Calculation, arXiv:1502.05193
Does Current Data Prefer a Non-minimally Coupled Inflaton?

Sermet Çağan

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- **Aim :** Understand the impact of a non-minimal coupling of the inflaton to the Ricci scalar, $\frac{1}{2}\xi R\phi^2$, on the inflationary predictions.
- **Study :** Focusing on the simplest inflationary model governed by the potential $V \propto \phi^2$
- **Data :** Planck 2018
- **Result :** Planck and BICEP2/Keck Array 2015; presence of a coupling ξ is favoured at a significance of 99% CL, $\xi \neq 0 \ \rightarrow 2\sigma$ level. Cross-correlation polarization spectra from BICEP2/Keck array and Planck,
  $r = 0.038^{+0.039}_{-0.030}$.
- **References :**
  - arXiv:1502.05193
  - arXiv:0002091

1 | Minimal coupled Inflaton in Jordan frame

For a minimal coupled inflaton, we set the $\xi = 0$. The action therefore takes the form,

$$S = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R - \frac{1}{2}\left(\partial\phi\right)^2 - V\left(\phi\right)\right] \tag{1.1}$$

The potential is taken as quadratic since it is the simplest one,

$$V\left(\phi\right) = \frac{1}{2}m^2\phi^2 \tag{1.2}$$

The derivatives are therefore,

$$V' = m^2\phi \quad V'' = m^2 \tag{1.3}$$

Slow-roll parameters are,

$$\epsilon = \frac{M_p^2}{16\pi}\left(\frac{V'}{V}\right)^2 = \frac{M_p^2}{4\pi\phi^2} \tag{1.4}$$

$$\eta = \frac{M_p^2}{8\pi}\frac{V''}{V} = \frac{M_p^2}{4\pi\phi^2} \tag{1.5}$$

Number of e-foldings can be calculated as shown below,

$$N = \int_{t_i}^{t_f} H dt = \int_{\phi_i}^{\phi_f} \frac{H}{\dot{\phi}}d\phi \tag{1.6}$$

$$= -\frac{24\pi}{3M_p^2}\int_{\phi_i}^{\phi_f}\frac{V}{V'}d\phi \tag{1.7}$$

$$= -\frac{2\pi}{M_p^2}\left(\phi_f^2 - \phi_i^2\right) \tag{1.8}$$

From this point, we can calculate the ϕ_f since $\epsilon = 1$ tell us that this is the end of inflation. Therefore,

$$\epsilon = 1 = \frac{M_p^2}{4\pi\phi_f^2} \quad \rightarrow \quad \phi_f^2 = \frac{M_p^2}{4\pi} \tag{1.9}$$

So the number of e-foldings is,

$$N = \frac{2\pi}{M_p^2}\phi_i^2 - \frac{1}{2} \tag{1.10}$$

Now, let us assume that the number of e-foldings N is equal to 60, thus we have the initial scalar field as,

$$\phi_i^2 = \left(60 + \frac{1}{2}\frac{M_p^2}{2\pi}\right) \tag{1.11}$$

Spectral index and tensor-to-scalar ratio expressions in terms of the slow-roll parameter are,

$$n_s = 1 + 2\eta - 6\epsilon \tag{1.12}$$

$$r = 16\epsilon \tag{1.13}$$

Inserting the found initial scalar field expression into the slow-roll parameter expressions and from there calculating the spectral index of the primordial scarlar perturbations n_s and tensor-to-scalar ratio r we have the following values for $N = 60$,

$$n_s = 0.96694214876 \qquad r = 0.132231404959 \tag{1.14}$$

and for $N = 50$ we have,

$$n_s = 0.960396039604 \qquad r = 0.158415841584 \tag{1.15}$$

Comparing the results with the Planck 2018 data,

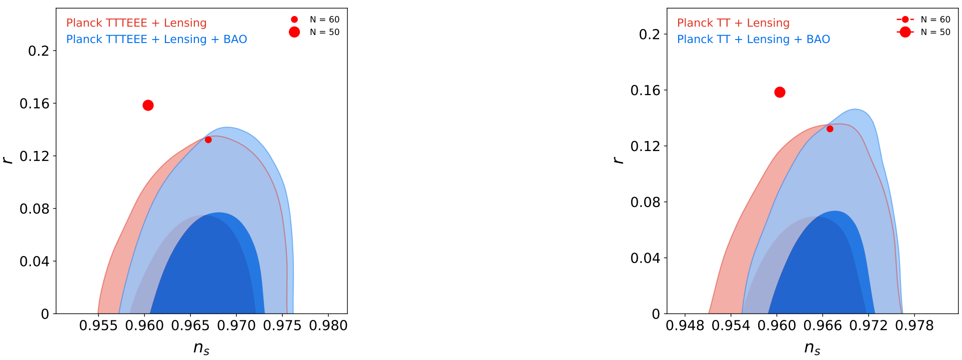

Figure 1. Tensor power spectrum amplitude (r)

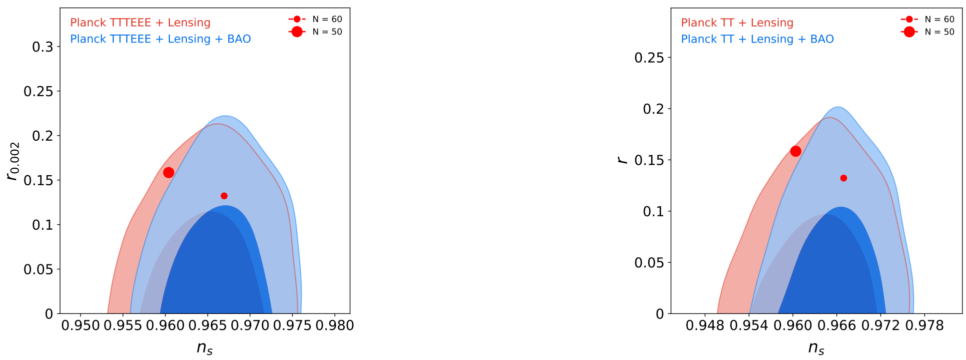

Figure 2. Running of the spectral index + Tensor power spectrum amplitude ($k = 0.05 Mpc^{-1}$)(nrun + r)

2 | Conformal transformation

Conformal transformations allows us to convert non-minimally coupled scalar field in Jordan frame into the minimal coupled scalar field in Einstein frame using the following transformation steps,

$$g_{\mu\nu} \quad \rightarrow \quad \tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \tag{2.1}$$

$$\Omega = \sqrt{1 - \kappa\xi\phi^2} \tag{2.2}$$

$$d\tilde{\phi} = \frac{\sqrt{1 - \kappa\xi\left(1 - 6\xi\right)\phi^2}}{1 - \kappa\xi\phi^2} \tag{2.3}$$

$$\tilde{V}\left[\tilde{\phi}\right] = \frac{V\left[\phi\left(\tilde{\phi}\right)\right]}{\left(1 - \kappa\xi\phi^2\right)^2} \tag{2.4}$$

The action which represents the dynamics of the non-minimally coupled scalar field in Jordan frame is given as,

$$S = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R + \frac{\xi}{2}R\phi^2 - \frac{1}{2}\left(\partial\phi\right)^2 - U\left(\phi\right)\right] \tag{2.5}$$

3 | Non-minimal coupling in Einstein frame