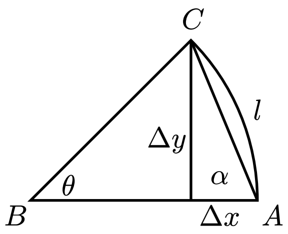

Given the measures of l and θ; AC is an arc; find Δx and Δy

The measures of l and θ are given and AC is an arc because they are given

$l = r\theta$ by the arc length formula in radians

$l = a\theta$ by substitution

$\frac{l}{\theta} = a$ by division

$\overline{AB} \cong \overline{BC}$ because radii $\cong$

$AB = BC$ by the definition of $\cong$ segments

$\triangle ABC$ is an isosceles $\triangle$ by the definition of an isosceles $\triangle$

Define the base $\angle$ other than α inside of $\triangle ABC$ to be γ

$\alpha \cong \angle BCA$ by base $\angle$'s $\cong$

$\alpha = m\angle BCA$ by the definition of $\cong$ $\angle$'s

$\pi = \theta + \alpha + m\angle BCA$ by the $\angle$ sum Th. in radians

$\pi = \theta + \alpha + \alpha$ by substitution

$\pi = \theta + 2\alpha$ by substitution

$\pi - \theta = 2\alpha$ by subtraction

$\frac{\pi-\theta}{2} = \alpha$ by subtraction

$\frac{\pi}{2} - \frac{\theta}{2} = \alpha$ by the distributive property

$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\theta)}$ by the Law of Sines

$\frac{\frac{l}{\theta}}{\sin\left(\frac{\pi}{2}-\frac{\theta}{2}\right)} = \frac{b}{\sin(\theta)}$ by substitution

$\cos\left(\frac{\pi}{2} - x\right) = \sin(x)$ and $\sin\left(\frac{\pi}{2} - x\right) = \cos(x)$ by the complementary-angle identities

$\frac{\frac{l}{\theta}}{\cos\left(\frac{\theta}{2}\right)} = \frac{b}{\sin(\theta)}$ by substitution

$\frac{l}{\cos\left(\frac{\theta}{2}\right)\theta} = \frac{b}{\sin(\theta)}$ by the associative property

$\sin(\theta)\left(\frac{l}{\cos\left(\frac{\theta}{2}\right)\theta}\right) = b$ by multiplication

$\frac{\sin(\theta)l}{\cos\left(\frac{\theta}{2}\right)\theta} = b$ by the distributive property

$\sin(2x) = 2\sin(x)\cos(x)$ by the sine double-angle formula

$\sin(x) = 2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)$ by division

$\frac{2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)l}{\cos\left(\frac{\theta}{2}\right)\theta} = b$ by substitution

$\frac{2\sin\left(\frac{\theta}{2}\right)l}{\theta} = b$ by division

$\sin(\alpha) = \frac{\text{opposite}}{\text{hypotenuse}}$ by the definition of sine in right angle trigonometry

$\cos(\alpha) = \frac{\text{adjacent}}{\text{hypotenuse}}$ by the definition of cosine in right angle trigonometry

$\sin(\alpha) = \frac{\Delta y}{b}$ and $\cos(\alpha) = \frac{-\Delta x}{b}$ by substitution

$b\sin(\alpha) = \Delta y$ and $b\cos(\alpha) = -\Delta x$ by multiplication

$\frac{2\sin\left(\frac{\theta}{2}\right)l}{\theta}\sin(\alpha) = \Delta y$ and $\frac{2\sin\left(\frac{\theta}{2}\right)l}{\theta}\cos(\alpha) = -\Delta x$ by substitution

$\frac{2\sin\left(\frac{\theta}{2}\right)l}{\theta}\sin\left(\frac{\pi}{2} - \frac{\theta}{2}\right) = \Delta y$ and $\frac{2\sin\left(\frac{\theta}{2}\right)l}{\theta}\cos\left(\frac{\pi}{2} - \frac{\theta}{2}\right) = -\Delta x$ by substitution

$\frac{2\sin\left(\frac{\theta}{2}\right)l}{\theta}\cos\left(\frac{\theta}{2}\right) = \Delta y$ and $\frac{2\sin\left(\frac{\theta}{2}\right)l}{\theta}\sin\left(\frac{\theta}{2}\right) = -\Delta x$ by substitution

$\frac{2\sin\left(\frac{\theta}{2}\right)l\cos\left(\frac{\theta}{2}\right)}{\theta} = \Delta y$ and $\frac{2\sin\left(\frac{\theta}{2}\right)l\sin\left(\frac{\theta}{2}\right)}{\theta} = -\Delta x$ by the associative property

$\sin(u)\cos(v) = \frac{1}{2}[\sin(u+v) + \sin(u-v)]$ by the product-to-sum formula

$\frac{2\sin^2\left(\frac{\theta}{2}\right)l}{\theta} = -\Delta x$ and $\frac{2\left(\frac{1}{2}\left[\sin\left(\frac{\theta}{2}+\frac{\theta}{2}\right)+\sin\left(\frac{\theta}{2}-\frac{\theta}{2}\right)\right]\right)l}{\theta} = \Delta y$ by substitution

$-\frac{2\sin^2\left(\frac{\theta}{2}\right)l}{\theta} = \Delta x$ by division

$\sin^2(x) = \frac{1-\cos(2x)}{2}$ by the sine lowering power formula

$-\frac{2\left(\frac{1-\cos(\theta)}{2}\right)l}{\theta} = \Delta x$ and $\frac{\sin(\theta)l}{\theta} = \Delta y$ by substitution

$-\frac{(1-\cos(\theta))l}{\theta} = \Delta x$ by the associative property

$\frac{l}{\theta}(-1 + \cos(\theta)) = \Delta x$ by substitution