



Given l and θ find Δx and Δy

l and θ are given because they are given

$l = r\theta$ by the arc length formula in radians

$l = a\theta$ by substitution

$\frac{l}{\theta} = a$ by division

Define the base \angle other than α inside of $\triangle ABC$ to be γ

$\alpha \cong \gamma$ by base \angle 's \cong

$\alpha = \gamma$ by the definition of $\cong \angle$'s

$\pi = \theta + \alpha + \gamma$ by the \angle sum Th. in radians

$\pi = \theta + \alpha + \alpha$ by substitution

$\pi = \theta + 2\alpha$ by substitution

$\pi - \theta = 2\alpha$ by subtraction

$\frac{\pi - \theta}{2} = \alpha$ by subtraction

$\frac{\pi}{2} - \frac{\theta}{2} = \alpha$ by the distributive property

$\frac{\pi}{2} - \frac{\theta}{2} = \alpha$ by substitution

$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\theta)}$ by the Law of Sines

$\frac{\frac{l}{\theta}}{\sin(\frac{\pi}{2} - \frac{\theta}{2})} = \frac{b}{\sin(\theta)}$ by substitution

$\cos(\frac{\pi}{2} - x) = \sin(x)$ and $\sin(\frac{\pi}{2} - x) = \cos(x)$ by the complementary-angle identities

$\frac{\frac{l}{\theta}}{\cos(\frac{\theta}{2})} = \frac{b}{\sin(\theta)}$ by substitution

$\frac{l}{\cos(\frac{\theta}{2})\theta} = \frac{b}{\sin(\theta)}$ by the associative property

$\sin(\theta) \left(\frac{l}{\cos(\frac{\theta}{2})\theta} \right) = b$ by multiplication

$\frac{\sin(\theta)l}{\cos(\frac{\theta}{2})\theta} = b$ by the distributive property

$\sin(2x) = 2 \sin(x) \cos(x)$ by the sine double-angle formula

$\sin(x) = 2 \sin(\frac{x}{2}) \cos(\frac{x}{2})$ by division

$\frac{2 \sin(\frac{\theta}{2}) \cos(\frac{\theta}{2}) l}{\cos(\frac{\theta}{2})\theta} = b$ by substitution

$\frac{2 \sin(\frac{\theta}{2}) l}{\theta} = b$ by division

$\sin(\alpha) = \frac{\text{opposite}}{\text{hypotenuse}}$ by the definition of sine in right angle trigonometry

$\cos(\alpha) = \frac{\text{adjacent}}{\text{hypotenuse}}$ by the definition of cosine in right angle trigonometry

$$\sin(\alpha) = \frac{\Delta y}{b} \text{ and } \cos(\alpha) = \frac{-\Delta x}{b} \text{ by substitution}$$

$$b \sin(\alpha) = \Delta y \text{ and } b \cos(\alpha) = -\Delta x \text{ by multiplication}$$

$$\frac{2 \sin(\frac{\theta}{2})l}{\theta} \sin(\alpha) = \Delta y \text{ and } \frac{2 \sin(\frac{\theta}{2})l}{\theta} \cos(\alpha) = -\Delta x \text{ by substitution}$$

$$\frac{2 \sin(\frac{\theta}{2})l}{\theta} \sin(\frac{\pi}{2} - \frac{\theta}{2}) = \Delta y \text{ and } \frac{2 \sin(\frac{\theta}{2})l}{\theta} \cos(\frac{\pi}{2} - \frac{\theta}{2}) = -\Delta x \text{ by substitution}$$

$$\frac{2 \sin(\frac{\theta}{2})l}{\theta} \cos(\frac{\theta}{2}) = \Delta y \text{ and } \frac{2 \sin(\frac{\theta}{2})l}{\theta} \sin(\frac{\theta}{2}) = -\Delta x \text{ by substitution}$$

$$\frac{2 \sin(\frac{\theta}{2})l \cos(\frac{\theta}{2})}{\theta} = \Delta y \text{ and } \frac{2 \sin(\frac{\theta}{2})l \sin(\frac{\theta}{2})}{\theta} = -\Delta x \text{ by the associative property}$$

$$\sin(u) \cos(v) = \frac{1}{2}[\sin(u+v) + \sin(u-v)] \text{ by the product-to-sum formula}$$

$$\frac{2 \sin^2(\frac{\theta}{2})l}{\theta} = -\Delta x \text{ and } \frac{2(\frac{1}{2}[\sin(\frac{\theta}{2} + \frac{\theta}{2}) + \sin(\frac{\theta}{2} - \frac{\theta}{2})])l}{\theta} = \Delta y \text{ by substitution}$$

$$-\frac{2 \sin^2(\frac{\theta}{2})l}{\theta} = \Delta x \text{ by division}$$

$$\sin^2(x) = \frac{1 - \cos(2x)}{2} \text{ by the sine lowering power formula}$$

$$-\frac{2(\frac{1 - \cos(\theta)}{2})l}{\theta} = \Delta x \text{ and } \frac{\sin(\theta)l}{\theta} = \Delta y \text{ by substitution}$$

$$-\frac{(1 - \cos(\theta))l}{\theta} = \Delta x \text{ by the associative property}$$

$$\frac{l}{\theta}(-1 + \cos(\theta)) = \Delta x \text{ by substitution}$$