



The parallel coordinate plot in action: design and use for geographic visualization

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Abstract

Implementations of interactive parallel coordinate plots in geographic visualization systems are presented. The plots represent spatial and spatio-temporal data, and are linked to maps and scatterplots. The interactive features of the parallel coordinate representations are discussed, with particular emphasis on their ability to facilitate geographic data exploration and understanding. The paper will present the systems as “case studies” to demonstrate the philosophical and practical advantages of highly interactive, multiple-perspective environments, including parallel coordinate plots, for the exploration of complex spatial and spatiotemporal information. Emphasis is placed on the encouragement and facilitation of creative thinking about geographic phenomena through the use of such data-rich graphical tools.

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1. Introduction

In the fall of 1998, over 10,000 people were killed in Honduras and Nicaragua in the worst weather disaster in over two centuries as Hurricane Mitch inched its way across Central America. Mitch reached category 5, the strongest hurricane classification on the Saffir–Simpson scale, which uses the maximum sustained wind as a yardstick for the strength of a storm. Yet it was after its demotion from a category 5 hurricane to a category 1 and eventually to merely a tropical storm, that Mitch did the majority of its damage. Persistent rains inundated the region for days, creating deadly floods and landslides in the mountainous terrain, wiping out families, farms, villages, and

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even whole cities. This discrepancy between the categorization of Mitch as a weak hurricane and the fact that it became one of the costliest natural disasters in recent history led the US National Hurricane Center to contemplate a restructuring of the classification of hurricanes to include more than just one variable—sustained wind speed (New York Times, 1998). The multitude of geographic questions that accompany such a restructuring constitutes a vast, interesting, and potentially controversial scientific problem. The undertaking would involve the association of loss of life or monetary damage with a host of quantifiable variables such as wind speed, water vapor content, velocity of forward motion, and sea surface temperature, as well as variables that are more difficult to quantify, such as quality of life, integrity of the buildings in the storm's path, communication and warning systems, and uncertain forecasts of landfall sites.

The investigation of problems such as these—ones that involve space, time, a host of attributes, and the interconnections among them—is one of the general missions of the GeoVISTA Center in the Department of Geography at the Pennsylvania State University. More specifically, GeoVISTA scientists develop ways of representing spatial and spatiotemporal phenomena and their analysis in a way that facilitates understanding of the phenomena through feature identification, pattern recognition, and ultimately knowledge construction (MacEachren et al., 1999). The introduction of the concept of the parallel coordinate representation (Inselberg, 1985) to the research group led to active brainstorming about applications of the multivariate representation system to problems such as hurricane reclassification, climate model inspection and analysis, and spatiotemporal health statistics exploration.

This paper demonstrates the utility and limitations of the parallel coordinate plot (PCP) in the context of geographic visualization (*geovisualization*) through its construction and application in environments where the PCP is linked to maps and other graphical displays. The PCP has been used in multiple-view geovisualization environments in other work (the cdv of Dykes (1997), the Descartes system of Andrienko and Andrienko (1999)). This paper adds to that body of work through the conceptual extension of the PCP to handle both spatial and temporal data sets, and through a discussion of specific interactive capabilities of the PCP that facilitate the exploration and understanding of complex spatio-temporal data. In addition, the paper demonstrates the representation's effectiveness as a spatiotemporal data exploration tool through "case studies" in two different problem domains: climate modeling and analysis and epidemiology.

2. The parallel coordinate representation

The details of the parallel coordinate representation are examined elsewhere (Inselberg, 1985; Wegman, 1990), though the PCP is simple enough to summarize very briefly here. Instead of points (as in a scatter plot), observations are represented on a PCP as a series of unbroken line segments, passing through parallel axes, each of which represents a different variable. Each line passes through an axis at a location that indicates the observation's value relative to all other values. The ends of the axis represent the maximum and minimum values of the axis variable for all observations

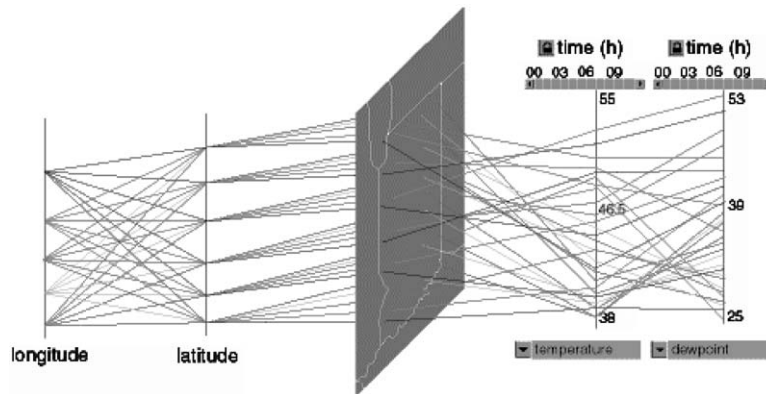


Fig. 1. The parallel plane concept.

under consideration. The result is a (possibly unique) multivariate *signature* for each observation, and a visual representation of relationships among many variables.

