

AI LAB 3 | [VND, Beam Search, Tabu search]

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Brief Description:

State space :

Every state is defined using class having literal values. Each variable is either 0 or 1.
In case of tabu search state representation is modified a bit and one more list to store tabu tenure values is added to state as below.

Variable Neighbourhood Descent(VND) Beam Search	<u>State:</u> ([a,b,c,d])
Tabu Search	<u>State:</u> ([a,b,c,d], [M1,M2,M3,M4])

Example: initial state with all literal equal to 0 and all literals are allowed to change is represented as : ([0,0,0,0]) for VND and beam search and ([0,0,0,0], [0,0,0,0]) for tabu search algorithm.

Start node:

We are starting with all literal values 0 and it is allowed to change any literal in the next move.

The state is represented as starting state: ([0,0,0,0])

In case of tabu search, initial tabu tenure list values are 0 for all literals and state is represented as starting state: ([0,0,0,0],[0,0,0,0])

Goal Node:

Goal node is a node with a literal list [a,b,c,d] such that boolean values of a,b,c,d when put into a given 4-SAT expression returns 1 i.e it satisfies the 4-SAT CNF expression.

MoveGen function

Pseudo code for the moveGen function is as below.

```
movGen():
    neighbours = []

    for each variable:
        change it's bit to negation of it (i.e change 0 to 1 and 1 to 0)
        add new node to neighbours

    return neighbours
```

Variable Neighbourhood Descent movGen

MovGen function for Variable Neighbourhood descent has n components for n-SAT formulas, each perturbing k number of bits and $k \in \{1, 2, 3, \dots, n\}$

```
movGen: #for VND
    neighbours = []
    index_list = #list containing all possible combinations of k indices

    for each combination in index_list:
        change all bits (i.e take negation) at each position in combination
        add resultant Node to neighbours

    return neighbours
```

Tabu Search movGen

```
movgen(): #Tabu search
    Define list neighbours to store possible moves
    for each literal in node(a,b,c,d):
        if M_literal is 0: ## i.e if hasn't been changed recently
            Change the value of literal ##change 0 to 1 and 1 to 0
            Change M_literal value to tabu_tenure
            Decrease all other M_x values to  $\max((M_x - 1), 0)$ 
        add modified node to neighbours list
    return neighbours
```

GoalTest():

Pseudo code for goalTest is as below.

```
goalTest():
    If [a,b,c,d] satisfies CNF expression :
        return true
    else :
        return false
```

Heuristic Function considered

Heuristic Function returns an integer equal to the total number of clauses satisfied in the formula.

Input Generation

Generated clauses are of the form $(_v_v_)^{\wedge}(_v_v_)^{\wedge}(_v_v_)^{\wedge}(_v_v_)^{\wedge}(_v_v_)$ with 4 variables a,b,c,d and 8 boolean values to choose from [a,~a,b,~b,c,~c,d,~d], each clause containing 3 literals and total 5 clauses.

Generating output :

- Run make file using command ``$make testcases`` you will be asked to type in total testcases to generate.
- All generated formulas will be stored in testcases.txt file in the same directory, one string per line as below.

```
1  (bvdva)^(~dvcvb)^(~avdvc)^(~bvav~d)^(~bv~dv~a)
2  (~avcv~d)^(avbvd)^(~cv~dv~a)^(~cv~av~b)^(~av~bv~d)
3  (dvbvc)^(avdvc)^(bv~dv~c)^(dv~avc)^(~cv~dv~a)
4  (~bvav~c)^(~dvavb)^(~cvbva)^(~av~bv~d)^(~dvcv~a)
5  (cv~avd)^(~bv~avd)^(~bv~avd)^(~bv~cv~a)^(~avdv~b)
6  (~cv~bv~a)^(bvavd)^(cvbv~d)^(cv~avd)^(cv~dva)
7  (dv~bva)^(dvbvc)^(~cvavb)^(~cvav~b)^(bvdvc)
```

To generate expressions with 4 literals per clauses, run testcase_generate-4cnf.py

Beam search analysis for different beam lengths

- When expressions have three literals in each clause and there are total 5 clauses, most of the expressions are getting satisfied in 0 or 1 iterations and very few takes 2 iterations.
- When analysed for different beam length, there is no change in total number of iterations required as irrespective of beam length expressions find solutions in the first move.
- Iterations required are tabulated for different beam lengths which are the same in this case.

