



Given the measures of  $l_l$ ,  $\theta_{r0}$ , and  $\theta_{r1}$ , the directed distance of  $x_r$ ,  $y_r$ ,  $x_{tl}$ ,  $y_{tl}$ ,  $x_{th}$ , and  $y_{th}$ , and  $\overrightarrow{P_{tl1}C_{tl1}}$  and  $\overrightarrow{P_{tl0}C_{tl0}}$  are  $\perp$  to the tangent lines at the at points  $C_{tl1}$  and  $C_{tl0}$  on  $\overline{C_{tr0}C_{tr1}}$ ; find the directed distance of  $\Delta x_r$  and  $\Delta y_r$

The measures of  $l_l$ ,  $\theta_{r0}$ , and  $\theta_{r1}$  and the directed distance of  $x_r$ ,  $y_r$ ,  $x_{tl}$ ,  $y_{tl}$ ,  $x_{th}$ , and  $y_{th}$  because they are given

Let  $\varphi_0$  be the angle  $\overline{B_rC_{tl0}}$  makes with the x-axis

Let  $\varphi_1$  be the angle  $\overline{B_rC_{tl1}}$  makes with the x-axis

$\Delta\theta_r + \varphi_0 = \varphi_1$  by the  $\angle$  add. post.

$\theta_{r1} = \varphi_1 + \frac{\pi}{2}$  and  $\theta_{r0} = \varphi_0 + \frac{\pi}{2}$  by the tangent radius theorem

$\Delta\theta_r = \varphi_1 - \varphi_0$ ,  $\varphi_1 = \theta_{r1} - \frac{\pi}{2}$ , and  $\varphi_0 = \theta_{r0} - \frac{\pi}{2}$  by subtraction

$\Delta\theta_r = \theta_{r1} - \frac{\pi}{2} - (\theta_{r0} - \frac{\pi}{2})$  by substitution

$\Delta\theta_r = \theta_{r1} - \frac{\pi}{2} - \theta_{r0} + \frac{\pi}{2}$  by multiplication

$\Delta\theta_r = \theta_{r1} - \theta_{r0}$  by substitution

$r_l = r_r + x_{tl}$  by the segment addition postulate

$r_r = r_l - x_{tl}$  by subtraction

$l_r = r_r \Delta\theta_r$  by the arc length formula

$\frac{l_r}{\Delta\theta_r} = r_r$  by division

$M_l A = \Delta x_l$  by the definition of  $\cong$  line segments

$AB_l = B_l M_l + M_l A$  by the segment addition postulate

$AB_l - M_l A = B_l M_l$  by subtraction

$AB_l - \Delta x_l = B_l M_l$  by substitution

$\frac{l_l}{\theta_l} - \Delta x_l = B_l M_l$  by substitution

$\Delta y_l$  is an opposite side to  $\angle\theta_l$  by the definition of an opposite side

$\overline{AB_l}$  is an adjacent side to  $\angle\theta_l$  by the definition of an adjacent side

$\overline{AB_l}$  is the hypotenuse of  $\triangle BMC$  by as the side opposite to the right  $\angle$  of a  $\triangle$  is the hypotenuse

$\sin(\theta_l) = \frac{\Delta y_l}{A_l B_l}, \cos(\theta_l) = \frac{B_l M_l}{AB_l}$  by the definition of sine and cosine in right angle trigonometry

$\sin(\theta_l) = \frac{\Delta y_l}{\frac{l_l}{\theta}}, \cos(\theta_l) = \frac{AB_l - \Delta x_l}{A} B_l$  by substitution

$\left(\frac{l_l}{\theta_l}\right) \sin(\theta_l) = \Delta y_l, AB_l \cos(\theta_l) = AB_l - \Delta x_l$  by multiplication

$AB_l \cos(\theta_l) - AB_l = -\Delta x_l$  by subtraction

$-AB_l \cos(\theta_l) + AB_l = \Delta x_l$  by division

$-AB_l(\cos(\theta_l) + 1) = \Delta x_l$  by substitution

$-\left(\frac{l_l}{\theta_l}\right)(\cos(\theta_l) + 1) = \Delta x_l$  by substitution

$\left(\frac{l_l}{\theta_l}\right)(\cos(\theta_l) + 1) = \Delta x_l$  by the magnitude of  $\overline{B_l A}$