Although its novel approach to representing multivariate information is useful, the PCP in static, non-interactive form has clear drawbacks that might have prevented its widespread acceptance as a useful statistical graphic in the years following its introduction. For example, the sheer number of observations and variables in a large (geographic) database quickly overloads the representation to the point where very little information can be perceived and extracted (Gahegan, 1998). The plot becomes a confusing jumble of lines resembling a haystack or an overworked telephone switchboard. In addition, the order in which the axes are lined up clearly influences the amount and quality of the insight gained from the representation. An additional problem particular to geographic databases is the necessary reduction of space to one dimension (to fit on one axis), or, better but not ideal, the imposition of a north–south/east–west structure on data so that the location can be represented as a line connecting two axes representing latitude and longitude, respectively. A useful variation of the PCP might be the incorporation of a third dimension that would allow parallel planes, one of which could be a map with latitude and longitude in a traditional manner (Fig. 1).

The parallel plane plot has not yet been implemented, but the alternative of a dynamic link between a separate map and a PCP has been developed as part of the research described here. The idea has been implemented for two different problem domains, described in Section 4. In the next section, features of the developed parallel coordinate representations to enhance the representation of geographic and temporal information are described.

3. Geovisualization and the design of the parallel coordinate plot

3.1. Geovisualization

Not unlike exploratory data analysis (EDA) in statistics, geovisualization grows out of research issues concerning the representation of and interaction with large amounts

of complex data, though in its case, the data are specifically geospatial (referenced to the earth's surface). It also grows out of a rejection of the goal of previous cartographic research to find single optimal ways of representing geographic information (MacEachren and Ganter, 1990). Following on the discussion of scientific visualization approaches for earth science data in general in DiBiase (1990), MacEachren (1994) constructed a theoretical framework that defines these goals according to three parameters relating to how a representation is *used*: the audience or user of the map, the objectives of the map (from “presenting knowns” to “revealing unknowns”), and the degree of interactivity of the map. Geovisualization is characterized by highly interactive representations designed for use by individuals, expert in the understanding of the mapped phenomenon, for exploratory analysis purposes. Research in geovisualization, thus, has focused on the unique characteristics of data, representations, and interactions that together are used for the exploratory analysis of geographic (spatial and spatiotemporal) data.

3.2. The parallel coordinate plot for spatiotemporal data

A dynamic, interactive, and customized version of the PCP is well suited for the type of expert-driven detective work necessary for the exploration of large spatial and spatiotemporal databases. This paper reports on two different applications of the PCP with different types and levels of interactivity: the first was constructed in Tcl/Tk and linked to IBM's Data Explorer as one of several display types for use with climatological data, and the second was developed within the ArcView[®] geographic information system (GIS) using its scripting language Avenue. Both development environments were selected first and foremost for their geographic representational abilities and their facility to *link* newly constructed graphical representations to maps. This link to spatial representations is a clear priority in geovisualization applications. These development environments were limited in their interactive capabilities, though some of the interactivity and graph manipulation techniques employed in other dynamic PCP applications were utilized in this project. Those features are described below.

A key variation of the PCP employed here is its ability to serve as tool to examine temporal, as well as spatial and attribute, patterns. In this case, two possibilities were identified. First, a single axis can serve as a time axis because time is an ordinal-level variable, as is developed in “case study I” below. Perhaps more insightful, however, is the assignment of each axis to a specific variable at a specific time, as in “case study II” below. This variation is required to visualize temporal trends, as long as the scales from axis to axis remained the same. Thus, in this use, the maximum and minimum values of the entire time series (and not just one temporal snapshot) would define the ends of the axes that represent the variable (Fig. 2).

3.3. Interactive features of the PCP designed for spatiotemporal visualization

Many features that have been incorporated in other dynamic versions of the plots (Wegman, 1990; Miller and Wegman, 1991; Chang and Yang, 1996; Inselberg, 1999) have been employed for the representations described here. Both versions described

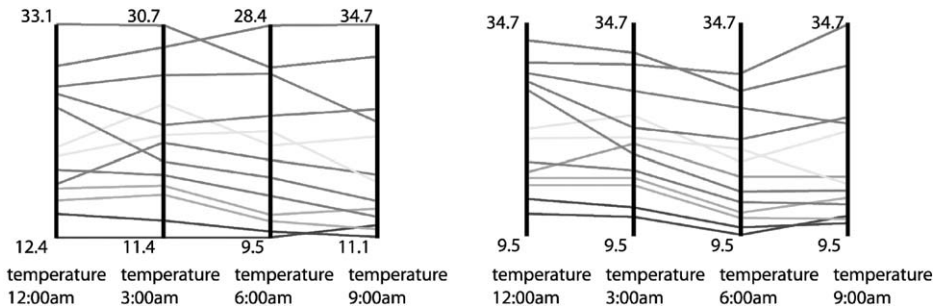


Fig. 2. Typical axis value designation hides temporal trends (left). Axis values that are consistent across time axes reveal temporal features (e.g., most observations reach their minimum temperature at 6:00 a.m.: right).

