Components of Positive and Negative Sequence Phasors in Various Rotating Reference Frames

This document outlines the steps involved in obtaining the components of Poisitve and Negative sequence voltages in RRF. The time-domain variation of voltages in three-phase domain can be represented in terms of sequence components as

$$\begin{aligned} v_{an} &= k_1 \left[V^0 \cos\left(\delta^0\right) + V^+ \cos\left(\omega_s t + \delta^+\right) + V^- \cos\left(\omega_s t + \delta^-\right)\right] \\ v_{bn} &= k_1 \left[V^0 \cos\left(\delta^0\right) + V^+ \cos\left(\omega_s t + \delta^+ - \phi_s\right) + V^- \cos\left(\omega_s t + \delta^- + \phi_s\right)\right] \\ v_{cn} &= k_1 \left[V^0 \cos\left(\delta^0\right) + V^+ \cos\left(\omega_s t + \delta^+ + \phi_s\right) + V^- \cos\left(\omega_s t + \delta^- - \phi_s\right)\right] \end{aligned} \tag{1}$$

where $\phi = 2\pi/3$.

A. *Components in positive RRF*

The components of the signal in the positive RRF can be computed as

$$\begin{bmatrix} v_d^+ \\ v_q^+ \end{bmatrix} = k_1 \begin{bmatrix} \cos(\omega_n t) & \cos(\gamma_1) & \cos(\gamma_2) \\ -\sin(\omega_n t) & -\sin(\gamma_1) & -\sin(\gamma_2) \end{bmatrix} \mathbf{v} \tag{2}$$

where $\mathbf{v} = [v_{an}, v_{bn}, v_{cn}]^t$, $\gamma_1 = \omega_n t - \phi_s$ and $\gamma_2 = \omega_n t + \phi_s$.

In order to simplify the derivation, the transformation of positive and negative sequence components to the positive RRF is outlined independently and are subsequently merged to give equation (4) in the main manuscript.

Let $\omega_n t = \theta_n$ and $\omega_s t = \theta_s$. Using (2), the d^+-component of positive sequence voltage can be computed as

$$\begin{aligned} v_d^+ &= \frac{2}{3} V^+ \left[\cos(\theta_n)\cos(\theta_s + \delta^+) + \cos(\theta_n - \phi_s)\cos(\theta_s + \delta^+ - \phi_s) + \cos(\theta_n + \phi_s)\cos(\theta_s + \delta^+ + \phi_s)\right] \\ v_d^+ &= \frac{V^+}{3} \left[\cos(\theta_n + \theta_s + \delta^+) + \cos(\theta_n - \theta_s - \delta^+) + \cos(\theta_n + \theta_s + \delta^+ - 2\phi_s) + \cos(\theta_n - \theta_s - \delta^+) \right. \\ &\qquad \left. + \cos(\theta_n + \theta_s + \delta^+ + 2\phi_s) + \cos(\theta_n - \theta_s - \delta^+)\right] \\ v_d^+ &= \frac{V^+}{3} \left[3(\cos(\theta_n - \theta_s - \delta^+)) + \cos(\theta_n + \theta_s + \delta^+) + \cos(\theta_n + \theta_s + \delta^+ - 2\phi_s) + \cos(\theta_n + \theta_s + \delta^+ + 2\phi_s)\right] \\ v_d^+ &= \frac{V^+}{3} \left[3(\cos(\theta_n - \theta_s - \delta^+)) + \cos(\theta_n + \theta_s + \delta^+) + 2(\cos(\theta_n + \theta_s + \delta^+)\cos(-2\phi_s))\right] \\ v_d^+ &= V^+ \cos(\theta_s - \theta_n + \delta^+) \;=\; V^+ \cos(\Delta\omega t + \delta^+) \end{aligned} \tag{3}$$

