Comparison of Drive Systems
Velocity and Force Derivations

SerrialError

1. Coordinate Definitions

Let:

- v_x: Robot velocity in x (strafe)
- v_y: Robot velocity in y (forward)
- ω: Angular velocity (yaw)
- v_{wheel}: Wheel tangential speed
- F_{wheel}: Force applied at wheel-ground contact

Assume:

- Ideal conditions: no slippage or losses
- All wheels provide the same torque τ and spin with speed v

2. X-Drive Derivation

2.1 Wheel Velocity Vectors

Each wheel is mounted at a 44° angle. Thus:

$$\vec{v}_{\text{wheel}} = \frac{v}{\sqrt{2}} \cdot \begin{bmatrix} \pm 1 \\ 1 \end{bmatrix}$$

2.2 Net Robot Velocity

To move forward:

$$v_y = 2 \cdot \frac{v}{\sqrt{2}} = \sqrt{2} \cdot v$$

Similarly for strafe velocity,

$$v_x = 2 \cdot \frac{v}{\sqrt{2}} = \sqrt{2} \cdot v$$

assuming wheels configured to strafe.

2.3 Force Projection

$$F_y = 2 \cdot \left(\frac{F}{\sqrt{2}} \right) = \sqrt{2} \cdot F \quad , \quad F_x = \sqrt{2} \cdot F$$

3. Mecanum Drive Derivation

3.1 Contact Velocity via Rollers

Roller directs velocity at 44°:

$$\vec{v}_{\text{contact}} = \frac{v}{\sqrt{2}} \cdot \begin{bmatrix} \pm 1 \\ 1 \end{bmatrix}$$

3.2 Force Perpendicular to Roller

$$F_{\perp} = F \cdot \cos(44^\circ) = \frac{F}{\sqrt{2}}, \quad F_y = \frac{F_{\perp}}{\sqrt{2}} = \frac{F}{2}$$

3.3 Net Force and Velocity

Total force and velocity contributions from 4 wheels:

$$F_{\text{robot}} = 4 \cdot \frac{F}{2} = 2F \quad (\text{total}) \quad , \quad v_y^{\text{robot}} = 4 \cdot \frac{v}{2} = 2v \quad (\text{total})$$

Assuming symmetrical contributions to v_x, similarly,

$$v_x^{\text{robot}} = 2v \quad , \quad F_x^{\text{robot}} = 2F$$

4. Tank (Differential) Drive Derivation

4.1 Geometry

Two wheels: Left (L) and Right (R), separated by width d

4.2 Velocity Equations

$$v_y = \frac{v_L + v_R}{2}, \quad \omega = \frac{v_R - v_L}{d}$$

4.3 Net Velocity and Force

Net forward velocity is directly average of wheel velocities:

$$v_{\text{net}} = v_y = \frac{v_L + v_R}{2}$$

Net force applied forward is sum of both wheels:

$$F_{\text{net}} = F_L + F_R = 2F$$

No strafe component (non-holonomic).

5. Grand Comparison Table

Drive	**Net Velocity**	**Net Force**
X-Drive	$\sqrt{2}v$	$\sqrt{2}F$
Mecanum	v	F
Tank	$\frac{v_L + v_R}{2}$	$2F$