Projective geometry and Panoramic image

Image Processing, Analysis and Classification

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Tuesday 2nd April, 2024

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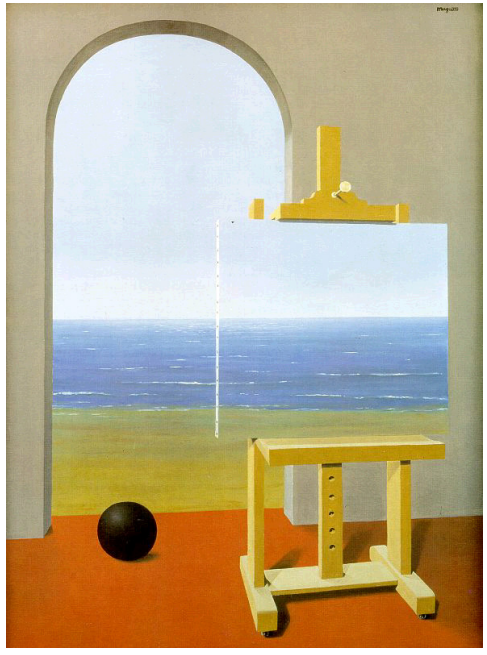
Table of Contents

Introduction

3D world $\rightarrow$ 2D content

An image is just a flat representation of the 3D world.



Even though we lose the depth component the image maintain the perspective.

Introduction

3D world $\rightarrow$ 2D content

Image are useful to store memories, but also we can extract information from them. For instance, we may want to know how tall is this bottle.

Introduction

3D world $\rightarrow$ 2D content

We need to represent the images in such a way that both humans and computers are able to deal with them. Thus matrix representation of images is defined. We can define an image $I \in M_{h,w}(\mathbb{Z}^n)$ as:

$$I = \begin{pmatrix} p_{1,1} & p_{1,2} & \cdots & p_{1,w} \\ p_{2,1} & p_{2,2} & \cdots & p_{2,w} \\ \vdots & \vdots & \ddots & \vdots \\ p_{h,1} & p_{h,2} & \cdots & p_{h,w} \end{pmatrix}_{h \times w}$$

Where each p_{ij} is a vector in $\mathbb{Z}^n$, n varies depending on the number of color channels.

Introduction

3D world $\rightarrow$ 2D content

Here we can see how the image is codified as we have defined before:

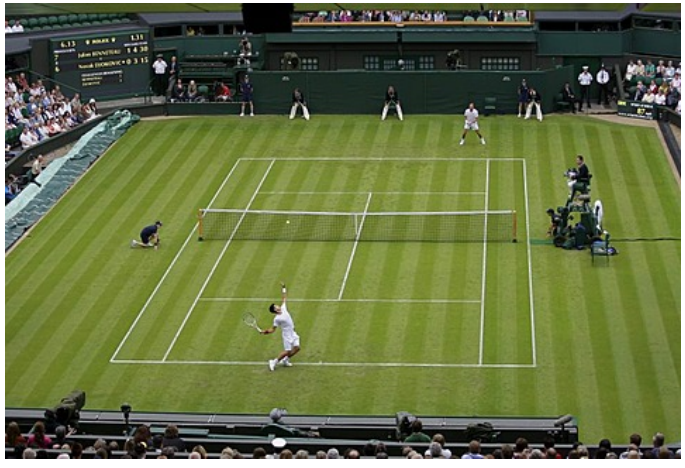
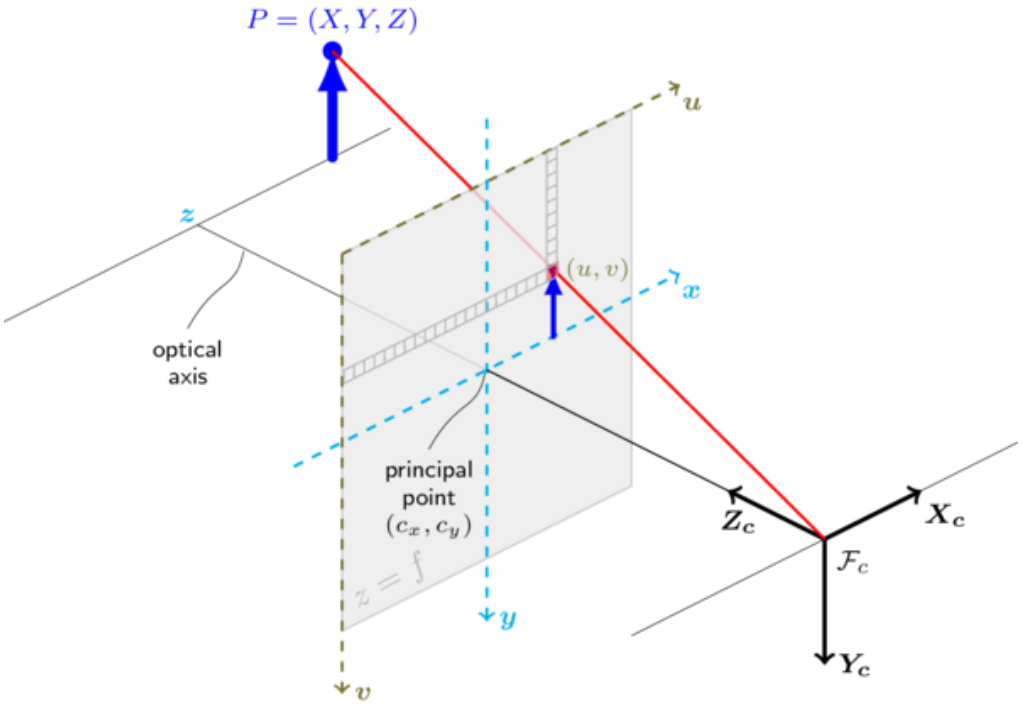


Table of Contents

Perspective

We can use angles to represent the perspective of a point in the image if we know the focal distance f of the camera.



$$\alpha = \arctan \frac{u}{f}$$

Perspective

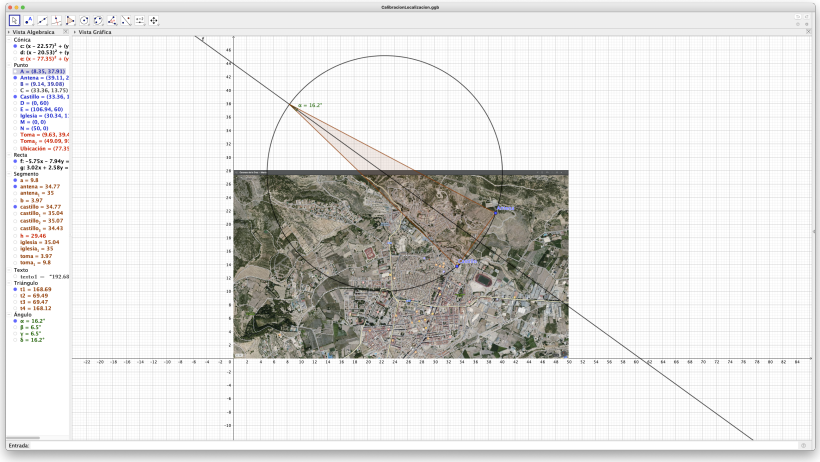
Use case: Locate the place from where a photo was taken

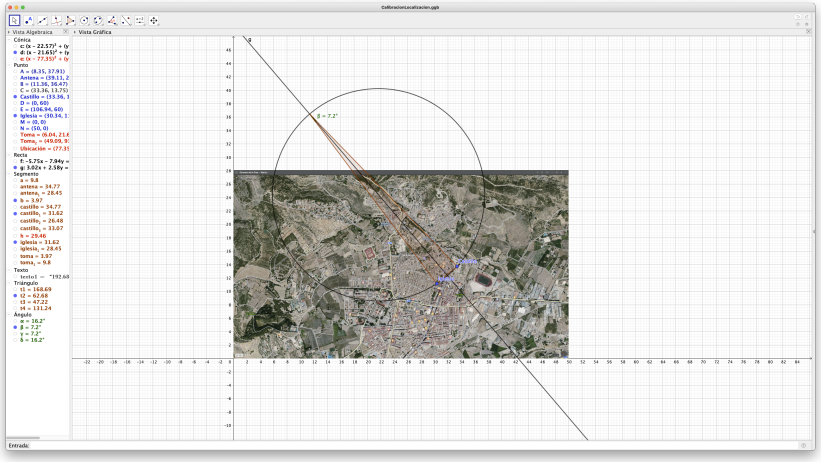
We can use this information to locate where a photo was taken.

