DC-DC Converter Modeling and Simulation using State Space Approach

Rodney H.G. Tan
Electrical & Electronics Engineering Department
UCSI University
Kuala Lumpur, Malaysia
rodneytan@ieee.org

Landon Y. H. Hoo
Electrical and Electronics Engineering Department
UCSI University
Kuala Lumpur, Malaysia
landon005@gmail.com

***Abstract*—This paper presents the DC-DC converter model, implementation and simulation using state space modeling approach. Three DC-DC converters including Buck, Boost and Buck-Boost converter are developed using state-space modeling approach and implemented in Simulink. Each converter is modeled based on its circuit topology and its state space matrix are derived in detail. The state space model simulation results are comparable with circuitry model with only up to 0.0015V of tolerance. The simulation computation time has improved up to 7.8 times faster as compared to circuitry model. The state space model embed DC-DC converter into a single block, thus allowing buck, boost or buck-boost to be easily implemented and integrate into other larger system.**

***Keywords*—DC-DC converter; buck converter; boost converter; buck-boost converter; state space modeling**

I. Introduction

Since the last two decades, DC-DC converters are widely used in many modern equipment's including power supply, battery charger, LED drive, DC motor drive, MPPT and etc. It is mean to manage and control the flow of electrical power to achieve the desire operation performance and efficiency. DC-DC converters are one of the essential topics in Power Electronics course. It basically covers the fundamental principle of buck, boost and buck-boost converters. In course delivery, modeling and simulation demonstration are commonly use to aid learning and understanding of DC-DC converter operation principle [1-4]. There are many approach to model and simulate a DC-DC converter, which include mathematical, circuitry, transfer function and state space approach [5-6]. The most common approach is circuit modeling, where converter circuit topology is drawn directly on modeling and simulation platform. However, there are not many literature on modeling and simulation using state space approach. This paper take the lead to present a comprehensive DC-DC converter modeling and simulation using state space approach.

There are many commercially available power electronic modeling and simulation software in the market. Few well known platform includes PSim, PLECS, PSCAD, Simulink and etc. Each software platform have its own merit [7]. In this paper, Simulink is chosen as a modeling and simulation platform for state space modeling of DC-DC converter for the following reasons. First, state space model is matrix base and Simulink is runs on top of MATLAB, where MATLAB is an acronym of MATric LABoratory, it uses matrix as basis for computation. This allows state space model to take advantage on the platform to improve the speed of simulation. Second, the basic Simulink blockset does not comes with DC-DC converter block. It require additional SimPowerSystems or SimElectronic blockset in order to model and simulate DC-DC converter. This paper presents method of developing DC-DC converter model block using basic Simulink building blocks without any additional blockset.

II. Modeling Approach

The DC-DC converters including Buck, Boost and Buck-Boost topology are modeled using state space modeling approach in the following sections. Simulink was chosen as modeling, implementation and test platform. To begin with, the state space modeling is fundamentally represent in (1), where A, B, C, D are the system matrix, *x* is the state variable, *x*' is the state variable derivative, *u* is the input, and *y* is the output.

$$x' = Ax + Bu \tag{1}$$

$$y = Cx + Du \tag{2}$$

A. Buck Converter State Space Model

The fundamental buck converter circuit and it's 'ON' and 'OFF' state equivalent circuits are shown in Figure 1.

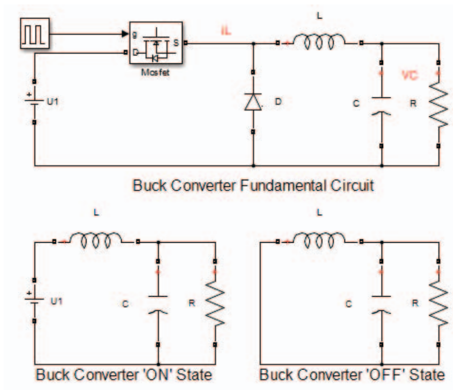

Fig. 1. Buck converter fundamental, ON and OFF state equivalent circuits

The buck converter state variables are V_C and i_L. During 'ON' state, V_C and i_L can be defined in (3) and (4) respectively.