Tabu search for different values of tabu tenure

- When expressions have three literals in each clause and there are total 5 clauses, most of the expressions are getting satisfied in 0 or 1 iterations and very few takes 2 iterations.
- When analysed for different tabu tenure, there is no change in total number of iterations required as irrespective of tabu tenure expressions find solutions in the first move.
- Iterations required are tabulated for different tabu tenures which are the same in this case.

Comparison of Variable neighborhood descent, Beam Search, Tabu Search: Nodes explored by each

- When expressions have three literals in each clause and there are total 5 clauses, most of the expressions are getting satisfied in 0 or 1 iterations and very few takes 2 iterations.
- All three algorithms explores 1 (i.e starting node) when initiated state is solution
- All three algorithms explores 5 (i.e starting node + 4 nodes with one bit perturbed) when solution is found in first iterations
- For few of the expressions solution is found in second iterations in that case
-

- **VND explores only 3 nodes** (i.e starting node, one node at level one and third solution node)

- **Beam search explores 4 nodes** (i.e starting node, two nodes at level two and third solution node) as the **beam width is increased** algorithm explores **more** nodes in vain.

- **Tabu search explores 3 nodes** (i.e starting node, one node at level one and third solution node)

Tabulating values:

<https://docs.google.com/spreadsheets/d/1T7h70UADdVpgBV2loeKQQR5rljc-8gBSGCT6WVv7YYY/edit?usp=sharing> (full table could be found at the link above)

M12	A	B	C	D	E	F	G	H	I	J	K	L
1	Expression	VND	1	2	3	4	6	8	1	2	3	4
