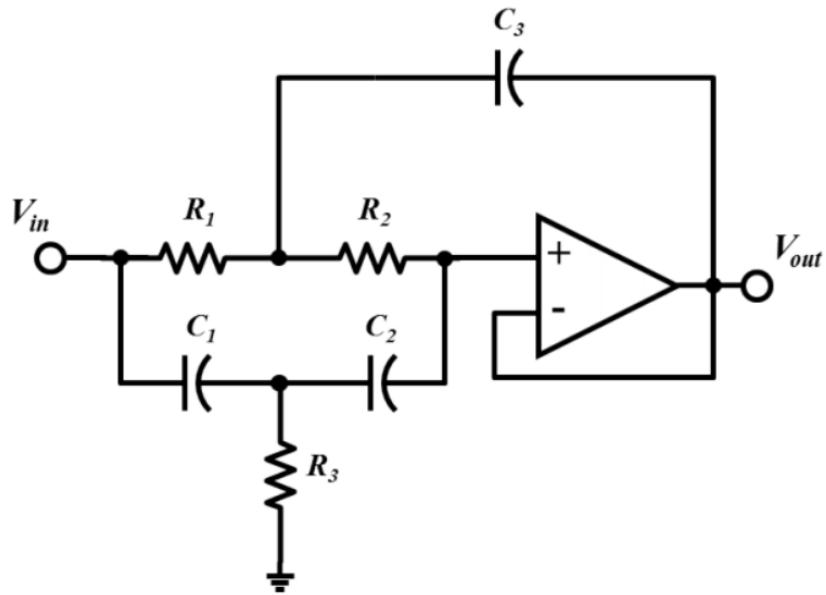


Luntrasu Sergiu Ioan

Proiect TS



1.

$$u = x_1 + x_2 + y$$

$$y = u - x_1 - x_2$$

$$D=1$$

$$U = x_1 + R_2 \cdot R_3$$

$$u_{rs} = u - x_1$$

$$x_3 + y = x_3 + u - x_1 - x_2$$

$$C_1 \frac{dx_1}{dt} = C_2 \frac{dx_2}{dt} + (u - R_1)/R_3$$

$$C_2 \frac{dx_2}{dt} + x_3/R_2 = 0$$

$$(U - x_3 - u + R_1 + R_2)/R_1 = C_3 \frac{dx_3}{dt} + x_3/R_2$$

$$(U - x_3 - u + R_1 + R_2)/R_1 - x_3/R_2 = C_3 \frac{dx_3}{dt}$$

$$C_1 \frac{dx_1}{dt} = (-x_3/R_2) + (u - R_1)/R_3$$

$$A = \begin{pmatrix} -1/(R_3 C_1) & 0 & -1/(R_2 C_1) \\ 0 & 0 & -1/(R_2 C_2) \\ 1/(R_1 C_3) & 1/(R_1 C_3) & -1/C_3 * (1/R_1 + 1/R_2) \end{pmatrix}$$

$$C = \begin{pmatrix} -1 & -1 & 0 \end{pmatrix}$$

$$D = 1$$

$$B = \begin{pmatrix} 1/(R_3 C_1) \\ 0 \\ 0 \end{pmatrix}$$

$$dx_1/dt = -(1/R_3 C_1) * x_1 - (1/R_2 C_1) * x_3 + (1/R_3 C_3) * x_3$$

$$dx_2/dt = -(1/R_2 C_2) * x_3$$

$$dx_3/dt = (1/R_1 C_3) * x_1 + (1/R_1 C_3) * x_2 - 1/R_1 C_3 - (1/R_2 C_3) * x_3$$

$$y = -x_1 - x_2 + u$$

$$2. \quad H(s) = \begin{pmatrix} -1 & -1 & 0 \end{pmatrix} * \begin{pmatrix} s + 1/R_3 C_1 & 0 & 1/R_2 C_1 \\ 0 & s & 1/R_2 C_2 \\ -1/R_1 C_3 & -1/R_1 C_3 & s + 1/R_1 C_3 + 1/R_2 C_3 \end{pmatrix} * \begin{pmatrix} 1/(R_3 C_1) \\ 0 \\ 0 \end{pmatrix}$$

