



$$F_{rxm} = F_{m2} + F_{m3} - F_{m1} - F_{m4}, F_{rxo} = 0$$

$$F_{rym} = F_{m1} + F_{m2} + F_{m3} + F_{m4}, F_{ryo} = F_{o1} + F_{o2}$$

$$\theta F_{rm} = \tan^{-1} \left( \frac{F_{rym}}{F_{rxm}} \right), \theta F_{ro} = 0$$

$$F_{rx} = F_{rxm}, F_{ry} = F_{rym} + F_{ryo}, \theta F_r = \tan^{-1} \left( \frac{F_{ry}}{F_{rx}} \right)$$

$$F_{rx} = F_{m2} + F_{m3} - F_{m1} - F_{m4}$$

$$F_{ry} = F_{m1} + F_{m2} + F_{m3} + F_{m4} + F_{o1} + F_{o2}$$

$$\theta F_r = \tan^{-1} \left( \frac{F_{m1} + F_{m2} + F_{m3} + F_{m4} + F_{o1} + F_{o2}}{F_{m2} + F_{m3} - F_{m1} - F_{m4}} \right)$$

$$F_{rim} = F_{m2} + F_{m4}$$

$$F_{lm} = F_{m1} + F_{m3}$$

$$\tau_{rm} = \frac{L+W}{4} (F_{rim} - F_{lm})$$

$$\tau_{ro} = \frac{W}{2} (F_{o2} - F_{o1})$$

$$\tau_r = \tau_{rm} + \tau_{ro}$$

$$\tau_r = \frac{L+W}{4} (F_{rim} - F_{lm}) + \frac{W}{2} (F_{o2} - F_{o1})$$

$$\tau_r = \frac{L+W}{4} (F_{m2} + F_{m4} - F_{m1} - F_{m3}) + \frac{W}{2} (F_{o2} - F_{o1})$$

Let  $T = \binom{m}{o}$ ,  $N_m = \binom{1}{2}{3}{4}$  and  $N_o = \binom{1}{2}$ .  $\forall t \in T$  and  $n \in N_t$ , we have

$$k_{tn}^{\min} \leq F_{tn} \leq k_{tn}^{\max}$$

Now we want to make some way to input  $\theta F_r$ ,  $\tau_r$ , and  $\forall t \in T$  and  $n \in N_t$ :  $k_{tn}^{\min}$  and  $k_{tn}^{\max}$  and then somehow get out  $\forall t \in T$  and  $n \in N_t$ :  $F_{tn}$  that maximizes  $F_{rx} + F_{ry}$

First let's find the "objective function". Substituting we get

$$F_{m2} + F_{m3} - F_{m1} - F_{m4} + F_{m1} + F_{m2} + F_{m3} + F_{m4} + F_{o1} + F_{o2}$$

Simplifying we then get

$$2F_{m2} + 2F_{m3} + F_{o1} + F_{o2}$$

We can then use the simplex algorithm to solve this. To input into the simplex algorithm it says to maximize  $c^T x$  subject to  $Ax \leq b$  and  $x \geq k$  where

- $A$  is an  $m \times n$  matrix
- $b \in \mathbb{R}^m$
- $c \in \mathbb{R}^n$
- $x \in \mathbb{R}^n$  are the decision variables
- $x$  is the decision variable vector
- $c$  is the objective-coefficient vector
- $c^T x$  is the objective function
- $A$  is the constraint matrix
- $b$  is the right-hand side vector
- $k \in \mathbb{R}$

First, let's put down what we know. We know the decision variable vector  $x$  would be  $\begin{pmatrix} F_{m1} \\ F_{m2} \\ F_{m3} \\ F_{m4} \\ F_{o1} \\ F_{o2} \end{pmatrix}$ .

Back from what we found before because of that we know the decision variable vector  $c$  is  $\begin{pmatrix} 0 \\ 2 \\ 2 \\ 0 \\ 1 \\ 1 \end{pmatrix}$ .

So  $c^T x = 2F_{m2} + 2F_{m3} + F_{o1} + F_{o2}$  (what we found earlier).

Now we are really close, all we have to worry about is  $A$ . So first we know a bunch of the inequalities:

$$\forall t \in T \text{ and } n \in N_t$$

$$k_{tn}^{\min} \leq F_{tn} \leq k_{tn}^{\max}$$

we need to then put it in the form  $Ax \leq b$  so we get

$$\forall t \in T \text{ and } n \in N_t$$

$$F_{tn} \leq k_{tn}^{\max}, -F_{tn} \leq -k_{tn}^{\min}$$

Ok but this isn't in the form with  $x$ , it is each of the forces separately. So since we only do one at a time the other forces would just be multiplied by 0. There would be 1 row for each force. This can be written as an identity matrix with the size of 6:  $I_6$ . This is done for both the minimum and maximum, with the minimum being negative:  $-I_6$ . Therefore we get the following inequalities:

$$\forall t \in T \text{ and } n \in N_t$$

$$I_6 x \leq k_{tn}^{\max}$$

$$-I_6 x \leq -k_{tn}^{\min}$$

So we know some of the constraints. The next constraint is the angle of force constraint. Now we need to linearize  $\theta F_r$ .

First we flip around the trig function to get

$$\tan(\theta F_r) = \frac{F_{m1} + F_{m2} + F_{m3} + F_{m4} + F_{o1} + F_{o2}}{F_{m2} + F_{m3} - F_{m1} - F_{m4}}$$

Then simplifying we get

$$F_{m1} + F_{m2} + F_{m3} + F_{m4} + F_{o1} + F_{o2} - \tan(\theta F_r)(F_{m2} + F_{m3} - F_{m1} - F_{m4}) = 0$$

we can then factorize to get

$$F_{m1}(1 + \tan(\theta F_r)) + F_{m2}(1 - \tan(\theta F_r)) + F_{m3}(1 - \tan(\theta F_r)) + F_{m4}(1 + \tan(\theta F_r)) + F_{o1} + F_{o2} = 0$$

We can then take the factors to the decision variables to get a vector

$$a_{\text{ang}} = \begin{pmatrix} 1 + \tan(\theta F_r) \\ 1 - \tan(\theta F_r) \\ 1 - \tan(\theta F_r) \\ 1 + \tan(\theta F_r) \\ 1 \\ 1 \end{pmatrix}$$

Which we can convert into two inequalities

$$a_{\text{ang}}^T x \leq 0$$

$$-a_{\text{ang}}^T x \leq 0$$

Now onto the last and final constraint we have is the torque constraint which we can apply the same steps and factorize getting the new vector

$$a_{\tau} = \begin{pmatrix} -\frac{L+W}{4} \\ \frac{L+W}{4} \\ -\frac{L+W}{4} \\ \frac{L+W}{4} \\ -\frac{W}{2} \\ \frac{W}{2} \end{pmatrix}$$

giving the following two inequalities

$$a_{\tau}^T x \leq \tau_r$$

$$-a_{\tau}^T x \leq \tau_r$$

This gives us the total matrix

$$A = \begin{pmatrix} I_6 \\ -I_6 \\ a_{\text{ang}}^T \\ -a_{\text{ang}}^T \\ a_\tau^T \\ -a_\tau^T \end{pmatrix}$$

and the vector

$$b = \begin{pmatrix} k_{\text{max}} \\ -k_{\text{min}} \\ 0 \\ 0 \\ \tau_r \\ \tau_r \end{pmatrix}$$

We can then put this into a simplex (or other algorithm) solver and it should output the correct answer!