in this paper allow a user to *assign a variable to any axis*. Relationships among a small number of variables would be difficult to discern if those variables were distant from one another on the representation. It is quite conceivable that two variables not adjacent to each other in an initial ordering (no matter how well conceived) might prove to have a surprising or unexpected association that, using the PCP, would be discerned only when the variables are moved to adjacent axes. This can be overcome by (a) selecting a variable for each axis from a drop-down menu—employed in the Tcl/Tk version, or (b) physically click-dragging an axis and repositioning it between two other axes of interest—employed in the ArcView[®] version.

The problem of line density in the PCP can be reduced dramatically by *focusing* on particular observations of interest (Fig. 3a). Focusing the PCP removes the lines of all other observations from the display to reduce the visual clutter and draws the focused observations in yellow or any other highlight color. Using this feature, a researcher may isolate observations that share a value or a range of values of a particular variable. Questions that might be answered easily using PCP focusing include “are the outliers of the barometric pressure variable also extreme values of other variables, such as precipitation?” and “how similar are the traces of all of the observations at this latitude?”

Another method of reducing the complexity of lines in the plot is through the application of the EDA concept of *brushing* (Fig. 3b). Brushing consists of highlighting a group of data observations by some method of selection; in a PCP, multiple line segments may be selected simultaneously by click-dragging a box around the bundle. Upon release of the mouse button, the observations passing through the box are highlighted. This can be a very effective method of drawing attention to the multivariate (or multiple-time) signatures of a group of observations that share similar values of one attribute (or one time).

Individual observations can be highlighted on the parallel coordinate plot by moving the mouse over the line segment. This is a special case of brushing; on a PCP, this recalls the image of a pick moving over strings of a guitar (the frets are the axes; the strings are the observation lines), thus it could be called *strumming* (Fig. 3c).

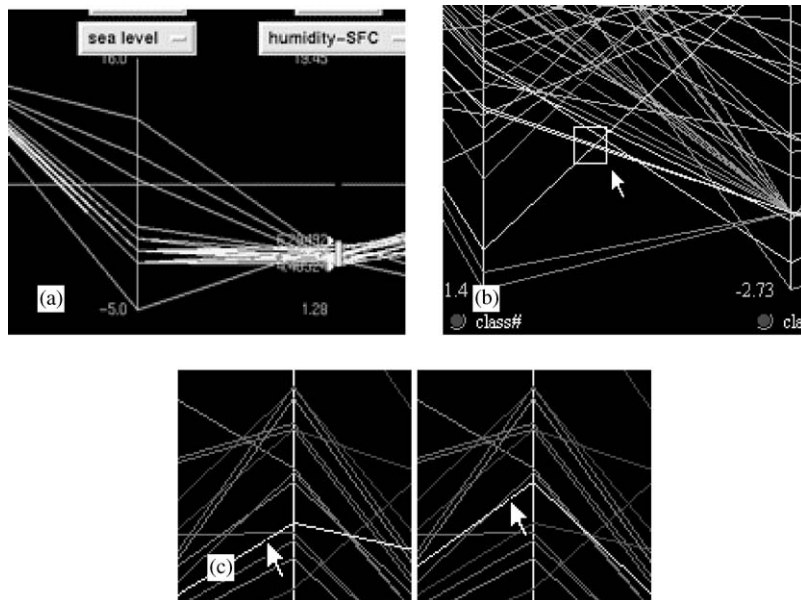


Fig. 3. (a) Focusing of PCP; (b) brushing of PCP; (c) strumming of PCP.

Strumming highlights the PCP trace across the whole plot, allowing users to identify interesting relationships in individual observations between a pair of variables (for example, an outlier with low surface humidity but significant rainfall) or a series of variables (such as a single observation that is higher than average “across the board”). Additionally, because this strumming action occurs in real time, users may be able to identify critical attribute values where significant multivariate relationship changes occur. For example, a meteorologist uses the thickness of the layer of atmosphere between 1000 mb (approximately the surface of the earth) and 500 mb as a predictor of either rain or snow: the rule-of-thumb is that snow (not rain) occurs in most cases when that thickness is less than 5430 m. Strumming along the “thickness” variable axis in a plot of a meteorological data set may reveal that point—5430 m—where there is a binary switch on the “precipitation type” variable axis from rain to snow.