$$H(s) = \begin{pmatrix} -1 & -1 & 0 \end{pmatrix} * \begin{pmatrix} s^2 + (1/R_1 C_3 + 1/R_2 C_3)s + 1/R_1 R_2 C_2 C_3 & x & x \\ 1/R_1 R_2 C_2 C_3 & x & x \\ x & x & x \end{pmatrix} *$$

$$* \begin{pmatrix} 1/(R_3 C_1) \\ 0 \\ 0 \end{pmatrix}$$

$$\text{Det}(sI - A)$$

$$\text{Det}(sI - A) = s^3 + (1/(R_1C_3) + 1/(R_2C_3) + 1/(R_3C_1)) * s^2 + (1/(R_1R_3C_1C_3) + 1/(R_2R_3C_1C_3) + 1/(R_1R_2C_1C_3) + 1/(R_1R_2C_2C_3)) * s + 1/(R_1R_2R_3C_1C_2C_3)$$

$$H(s) = \frac{s^3 + (1/(R_1C_3) + 1/(R_2C_3)) * s^2 + (1/(R_1R_3C_1C_3) + 1/(R_2R_3C_1C_3)) * s + 1/(R_1R_2R_3C_1C_2C_3)}{s^3 + (1/(R_1C_3) + 1/(R_2C_3) + 1/(R_3C_1)) * s^2 + (1/(R_1R_3C_1C_3) + 1/(R_2R_3C_1C_3) + 1/(R_1R_2C_1C_3) + 1/(R_1R_2C_2C_3)) * s + 1/(R_1R_2R_3C_1C_2C_3)}$$

3.

```
R1=10 * 10e3;
R2=10 * 10e3;
R3=4.7 * 10e3;
C1=2.2 * 10e-6;
C2=2.2 * 10e-6;
C3=4.7 * 10e-6;
A=[-1/(R3*C1), 0, -1/(R2*C1); 0, 0, -1/(R2*C2); 1/(R1*C3),
1/(R1*C3), (-1/C3)*(1/R1+1/R2)];
B=[1/(R3*C1); 0; 0];
C=[-1, -1, 0];
D=[1];
sys=ss(A,B,C,D);
[num,den]=ss2tf(A,B,C,D);
H=tf(num,den)
pole(H)
zero(H);
```

Polii sistemului:

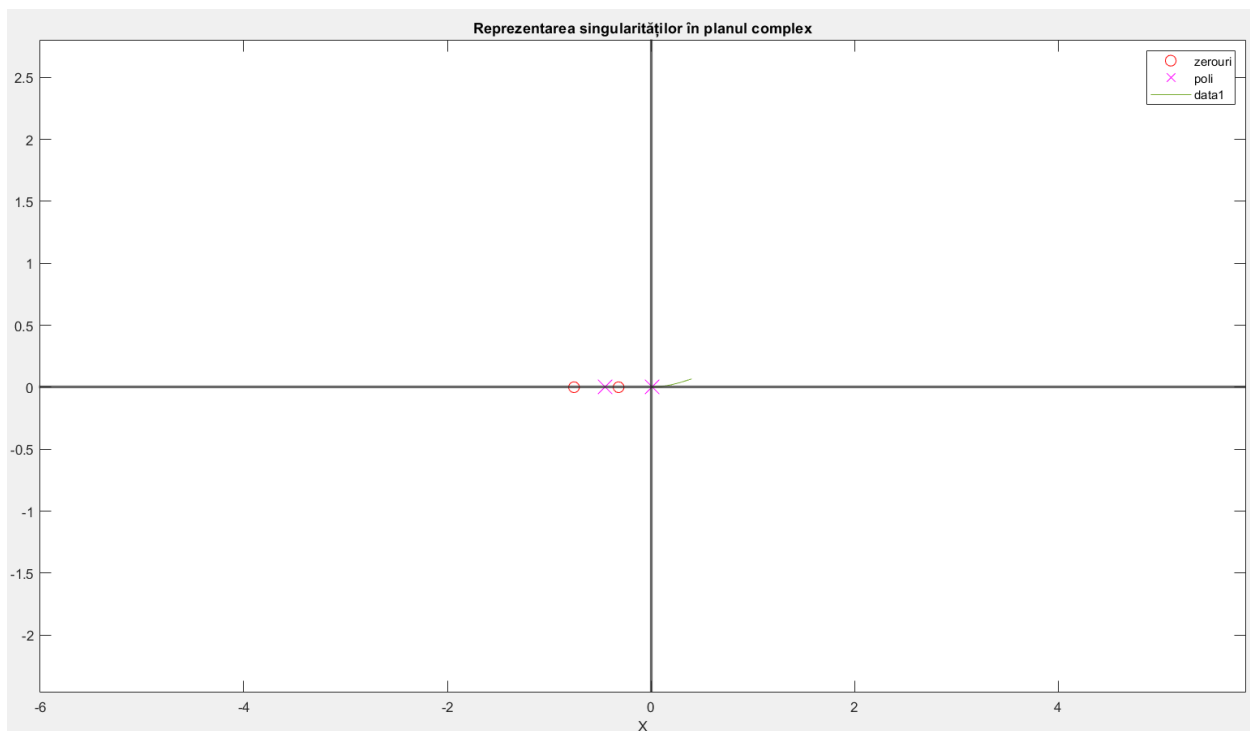
```
-0.7562 + 0.0000i
-0.3182 + 0.1497i
-0.3182 - 0.1497i
```

