Calculation, arXiv:1502.05193 Does Current Data Prefer a Non-minimally Coupled Inflaton?

Sermet Çağan

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- Aim: Understand the impact of a non-minimal coupling of the inflaton to the Ricci scalar, $\frac{1}{2}\xi R\phi^2$, on the inflationary predictions.
- Study : Focusing on the simplest inflationary model governed by the potential $V \propto \phi^2$
- Data: Planck and BICEP2/Keck Array 2015
- Result: Presence of a coupling ξ is favoured at a significance of 99% CL, $\xi \neq 0 \rightarrow 2\sigma$ level. Cross-correlation polarization spectra from BICEP2/Keck array and Planck, $r = 0.038^{+0.039}_{-0.030}$.

1 | Minimal coupled Inflaton in Jordan frame

For a minimal coupled inflaton, we set the $\xi = 0$. The action therefore takes the form,

$$S = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R - \frac{1}{2} \left(\partial \phi \right)^2 - V(\phi) \right]$$
 (1.1)

The potential is taken as quadratic since it is the simplest one,

$$V\left(\phi\right) = \frac{1}{2}m^{2}\phi^{2} \tag{1.2}$$

The derivatives are therefore,

$$V' = m^2 \phi \tag{1.3}$$

$$V'' = m^2 \tag{1.4}$$

Slow-roll parameters are,

$$\epsilon = \frac{M_p^2}{16\pi} \left(\frac{V'}{V}\right)^2 = \frac{M_p^2}{4\pi\phi^2} \tag{1.5}$$

$$\eta = \frac{M_p^2 V''}{8\pi V} = \frac{M_p^2}{4\pi \phi^2} \tag{1.6}$$

Number of e-foldings can be calculated as shown below,

$$N = \int_{t_{-}}^{t_{f}} H dt \tag{1.7}$$

$$= \int_{\phi_i}^{\phi_f} \frac{H}{\dot{\phi}} d\phi \tag{1.8}$$

$$= -\frac{24\pi}{3M_p^2} \int_{\phi_i}^{\phi_f} \frac{V}{V'} d\phi$$
 (1.9)

$$= -\frac{2\pi}{M_p^2} \left(\phi_f^2 - \phi_i^2 \right) \tag{1.10}$$

From this point, we can calculate the ϕ_f since $\epsilon = 1$ tell us that this is the end of inflation. Therefore,

$$\epsilon = 1 = \frac{M_p^2}{4\pi\phi_f^2} \tag{1.11}$$

$$\phi_f^2 = \frac{M_p^2}{4\pi} \tag{1.12}$$

So the number of e-foldings is,

$$N = \frac{2\pi}{M_p^2} \phi_i^2 - \frac{1}{2} \tag{1.13}$$

Now, let us assume that the number of e-foldings N is equal to 60, thus we have the initial scalar field as,

$$\phi_i^2 = \left(60 + \frac{1}{2} \frac{M_p^2}{2\pi}\right) \tag{1.14}$$

Inserting the found initial scalar field expression into the slow-roll parameter expressions and from there calculating the spectral index of the primordial scarlar perturbations n_s and tensor-to-scalar ratio r we have the following values for N = 60,

$$n_s = 0.96694214876 \tag{1.15}$$

$$r = 0.132231404959 \tag{1.16}$$

and for N = 50 we have,

$$n_s = 0.960396039604 \tag{1.17}$$

$$r = 0.158415841584 \tag{1.18}$$

2 | Conformal transformation

3 | Non-minimal coupling in Einstein frame