Visual and analytical cartographic research has been incorporated in the applications of the PCP described here through *customizing color and classification*. Observations may be classified, for example, according to one variable (in a choice of classification schemes) and colored according to this classification. In Fig. 2, the observations are colored according to a classification on temperatures at 12:00 a.m. This allows the visualization of the consistency (or persistence) of observations across variables—for example, do cold observations remain cold over the entire time period of interest? Do observations that are low in median income also have low values in most other socioeconomic indexes? Choice of classification schemes are driven in part by a knowledge of the variable’s distribution. One primary advantage of the PCP is that this

distribution is presented in a straightforward and intuitive way. A variable with a linear distribution, for example, is shown with an even distribution of line segments crossing that axis. A simple classification method such as quantiles or equal intervals is just as effective as a more complex method, such as natural breaks. However, a variable with an irregular, clustered, or skewed distribution is shown with clustered bundles of line segments crossing at particular points with empty space in other ranges of the distribution. For such a distribution, a researcher might wish to customize the classification scheme by inserting class breaks at natural breaks in the distribution, or by bunching several class breaks within a cluster of lines crossing the axis.

Applications of these capabilities of the PCP incorporated within geographic visualization environments are demonstrated in the next section.

4. Parallel coordinates in action: being a geographical multidimensional detective

The parallel coordinate plots integrated into geographic visualization environments proved useful in practice in at least two applications. First, the PCP was employed in the examination of climate model output after the output's automated classification using data mining software. Second, the PCP was part of a GIS environment for the exploration of multivariate health statistics. These "case studies" serve as proof of the concepts that led to the design of the interactive features of the spatiotemporal PCP described above.

4.1. Case study I: using parallel coordinates in conjunction with knowledge discovery in databases

Creative exploration of multivariate databases, using such tools as the PCP described above, becomes not only useful but necessary when the data sets are so large and multivariate that relationships among variables and features within the data sets are completely unknown and unrecognizable without the aid of computational algorithms that sort through the data set to extract patterns of interest. The theoretical underpinnings of these methods falls in the interdisciplinary field of knowledge discovery in databases (KDD). When applied to geographic data, these computational methods have come to be known as *geocomputation* and are patterned after inductive reasoning, emphasizing learning by trial-and-error, with little or no a priori knowledge of the information contained in the data set being analyzed. With the increase in computing power, memory, and accessibility over the last several decades, methods for modeling large data sets have evolved toward this inductive approach.

An environment that merges KDD and geovisualization was designed and implemented (MacEachren et al., 1999). This environment, GKConstruct (Geographic Knowledge Construction), was used to examine a sample, gridded data set of daily predicted observations from a model of regional climate in Northern Mexico and Southern Texas (Fig. 4). The search was for space–time–attribute features in this large data set. A key step in the process, which includes initial data selection, preprocessing, and transformation, is data mining. The public domain data mining package AutoClass was used

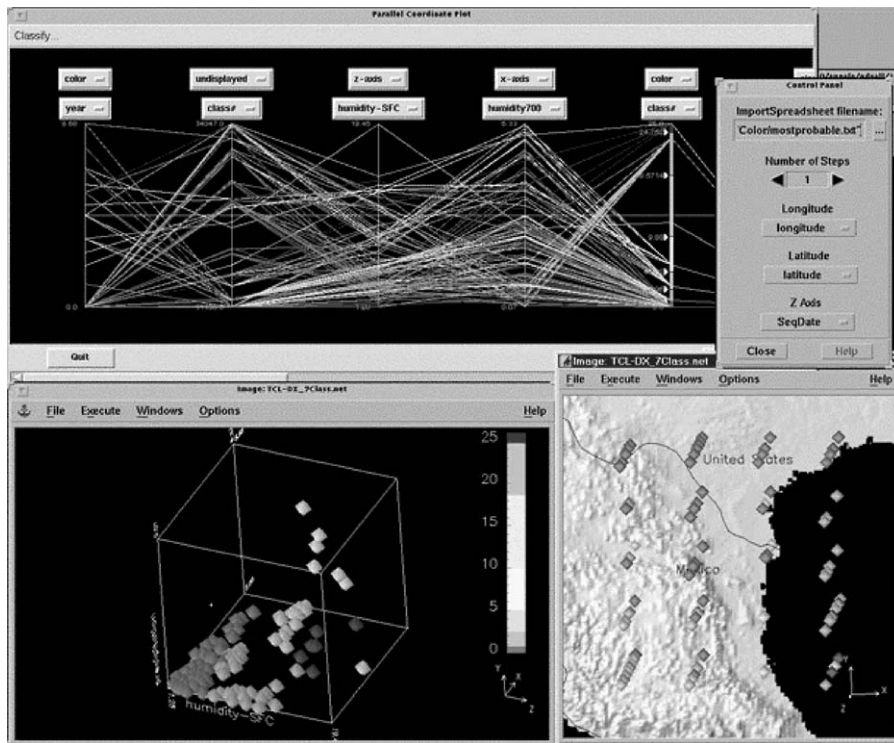


Fig. 4. A typical screen display from the GKConstruct tool. The PCP is seen in the upper half of the figure. The three-dimensional displays (the GeoView and the scatterplot, lower half) are produced using IBM's Data Explorer, and were linked to the PCP, developed in Tcl/Tk. See MacEachren et al. (1999) for more detail.

