포트폴리오의 VaR 추정

금융통계 기말과제

통계학과 2018580028 정세린

포트폴리오 소개

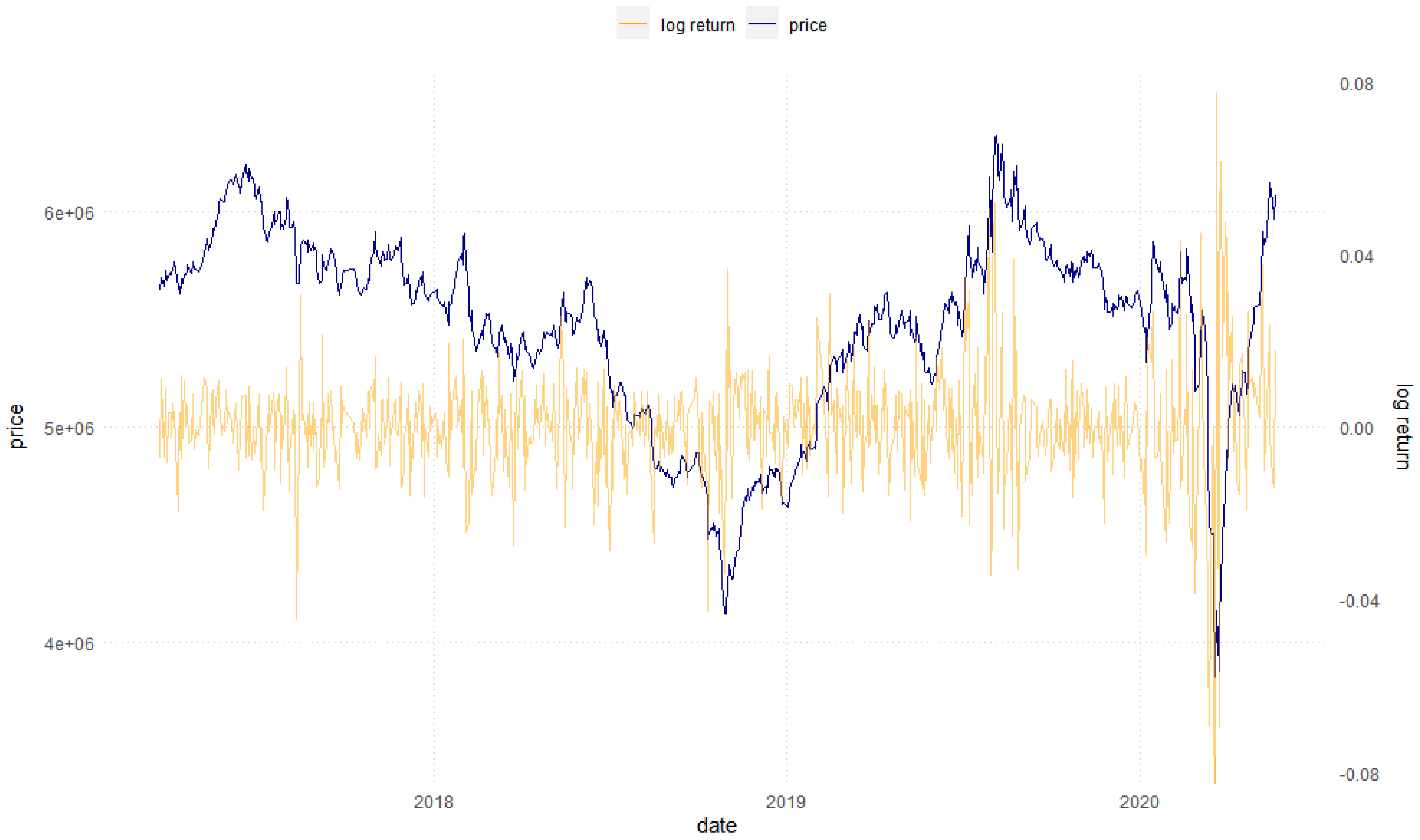
- 분석기간 : 2014년 5월 21일 ~ 2020년 5월 20일

- 구성 종목 : 하이트진로, 제일기획, 농심, NHN, 한국제지 , 모나미
  (데이터 출처 : 한국거래소)

- $V_{P,t} = 42 \times S_{1,t} + 52 \times S_{2,t} + 3 \times S_{3,t} + 14 \times S_{4,t} + 34 \times S_{5,t} + 253 \times S_{6,t}$

- 2017년 3월 23일부터 2020년 5월 20일까지의 VaR, ES를 추정하였다.
  (총 772일)

포트폴리오 소개



log return
price
6e+06
5e+06
4e+06
price
0.08
0.04
0.00
-0.04
-0.08
log return
2018
2019
2020
date

VaR 추정

- $-R_t = \sigma_t z_t$로 둘 때,

$$\widehat{VaR}_t^{1-p} = \hat{\sigma}_t \hat{z}_{1-p}, \qquad \widehat{ES}_t^{1-p} = \hat{\sigma} \times \frac{1}{p} \int_{1-p}^{1} \hat{z}_u \, du$$

- 변동성 모형 : GARCH(1, 1), GJR-GARCH(1, 1)

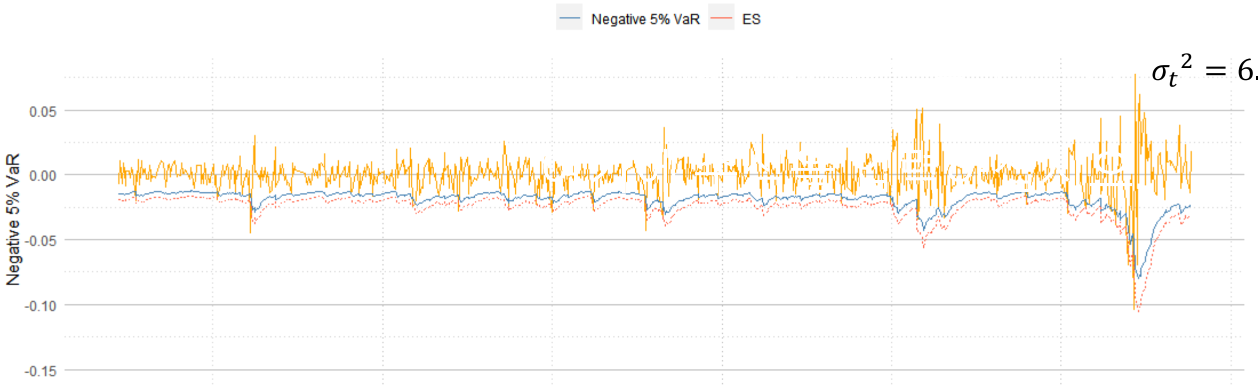
- $\hat{z}_{1-p}$ 추정 방법 : 표준정규분포, standardized t분포, asymmetric t분포 가정.
  또는 Peak Over Threshold Model(EVT), FHS를 이용하여 추정.

Standard Normal Distribution

$p = 0.05$

$\hat{z}_{1-p}$: 표준정규분포로 가정, 추정

GARCH(1, 1)

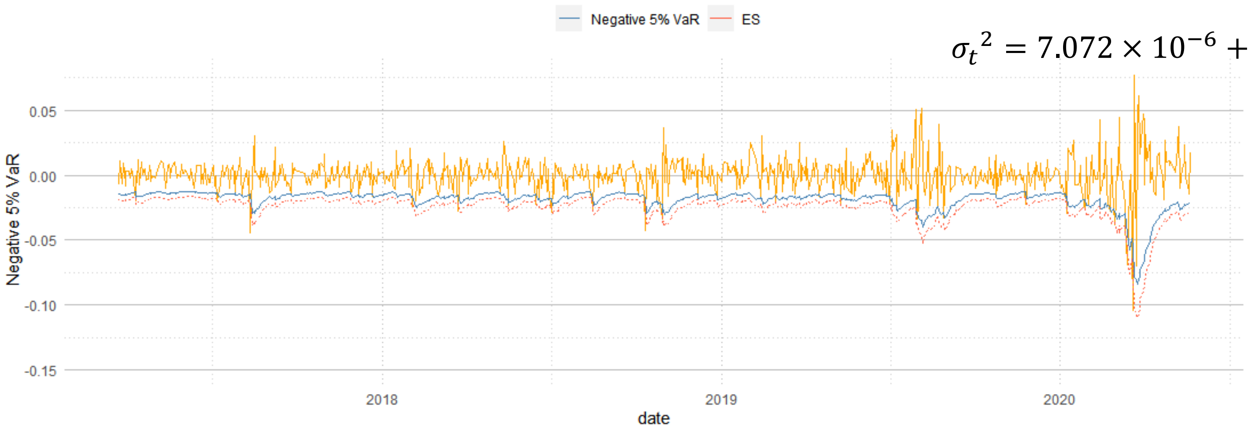


$$\sigma_t^{\ 2} = 6.783 \times 10^{-6} + 0.100R_{t-1} + 0.851\sigma_{t-1}^{\ 2}$$

$$-\widehat{VaR_t^{1-p}} = -\hat{\sigma}_t \times 1.645$$

$$-\widehat{ES_t^{1-p}} = -\hat{\sigma}_t \times 2.063$$

GJR-GARCH(1, 1)

