1.1 **(a)** B, E, F, H, I

(b) B, E, F, H, I

(c) B, E, F, I

(d) E, F, I

1.2 **(a)** $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$

1.3 **(a)** $\begin{bmatrix} 4 & 1 & 5 \\ -2 & 1 & -1 \end{bmatrix}$

(b) $\begin{bmatrix} 8 & 2 & 10 \\ -4 & 2 & -2 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 1 & -4 \\ 2 & -1 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 3 & 2 & -15 \\ 11 & 2 & -2 \end{bmatrix}$

1.4 $X = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & 2 \\ 2 & 2 & 3 \end{bmatrix}$

1.5 $AB = [-1]$, $BA = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 2 & -1 \\ 3 & 6 & -3 \end{bmatrix}$

1.6 $(AB)_{23} = 7$, $(BA)_{12} = 7$, $(AB)_{22} = 11$, $(BA)_{22} = 8$

1.7 **(a)** $\begin{bmatrix} 2 & 5 \end{bmatrix}$

(b) Não está definido

(c) $\begin{bmatrix} -2 & 2 & 1 \\ 1 & -1 & 0 \\ -2 & 2 & 2 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix}$

1.12 $\left\{\begin{bmatrix} \alpha & \beta \\ 0 & \alpha \end{bmatrix} : \alpha, \beta \in \mathbb{R}\right\}$

1.15 Se $D = \operatorname{diag}(d_1, \ldots, d_n)$ então $D^k = \operatorname{diag}\left({d_1}^k, \ldots, {d_n}^k\right)$

1.16 **(a)** Por exemplo, $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(b) Por exemplo, $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

(c) Por exemplo, $A = \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$ e $B = \begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix}$

1.20 **(a)** Elementos da diagonal principal não nulos

(b) Se $D = \operatorname{diag}(d_1, \ldots, d_n)$ então $D^{-1} = \operatorname{diag}\left({d_1}^{-1}, \ldots, {d_n}^{-1}\right)$

1.21 **(a)** Por exemplo, para $n = 2$, $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

(b) Por exemplo, para $n = 2$, $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

1.22 **(a)** $A^{-1} = A^2$

(b) $A^{-1} = A + 2I_n$

(c) $A^{-1} = -\frac{1}{\beta}(A + \alpha I_n)$

1.25 **(a)** $X = A^{-1}C$

(b) $X = A^{-1}CB^{-1}$

(c) $X = AB^{-1}A^{-1}C$

1.26 **(a)** $\begin{bmatrix} 9 & 5 \\ 12 & 4 \\ 8 & 11 \end{bmatrix}$

(b) $C = I_3 + 2A^{-1} = \begin{bmatrix} 3 & 2 & 4 \\ 0 & 3 & 6 \\ 8 & 4 & 3 \end{bmatrix}$

1.27 $A^{-1} = \begin{bmatrix} 2 & 2 \\ 4 & 2 \end{bmatrix}$

1.33 **(a)** $a = -1 \land b = -3 \land c = 0$

(b) $\begin{bmatrix} 4 & 1 & -3 \\ 3 & -1 & 0 \\ 3 & -1 & -1 \end{bmatrix}$

1.34 **(a)** A, C

(b) A, E

1.40 **(a)** A, E

(b) B

1.42 **(a)** Sim, do tipo III

(b) Sim, do tipo II

(c) Não

(d) Não

(e) Sim, do tipo II ou do tipo III

1.43 **(a)** $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 6 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{1}{5} & 1 \end{bmatrix}$

1.44 **(a)** $\begin{bmatrix} e & f & g & h \\ a & b & c & d \\ i & j & k & l \end{bmatrix}$

(b) $\begin{bmatrix} 5e & 5f & 5g & 5h \\ a & b & c & d \\ i & j & k & l \end{bmatrix}$

(c) $\begin{bmatrix} a & b & c & d+3c \\ e & f & g & h+3g \\ i & j & k & l+3k \end{bmatrix}$

(d) $\begin{bmatrix} 2a & 2b & 2c-10b \\ d & e & f-5e \end{bmatrix}$

1.45 **(a)** Por exemplo,
$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(b) Por exemplo,
$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

1.46 **(a)** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{5} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$

