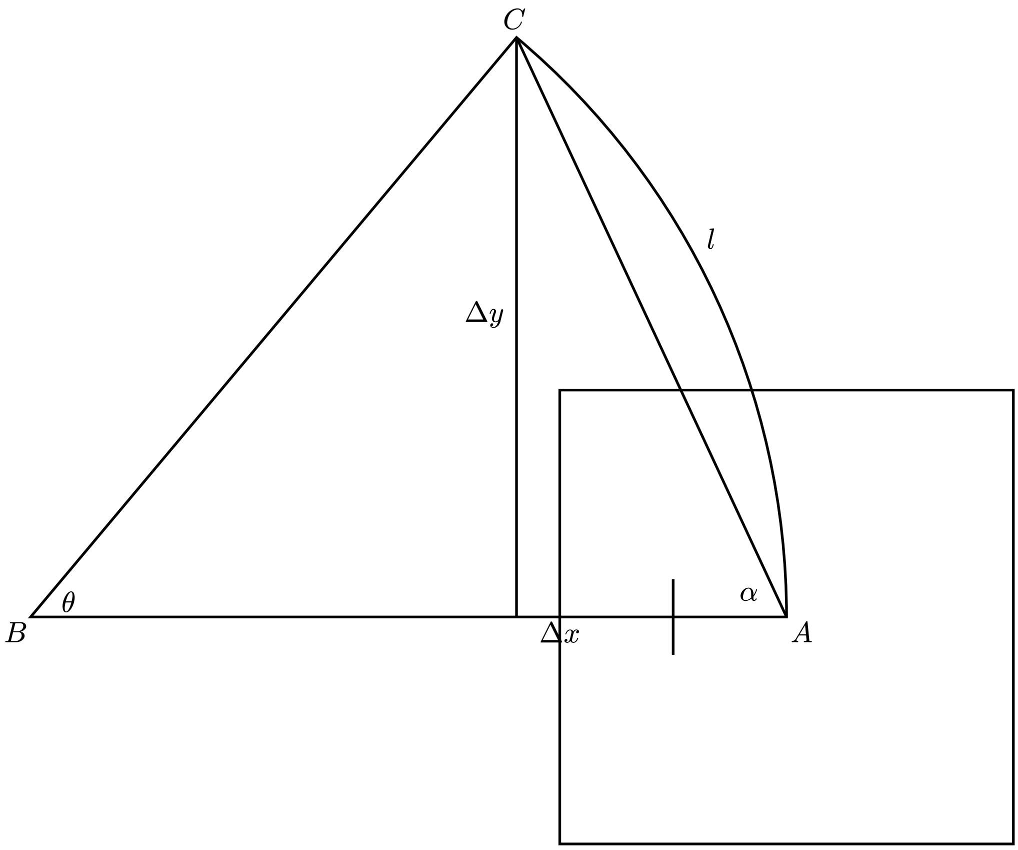


Given the measures of $l$ and $\theta$; AC is an arc; find $\Delta x$ and $\Delta y$

The measures of $l$ and $\theta$ and AC is an arc because they are given

$l = r\theta$ by the arc length formula in radians

$l = (c + \Delta x_v)\theta$ by substitution

$\frac{l}{\theta} = c + \Delta x_v$ by division

$\frac{l}{\theta} - \Delta x_v = c$ by subtraction

$\overline{AB} \cong \overline{BC}$ because radii $\cong$

$AB = BC$ by the definition of $\cong$ segments

$\triangle\ ABC$ is an isosceles $\triangle$ by the definition of an isosceles $\triangle$

$\alpha \cong \angle BCA$ by base $\angle$'s $\cong$

$\alpha = m\angle BCA$ by the definition of $\cong$ $\angle$'s

$\pi = \theta + \alpha + m\angle BCA$ by the $\angle$ sum Th. in radians

$\pi = \theta + \alpha + \alpha$ by substitution

$\pi = \theta + 2\alpha$ by substitution

$\pi - \theta = 2\alpha$ by subtraction

$\frac{\pi - \theta}{2} = \alpha$ by subtraction

$\frac{\pi}{2} - \frac{\theta}{2} = \alpha$ by the distributive property

$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\theta)}$ by the Law of Sines