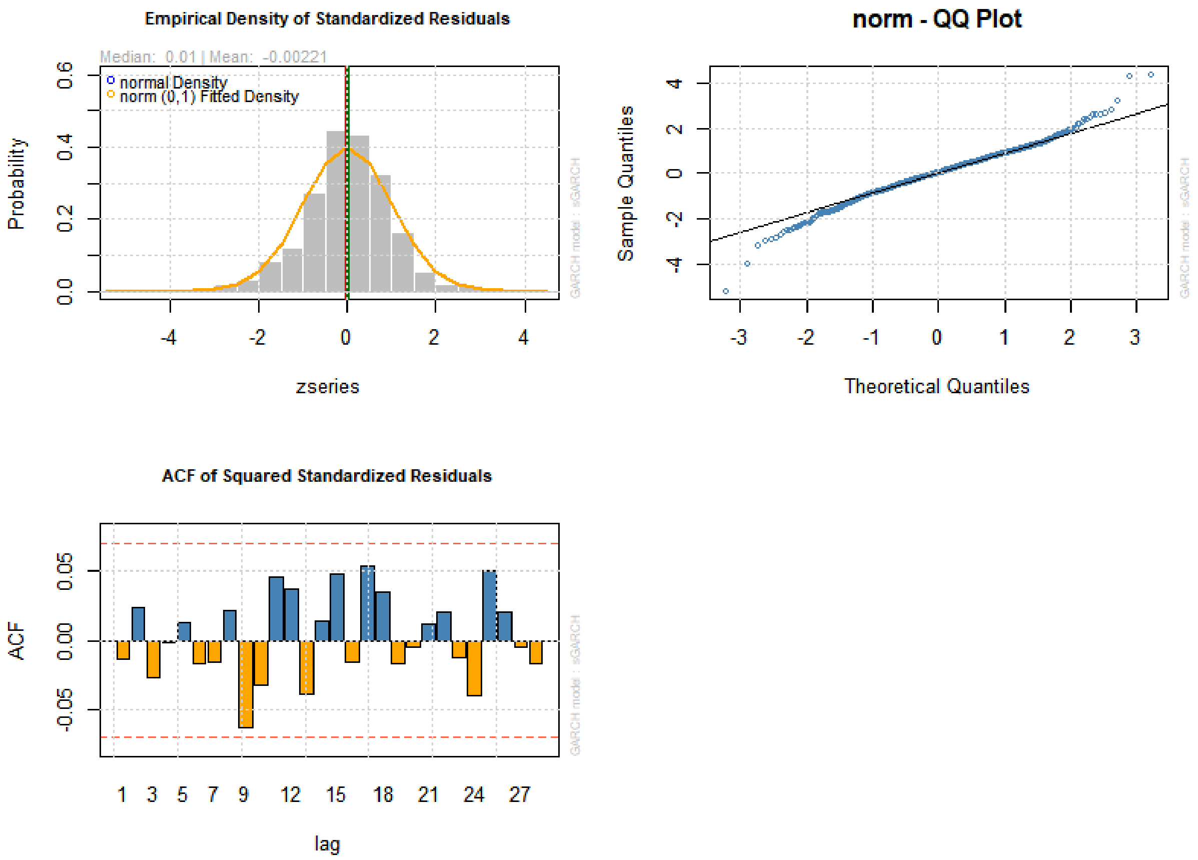


$$\sigma_t^{\ 2} = 7.072 \times 10^{-6} + 0.067(1 + 0.065I_{t-1})R_{t-1} + 0.848\sigma_{t-1}^{\ 2}$$

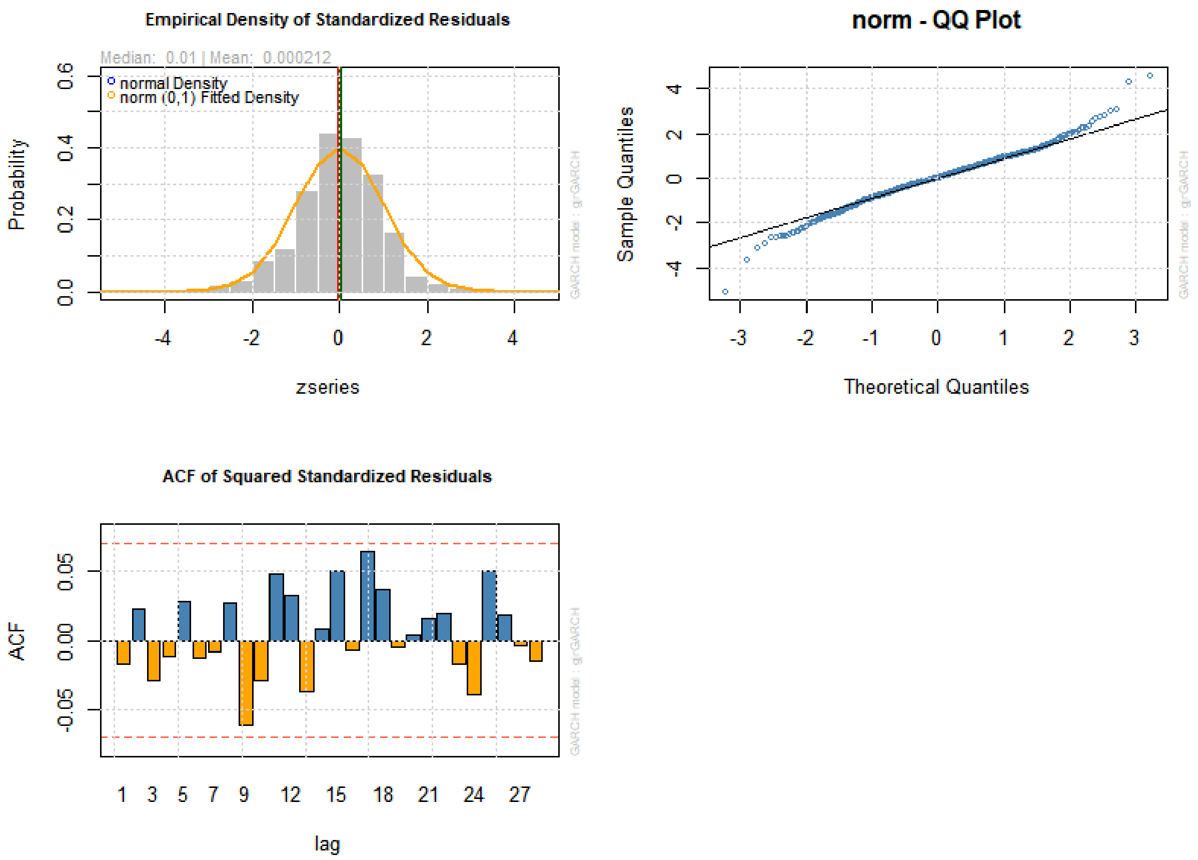
$$-\widehat{VaR_t^{1-p}} = -\hat{\sigma}_t \times 1.645$$

