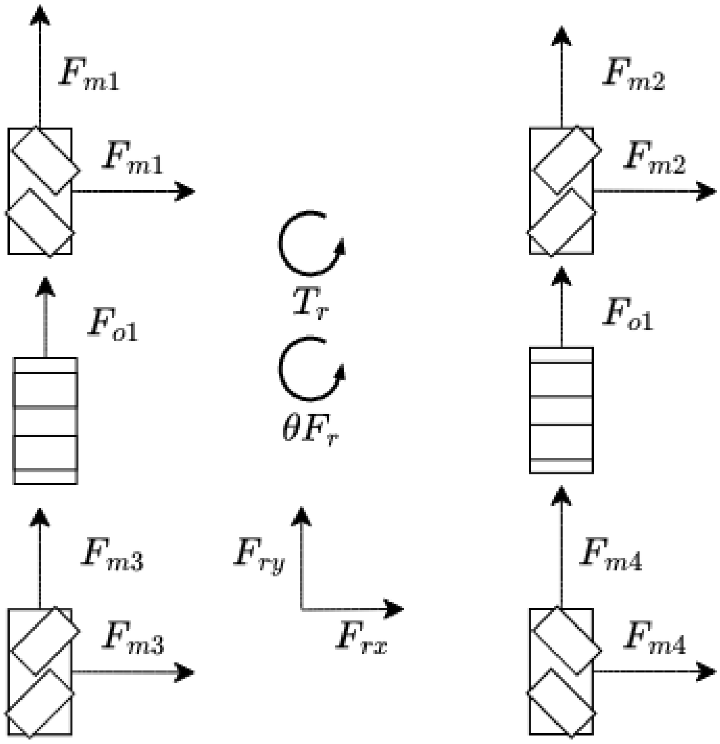

Given

$v_{xm} = v_{m2} + v_{m3} - v_{m1} - v_{m4}$, $v_{xo} = 0$

$v_{ym} = v_{m1} + v_{m2} + v_{m3} + v_{m4}$, $v_{yo} = v_{o1} + v_{o2}$

$v_x = v_{xm}$, $v_y = v_{ym} + v_{yo}$, $\tan(\theta) = \frac{v_y}{v_x}$

$$v_x = v_{m2} + v_{m3} - v_{m1} - v_{m4}$$

$$v_y = v_{m1} + v_{m2} + v_{m3} + v_{m4} + v_{o1} + v_{o2}$$

$$\tan(\theta) = \frac{v_y}{v_x}$$

$v_{im} = v_{m2} + v_{m4}$

$v_{lm} = v_{m1} + v_{m3}$

$\omega_m = \frac{L+W}{4}(v_{im} - v_{lm})$

$\omega_o = \frac{W}{2}(v_{o2} - v_{o1})$

$\omega = \omega_m + \omega_o$

$\omega = \frac{L+W}{4}(v_{im} - v_{lm}) + \frac{W}{2}(v_{o2} - v_{o1})$

$$\omega = \frac{L+W}{4}(v_{m2} + v_{m4} - v_{m1} - v_{m3}) + \frac{W}{2}(v_{o2} - v_{o1})$$

$$\frac{L+W}{4}(v_{m2} + v_{m4} - v_{m1} - v_{m3}) = \frac{W}{2}(v_{o2} - v_{o1})$$

$v_r = \sqrt{v_x^2 + v_y^2}$

$T = (m, o)$, $N^m = (1, 2, 3, 4)$ and $N^o = (1, 2)$,

$\forall T \forall N : T_n \in \left[T_{N_{\text{min}}}, T_{N_{\text{max}}}\right]$

$\forall T \forall N : T_n \in \left[T_{N_{\text{min}}}, T_{N_{\text{max}}}\right]$ and $T_{N_{\text{nom}}} = \frac{T_{N_{\text{min}}} + T_{N_{\text{max}}}}{2}$

Find $\text{motor}_{\text{opt}} = \left((t_n : n \in N, t \in T)\right)$ st. $\bigcup_{i=1}^{k} \text{motor}_{\text{poss } i} = \text{motor}_{\text{total poss}}$ st. $k \in \mathbb{N}$ st. $\text{motor}_{\text{opt}} \in \text{motor}_{\text{total poss}}$ $\forall N \forall T : T_n \in \text{motor}_{\text{opt}} \in \min_{t,n}\left(\sum_{t \in T} \sum_{n \in N} |t_n - t_{n \text{ nom}}|\right) = S_{\text{abs diff}}$

Proof

let $a \subseteq \mathbb{R}$ and $y \subseteq \mathbb{R}$ if $\min(a) \nless \min(y)$ and $\max(a) \geq \max(y)$ and $\min(y) \in a$ then $\min(a) = \min(y)$

Therefore if $\forall N \forall T : d_n^T = |T_n - T_{n \text{ nom}}|$ and $\min_{t,n}\left(\sum_{t \in T} \sum_{n \in N} d_n^t\right) = S_{\text{abs diff}}$ then if $u_n^T \geq |T_n - T_{n \text{ nom}}|$ then $\min_{t,n}\left(\sum_{t \in T} \sum_{n \in N} u_n^t\right) = S_{\text{abs diff}}$

Let $\forall N \forall T : d_n^T \geq |T_n - T_{n \text{ nom}}|$

therefore $\forall N \forall T : d_n^T \geq T_n - T_{n \text{ nom}}; d_n^T \geq -(T_n - T_{n \text{ nom}}); d_n^T \geq 0$

therefore $\min_{t,n}\left(\sum_{t \in T} \sum_{n \in N} d_n^t\right) = S_{\text{abs diff}}$