$$V_C = u_1 - L\frac{di_L}{dt} \tag{3}$$

$$i_L = C\frac{dV_C}{dt} + \frac{V_C}{R} \tag{4}$$

By mapping state variables $i_L = x_1$ and $V_C = x_2$. Its derivative x_1' and x_2' in (5) and (6) can be obtained by rearranging (3) and (4). The state space matrix A and B in (7) for buck converter in 'ON' state can be formulated using (5) and (6).

$$x_1' = -\frac{1}{L}x_2 + \frac{1}{L}u_1 \tag{5}$$

$$x_2' = \frac{1}{C}x_1 - \frac{1}{RC}x_2 \tag{6}$$

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}u_1 \tag{7}$$

During 'OFF' state, where u_1 is zero and its derivative x_1' is shown in (8) and derivative x_2' is same as (6). Similarly, the state space matrix A and B in (9) for buck converter in 'OFF' state can be formulated using (8) and (6).

$$x_1' = -\frac{1}{L}x_2 \tag{8}$$

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}u_1 \tag{9}$$

After derived the buck converter state space A and B matrix for its 'ON' and 'OFF' state. It is require to find its average A and B matrix with the account of switching duty cycle d. The average A and B matrix are shown (10 and (11) respectively.

$$\overline{A} = A_{(ON)}d + A_{(OFF)}(1-d)$$

$$\overline{A} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix}d + \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix}(1-d) = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \tag{10}$$

$$\overline{B} = B_{(ON)}d + B_{(OFF)}(1-d)$$

$$\overline{B} = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}d + \begin{bmatrix} 0 \\ 0 \end{bmatrix}(1-d) = \begin{bmatrix} \frac{d}{L} \\ 0 \end{bmatrix} \tag{11}$$

To complete the buck converter model, the average matrix of (10) and (11) are substitute into (1). The completed buck converter state space model is shown in (12).

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{d}{L} \\ 0 \end{bmatrix}u_1 \tag{12}$$

Lastly, to obtain the output state of V_C and i_L, the output state space for C and D matrix is shown in (13).

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} i_L \\ V_C \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}u_1 \tag{13}$$

B. Boost Converter State Space Model

The fundamental boost converter circuit and its 'ON' and 'OFF' state circuits are shown in Figure 2.

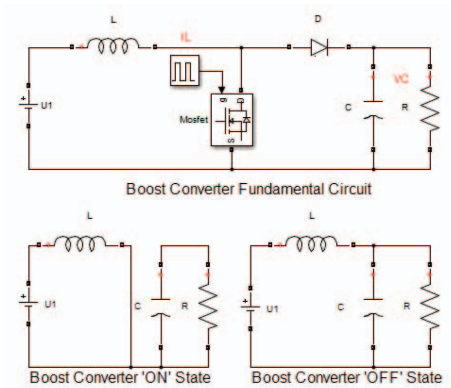

Fig. 2. Boost converter fundamental, ON and OFF state equivalent circuits

During 'ON' state, the inductor is charge through u_1 defined in (14). There is no current flow to the capacitor and resistor in this state, where i_L is zero as defined in (15).

$$u_1 = L\frac{di_L}{dt} \tag{14}$$

$$0 = C\frac{dV_C}{dt} + \frac{V_C}{R} \tag{15}$$

The state derivative of x_1' and x_2' in (16) and (17) can be obtained by rearranging (14) and (15). The state space matrix A and B in (18) for boost converter in 'ON' state can be formulated using (16) and (17).

$$x_1' = \frac{1}{L}u_1 \tag{16}$$

$$x_2' = -\frac{x_2}{RC} \tag{17}$$

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}u_1 \tag{18}$$

When Boost converter enter 'OFF' state condition, where its equivalent circuit is similar to Buck converter in the 'ON' state. Therefore, state space matrix A and B for Boost converter 'OFF' state is similar with (7).