1.47 Por exemplo,
$A^{-1} = \begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$

1.48 **(a)** Sim

(b) Não

(c) Sim

(d) Não

1.49 **(a)** Por exemplo,
$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

(b) Por exemplo,
$\begin{bmatrix} 2 & 4 & -2 & 6 & 0 \\ 0 & 0 & 0 & -5 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(c) Por exemplo,
$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 5 \\ 0 & 0 & 0 \end{bmatrix}$

1.50 **(a)** Por exemplo, $\begin{bmatrix} 2 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ e $\begin{bmatrix} 1 & 1 & \frac{1}{2} \\ 0 & 0 & 0 \end{bmatrix}$

(b) Por exemplo, $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 5 & 0 \end{bmatrix}$ e $\begin{bmatrix} 1 & 0 & -3 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

1.51 **(a)** Sim

(b) Sim

(c) Não

(d) Sim

(e) Sim

1.52 **(a)** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 & -1 & 0 & 3 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

1.53 **(a)** $[0], [1]$

(b) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & * \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & * \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

onde $*$ representa um elemento arbitrário de $\mathbb{K}$

1.55 **(a)** Não

(b) Sim

1.56 **(a)** Por exemplo,
$(\frac{1}{2}l_1, l_2 + (-1)l_1, l_1 + 2l_2, 4l_2,$
$l_2 + (-1)l_1)$

(b) Por exemplo,
$(l_2 + l_1, \frac{1}{4}l_2, l_1 + (-2)l_2, l_2 + l_1, 2l_1)$

1.57 $\mathrm{r}(A_1) = 3$, $\mathrm{r}(A_2) = 3$, $\mathrm{r}(A_3) = 2$, $\mathrm{r}(A_4) = 3$

1.58 $\mathrm{r}(A_\alpha) = \begin{cases} 2, & \text{se } \alpha = 2 \\ 3, & \text{se } \alpha \neq 2 \end{cases}$

$\mathrm{r}(B_\alpha) = \begin{cases} 3, & \text{se } \alpha = 2 \\ 4, & \text{se } \alpha \neq 2 \end{cases}$

$\mathrm{r}(C_{\alpha,\beta}) = \begin{cases} 2, & \text{se } \alpha = 0 \text{ ou } \beta = 0 \\ 3, & \text{se } \alpha \neq 0 \text{ e } \beta \neq 0 \end{cases}$

$\mathrm{r}(D_{\alpha,\beta}) = \begin{cases} 3, & \text{se } \beta = 0 \text{ e } \alpha \in \mathbb{R} \\ 4, & \text{se } \beta \neq 0 \text{ e } \alpha \in \mathbb{R} \end{cases}$

1.59 $\mathrm{r}\left(\begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}\right) = 1$ e $\mathrm{r}\left(\begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}\right) = 2$

1.60 **(a)** $\mathrm{r}(A) = \mathrm{r}(-A)$

(b) $\mathrm{r}(\alpha A) = \begin{cases} \mathrm{r}(A), & \text{se } \alpha \neq 0 \\ 0, & \text{se } \alpha = 0 \end{cases}$

1.61 **(a)** Por exemplo, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ e $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(b) Por exemplo, $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ e $\begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

1.62 **(a)** $\alpha \in \mathbb{R} \setminus \{-5\}$

(b) $\alpha \in \mathbb{R} \setminus \{-1\}$ e $\beta \in \mathbb{R} \setminus \{2\}$

1.65 **(a)** $\begin{bmatrix} 0 & \frac{1}{2} \\ -1 & \frac{1}{2} \end{bmatrix}$

(b) Por exemplo,

$A^{-1} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

1.66 $B^{-1} = \frac{1}{5} \begin{bmatrix} 3 & 1 & 2 \\ -2 & 1 & 2 \\ -2 & 1 & -3 \end{bmatrix}$

$C^{-1} = \begin{bmatrix} -i & -1+i \\ 1 & -i \end{bmatrix}$

$D^{-1} = \frac{1}{2} \begin{bmatrix} 0 & 2 & -4 & -1 \\ 1 & 1 & -5 & -1 \\ 1 & 1 & -3 & 0 \\ 1 & -1 & 1 & 0 \end{bmatrix}$

1.70 (a) $\{0\}$

(b) $\emptyset$

1.71 (a) E, F

(b) F

(c) A, D, F

(d) A, B, D, F

(e) A, D, F

(f) A, F

1.72 $\begin{bmatrix} a & b & 0 & 0 \\ c & a & b & 0 \\ 0 & c & a & b \\ 0 & 0 & c & a \end{bmatrix}$