Zerourile sistemului:

$-0.4536 + 0.0000i$
 $0.0141 + 0.4539i$
 $0.0141 - 0.4539i$

Reprezentarea singularitatilor in planul complex:

```
p1=plot(-0.7562,0,'ro','MarkerSize',8);grid
hold on
plot(-0.3182,0.1497i,'ro','MarkerSize',8)
plot(-0.3182,-0.1497i,'ro','MarkerSize',8);
xlim([-35 10])
ylim([-10 10])
plot(-100:100,1,'k')
xline(0,'LineWidth',2)
yline(0,'LineWidth',2)
p2=plot(-0.4536,0,'mx','MarkerSize',15);
plot(0.0141,0.4539i,'mx','MarkerSize',15)
plot(0.0141,-0.4539i,'mx','MarkerSize',15)
legend([p1 p2],'zerouri','poli')
title("Reprezentarea singularitat??ilor în planul complex")
xlabel('X');
ylabel('jY');
```

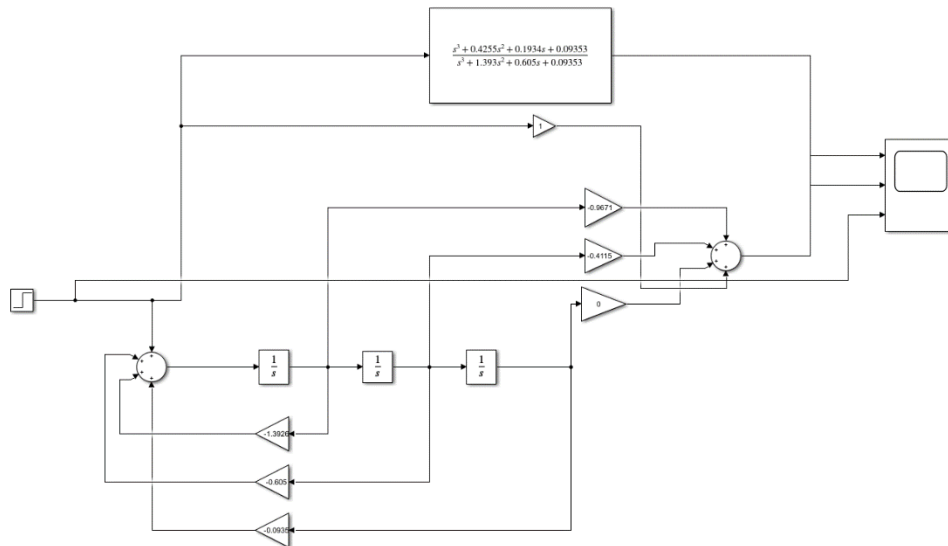


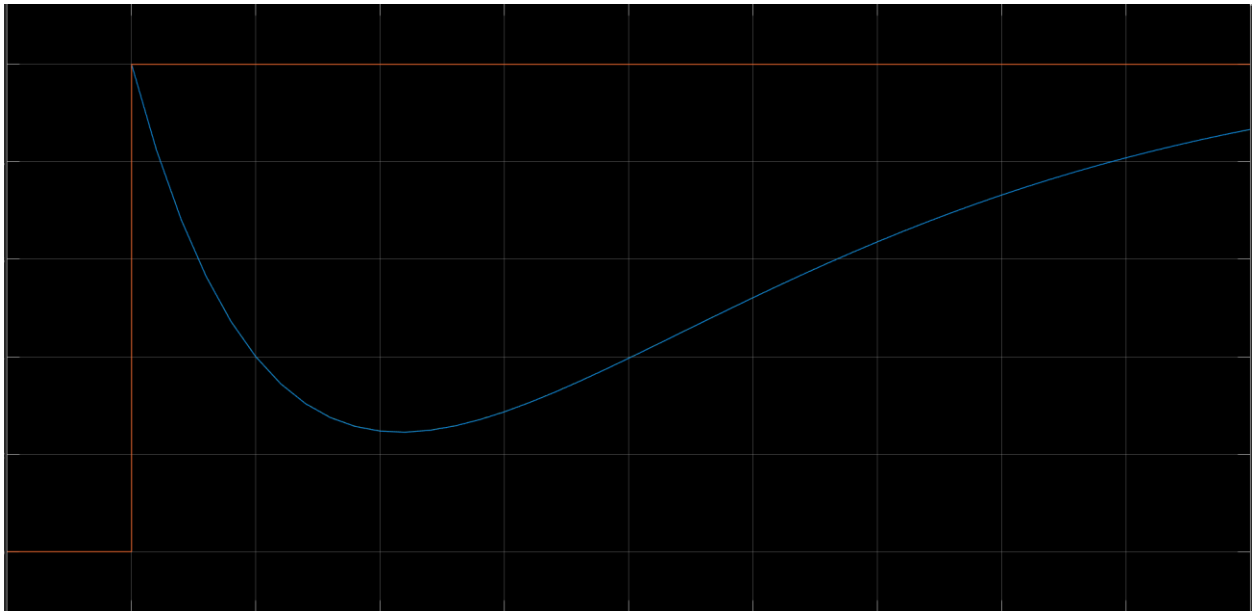
4.

```
[num,den] = tfdata(min, 'v');
[A_FCC, B_FCC, C_FCC, D] = tf2ss(num,den);
A_FCO = A_FCC';
B_FCO = C_FCC';
C_FCO = B_FCC';

sys_fco = ss(A_FCO, B_FCO, C_FCO, D)
```

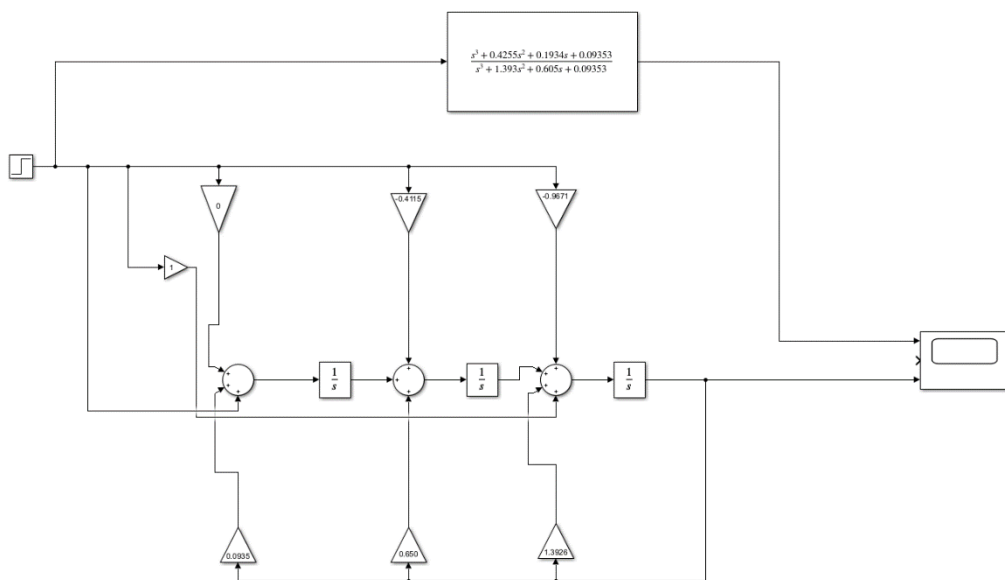
FCC:

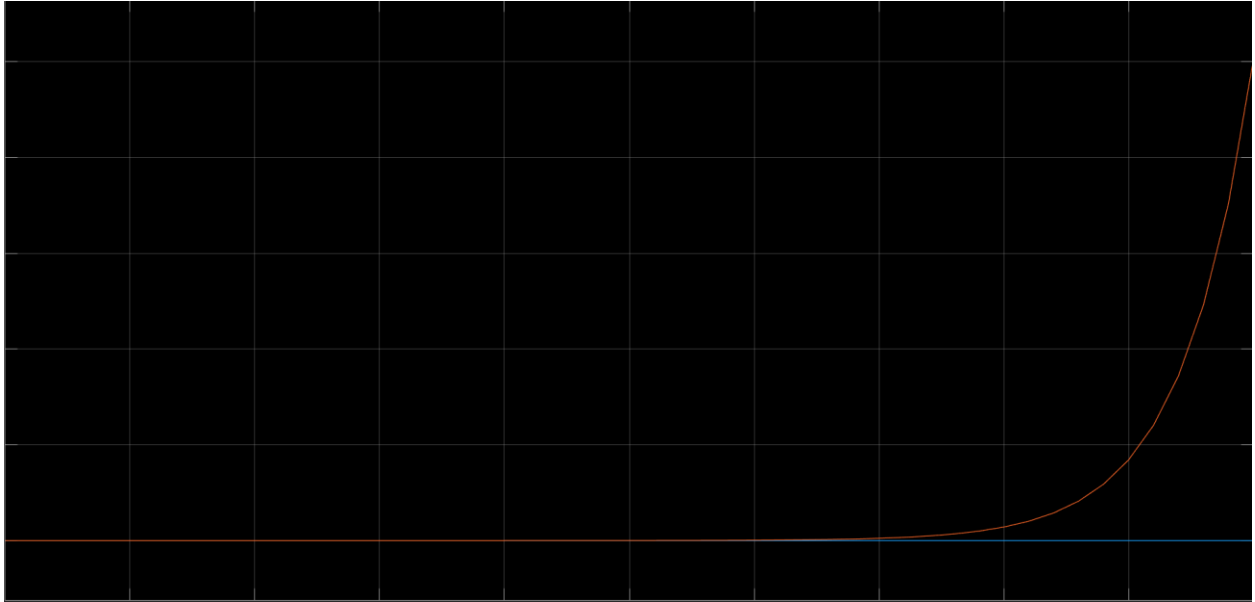




Este correct deoarece, semnalul de intrare este suprapus cu cel al functiei initiale.

FCO:





Este correct deoarece, semnalul de intrare este suprapus cu cel al functiei initiale.

5.

```
num_ext = [num zeros(1,5)];
gamma = deconv(num_ext,den);
min = zpk(minreal(H));
```

```
Hmin = (s + 0.4536) * (s^2 - 0.002811 * s + 0.2062) / (s +
0.7562) * (s^2 + 0.06364 * s + 0.1237)
```

6.