for this step (Cheeseman and Stutz, 1996). As a Bayesian classifier, AutoClass describes the classes probabilistically, so that an object (in this case, an observation) can have partial membership in the different classes, and the class definitions can overlap.¹ All observations sharing a similar multivariate signature would be grouped within one class. Classification of this type fits well with the types of applications faced by climatologists concerned about describing and predicting the damage potential for storms or drought or other climate features (Yarnal, 1993).

The focus here is not on the data mining procedure, but on creating visual tools that connect the user to the methods. Fig. 4 shows a typical screen display from the GKConstruct tool. The PCP is seen in the upper half of the figure. This implementation of the PCP, created with in the rapid prototyping development language Tcl/Tk, is

¹ This was an intriguing product to the Penn State team because of the possible links between statistical and visual representations of uncertainty, a theme in geovisualization research (Howard and MacEachren, 1996; Ehlschlaeger et al., 1997).

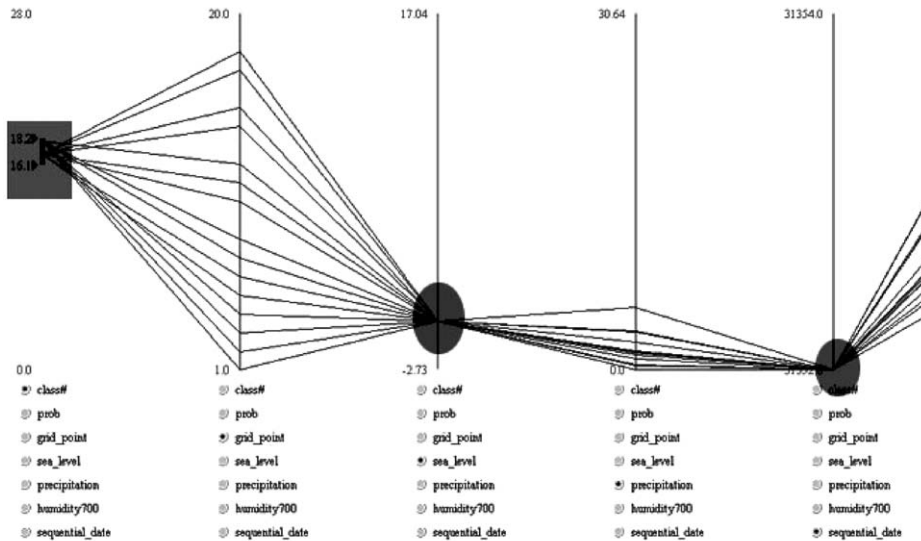


Fig. 5. PCP focused on classes 17 and 18, revealing errors in the raw data.

among the first linked to a map, and likely the first to target specifically spatiotemporal information.²

As an example of the power of the PCP representation for this type of information and analysis, the data set was examined after it had been classified by AutoClass. Two output classes were focused—classes 17 and 18 of 30 (boxed in Fig. 5)—and the multivariate signatures of all observations that fell within each class were examined. The classes were dominated by two features (circled in Fig. 5), each on a separate variable axis: first, all observations in class 17 and 18 occurred on the same date. In addition, all observations shared the same value for the axis labeled “sea level”, which stood for the *change* in sea-level-adjusted barometric pressure between the day of the observation and the previous day (a rough measure of pressure tendency). Upon interaction, it was discovered that the value shared by all observations in “sea level” was zero, and that the value shared by all observations in “seq date” (sequential date) was 31,352 (days since 01/01/1900)—the first day of the model run. The “sea level” value, rather than zero, should have been considered null, or undefined, since there was no “previous day” with which to determine pressure tendency. However, that particular AutoClass model had coded that value as zero (no pressure change), a value that produced spurious classification results. The multivariate feature recognition that led to this conclusion would have been very difficult, if not impossible, without the aid of the parallel coordinate plot.

² A PCP was used in a data mining context by Keim and Kriegel (1996), and in an interactive geographic system (at about the same time as our implementation) by Dykes (1997). Dykes has led the way for the implementation of the Tcl/Tk language for geovisualization.

4.2. Case Study II: using parallel coordinates for exploration of multidimensional health statistics data

Epidemiologists, like climatologists, are among the many researchers and scientists who examine large, complex data sets for patterns and trends across space and time. These data sets can be highly multivariate, with each location in space and time associated with many different socioeconomic, demographic, and health-related variables. Providing tools and methods to explore the multitude of possible relationships among these variables visually may enable a researcher to discover important characteristics of the data set that would be difficult, if not impossible, to detect with non-visual statistical methods alone.