Perspective

Use case: Locate the place from where a photo was taken

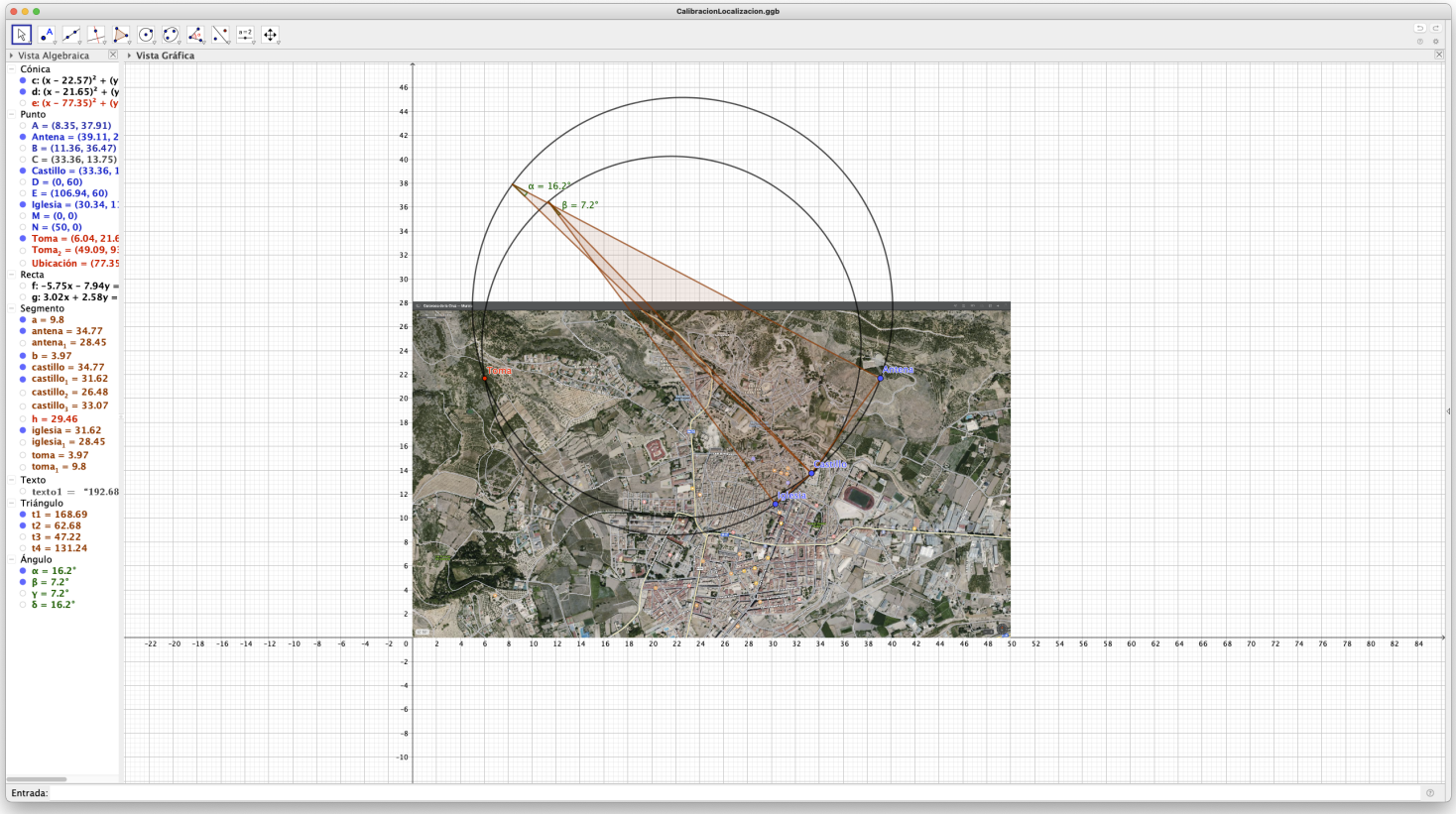
We only need to use some trigonometry.





Perspective

Use case: Locate the place from where a photo was taken



Perspective

Use case: Locate the place from where a photo was taken

The precision is quite accurate even though the methods are not so precise.

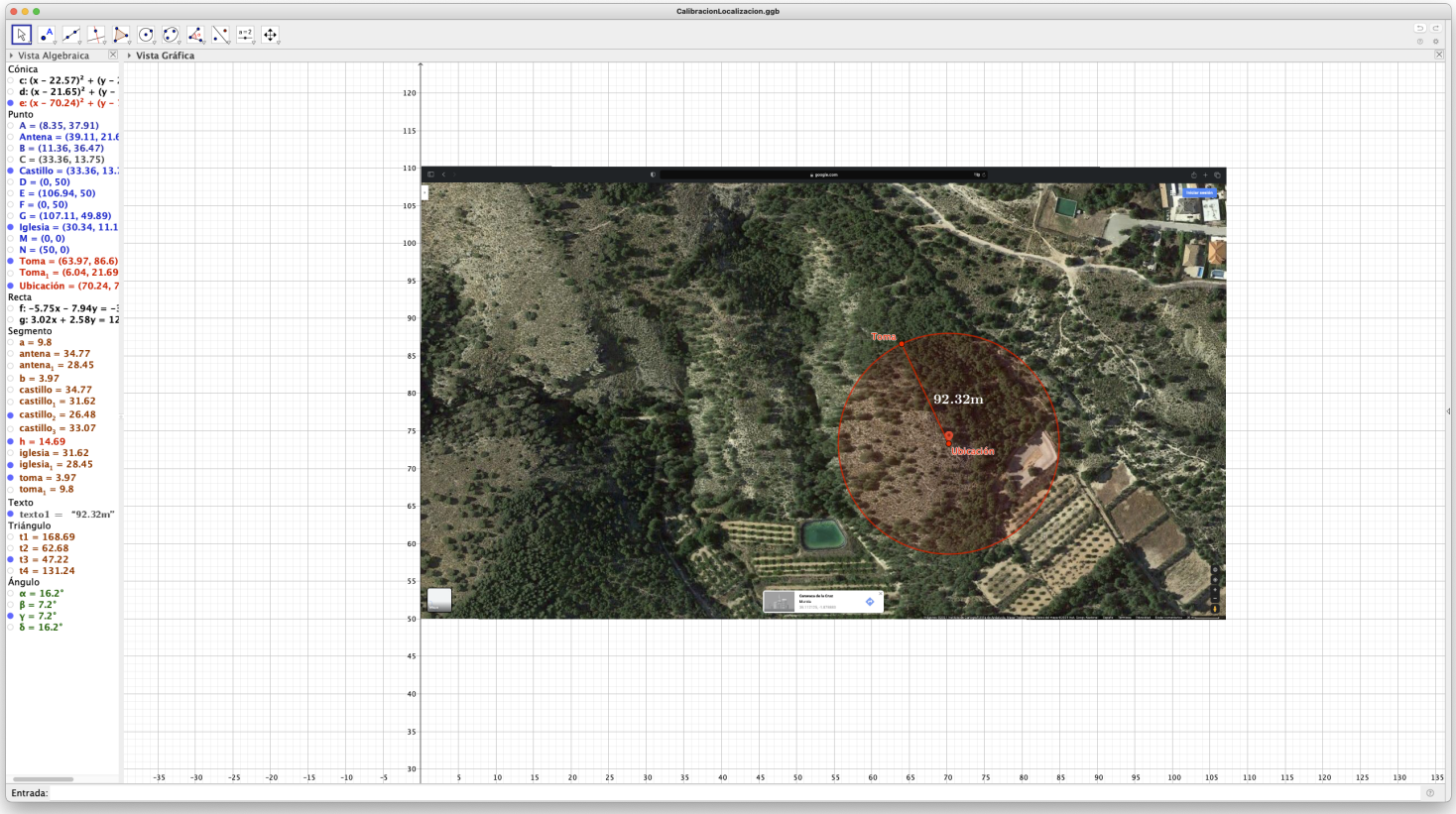
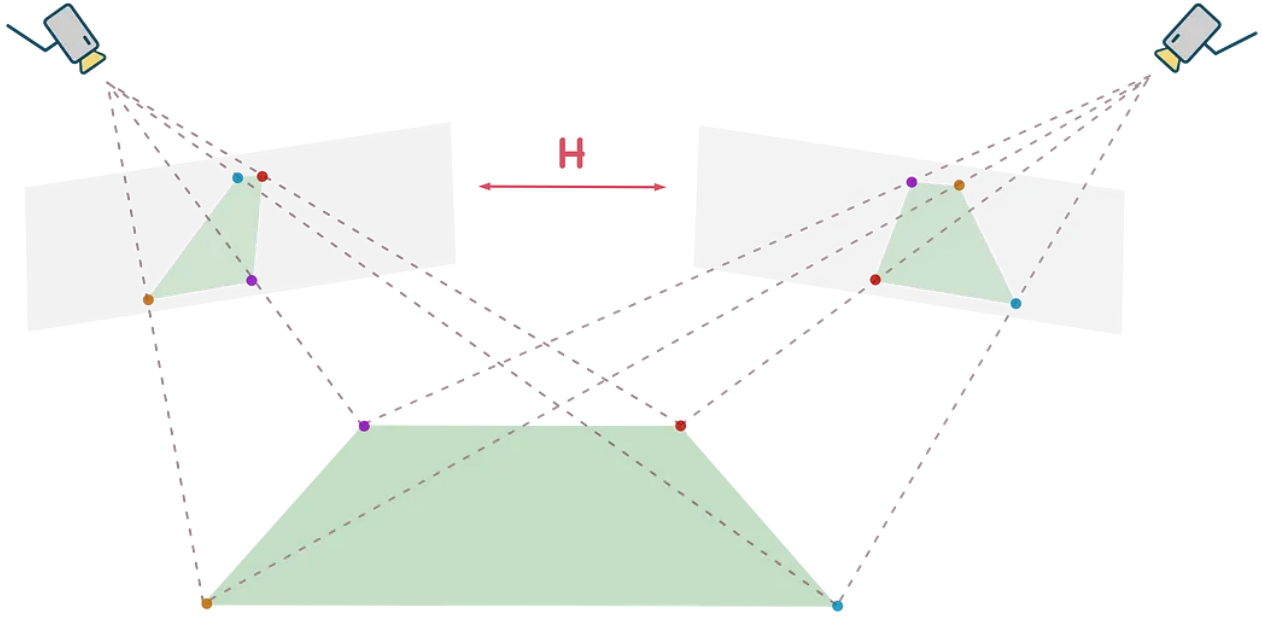


