

Projective geometry and Panoramic image

Image Processing, Analysis and Classification

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Introduction

3D world → 2D content

An image is just a flat representation of the 3D world.



Even though we lose the depth component the image maintain the perspective.



Introduction

3D world → 2D content

Images are useful to store memories, but also we can extract information from them. For instance, we may want to know how tall is this bottle.





Introduction

3D world → 2D content

We need to represent the images in such a way that both humans and computers are able to deal with them. Thus matrix representation of images is defined. We can define an image $I \in M_{h,w}(\mathbb{R}^n)$ as:

$$I = \begin{pmatrix} p_{1,1} & p_{1,2} & \dots & p_{1,w} \\ p_{2,1} & p_{2,2} & \dots & p_{2,w} \\ \vdots & \vdots & \ddots & \vdots \\ p_{h,1} & p_{h,2} & \dots & p_{h,w} \end{pmatrix}_{h \times w}$$

Where each p_{ij} is a vector in \mathbb{R}^n , n varies depending on the number of color channels.



Introduction

3D world → 2D content

Here we can see how the image is codified as we have defined before:



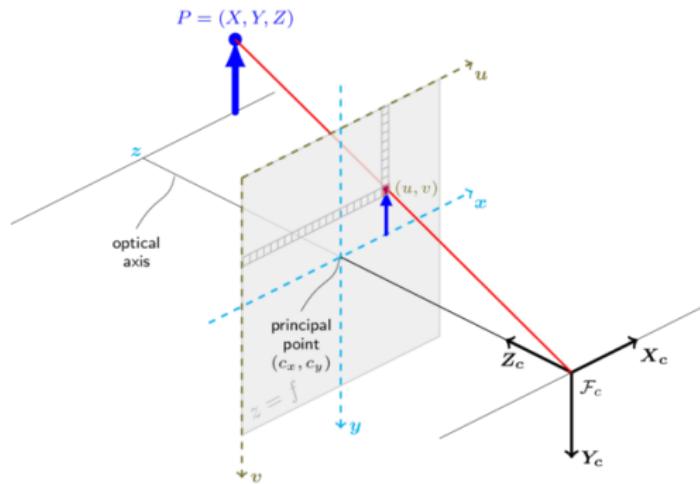


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Perspective

We can use angles to represent the perspective of a point in the image if we know the focal distance f of the camera.



$$\alpha = \arctan \frac{u}{f}$$



Perspective

Use case: Locate the place from where a photo was taken

We can use this information to locate where a photo was taken.

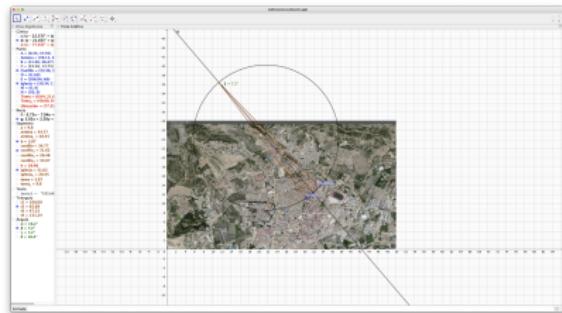
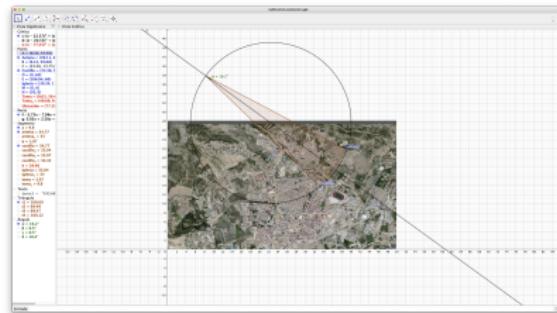




