

Calculation, arXiv:1502.05193
**Does Current Data Prefer a Non-minimally Coupled
Inflaton?**

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- **Aim** : Understand the impact of a non-minimal coupling of the inflaton to the Ricci scalar, $\frac{1}{2}\xi R\phi^2$, on the inflationary predictions.
- **Study** : Focusing on the simplest inflationary model governed by the potential $V \propto \phi^2$
- **Data** : Planck and BICEP2/Keck Array 2015
- **Result** : Presence of a coupling ξ is favoured at a significance of 99% CL, $\xi \neq 0 \rightarrow 2\sigma$ level. Cross-correlation polarization spectra from BICEP2/Keck array and Planck, $r = 0.038^{+0.039}_{-0.030}$.

1 | Minimal coupled Inflaton in Jordan frame

For a minimal coupled inflaton, we set the $\xi = 0$. The action therefore takes the form,

$$S = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right] \quad (1.1)$$

The potential is taken as quadratic since it is the simplest one,

$$V(\phi) = \frac{1}{2} m^2 \phi^2 \quad (1.2)$$

The derivatives are therefore,

$$V' = m^2 \phi \quad (1.3)$$

$$V'' = m^2 \quad (1.4)$$

Slow-roll parameters are,

$$\epsilon = \frac{M_p^2}{16\pi} \left(\frac{V'}{V} \right)^2 = \frac{M_p^2}{4\pi\phi^2} \quad (1.5)$$

$$\eta = \frac{M_p^2}{8\pi} \frac{V''}{V} = \frac{M_p^2}{4\pi\phi^2} \quad (1.6)$$

Number of e-foldings can be calculated as shown below,

$$N = \int_{t_i}^{t_f} H dt \quad (1.7)$$

$$= \int_{\phi_i}^{\phi_f} \frac{H}{\dot{\phi}} d\phi \quad (1.8)$$

$$= -\frac{24\pi}{3M_p^2} \int_{\phi_i}^{\phi_f} \frac{V}{V'} d\phi \quad (1.9)$$

$$= -\frac{2\pi}{M_p^2} (\phi_f^2 - \phi_i^2) \quad (1.10)$$

From this point, we can calculate the ϕ_f since $\epsilon = 1$ tell us that this is the end of inflation. Therefore,

$$\epsilon = 1 = \frac{M_p^2}{4\pi\phi_f^2} \quad (1.11)$$

$$\phi_f^2 = \frac{M_p^2}{4\pi} \quad (1.12)$$

So the number of e-foldings is,

$$N = \frac{2\pi}{M_p^2} \phi_i^2 - \frac{1}{2} \quad (1.13)$$

Now, let us assume that the number of e-foldings N is equal to 60, thus we have the initial scalar field as,

$$\phi_i^2 = \left(60 + \frac{1}{2} \frac{M_p^2}{2\pi} \right) \quad (1.14)$$

Inserting the found initial scalar field expression into the slow-roll parameter expressions and from there calculating the spectral index of the primordial scalar perturbations n_s and tensor-to-scalar ratio r we have the following values for $N = 60$,

$$n_s = 0.96694214876 \quad (1.15)$$

$$r = 0.132231404959 \quad (1.16)$$

and for $N = 50$ we have,

$$n_s = 0.960396039604 \quad (1.17)$$

$$r = 0.158415841584 \quad (1.18)$$

2 | Conformal transformation

3 | Non-minimal coupling in Einstein frame