Similarly the q^+ component of the positive sequence voltage can be computed as

$$\begin{aligned} v_q^+ &= -\frac{2}{3} V^+ \left[(\sin(\theta_n)\cos(\theta_s + \delta^+) + \sin(\theta_n - \phi_s)\cos(\theta_s + \delta^+ - \phi_s) + \sin(\theta_n + \phi_s)\cos(\theta_s + \delta^+ + \phi_s))\right] \\ v_q^+ &= \frac{-V^+}{3} \left[\sin(\theta_n + \theta_s + \delta^+) + \sin(\theta_n - \theta_s - \delta^+) + \sin(\theta_n + \theta_s + \delta^+ - 2\phi_s) + \sin(\theta_n - \theta_s - \delta^+) \right. \\ &\qquad \left. + \sin(\theta_n + \theta_s + \delta^+ + 2\phi_s) + \sin(\theta_n - \theta_s - \delta^+)\right] \\ v_q^+ &= \frac{-V^+}{3} \left[3(\sin(\theta_n - \theta_s - \delta^+)) + \sin(\theta_n + \theta_s + \delta^+) + \sin(\theta_n + \theta_s + \delta^+ - 2\phi_s) + \sin(\theta_n + \theta_s + \delta^+ + 2\phi_s)\right] \\ v_q^+ &= \frac{-V^+}{3} \left[3(\sin(\theta_n - \theta_s - \delta^+)) + \sin(\theta_n + \theta_s + \delta^+) + 2(\sin(\theta_n + \theta_s + \delta^+)\cos(-2\phi_s))\right] \\ v_q^+ &= V^+ \sin(\theta_s - \theta_n + \delta^+) \;=\; V^+ \sin(\Delta\omega t + \delta^+) \end{aligned} \tag{4}$$

The d^+-component of negative sequence voltage can be computed as

$$
\begin{aligned}
v_d^- &= \frac{2}{3}V^- \left[\cos(\theta_n) * \cos(\theta_s + \delta^-) + \cos(\theta_n - \phi_s) * \cos(\theta_s + \delta^- + \phi_s) + \cos(\theta_n + \phi_s) * \cos(\theta_s + \delta^- - \phi_s)\right] \\
v_d^- &= \frac{V^-}{3} \left[\cos(\theta_n + \theta_s + \delta^-) + \cos(\theta_n - \theta_s - \delta^-) + \cos(\theta_n + \theta_s + \delta^-) + \cos(\theta_n - \theta_s - \delta^- - 2\phi_s) \right. \\
&\qquad \left. + \cos(\theta_n + \theta_s + \delta^-) + \cos(\theta_n - \theta_s - \delta^- + 2\phi_s)\right] \\
v_d^- &= \frac{V^-}{3} \left[3(\cos(\theta_n + \theta_s + \delta^-)) + \cos(\theta_n - \theta_s - \delta^-) + \cos(\theta_n - \theta_s - \delta^- - 2\phi_s) + \cos(\theta_n - \theta_s - \delta^- + 2\phi_s)\right] \\
v_d^- &= \frac{V^-}{3} \left[3(\cos(\theta_n + \theta_s + \delta^-)) + \cos(\theta_n - \theta_s - \delta^-) + 2(\cos(\theta_n - \theta_s - \delta^-) \cos(-2\phi_s))\right] \\
v_d^- &= V^- \cos(\theta_s + \theta_n + \delta^-) \; = \; V^- \cos(\omega_s t + \omega_n t + \delta^-)
\end{aligned}
\tag{5}
$$

The q^+-component of negative sequence voltage can be computed as

$$
\begin{aligned}
v_q^- &= -\frac{2}{3}V^- \left[\sin(\theta_n) \cos(\theta_s + \delta^-) + \sin(\theta_n - \phi_s) \cos(\theta_s + \delta^- + \phi_s) + \sin(\theta_n + \phi_s) \cos(\theta_s + \delta^- - \phi_s)\right] \\
v_q^- &= \frac{-V^-}{3} \left[\sin(\theta_n + \theta_s + \delta^-) + \sin(\theta_n - \theta_s - \delta^-) + \sin(\theta_n + \theta_s + \delta^-) + \sin(\theta_n - \theta_s - \delta^- - 2\phi_s) \right. \\
&\qquad \left. + \sin(\theta_n + \theta_s + \delta^-) + \sin(\theta_n - \theta_s - \delta^- + 2\phi_s)\right] \\
v_q^- &= \frac{-V^-}{3} \left[3(\sin(\theta_n + \theta_s + \delta^-)) + \sin(\theta_n - \theta_s - \delta^-) + \sin(\theta_n - \theta_s - \delta^- - 2\phi_s) + \sin(\theta_n - \theta_s - \delta^- + 2\phi_s)\right] \\
v_q^- &= \frac{-V^-}{3} \left[3(\sin(\theta_n + \theta_s + \delta^-)) + \sin(\theta_n - \theta_s - \delta^-) + 2(\sin(\theta_n - \theta_s - \delta^-) \cos(-2\phi_s))\right] \\
v_q^- &= -V^- \sin(\theta_s + \theta_n + \delta^-) \; = \; - V^- \sin(\omega_s t + \omega_n t + \delta^-)
\end{aligned}
\tag{6}
$$