$$-\widehat{ES_t^{1-p}} = -\hat{\sigma}_t \times 2.063$$

Standard Normal Distribution



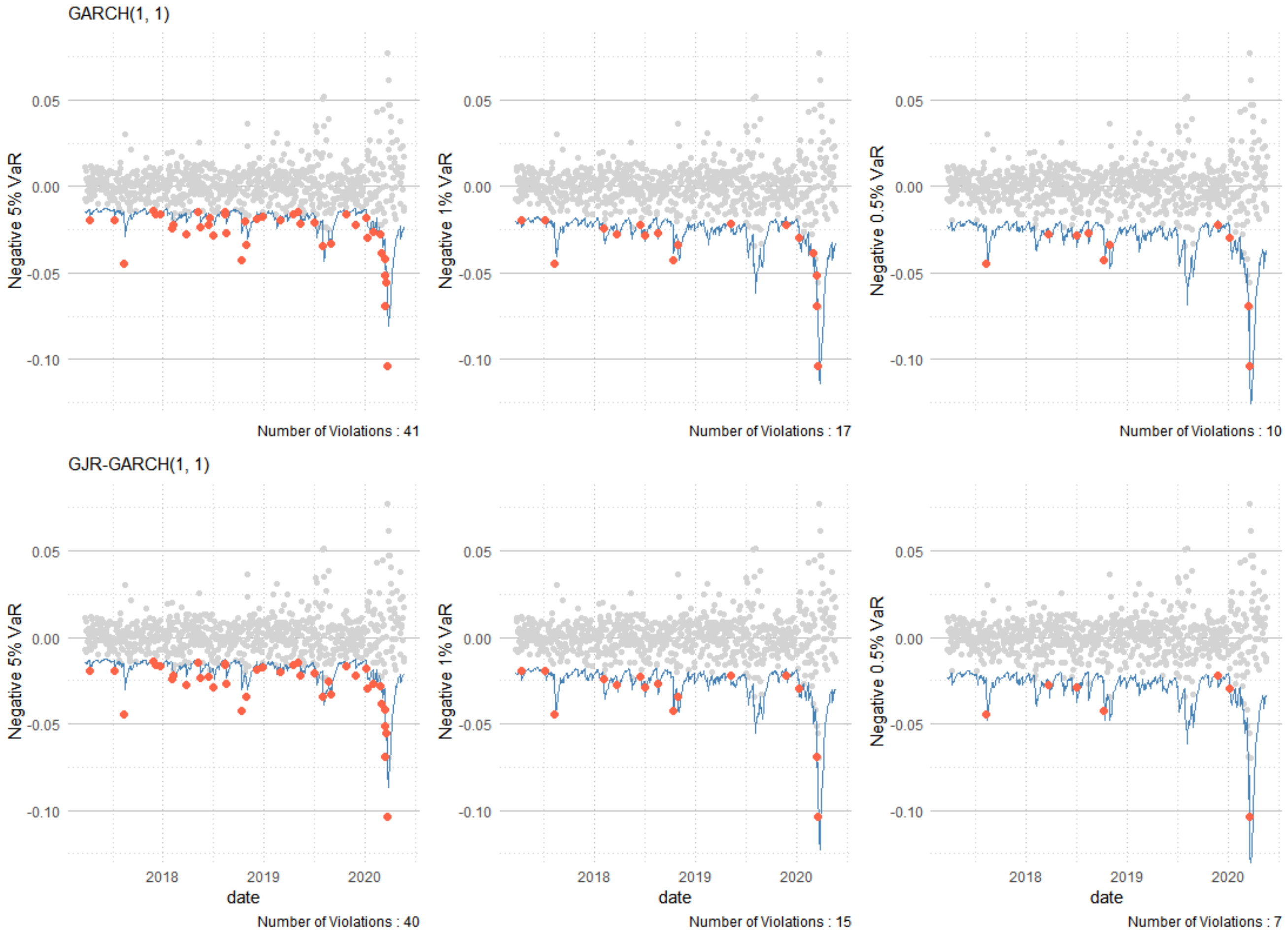
↑ GARCH(1, 1) 모형에서의 잔차 분석



↑ GJR-GARCH(1, 1) 모형에서의 잔차 분석

Standard Normal Distribution

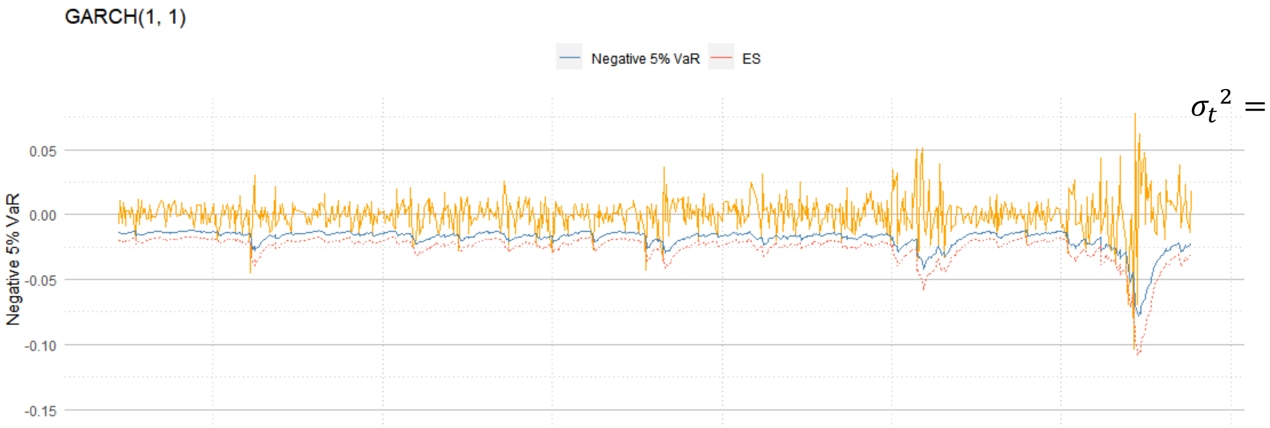
$p = 0.05,\ 0.01,\ 0.005$일 때 각각의 초과손실



Standardized t Distribution

$p = 0.05$

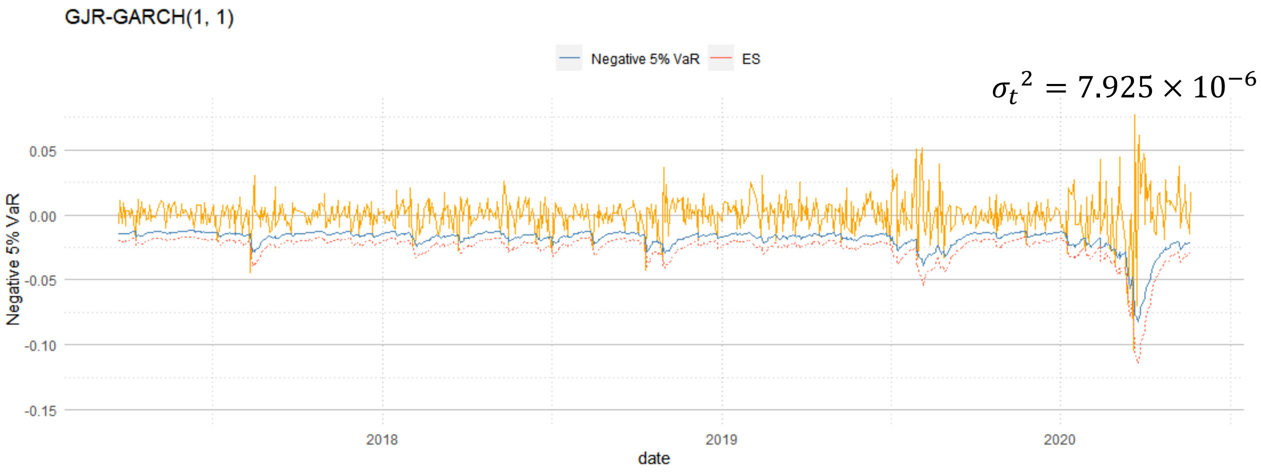
$\hat{z}_{1-p}$: standardized t 분포로 가정, 추정



$$\sigma_t{}^2 = 7.518 \times 10^{-6} + 0.102 R_{t-1} + 0.844 \sigma_{t-1}{}^2$$

$$-\widehat{VaR_t^{1-p}} = -\hat{\sigma}_t \times 1.586$$

$$-\widehat{ES_t^{1-p}} = -\hat{\sigma}_t \times 2.214$$

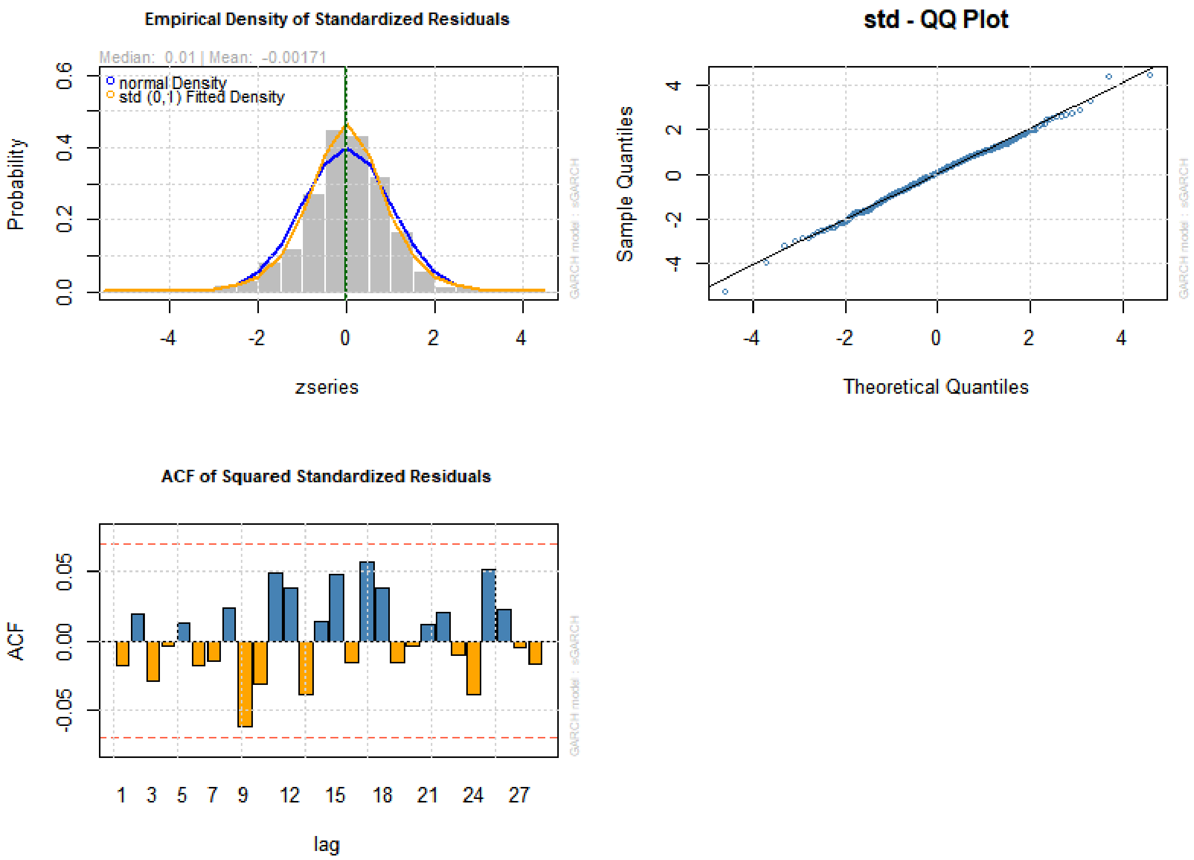


$$\sigma_t{}^2 = 7.925 \times 10^{-6} + 0.080(1 + 0.047 I_{t-1}) R_{t-1} + 0.839 \sigma_{t-1}{}^2$$

