Universidad Politécnica de Madrid

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Projective geometry and Panoramic image

Image Processing, Analysis and Classification

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Contents

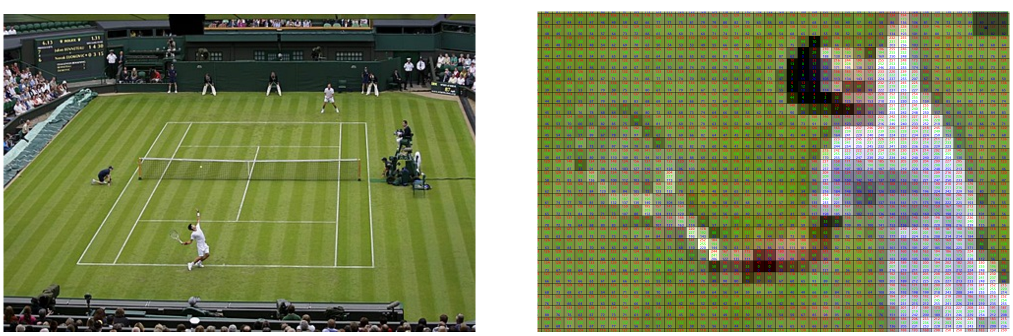
1 Introduction

We live in a 3 dimensional world, but images are usually represented in 2 dimensional surfaces, such as screens, pictures, etc. It is necessary to remove one of this dimensions, and logically it has to be the depth component.

Digital images are visual representations stored in electronic form, composed of discrete picture elements or pixels. To enable interpretation by both humans and computers, images are often represented as matrices. This matrix representation allows for manipulation and processing using computational algorithms. In the context of digital image representation, an image $I \in M_{h,w}\left(\mathbb{Z}^n\right)$ can be defined as:

$$I = \begin{pmatrix} p_{1,1} & p_{1,2} & \cdots & p_{1,w} \\ p_{2,1} & p_{2,2} & \cdots & p_{2,w} \\ \vdots & \vdots & \ddots & \vdots \\ p_{h,1} & p_{h,2} & \cdots & p_{h,w} \end{pmatrix}_{h \times w}$$

Here, each p_{ij} represents a pixel, typically containing a vector in $\mathbb{Z}^n$, where n varies based on the number of color channels. This mathematical representation serves as the foundation for various image processing and analysis techniques, facilitating both human perception and machine understanding. In Fig. 1 it can be seen how the image is encoded using RGB color space.



(a) Original image (b) Pixel matrix

Figure 1: Digital image representation

2 Perspective

Perspective in digital images is a critical aspect that influences the perception of depth and spatial relationships within a scene. It is primarily manipulated through various parameters, with focal distance being one of the key elements.

The focal distance, often denoted as f, is a fundamental parameter in digital imaging that determines the magnification and perspective of the captured scene. It refers to the distance between the lens and the image sensor when the subject is in focus. A shorter focal distance results in a wider field of view, allowing more of the scene to be captured within the frame, while a longer focal distance narrows the field of view, resulting in magnification and compression of distant objects.

By using this focal distance, we can establish different measurements (angles) inside of the image. A potential use case for these measurements is triangulate locations. Firstly, some reference points are selected. Then, calculate the angle between them, two by two. Finally, these angles are placed over a map, and the intersection between them would indicate the actual position where the photo was taken. This method only requires some basic trigonometrics, as it can be seen in Fig. 2.

