



Given

Note: These are the correct formulas for velocity. Incorrectly throughout the proof I used velocity instead of force. Everything in the proof should be replaced with velocity. However as in this case force and velocity are proportional it doesn't change anything

$$v_x = \frac{1}{4}(v_{m1} + v_{m4} - v_{m2} - v_{m3})$$

$$v_y = \frac{1}{6}(v_{m1} + v_{m2} + v_{m3} + v_{m4} + v_{o1} + v_{o2})$$

$$\tan(\theta) = \frac{v_y}{v_x}$$

$$\omega = \frac{L+W}{24}(v_{m2} + v_{m4} - v_{m1} - v_{m3}) + \frac{W}{12}(v_{o2} - v_{o1})$$

$$\frac{L+W}{24}(v_{m2} + v_{m4} - v_{m1} - v_{m3}) = \frac{W}{12}(v_{o2} - v_{o1})$$

The incorrect velocities

$$v_{xm} = v_{m1} + v_{m4} - v_{m2} - v_{m3}, v_{xo} = 0$$

$$v_{ym} = v_{m1} + v_{m2} + v_{m3} + v_{m4}, v_{yo} = v_{o1} + v_{o2}$$

$$v_x = v_{xm}, v_y = v_{ym} + v_{yo}, \tan(\theta) = \frac{v_y}{v_x}$$

$$v_x = v_{m1} + v_{m4} - v_{m2} - v_{m3}$$

$$v_y = v_{m1} + v_{m2} + v_{m3} + v_{m4} + v_{o1} + v_{o2}$$

$$\tan(\theta) = \frac{v_y}{v_x}$$

$$v_{im} = v_{m2} + v_{m4}$$

$$v_{lm} = v_{m1} + v_{m3}$$

$$\omega_m = \frac{L+W}{4}(v_{im} - v_{lm})$$

$$\omega_o = \frac{W}{2}(v_{o2} - v_{o1})$$

$$\omega = \omega_m + \omega_o$$

$$\omega = \frac{L+W}{4}(v_{im} - v_{lm}) + \frac{W}{2}(v_{o2} - v_{o1})$$

$$\omega = \frac{L+W}{4}(v_{m2} + v_{m4} - v_{m1} - v_{m3}) + \frac{W}{2}(v_{o2} - v_{o1})$$

$$\frac{L+W}{4}(v_{m2} + v_{m4} - v_{m1} - v_{m3}) = \frac{W}{2}(v_{o2} - v_{o1})$$

$$v_r = \sqrt{v_x^2 + v_y^2}$$

$$T = (m, o), N^m = (1, 2, 3, 4) \text{ and } N^o = (1, 2),$$

$$\forall T \forall N : T_n \in [T_{N_{\min}}, T_{N_{\max}}]$$

$$\forall T \forall N : T_n \in [T_{N_{\min}}, T_{N_{\max}}] \text{ and } T_{N_{\text{nom}}} = \frac{T_{N_{\min}} + T_{N_{\max}}}{2}$$

$$\text{Find motor}_{\text{opt}} = ((t_n : n \in N, t \in T)) \text{ st. } \bigcup_{i=1}^k \text{motor}_{\text{poss } i} = \text{motor}_{\text{total poss}} \text{ st. } k \in \mathbb{N} \text{ st. } \text{motor}_{\text{opt}} \in \text{motor}_{\text{total poss}} \forall N \forall T : T_n \in \text{motor}_{\text{opt}} \in \min_{t,n} \left(\sum_{t \in T} \sum_{n \in N} |t_n - t_{n \text{ nom}}| \right) = S_{\text{abs diff}}$$

Proof

let $a \subseteq \mathbb{R}$ and $y \subseteq \mathbb{R}$ if $\min(a) \not\leq \min(y)$ and $\max(a) \geq \max(y)$ and $\min(y) \in a$ then $\min(a) = \min(y)$

Therefore if $\forall N \forall T : d_n^T = |T_n - T_{n \text{ nom}}|$ and $\min_{t,n} \left(\sum_{t \in T} \sum_{n \in N} d_n^t \right) = S_{\text{abs diff}}$ then if $u_n^T \geq |T_n - T_{n \text{ nom}}|$ then $\min_{t,n} \left(\sum_{t \in T} \sum_{n \in N} u_n^t \right) = S_{\text{abs diff}}$

$$\text{Let } \forall N \forall T : d_n^T \geq |T_n - T_{n \text{ nom}}|$$

$$\text{therefore } \forall N \forall T : d_n^T \geq T_n - T_{n \text{ nom}}; d_n^T \geq -(T_n - T_{n \text{ nom}}); d_n^T \geq 0$$

$$\text{therefore } \min_{t,n} \left(\sum_{t \in T} \sum_{n \in N} d_n^t \right) = S_{\text{abs diff}}$$

We can then use the simplex algorithm to solve this. To input into the simplex algorithm it says to maximize $c^T x$ subject to $Ax \leq b$ and $x \geq k$ where

