Given

$V = IR + E_{\text{mf}}$

$E_{\text{mf}} = \omega\kappa_e$

$\tau = I\kappa_t$

$\tau = J\alpha + B\omega + \tau_{\text{load}}$

Find V voltage at some initial angular velocity ω, the wanted angular acceleration α

We know I, J, τ_{load}, R, B, ω, α, κ_t, and κ_e. So therefore we only need to find V as it is the only remaining variable.

$I\kappa_t = J\alpha + B\omega + \tau_{\text{load}}$ By substitution

$I = \frac{J\alpha+B\omega+\tau_{\text{load}}}{\kappa_t}$ By Division

$-IR = E_{\text{mf}} - V$ By Subtraction

$I = \frac{E_{\text{mf}}-V}{-R}$ By Division

$\frac{E_{\text{mf}}-V}{-R} = \frac{J\alpha+B\omega+\tau_{\text{load}}}{\kappa_t}$ By substitution

$\frac{\omega\kappa_e-V}{-R} = \frac{J\alpha+B\omega+\tau_{\text{load}}}{\kappa_t}$ By substitution

$\omega\kappa_e - V = \frac{J\alpha+B\omega+\tau_{\text{load}}}{\kappa_t}(-R)$ By multiplication

$-V = \frac{J\alpha+B\omega+\tau_{\text{load}}}{\kappa_t}(-R) - \omega\kappa_e$ By subtraction

$V = -\left(\frac{J\alpha+B\omega+\tau_{\text{load}}}{\kappa_t}(-R) - \omega\kappa_e\right)$ By division

$V = \left(\frac{J\alpha+B\omega+\tau_{\text{load}}}{\kappa_t}(R) + \omega\kappa_e\right)$ By substitution

Define a function $V : \mathbb{R} \to \mathbb{R}$ by $V(\omega, \alpha) = \left(\frac{J\alpha+B\omega+\tau_{\text{load}}}{\kappa_t}(R) + \omega\kappa_e\right)$

End of Proof

Find the maximum α angular acceleration at some angular velocity

Note, the VEX motor controller has current limits so we must also account for those.

Define I_{min} and I_{max} as the current limits and define I_{clamp} as the clamped value of I

$I_{\text{clamp}} = \min(\max(I, I_{\text{min}}), I_{\text{max}})$ by the definition of clamping

$I_{\text{clamp}} = \min\left(\max\left(\left(\frac{E_{\text{mf}}-V}{-R}\right), I_{\text{min}}\right), I_{\text{max}}\right)$ by substitution

$I_{\text{clamp}} = \min\left(\max\left(\left(\frac{\omega\kappa_e-V}{-R}\right), I_{\text{min}}\right), I_{\text{max}}\right)$ by substitution

Define $V_{\text{max}} = V$ as the max voltage

$I_{\text{clamp}} = \min\left(\max\left(\left(\frac{\omega\kappa_e-V_{\text{max}}}{-R}\right), I_{\text{min}}\right), I_{\text{max}}\right)$ by substitution

Restrict I by $I_{\text{clamp}} = I$ by the note

$-J\alpha = B\omega + \tau_{\text{load}} - \tau$ by Subtraction

$\alpha = \frac{B\omega+\tau_{\text{load}}-\tau}{-J}$ by Division

$\alpha = \frac{B\omega+\tau_{\text{load}}-I\kappa_t}{-J}$ by Substitution

$\alpha = \frac{B\omega+\tau_{\text{load}}-I_{\text{clamp}}\kappa_t}{-J}$ by Substitution

Define a function α by $\alpha(\omega) = \frac{B\omega+\tau_{\text{load}}-I_{\text{clamp}}\kappa_t}{-J}$