MacEachren et al. (1998) identify a general domain goal in the use of GIS and mapping for visualizing health statistics: “to understand the spatially varying factors that lead to mortality and disease and the variation in those factors for different at-risk groups in the population” (p. 10). The interactive system developed and tested in that report, as well as that developed in Plaisant (1993) emphasize the need for visual representations and dynamic queries in spatiotemporal health data analysis.

Recognizing this need, and the potential for interactive multidimensional visualization environments to fulfill it, the National Cancer Institute (NCI) contracted with the

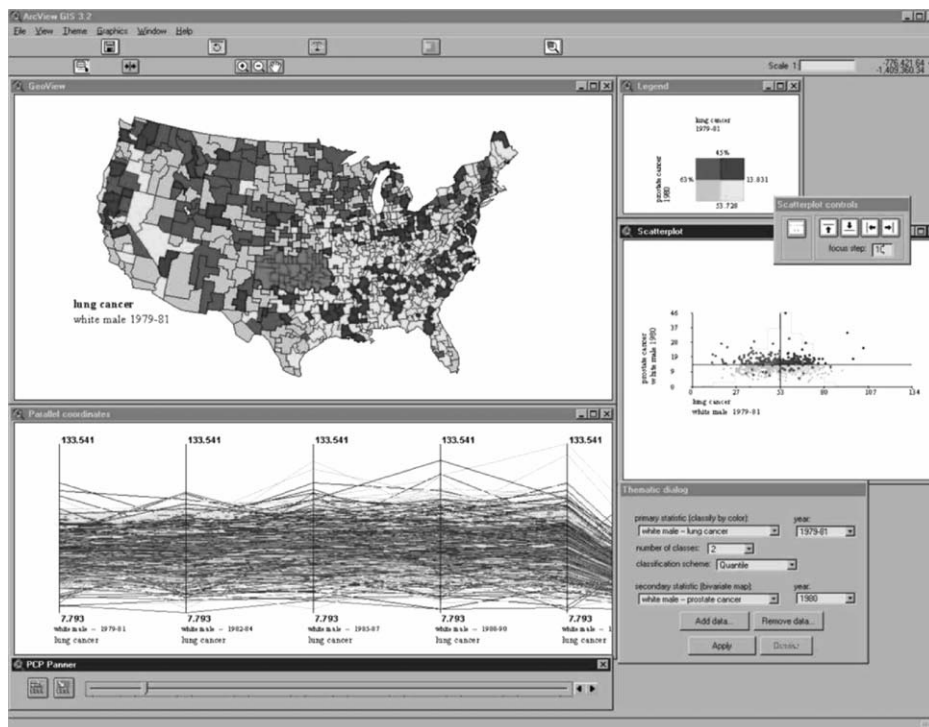


Fig. 6. The HealthVisPCP environment.

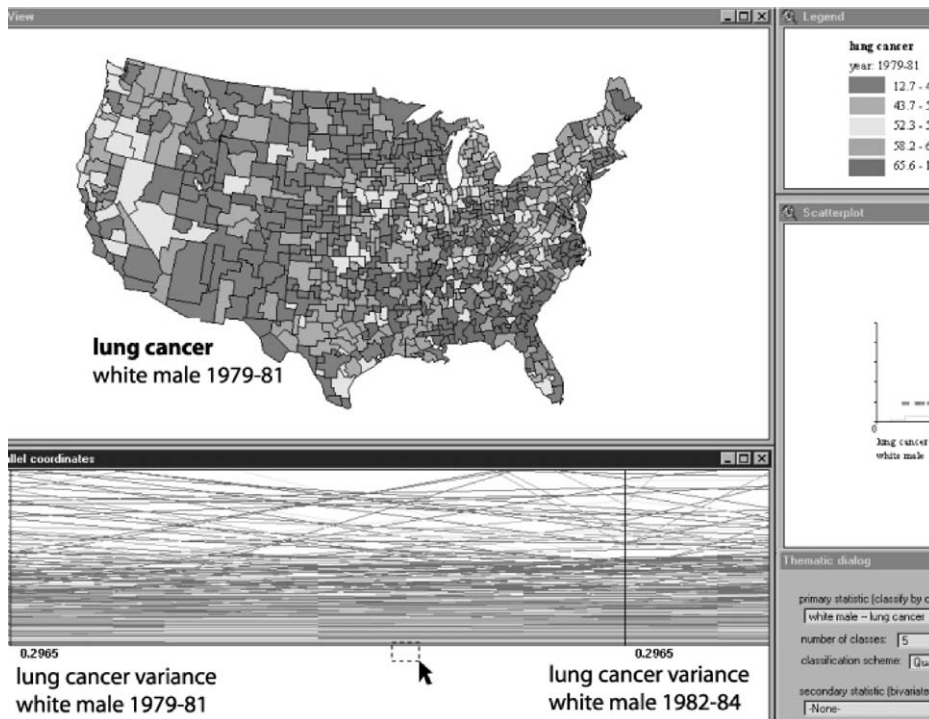


Fig. 7. Brushing the PCP to show areas of low uncertainty of mortality rates.

