



Given the measures of  $l$  and  $\theta$ ;  $AC$  is an arc; find  $\Delta x$  and  $\Delta y$

The measures of  $l$  and  $\theta$  are given and  $AC$  is an arc because they are given

$l = r\theta$  by the arc length formula in radians

$l = a\theta$  by substitution

$\frac{l}{\theta} = a$  by division

$\overline{AB} \cong \overline{BC}$  because radii  $\cong$

$AB = BC$  by the definition of  $\cong$  segments

$\triangle ABC$  is an isosceles  $\triangle$  by the definition of an isosceles  $\triangle$

Define the base  $\angle$  other than  $\alpha$  inside of  $\triangle ABC$  to be  $\gamma$

$\alpha \cong \angle BCA$  by base  $\angle$ 's  $\cong$

$\alpha = m\angle BCA$  by the definition of  $\cong \angle$ 's

$\pi = \theta + \alpha + m\angle BCA$  by the  $\angle$  sum Th. in radians

$\pi = \theta + \alpha + \alpha$  by substitution

$\pi = \theta + 2\alpha$  by substitution

$\pi - \theta = 2\alpha$  by subtraction

$\frac{\pi - \theta}{2} = \alpha$  by subtraction

$\frac{\pi}{2} - \frac{\theta}{2} = \alpha$  by the distributive property

$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\theta)}$  by the Law of Sines

$\frac{\frac{l}{\theta}}{\sin(\frac{\pi}{2} - \frac{\theta}{2})} = \frac{b}{\sin(\theta)}$  by substitution

$\cos(\frac{\pi}{2} - x) = \sin(x)$  and  $\sin(\frac{\pi}{2} - x) = \cos(x)$  by the complementary-angle identities

$\frac{\frac{l}{\theta}}{\cos(\frac{\theta}{2})} = \frac{b}{\sin(\theta)}$  by substitution

$\frac{l}{\cos(\frac{\theta}{2})\theta} = \frac{b}{\sin(\theta)}$  by the associative property

$\sin(\theta) \left( \frac{l}{\cos(\frac{\theta}{2})\theta} \right) = b$  by multiplication

$\frac{\sin(\theta)l}{\cos(\frac{\theta}{2})\theta} = b$  by the distributive property

$\sin(2x) = 2 \sin(x) \cos(x)$  by the sine double-angle formula

$\sin(x) = 2 \sin(\frac{x}{2}) \cos(\frac{x}{2})$  by division

$\frac{2 \sin(\frac{\theta}{2}) \cos(\frac{\theta}{2}) l}{\cos(\frac{\theta}{2})\theta} = b$  by substitution

$\frac{2 \sin(\frac{\theta}{2}) l}{\theta} = b$  by division

$\sin(\alpha) = \frac{\text{opposite}}{\text{hypotenuse}}$  by the definition of sine in right angle trigonometry

$\cos(\alpha) = \frac{\text{adjacent}}{\text{hypotenuse}}$  by the definition of cosine in right angle trigonometry

$\sin(\alpha) = \frac{\Delta y}{b}$  and  $\cos(\alpha) = \frac{-\Delta x}{b}$  by substitution

$b \sin(\alpha) = \Delta y$  and  $b \cos(\alpha) = -\Delta x$  by multiplication

$\frac{2 \sin(\frac{\theta}{2})l}{\theta} \sin(\alpha) = \Delta y$  and  $\frac{2 \sin(\frac{\theta}{2})l}{\theta} \cos(\alpha) = -\Delta x$  by substitution

$\frac{2 \sin(\frac{\theta}{2})l}{\theta} \sin(\frac{\pi}{2} - \frac{\theta}{2}) = \Delta y$  and  $\frac{2 \sin(\frac{\theta}{2})l}{\theta} \cos(\frac{\pi}{2} - \frac{\theta}{2}) = -\Delta x$  by substitution

$\frac{2 \sin(\frac{\theta}{2})l}{\theta} \cos(\frac{\theta}{2}) = \Delta y$  and  $\frac{2 \sin(\frac{\theta}{2})l}{\theta} \sin(\frac{\theta}{2}) = -\Delta x$  by substitution

$\frac{2 \sin(\frac{\theta}{2})l \cos(\frac{\theta}{2})}{\theta} = \Delta y$  and  $\frac{2 \sin(\frac{\theta}{2})l \sin(\frac{\theta}{2})}{\theta} = -\Delta x$  by the associative property

$\frac{\sin(\theta)l}{\theta} = \Delta y$  and  $\frac{2 \sin^2(\frac{\theta}{2})l}{\theta} = -\Delta x$  by substitution

$-\frac{2 \sin^2(\frac{\theta}{2})l}{\theta} = \Delta x$  by division

$\sin^2(x) = \frac{1 - \cos(2x)}{2}$  by the sine lowering power formula

$-\frac{2 \left( \frac{1 - \cos(\theta)}{2} \right) l}{\theta} = \Delta x$  by substitution

$-\frac{(1 - \cos(\theta))l}{\theta} = \Delta x$  by the associative property

$\frac{l}{\theta}(-1 + \cos(\theta)) = \Delta x$  by substitution