Perspective

Use case: Locate the place from where a photo was taken

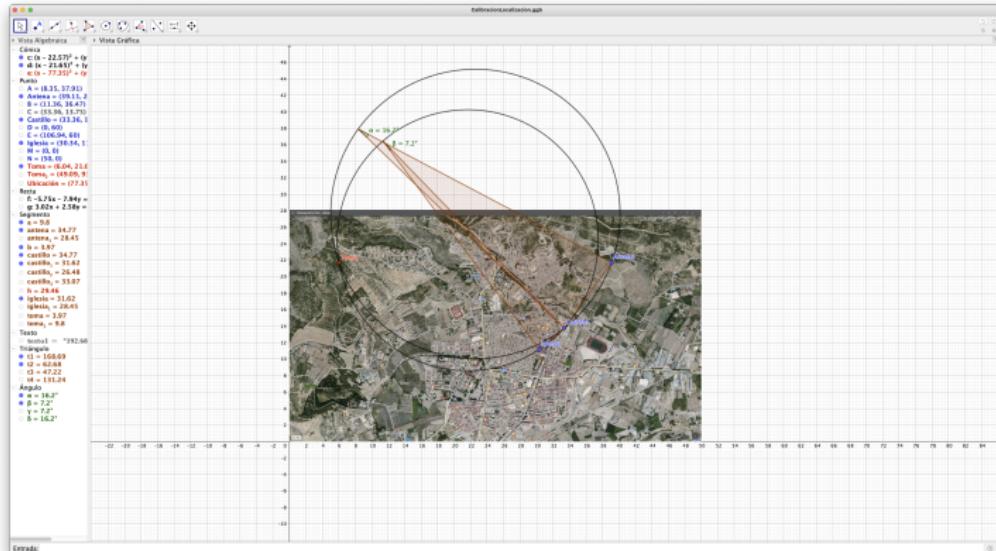
We only need to use some trigonometry.





Perspective

Use case: Locate the place from where a photo was taken



Perspective

Use case: Locate the place from where a photo was taken

The precision is quite accurate even though the methods are not so precise.

