Random Field Ising Model
Term Paper

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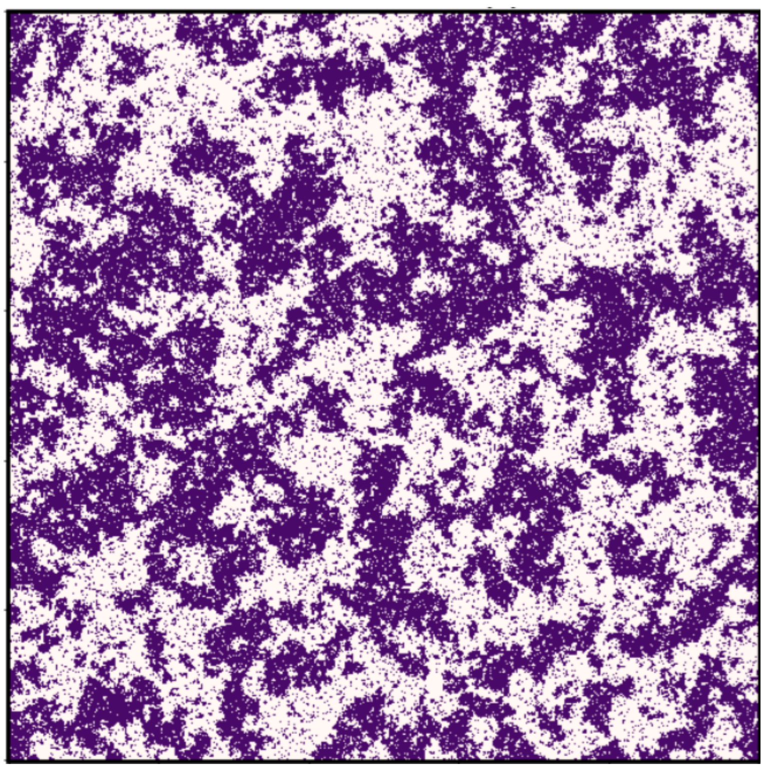

Figure 1: Regular Ising Model for $T = T_{critical}$

Contents

1 Introduction

Ising model is the key to understand criticality. This term paper is consisted of three main sections. In the second section I will take a review on one previous paper [3], in which they evaluated the Hamiltonian and phase transition curve in a Gaussian RFIM with infinite-range of interactions. In the Third section I will discuss a fast and accurate method of developing a Gaussian RFIM and the results of this program. In the Forth section I will have compare the RFIM with Constant Field Ising Model (CFIM) with the data extracted from my codes [1]. The Fifth section will illustrate why we are interested in $\langle log(z)\rangle$ instead of $\langle z\rangle$.

2 Review on previous papers

In this section, I will give a review on [3]. In this paper, it is considered that each site can interact with all of the other sites. Each site is statistically independent and an external field with Gaussian distribution is exerted on the lattice. The model is solved exactly and exhibits both an independent spin phase and a ferromagnetic phase, separated by a line of second-order phase transitions. Eventually, it is shown that the replica technique yields exact results in the present model but not in the related random exchange system.

2.1 Calculations

We consider N Ising spins interacting through an infinite ranged exchange interaction. The Ising spins are coupled to a random field with a Gaussian distribution. The Hamiltonian is,

$$H = -\frac{J}{N}\sum_{i=/=j} S_i S_j - \sum_i h_i S_i \tag{1}$$

where h_i is distributed according to,

$$P(h) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp -\frac{h^2}{2\sigma^2} \tag{2}$$

One can calculate the free energy for random field ising model with replica trick. The result is derived similar to [4].

$$\frac{\langle F\rangle}{N} = jm^2 - \frac{1}{\beta}\int dhP(h)ln(2)cosh[\beta(2Jm+h)] \tag{3}$$

where m is,

$$m = \langle S_i\rangle = \int dhP(h)tanh[\beta(2Jm+h)] \tag{4}$$

I found the details of the calculations in the appendix of [3].

