



Given the measures of  $l$  and  $\theta$ ; AC is an arc; find  $\Delta x$  and  $\Delta y$

The measures of  $l$  and  $\theta$  are given and AC is an arc because they are given

$l = r\theta$  by the arc length formula in radians

$l = (c + \Delta x_v)\theta$  by substitution

$\frac{l}{\theta} = c + \Delta x_v$  by division

$\frac{l}{\theta} - \Delta x_v = c$  by subtraction

$\overline{AB} \cong \overline{BC}$  because radii  $\cong$

$AB = BC$  by the definition of  $\cong$  segments

$\triangle ABC$  is an isosceles  $\triangle$  by the definition of an isosceles  $\triangle$

$\alpha \cong \angle BCA$  by base  $\angle$ 's  $\cong$

$\alpha = m\angle BCA$  by the definition of  $\cong \angle$ 's

$\pi = \theta + \alpha + m\angle BCA$  by the  $\angle$  sum Th. in radians

$\pi = \theta + \alpha + \alpha$  by substitution

$\pi = \theta + 2\alpha$  by substitution

$\pi - \theta = 2\alpha$  by subtraction

$\frac{\pi - \theta}{2} = \alpha$  by subtraction

$\frac{\pi}{2} - \frac{\theta}{2} = \alpha$  by the distributive property

$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\theta)}$  by the Law of Sines

$$\frac{\frac{l}{\theta} - \Delta x_v}{\sin\left(\frac{\pi}{2} - \frac{\theta}{2}\right)} = \frac{b}{\sin(\theta)} \text{ by substitution}$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin(x) \text{ and } \sin\left(\frac{\pi}{2} - x\right) = \cos(x) \text{ by the complementary-angle identities}$$

$$\frac{\frac{l}{\theta} - \Delta x_v}{\cos\left(\frac{\theta}{2}\right)} = \frac{b}{\sin(\theta)} \text{ by substitution}$$

$$\frac{l}{\cos\left(\frac{\theta}{2}\right)\theta} = \frac{b}{\sin(\theta)} \text{ by the associative property}$$

$$\sin(\theta) \left( \frac{l}{\cos\left(\frac{\theta}{2}\right)\theta} \right) = b \text{ by multiplication}$$

$$\frac{\sin(\theta)l}{\cos\left(\frac{\theta}{2}\right)\theta} = b \text{ by the distributive property}$$

$$\sin(2x) = 2 \sin(x) \cos(x) \text{ by the sine double-angle formula}$$

$$\sin(x) = 2 \sin\left(\frac{x}{2}\right) \cos\left(\frac{x}{2}\right) \text{ by division}$$

$$\frac{2 \sin\left(\frac{\theta}{2}\right) \cos\left(\frac{\theta}{2}\right) l}{\cos\left(\frac{\theta}{2}\right)\theta} = b \text{ by substitution}$$

$$\left(2 \sin\left(\frac{\theta}{2}\right)\right) \left(\frac{l}{\theta} - \Delta x_v\right) = b \text{ by division}$$

$$\sin(\alpha) = \frac{\text{opposite}}{\text{hypotenuse}} \text{ by the definition of sine in right angle trigonometry}$$

$$\cos(\alpha) = \frac{\text{adjacent}}{\text{hypotenuse}} \text{ by the definition of cosine in right angle trigonometry}$$

$$\sin(\alpha) = \frac{\Delta y}{b} \text{ and } \cos(\alpha) = \frac{-\Delta x}{b} \text{ by substitution}$$

$$b \sin(\alpha) = \Delta y \text{ and } b \cos(\alpha) = -\Delta x \text{ by multiplication}$$

$$\left(2 \sin\left(\frac{\theta}{2}\right)\right) \left(\frac{l}{\theta} - \Delta x_v\right) \sin(\alpha) = \Delta y \text{ and } \left(2 \sin\left(\frac{\theta}{2}\right)\right) \left(\frac{l}{\theta} - \Delta x_v\right) \cos(\alpha) = -\Delta x \text{ by substitution}$$

$$\left(2 \sin\left(\frac{\theta}{2}\right)\right) \left(\frac{l}{\theta} - \Delta x_v\right) \sin\left(\frac{\pi}{2} - \frac{\theta}{2}\right) = \Delta y \text{ and } \left(2 \sin\left(\frac{\theta}{2}\right)\right) \left(\frac{l}{\theta} - \Delta x_v\right) \cos\left(\frac{\pi}{2} - \frac{\theta}{2}\right) = -\Delta x \text{ by substitution}$$

$$\left(2 \sin\left(\frac{\theta}{2}\right)\right) \left(\frac{l}{\theta} - \Delta x_v\right) \cos\left(\frac{\theta}{2}\right) = \Delta y \text{ and } \left(2 \sin\left(\frac{\theta}{2}\right)\right) \left(\frac{l}{\theta} - \Delta x_v\right) \sin\left(\frac{\theta}{2}\right) = -\Delta x \text{ by substitution}$$

$$\left(2 \sin\left(\frac{\theta}{2}\right) \cos\left(\frac{\theta}{2}\right)\right) \left(\frac{l}{\theta} - \Delta x_v\right) = \Delta y \text{ and } \left(2 \sin\left(\frac{\theta}{2}\right) \sin\left(\frac{\theta}{2}\right)\right) \left(\frac{l}{\theta} - \Delta x_v\right) = -\Delta x \text{ by the associative property}$$

$$\sin(\theta) \left(\frac{l}{\theta} - \Delta x_v\right) = \Delta y \text{ and } 2 \sin^2\left(\frac{\theta}{2}\right) \left(\frac{l}{\theta} - \Delta x_v\right) = -\Delta x \text{ by substitution}$$

$$-2 \sin^2\left(\frac{\theta}{2}\right) \left(\frac{l}{\theta} - \Delta x_v\right) = \Delta x \text{ by division}$$

$$\sin^2(x) = \frac{1 - \cos(2x)}{2} \text{ by the sine lowering power formula}$$

$$-2 \left( \frac{1 - \cos(\theta)}{2} \right) \left(\frac{l}{\theta} - \Delta x_v\right) = \Delta x \text{ by substitution}$$

$$-((1 - \cos(\theta)) \left(\frac{l}{\theta} - \Delta x_v\right)) = \Delta x \text{ by the associative property}$$

$$\left(\frac{l}{\theta} - \Delta x_v\right) (-1 + \cos(\theta)) = \Delta x \text{ by substitution}$$