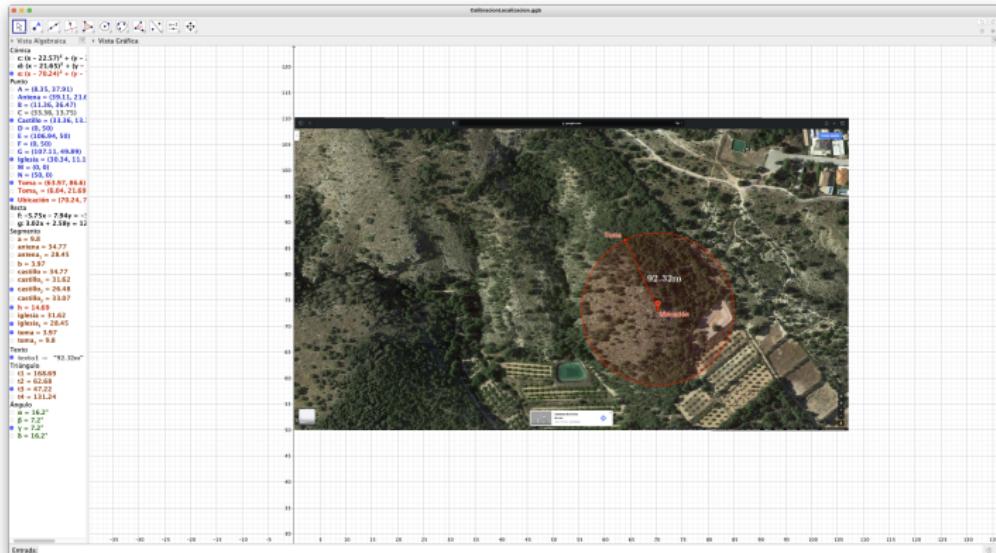


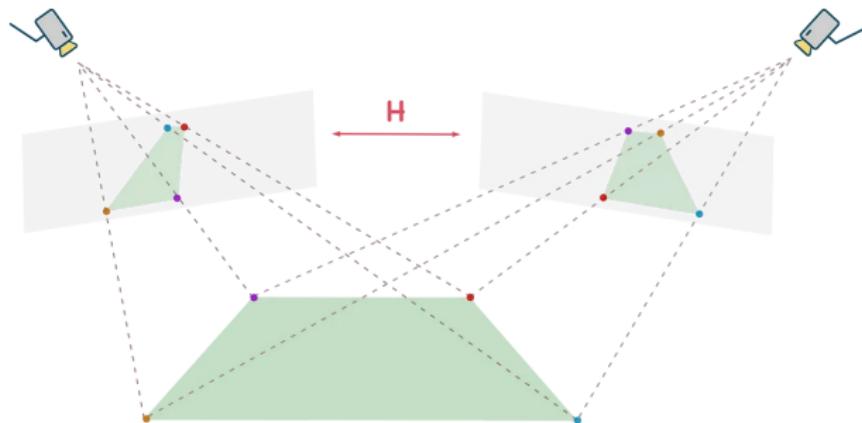


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Homographies

“The homograph is a mapping between two perspective images of a planar surface in a scene” [1]





Homographies

An homography is just a matrix that convert some points into others.

$$\lambda \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} H_{1,1} & H_{1,2} & H_{1,3} \\ H_{2,1} & H_{2,2} & H_{2,3} \\ H_{3,1} & H_{3,2} & H_{3,3} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



Homographies

Use case: How far was this shot?

By using homographies we can remove the perspective from an image, and after that we can measure in the image.





Homographies

Use case: How far was this shot?

By using homographies we can remove the perspective from an image.

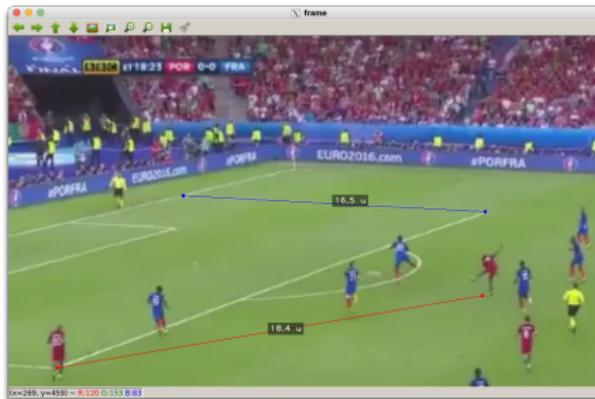




Homographies

Use case: How far was this shot?

We can establish measurements in the image.



Homographies

Use case: How far was this shot?

Using trigonometry, as before, we can estimate the shot distance.



Homographies

Use case: How far was this shot?

Using trigonometry, as before, we can estimate the shot distance.





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Panoramic image

Panoramic image reconstruction uses the techniques explained before.

Rotating camera, arbitrary world

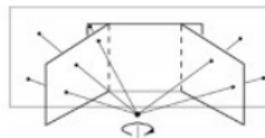
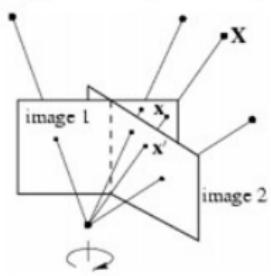


Figure: Panoramic image reconstruction [2]



Panoramic image

The key points have to be automatically calculated.

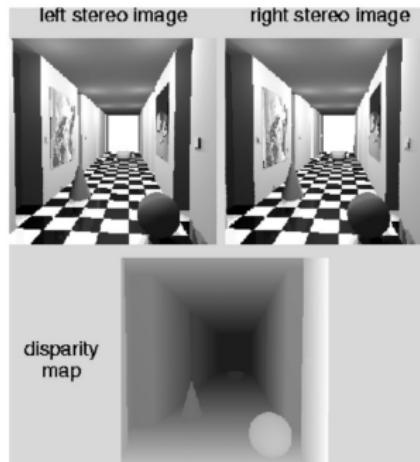


Figure: Stereo images and related disparity map [1]



Panoramic image

We can use techniques explain during the lessons (Key point feature extraction), such as SIFT, FAST, etc.