Similarly, the average of the boost converter state space A and B matrix for its 'ON' and 'OFF' state can be formulated with the account of switching duty cycle d. The average A and B matrix are shown (19) and (20) respectively.

$$\overline{A} = A_{(ON)}d + A_{(OFF)}(1-d)$$

$$\overline{A} = \begin{bmatrix} 0 & 0 \\ 0 & -\dfrac{1}{RC} \end{bmatrix} d + \begin{bmatrix} 0 & -\dfrac{1}{L} \\ \dfrac{1}{C} & -\dfrac{1}{RC} \end{bmatrix} (1-d) = \begin{bmatrix} 0 & -\dfrac{1-d}{L} \\ \dfrac{1-d}{C} & -\dfrac{1}{RC} \end{bmatrix} \quad (19)$$

$$\overline{B} = B_{(ON)}d + B_{(OFF)}(1-d)$$

$$\overline{B} = \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix} d + \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix} (1-d) = \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix} \quad (20)$$

To complete the boost converter model, the average matrix of (19) and (20) are substitute into (1). The completed boost converter state space model is shown in (21).

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 0 & -\dfrac{(1-d)}{L} \\ \dfrac{(1-d)}{C} & -\dfrac{1}{RC} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix} u_1 \quad (21)$$

To obtain the output state of V_C and i_L, the output state space for C and D matrix is similar with (13).

C. Buck-Boost Converter State Space Model

The fundamental buck-boost converter circuit and its 'ON' and 'OFF' state circuits are shown in Figure 3

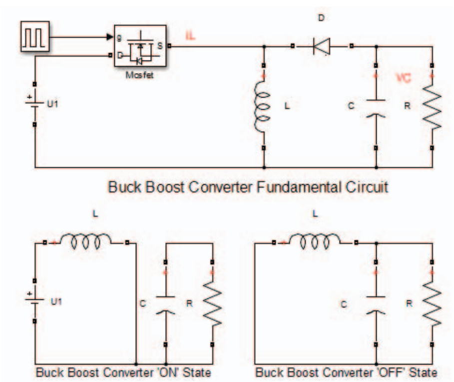

Fig. 3. Buck Boost converter fundamental, ON and OFF state equivalent circuits

The buck-boost converter 'ON' state is similar to boost converter in the 'ON' state. Therefore, state space matrix A and B for buck-boost converter 'OFF' state is similar with (18).

During 'OFF' state, it is similar to buck converter in 'OFF' state where u_1 is zero, but its output state V_C and i_L are reverse in polarity due to inductor discharging as shown in (22) and (23) respectively.

$$-V_C = -L\frac{di_L}{dt} \quad (22)$$

$$-i_L = C\frac{dV_C}{dt} + \frac{V_C}{R} \quad (23)$$

Again by mapping state variables $V_C = x_1$ and $i_L = x_2$. Its derivative x_1 and x_2 in (24) and (25) can be obtained by rearranging (22) and (23). The state space matrix A and B in (26) for buck-boost converter in 'OFF' state can be formulated using (24) and (25).

$$x_1' = \frac{1}{L}x_2 \quad (24)$$

$$x_2' = -\frac{1}{C}x_1 - \frac{1}{RC}x_2 \quad (25)$$

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 0 & \dfrac{1}{L} \\ -\dfrac{1}{C} & -\dfrac{1}{RC} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u_1 \quad (26)$$

Similarly, the average of the buck-boost converter state space A and B matrix for its 'ON' and 'OFF' state can be formulated with the account of switching duty cycle d. The average A and B matrix are shown (27) and (28) respectively.

$$\overline{A} = A_{(ON)}d + A_{(OFF)}(1-d)$$

$$\overline{A} = \begin{bmatrix} 0 & 0 \\ 0 & -\dfrac{1}{RC} \end{bmatrix} d + \begin{bmatrix} 0 & \dfrac{1}{L} \\ -\dfrac{1}{C} & -\dfrac{1}{RC} \end{bmatrix} (1-d) = \begin{bmatrix} 0 & \dfrac{1-d}{L} \\ -\dfrac{1-d}{C} & -\dfrac{1}{RC} \end{bmatrix} \quad (27)$$

$$\overline{B} = B_{(ON)}d + B_{(OFF)}(1-d)$$

$$\overline{B} = \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix} d + \begin{bmatrix} 0 \\ 0 \end{bmatrix} (1-d) = \begin{bmatrix} \dfrac{d}{L} \\ 0 \end{bmatrix} \quad (28)$$

To complete the buck-boost converter model, the average matrix of (27) and (28) are substitute into (1). The completed buck-boost converter state space model is shown in (29).