Sensitivitatea interna:

$$\text{Det}(\lambda I - A) = 0$$

$$\begin{pmatrix} \lambda + 96,71 & 0 & -45,45 \\ 0 & \lambda & -45,45 \\ 21,27 & 21,27 & \lambda + 42,55 \end{pmatrix} = \lambda^3 + 139,26 * \lambda^2 + 1\,933,44 * \lambda + 97\,606,5$$

$$\begin{array}{lcl} \lambda^3 & 1, & 1\,933,44 \\ \lambda^2 & 139,26 & 97\,606,5 \\ \lambda & x & 0 \\ \lambda^0 & y & 0 \end{array}$$

$$x = - \frac{\begin{vmatrix} 1 & 1\,933,44 \\ 139,26 & 97\,606,5 \end{vmatrix}}{139,26}$$

$$x = 1\,232,54$$

$$y = - \frac{\begin{vmatrix} 139,26 & 97\,606,5 \\ 1\,232,54 & 0 \end{vmatrix}}{1\,232,54}$$

$$y = 97\,606,5$$

In polinomul caracteristic, nu exista nici o schimbare de sens la radacini, rezulta un sistem stabil intern.

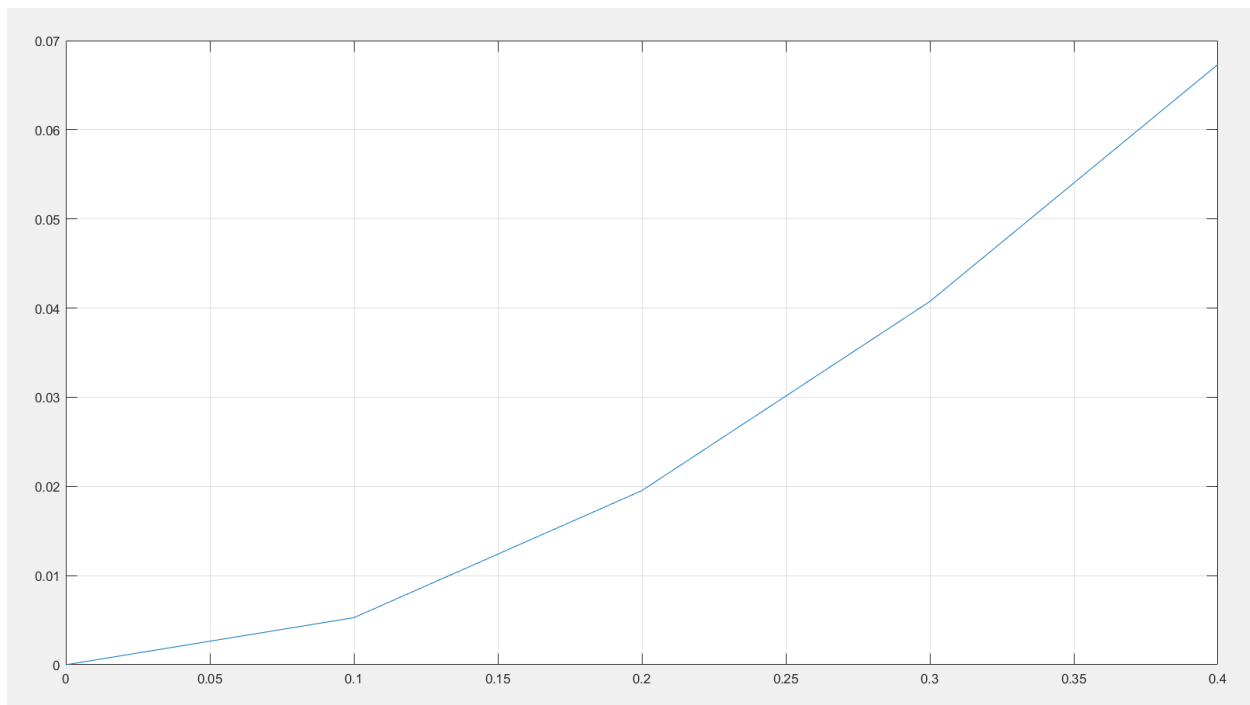
Sensitivitatea externa:

Stabilitatea internă impică stabilitatea externă, deci sistemul este de asemenea stabil extern.

7.

```
Q = eye(length(A));
P = lyap(A', Q)
eig(P)
sys = ss(A, B, C, D);
t = 0:0.1:0.4;
step_f = @(t) (t>=0);
st = step_f(t);
[~, time, x] = lsim(sys, st, t);

Vx = zeros(length(t), 1);
for i = 1:length(t)
    Vx(i) = x(i,:) * P * x(i,:)';
end
plot(t, Vx);
grid;
```



8.

