F_{m1}

F_{m1}

F_{m2}

F_{m2}

T_r

F_{o1}

F_{o1}

θF_r

F_{m3}

F_{ry}

F_{m4}

F_{m3}

F_{rx}

F_{m4}

Given

$v_{xm} = v_{m2} + v_{m3} - v_{m1} - v_{m4},\ v_{xo} = 0$

$v_{ym} = v_{m1} + v_{m2} + v_{m3} + v_{m4},\ v_{yo} = v_{o1} + v_{o2}$

$v_x = v_{xm},\ v_y = v_{ym} + v_{yo},\ \tan(\theta) = \frac{v_y}{v_x}$

$$v_x = v_{m2} + v_{m3} - v_{m1} - v_{m4}$$

$$v_y = v_{m1} + v_{m2} + v_{m3} + v_{m4} + v_{o1} + v_{o2}$$

$$\tan(\theta) = \frac{v_y}{v_x}$$

$v_{im} = v_{m2} + v_{m4}$

$v_{lm} = v_{m1} + v_{m3}$

$\omega_m = \frac{L+W}{4}(v_{im} - v_{lm})$

$\omega_o = \frac{W}{2}(v_{o2} - v_{o1})$

$\omega = \omega_m + \omega_o$

$\omega = \frac{L+W}{4}(v_{im} - v_{lm}) + \frac{W}{2}(v_{o2} - v_{o1})$

$$\omega = \frac{L+W}{4}(v_{m2} + v_{m4} - v_{m1} - v_{m3}) + \frac{W}{2}(v_{o2} - v_{o1})$$

$$\frac{L+W}{4}(v_{m2} + v_{m4} - v_{m1} - v_{m3}) = \frac{W}{2}(v_{o2} - v_{o1})$$

$T = (m, o),\ N^m = (1, 2, 3, 4)$ and $N^o = (1, 2)$,

$\forall T \forall N : T_n \in \left[T_{N_{\min}}, T_{N_{\max}} \right]$

$\forall T \forall N : T_n \in \left[T_{N_{\min}}, T_{N_{\max}}\right]$ and $T_{N_{\text{nom}}} = \frac{T_{N_{\min}} + T_{N_{\max}}}{2}$

Find $\text{motor}_{\text{opt}} = ((t_n {:} n \in N, t \in T))$ st. $\bigcup_{i=1}^{k} \text{motor}_{\text{poss } i} = \text{motor}_{\text{total poss}}$ st. $k \in \mathbb{N}$ st. $\text{motor}_{\text{opt}} \in \text{motor}_{\text{total poss}}$ $\forall N \forall T : T_n \in \text{motor}_{\text{opt}} \in \min_{t,n}\left(\sum_{t \in T} \sum_{n \in N} |t_n - t_{n \text{ nom}}|\right) = S_{\text{abs diff}}$

Proof

let $a \subseteq \mathbb{R}$ and $y \subseteq \mathbb{R}$ if $\min(a) \not< \min(y)$ and $\max(a) \geq \max(y)$ and $\min(y) \in a$ then $\min(a) = \min(y)$

Therefore if $\forall N \forall T : d_n^T = |T_n - T_{n \text{ nom}}|$ and $\min_{t,n}\left(\sum_{t \in T} \sum_{n \in N} d_n^t\right) = S_{\text{abs diff}}$ then if $u_n^T \geq |T_n - T_{n \text{ nom}}|$ then $\min_{t,n}\left(\sum_{t \in T} \sum_{n \in N} u_n^t\right) = S_{\text{abs diff}}$

Let $\forall N \forall T : d_n^T \geq |T_n - T_{n \text{ nom}}|$

therefore $\forall N \forall T : d_n^T \geq T_n - T_{n \text{ nom}}; d_n^T \geq -(T_n - T_{n \text{ nom}}); d_n^T \geq 0$

therefore $\min_{t,n}\left(\sum_{t \in T} \sum_{n \in N} d_n^t\right) = S_{\text{abs diff}}$

We can then use the simplex algorithm to solve this. To input into the simplex algorithm it says to maximize c^T x subject to $Ax \leq b$ and $x \geq k$ where

- A is an $m \times n$ matrix
- $b \in \mathbb{R}^m$
- $c \in \mathbb{R}^n$
- $x \in \mathbb{R}^n$ are the decision variables
- x is the decision variable vector
- c is the objective-coefficient vector
- $c^T x$ is the objective function
- A is the constraint matrix
- b is the right-hand side vector
- $k \in \mathbb{R}$

$x = \left(((t_n {:} n \in N, t \in T); (d_n^t {:} n \in N, t \in T))\right)$

$$A = \begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{L+W}{4} & \frac{L+W}{4} & -\frac{L+W}{4} & \frac{L+W}{4} & -\frac{W}{2} & \frac{W}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & -1 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{L+W}{4} & -\frac{L+W}{4} & \frac{L+W}{4} & -\frac{L+W}{4} & \frac{W}{2} & -\frac{W}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{L+W}{4} & \frac{L+W}{4} & -\frac{L+W}{4} & \frac{L+W}{4} & -\frac{W}{2} & \frac{W}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{L+W}{4} & -\frac{L+W}{4} & \frac{L+W}{4} & -\frac{L+W}{4} & \frac{W}{2} & -\frac{W}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
I^6 \mid 0_{6x6} & & & & & & & & & & & \\
-I^6 \mid 0_{6x6} & & & & & & & & & & & \\
-1_{12x6} & & & & & & & & & & & \\
1_{12x6} & & & & & & & & & & & \\
-1_{12x1} & & & & & & & & & & & \\
\end{pmatrix}$$

$$
b = \begin{pmatrix}
v_y \\
v_x \\
\omega \\
-v_y \\
-v_x \\
-\omega \\
0 \\
0 \\
\forall T \forall N {:} T_{N_{\max}} \\
\forall T \forall N {:} T_{N_{\min}} \\
\forall T \forall N {:} T_{N_{\mathrm{nom}}} \\
\forall T \forall N {:} {-}T_{N_{\mathrm{nom}}} \\
0
\end{pmatrix}
$$