- A is an $m \times n$ matrix
- $b \in \mathbb{R}^m$
- $c \in \mathbb{R}^n$
- $x \in \mathbb{R}^n$ are the decision variables
- x is the decision variable vector
- c is the objective-coefficient vector
- $c^T x$ is the objective function
- A is the constraint matrix
- b is the right-hand side vector
- $k \in \mathbb{R}$

$$x = ((t_n : n \in N, t \in T); (d_n^t : n \in N, t \in T))$$

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{L+W}{4} & \frac{L+W}{4} & -\frac{L+W}{4} & \frac{L+W}{4} & -\frac{W}{2} & \frac{W}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{L+W}{4} & -\frac{L+W}{4} & \frac{L+W}{4} & -\frac{L+W}{4} & \frac{W}{2} & -\frac{W}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{L+W}{4} & \frac{L+W}{4} & -\frac{L+W}{4} & \frac{L+W}{4} & -\frac{W}{2} & \frac{W}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{L+W}{4} & -\frac{L+W}{4} & \frac{L+W}{4} & -\frac{L+W}{4} & \frac{W}{2} & -\frac{W}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ I^6 \mid 0_{6 \times 6} \\ -I^6 \mid 0_{6 \times 6} \\ -1_{12 \times 6} \\ 1_{12 \times 6} \\ O_{6 \times 6} \mid -1_{6 \times 6} \end{pmatrix}$$

$$b = \begin{pmatrix} v_y \\ v_x \\ \omega \\ -v_y \\ -v_x \\ -\omega \\ 0 \\ 0 \\ \forall T \forall N: T_{N_{\max}} \\ \forall T \forall N: T_{N_{\min}} \\ \forall T \forall N: T_{N_{\text{nom}}} \\ \forall T \forall N: -T_{N_{\text{nom}}} \\ 0 \end{pmatrix}$$

$$c = (0_{6 \times 1} \mid -1_{6 \times 1})$$

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$$v = A_{1:3}x = \begin{pmatrix} v_x \\ v_y \\ \omega \end{pmatrix}$$

$$v_{xy} = A_{1:2}x = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

$$F = \{x \in \mathbb{R}^6 : x^{\min} \leq x \leq x^{\max}, A_3 : x = \omega_{\text{des}}\}$$

$$\text{Given } \omega_{\text{des}} \text{ and } \theta_{\text{des}} \text{ st. } \omega_{\text{des}} = \omega \text{ and } \theta_{\text{des}} = \theta$$

$$\text{Find } \max_{r(x)} v_{r(x)} := \|v_{xy}\|_2 = \sqrt{v_x^2 + v_y^2} \text{ st. } x \in F$$

Proof

$$\text{For any vector } l \in \mathbb{R}^2$$

$$\|l\|_2 = \max_{u \in \mathbb{R}^2, \|u\|_2 = 1} u^T l \text{ by Cauchy-Schwarz, for all unit vectors } u \text{ we have } u^T l \leq \|u\|_2 \|l\|_2 = \|l\|_2. \text{ Equality is achieved by taking } u = \frac{l}{\|l\|_2} \text{ if } (l \neq 0). \text{ Hence the max over unit } u \text{ equals } \|l\|_2.$$

$$l = v_{xy} = A_{1:2}x$$

$$\max_{x \in F} \|A_{1:2}x\|_2 = \max_{x \in F} \max_{\|u\|=1} u^T (A_{1:2}x) \text{ by substitution}$$

$$\max_{\|u\|=1} \max_{x \in F} u^T (A_{1:2}x) = \max_{x \in F} \max_{\|u\|=1} u^T (A_{1:2}x) \text{ because both } F \text{ and the unit circle } U = \{u \in \mathbb{R}^2 : \|u\| = 1\} \text{ are compact and the function } f(u, t) := u^T (A_{1:2}x) \text{ is continuous on the compact set } U \times F \text{ and the extreme value theorem guarantees a maximum of } f \text{ on the product set and both maximizations are finite and over compact sets}$$

$$u^T (A_{1:2}x) = (A_{1:2}^T u)^T x = c(u)^T x$$

$$\text{where } c(u) := A_{1:2}^T u \in \mathbb{R}^6$$

$$\max_{x \in F} \max_{\|u\|=1} u^T (A_{1:2}x) = \max_{\|u\|=1} \max_{x \in F} c(u)^T x \text{ by substitution}$$

$$u = \begin{pmatrix} \cos(\theta_{\text{des}}) \\ \sin(\theta_{\text{des}}) \end{pmatrix}$$

$$c(u) := A_{1:2}^T u = \cos(\theta_{\text{des}}) A_{1:2}^T e_1 + \sin(\theta_{\text{des}}) A_{1:2}^T e_2$$

$$p := \max_{x \in F} c^T x$$

$$\max_{\|u\|=1} \max_{x \in F} c(u)^T x = \max_{\theta_{\text{des}} \in [0, 2\pi)} p$$

$$x_{\text{new}} = x$$

$$A_{\text{new}} = A_{1:10}$$

$$b_{\text{new}} = b_{1:10}$$

$$c_{\text{new}} = \left(\forall x: \cos(\theta_{\text{des}}) A_{(\text{new})_x} + \sin(\theta_{\text{des}}) A_{(\text{new})_y} \right)$$

This then outputs the optimized vector x_{new}^* we then plug in that vector into v_x and v_y to get v_r