And from the [4] the energy can be calculated as,

$$\frac{\langle H\rangle}{N} = -Jm^2 + \beta\sigma^2(1-q_h) \tag{5}$$

where,

$$q_h = \langle\langle S_i\rangle^2\rangle \tag{6}$$

In the independent spin phase, for $2J/\sigma < \sqrt{(\pi/2)}$, the specific heat and susceptibility are smooth functions of T. In fact at low temperature the specific heat is given by

$$C = \sqrt{\frac{2}{pi}}\frac{\pi^2}{12}k_B^2 T/\sigma \tag{7}$$

The result is shown in 2 and the trend of this plot is similar to the one simulated by me and shown in 7. For further and more accurate comparison, one must compare the numerical results, which is not of the scope of this term paper.

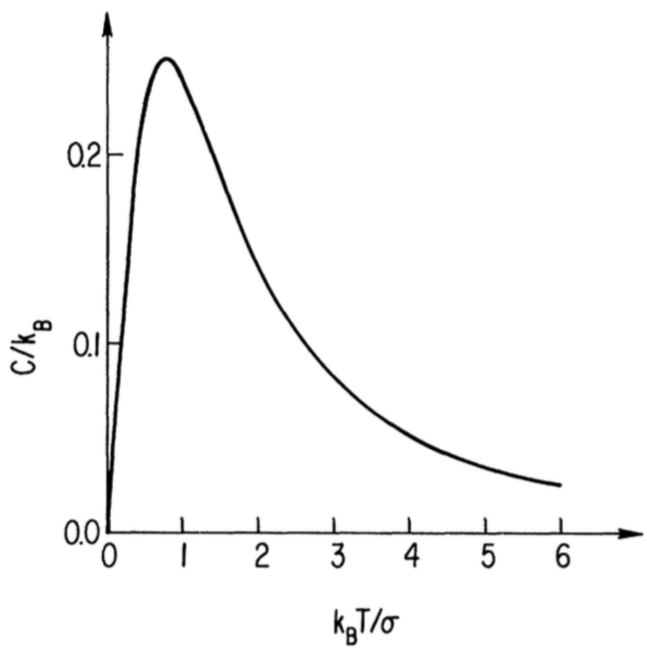

Figure 2: This plot is adopted from [3]. The heat capacity is plotted versus k_BT for Gaussian RFIM.

3 Computational results

In this section I will introduce a very efficient Monte Carlo method in which we can simulate the Random Field Ising Model, and subsequently find the relative plots and results. I programmed this code [1] in Python language and took advantage of the Numpy package. For the simplification only nearest neighbors can interact with each other. For each plot I considered 3 separate Ising models for comparison. The first one is an Ising model with constant external magnetic field $h = 0.3$. For the second one I set a normal (Gaussian) random external field on a 100*100 lattice, with $\langle h \rangle = 0.3$ and $\langle \sigma \rangle = 0.3$. And the third one is a regular Ising model with no external field at all. Afterwards, we can run the code for the lattice and extract data for different properties of Ising model, four of which magnetisation, magnetic susceptibility, total energy and heat capacity. In the following sections the results are included.

3.1 Magnetisation

Magnetisation is nothing but the average of σ over every site of lattice.