Table of Contents

Homographies

“The homograph is a mapping between two perspective images of a planar surface in a scene” [1]



Homographies

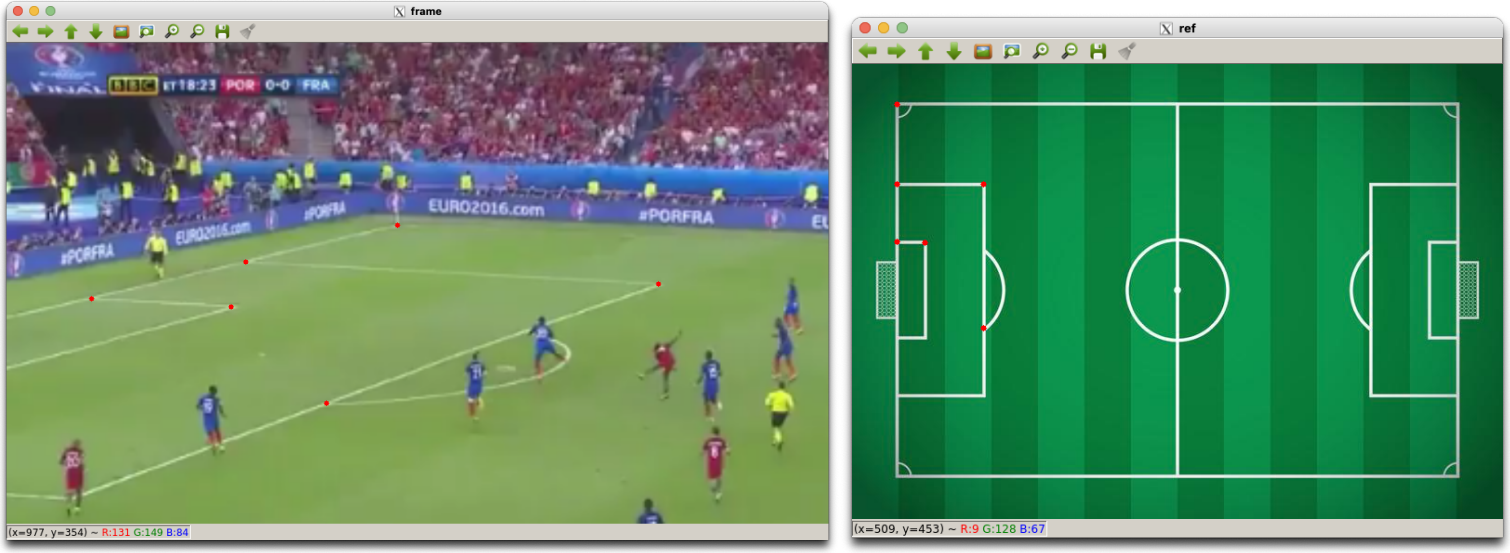
An homography is just a matrix that convert some points into others.

$$\lambda \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} H_{1,1} & H_{1,2} & H_{1,3} \\ H_{2,1} & H_{2,2} & H_{2,3} \\ H_{3,1} & H_{3,2} & H_{3,3} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Homographies

Use case: How far was this shot?

By using homographies we can remove the perspective from an image, and after that we can measure in the image.



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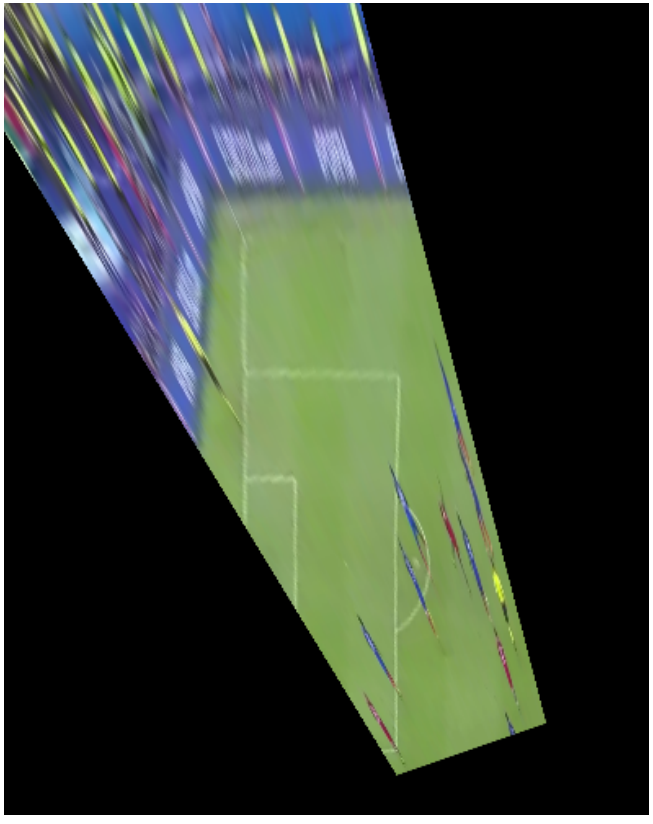
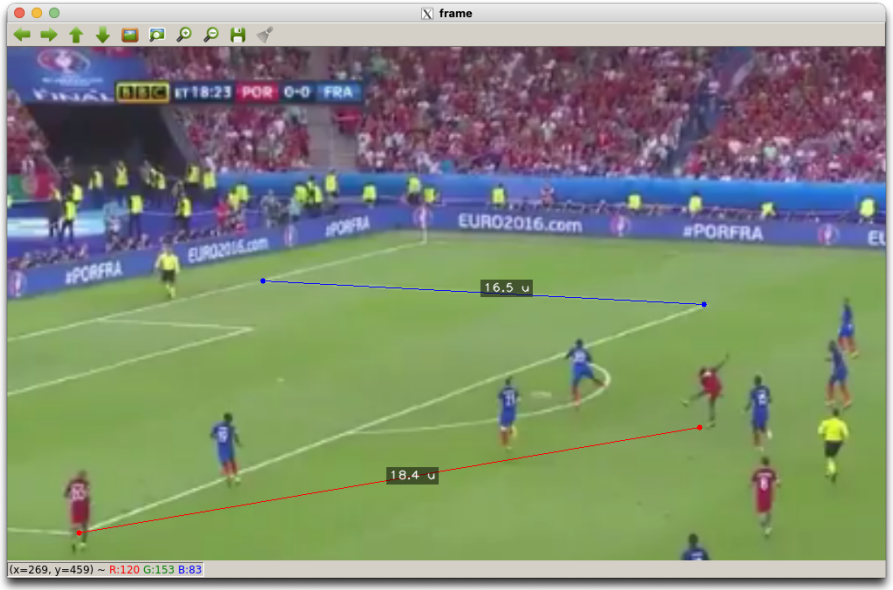


