Given (for all proofs below)

$V = IR + E_{\text{mf}}$ by Ohm's Law with back EMF for a DC motor

$E_{\text{mf}} = \omega\kappa_e$ by the Back EMF equation of a motor

$\tau = I\kappa_t$ by the Motor torque equation

$\tau = J\alpha + B\omega + \tau_{\text{load}}\ \text{sign}(\omega)$ by the Rotational equation of motion

The value of ω and α (Only for the proof directly below this)

Find V voltage

$V = IR + E_{\text{mf}}$, $E_{\text{mf}} = \omega\kappa_e$, $\tau = I\kappa_t$, $\tau = J\alpha + B\omega + \tau_{\text{load}}\ \text{sign}(\omega)$, and the value of ω and α because they are given

$I\kappa_t = J\alpha + B\omega + \tau_{\text{load}}\ \text{sign}(\omega)$ By substitution

$I = \frac{J\alpha + B\omega + \tau_{\text{load}}\ \text{sign}(\omega)}{\kappa_t}$ By Division

$-IR = E_{\text{mf}} - V$ By Subtraction

$I = \frac{E_{\text{mf}} - V}{-R}$ By Division

$\frac{E_{\text{mf}} - V}{-R} = \frac{J\alpha + B\omega + \tau_{\text{load}}\ \text{sign}(\omega)}{\kappa_t}$ By substitution

$\frac{\omega\kappa_e - V}{-R} = \frac{J\alpha + B\omega + \tau_{\text{load}}\ \text{sign}(\omega)}{\kappa_t}$ By substitution

$\omega\kappa_e - V = \frac{J\alpha + B\omega + \tau_{\text{load}}\ \text{sign}(\omega)}{\kappa_t}(-R)$ By multiplication

$-V = \frac{J\alpha + B\omega + \tau_{\text{load}}\ \text{sign}(\omega)}{\kappa_t}(-R) - \omega\kappa_e$ By subtraction

$V = -\left(\frac{J\alpha + B\omega + \tau_{\text{load}}\ \text{sign}(\omega)}{\kappa_t}(-R) - \omega\kappa_e\right)$ By division

$V = \frac{J\alpha + B\omega + \tau_{\text{load}}\ \text{sign}(\omega)}{\kappa_t}(R) + \omega\kappa_e$ By substitution

$V = \frac{J\alpha R}{\kappa_t} + \frac{B\omega R}{\kappa_t} + \frac{\tau_{\text{load}}\ \text{sign}(\omega) R}{\kappa_t} + \frac{\omega\kappa_e\kappa_t}{\kappa_t}$ By substitution

$V = \frac{J\alpha R}{\kappa_t} + \frac{B\omega R + \omega\kappa_e\kappa_t}{\kappa_t} + \frac{\tau_{\text{load}}\ \text{sign}(\omega) R}{\kappa_t}$ By substitution

$V = \frac{J\alpha R}{\kappa_t} + \frac{\omega(BR + \kappa_e\kappa_t)}{\kappa_t} + \frac{\tau_{\text{load}}\ \text{sign}(\omega) R}{\kappa_t}$ By substitution

Define K_v by $K_v = \frac{BR + \kappa_e\kappa_t}{\kappa_t}$

Define K_a by $K_a = \frac{JR}{\kappa_t}$

Define K_s by $K_s = \frac{\tau_{\text{load}} R}{\kappa_t}$

$V = K_a\alpha + K_v\omega + K_s\ \text{sign}(\omega)$

Define a function V by $V(\omega, \alpha) = K_a\alpha + K_v\omega + K_s\ \text{sign}(\omega)$

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Given (Only for the proof directly below this)

The value of ω

Find the maximum α angular acceleration at some angular velocity ω

$V = IR + E_{\text{mf}}$, $E_{\text{mf}} = \omega\kappa_e$, $\tau = I\kappa_t$, $\tau = J\alpha + B\omega + \tau_{\text{load}}\ \text{sign}(\omega)$, and the value of ω because they are given

Note: The VEX motor controller has current limits so we must also account for those.

Define I_{min} and I_{max} as the current limits and define I_{clamp} as the clamped value of I

Define V_{max} as the maximum voltage and set it to V when finding the max acceleration

Define α_{max} as the maximum voltage and set it to α when finding the max acceleration

$I_{\text{clamp}} = \min(\max(I, I_{\text{min}}), I_{\text{max}})$ by the definition of clamping

$I_{\text{clamp}} = \min\left(\max\left(\left(\frac{E_{\text{mf}} - V_{\text{max}}}{-R}\right), I_{\text{min}}\right), I_{\text{max}}\right)$ by substitution

$I_{\text{clamp}} = \min\left(\max\left(\left(\frac{\omega\kappa_e - V_{\text{max}}}{-R}\right), I_{\text{min}}\right), I_{\text{max}}\right)$ by substitution

Define $V_{\text{max}} = V$ as the max voltage

$I_{\text{clamp}} = \min\left(\max\left(\left(\frac{\omega\kappa_e - V_{\text{max}}}{-R}\right), I_{\text{min}}\right), I_{\text{max}}\right)$ by substitution

Restrict I by $I_{\text{clamp}} = I$ by the note

$-J\alpha_{\text{max}} = B\omega + \tau_{\text{load}} \operatorname{sign}(\omega) - \tau$ by Subtraction

$\alpha_{\text{max}} = \frac{B\omega + \tau_{\text{load}} \operatorname{sign}(\omega) - \tau}{-J}$ by Division

$\alpha_{\text{max}} = \frac{B\omega + \tau_{\text{load}} \operatorname{sign}(\omega) - I\kappa_t}{-J}$ by Substitution

$\alpha_{\text{max}} = \frac{B\omega + \tau_{\text{load}} \operatorname{sign}(\omega) - I_{\text{clamp}}\kappa_t}{-J}$ by Substitution

Define a function α by $\alpha_{\text{max}}(\omega) = \frac{B\omega + \tau_{\text{load}} \operatorname{sign}(\omega) - I_{\text{clamp}}\kappa_t}{-J}$

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Find V voltage of wheel n at some initial linear velocity v and the wanted linear acceleration a

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Find the maximum α angular acceleration of wheel n at some linear velocity v

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