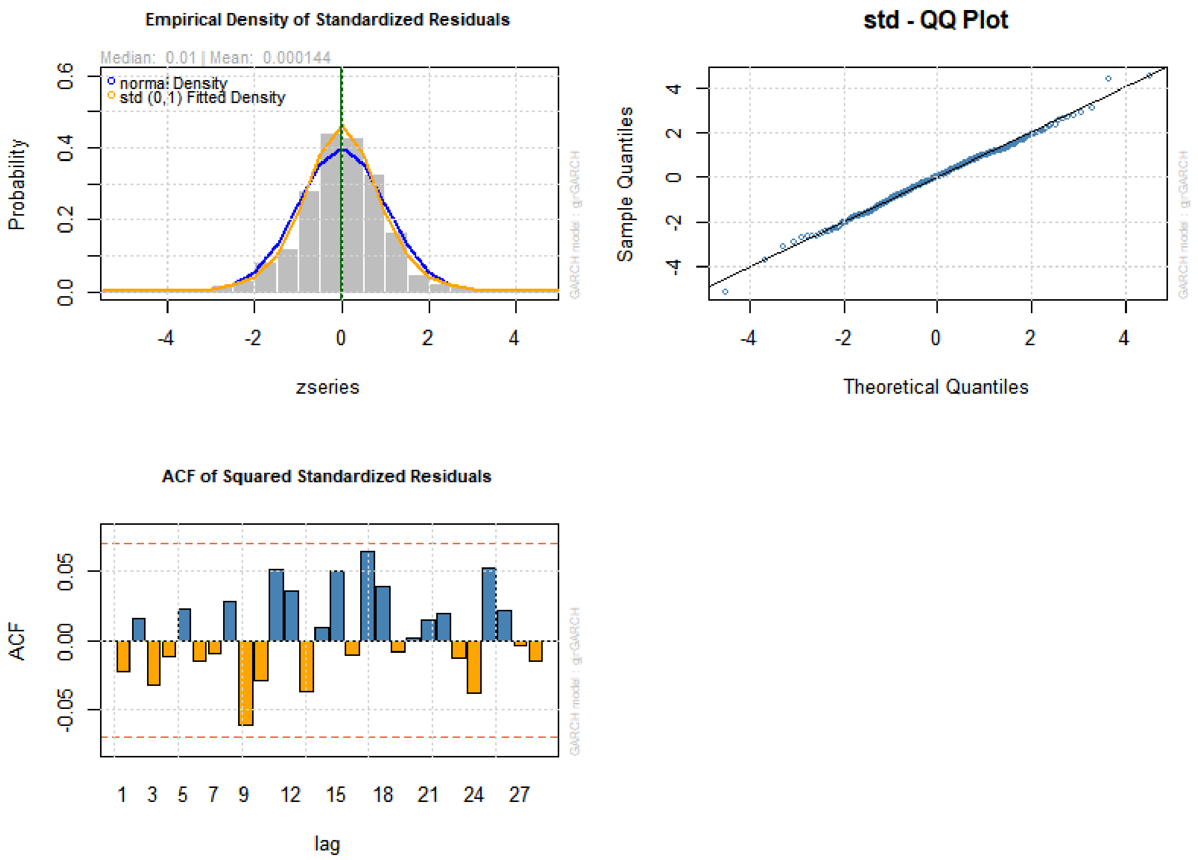
$$-\widehat{VaR_t^{1-p}} = -\hat{\sigma}_t \times 1.588$$

$$-\widehat{ES_t^{1-p}} = -\hat{\sigma}_t \times 2.211$$

Standardized t Distribution



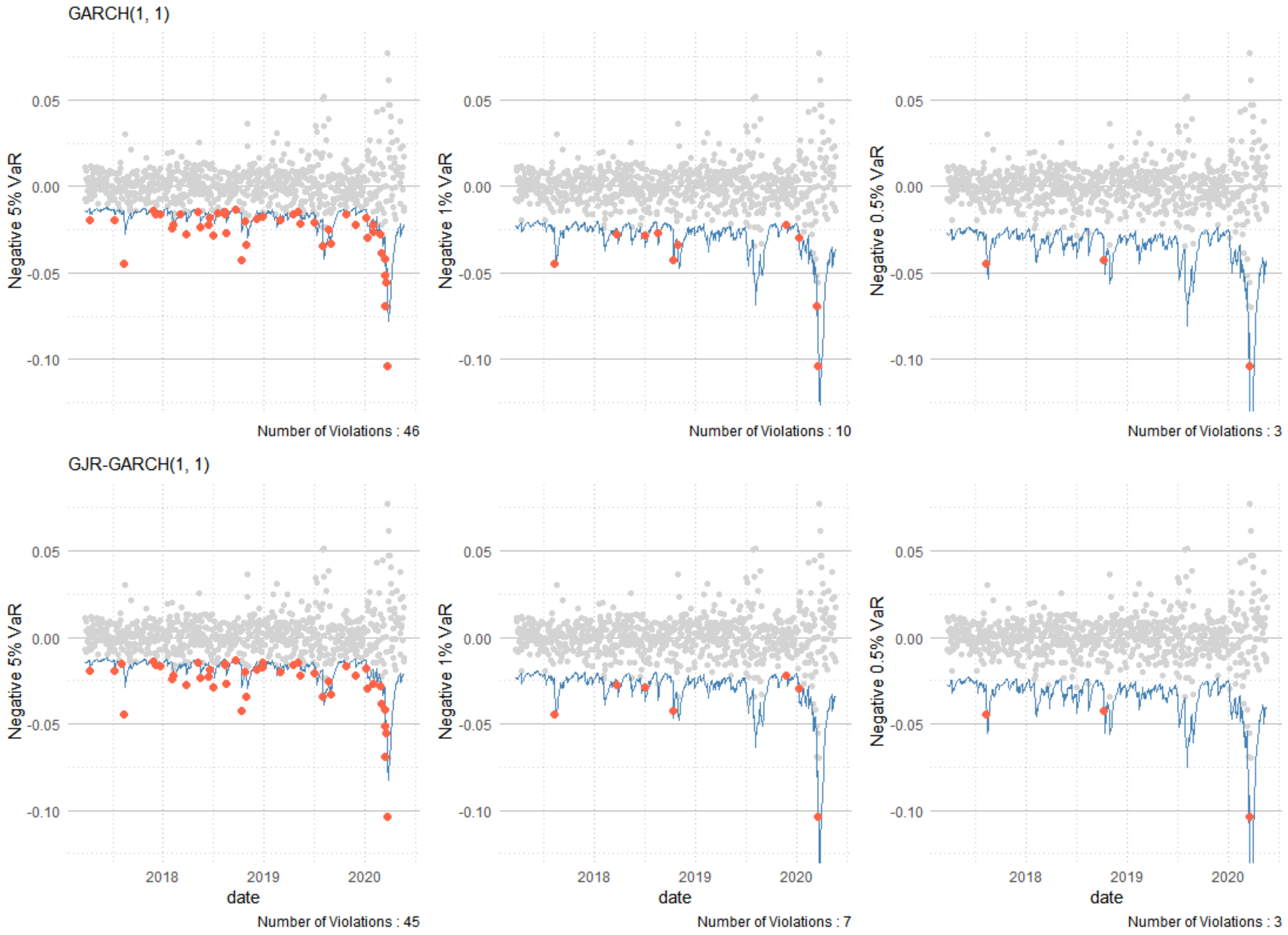
↑ GARCH(1, 1) 모형에서의 잔차 분석



↑ GJR-GARCH(1, 1) 모형에서의 잔차 분석

Standardized t Distribution

$p = 0.05,\ 0.01,\ 0.005$일 때 각각의 초과손실

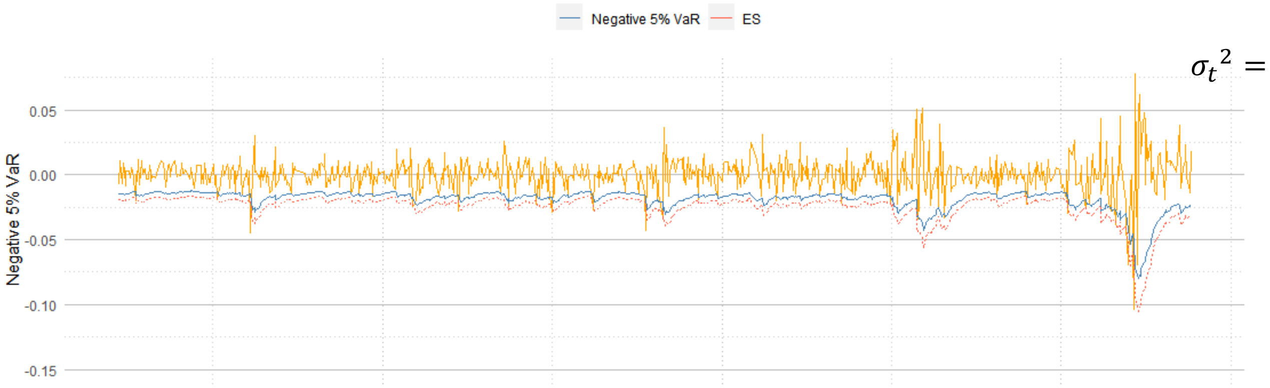


Asymmetric t Distribution

$p = 0.05$

$\hat{z}_{1-p}$: asymmetric t 분포로 가정, 추정

GARCH(1, 1)

