Given

$F_y = m_1 + m_2 + o_1 + o_2 + m_3 + m_4$

$F_x = m_1 - m_2 - m_3 + m_4$

The bounds of m_i are denoted by $[l_i, u_i]$ which are constants

The bounds of o_i are denoted by $[l_{oi}, u_{oi}]$ which are constants

Goal:

Find the bounds of F_y based upon F_x

Let $A = m_1 + m_4$, $B = m_2 + m_3$, $O = o_1 + o_2$, and $R = F_x$

Then the bounds of A are $[l_1 + l_4, u_1 + u_4]$, the bounds of B are $[l_2 + l_3, u_2 + u_3]$, and the bounds of O are $[l_{o1} + l_{o2}, u_{o1} + u_{o2}]$

Let $l_1 + l_4 = A_{\text{min}}$, $u_1 + u_4 = A_{\text{max}}$, $l_2 + l_3 = B_{\text{min}}$, $u_2 + u_3 = B_{\text{max}}$, $l_{o1} + l_{o2} = O_{\text{min}}$, and $u_{o1} + u_{o2} = O_{\text{max}}$

$A \in [A_{\text{min}}, A_{\text{max}}]$, $B \in [B_{\text{min}}, B_{\text{max}}]$, and $[O_{\text{min}}, O_{\text{max}}]$ by substitution

$F_y = A + B + O$ and $F_x = A - B$ by substitution

$R = A - B$ by substitution

$-A = -R - B$ by subtraction

$A = R + B$ by division

$F_y = 2B + R + O$ by the transitive property

$R + B \in [A_{\text{min}}, A_{\text{max}}]$ by substitution

Therefore $A_{\text{min}} \leq R + B \leq A_{\text{max}}$

$A_{\text{min}} - R \leq B \leq A_{\text{max}} - R$

Therefore $B \in [B_{\text{min}}, B_{\text{max}}] \cap [A_{\text{min}} - R, A_{\text{max}} - R]$

Let $B_{\text{feas,min}} = \max\{B_{\text{min}}, A_{\text{min}} - R\}$ and $B_{\text{feas,max}} = \min\{B_{\text{max}}, A_{\text{max}} - R\}$

Therefore $B \in \left[B_{\text{feas,min}}, B_{\text{feas,max}}\right]$

$F_y = 2B + F_x + O$ by substitution

Define function F_y as $F_y(F_x) = 2B + F_x + O$

Therefore $F_{\text{y,min}} = 2B_{\text{feas,min}} + R + O_{\text{min}}$ and $F_{\text{y,max}} = 2B_{\text{feas,max}} + R + O_{\text{max}}$

Therefore $F_y \in \left[F_{\text{y,min}}, F_{\text{y,max}}\right]$

Goal:

Find the bounds of F_x based upon F_y

Let S be F_y and T be $A + B$

$S = A + B + O, T = A + B$, $F_x = A - B$ by substitution

Define F_x as $F_x(S) = A - B$

$S - O = A + B$ by substraction

$S - O = T$ by substitution

Therefore the bounds of T are $[S - O_{\text{max}}, S - O_{\text{min}}]$ and $A + B \in [A_{\text{min}} + B_{\text{min}}, A_{\text{max}} + B_{\text{max}}]$

$T \in [A_{\text{min}} + B_{\text{min}}, A_{\text{max}} + B_{\text{max}}]$

Therefore $T \in [S - O_{\text{max}}, S - O_{\text{min}}] \cap [A_{\text{min}} + B_{\text{min}}, A_{\text{max}} + B_{\text{max}}]$

Let $T_{\text{min}} = \max\{A_{\text{min}} + B_{\text{min}}, S - O_{\text{max}}\}$ and $T_{\text{max}} = \max\{A_{\text{max}} + B_{\text{max}}, S - O_{\text{min}}\}$

Therefore the bounds of $A + B$ are $[T_{\text{min}}, T_{\text{max}}]$

By extremizing $F_x = A - B$ we find $F_{\text{x,max}}(S) = \max\{T_{\text{max}} - 2B_{\text{min}}, 2A_{\text{max}} - T_{\text{min}}\}$ and $F_{\text{x,min}}(S) = \min\{T_{\text{min}} - 2B_{\text{max}}, 2A_{\text{min}} - T_{\text{max}}\}$

$F_x = A - B$ we find $F_{\text{x,max}}(F_y) = \max\{T_{\text{max}} - 2B_{\text{min}}, 2A_{\text{max}} - T_{\text{min}}\}$ and $F_{\text{x,min}}(F_y) = \min\{T_{\text{min}} - 2B_{\text{max}}, 2A_{\text{min}} - T_{\text{max}}\}$ by substitution

Therefore $F_x \in [F_{\text{x,min}}(F_y), F_{\text{x,max}}(F_y)]$