H=

$$\frac{(s+0.4536) (s^2 - 0.02811s + 0.2062)}{(s+0.7562) (s^2 + 0.6364s + 0.1237)}$$

Linia parabolică
 Ecuația naturală
 Răspunsul indicial
 Răspunsul la rampă

$$n^2 + 0.6364n + 0.1237$$

$$n + 0.7562$$

$$\frac{A}{n + 0.7562} + \frac{Bn + C}{n^2 + 0.6364n + 0.1237} = \frac{(n + 0.4535)(n^2 - 0.002811n + 0.2062)}{0.2062}$$

$$\underline{A}n^2 + \underline{0.6364A}n + \boxed{0.1237A} + \underline{B}n^2 + \underline{C}n + \underline{0.7562nB} + \boxed{0.7562C}$$

$$\begin{cases} A + B = 0.4255 \\ 0.6364A + C + 0.7562nB = 0.1934 \\ 0.1237A + 0.7562C = 0.09353 \end{cases}$$

$$B = 0.4255 - A$$

$$C = \frac{0.09353 - 0.1237A}{0.7562}$$

$$\begin{aligned} & \frac{0.6364A + \frac{0.09353 - 0.1237A}{0.7562} + 0.7562(0.4255 - A)}{0.7562} + 0.3217 - 0.7562A = 0.1934 \end{aligned}$$

$$0.4812A + 0.09353 - 0.1237A + 0.2432 - 1.512A = 0.146$$

$$-1.15A + 0.3358 = 0.146$$

$$-1.15A = -0.189$$

$$\boxed{A = 0.16}$$

$$\boxed{A = 0.1643}$$

$$\boxed{B = 0.2612}$$

$$\boxed{C = 0.0958}$$

$$\begin{cases} A+B+D=1 \\ 0.636A+C+0.7562B+0.6364D+0.7562D=0.4255 \\ 0.1237A+0.7562C+0.1237D+0.7562D=0.1934 \\ 0.0935D=0.0935 \end{cases}$$

$$\boxed{D=1}$$

$$A+B=0$$

$$A=-B$$

$$0.1237A+0.7562C+0.8799=0.1934$$

$$0.7562C = \frac{-0.6865 - 0.1237A}{0.7562}$$

$$0.6364A - 0.7562A + \frac{0.6865 + 0.1237A}{0.7562} + 0.8799 = 0.1934$$

$$-0.1198A - \frac{0.6865 + 0.1237A}{0.7562} + 0.8799 = 0.1934$$

$$-0.0905A - 0.6865 + 0.1237A + 0.6653 = 0.1462$$

$$0.0332A = 0.1674$$

$$\boxed{A = 5.0421}$$

$$\boxed{B = -5.0421}$$

$$\boxed{C = -1.7326}$$

$$\frac{0.1643}{s+0.7562} + \frac{0.2612 \cdot s + 0.0968}{s^2 + 0.696 \cdot s + 0.1237}$$

$$\begin{aligned} \mathcal{L}(\omega) &= \mathcal{L}^{-1} \left\{ \frac{0.1643}{s+0.7562} \right\} + \mathcal{L}^{-1} \left\{ \frac{0.2612 \cdot s + 0.0968}{s^2 + 0.696 \cdot s + 0.1237} \right\} \\ &= 0.1643 \cdot e^{-0.7562t} + \mathcal{L}^{-1} \left\{ \frac{0.2612 \cdot (s+0.318)}{(s+0.318)^2 + 0.0226} + \frac{0.0138}{(s+0.318)^2 + 0.0226} \right\} \end{aligned}$$

$$\begin{aligned} &= 0.1643 \cdot e^{-0.7562t} + 0.2612 \cdot (\cos(0.1503t) \cdot e^{-0.318t}) \\ &\quad + \sin(0.1503t) \cdot e^{-0.318t} \end{aligned}$$