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 0 & \dfrac{(1-d)}{L} \\ -\dfrac{(1-d)}{C} & -\dfrac{1}{RC} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \dfrac{d}{L} \\ 0 \end{bmatrix} u_1 \quad (29)$$

The output state space for C and D matrix is similar with (13).

III. Model Implementation

The DC-DC converter model implementation in Simulink is shown in Figure 4. The constant block is use to model the DC source in volt, the gain block is used to model the converter efficiency caused by switching device losses. State-Space block model the desire converter by entering the state-space A, B, C, D matrix into the block parameters as shown in Figure 5. By entering equation (12) into matric A and B for buck, (21) for boost and (29) for buck-boost converter. Matrix

C and D from equation (13) are the same for all three DC-DC converters. The demultiplexer block split the output variable of V_C and i_L to the scope for simulation output.

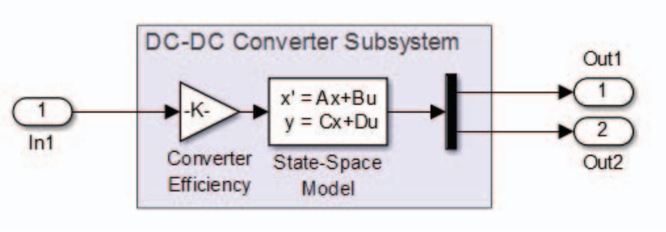

Fig. 4. DC-DC converter subsystem in Simulink

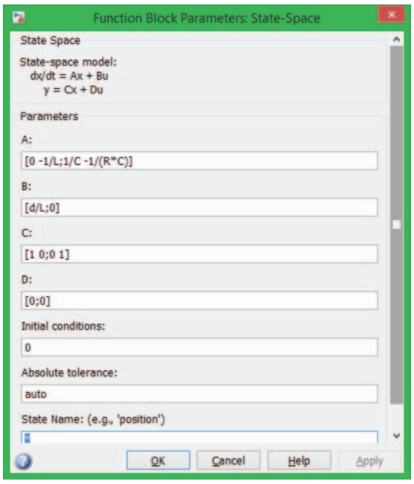

Fig. 5. State-space block A,B,C,D matrix parameters for buck converter model

The above implementation require the user to enter the chosen converter state-space matrix and prepare the inductance, capacitance, resistance and duty cycle values in workspace before simulation, which is tedious and prone to error. To improve user friendliness the DC-DC converter subsystem in Figure 4 can be masked into a single DC-DC converter block as shown in Figure 6 to embed the state-space matrix for all converters to create a selection menu of buck, boost and buck-boost converter. The inductance, capacitance, resistance, duty cycle and converter efficiency parameters are also brought out allowing user to directly enter the values respectively. On top of that, to take advantage of state-space model, dynamic system analysis function such as bode plot, pole-zero map and step response can be easily implemented into the masked block. The masked DC-DC converter function block parameters is shown in Figure 7.