3	(bvdva)^(dvcvb)^(avdvc)^(bvav-d)^(bv-dv-a)	1	1	1	1	1	1	1	1	1	1	1
4	(-avcv-d)^(avbvd)^(cv-dv-a)^(cv-av-b)^(av-bv-d)	1	1	1	1	1	1	1	1	1	1	1
5	(dvbvc)^(avdvc)^(bv-dv-c)^(dv-avc)^(cv-dv-a)	1	1	1	1	1	1	1	1	1	1	1
6	(-bvav-c)^(dvavb)^(cvbva)^(av-bv-d)^(dvcv-a)	0	0	0	0	0	0	0	0	0	0	0
7	(cv-avd)^(bv-avd)^(bv-avd)^(bv-cv-a)^(avdv-b)	0	0	0	0	0	0	0	0	0	0	0
8	(-cv-bv-a)^(bvavd)^(cvbv-d)^(cv-avd)^(cv-dva)	1	1	1	1	1	1	1	1	1	1	1
9	(dv-bva)^(dvbvc)^(cvavb)^(cvav-b)^(bvdvc)	1	1	1	1	1	1	1	1	1	1	1
10	(-bvdva)^(dvcvb)^(dv-av-c)^(bvav-d)^(av-bvd)	1	1	1	1	1	1	1	1	1	1	1
11	(-cvbv-d)^(dv-av-b)^(cvbva)^(avcvc)^(avdv-b)	1	1	1	1	1	1	1	1	1	1	1
12	(-dvbva)^(bvcva)^(av-bv-d)^(dvav-b)^(avdv-c)	1	1	1	1	1	1	1	1	1	1	1
13	(cvbvd)^(dv-cv-a)^(dvavb)^(bvcv-d)^(av-cv-b)	1	1	1	1	1	1	1	1	1	1	1
14	(-avdvc)^(cv-av-d)^(bv-cv-a)^(av-cv-d)^(bvav-d)	0	0	0	0	0	0	0	0	0	0	0
15	(dv-av-b)^(avbv-c)^(avdv-b)^(av-cv-d)^(cv-dv-a)	0	0	0	0	0	0	0	0	0	0	0
16	(-av-bv-c)^(cv-bv-d)^(av-cv-b)^(avdvc)^(avdvb)	1	1	1	1	1	1	1	1	1	1	1
17	(-dv-bv-c)^(cvbv-d)^(avdvc)^(bv-dv-c)^(avdv-b)	0	0	0	0	0	0	0	0	0	0	0
18	(-cv-bv-a)^(dvcvb)^(bvdva)^(avdvc)^(cvbva)	1	1	1	1	1	1	1	1	1	1	1
19	(-dvbv-c)^(bv-cv-a)^(dv-av-b)^(cv-avd)^(bv-cva)	0	0	0	0	0	0	0	0	0	0	0
20	(cvav-d)^(cvav-d)^(cvav-d)^(cv-bv-d)^(avbv-d)	0	0	0	0	0	0	0	0	0	0	0
21	(dvcva)^(cvav-d)^(cvav-d)^(cvav-d)^(av-bv-d)	1	1	1	1	1	1	1	1	1	1	1
22	(-avdvc)^(av-bv-d)^(av-dv-c)^(dvav-b)^(dvavc)	1	1	1	1	1	1	1	1	1	1	1
23	(cvavd)^(dv-bv-c)^(dv-bv-c)^(avdvc)^(cv-dva)	1	1	1	1	1	1	1	1	1	1	1
24	(avbvd)^(dv-bva)^(avcvc)^(bv-avd)^(avdv-b)	1	1	1	1	1	1	1	1	1	1	1
25	(cvdv-a)^(bv-cv-a)^(dvav-c)^(cv-av-d)^(avbvd)	1	1	1	1	1	1	1	1	1	1	1
26	(-cv-av-d)^(dv-avc)^(avcvc)^(dv-bva)^(cvdv-a)	1	1	1	1	1	1	1	1	1	1	1
27	(-dv-bv-c)^(av-cv-d)^(cv-dv-a)^(av-bvd)^(bv-cv-d)	0	0	0	0	0	0	0	0	0	0	0

