Universidad Politécnica de Madrid

Escuela Técnica Superior de Ingenieros Informáticos

Basic Exercises - Theoretical exercises

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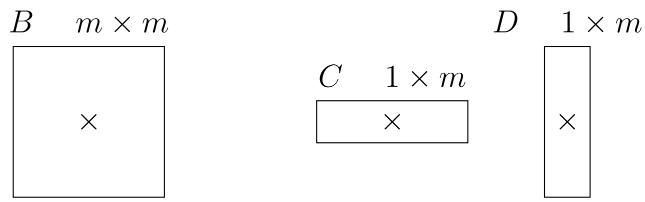
Contents

1 Exercise 2b

Estimate the number of operations of operations to process a $n \times n$ input image using: $\delta_B(I)$ or $\delta_C\left(\delta_D\left(I\right)\right)$.



First of all, it is defined the function $op(f)$ that counts the number of operations required to compute f. It is also defined that computing the maximum of two numbers $a, b \in \mathbb{Z}$ needs only 1 operation, thus, $op\left(\max\{a, b\}\right) = 1$.

Using these assumptions computing the maximum of n numbers (x_i) will require $n-1$ operations.

$$op(\underbrace{\max\{\cdots\max\{\max\{}_{\max\ \text{is repeated } n-1 \text{ times}} x_1, x_2\}, x_3\}, \cdots, x_n\}) = n-1$$

The number of operations required to compute $\delta_B(I)$ will be computed as the maximum among the structuring element B that has m^2 elements, so it will require $m^2 - 1$ operations. This is performed in each pixel of the image, that is n^2 times.

The number of operations required to compute $\delta_C\left(\delta_D\left(I\right)\right)$ will be computed as the maximum among the structuring element C that has m elements, so it will require $m-1$ operations, plus computing the maximum among the other structuring element D that will also require $m-1$ operations because it has m elements too. This is performed in each pixel of the image, that is n^2 times.

In conclusion,

$$n^2(m^2-1) = op\left(\delta_B(I)\right) \geq op\left(\delta_C\left(\delta_D\left(I\right)\right)\right) = n^2(m-1) + n^2(m-1) = 2n^2(m-1)$$

.

2 Exercise 9a

Proof the idempotence of the ‘`closing-opening`’ alternated filter.

To prove the idempotence of the closing-opening ($\gamma_B\varphi_B$) we first need to assume some statements:

- Both γ_B (opening) and φ_B (closing) are increasing and idempotent.
- γ_B (opening) is antiextensive.
- φ_B (closing) is extensive.

Theorem 1. *Idempotence of the ‘closing-opening’ alternated filter:*

$$\gamma_B\varphi_B\gamma_B\varphi_B(I) = \gamma_B\varphi_B(I)$$

Proof. The proof is done showing that both of the inequalities are satisfied.

$\boxed{\gamma_B\varphi_B\gamma_B\varphi_B(I) \leq \gamma_B\varphi_B(I)}$

$$\begin{aligned} \gamma_B\varphi_B(I) = \gamma_B\varphi_B(I) \Rightarrow \\ \gamma_B\varphi_B\varphi_B\varphi_B(I) = \gamma_B\varphi_B(I) \Rightarrow \\ \gamma_B\varphi_B\gamma_B\varphi_B(I) \leq \gamma_B\varphi_B(I) \end{aligned} \tag{1}$$

$\boxed{\gamma_B\varphi_B\gamma_B\varphi_B(I) \geq \gamma_B\varphi_B(I)}$

$$\begin{aligned} \gamma_B\varphi_B(I) = \gamma_B\varphi_B(I) \Rightarrow \\ \gamma_B\gamma_B\gamma_B\varphi_B(I) = \gamma_B\varphi_B(I) \Rightarrow \\ \gamma_B\varphi_B\gamma_B\varphi_B(I) \geq \gamma_B\varphi_B(I) \end{aligned} \tag{2}$$

By (1) and (2) it can be concluded that the equality $\gamma_B\varphi_B\gamma_B\varphi_B(I) = \gamma_B\varphi_B(I)$ holds. $\square$