1.73 (a) $\begin{bmatrix} 0 & 2 & 4 \\ 6 & 3 & 9 \end{bmatrix}$

(b) $\begin{bmatrix} 4 & 8 \\ -4 & 0 \\ 0 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 5 & 4 & 3 \\ 2 & 16 & 3 \end{bmatrix}$

1.75 (a) 3×3

(b) 5×5

(c) 3×1

(d) 3×1

1.77 $A_1B_1 = \begin{bmatrix} -2 & 2 & 6 \\ 0 & 0 & 0 \\ -5 & -1 & 1 \end{bmatrix}$

$A_2B_2 = \begin{bmatrix} 2 & 2 \\ 0 & 2 \\ 3 & -1 \end{bmatrix}$

$A_3B_3 = \begin{bmatrix} 3 & 3 & 0 \\ -2 & 4 & 0 \end{bmatrix}$

1.78 $[A - BA + 2I_n]_{ij} = a_{ij} - \sum_{k=1}^{n} b_{ik}a_{kj} + 2(I_n)_{ij}$

1.82 (a) $npq + mnq$ multiplicações

(b) $mnp + mpq$ multiplicações

1.88 (a) $A^3 + ABA + BA^2 + B^2A + A^2B + AB^2 + BAB + B^3$

(b) $A^3 + 3A^2B + 3B^2A + B^3$

1.89 (a) $A^2 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$, $A^3 = \begin{bmatrix} 1 & 3 & 6 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

1.93 (a) Por exemplo,

$J_n = \begin{bmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{bmatrix}$ e $0_{n\times n}$

1.99 (b) Por exemplo, $\operatorname{diag}(1, 0, \ldots, 0)$

1.107 (b) $A_p{}^{-1} = A_{-p}$

1.109 $A^{-1} = \frac{1}{5}A^2$

1.120 (c) Por exemplo, $Y = \begin{bmatrix} 1 \\ i \\ 0 \\ \vdots \\ 0 \end{bmatrix}$

1.123 (a) B, D

(b) D, E

(c) D

1.124 (a) Todas

(b) Apenas a matriz nula

1.125 (a) $b = 2$ e $a, c \in \mathbb{R}$

(b) $a = 1$ e $b = -2$ e $c = -3$

1.129 (d) $B = \begin{bmatrix} 1 & 3 & 5 \\ 3 & 5 & 7 \\ 5 & 7 & 9 \end{bmatrix}$, $C = \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$

1.132 (a) A, C, G

(b) D, G

(c) G

1.133 $\alpha \in \mathbb{R}$

1.140 (a) $A_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \beta & 1 \end{bmatrix} \begin{bmatrix} \alpha & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

(b) $A_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \beta & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

(c) $B = E_2E_1AC_1C_2C_3$, com

$E_1 = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \beta & 1 \end{bmatrix}$,

$C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$, $C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \gamma & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$,

$C_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \delta & 0 & 1 \end{bmatrix}$

1.141 **(a)** $\begin{bmatrix} a_{21} & a_{22} & a_{23} \\ a_{11} & a_{12} & a_{13} \end{bmatrix}$

(b) $\begin{bmatrix} a_{21}+\alpha a_{11} & a_{22}+\alpha a_{12} & a_{23}+\alpha a_{13} \\ a_{11} & a_{12} & a_{13} \end{bmatrix}$

(c) $\begin{bmatrix} a_{11} & \alpha a_{12} & \beta a_{13} \\ a_{21} & \alpha a_{22} & \beta a_{23} \end{bmatrix}$

(d) $\begin{bmatrix} \beta a_{12} & \alpha a_{11} & \gamma a_{13} \\ \beta a_{22} & \alpha a_{21} & \gamma a_{23} \end{bmatrix}$

1.143 **(a)** $A^{-1} = A$, $B^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$,

$C^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

(b) $S = \begin{bmatrix} 1 & \beta \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha & 0 \\ 0 & 1 \end{bmatrix}$, $S^{-1} = \begin{bmatrix} \frac{1}{\alpha} & -\frac{\beta}{\alpha} \\ 0 & 1 \end{bmatrix}$

1.144 **(a)** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -6 & 1 \\ 0 & 3 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & \frac{1}{3} \\ 0 & 1 & 2 \end{bmatrix}$

1.146 **(b)** (i) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$

$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

1.148 **(a)** $A^{-1} \xrightarrow[c_i \leftrightarrow c_j]{} B^{-1}$.