Homographies

Use case: How far was this shot?

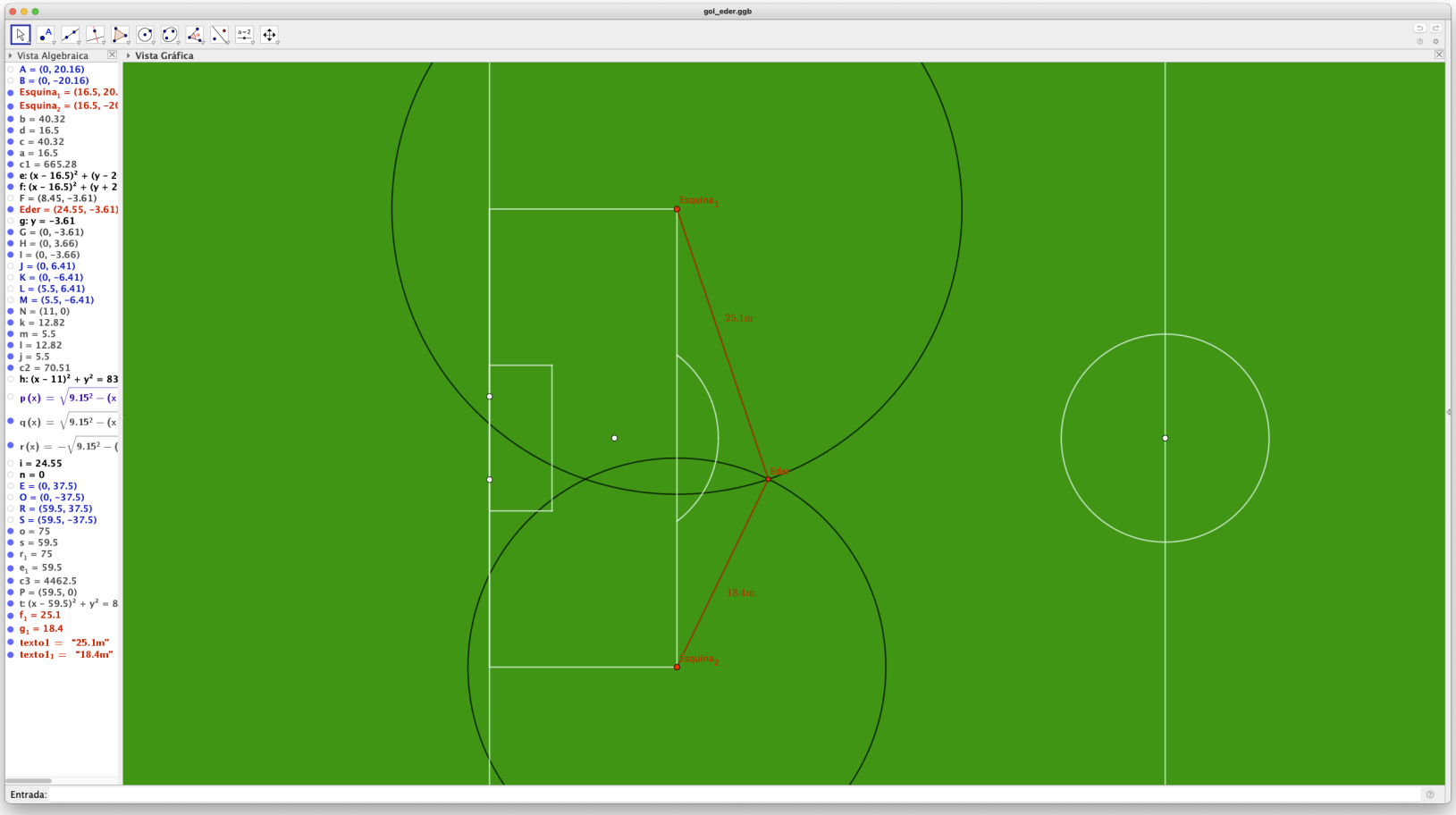
We can establish measurements in the image.



Homographies

Use case: How far was this shot?

Using trigonometry, as before, we can estimate the shot distance.



Homographies

Use case: How far was this shot?

Using trigonometry, as before, we can estimate the shot distance.

Table of Contents

Panoramic image

Panoramic image reconstruction uses the techniques explain before.

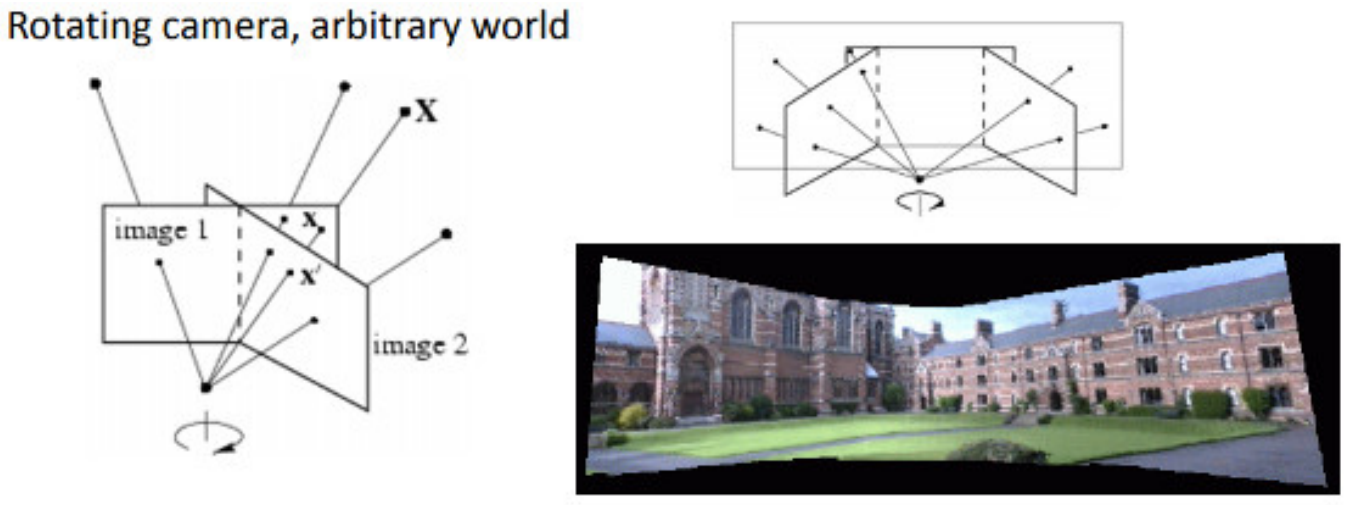


Figure: Panoramic image reconstruction [2]

Panoramic image

The key points have to be automatically calculated.