$$M = \langle \sigma \rangle_{sites} \tag{8}$$

The result for magnetisation is shown in 3

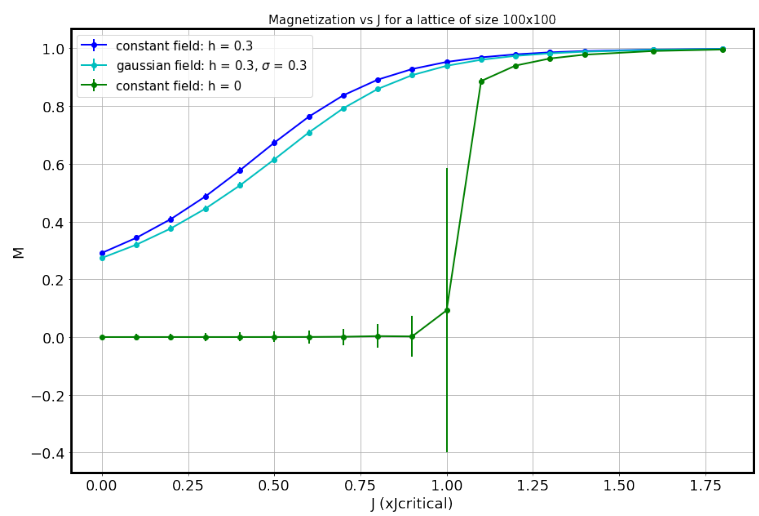

Figure 3: The Magnetisation is plotted with respect to J. As we expected, around $J_{critical}$ we see a phase transition when no external field was exerted.

3.2 Magnetic susceptibility

The derivation of magnetic susceptibility is similar to C_v. One can find is as below:

$$\chi = \frac{d\langle M \rangle}{dT} = \beta M^2 \tag{9}$$

Therefore we can use the variance of magnetic susceptibility as the error of magnetisation. The result is shown in 4. As we expect, we can see the phase transition in $J = J_{critical}$ for case $h = 0$. However, this phase transition does not occur for the other 2 external field distributions.

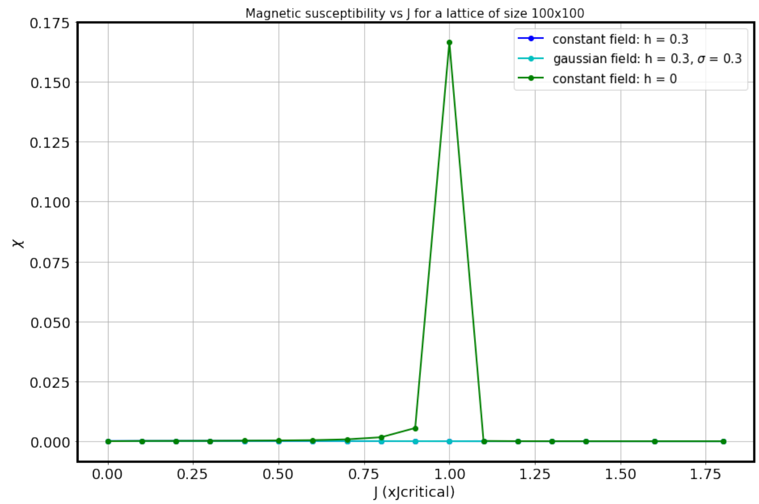

Figure 4: Susceptibility for three different h distributions is plotted with respect to J, with the method defined in previous section. χ peaks at $J_{critical}$ for h = 0. However the trends of Gaussian and constant non-zero h distributions are not evident.5 is the same plot except for excluding the zero external field from this plot

3.3 Energy

Energy in this model is easily calculated. We only need to make sure that our data is enough for taking average. Besides one must evaluate the energy value after the system is relaxed. The plot is shown in 6

3.4 Heat Capacity

Heat capacity is defined as below.

$$C = \frac{d\langle E \rangle}{dT} \tag{10}$$

Therefore is can be written as,

$$C = \beta^2 \sigma_E^2 \tag{11}$$

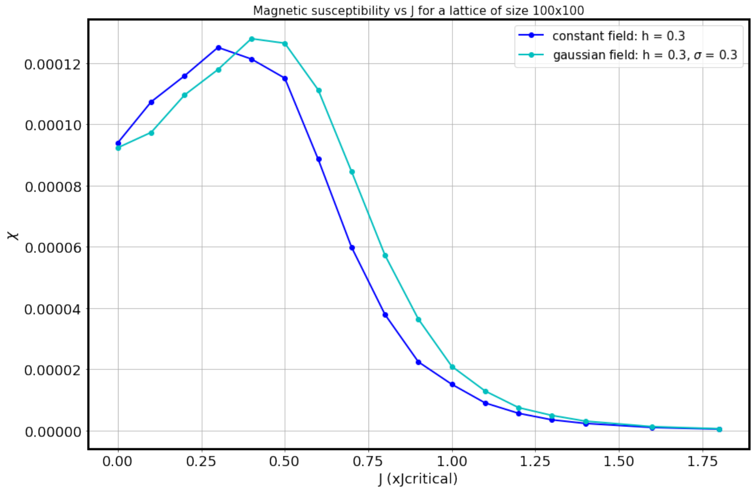

Figure 5: magnetic susceptibility only for Gaussian and non-zero external fields is plotted with respect to J, with the method defined in previous section