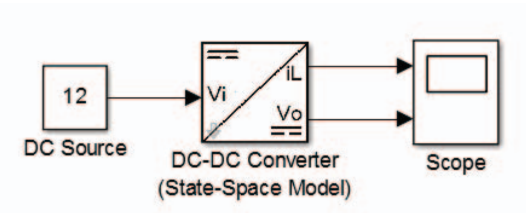

Fig. 6. Masked DC-DC converter state-space block in Simulink

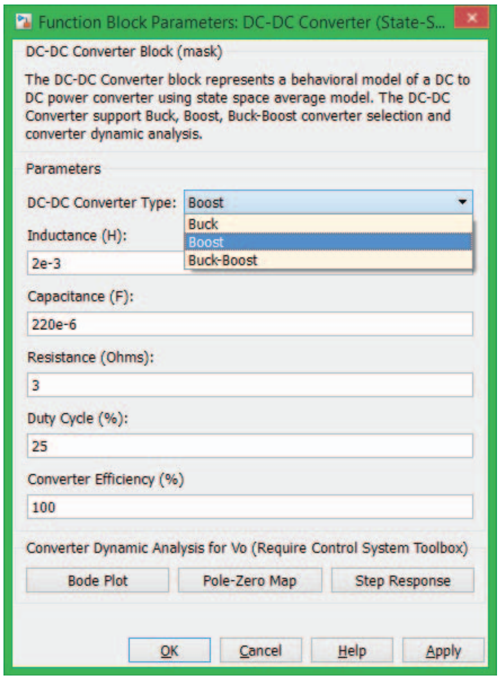

Fig. 7. Masked DC-DC Converter block parameters user interface

The converter dynamic analysis for a boost converter output voltage with above setting is shown in Figure 8. It shows the converter dynamic behavior which is vital for DC-DC converter closed loop controller design.

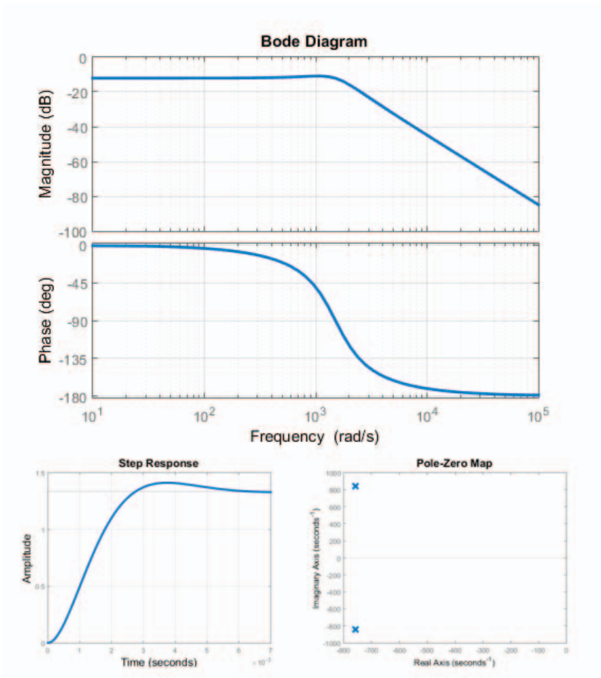

Fig. 8. Bode plot, pole-zero map and step response of a boost converter

IV. RESULTS AND DISCUSSION

To demonstrate the state space model simulation output is comparable to circuitry model, the simulation results for all three converters are simulate based on the following parameters, L=2mH, C=220μF, R=3Ω, switching frequency of 10kHz, duty cycle=25% and input voltage of 12V. The simulation results are compared with converter model using SimPowerSystems blockset. The SimPowerSystems buck, boost and buck-boost converter circuits are similar with Figure 1, 2 and 3 as it is drawn using SimPowerSystems. Both models are run in Simulink using the same ordinary differential equation ode45 (Dormand-Prince) solver to ensure fair comparison. The computer hardware is 4th Generation Intel Core i3-4010U 1.7GHz processor with 4GB RAM and Windows 8.1 operating system.

A. Buck Converter Result

With the above parameters, the state space model simulation gives a theoretical output voltage of 3V. This is true for state space model under ideal condition without losses. However circuitry model simulate by SimPowerSystems are take switching device losses into account which give an output of 2.3782V. To simulate the same switching device losses in state space model, the converter efficiency block is set to 79.27% which is the ratio of SimPowerSystems output voltage and the theoretical state space model output voltage. Figure 9 shows the simulation output of buck converter inductor current i_L and output voltage V_C from state space and circuitry models. It can be clearly see that both models output are almost overlapped with each other, except state space model is not able to simulate the ripple effect on the inductor current and output voltage, due to the absent of switching frequency in state space model. The state space model output voltage is 2.3767V and the circuitry model output voltage is 2.3782V which gives an error of 0.0015V. The simulation computation time speed test was carried out 10 times and the average computation time for circuitry and state space model are 712.6ms and 91.3ms respectively. The state space model is 7.8 times faster than the circuitry model.