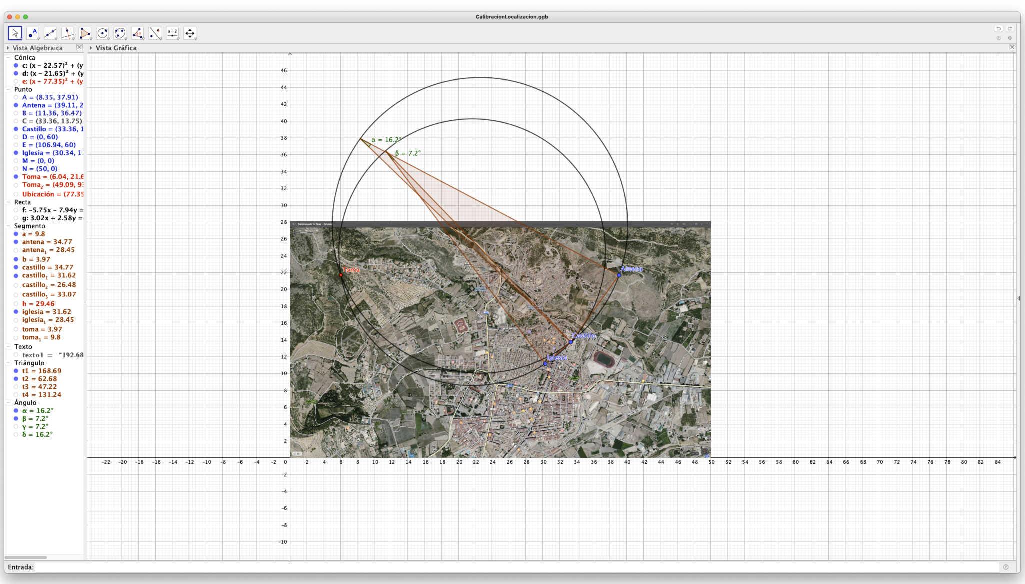


Figure 2: Example of location triangulation

3 Homographies

Sometimes there are photographies of the same object but taken from different perspectives (Fig. 3).

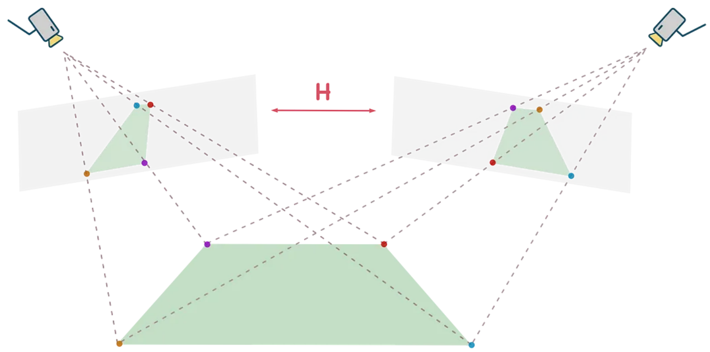


Figure 3: Different perspectives of the same object [1]

The concept of homography raises with the aim of represent how each point can be converted into other (because in reality they are the same point). To represent this spacial translation, a function can be used, and this function can be represented as a matrix.

$$\lambda \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} H_{1,1} & H_{1,2} & H_{1,3} \\ H_{2,1} & H_{2,2} & H_{2,3} \\ H_{3,1} & H_{3,2} & H_{3,3} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

4 Panoramic image reconstruction

Panoramic images offer a wide-angle perspective of scenes through the seamless merging of multiple images taken from different viewpoints. This process involves several key steps, including the extraction of key point features, their matching, and the estimation of homographies.

Key point feature extraction techniques like Scale-Invariant Feature Transform (SIFT) or Features from Accelerated Segment Test (FAST) play crucial roles in panoramic image stitching. These techniques identify distinctive points or regions in images that remain invariant to changes in scale, rotation, and illumination. Robust features are extracted from each image to enable subsequent matching of corresponding points across overlapping images.

Following key point extraction, the matching process aims to establish correspondences between these points across different images (Fig. 4).

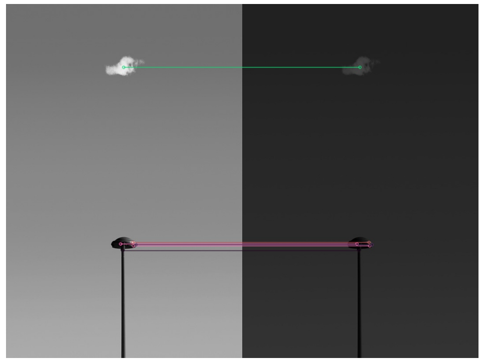


Figure 4: Example of key point matching

Homography estimation is then performed to calculate the transformation necessary to align matched key points between images. Estimating the homography matrix enables effective warping and blending of images to produce a seamless panoramic composition (Fig. 5).