Almost similar result is obtained when each clause contains 4 literals (4SAT) from 8

Result is tabulated below.

<https://docs.google.com/spreadsheets/d/1T7h70UADdVpgBV2loeKOQRR5rljc-8gBSGCT6WVy7YY/edit?usp=sharing>

	A	B	C	D	E	F	G	H	I	J
1			Iterations Required (for tabu tenures listed in columns)						Beam Search	
2	Expression	VND	1	2	3	4	6	8	2	3
3	(~avbvcv-d)^(~avbv-cv-d)^(av~bv-cvd)^(~avbvcvd)^(av~bvcvd)	0	0	0	0	0	0	0	0	0
4	(avbv~cvd)^(~av~bvcv-d)^(avbv~cv-d)^(~av~bvcv-d)^(~av~bvcvd)	0	0	0	0	0	0	0	0	0
5	(~av~bvcv-d)^(av~bv-cv-d)^(~avbv~cvd)^(~av~bv-cv-d)^(~avbv~cv-d)	0	0	0	0	0	0	0	0	0
6	(~av~bv~cvd)^(av~bv~cv-d)^(avbvcvd)^(~av~bv~cv-d)^(~av~bvcv-d)	1	1	1	1	1	1	1	1	1
7	(av~bv~cvd)^(avbvcvd)^(~av~bvcvd)^(av~bv~cvd)^(av~bvcvd)	1	1	1	1	1	1	1	1	1
8	(~av~bv~cvd)^(av~bvcvd)^(av~bvcvd)^(~av~bvcvd)^(avbvcvd)	1	1	1	1	1	1	1	1	1
9	(av~bvcvd)^(~avbv~cvd)^(avbv~cvd)^(av~bvcvd)^(~av~bv~cv-d)	0	0	0	0	0	0	0	0	0
10	(avbvcvd)^(~avbv~cv-d)^(av~bvcvd)^(avbvcvd)^(avbvcv-d)	1	1	1	1	1	1	1	1	1
11	(~av~bv~cv-d)^(~av~bv~cv-d)^(avbvcvd)^(avbv~cvd)^(~av~bv~cv-d)	1	1	1	1	1	1	1	1	1
12	(~avbv~cvd)^(~avbv~cv-d)^(avbv~cvd)^(~av~bvcv-d)^(~av~bv~cvd)	0	0	0	0	0	0	0	0	0
13	(avbvcvd)^(avbvcvd)^(avbvcvd)^(av~bvcvd)^(avbv~cvd)	1	1	1	1	1	1	1	1	1
14	(~avbvcv-d)^(~av~bv~cv-d)^(avbv~cvd)^(~av~bv~cvd)^(~avbvcvd)	0	0	0	0	0	0	0	0	0
15	(~av~bvcv-d)^(~av~bvcvd)^(avbv~cv-d)^(avbvcvd)^(~av~bvcvd)	1	1	1	1	1	1	1	1	1
16	(~avbv~cv-d)^(avbvcvd)^(av~bvcvd)^(av~bv~cv-d)^(av~bvcv-d)	1	1	1	1	1	1	1	1	1
17	(av~bv~cv-d)^(av~bv~cv-d)^(av~bvcvd)^(avbv~cvd)^(av~bv~cvd)	0	0	0	0	0	0	0	0	0
18	(avbvcv-d)^(av~bvcv-d)^(~avbv~cv-d)^(~av~bv~cv-d)^(av~bvcv-d)	0	0	0	0	0	0	0	0	0
19	(~avbv~cv-d)^(av~bv~cv-d)^(~avbv~cvd)^(~av~bvcvd)^(av~bv~cvd)	0	0	0	0	0	0	0	0	0
20	(~avbvcvd)^(av~bv~cvd)^(~av~bvcv-d)^(avbv~cv-d)^(av~bv~cvd)	0	0	0	0	0	0	0	0	0
21	(~avbv~cvd)^(av~bv~cv-d)^(~av~bvcvd)^(av~bv~cv-d)^(avbvcv-d)	0	0	0	0	0	0	0	0	0
22	(av~bv~cvd)^(~av~bv~cvd)^(~avbvcv-d)^(~av~bv~cvd)^(~avbvcvd)	0	0	0	0	0	0	0	0	0
23	(~av~bv~cv-d)^(~av~bvcv-d)^(~av~bvcvd)^(~av~bvcv-d)^(~av~bvcv-d)	0	0	0	0	0	0	0	0	0
24	(~avbv~cvd)^(~avbv~cv-d)^(avbv~cv-d)^(~av~bvcv-d)^(~av~bv~cv-d)	0	0	0	0	0	0	0	0	0
25	(avbv~cvd)^(~avbv~cvd)^(~avbvcv-d)^(~av~bv~cvd)^(~av~bvcv-d)	0	0	0	0	0	0	0	0	0
26	(av~bv~cv-d)^(~av~bvcvd)^(av~bvcvd)^(avbvcvd)^(av~bv~cvd)	1	1	1	1	1	1	1	1	1
27	(avbv~cv-d)^(~av~bv~cvd)^(av~bvcvd)^(avbv~cvd)^(avbv~cv-d)	0	0	0	0	0	0	0	0	0

7 Variables - 5 clauses - four literals per clause

<https://docs.google.com/spreadsheets/d/1T7h70UADdVpgBV2loeKOQRR5rljc-8gBSGCT6WVy7YY/edit?usp=sharing>

[illegible]

7 literals - 5 clauses - 4 literals per clause - negation not included for each variable

<https://docs.google.com/spreadsheets/d/1T7h70UADdVpgBV2loeKOQRR5rljc-8gBSGCT6WVy7YY/edit?usp=sharing>

