Universidad Politécnica de Madrid

Escuela Técnica Superior de Ingenieros Informáticos
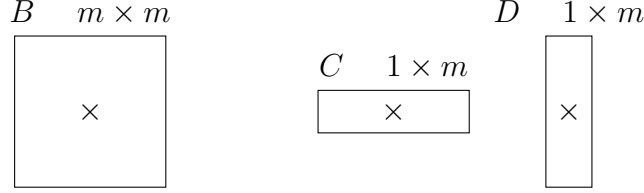
Basic Exercises - Theoretical exercises

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Exercise 2d

Estimate the number of operations of operations to process a $n \times n$ input image using: $\delta_B(I)$ **or** $\delta_C\left(\delta_D\left(I\right)\right)$**.**

$$B \quad m \times m$$

$$C \quad 1 \times m$$

$$D \quad 1 \times m$$

First of all, it is defined the function $op(f)$ that counts the number of operations required to compute f. It is also defined that computing the maximum of two numbers $a, b \in \mathbb{Z}$ needs only 1 operation, thus, $op\left(\max\{a, b\}\right) = 1$.

Using these assumptions computing the maximum of n numbers (x_i) will require $n-1$ operations.

$$op(\underbrace{\max\{\cdots\max\{\max\{}_{\text{max is repeated } n-1 \text{ times}} x_1, x_2\}, x_3\}, \cdots, x_n\}) = n - 1$$

The number of operations required to compute $\delta_B(I)$ will be computed as the maximum among the structuring element B that has m^2 elements, so it will require $m^2 - 1$ operations. This is performed in each pixel of the image, that is n^2 times.

The number of operations required to compute $\delta_C\left(\delta_D\left(I\right)\right)$ will be computed as the maximum among the structuring element C that has m elements, so it will require $m - 1$ operations, plus computing the maximum among the other structuring element D that will also require $m - 1$ operations because it has m elements too. This is performed in each pixel of the image, that is n^2 times.

In conclusion,

$$n^2(m^2 - 1) = op\left(\delta_B(I)\right) \geq op\left(\delta_C\left(\delta_D\left(I\right)\right)\right) = n^2(m - 1) + n^2(m - 1) = 2n^2(m - 1)$$
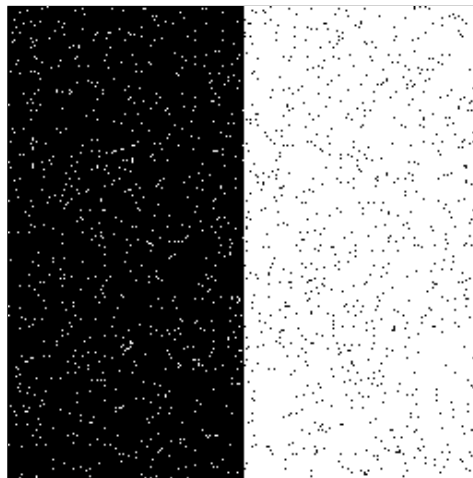
Exercise 8a

Let I be the input image in file `isn_256.pgm`, which has a binary impulsive noise added("salt-and-pepper" noise). Let B be a structuring element square of size 3×3.

The results of executing the respective filters can be seen in Fig. 1. The best results are obtained using Filter 4 (Opening-Closing $\gamma\varphi$) where it is easy to see there is only one artifact left in the image (on top of the black part).