(a) Original images before the mosaic [2]

(b) Two images stitched together [2]

Figure 5: Example of panoramic image reconstruction

5 Use case: Dental images

An use case for this panoramics is dental health images. In this context, X-ray images could have hidden teeth by other ones, as shown in Fig. 6.

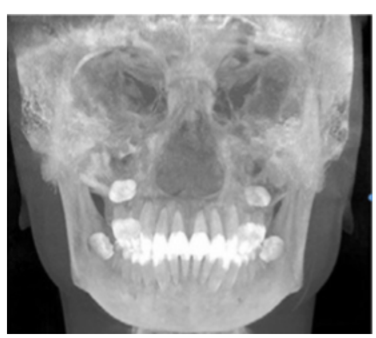


Figure 6: Some teeth are hidden

Yun *et al.* [3] came up with a solution for this problem using panoramic images. Their proposal is shown in Fig. 7.

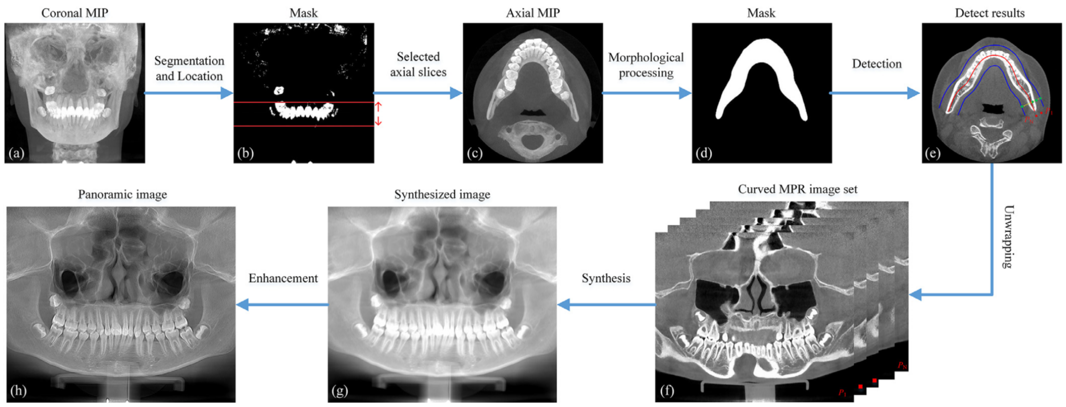


Figure 7: Proposed methodology

Firstly, they use thresholding based on the histogram of the image to build an accurate mask of the whole teething. After that, they estimate the dental arch using the following approximation:

$$I_0(i,j) = S \cdot \log\left(\sum_{n=1}^{N} e^{\frac{P_n(i,j)}{S}}\right)$$

This arch approximation can be used together with a lot of images around the whole head, and homographies to create an unwrapped version of the teething (Fig. 8).

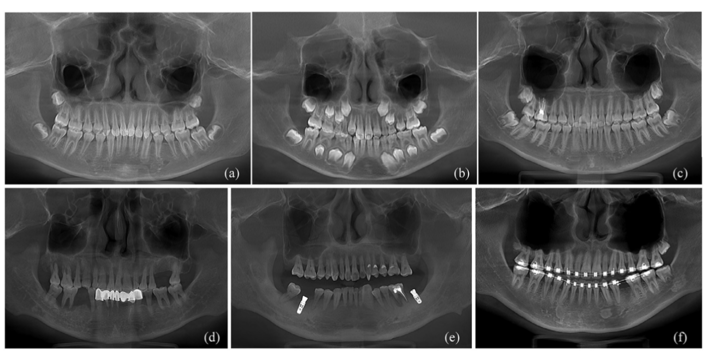


Figure 8: Unwrapped images

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