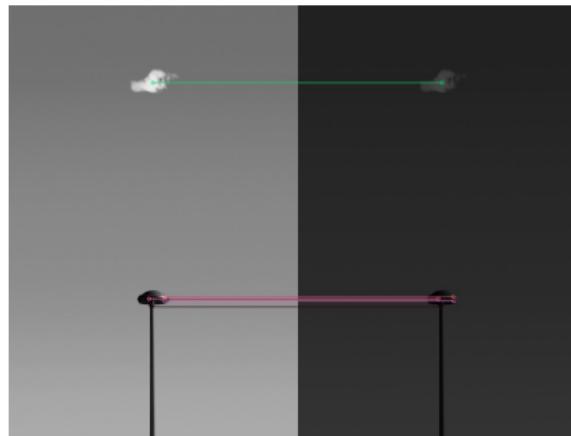


Figure: Key points from practical exercise



Panoramic image

If the key points are good enough selected, the results could be quite accurate.



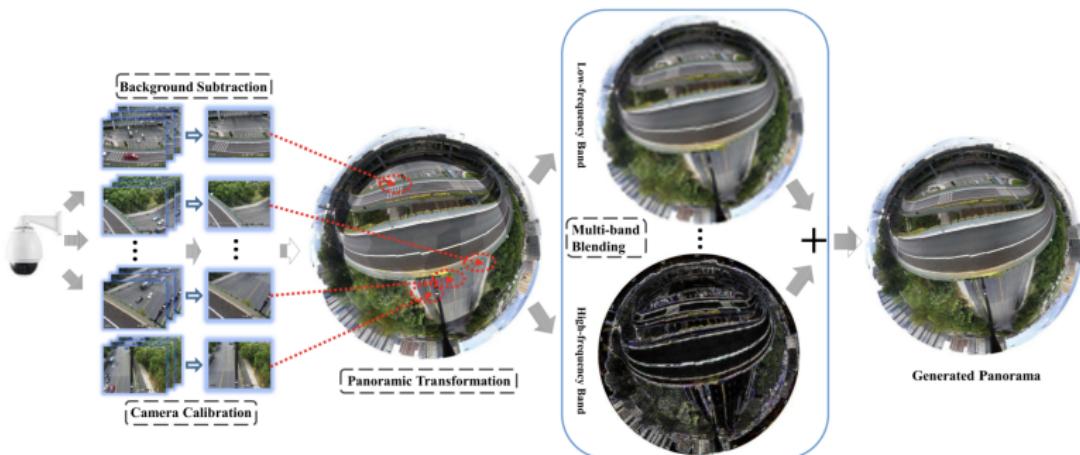
(a) Original images before the mosaic [1]



(b) Two images stitched together [1]

Panoramic image

This technique can also be used in the reconstruction of non-planar images. [3]





Panoramic image

Another example with a 360° image. [4]

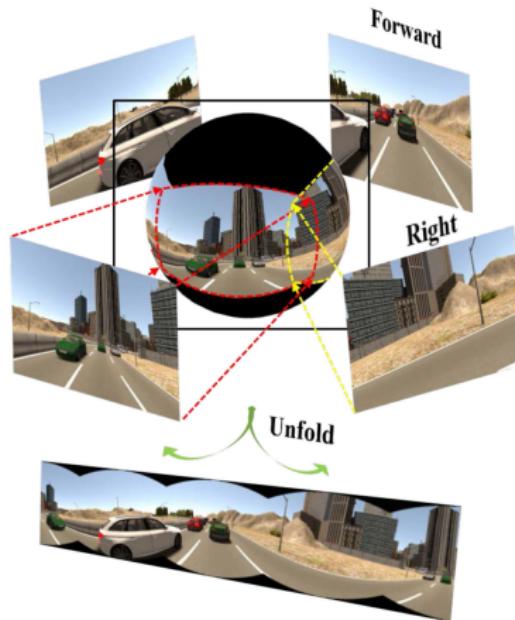




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Use case: Dental images

Problem description

In the context of dental health, as teeth are bones, the dentist needs to take X-rays.

