



Given the measures of l_l , θ_{r0} , and θ_{r1} , the directed distance of x_r , y_r , x_{tl} , y_{tl} , x_{th} , and y_{th} , and $\overrightarrow{P_{tl1}C_{tl1}}$ and $\overrightarrow{P_{tl0}C_{tl0}}$ are \perp to the tangent lines at the points C_{tl1} and C_{tl0} on $\overline{C_{tr0}C_{tr1}}$; find the directed distance of Δx_r and Δy_r

The measures of l_l , θ_{r0} , and θ_{r1} and the directed distance of x_r , y_r , x_{tl} , y_{tl} , x_{th} , and y_{th} because they are given

Let φ_0 be the angle $\overline{B_rC_{tl0}}$ makes with the x-axis

Let φ_1 be the angle $\overline{B_rC_{tl1}}$ makes with the x-axis

$\Delta\theta_r + \varphi_0 = \varphi_1$ by the add. post.

$\theta_{r1} = \varphi_1 + \frac{\pi}{2}$ and $\theta_{r0} = \varphi_0 + \frac{\pi}{2}$ by the tangent radius theorem

$\Delta\theta_r = \varphi_1 - \varphi_0$, $\varphi_1 = \theta_{r1} - \frac{\pi}{2}$, and $\varphi_0 = \theta_{r0} - \frac{\pi}{2}$ by subtraction

$\Delta\theta_r = \theta_{r1} - \frac{\pi}{2} - (\theta_{r0} - \frac{\pi}{2})$ by substitution

$\Delta\theta_r = \theta_{r1} - \frac{\pi}{2} - \theta_{r0} + \frac{\pi}{2}$ by multiplication

$\Delta\theta_r = \theta_{r1} - \theta_{r0}$ by substitution

$r_l = r_r + x_{tl}$ by the segment addition postulate

$r_r = r_l - x_{tl}$ by subtraction

$l_r = r_r \Delta\theta_r$ by the arc length formula

$\frac{l_r}{\Delta\theta_r} = r_r$ by division

$M_l A = \Delta x_l$ by the definition of \cong line segments

$AB_l = B_l M_l + M_l A$ by the segment addition postulate

$AB_l - M_l A = B_l M_l$ by subtraction

$AB_l - \Delta x_l = B_l M_l$ by substitution

$\frac{l_l}{\theta_l} - \Delta x_l = B_l M_l$ by substitution

Δy_l is an opposite side to $\angle\theta_l$ by the definition of an opposite side

$\overline{AB_l}$ is an adjacent side to $\angle\theta_l$ by the definition of an adjacent side

$\overline{AB_l}$ is the hypotenuse of $\triangle BMC$ by as the side opposite to the right \angle of a \triangle is the hypotenuse

$\sin(\theta_l) = \frac{\Delta y_l}{A_l B_l}$, $\cos(\theta_l) = \frac{B_l M_l}{AB_l}$ by the definition of sine and cosine in right angle trigonometry

$\sin(\theta_l) = \frac{\Delta y_l}{\frac{l_l}{\theta_l}}$, $\cos(\theta_l) = \frac{AB_l - \Delta x_l}{A} B_l$ by substitution

$\left(\frac{l_l}{\theta_l}\right) \sin(\theta_l) = \Delta y_l$, $AB_l \cos(\theta_l) = AB_l - \Delta x_l$ by multiplication

$AB_l \cos(\theta_l) - AB_l = -\Delta x_l$ by subtraction

$-AB_l \cos(\theta_l) + AB_l = \Delta x_l$ by division

$-AB_l (\cos(\theta_l) + 1) = \Delta x_l$ by substitution

$-\left(\frac{l_l}{\theta_l}\right) (\cos(\theta_l) + 1) = \Delta x_l$ by substitution

$\left(\frac{l_l}{\theta_l}\right) (\cos(\theta_l) + 1) = \Delta x_l$ by the magnitude of $\overline{B_l A}$