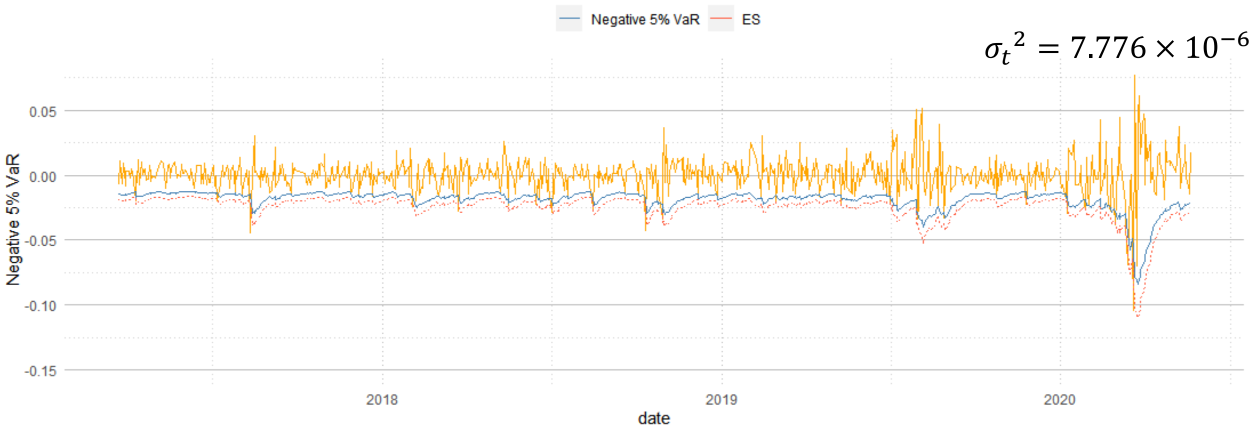


$$\sigma_t{}^2 = 7.435 \times 10^{-6} + 0.103 R_{t-1} + 0.844 \sigma_{t-1}{}^2$$

$$-\widehat{VaR_t^{1-p}} = -\hat{\sigma}_t \times 1.624$$

$$-\widehat{ES_t^{1-p}} = -\hat{\sigma}_t \times 2.137$$

GJR-GARCH(1, 1)

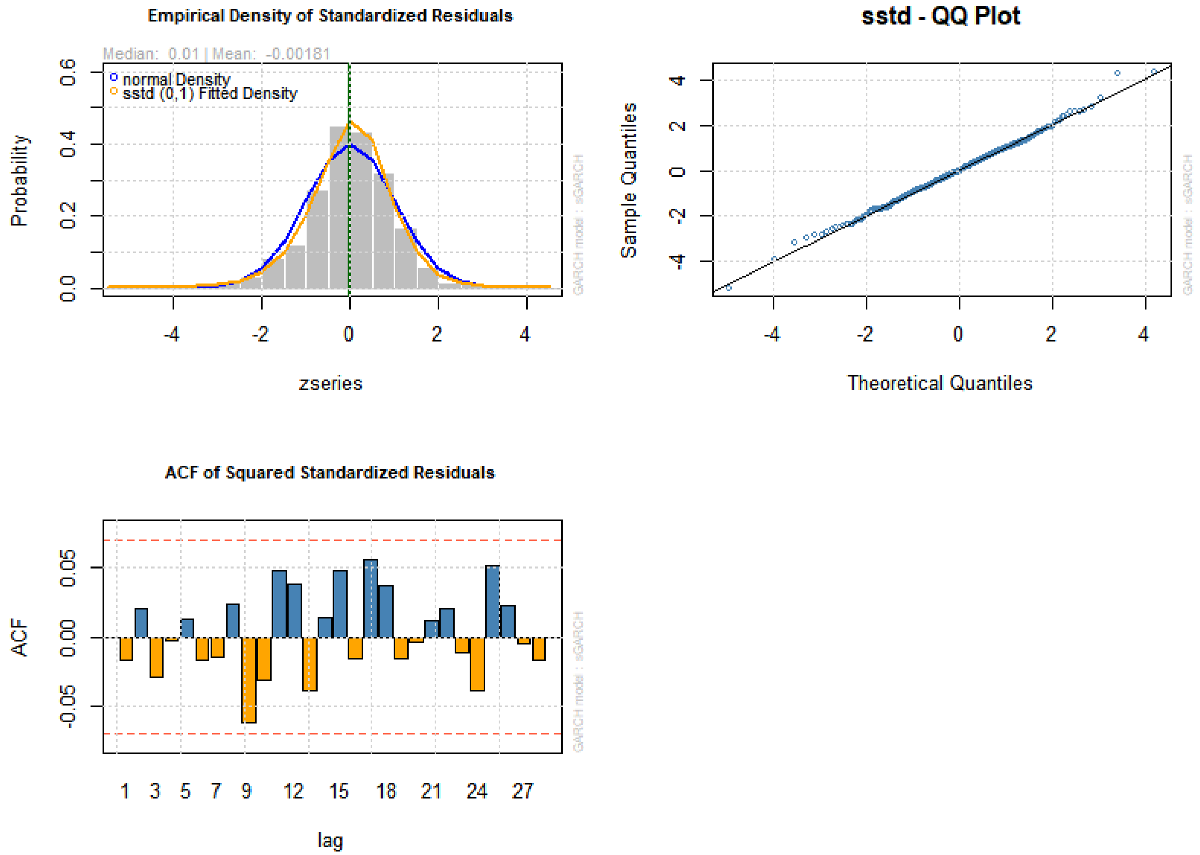


$$\sigma_t{}^2 = 7.776 \times 10^{-6} + 0.080(1 + 0.047 I_{t-1}) R_{t-1} + 0.840 \sigma_{t-1}{}^2$$

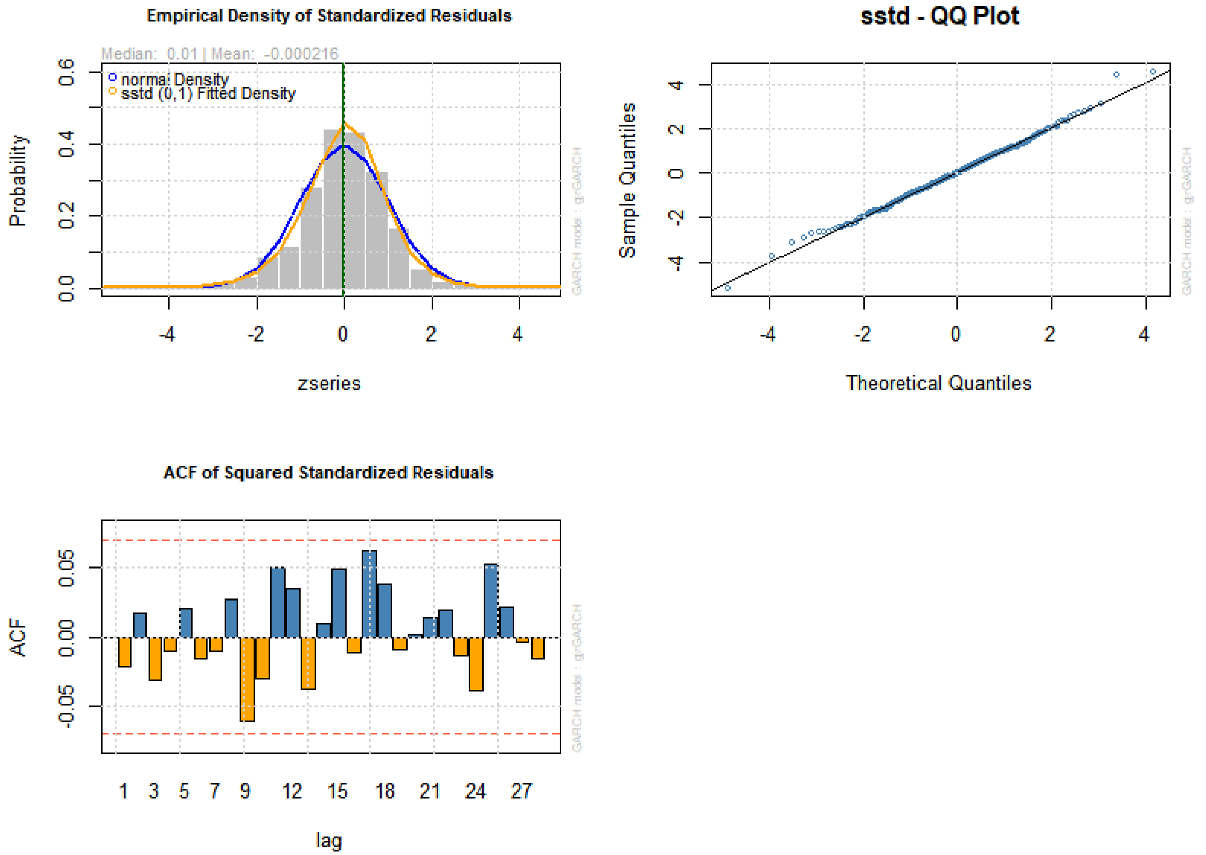
$$-\widehat{VaR_t^{1-p}} = -\hat{\sigma}_t \times 1.626$$