Funcția pondere

$$y(t) = \mathcal{L}^{-1} \left\{ H(s) \cdot \frac{1}{s} \right\}$$

$$\begin{aligned} &\underline{A s^3 + 0.6964 \cdot s^2 \cdot A + 0.1237 \cdot s \cdot A + B s^3 + C \cdot s^2 + 0.7562 \cdot s \cdot B +} \\ &0.7562 \cdot s \cdot C + \underline{0.1237 \cdot s \cdot D + 0.6964 \cdot s^2 \cdot D + 0.1237 \cdot s \cdot D + 0.7562 \cdot s^2 \cdot D +} \\ &0.7562 \cdot s \cdot D + 0.0935 D = \text{Numărătorul funcției de transfer} \end{aligned}$$

$$\begin{cases} A + B + D = 1 \\ 0.6364A + C + 0.7562B + 0.6364D + 0.7562D = 0.4255 \\ 0.1237A + 0.7562C + 0.1237D + 0.7562D = 0.1934 \\ 0.0935D = 0.0935 \end{cases}$$

$$\boxed{D=1}$$

$$A + B = 0$$

$$A = -B$$

$$0.1237A + 0.7562C + 0.8799 = 0.1934$$

$$0.7562C = \frac{-0.6865 - 0.1237A}{0.7562}$$

$$0.6364A - 0.7562A + \frac{0.6865 + 0.1237A}{0.7562} + 0.8799 = 0.1934$$

$$-0.1198A - \frac{0.6865 + 0.1237A}{0.7562} + 0.8799 = 0.1934$$

$$-0.0905A - 0.6865 + 0.1237A + 0.5653 = 0.1462$$

$$0.0332A = 0.1674$$

$$\boxed{A = 5.0421}$$

$$\boxed{B = -5.0421}$$

$$\boxed{C = -1.7326}$$

$$\frac{5.0421}{1+0.7562} + \frac{-5.0421 \cdot 0.7562}{1^2+0.636 \cdot 1+0.1237} + \frac{1}{0} = y(t)$$

$$\mathcal{L}^{-1}\{y(s)\} = 5.0421 \cdot e^{-0.7562t} - \mathcal{L}^{-1}\left\{ \frac{5.0421 \cdot (1+0.7562)}{(1+0.17)^2+0.0226} - \frac{0.025}{0.0226+0.17} \right\}$$

$$y(t) = 5.0421 \cdot e^{-0.7562t} - 5.0421 \cdot (\cos(0.4753t) \cdot e^{-3.18t})$$

$$- \sin(0.025 \cdot t) \cdot e^{-3.18t}$$

Răspunsul
inicial

Răspunsul în rampă:

$$y_r(t) = \mathcal{L}^{-1}\left\{ H(s) \cdot \frac{1}{s^2} \right\}$$

9.

$$\frac{(T^0 + 1) (s^2 + 2 \cdot \zeta \cdot \omega_n \cdot s + \omega_n^2)}{(T^0 + 1) (s^2 + 2 \cdot \zeta \cdot \omega_n \cdot s + \omega_n^2)}$$

$$(T^0 + 1) (s^2 + 2 \cdot \zeta \cdot \omega_n \cdot s + \omega_n^2)$$

Rezultam de aici;

$$T^0 = 1/0.4536$$

$$T^{\wedge} = 1/0.7562$$

$$w_n^0 = \sqrt{0.2062} = 0.4540$$

$$w_n^{\wedge} = \sqrt{0.1237} = 0.3517$$

$$2 * \zeta^0 * w_n = -0.02811 \quad \text{rezulta: } \zeta^0 = -0.0309$$

$$2 * \zeta^{\wedge} * w_n = 0.6364 \quad \text{rezulta: } \zeta^{\wedge} = 0.9047$$

Factorul de proportionalitate:

$$K = H(0) = (0.4536 * 0.2062) / (0.7562 * 0.1237) = 1$$

Eroarea stationara de pozitie:

$$\epsilon_{ssp} = 1 - H(0) = 1 - 1 = 0$$

Eroarea stationara de viteza(in cazul finit, deoarece $\epsilon_{ssp}=0$):

$$\epsilon_{ssv} = \lim_{t \rightarrow \infty} (u(t) - y(t)) \in \mathbb{R}$$