



Given the measures of l and θ ; AC is an arc; find Δx and Δy

The measures of l and θ are given and AC is an arc because they are given

$l = r\theta$ by the arc length formula in radians

$l = (c + \Delta x_v)\theta$ by substitution

$\frac{l}{\theta} - \Delta x_v = c + \Delta x_v$ by division

$\frac{l}{\theta} - \Delta x_v - \Delta x_v = c$ by subtraction

$\overline{AB} \cong \overline{BC}$ because radii \cong

$AB = BC$ by the definition of \cong segments

$\triangle ABC$ is an isosceles \triangle by the definition of an isosceles \triangle

Define the base \angle other than α inside of $\triangle ABC$ to be γ

$\alpha \cong \angle BCA$ by base \angle 's \cong

$\alpha = m\angle BCA$ by the definition of $\cong \angle$'s

$\pi = \theta + \alpha + m\angle BCA$ by the \angle sum Th. in radians

$\pi = \theta + \alpha + \alpha$ by substitution

$\pi = \theta + 2\alpha$ by substitution

$\pi - \theta = 2\alpha$ by subtraction

$\frac{\pi - \theta}{2} = \alpha$ by subtraction

$\frac{\pi}{2} - \frac{\theta}{2} = \alpha$ by the distributive property

$$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\theta)} \text{ by the Law of Sines}$$

$$\frac{\frac{l}{\theta} - \Delta x_v}{\sin(\frac{\pi}{2} - \frac{\theta}{2})} = \frac{b}{\sin(\theta)} \text{ by substitution}$$

$$\cos(\frac{\pi}{2} - x) = \sin(x) \text{ and } \sin(\frac{\pi}{2} - x) = \cos(x) \text{ by the complementary-angle identities}$$

$$\frac{\frac{l}{\theta} - \Delta x_v}{\cos(\frac{\theta}{2})} = \frac{b}{\sin(\theta)} \text{ by substitution}$$

$$\frac{l}{\cos(\frac{\theta}{2})\theta} = \frac{b}{\sin(\theta)} \text{ by the associative property}$$

$$\sin(\theta) \left(\frac{l}{\cos(\frac{\theta}{2})\theta} \right) = b \text{ by multiplication}$$

$$\frac{\sin(\theta)l}{\cos(\frac{\theta}{2})\theta} = b \text{ by the distributive property}$$

$$\sin(2x) = 2 \sin(x) \cos(x) \text{ by the sine double-angle formula}$$

$$\sin(x) = 2 \sin(\frac{x}{2}) \cos(\frac{x}{2}) \text{ by division}$$

$$\frac{2 \sin(\frac{\theta}{2}) \cos(\frac{\theta}{2}) l}{\cos(\frac{\theta}{2})\theta} = b \text{ by substitution}$$

$$(2 \sin(\frac{\theta}{2})) \left(\frac{l}{\theta} - \Delta x_v \right) = b \text{ by division}$$

$$\sin(\alpha) = \frac{\text{opposite}}{\text{hypotenuse}} \text{ by the definition of sine in right angle trigonometry}$$

$$\cos(\alpha) = \frac{\text{adjacent}}{\text{hypotenuse}} \text{ by the definition of cosine in right angle trigonometry}$$

$$\sin(\alpha) = \frac{\Delta y}{b} \text{ and } \cos(\alpha) = \frac{-\Delta x}{b} \text{ by substitution}$$

$$b \sin(\alpha) = \Delta y \text{ and } b \cos(\alpha) = -\Delta x \text{ by multiplication}$$

$$(2 \sin(\frac{\theta}{2})) \left(\frac{l}{\theta} - \Delta x_v \right) \sin(\alpha) = \Delta y \text{ and } (2 \sin(\frac{\theta}{2})) \left(\frac{l}{\theta} - \Delta x_v \right) \cos(\alpha) = -\Delta x \text{ by substitution}$$

$$(2 \sin(\frac{\theta}{2})) \left(\frac{l}{\theta} - \Delta x_v \right) \sin(\frac{\pi}{2} - \frac{\theta}{2}) = \Delta y \text{ and } (2 \sin(\frac{\theta}{2})) \left(\frac{l}{\theta} - \Delta x_v \right) \cos(\frac{\pi}{2} - \frac{\theta}{2}) = -\Delta x \text{ by substitution}$$

$$(2 \sin(\frac{\theta}{2})) \left(\frac{l}{\theta} - \Delta x_v \right) \cos(\frac{\theta}{2}) = \Delta y \text{ and } (2 \sin(\frac{\theta}{2})) \left(\frac{l}{\theta} - \Delta x_v \right) \sin(\frac{\theta}{2}) = -\Delta x \text{ by substitution}$$

$$(2 \sin(\frac{\theta}{2}) \cos(\frac{\theta}{2})) \left(\frac{l}{\theta} - \Delta x_v \right) = \Delta y \text{ and } (2 \sin(\frac{\theta}{2}) \sin(\frac{\theta}{2})) \left(\frac{l}{\theta} - \Delta x_v \right) = -\Delta x \text{ by the associative property}$$

$$\sin(\theta) \left(\frac{l}{\theta} - \Delta x_v \right) = \Delta y \text{ and } (2 \sin^2(\frac{\theta}{2})) \left(\frac{l}{\theta} - \Delta x_v \right) = -\Delta x \text{ by substitution}$$

$$-2 \sin^2(\frac{\theta}{2}) \left(\frac{l}{\theta} - \Delta x_v \right) = \Delta x \text{ by division}$$

$$\sin^2(x) = \frac{1 - \cos(2x)}{2} \text{ by the sine lowering power formula}$$

$$-2 \left(\frac{1 - \cos(\theta)}{2} \right) \left(\frac{l}{\theta} - \Delta x_v \right) = \Delta x \text{ by substitution}$$

$$-((1 - \cos(\theta)) \left(\frac{l}{\theta} - \Delta x_v \right)) = \Delta x \text{ by the associative property}$$

$$\left(\frac{l}{\theta} - \Delta x_v \right) (-1 + \cos(\theta)) = \Delta x \text{ by substitution}$$