The second best filter is Filter 3 (Closing-Opening $\varphi\gamma$) where a few more artifacts remain in the image, in this case, in the white part.
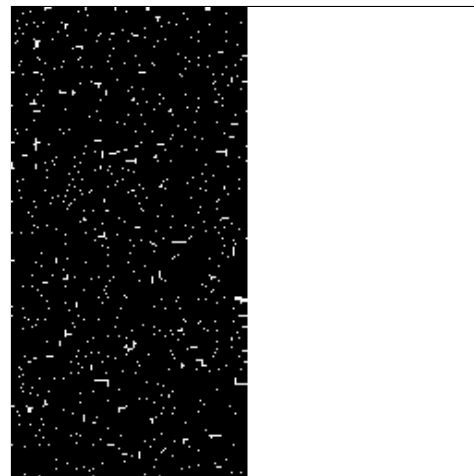
As we can see in Filters 1 (Opening γ) and 2 (Closing φ), each operation removes artifacts from one of the parts, and thus, the combinations of both are the ones that obtain better results.
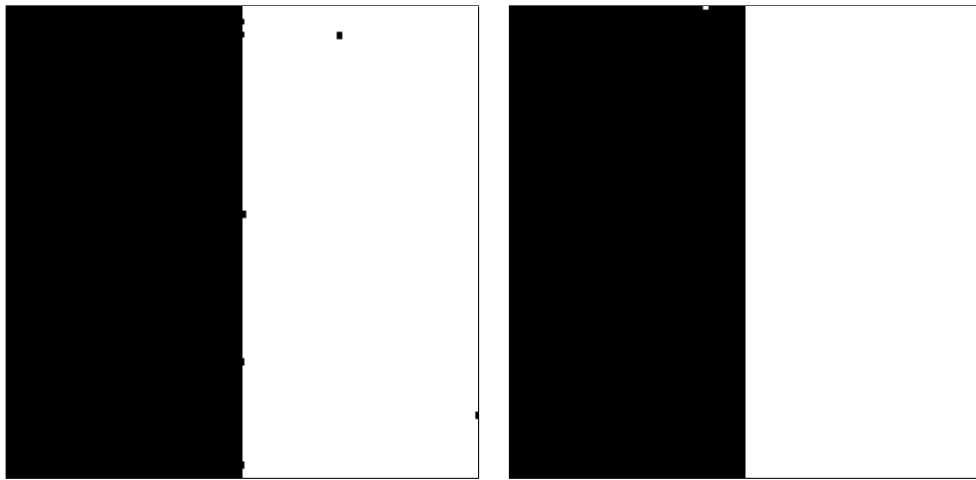

Orignal image: `isn_256.pgm`
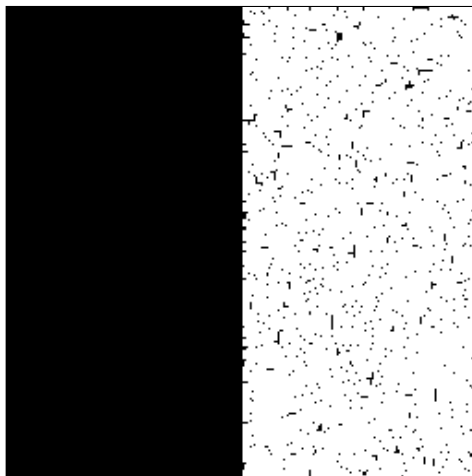
Filter 1: Opening $\gamma_B(I)$

Filter 2: Closing $\varphi_B(I)$

Filter 3: Closing-Opening $\varphi_B \gamma_B(I)$ Filter 4: Opening-Closing $\gamma_B \varphi_B(I)$

Figure 1: Images from Exercise 8a

Exercise 9a

Proof the idempotence of the 'closing–opening' alternated filter.

To prove the idempotence of the closing-opening ($\varphi_B \gamma_B$) we first need to assume some statements:

- Both γ_B (opening) and φ_B (closing) are increasing and idempotent.

- γ_B (opening) is anti-extensive.

- φ_B (closing) is extensive.

Theorem 1. *Idempotence of the 'closing-opening' alternated filter:*

$$\varphi_B \gamma_B \varphi_B \gamma_B (I) = \varphi_B \gamma_B (I)$$

Proof. The proof is done showing that both of the inequalities are satisfied.

$\boxed{\varphi_B \gamma_B \varphi_B \gamma_B (I) \leq \varphi_B \gamma_B (I)}$

$$
\begin{aligned}
\varphi_B \gamma_B (I) &= \varphi_B \gamma_B (I) \Rightarrow \\
\varphi_B \varphi_B \varphi_B \gamma_B (I) &= \varphi_B \gamma_B (I) \Rightarrow \\
\varphi_B \gamma_B \varphi_B \gamma_B (I) &\leq \varphi_B \gamma_B (I)
\end{aligned}
\tag{1}
$$

$\boxed{\varphi_B \gamma_B \varphi_B \gamma_B (I) \geq \varphi_B \gamma_B (I)}$

$$
\begin{aligned}
\varphi_B \gamma_B (I) &= \varphi_B \gamma_B (I) \Rightarrow \\
\varphi_B \gamma_B \gamma_B \gamma_B (I) &= \varphi_B \gamma_B (I) \Rightarrow \\
\varphi_B \gamma_B \varphi_B \gamma_B (I) &\geq \varphi_B \gamma_B (I)
\end{aligned}
\tag{2}
$$

By (1) and (2) it can be concluded that the equality $\varphi_B \gamma_B \varphi_B \gamma_B (I) = \varphi_B \gamma_B (I)$ holds.

$\square$