Using (3)-(6), the voltage in the positive RRF can be written as

$$
\boxed{
\begin{aligned}
v_d^+ &= V^+ \cos\left(\Delta\omega t + \delta^+\right) + V^- \cos\left(\omega_1 t + \delta^-\right) \\
v_q^+ &= V^+ \sin\left(\Delta\omega t + \delta^+\right) - V^- \sin\left(\omega_1 t + \delta^-\right)
\end{aligned}
}
\tag{7}
$$

B. Components in negative RRF

The components of the signal in the negative RRF can be computed as

$$
\begin{bmatrix} v_d^- \\ v_q^- \end{bmatrix} = k_1 \begin{bmatrix} \cos(\omega_n t) & \cos(\gamma_2) & \cos(\gamma_1) \\ -\sin(\omega_n t) & -\sin(\gamma_2) & -\sin(\gamma_1) \end{bmatrix} \mathbf{v}
\tag{8}
$$

where $\mathbf{v} = [v_{an}, v_{bn}, v_{cn}]^t$, $\gamma_1 = \omega_n t - \phi_s$ and $\gamma_2 = \omega_n t + \phi_s$. Using (8), the d^--component of positive sequence voltage can be computed as

$$
\begin{aligned}
v_d^- &= \frac{2}{3}V^+ \left[\cos(\theta_n) \cos(\theta_s + \delta^+) + \cos(\theta_n + \phi_s) \cos(\theta_s + \delta^+ - \phi_s) + \cos(\theta_n - \phi_s) \cos(\theta_s + \delta^+ + \phi_s)\right] \\
v_d^- &= \frac{V^+}{3} \left[\cos(\theta_n + \theta_s + \delta^+) + \cos(\theta_n - \theta_s - \delta^+) + \cos(\theta_n + \theta_s + \delta^+) + \cos(\theta_n - \theta_s - \delta^+ + 2\phi_s) \right. \\
&\qquad \left. + \cos(\theta_n + \theta_s + \delta^+) + \cos(\theta_n - \theta_s - \delta^+ - 2\phi_s)\right] \\
v_d^- &= \frac{V^+}{3} \left[3(\cos(\theta_n + \theta_s + \delta^+)) + \cos(\theta_n - \theta_s - \delta^+) + \cos(\theta_n - \theta_s - \delta^+ - 2\phi_s) + \cos(\theta_n - \theta_s - \delta^+ + 2\phi_s)\right] \\
v_d^- &= \frac{V^+}{2} \left[3(\cos(\theta_n + \theta_s + \delta^+)) + \cos(\theta_n - \theta_s - \delta^+) + 2(\cos(\theta_n - \theta_s - \delta^+) \cos(-2\phi_s))\right] \\
v_d^- &= \frac{3V^+}{2} \cos(\theta_s + \theta_n + \delta^+)
\end{aligned}
\tag{9}
$$

Similarly, the q^- component of positive sequence voltage can be computed as

$$\begin{aligned}
v_q^- &= \frac{2}{3}V^+ - \big[\sin(\theta_n)\cos(\theta_s+\delta^+) + \sin(\theta_n+\phi_s)\cos(\theta_s+\delta^+-\phi_s) + \sin(\theta_n-\phi_s)\cos(\theta_s+\delta^++\phi_s)\big] \\
v_q^- &= \frac{-V^+}{3}\big[\sin(\theta_n+\theta_s+\delta^+) + \sin(\theta_n-\theta_s-\delta^+) + \sin(\theta_n+\theta_s+\delta^+) + \sin(\theta_n-\theta_s-\delta^++2\phi_s) \\
&\qquad\qquad + \sin(\theta_n+\theta_s+\delta^+) + \sin(\theta_n-\theta_s-\delta^+-2\phi_s)\big] \\
v_q^- &= \frac{-V^+}{3}\big[3(\sin(\theta_n+\theta_s+\delta^+)) + \sin(\theta_n-\theta_s-\delta^+) + \sin(\theta_n-\theta_s-\delta^+-2\phi_s) + \sin(\theta_n-\theta_s-\delta^++2\phi_s)\big] \\
v_q^- &= \frac{-V^+}{3}\big[3(\sin(\theta_n+\theta_s+\delta^+)) + \sin(\theta_n-\theta_s-\delta^+) + 2(\sin(\theta_n-\theta_s-\delta^+)\cos(-2\phi_s))\big] \\
v_q^- &= -V^+\sin(\theta_s+\theta_n+\delta^+) \;=\; -V^+\sin(\omega_s t+\omega_n t+\delta^+)
\end{aligned} \tag{10}$$