We can then use the simplex algorithm to solve this. To input into the simplex algorithm it says to maximize c^T x subject to $Ax \leq b$ and $x \geq k$ where

- A is an $m \times n$ matrix
- $b \in \mathbb{R}^m$
- $c \in \mathbb{R}^n$
- $x \in \mathbb{R}^n$ are the decision variables
- x is the decision variable vector
- c is the objective-coefficient vector
- $c^T x$ is the objective function
- A is the constraint matrix
- b is the right-hand side vector
- $k \in \mathbb{R}$

$$x = \left(((t_n : n \in N, t \in T); (d_n^t : n \in N, t \in T))\right)$$

$$A = \begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{L+W}{4} & \frac{L+W}{4} & -\frac{L+W}{4} & \frac{L+W}{4} & -\frac{W}{2} & \frac{W}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & -1 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{L+W}{4} & -\frac{L+W}{4} & \frac{L+W}{4} & -\frac{L+W}{4} & \frac{W}{2} & -\frac{W}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{L+W}{4} & \frac{L+W}{4} & -\frac{L+W}{4} & \frac{L+W}{4} & -\frac{W}{2} & \frac{W}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{L+W}{4} & -\frac{L+W}{4} & \frac{L+W}{4} & -\frac{L+W}{4} & \frac{W}{2} & -\frac{W}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
I^6 \mid 0_{6\times 6} \\
-I^6 \mid 0_{6\times 6} \\
-1_{12\times 6} \\
1_{12\times 6} \\
-1_{12\times 1}
\end{pmatrix}$$

$$b = \begin{pmatrix} v_y \\ v_x \\ \omega \\ -v_y \\ -v_x \\ -\omega \\ 0 \\ 0 \\ \forall T \forall N{:}T_{N_{\max}} \\ \forall T \forall N{:}T_{N_{\min}} \\ \forall T \forall N{:}T_{N_{\text{nom}}} \\ \forall T \forall N{:}-T_{N_{\text{nom}}} \\ 0 \end{pmatrix}$$

$c = (0_{6\times 1} \mid -1_{6\times 1})$

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$v = A_{1:3}x = \begin{pmatrix} v_x \\ v_y \\ \omega \end{pmatrix}$

$v_{xy} = A_{1:2}x = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$

$F = \{x \in \mathbb{R}^6 : x^{\min} \leq x \leq x^{\max}, A_3 : x = \omega_{\text{des}}\}$

Given ω_{des} and θ_{des} st. $\omega_{\text{des}} = \omega$ and $\theta_{\text{des}} = \theta$

Find max $v_{r(x)} \coloneqq \|v_{xy}\|_2 = \sqrt{v_x^2 + v_y^2}$ st. $x \in F$

Proof

For any vector $l \in \mathbb{R}^2$

$\|l\|_2 = \max_{u \in \mathbb{R}^2,\ \|u\|_2 = 1} u^T l$ by Cauchy-Schwarz, for all unit vectors u we have $u^T l \leq \|u\|_2\ \|l\|_2 = \|l\|_2$. Equality is achieved by taking $u = \frac{l}{\|l\|_2}$ if $(l \neq 0)$. Hence the max over unit u equals $\|l\|_2$.

$l = v_{xy} = A_{1:2}x$

$\max_{x \in F} \|A_{1:2}x\|_2 = \max_{x \in F} \max_{\|u\|=1} u^T(A_{1:2}x)$ by substitution

$\max_{\|u\| = 1} \max_{x \in F} u^T(A_{1:2}x) = \max_{x \in F} \max_{\|u\|=1} u^T(A_{1:2}x)$ because both F and the unit circle $U = \{u \in \mathbb{R}^2 : \|u\| = 1\}$ are compact and the function $f(u,t) \coloneqq u^T(A_{1:2}x)$ is continuous on the compact set $U \times F$ and the extreme value theorem guarantees a maximum of f on the product set and both maximizations are finite and over compact sets

$$u^T(A_{1:2}x) = \left(A_{1:2}^T u\right)^T x = c(u)^T x$$

where $c(u) \coloneqq A_{1:2}^T u \in \mathbb{R}^6$

$\max_{x \in F} \max_{\|u\|=1} u^T(A_{1:2}x) = \max_{\|u\|=1} \max_{x \in F} c(u)^T x$ by substitution

$u = \begin{pmatrix} \cos(\theta_{\text{des}}) \\ \sin(\theta_{\text{des}}) \end{pmatrix}$

$c(u) \coloneqq A_{1:2}^T u = \cos(\theta_{\text{des}}) A_{1:2}^T e_1 + \sin(\theta_{\text{des}}) A_{1:2}^T e_2$

$p \coloneqq \max_{x \in F} c^T x$

$\max_{\|u\|=1} \max_{x \in F} c(u)^T x = \max_{\theta_{\text{des}} \in [0, 2\pi)} p$

$x_{\text{new}} = x$

$A_{\text{new}} = A_{1:10}$

$b_{\text{new}} = b_{1:10}$

$c_{\text{new}} = \left(\forall x\colon \cos(\theta_{\text{des}}) A_{(\text{new})_x} + \sin(\theta_{\text{des}}) A_{(\text{new})_y}\right)$

This then outputs the optimized vector x^*_{new} we then plug in that vector into v_x and v_y to get v_r