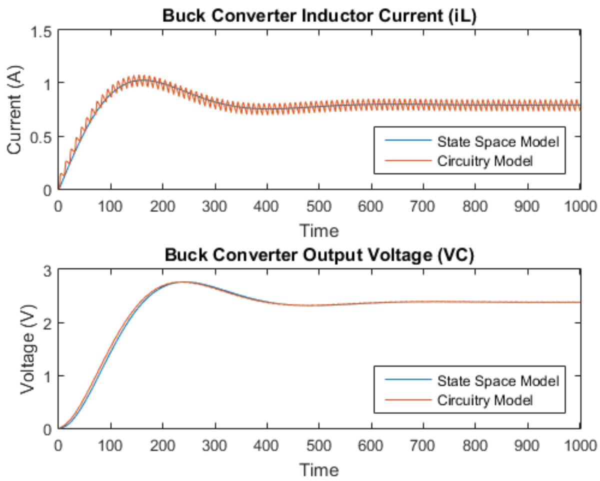

Fig. 9. Simulation output of buck converter

B. Boost Converter Result

For boost converter, the state space model simulation gives a theoretical output voltage of 16V. The circuitry model simulate an output of 14.9670V. The converter efficiency in state space model is set to 93.56%. Figure 10 shows the simulation output of buck converter inductor current i_L and output voltage V_C from state space and circuitry models. In this boost converter, since the ripple is not noticeable, both models output are seem to be perfectly overlapped with each other. The state space model output voltage is 14.9676V and the circuitry model output voltage is 14.9670V which gives an error of 0.0006V. The average simulation computation time speed test for circuitry and state space model are 591.4ms and 90.2ms respectively. The state space model is 6.6 times faster than the circuitry model.

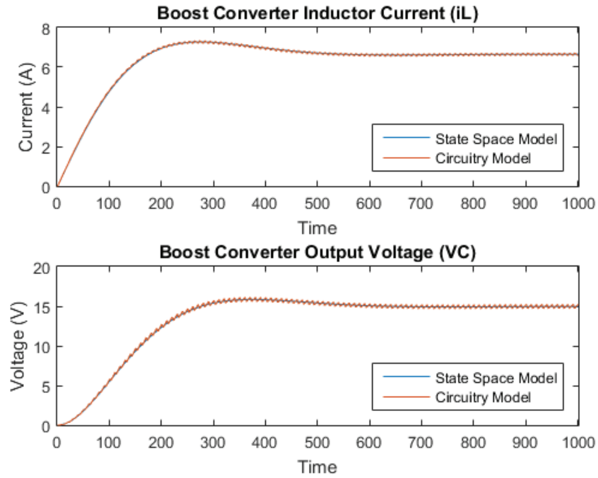

Fig. 10. Simulation output of boost converter

C. Buck-Boost Converter Result

For buck-boost converter, the state space model simulation gives a theoretical output voltage of -4V. The circuitry model simulate an output of -3.1502V. The converter efficiency in state space model is set to 78.75%. Figure 11 shows the simulation output of buck-boost converter inductor current i_L and output voltage V_C from state space and circuitry models. Again, it can be clearly see that both models output are almost overlapped with each other, except state space model is not able to simulate the ripple effect on the inductor current and output voltage. The state space model output voltage is -3.1496V and the circuitry model output voltage is -3.1502V which gives an error of 0.0006V. The average simulation computation time speed test for circuitry and state space model are 633.5ms and 138.1ms respectively. The state space model is 4.6 times faster than the circuitry model.