On the other hand, the d^--component of negative sequence voltage can be computed as

$$\begin{aligned}
v_d^- &= \frac{2}{3}V^-\big[\cos(\theta_n)\cos(\theta_s+\delta^-) + \cos(\theta_n+\phi_s)\cos(\theta_s+\delta^-+\phi_s) + \cos(\theta_n-\phi_s)\cos(\theta_s+\delta^--\phi_s)\big] \\
v_d^- &= \frac{V^-}{3}\big[\cos(\theta_n-\theta_s-\delta^-) + \cos(\theta_n+\theta_s-+\delta^-) + \cos(\theta_n-\theta_s-\delta^-) + \cos(\theta_n+\theta_s+\delta^-+2\phi_s) \\
&\qquad\qquad + \cos(\theta_n-\theta_s-\delta^-) + \cos(\theta_n+\theta_s+\delta^--2\phi_s)\big] \\
v_d^- &= \frac{V^-}{3}\big[3(\cos(\theta_n-\theta_s-\delta^-)) + \cos(\theta_n+\theta_s+\delta^-) + \cos(\theta_n+\theta_s+\delta^-+2\phi_s) + \cos(\theta_n+\theta_s+\delta^--2\phi_s)\big] \\
v_d^- &= \frac{V^-}{3}\big[3(\cos(\theta_n-\theta_s-\delta^-)) + \cos(\theta_n+\theta_s+\delta^-) + 2(\cos(\theta_n+\theta_s+\delta^-)\cos(2\phi_s))\big] \\
v_d^- &= V^-\cos(\theta_s-\theta_n+\delta^-) \;=\; V^-\cos(\Delta\omega t+\delta^-)
\end{aligned} \tag{11}$$

The q^--component of negative sequence voltage can be computed as

$$\begin{aligned}
v_q^- &= -\frac{2}{3}V^-\big[\sin(\theta_n)\cos(\theta_s+\delta^-) + \sin(\theta_n+\phi_s)\cos(\theta_s+\delta^-+\phi_s) + \sin(\theta_n-\phi_s)\cos(\theta_s+\delta^--\phi_s)\big] \\
v_q^- &= \frac{-V^-}{3}\big[\sin(\theta_n-\theta_s-\delta^-) + \sin(\theta_n+\theta_s-+\delta^-) + \sin(\theta_n-\theta_s-\delta^-) + \sin(\theta_n+\theta_s+\delta^-+2\phi_s) \\
&\qquad\qquad + \sin(\theta_n-\theta_s-\delta^-) + \sin(\theta_n+\theta_s+\delta^--2\phi_s)\big] \\
v_q^- &= \frac{-V^-}{3}\big[3(\sin(\theta_n-\theta_s-\delta^-)) + \sin(\theta_n+\theta_s+\delta^-) + \sin(\theta_n+\theta_s+\delta^-+2\phi_s) + \sin(\theta_n+\theta_s+\delta^--2\phi_s)\big] \\
v_q^- &= \frac{-V^-}{3}\big[3(\sin(\theta_n-\theta_s-\delta^-)) + \sin(\theta_n+\theta_s+\delta^-) + 2(\sin(\theta_n+\theta_s+\delta^-)\cos(2\phi_s))\big] \\
v_q^- &= V^-\sin(\theta_s-\theta_n+\delta^-) \;=\; V^-\sin(\Delta\omega t+\delta^-)
\end{aligned} \tag{12}$$

Using (9)-(12), the voltage in the negative RRF can be written as

$$\boxed{\begin{aligned}
v_d^- &= V^+\cos\left(\omega_1 t+\delta^+\right) + V^-\cos\left(\Delta\omega t+\delta^-\right) \\
v_q^- &= -V^+\sin\left(\omega_1 t+\delta^+\right) + V^-\sin\left(\Delta\omega t+\delta^-\right)
\end{aligned}} \tag{13}$$