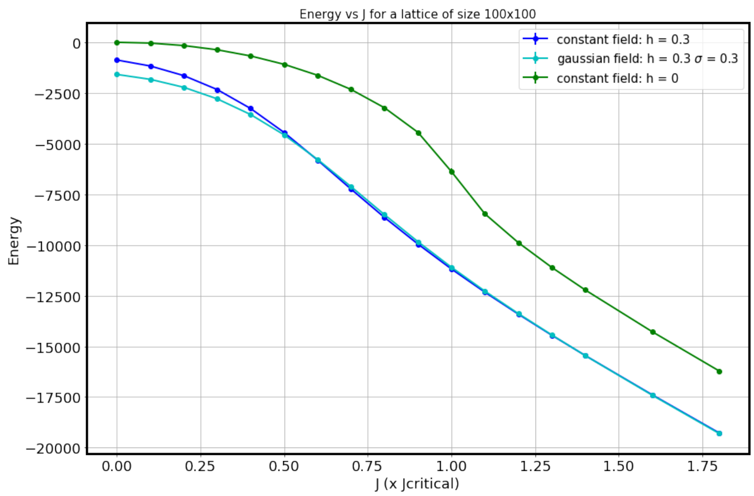

Figure 6: Energy for three different h distributions is plotted with respect to J.

Thus to calculate the heat capacity we only need to find the variance of energy. The result is shown in 7

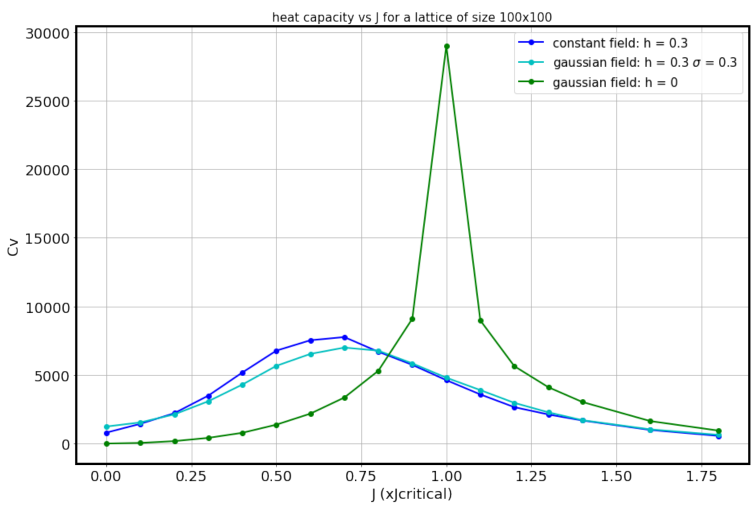

Figure 7: Heat capacity for three different h distributions is plotted with respect to J, with the method defined in section 3

4 Results

4.1 Four important parameters

Now we reach at the point where we can compare the results of RFIM and Constant Field Ising Model (CFIM). Let's take a look at all the figures again. For all the plots given in this paper, including 3, 4, 6, 7, it is obvious that RFIM and CFIM have very similar behaviors at $J \longrightarrow \infty$ or similarly $T \Longrightarrow 0$, and also at $J \longrightarrow 0$. Because at $T \longrightarrow \infty$ the effects of external field vanishes such that the external field does not matter much anymore. However, to come to a more precise conclusion, we will need more time and considerations.