	A	B	C	D	E	F	G	H	I	J	K	L	M
1			Tabu tenures						Beam Widths				
2	Expression	VND	1	2	3	4	6	8	1	2	3	4	6
3	(evdfrfb)^(evcbvba)^(bvbvgvd)^(bvcvfcf)^(avavavc)	2	2	2	2	2	2	2	2	2	2	2	2
4	(bvdveva)^(cvfvdvg)^(avgvdc)^(cvgvgvg)^(bvavavc)	2	2	2	2	2	2	2	2	2	2	2	2
5	(dvgvbe)^(fvevevc)^(dvfcve)^(bvfvdvf)^(bvbvava)	2	2	2	2	2	2	2	2	2	2	2	2
6	(dvavevb)^(dvgvdvg)^(bvbvgva)^(gfbvfrd)^(dvdvbc)	2	2	2	2	2	2	2	2	2	2	2	2
7	(cvfvrfb)^(cvbvdvd)^(bvbvave)^(dvbcve)^(fvgvgvd)	2	2	2	2	2	2	2	2	2	2	2	2
8	(avavgvc)^(fbvgve)^(dvevgvg)^(cvdvdvd)^(dvgvgvd)	2	2	2	2	2	2	2	2	2	2	2	2
9	(bvgvfrd)^(cvavcve)^(cvevbvc)^(avfvavf)^(bvcvdc)	2	2	2	2	2	2	2	2	2	2	2	2
10	(cvbvava)^(evevdc)^(cvdvgb)^(dvdvfg)^(gvgvgvc)	2	2	2	2	2	2	2	2	2	2	2	2
11	(evavcb)^(dvevag)^(fbvbvc)^(evfvevc)^(evbvca)	2	2	2	2	2	2	2	2	2	2	2	2
12	(gvevavf)^(evbvave)^(cvgvfc)^(cvgvbve)^(gfvvcvg)	2	2	2	2	2	2	2	2	2	2	2	2
13	(evcbvba)^(bvevavf)^(gvcvdc)^(cvfvdve)^(fvfvcvd)	2	2	2	2	2	2	2	2	2	2	2	2
14	(cvbvfva)^(bvfvrvc)^(bvdvevg)^(bvbvcva)^(cvavdvb)	1	1	1	1	1	1	1	1	1	1	1	1
15	(avbvca)^(cvcvfrd)^(fvevcvc)^(bvbvevd)^(dvevgvd)	2	2	2	2	2	2	2	2	2	2	2	2
16	(evgvgvg)^(bvdvavg)^(cvavdv)^(dvavde)^(cvevcvc)	2	2	2	2	2	2	2	2	2	2	2	2
17	(evbvdvd)^(gfvavg)^(evevgva)^(avbvgvd)^(bvevbva)	2	2	2	2	2	2	2	2	2	2	2	2
18	(cvcvcvc)^(dvgvfrf)^(dvdvgvd)^(dvgvbvd)^(evgvgvf)	2	2	2	2	2	2	2	2	2	2	2	2
19	(avfvge)^(cvevdva)^(bvcvdrf)^(avbvvc)^(avavevf)	2	2	2	2	2	2	2	2	2	2	2	2
20	(fvdvgvb)^(bvdvevc)^(cvdvca)^(dvcvfrg)^(dvfeva)	1	1	1	1	1	1	1	1	1	1	1	1
21	(evavrb)^(evfvbg)^(dvbvdvf)^(fvfrd)^(dvcvgvg)	2	2	2	2	2	2	2	2	2	2	2	2
22	(cvdvvgc)^(fvgvcvd)^(fvcvcve)^(dvcvdvg)^(fvevavd)	2	2	2	2	2	2	2	2	2	2	2	2
23	(dvevdvg)^(fbvavg)^(dvfrd)^(avdvavf)^(bvgveve)	2	2	2	2	2	2	2	2	2	2	2	2
24	(bvbvevd)^(fvcvbvc)^(gvevdvg)^(cvcvgva)^(bvgvavf)	2	2	2	2	2	2	2	2	2	2	2	2
25	(bvdvdc)^(avgvfrg)^(cvgvrb)^(evbvave)^(gvavbvc)	2	2	2	2	2	2	2	2	2	2	2	2
26	(evcavg)^(fvdveve)^(gfbvdvf)^(bvdvdc)^(avbvgvf)	2	2	2	2	2	2	2	2	2	2	2	2
27	(fvbfrd)^(gvavevb)^(cvdvvg)^(fvdvevc)^(avbvfva)	2	2	2	2	2	2	2	2	2	2	2	2

Conclusion:

Total Variables	Performance Comparison
4	All three algorithms perform the same irrespective of tabu tenure or beam width.
7	Even with 7 variables all algorithms performed almost similar and thus optimal tabu tenure is 1 and beam width is 1.
7 (no negation allowed)	To introduce more strict criteria, negations were not included in expressions i.e literals were randomly drawn from (a,b,c,d,e,f,g) only. Total number of iterations to reach the goal state increased compared to cases when negations were taken into consideration.

Thus optimal tabu tenure is 1 and optimal beam width is 1 for SAT problems with 4 variables.