$$-\widehat{ES_t^{1-p}} = -\hat{\sigma}_t \times 2.135$$

Asymmetric t Distribution



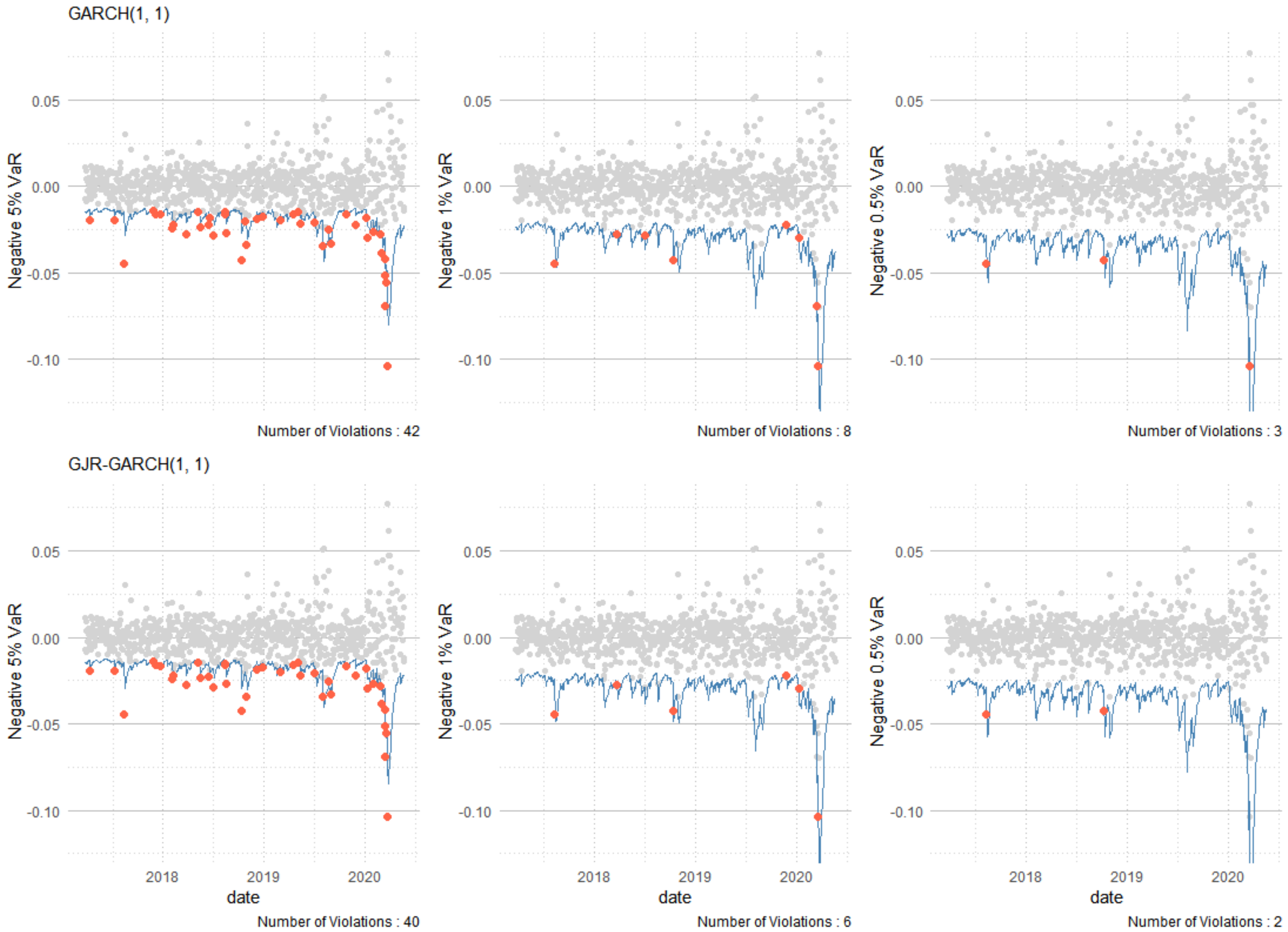
↑ GARCH(1, 1) 모형에서의 잔차 분석



↑ GJR-GARCH(1, 1) 모형에서의 잔차 분석

Asymmetric t Distribution

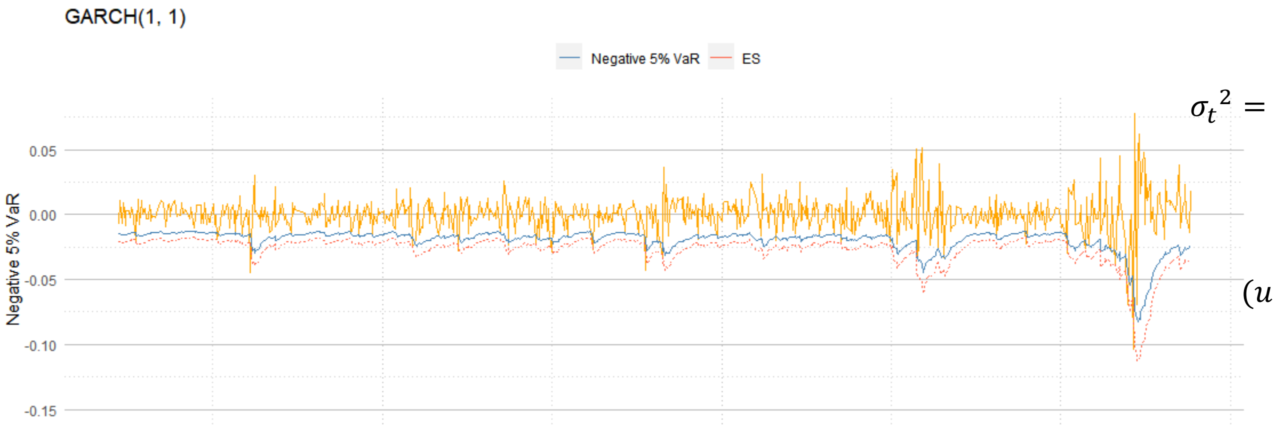
$p = 0.05,\ 0.01,\ 0.005$일 때 각각의 초과손실



Peak Over Threshold Model(EVT)

$p = 0.05$

$\hat{z}_{1-p}$: Generalized Pareto Distribution이용, 추정

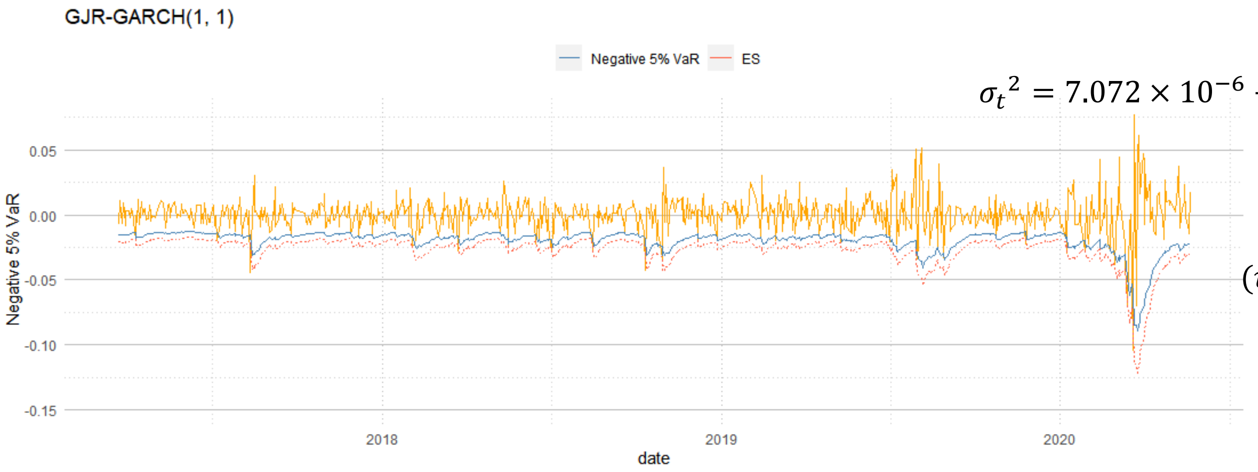


$$\sigma_t^2 = 6.783 \times 10^{-6} + 0.100R_{t-1} + 0.851\sigma_{t-1}^2$$

$$-\widehat{VaR_t^{1-p}} = -\hat{\sigma}_t \times 1.687$$

$$-\widehat{ES_t^{1-p}} = -\hat{\sigma}_t \times 2.326$$

$$(u = 1.687,\ \widehat{p_u} = 0.05,\ \xi = 0.124,\ \beta = 0.561)$$



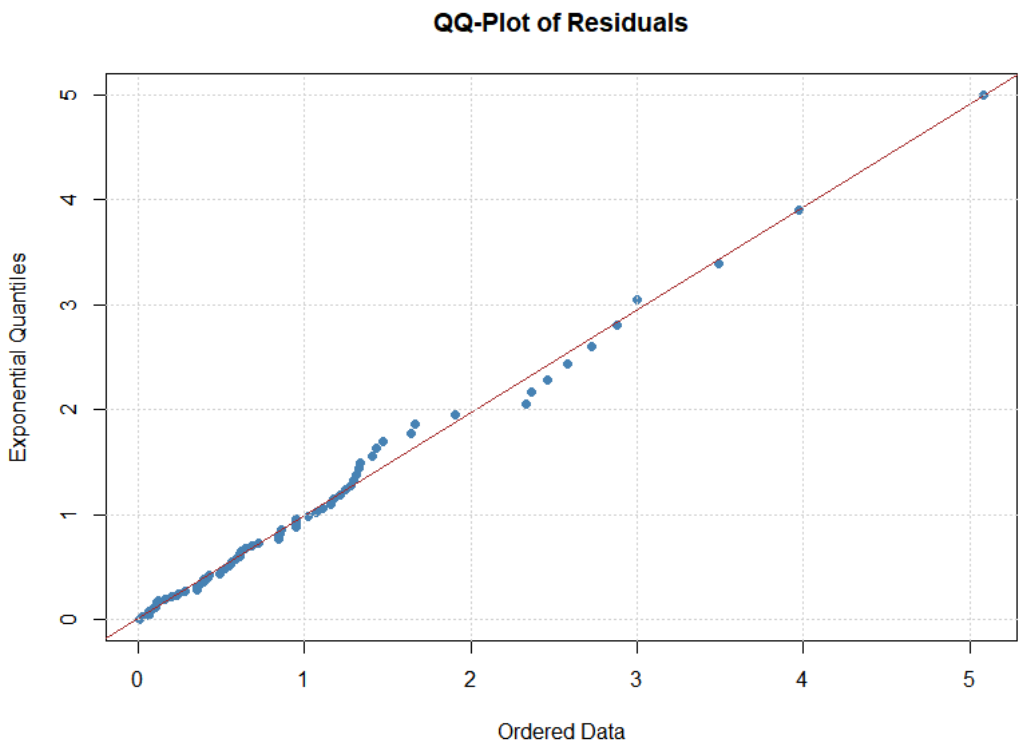
$$\sigma_t^2 = 7.072 \times 10^{-6} + 0.067(1 + 0.065I_{t-1})R_{t-1} + 0.848\sigma_{t-1}^2$$