4.2 Can we comment on δ?

From our knowledge we are aware that in CFIM $M \approx h^{\frac{1}{\delta}}$. This made me think of finding the same equation for RFIM. Are they similar? To address this question, I plotted magnetisation versus external field. This plot is given for three different temperatures for two models: first, a CFIM with h = 0.3. Second, a Gaussian RFIM with $h = 0.3$ and $\sigma = 0.3$. The result is shown in **??**. Again, RFIM diverges from CFIM with similar h at medium external fields. At $h \longrightarrow 0$ and $h \longrightarrow$ inf the similar behavior is observed. This makes us think of the indication from [2]: "A lot of progress has been made recently in our understanding of the random–field Ising model thanks to large–scale numerical simulations. In particular, it has been shown that, contrary to previous statements: the critical exponents for different probability distributions of the random fields and for diluted anti-ferromagnets in a field are the same. Therefore, critical universality, which is a perturbative renormalization–group prediction, holds beyond the validity regime of perturbation theory."

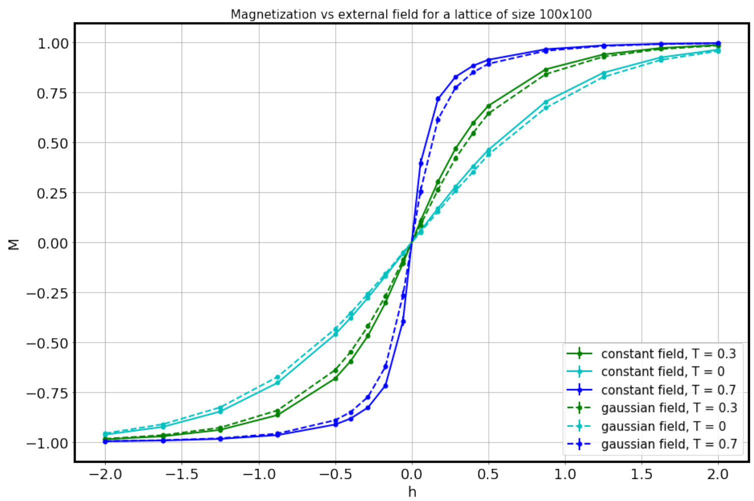

Figure 8: Heat capacity for three different h distributions is plotted with respect to J, with the method defined in section 3

5 Why $\langle log(z)\rangle$?

As I previously illustrated, the function $\langle log(z)\rangle$ is deterministic, for a Gaussian RFIM. And it is of more importance considering its relation with free energy. To make sure of the difference between $\langle log(z)\rangle$ and $log\langle z\rangle$, we can take a look into the plot 9. The results are comparable, however it seems that $log\langle z\rangle$ gives an upper approximation for $\langle log(z)\rangle$.

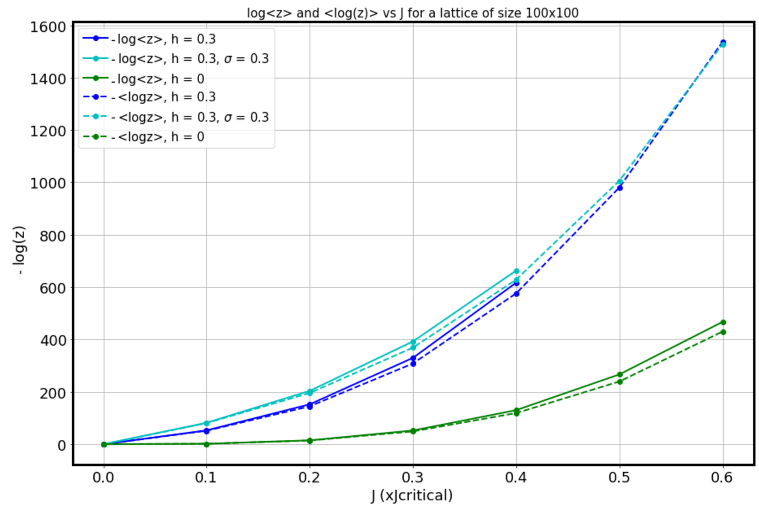

Figure 9: - log(Z) versus J is plotted

References

[1] Setareh Foroozan. "Source Code". In: (2020). DOI: https://github.com/SetarehForoozan/RFIM/blob/master/RFIM.ipynb.

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