(b) $A^{-1} \xrightarrow[\alpha^{-1} c_i]{} B^{-1}$.

(c) $A^{-1} \xrightarrow[c_j + (-\beta) c_i]{} B^{-1}$.

1.149 **(a)** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (f.e.r.)
Por exemplo,
$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (f.e.)

(b) $\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (f.e.r.)
Por exemplo,
$\begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (f.e.)

(c) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ (f.e.r.)
Por exemplo,
$\begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$ (f.e.)

(d) $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ (f.e.r.)
Por exemplo,
$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & -1 & -1 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ (f.e.)

(e) I_n (f.e.r.)
Por exemplo,
$5I_n$ (f.e.)

1.150 **(a)** Por exemplo,
$(l_1 \leftrightarrow l_2, l_3 + (-2)l_1, l_1 + 2l_3, \frac{1}{4}l_2)$

(b) Por exemplo,
$(4l_2, l_1 + (-2)l_3, l_3 + 2l_1, l_1 \leftrightarrow l_2)$

1.151 Por exemplo,
$(l_2 + l_1, l_3 + (-2)l_1, l_2 + 3l_1, l_1 \leftrightarrow l_3)$

1.152 $\mathrm{r}(A) = 2$, $\mathrm{r}(B) = 3$, $\mathrm{r}(C) = 1$,
$\mathrm{r}(D) = 2$, $\mathrm{r}(I_n) = n$

1.153 **(a)** $\mathrm{r}(J_n) = 1$

(b) $\mathrm{r}\left((n-2)I_n + J_n\right) =$
$$\begin{cases} 0, & \text{se } n = 1 \\ 1, & \text{se } n = 2 \\ n, & \text{se } n > 2 \end{cases}$$

1.154 **(a)** $\mathrm{r}(A) = \begin{cases} 1, & \text{se } \alpha \in \{-1, 1\} \\ 4, & \text{caso contrário} \end{cases}$

(b) $\mathrm{r}(A) = \begin{cases} 1, & \text{se } \alpha \in \{-1, 1, -i, i\} \\ 4, & \text{caso contrário} \end{cases}$

1.155 $\mathrm{r}(C) = \begin{cases} 2, & \text{se } \alpha = \frac{1}{2} \text{ e } \beta = 2 \\ 3, & \text{caso contrário} \end{cases}$

1.156 $\mathrm{r}\left(A_{\alpha,\beta}\right) = \begin{cases} 3, & \text{se } \alpha\beta \neq 1 \text{ e } \alpha \neq 0 \\ 2, & \text{caso contrário} \end{cases}$

$$\mathrm{r}\left(B_{\alpha,\beta}\right) = \begin{cases} 3, & \text{se } \beta \neq 1 \\ 2, & \text{se } \beta = 1 \text{ e } \alpha \neq 1 \\ 1, & \text{se } \beta = 1 \text{ e } \alpha = 1 \end{cases}$$

$$\mathrm{r}\left(C_{\alpha,\beta}\right) = \begin{cases} 4, & \text{se } (\alpha \neq 0 \text{ ou } \beta \neq 0) \\ & \quad \text{e } \beta \neq -\alpha \\ 3, & \text{se } (\alpha \neq 0 \text{ ou } \beta \neq 0) \\ & \quad \text{e } \beta = -\alpha \\ 0, & \text{se } \alpha = \beta = 0 \end{cases}$$

$$\mathrm{r}\left(D_{\alpha,\beta}\right) = \begin{cases} 4, & \text{se } \alpha \neq 1 \text{ e } \beta \neq 3\alpha - 2 \\ 3, & \text{se } (\alpha = 1 \text{ e } \beta \neq 3\alpha - 2) \\ & \quad \text{ou } (\alpha \neq 1 \text{ e } \beta = 3\alpha - 2) \\ 2, & \text{se } \alpha = 1 \text{ e } \beta = 3\alpha - 2 \end{cases}$$

1.157 $\mathrm{r}(B_t) = 1$, para qualquer $t \in \mathbb{R}$

1.161 Por exemplo, $A_1 = \begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix}$, $A_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$,
$A_3 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

1.162 **(a)** $A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

(b) B não é invertível

(c) $C^{-1} = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$

(d) $D^{-1} = \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

1.163 $A = \begin{bmatrix} -1 & -\frac{1}{2} & 1 \\ -1 & -\frac{1}{2} & 1 \\ -1 & 0 & -1 \end{bmatrix}$