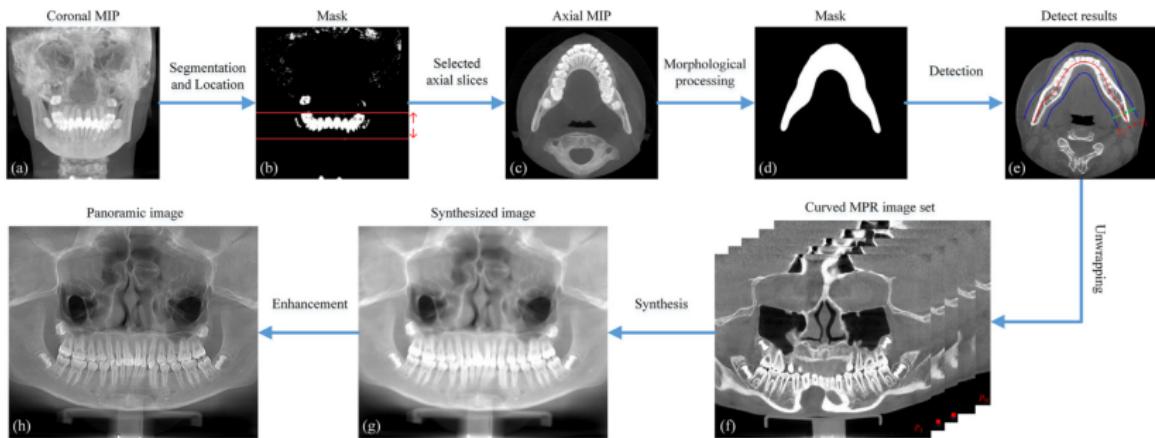


Figure: Some teeth hide others [5]

Use case: Dental images

Proposed solution

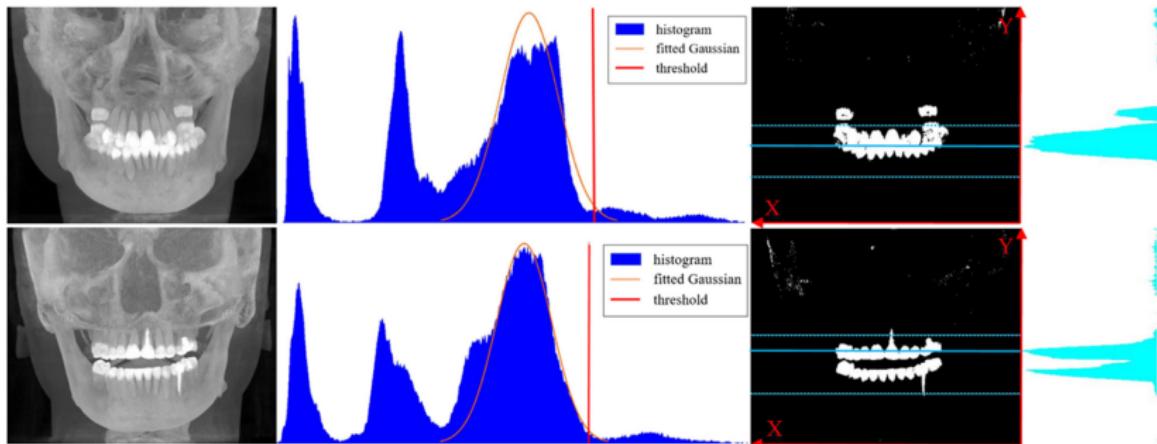
Yun *et al.* [5] came up with a solution based on the contents of this subject.



