

Given (for all proofs below)

$V = IR + E_{mf}$  by Ohm's Law with back EMF for a DC motor

$E_{mf} = \omega\kappa_e$  by the Back EMF equation of a motor

$\tau = I\kappa_t$  by the Motor torque equation

$\tau = J\alpha + B\omega + \tau_{load} \text{sign}(\omega)$  by the Rotational equation of motion

The value of  $\omega$  and  $\alpha$  (Only for the proof directly below this)

Find  $V$  voltage

$V = IR + E_{mf}$ ,  $E_{mf} = \omega\kappa_e$ ,  $\tau = I\kappa_t$ ,  $\tau = J\alpha + B\omega + \tau_{load} \text{sign}(\omega)$ , and the value of  $\omega$  and  $\alpha$  because they are given

$I\kappa_t = J\alpha + B\omega + \tau_{load} \text{sign}(\omega)$  By substitution

$I = \frac{J\alpha + B\omega + \tau_{load} \text{sign}(\omega)}{\kappa_t}$  By Division

$-IR = E_{mf} - V$  By Subtraction

$I = \frac{E_{mf} - V}{-R}$  By Division

$\frac{E_{mf} - V}{-R} = \frac{J\alpha + B\omega + \tau_{load} \text{sign}(\omega)}{\kappa_t}$  By substitution

$\frac{\omega\kappa_e - V}{-R} = \frac{J\alpha + B\omega + \tau_{load} \text{sign}(\omega)}{\kappa_t}$  By substitution

$\omega\kappa_e - V = \frac{J\alpha + B\omega + \tau_{load} \text{sign}(\omega)}{\kappa_t}(-R)$  By multiplication

$-V = \frac{J\alpha + B\omega + \tau_{load} \text{sign}(\omega)}{\kappa_t}(-R) - \omega\kappa_e$  By subtraction

$V = -\left(\frac{J\alpha + B\omega + \tau_{load} \text{sign}(\omega)}{\kappa_t}(-R) - \omega\kappa_e\right)$  By division

$V = \frac{J\alpha + B\omega + \tau_{load} \text{sign}(\omega)}{\kappa_t}(R) + \omega\kappa_e$  By substitution

$V = \frac{J\alpha R}{\kappa_t} + \frac{B\omega R}{\kappa_t} + \frac{\tau_{load} \text{sign}(\omega) R}{\kappa_t} + \frac{\omega\kappa_e \kappa_t}{\kappa_t}$  By substitution

$V = \frac{J\alpha R}{\kappa_t} + \frac{B\omega R + \omega\kappa_e \kappa_t}{\kappa_t} + \frac{\tau_{load} \text{sign}(\omega) R}{\kappa_t}$  By substitution

$V = \frac{J\alpha R}{\kappa_t} + \frac{\omega(BR + \kappa_e \kappa_t)}{\kappa_t} + \frac{\tau_{load} \text{sign}(\omega) R}{\kappa_t}$  By substitution

Define  $K_v$  by  $K_v = \frac{BR + \kappa_e \kappa_t}{\kappa_t}$

Define  $K_a$  by  $K_a = \frac{JR}{\kappa_t}$

Define  $K_s$  by  $K_s = \frac{\tau_{load} R}{\kappa_t}$

$V = K_a\alpha + K_v\omega + K_s \text{sign}(\omega)$

Define a function  $V$  by  $V(\omega, \alpha) = K_a\alpha + K_v\omega + K_s \text{sign}(\omega)$

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Given (Only for the proof directly below this)

The value of  $\omega$

Find the maximum  $\alpha$  angular acceleration at some angular velocity  $\omega$

$V = IR + E_{mf}$ ,  $E_{mf} = \omega\kappa_e$ ,  $\tau = I\kappa_t$ ,  $\tau = J\alpha + B\omega + \tau_{load} \text{sign}(\omega)$ , and the value of  $\omega$  because they are given

Note: The VEX motor controller has current limits so we must also account for those.