$\frac{\frac{l}{\theta}-\Delta x_v}{\sin\left(\frac{\pi}{2}-\frac{\theta}{2}\right)} = \frac{b}{\sin(\theta)}$ by substitution

$\cos\left(\frac{\pi}{2}-x\right) = \sin(x)$ and $\sin\left(\frac{\pi}{2}-x\right) = \cos(x)$ by the complementary-angle identities

$\frac{\frac{l}{\theta}-\Delta x_v}{\cos\left(\frac{\theta}{2}\right)} = \frac{b}{\sin(\theta)}$ by substitution

$\frac{l}{\cos\left(\frac{\theta}{2}\right)\theta} = \frac{b}{\sin(\theta)}$ by the associative property

$\sin(\theta)\left(\frac{l}{\cos\left(\frac{\theta}{2}\right)\theta}\right) = b$ by multiplication

$\frac{\sin(\theta)l}{\cos\left(\frac{\theta}{2}\right)\theta} = b$ by the distributive property

$\sin(2x) = 2\sin(x)\cos(x)$ by the sine double-angle formula

$\sin(x) = 2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)$ by division

$\frac{2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)l}{\cos\left(\frac{\theta}{2}\right)\theta} = b$ by substitution

$\left(2\sin\left(\frac{\theta}{2}\right)\right)\left(\frac{l}{\theta}-\Delta x_v\right) = b$ by division

$\sin(\alpha) = \frac{\text{opposite}}{\text{hypotenuse}}$ by the definition of sine in right angle trigonometry

$\cos(\alpha) = \frac{\text{adjacent}}{\text{hypotenuse}}$ by the definition of cosine in right angle trigonometry

$\sin(\alpha) = \frac{\Delta y}{b}$ and $\cos(\alpha) = \frac{-\Delta x}{b}$ by substitution

$b\sin(\alpha) = \Delta y$ and $b\cos(\alpha) = -\Delta x$ by multiplication

$\left(2\sin\left(\frac{\theta}{2}\right)\right)\left(\frac{l}{\theta}-\Delta x_v\right)\sin(\alpha) = \Delta y$ and $\left(2\sin\left(\frac{\theta}{2}\right)\right)\left(\frac{l}{\theta}-\Delta x_v\right)\cos(\alpha) = -\Delta x$ by substitution

$\left(2\sin\left(\frac{\theta}{2}\right)\right)\left(\frac{l}{\theta}-\Delta x_v\right)\sin\left(\frac{\pi}{2}-\frac{\theta}{2}\right) = \Delta y$ and $\left(2\sin\left(\frac{\theta}{2}\right)\right)\left(\frac{l}{\theta}-\Delta x_v\right)\cos\left(\frac{\pi}{2}-\frac{\theta}{2}\right) = -\Delta x$ by substitution

$\left(2\sin\left(\frac{\theta}{2}\right)\right)\left(\frac{l}{\theta}-\Delta x_v\right)\cos\left(\frac{\theta}{2}\right) = \Delta y$ and $\left(2\sin\left(\frac{\theta}{2}\right)\right)\left(\frac{l}{\theta}-\Delta x_v\right)\sin\left(\frac{\theta}{2}\right) = -\Delta x$ by substitution

$\left(2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\right)\left(\frac{l}{\theta}-\Delta x_v\right) = \Delta y$ and $\left(2\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)\right)\left(\frac{l}{\theta}-\Delta x_v\right) = -\Delta x$ by the associative property

$\sin(\theta)\left(\frac{l}{\theta}-\Delta x_v\right) = \Delta y$ and $(2\sin^2\left(\frac{\theta}{2}\right)\left(\frac{l}{\theta}-\Delta x_v\right) = -\Delta x$ by substitution

$-2\sin^2\left(\frac{\theta}{2}\right)\left(\frac{l}{\theta}-\Delta x_v\right) = \Delta x$ by division

$\sin^2(x) = \frac{1-\cos(2x)}{2}$ by the sine lowering power formula

$-2\left(\frac{1-\cos(\theta)}{2}\right)\left(\frac{l}{\theta}-\Delta x_v\right) = \Delta x$ by substitution

$-((1-\cos(\theta))\left(\frac{l}{\theta}-\Delta x_v\right) = \Delta x$ by the associative property

$\left(\frac{l}{\theta}-\Delta x_v\right)(-1+\cos(\theta)) = \Delta x$ by substitution