Use case: Dental images

Thresholding

They use histograms to establish a threshold value.

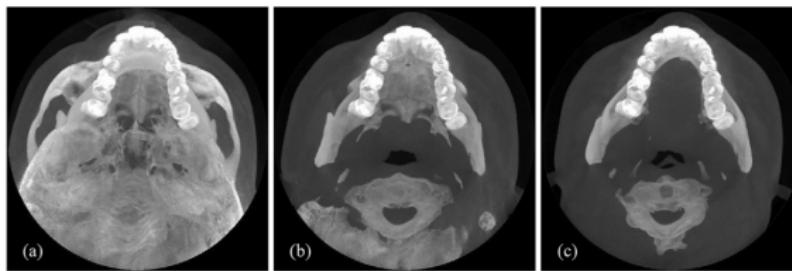




Use case: Dental images

Morphological processing

With the threshold calculated before, they managed to create a mask.

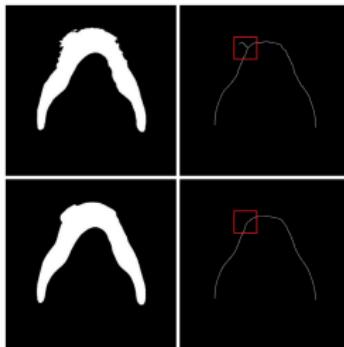




Use case: Dental images

Morphological processing

With the threshold calculated before, they managed to create a mask.



Also perform Gaussian filtering



Use case: Dental images

Arch approximation

Dental arch approximation using the following formula:

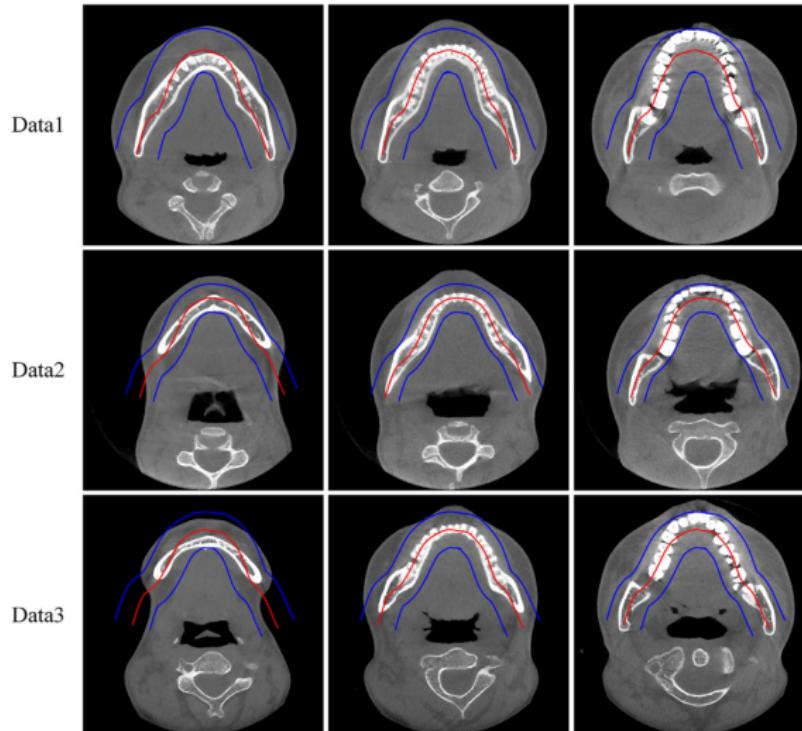
$$I_0(i,j) = S \cdot \log \left(\sum_{n=1}^N e^{\frac{P_n(i,j)}{S}} \right)$$