Define  $I_{\min}$  and  $I_{\max}$  as the current limits and define  $I_{\text{clamp}}$  as the clamped value of  $I$

Define  $V_{\max}$  as the maximum voltage and set it to  $V$  when finding the max acceleration

Define  $\alpha_{\max}$  as the maximum voltage and set it to  $\alpha$  when finding the max acceleration

$I_{\text{clamp}} = \min(\max(I, I_{\min}), I_{\max})$  by the definition of clamping

$I_{\text{clamp}} = \min\left(\max\left(\left(\frac{E_{\text{mf}} - V_{\max}}{-R}\right), I_{\min}\right), I_{\max}\right)$  by substitution

$I_{\text{clamp}} = \min\left(\max\left(\left(\frac{\omega \kappa_e - V_{\max}}{-R}\right), I_{\min}\right), I_{\max}\right)$  by substitution

Define  $V_{\max} = V$  as the max voltage

$I_{\text{clamp}} = \min\left(\max\left(\left(\frac{\omega \kappa_e - V_{\max}}{-R}\right), I_{\min}\right), I_{\max}\right)$  by substitution

Restrict  $I$  by  $I_{\text{clamp}} = I$  by the note

$-J\alpha_{\max} = B\omega + \tau_{\text{load}} \text{sign}(\omega) - \tau$  by Subtraction

$\alpha_{\max} = \frac{B\omega + \tau_{\text{load}} \text{sign}(\omega) - \tau}{-J}$  by Division

$\alpha_{\max} = \frac{B\omega + \tau_{\text{load}} \text{sign}(\omega) - I\kappa_t}{-J}$  by Substitution

$\alpha_{\max} = \frac{B\omega + \tau_{\text{load}} \text{sign}(\omega) - I_{\text{clamp}}\kappa_t}{-J}$  by Substitution

Define a function  $\alpha$  by  $\alpha_{\max}(\omega) = \frac{B\omega + \tau_{\text{load}} \text{sign}(\omega) - I_{\text{clamp}}\kappa_t}{-J}$

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Find  $\omega(t)$  given that the motor is constantly accelerating

$\alpha_{\max}(\omega) = \frac{V_{\max} - K_v\omega - \text{sgn}(\omega)K_s}{K_a}$

Let  $s = \text{sgn}(\omega) \in \{1, -1\}$

$\frac{d\omega}{dt} = \frac{V_{\max} - K_v\omega - sK_s}{K_a}$

$\frac{d\omega}{dt} + \frac{K_v}{K_a}\omega = \frac{V_{\max} - sK_s}{K_a}$

$I(t) = e^{\int \frac{K_v}{K_a} dt}$

$I(t) = e^{\frac{K_v}{K_a} t}$

$\omega(t) = \frac{1}{I(t)} \left( \int_0^t I(\tau) Q d\tau + I(0)\omega_0 \right)$

$\omega(t) = \frac{1}{e^{\frac{K_v}{K_a} t}} \left( \int_0^t e^{\frac{K_v}{K_a} \tau} \frac{V_{\max} - sK_s}{K_a} d\tau + \omega_0 \right)$

$\omega(t) = \frac{1}{e^{\frac{K_v}{K_a} t}} \left( \frac{V_{\max} - sK_s}{K_a} \frac{K_a}{K_v} \left( e^{\frac{K_v}{K_a} \tau} - 1 \right) + \omega_0 \right)$

$\omega(t) = \frac{V_{\max} - sK_s}{K_v} \frac{e^{\frac{K_v}{K_a} \tau} - 1}{e^{\frac{K_v}{K_a} t}} + \omega_0 \left( e^{-\frac{K_v}{K_a} t} \right)$

$\omega(t) = \frac{V_{\max} - sK_s}{K_v} + \left( \omega_0 - \frac{V_{\max} - sK_s}{K_v} \right) e^{-\frac{K_v}{K_a} t}$

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Find the maximum  $\alpha$  angular acceleration of wheel  $n$  at some linear velocity  $v$

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