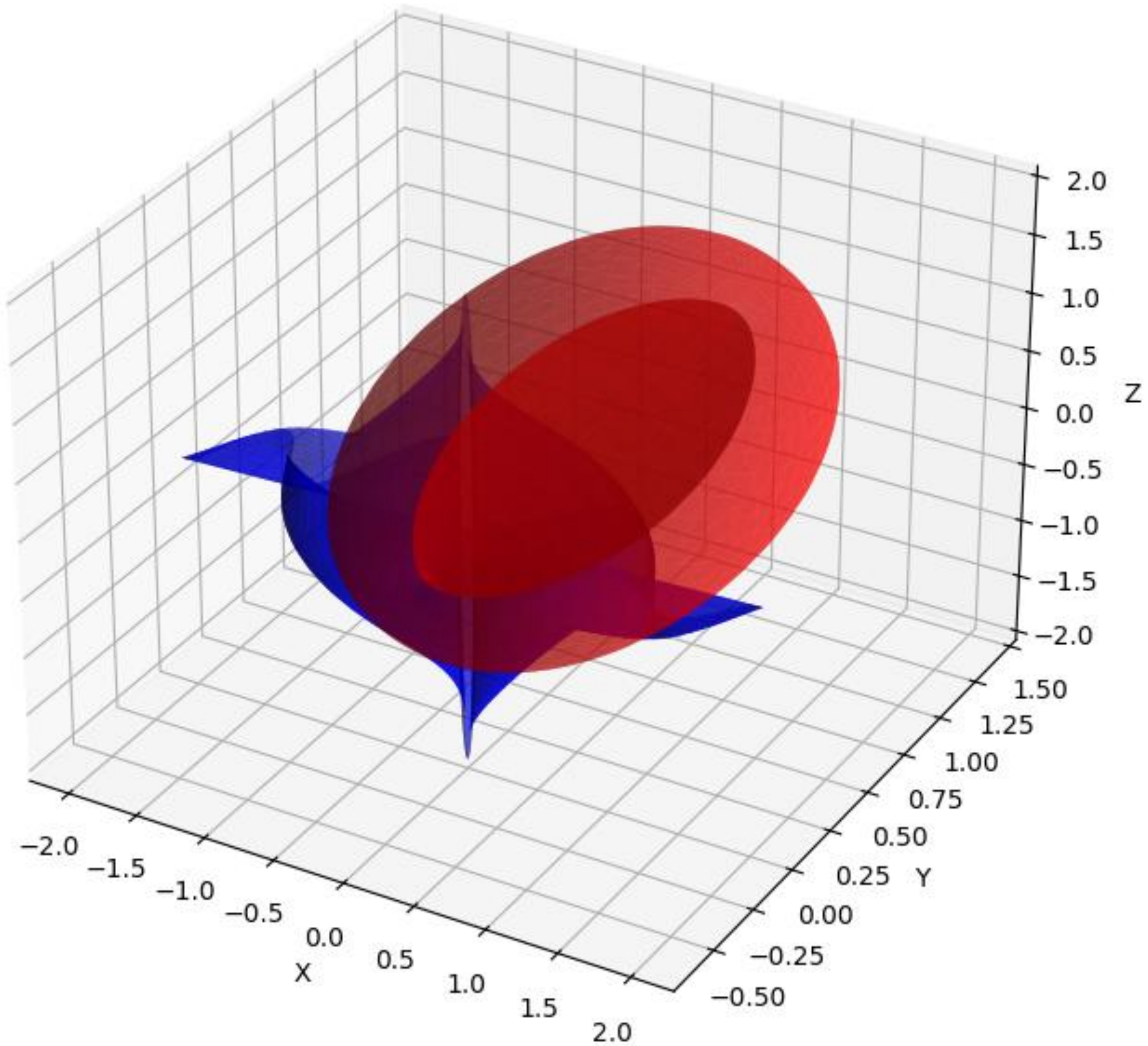
1. Побудувати на одному рисунку 2 поверхні, що задані параметрично:

$$\begin{cases} x = \sin u\left(1+0.2v\right), \\ y = 0.2\sin u\cos v, \qquad u,v\in\left[-5;5\right] \\ z = \cos u\left(1+0.2v\right), \end{cases} \qquad \begin{cases} x = 0.4+\left(0.3+\cos v\right)\cos u, \\ y = 0.4+\sin v, \qquad u,v\in\left[0;2\pi\right] \\ z = 0.4+\left(0.3+\cos v\right)\sin u, \end{cases}$$

12.2Solution code

```
import sympy as sp
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

u, v = sp.symbols('u v')

x1 = sp.sin(u) * (1 - 0.2 * v)
y1 = 0.2 * sp.sin(u) * sp.cos(v)
z1 = sp.cos(u) * (1 + 0.2 * v)

x2 = 0.4 + (0.3 + sp.cos(v)) * sp.cos(u)
y2 = 0.4 + sp.sin(v)
z2 = 0.4 + (0.3 + sp.cos(v)) * sp.sin(u)

f_x1 = sp.lambdify((u, v), x1)
f_y1 = sp.lambdify((u, v), y1)
f_z1 = sp.lambdify((u, v), z1)

f_x2 = sp.lambdify((u, v), x2)
f_y2 = sp.lambdify((u, v), y2)
f_z2 = sp.lambdify((u, v), z2)

u_vals, v_vals = np.meshgrid(np.linspace(-5, 5, 400), np.linspace(-5, 5, 400))

x1_vals = f_x1(u_vals, v_vals)
y1_vals = f_y1(u_vals, v_vals)
z1_vals = f_z1(u_vals, v_vals)

u_vals2, v_vals2 = np.meshgrid(np.linspace(0, 2 * np.pi, 400), np.linspace(0, 2 * np.pi,
400))

x2_vals = f_x2(u_vals2, v_vals2)
y2_vals = f_y2(u_vals2, v_vals2)
z2_vals = f_z2(u_vals2, v_vals2)

fig = plt.figure(figsize=(10, 8))
ax = fig.add_subplot(111, projection='3d')

ax.plot_surface(x1_vals, y1_vals, z1_vals, color='b', alpha=0.5)

ax.plot_surface(x2_vals, y2_vals, z2_vals, color='r', alpha=0.5)

ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')

plt.show()
```

12.3 Output



X
Y
Z
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
1.25
1.50