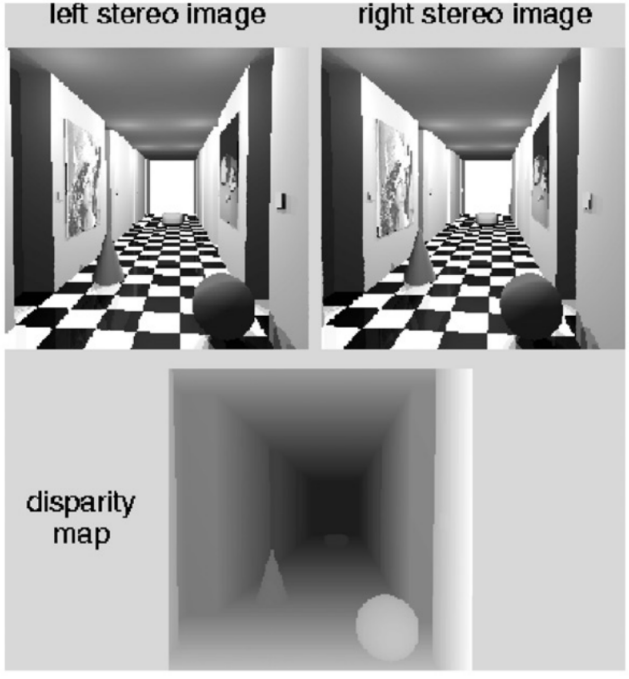


Figure: Stereo images and related disparity map [1]

Panoramic image

We can use techniques explain during the lessons (Key point feature extraction), such as SIFT, FAST, etc.

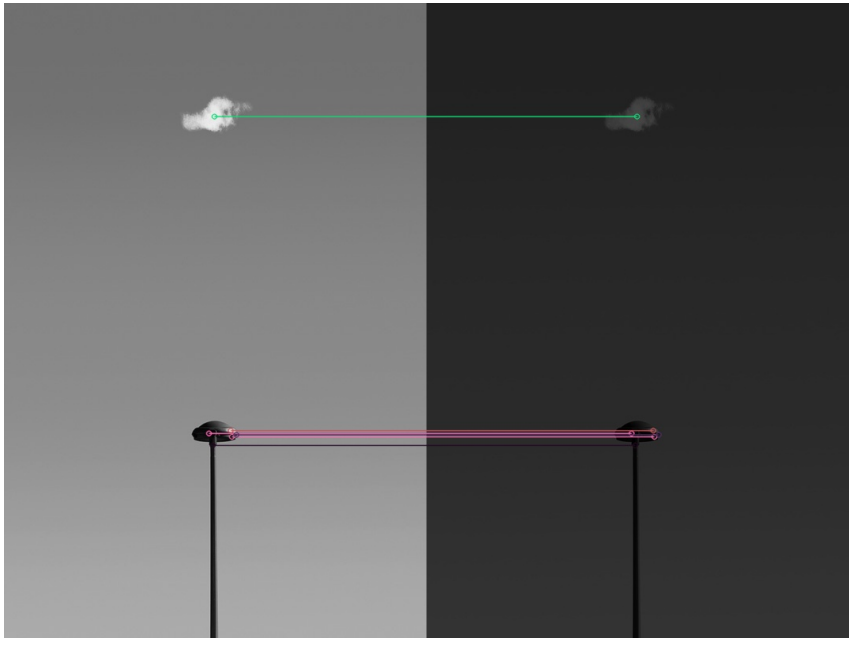


Figure: Key points from practical exercise

Panoramic image

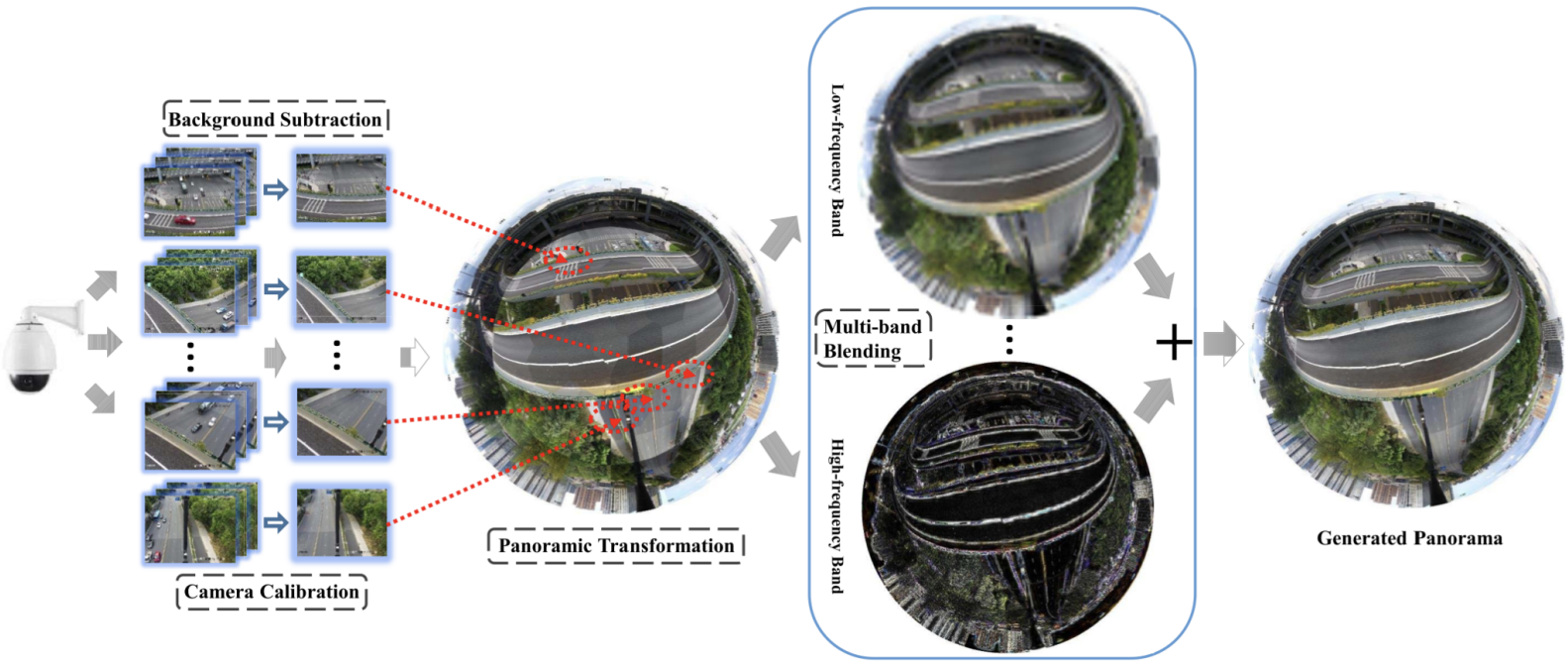
If the key points are good enough selected, the results could be quite accurate.