1.164 $M^{-1} = \begin{bmatrix} 2+2i & -2-i & -1-2i \\ -2-i & 2 & 2+i \\ -1 & 1 & 1 \end{bmatrix}$

1.165 **(a)** $A^{-1} = \frac{1}{1-ab}\begin{bmatrix} 1 & 0 & -b \\ 0 & 1-ab & 0 \\ -a & 0 & 1 \end{bmatrix}$

(b) $B^{-1} = \begin{bmatrix} \alpha^{-1} & 0 & 0 \\ -\alpha^{-2} & \alpha^{-1} & 0 \\ \alpha^{-3} & -\alpha^{-2} & \alpha^{-1} \end{bmatrix}$

(c) $C^{-1} = \begin{bmatrix} 1 & -a & 0 & 0 \\ 0 & 1 & -a & 0 \\ 0 & 0 & 1 & -a \\ 0 & 0 & 0 & 1 \end{bmatrix}$

1.166 **(a)** $A^{-1} = \begin{bmatrix} 1 & 10 & 10^2 & \cdots & 10^{n-2} & 10^{n-1} \\ 0 & 1 & 10 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ & & & \cdots & & \\ 0 & 0 & 0 & \cdots & 1 & 10 \\ 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix}$

(b) $B^{-1} = \begin{bmatrix} 0 & \cdots & b_n^{-1} \\ \vdots & \iddots & \vdots \\ b_1^{-1} & \cdots & 0 \end{bmatrix}$

1.168 $A^{-1} = \frac{1}{2}B^\top$

$B^{-1} = \frac{1}{2}A^\top$

$\left(B^2\right)^{-1} = \frac{1}{4}\left(A^2\right)^\top$

Soluções dos exercícios

Abreviaturas utilizadas:

S.P.D.
Sistema **P**ossível **D**eterminado
S.I.
Sistema **I**mpossível
S.P.I.
Sistema **P**ossível **I**ndeterminado
g.i.
grau de **i**ndeterminação

2.1 $\begin{cases} x_2 + 2x_3 = 2 \\ 2x_1 = 1 \\ -x_1 + 2x_3 = -1 \end{cases}$

2.3 Basta tomar a matriz
$B = A\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ 19 \\ 5 \\ 17 \end{bmatrix}$
resultando o sistema, nas incógnitas x, y, z, sobre $\mathbb{R}$,
$\begin{cases} x - z = -2 \\ 2x + 4y + 3z = 19 \\ -x + 2z = 5 \\ 3x + 4y + 2z = 17 \end{cases}$

2.7 (S_1) S.P.D.

Conjunto das soluções de (S_1):
$\{(1, -1, 0)\}$

(S_2) S.P.I. com g.i. 1

Conjunto das soluções de (S_2):
$\left\{\left(\frac{1}{7} + \frac{1}{7}\alpha, \frac{5}{7} - \frac{2}{7}\alpha, \alpha\right) : \alpha \in \mathbb{R}\right\}$

(S_3) S.I.

2.8 **(a)** S.P.D.
(b) S.I.
(c) S.P.I. com g.i. 2
(d) S.P.I. com g.i. 4
(e) S.I.
(f) S.P.D.
(g) S.P.I. com g.i. 4

2.9 **(a)** Por exemplo, $\begin{cases} x - 2z = 1 \\ y - z = -4 \end{cases}$

(b) Por exemplo, $\begin{cases} x - y + 2z = 0 \\ 2x - 2y + 4z = 0 \end{cases}$

Sim, basta tomar o sistema
$0x + 0y + 0z = 0$

2.10 **(a)** $\{(2, 3, 4)\}$
(b) $\emptyset$
(c) $\{(-1 - 5\alpha_2 - 2\alpha_5, \alpha_2, 2 - \alpha_5, 4 + 3\alpha_5, \alpha_5) : \alpha_2, \alpha_5 \in \mathbb{R}\}$

2.11 **(a)** $C = \mathbb{R} \setminus \{3\}$
(b) $C = \emptyset$
(c) $C = \{3\}$

2.12 **(a)** (S): $\{-1\}$
(S'): $\{-1\}$
(S''): $\emptyset$

(b) (S): $\mathbb{R} \setminus \{-1, 1\}$
(S'): $\mathbb{R} \setminus \{-1, 1\}$
(S''): $\mathbb{R} \setminus \{-1, 1\}$

