

The arc-length s formula in degrees is

$$s = \frac{\theta}{360^\circ} \pi d.$$

If the timestep of $\Delta\theta$ (in degrees) is one second, then the formula for RPM is

$$\text{RPM} = \frac{\Delta\theta}{360^\circ} \times 60 = \frac{\Delta\theta}{6}.$$

You can then take the derivative of the arc-length formula with respect to time to give the linear velocity v . Substituting the definition of RPM, you get

$$v = \frac{\text{RPM}}{60} \pi d = \frac{\text{RPM } g_r \pi d}{60},$$

where g_r is the gear ratio.

Next we can include motor dynamics to develop a more robust model of the robot. First we take the definition of voltage (Ohm's Law) as it relates to a motor (the motor also acting as a generator in some sense):

$$V = IR + E_{\text{back}},$$

where V is the applied voltage, I the current, R the winding resistance, and E_{back} the back-electromotive force (EMF).

$$E_{\text{back}} = k_e \omega.$$

We also know that the applied voltage is countered by the back EMF; therefore,

$$\omega_0 = \frac{V}{k_e}.$$

Also, the more current you put in, the more torque is output, hence:

$$\tau = k_t I,$$

with $k_t = k_e$ by Faraday's law. Next we rearrange Ohm's Law to get the no-load current:

$$I_0 = \frac{V}{R}.$$

Substituting into the torque equation gives:

$$\tau = k_t \frac{V}{R}.$$

Returning to the motor voltage equation and substituting E_{back} ,

$$V = I R + k_e \omega.$$

Substitute $I = \tau/k_t$ to obtain:

$$V = \frac{\tau}{k_t} R + k_e \omega.$$

Rearranging,

$$k_e \omega = V - \frac{\tau}{k_t} R.$$

Dividing both sides by k_e :

$$\omega = \frac{V}{k_e} - \frac{\tau R}{k_t k_e}.$$

Noting that $\omega_0 = V/k_e$ and $\tau_0 = k_t I_0 = k_t V/R$, we can write:

$$\omega = \omega_0 - \frac{\tau}{\tau_0} \omega_0 = \omega_0 \left(1 - \frac{\tau}{\tau_0}\right).$$

Now ω is the no-load speed that you can find using the gear ratio's and RPM. $1 - \frac{\tau}{\tau_0}$ is the load factor which can be found where τ is the torque of the wheel and τ_0 is the available torque, from the motor. To find the available torque you can use the gear ratio, radius of the wheel, and stall torque, using this formula (where η is efficiency and is around 80% for a vex motor and gear box)

$$\tau_{\text{avail}} = \tau_0 \cdot g_r \cdot \eta$$

To find the torque on the wheel we can use the definition of force as it relates to friction (where F_f is friction force).

$$\tau_f = F_f \cdot r$$

For finding rolling resistance which in this case is friction force you can use the formula

$$F_{rr} = C_{rr} \cdot N$$

For this field and robot we will use around 0.01 for C_{rr} .

Finally we can plug in this formula to our formula to find linear velocity and we get the total equation:

$$v = \frac{RPM \cdot g_r \cdot \pi \cdot d}{60} \left(1 - \frac{\tau_0 \cdot g_r \cdot \eta}{C_{rr} \cdot N \cdot r}\right)$$