(a) Original images before the mosaic [1]

(b) Two images stitched together [1]

Panoramic image

This technique can also be used in the reconstruction of non-planar images. [3]



Panoramic image

Another example with a 360° image. [4]

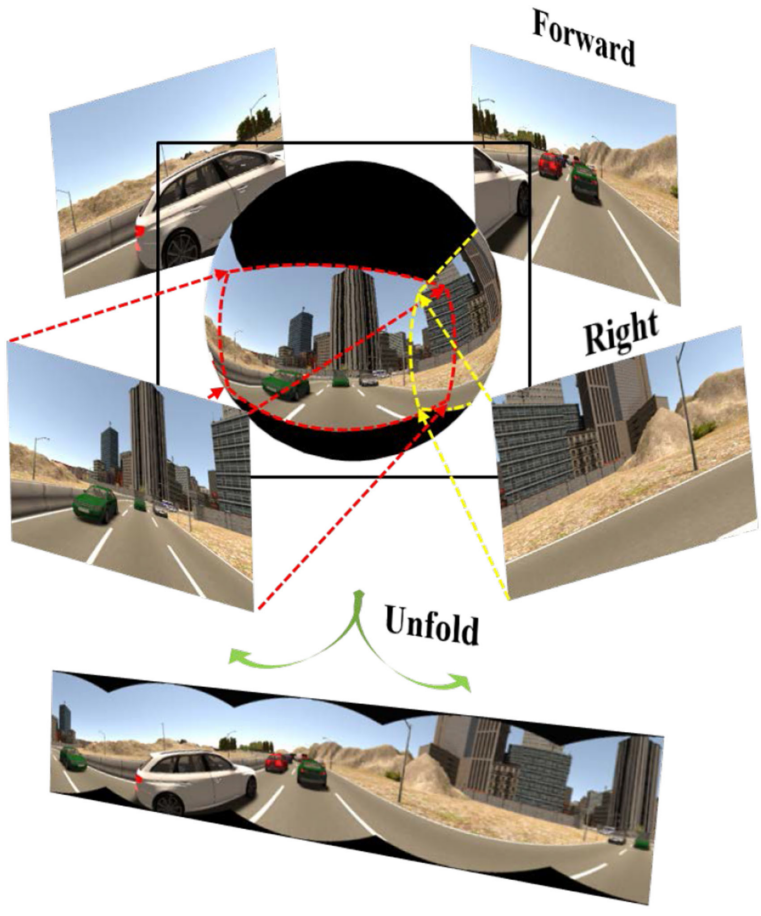


Table of Contents

Use case: Dental images

Problem description

In the context of dental health, as teeth are bones, the dentist needs to take X-rays.

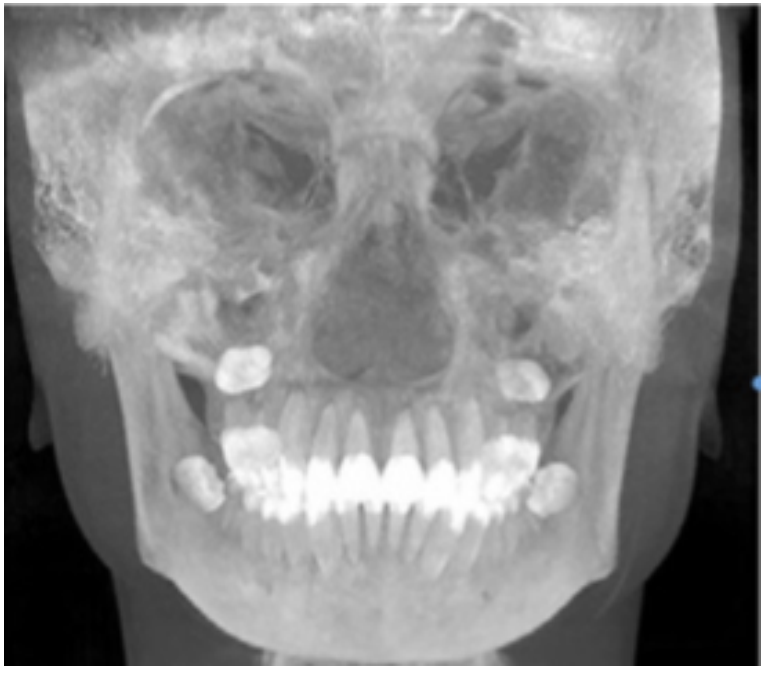
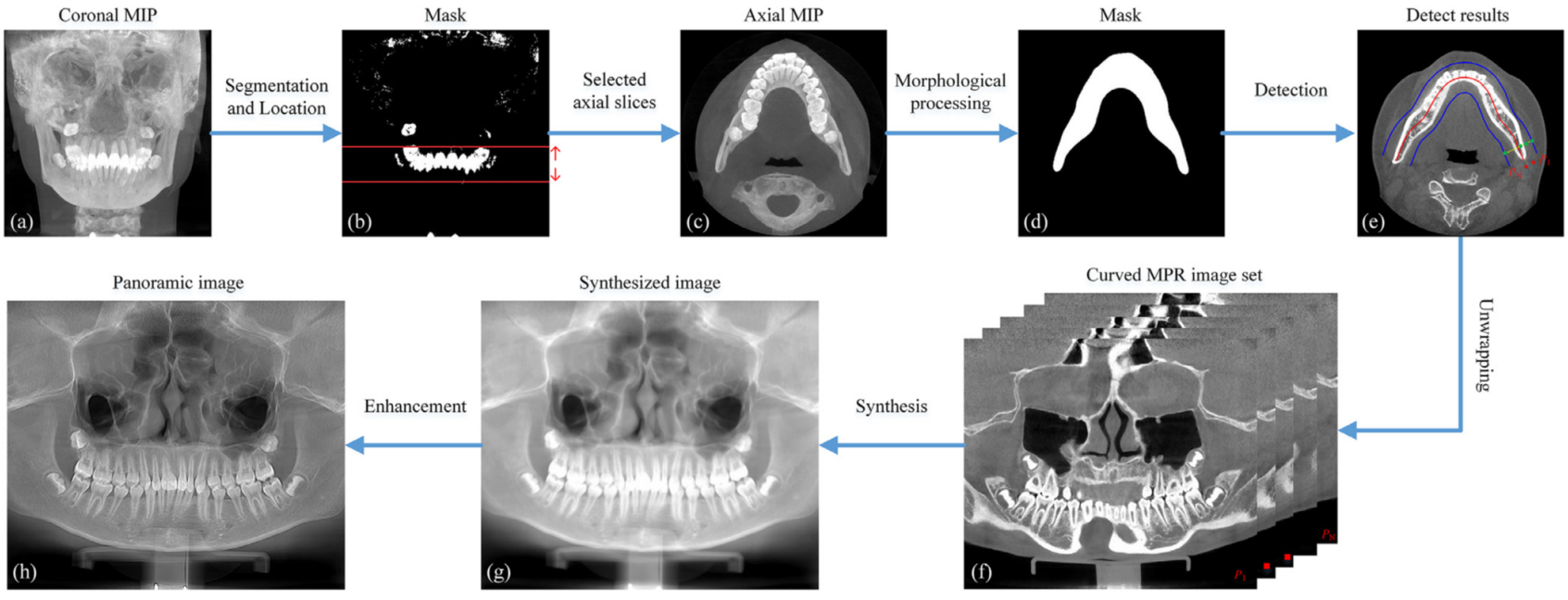


Figure: Some teeth hide others [5]

Use case: Dental images

Proposed solution

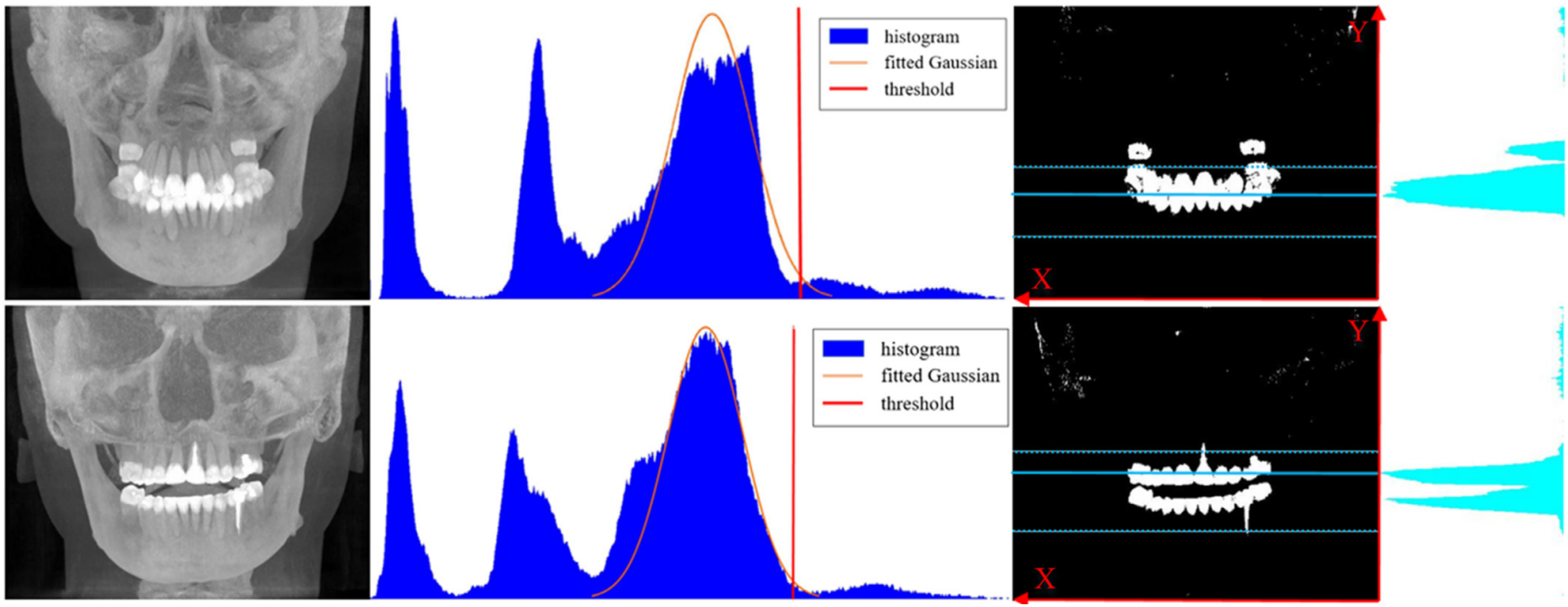
Yun *et al*. [5] came up with a solution based on the contents of this subject.