(c) (S): $\{1\}$
(S'): $\{1\}$
(S''): $\{-1, 1\}$

2.15 Conjunto das soluções:
$\left\{\left(-\frac{1}{4}\alpha + \frac{3}{8}\beta + \frac{1}{2}\gamma, \frac{1}{2}\beta + \gamma, -\frac{1}{4}\alpha - \frac{1}{8}\beta + \frac{1}{2}\gamma\right) : \alpha, \beta, \gamma \in \mathbb{R}\right\}$

2.16 **(a)** Sendo $A = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 0 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$, tem-se

$AX = \begin{bmatrix} 1 \\ 0 \\ \cdots \\ 0 \end{bmatrix}$, $AX = \begin{bmatrix} 2 \\ 1 \\ \cdots \\ 0 \end{bmatrix}$, ...,
$AX = \begin{bmatrix} n \\ n-1 \\ \cdots \\ 1 \end{bmatrix}$

(b) $Y = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 0 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$

2.23 $z = -1 - i$ e $w = 1$

2.24 (S_1) S.P.D.

Conjunto das soluções de (S_1):
$\left\{\left(\frac{3}{5}, -\frac{7}{5}, -\frac{4}{5}\right)\right\}$

(S_2) S.P.I. com g.i. 1

Conjunto das soluções de (S_2):
$\{(-1 + 2\alpha, -\alpha, \alpha) : \alpha \in \mathbb{R}\}$

(S_3) S.I.

(S_4) S.P.I. com g.i. 1

Conjunto das soluções de (S_4):
$\left\{\left(-\frac{2}{5}-\frac{3}{5}\alpha, \frac{1}{5}-\frac{1}{5}\alpha, \alpha\right) : \alpha \in \mathbb{R}\right\}$

(S_5) S.P.I. com g.i. 1

Conjunto das soluções de (S_5):
$\{(1,0,\alpha) : \alpha \in \mathbb{R}\}$

(S_6) S.I.

(S_7) S.P.I. com g.i. 1

Conjunto das soluções de (S_7):
$\{(1-\alpha, -1+2\alpha, \alpha) : \alpha \in \mathbb{R}\}$

2.25 $\alpha = \frac{\pi}{2} + 2k\pi,\ k \in \mathbb{Z}$
$\beta = \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}$
$\gamma = k\pi,\ k \in \mathbb{Z}$

2.27 **(a)** $(2-b-c)x^2 + bx + c$, com $b, c \in \mathbb{R}$
(b) $(4-c)x^2 - 2x + c$, com $c \in \mathbb{R}$
(c) $x^2 - 2x + 3$

2.28 **(a)** $u \in \mathcal{L}$
(b) $u \notin \mathcal{L}$

2.32 **(a)** Se $a \neq 0$ e $a \neq 1$, S.I.
Se $a = 0$ ou $a = 1$, S.P.D.
(b) Se $a \neq 2$, S.P.I. com g.i. 1
Se $a = 2$, S.I.

2.33 Se $\alpha = 7$, S.P.I. com g.i. 2
Se $\alpha = 0$, S.P.I. com g.i. 1
Se $\alpha \neq 7$ e $\alpha \neq 0$, S.P.D.

2.34 Se $|\alpha| \neq 1$, S.P.D.
Conjunto das soluções: $\{(0,0,0)\}$
Se $|\alpha| = 1$, S.P.I. com g.i. 2
Conjunto das soluções:
$\left\{\left(-\alpha b - \alpha^2 c, b, c\right) :\ b, c \in \mathbb{C}\right\}$

2.35 **(a)** Se $\alpha \neq 0$ e $\alpha \neq 1$ e $\beta \in \mathbb{R}$, S.P.D.
Se $\alpha = 1$ e $\beta \in \mathbb{R}$, S.P.I. com g.i. 1
Se $\alpha = 0$ e $\beta \neq 1$, S.I.
Se $\alpha = 0$ e $\beta = 1$, S.P.I. com g.i. 1
(b) Conjunto das soluções:
$\{(1+\gamma, 0, \gamma) : \gamma \in \mathbb{R}\}$