GeoVISTA Center to develop the PCP and integrate it with a health visualization system. The result was HealthVisPCP, a customized interface and tool set in ArcView[®] GIS. In it, the PCP is linked to a scatter plot and choropleth map in a similar (conceptual) way to the linkages made in the GKConstruct system described in the previous section (Fig. 6). The choropleth map is colored according to a statistic (such as “prostate cancer mortality, white male”) and a year (such as “1982–1984”; in the data sets obtained, the individual rates for each HSA are in fact 3-year averages). The statistic and the year can be selected by the user in a dialog box known as the Thematic Mapper. Also in this dialog box, a user can specify the number of classes (2, 5, or 7). He or she also is given the option of creating a bivariate map, plotting two variables at once, using the four-color scheme described above. Any change in classification of the choropleth map also changes the characteristics (e.g., color) of the corresponding objects in the PCP and the scatter plot.

This highly interactive exploratory system yields interesting revelations about the complex health statistics information that can be displayed. Examples include the observation that areas with the least uncertainty in the age-adjusted mortality rates in the database also are those that have the highest population: brushing observations with close to zero uncertainty rates (by dragging a box around areas of the PCP) highlights metro areas on the choropleth map (Fig. 7). The classification and coloring on the PCP

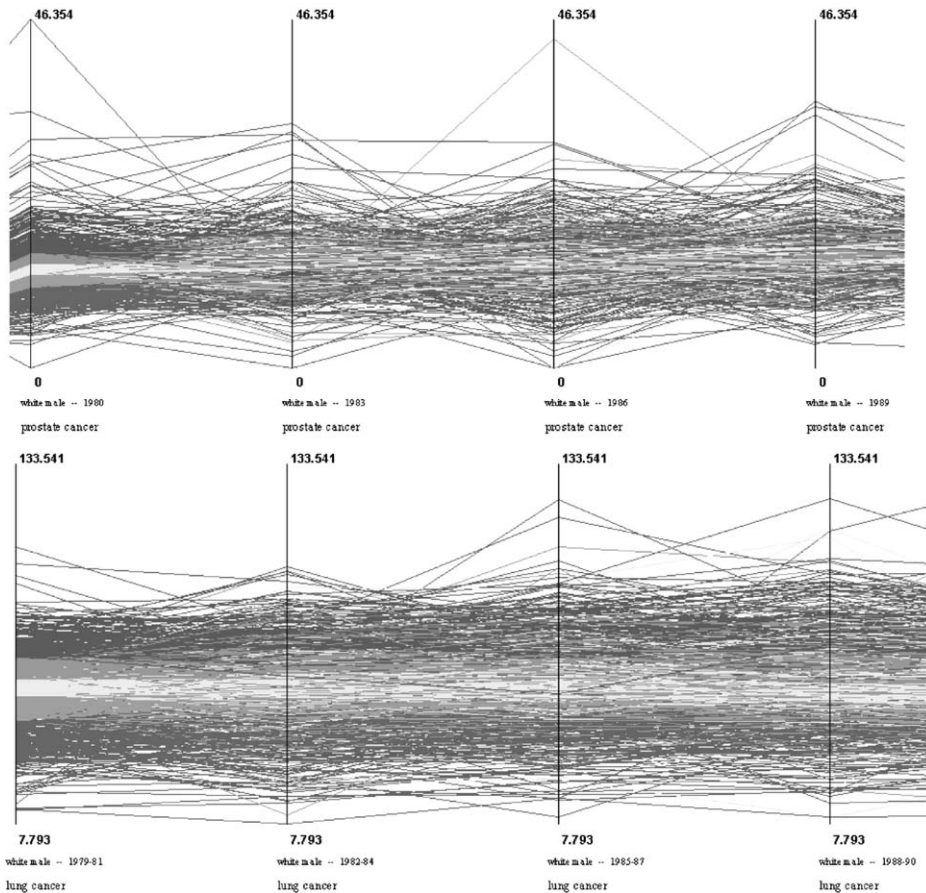


Fig. 8. Spatiotemporal persistence: lines represent spatial locations. Spatiotemporal persistence is low in prostate cancer (top: purples, greens, grays get mixed in proceeding from left to right—over time); high in lung cancer (bottom: purple remains high, green remains low in PCP from left to right).

shows that lung cancer mortality is persistent over space and time—those areas with high lung cancer rates tended to remain high over time—while prostate cancer did not show that type of regularity (Fig. 8). Outliers within a conditioned subset of the overall data set can also be detected with multi-step interactions: of those (metropolitan) regions with low uncertainty, Salt Lake City stands out as maintaining a consistently low lung cancer rate over time (Fig. 9).