Use case: Dental images

Thresholding

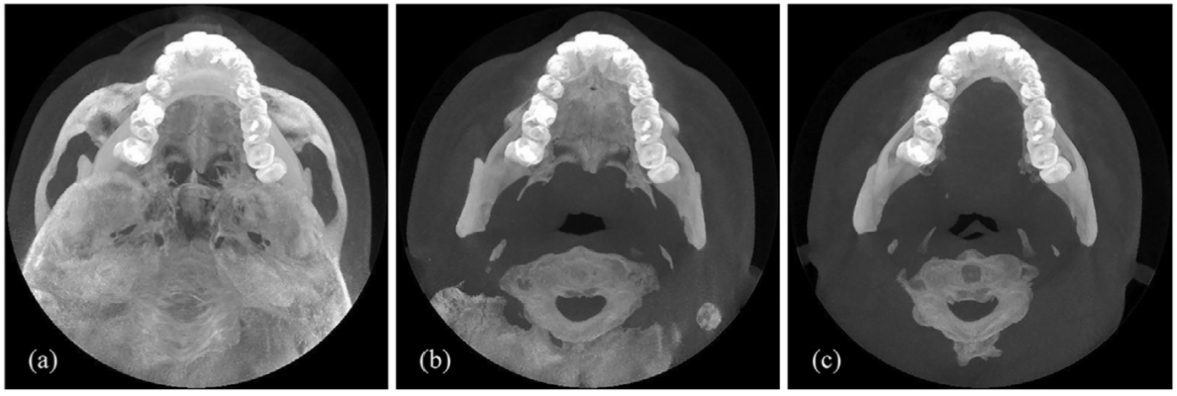
They use histograms to establish a threshold value.



Use case: Dental images

Morphological processing

With the threshold calculated before, they managed to create a mask.



Use case: Dental images

Morphological processing

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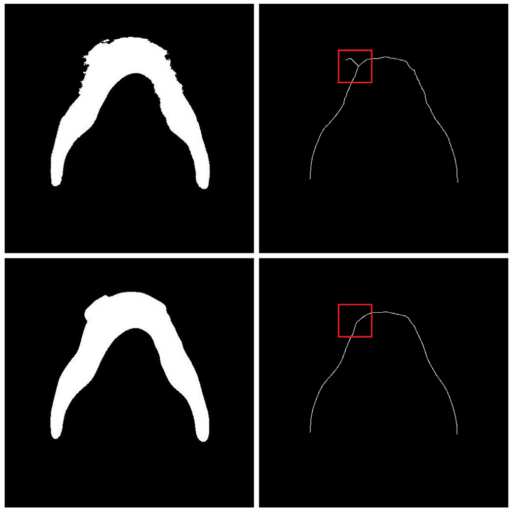


Also perform Gaussian filtering

Use case: Dental images

Arch approximation

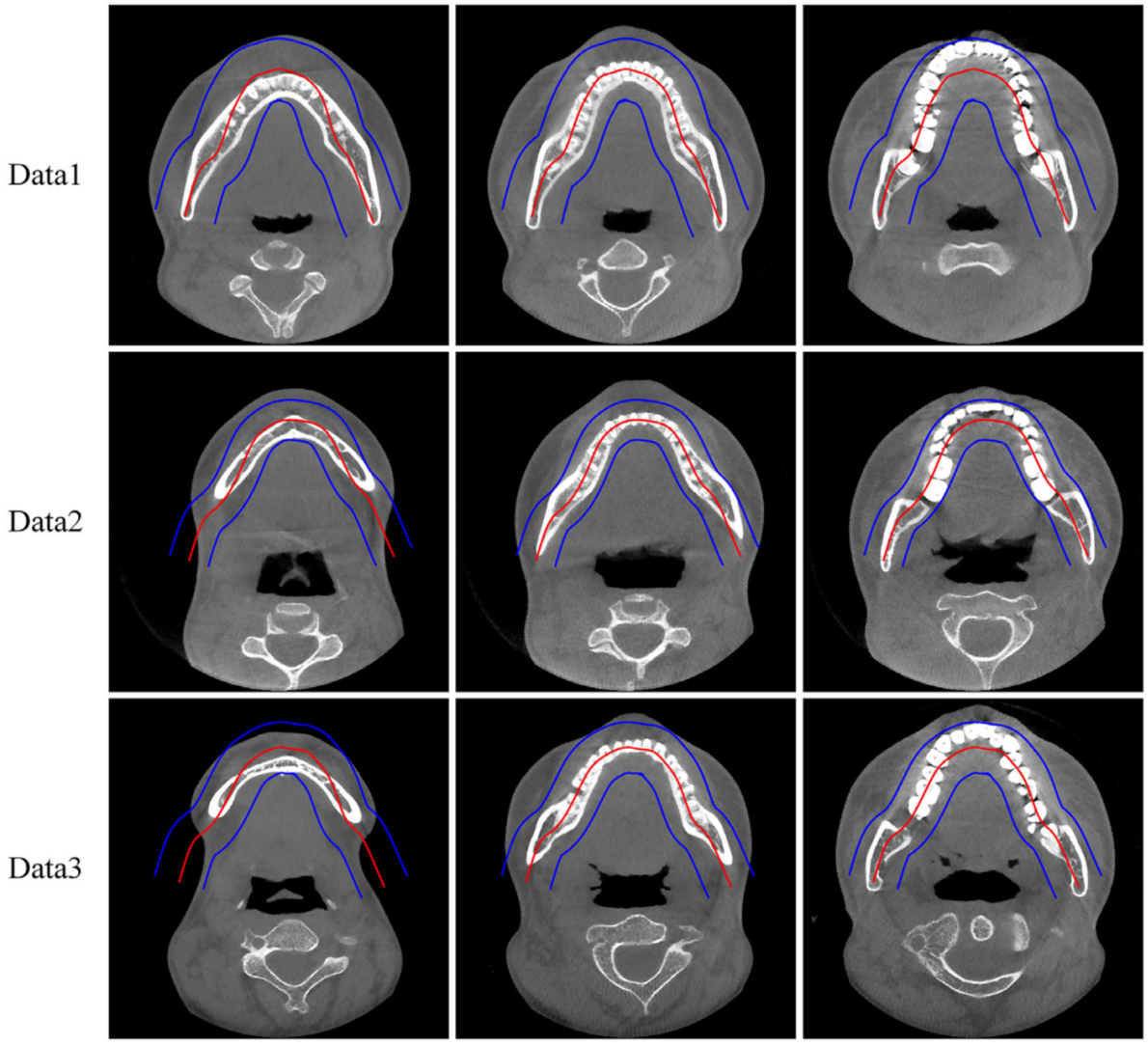
Dental arch approximation using the following formula:

$$I_0(i,j) = S \cdot \log\left(\sum_{n=1}^{N} e^{\frac{P_n(i,j)}{S}}\right)$$

Use case: Dental images

Arch approximation

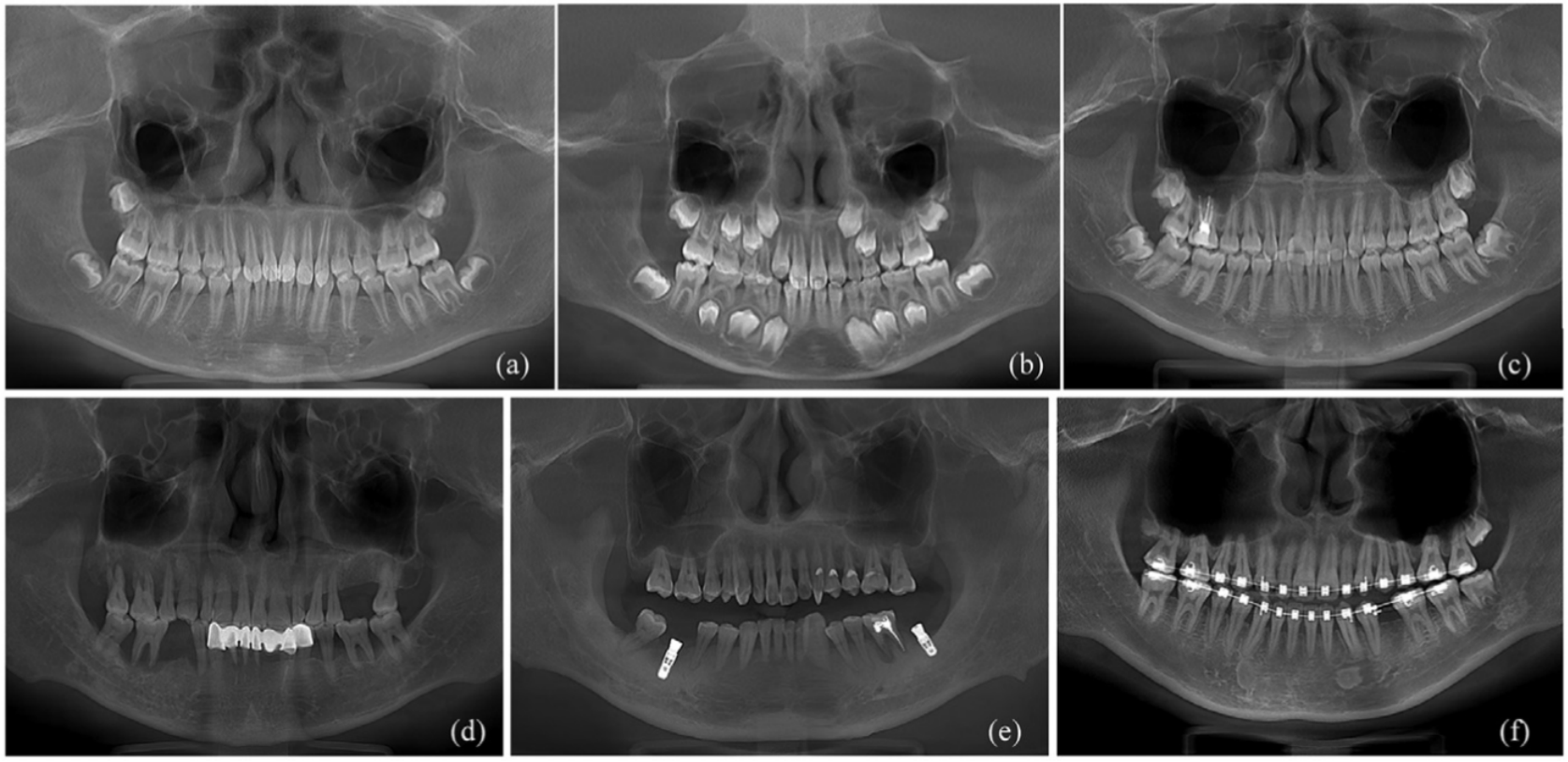
Dental arch approximations are quite accurate:



Use case: Dental images

Panoramic images generation

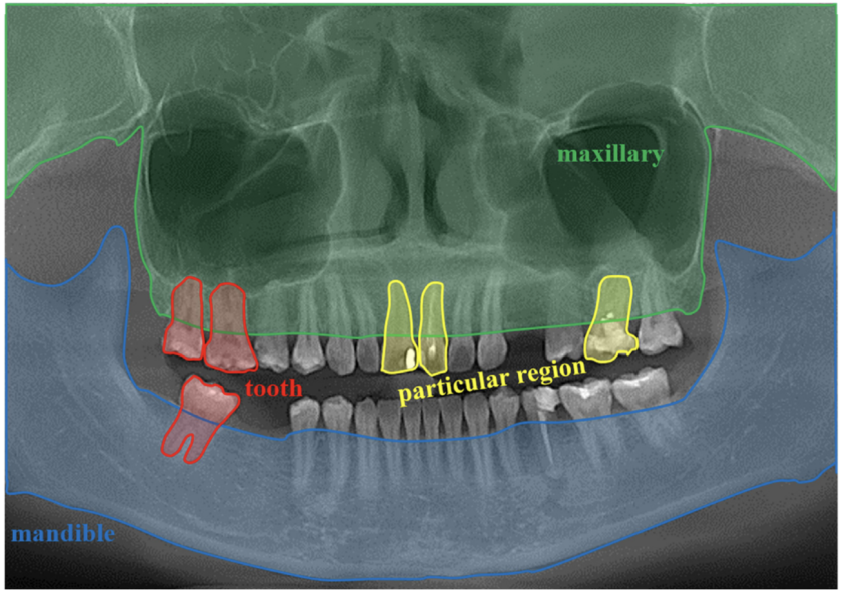
Using homographies and the arch estimation in each tooth, they are able to reconstruct a panoramic image of the whole teething.



Use case: Dental images

Applications

Using techniques similar as the ones seen during the lessons, dentists can obtain automatic tooth identification.



Use case: Dental images

Known problems

One future path of this method is dealing with mental objects.

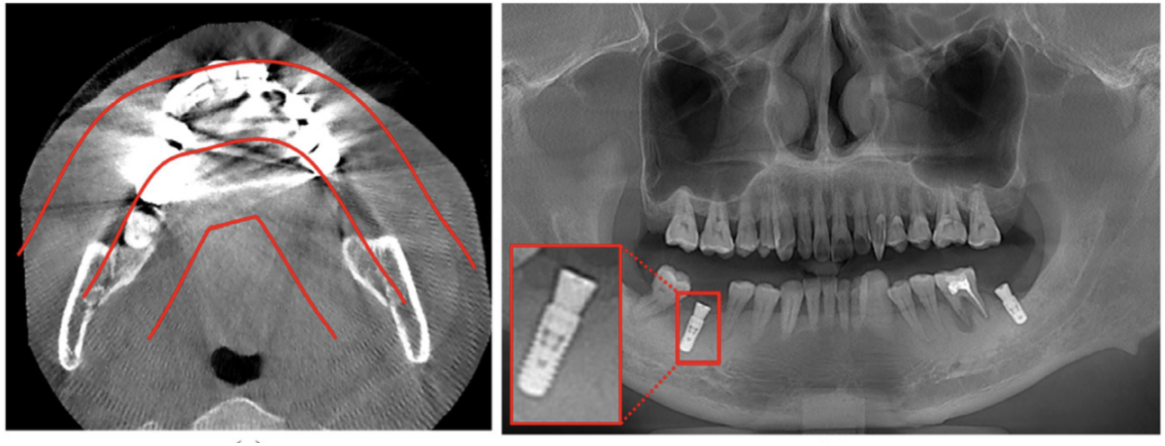


Table of Contents

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[1] D. Gledhill, G. Y. Tian, D. Taylor, and D. Clarke, "Panoramic imaging—a review," *Computers & Graphics*, vol. 27, no. 3, pp. 435–445, Jun. 1, 2003. DOI: 10.1016/S0097-8493(03)00038-4.

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