2.36 **(a)** Se $\alpha \neq 0$ e $\beta \neq 3$, S.P.D.
Se $\alpha = 0$ e $\beta \in \mathbb{R}$, S.I.
Se $\alpha = 1$ e $\beta = 3$, S.P.I. com g.i. 1
Se $\alpha \neq 0$ e $\alpha \neq 1$ e $\beta = 3$, S.I.
(b) Conjunto das soluções:
$\{(2, \gamma, 1) :\ \gamma \in \mathbb{R}\}$

2.37 **(a)** Se $\alpha \neq 0$ e $\beta \neq 1$, S.P.D.
Se $\alpha = 0$ e $\beta = 1$, S.P.I. com g.i. 2
Se $\alpha = 0$ e $\beta \neq 1$, S.P.I. com g.i. 1
Se $\alpha \neq 0$ e $\beta = 1$, S.I.
(b) (iii) $(5, 1, -3)$

2.38 **(a)** Se $b \neq 6$ e $a \in \mathbb{R}$, S.I.
Se $b = 6$ e $a = 1$, S.P.I. com g.i. 1
Se $b = 6$ e $a \neq 1$, S.P.D.
(b) Se $b \neq 0$ e $a \in \mathbb{R}$, S.P.D.
Se $b = 0$ e $a = -1$, S.P.I. com g.i. 1
Se $b = 0$ e $a \neq 1$, S.I.
(c) Se $3a + b \neq 0$ e $b \neq a$, S.I.
Se $3a+b \neq 0$ e $b = a$, S.P.I. com g.i. 2
Se $3a + b = 0$ e $a \neq 0$, S.P.D.

2.39 **(a)** Se $a \neq -1$ e $b \in \mathbb{R}$, S.P.I. com g.i. 1
Se $a = -1$ e $b = 1$, S.P.I. com g.i. 2
Se $a = -1$ e $b \neq 1$, S.I.
(b) Se $a \neq 3$ e $b \in \mathbb{R}$, S.P.I. com g.i. 1
Se $a = 3$ e $b \neq 3$, S.I.
Se $a = 3$ e $b = 3$, S.P.I. com g.i. 2
(c) Se $a \neq 0$ e $b \neq 0$, S.P.D.
Se $a = 0$ e $b \neq 0$, S.P.I. com g.i. 1
Se $a \neq 0$ e $b = 0$, S.I.
Se $a = 0$ e $b = 0$, S.P.I. com g.i. 2

2.40 $(a, 2b, 3c)$

3.1 **(a)** 1
(b) -3
(c) 0

3.2 2

3.3 **(a)** 0
(b) -7
(c) -1

3.4 **(a)** -1
(b) -4
(c) 3

3.5 $-uvxyz$

3.6 $\{0, 1, 3\}$

3.8 $\overline{\alpha}$

3.9 $|A| = |B| = |C| = -6$
$|D| = 6$

3.10 **(a)** γ
(b) -12γ
(c) γ
(d) -3γ
(e) $-\gamma$

3.14 312

3.15 $k \in \{-2, 1\}$

3.17 $x \in \{-3a, a\}$

3.18 **(a)** $\mathbb{R} \setminus \{-3, 2\}$
(b) $\emptyset$

3.19 $t \in \mathbb{R} \setminus \{0, 2\}$

3.20 $|AB^\top C| = -40$
$|3B| = 3^n(-5)$
$|B^2C| = (-5)^2 \cdot 4$

3.24 $\det\left(C^{-1}A^\top B^{-1}\right) = -\frac{1}{10}$

3.25 **(a)** $|A| = -32$
$|B| = 0$
(b) $|A^{-1}| = -\frac{1}{32}$
(c) (i) Sim
(ii) Não

3.28 **(a)** $A^{-1} = \begin{bmatrix} 1 & 1 & -\frac{3}{2} \\ 0 & 1 & -\frac{1}{2} \\ -1 & -2 & \frac{5}{2} \end{bmatrix}$
(b) $V_\alpha^{-1} = \left[\begin{smallmatrix} \cos\alpha & \operatorname{sen}\alpha \\ -\operatorname{sen}\alpha & \cos\alpha \end{smallmatrix}\right] = V_{-\alpha}$
(c) $A^{-1} = \frac{1}{|z|^2+|w|^2}\left[\begin{smallmatrix} \overline{z} & -w \\ \overline{w} & z \end{smallmatrix}\right]$

