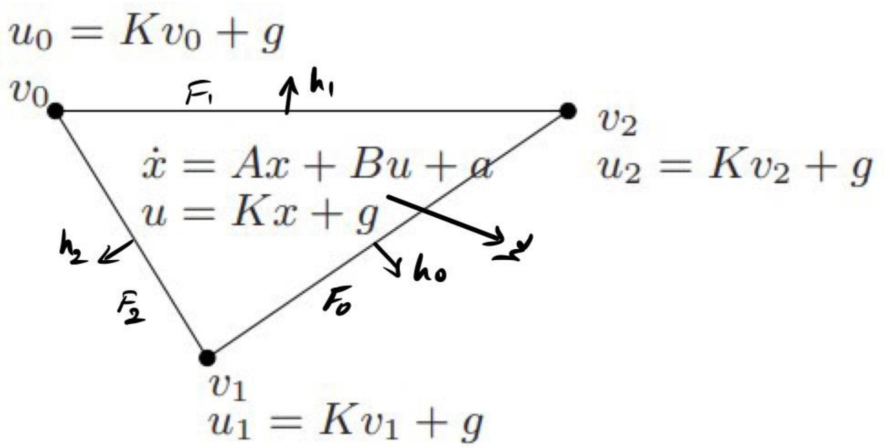
Reach Control Problem:



On Vertex v_i:

- Invariance Conditions:

$$h_j^T\,(Av_i + Bu_i + a) = h_j^T \alpha_i < 0 \qquad i \in I/\{i\}$$

- Flow Condition:

$$\xi^T(Av_i + Bu_i + a) = \xi^T \alpha_i > 0$$

$$\underbrace{\begin{bmatrix} v_0^T & 1 \\ \vdots & \\ v_n^T & 1 \end{bmatrix}}_{\text{invertible}} \begin{bmatrix} K^T \\ g^T \end{bmatrix} = \begin{bmatrix} u_0^T \\ \vdots \\ u_n^T \end{bmatrix}$$

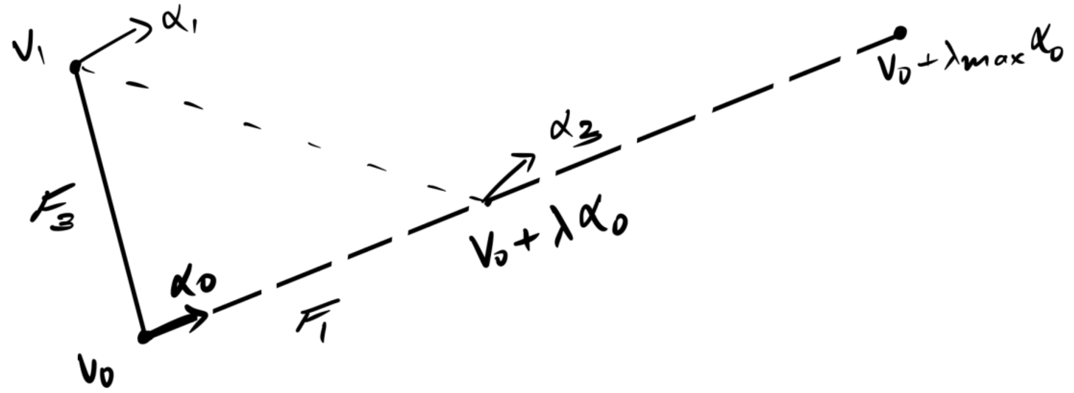
L.C.G.J.M. Habets and J.H. van Schuppen. Automatica 2004.

Notation Conventions:

- F_i is the facet opposite to the vertex v_i.
- h_i is the outward normal of the facet F_i.
- F_0 $-$ Exit Facet.
- $\{F_1, F_2, \ldots, F_n\}$ $-$ Restricted Facet.
- v_0 $-$ Restricted Vertex.
- $\{v_1, v_2, \ldots, v_n\}$ $-$ Flow Vertices
- ξ $-$ Flow Vector
- $\alpha_i = A\,v_i + Bu_i + a$
- $I = \{1, 2, \ldots, n\}$ $-$ Flow index set

Simplex Generation Problem:

Find the final vertex of the simplex such that the Reach Control Problem is solvable on it given the first two vertices and the direction of flow.



- $\boxed{Maximize \quad J(u_i) + \lambda + \left(\sum_i \xi^T \alpha_i\right) \quad \forall\, i \in I}$
- $(where\ v_n = v_0 + \lambda\, \alpha_0)$
  - $Subject\ to$:
    - $Invariance\ and\ flow\ conditions\ on\ flow\ vertices$:
      - $h_j^T\,(Av_i + Bu_i + a) = h_j^T \alpha_i < 0 \quad j \in \frac{I}{\{i\}}\ , \quad i \in I$
      - $\xi^T(Av_i + Bu_i + a) = \xi^T \alpha_i > 0 \quad i \in I$
    - $0 < \lambda \le \lambda_{\{max\}}$
    - $u_{\{min\}} \le u_i \le u_{\{max\}}$

Demonstration - Case of an affine linear system:

$$\dot{\boldsymbol{x}} = \begin{bmatrix} -0.1 & 0.2 \\ -1 & 0.4 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \boldsymbol{u} + \begin{bmatrix} -0.2 \\ -0.1 \end{bmatrix}$$

Limitations:

1. B matrix should be full row rank.
2. The propagation of simplices using the method described cannot guarantee convexity in all the cases.
3. The simplex chain is not unique but dependent on the support curve (which need not be feasible)