$$-\widehat{VaR_t^{1-p}} = -\hat{\sigma}_t \times 1.703$$

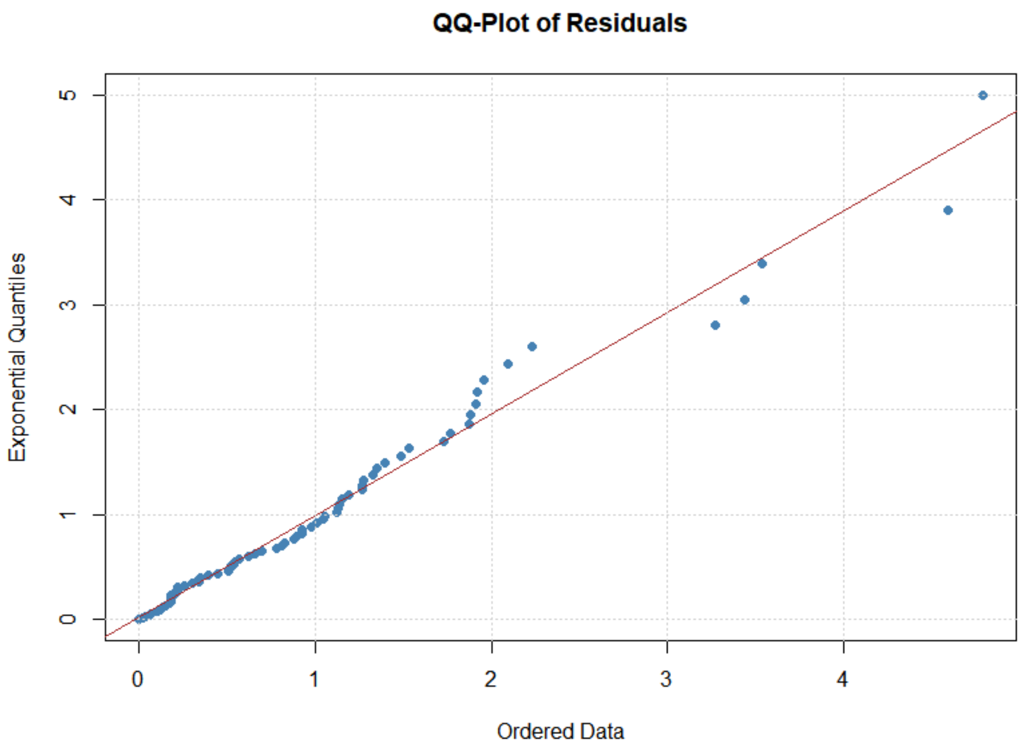
$$-\widehat{ES_t^{1-p}} = -\hat{\sigma}_t \times 2.315$$

$$(u = 1.703,\ \widehat{p_u} = 0.05,\ \xi = 0.067,\ \beta = 0.571)$$

Peak Over Threshold Model(EVT)



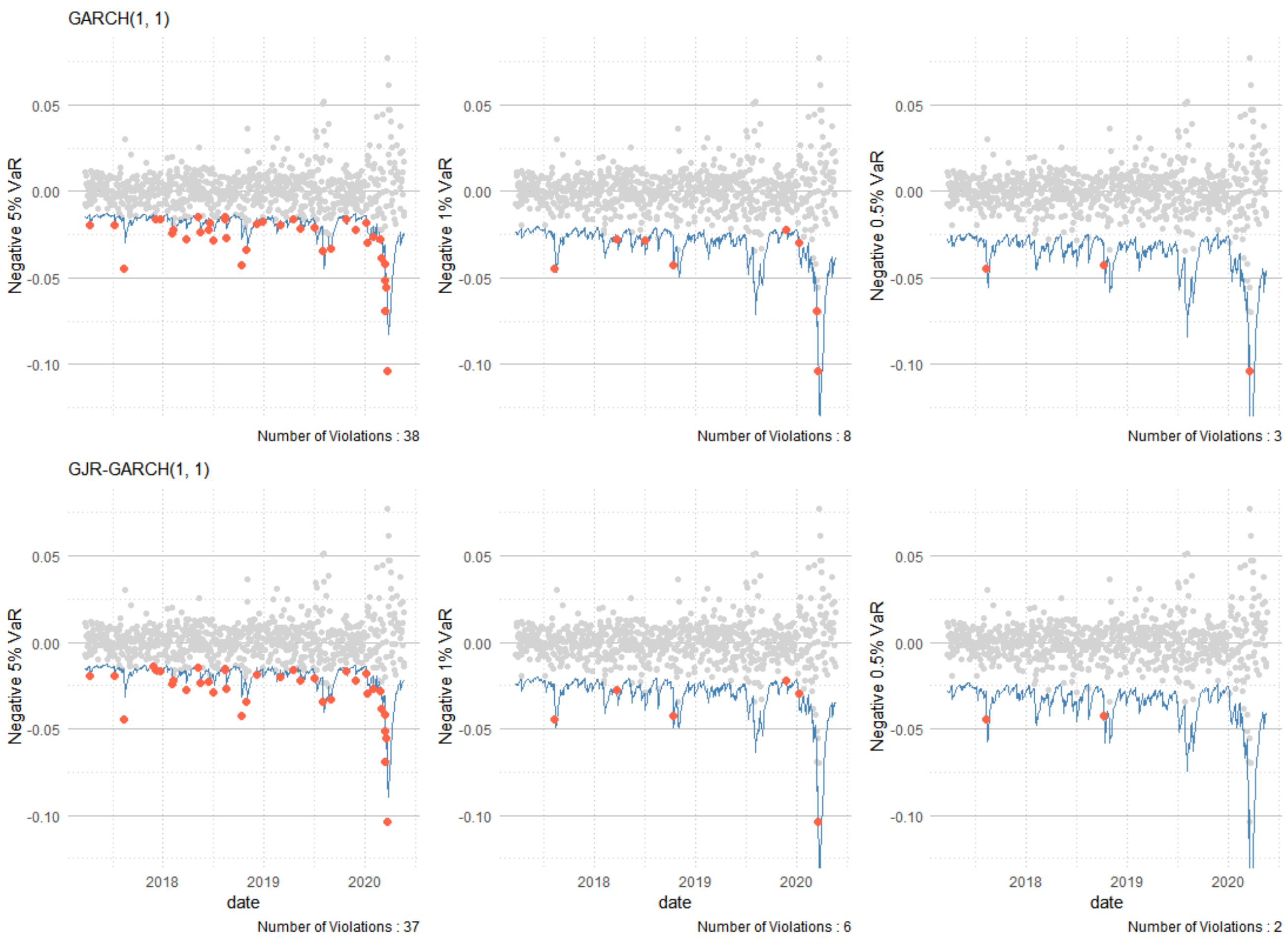
↑ GARCH(1, 1) 모형에서의 QQ plot



↑ GJR-GARCH(1, 1) 모형에서의 QQ plot

Peak Over Threshold Model(EVT)