Use case: Dental images

Arch approximation

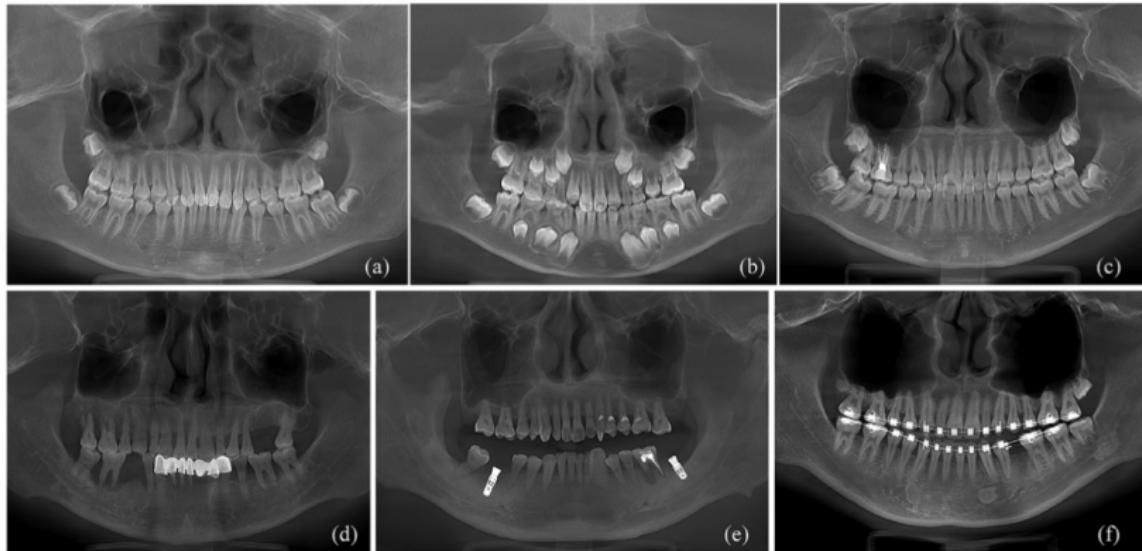
Dental arch approximation are quite accurate:



Use case: Dental images

Panoramic images generation

Using homographies and the arch estimation in each tooth, they are able to reconstruct a panoramic image of the whole teething.

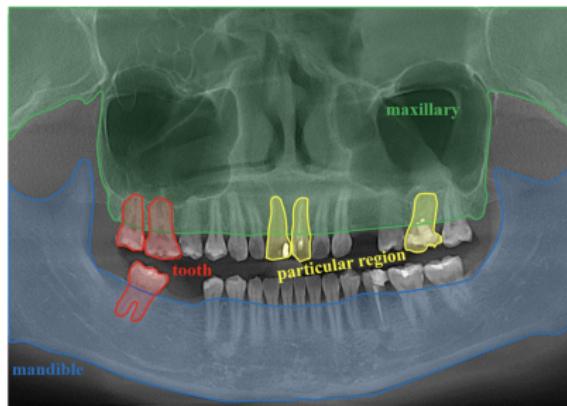




Use case: Dental images

Applications

Using techniques similar as the ones seen during the lessons, dentists can obtain automatic tooth identification.



Use case: Dental images

Known problems

One future path of this method is dealing with mental objects.

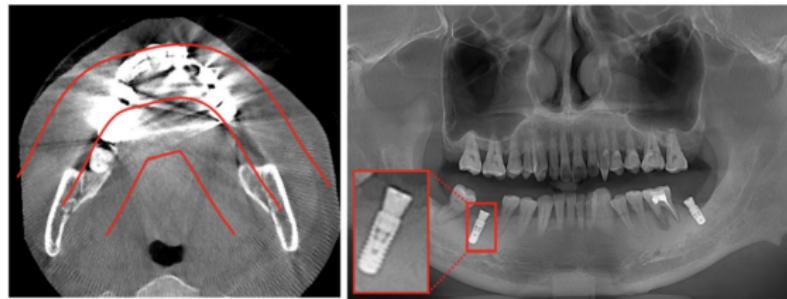




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