5. Conclusion

In a pair of highly interactive applications, the PCP has been shown to be effective in the understanding of complex spatiotemporal data sets. The application of interactive

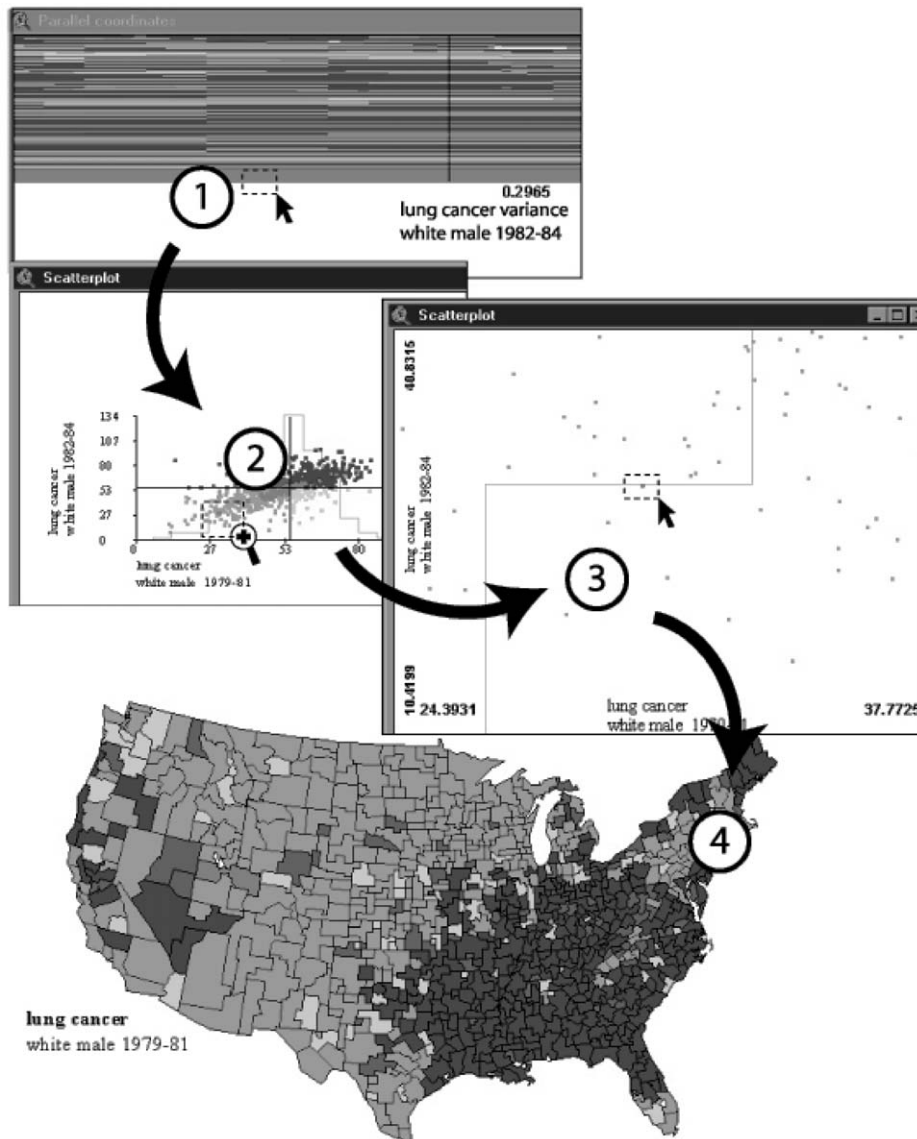


Fig. 9. Complex interaction with HealthVisPCP: observations with low uncertainty of lung cancer for 1982–1984 are brushed (1). This brushing is linked to corresponding points in the scatterplot (2). The user notes an unusual point low in lung cancer for both 1979–1981 and 1982–1984, and zooms into that region of the scatterplot (3). The user selects just that point after zooming in, and notes on the map (4) that it corresponds to the health service area that includes Provo and Salt Lake City, Utah.

data analysis techniques such as linking, brushing, focusing, and zooming all represent important features of the PCP to represent and examine geographic data sets such as those found in climatology and epidemiology, as well as a host of other problem

domains where an emphasis must be placed on spatial, temporal, and attribute-based relationships. Conceptual extensions of the PCP for representing data common in geovisualization applications include the parallel plane plot (within which a two-dimensional map may be inserted in place of an axis) and the conversion of the parallel axes to represent temporal snapshots of the same attribute variable. Cartographic concepts of classification and color theory are also integrated into the design of the representation. This work represents necessary interaction between the communities of geovisualization and computational statistics, both of which maintain a fundamental goal of making connections between complex data sets and human analysts.

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