3.29 **(a)** $\operatorname{adj} M = \begin{bmatrix} m^2-1 & 1-m & 1-m \\ 1-m & m^2-1 & 1-m \\ 1-m & 1-m & m^2-1 \end{bmatrix}$
(b) $m \in \mathbb{R} \setminus \{-2, 1\}$
(c) $M^{-1} =$
$\frac{1}{(m+2)(m-1)}\begin{bmatrix} m+1 & -1 & -1 \\ -1 & m+1 & -1 \\ -1 & -1 & m+1 \end{bmatrix}$

3.30 $(\operatorname{adj} D)_{ij} = \begin{cases} 0 & , \text{ se } i \neq j \\ \Pi_{\substack{k=1 \\ k\neq i}}^{n} d_k & , \text{ se } i = j \end{cases}$

3.32 **(a)** $|A| = -3$
(b) $(1, 2, 3)$

3.33 **(a)** $k \in \mathbb{R} \setminus \{0, -3\}$
(b) $(1, -\frac{1}{2}, \frac{1}{2})$

3.35 Número de inversões de σ_1: 3
Número de inversões de σ_2: 4
Número de inversões de σ_3: 4

3.36 $\{(1, 4)\}$

3.37 $\begin{cases} \text{par}, & \text{se } n = 4k \text{ ou } n = 4k+1 \\ \text{ímpar}, & \text{caso contrário} \end{cases}$

3.41 **(c)** Por exemplo, $B = \left[\begin{smallmatrix} 0 & 2 \\ -2 & 0 \end{smallmatrix}\right]$

3.43 **(a)** $\alpha\beta\gamma k$
(b) k
(c) $-k$

3.51 $(1 - x^{n+1})^n$

3.56 **(a)** 0

(b) $x^n + (-1)^{n+1}y^n$

3.58 $a \neq b$ e $a \neq -b$

3.64 $x = -1$, $y = 3$

3.71 **(b)** $\det A \in \{-1, 1\}$, se n é par
$\det A \in \{-i, i\}$, se n é ímpar

3.72 $\det A \in \{0, 1\}$, se n é par
$\det A \in \{0, -1\}$, se n é ímpar

3.74 **(a)** $\frac{1}{\alpha r}$

(b) $\frac{\alpha}{r}$

(c) $\frac{\alpha}{r}$

(d) $\frac{1}{\alpha r}$

3.80 $\widehat{\alpha A} = \alpha^{n-1}\widehat{A}$

3.82 É também triangular superior

3.84 **(b)** $A_\alpha^{-1} = \begin{bmatrix} 1+\alpha & \alpha \\ -\alpha & 1-\alpha \end{bmatrix} = A_{-\alpha}$

3.85 $A^{-1} = \frac{1}{\alpha^2\varepsilon}\begin{bmatrix} \alpha\varepsilon & \gamma\delta-\beta\varepsilon & \alpha\gamma \\ 0 & \alpha\varepsilon & 0 \\ 0 & -\alpha\delta & \alpha^2 \end{bmatrix}$

3.86 **(b)** $(\operatorname{adj} C)_{44} = -2$, $\left(A^{-1}\right)_{44} = \frac{1}{2}$
$(\operatorname{adj} C)_{23} = -4$, $\left(A^{-1}\right)_{23} = 1$

3.87 **(a)** $\operatorname{adj} A = (ad - bc)\begin{bmatrix} 0 & d & 0 & -c \\ d & 0 & -b & 0 \\ 0 & -b & 0 & a \\ -c & 0 & a & 0 \end{bmatrix}$

(b) $ad \neq bc$

(c) $A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} 0 & d & 0 & -c \\ d & 0 & -b & 0 \\ 0 & -b & 0 & a \\ -c & 0 & a & 0 \end{bmatrix}$

3.91 **(a)** $A = \begin{bmatrix} 3 & \frac{1}{2} & -1 \\ 2 & 1 & 0 \\ -1 & -\frac{1}{2} & 1 \end{bmatrix}$

3.94 Solução do sistema: $\left(\frac{b_1 d - b_2 b}{ad-bc}, \frac{b_2 a - b_1 c}{ad-bc}\right)$

3.95 Solução de (S): $(0, 0, 1)$

3.96 Solução do sistema:
$\left(\frac{(b-m)(c-m)}{(b-a)(c-a)}, \frac{(m-a)(c-m)}{(b-a)(c-b)}, \frac{(m-a)(m-b)}{(c-a)(c-b)}\right)$

3.98 **(a)** $(n-1)!$ inversões

(b) $n!$ inversões

3.99 $|A| = a_{11} \cdots a_{nn}$