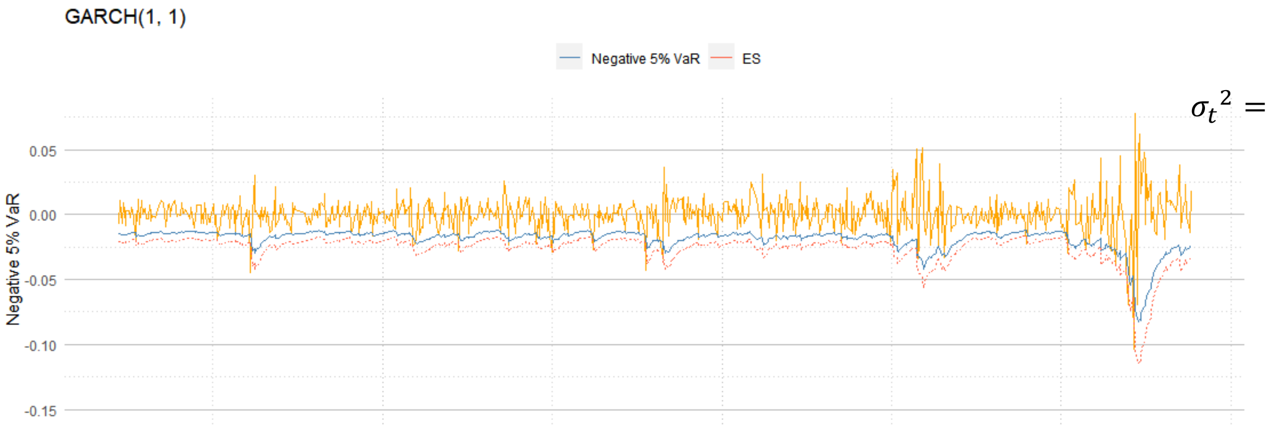
$p = 0.05,\ 0.01,\ 0.005$일 때 각각의 초과손실



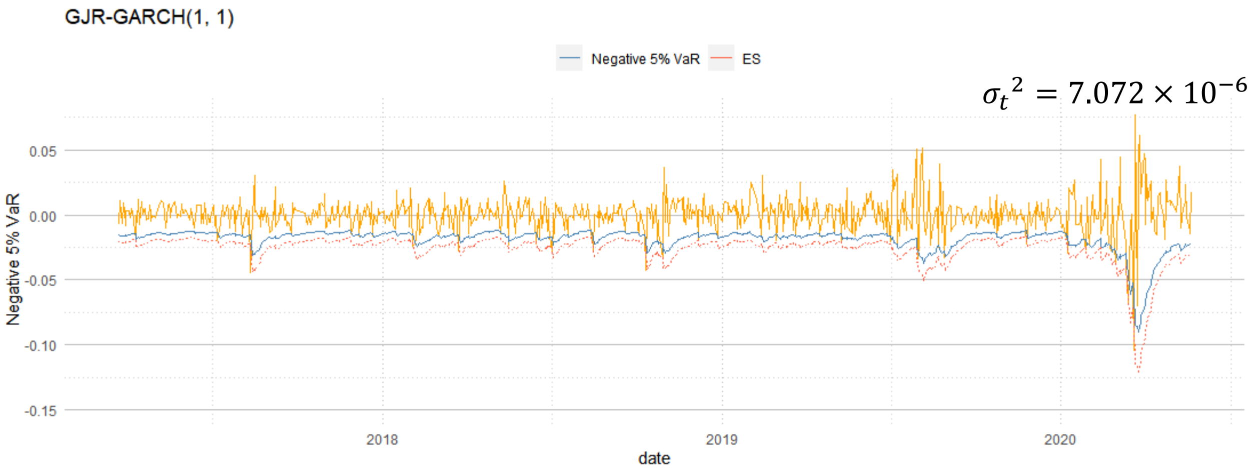
Filtered Historical Simulation(FHS)

$p = 0.05$

$\hat{z}_{1-p}$: FHS with Moving Window Method를 이용, 추정(size = 700)



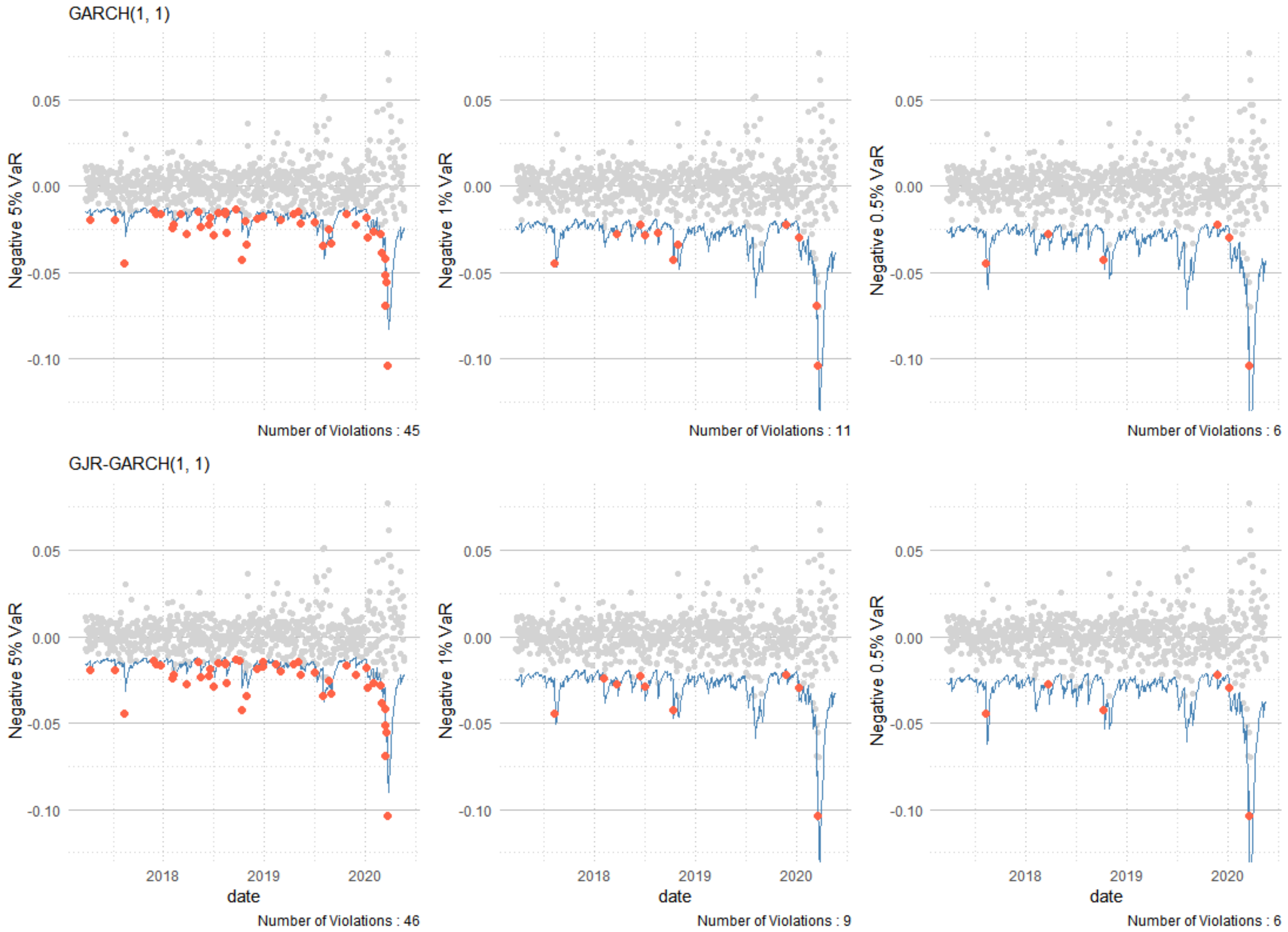
$$\sigma_t^{\ 2} = 6.783 \times 10^{-6} + 0.100R_{t-1} + 0.851\sigma_{t-1}^{\ 2}$$



$$\sigma_t^{\ 2} = 7.072 \times 10^{-6} + 0.067(1 + 0.065I_{t-1})R_{t-1} + 0.848\sigma_{t-1}^{\ 2}$$

Filtered Historical Simulation(FHS)

$p = 0.05,\ 0.01,\ 0.005$일 때 각각의 초과손실



적절성 검증

- 초과손실 횟수

Model	Distribution	0.95 (38.6)	0.99 (7.72)	0.995 (3.86)
GARCH	Normal	41	17	10
	Standardized-t	46	10	3
	Asymmetric-t	42	8	3
	EVT	38	8	3
	FHS	45	11	6
GJR-GARCH	Normal	40	15	7
	Standardized-t	45	7	3
	Asymmetric-t	40	6	2
	EVT	37	6	2
	FHS	46	10	6

적절성 검증

- 총 10가지 방법으로 추정한 VaR 중, 초과손실 횟수를 이용하여 선정한 6가지에 대해서 다음 네가지 test를 진행한다.

- Unconditional coverage test
  Independene test
  Conditional coverage test
  Dynamic quantile test

적절성 검증

- p-value

Model	Distribution	Test	0.95	0.99	0.995
GARCH	Standardized-t	LR_{uc}	0.235	0.430	0.648
		LR_{ind}	0.065	0.680	0.878
		LR_{cc}	0.091	0.642	0.890
		DQ	0.070	0.005	0.324
	Asymmetric-t	LR_{uc}	0.580	0.920	0.648
		LR_{ind}	0.027	0.682	0.878
		LR_{cc}	0.075	0.915	0.890
		DQ	0.011	0.004	0.324
	EVT	LR_{uc}	0.921	0.920	0.648
		LR_{ind}	0.010	0.682	0.878
		LR_{cc}	0.035	0.915	0.890
		DQ	0.049	0.004	0.324

적절성 검증

- p-value

Model	Distribution	Test	0.95	0.99	0.995
GJR-GARCH	Standardized-t	LR_{uc}	0.303	0.791	0.648
		LR_{ind}	0.015	0.720	0.878
		LR_{cc}	0.030	0.906	0.890
		DQ	0.006	0.911	0.322
	Asymmetric-t	LR_{uc}	0.818	0.517	0.295
		LR_{ind}	0.016	0.759	0.919
		LR_{cc}	0.055	0.774	0.576
		DQ	0.033	0.877	0.994
	EVT	LR_{uc}	0.790	0.517	0.295
		LR_{ind}	0.007	0.759	0.919
		LR_{cc}	0.026	0.774	0.576
		DQ	0.030	0.878	0.994

적절성 검증

- 손실함수 비교

Model	Distribution	0.95	0.99	0.995
GARCH	Standardized-t	108.021	30.983	17.594
	Asymmetric-t	107.678	30.649	17.520
	EVT	107.409	30.631	17.526
GJR-GARCH	Standardized-t	106.8464	29.466	16.555
	Asymmetric-t	106.5567	29.511	16.744
	EVT	106.2216	29.492	16.689

- GJR-GARCH 모형에서 Asymmetric-t분포를 가정하여 추정한 VaR이 유의수준을 $\alpha = 0.01$로 둘 때 모든 검정을 통과하고, 대체로 낮은 손실함수 총합을 가진다.