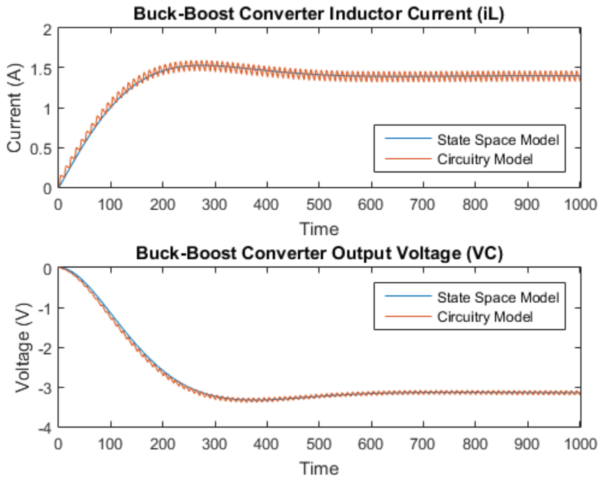

Fig. 11. Simulation output of buck-boost converter

The performance of state space and circuitry model for buck, boost and buck-boost converter are summarized in Table 1. It can be clearly see that state space model is able to model and simulate DC-DC converter with high accuracy of only 0.0015V error. The simulation computation time of state space model has improved up to 7.8 times faster, this is because the state space model take advantage of the native matrix computation engine in MATLAB/Simulink. It also has the advantage to simulate ideal theoretical model. The state space model does not require SimPowerSystems or SimElectronics add-on blockset to model and simulate DC-DC converter.

The shortfall of state space model is not able to simulate ripple effect on the inductor current and output voltage due to the absent of switching frequency parameter. It also does not allow closed loop controlled simulation due to the absent of switching device duty cycle gate input. The other shortfall is the converter efficiency cannot directly model using switching device data such as forward voltage, turn-on resistance and etc. It can only indirectly translate using the converter efficiency parameter ranging between 0 is 100%, where 100% is ideal model with no loss.

The state-space DC-DC converter model and circuitry models presented in this paper are made available for download by the author on the official Mathworks file exchange community link below. All results presented in this paper are reproducible.

http://www.mathworks.com/matlabcentral/fileexchange/52546
http://www.mathworks.com/matlabcentral/fileexchange/52948

TABLE I. DC-DC CONVERTER STAE SPACE AND CIRCUITRY MODEL PERFORMANCE COMPARISON

		Circuitry Model	State Space Model
Buck Converter	Output Voltage	2.3782	2.3767
	Inductor Current	0.7930	0.7924
	Computation Time	712.6ms	91.3ms
Boost Converter	Output Voltage	14.9670	14.9676
	Inductor Current	6.6546	6.6552
	Computation Time	591.4ms	90.2ms
Buck-Boost Converter	Output Voltage	-3.1502	-3.1496
	Inductor Current	1.4007	1.4004
	Computation Time	633.5ms	138.1ms
Ideal Model Simulation		No	Yes
Converter Dynamic Analysis		No	Yes
Ripple Simulation		Yes	No
Closed Loop Control		Yes	No

V. CONCLUSION

DC-DC converter including Buck, Boost and Buck-Boost topology are modeled and implemented using state space approach in Simulink. The state space model simulation results are comparable with circuitry model with only up to 0.0015V of error. The simulation computation time has improved up to 7.8 times faster as compared to circuitry model. On top of that it does not require any additional Simulink add-on blockset such as SimPowerSystems or SimElectronics to model DC-DC converter. The only shortfall of state space model is not able to simulate ripple effect on the inductor current and output voltage due to the absent of frequency switching input element. The state space model embed DC-DC converter into a single block, thus allowing buck, boost or buck-boost to be easily simulate and analyze.

REFERENCES

[1] I. Batarseh, D.A. Kemnitz, "Undergraduate Education in Power Electronics", Southcon/94, 1994, pp. 207-213.

[2] R.A. Kordkheili, M. Yazdani-Asrami, A.M. Sayidi, "Making DC-DC Converters Easy to Understand for Undergraduate Students", IEEE Conference on Open Systmes (ICOS), 2010, pp. 28-33.

[3] L.S. Patil, K.D. Patil, A.G. Thosar,'The Role of Computer Modeling and Simulation in Power Electronics Education", IEEE 2nd International Conference on Emerging Trends in Engineering and Technology (ICETET), 2009, pp.416-419.

[4] C.A. Canesin, F.A.S. Goncalves, L.P Sampaio, "Simulation Tools of DC-DC Converters for Power Electronics Education", 13th European Conference on Power Electronics and Applications (EPE), 2009, pp. 1-10.

[5] Rodney H.G. Tan, Matthew Teow Yok Wooi, "A Comprehensive Modeling, Simulation and Computational Implementation of Buck Converter Using MATLAB/Simulink", IEEE Conference on Energy Conversion (CENCON) 2014, 13-14 Oct 2014, Johor Bahru, Malaysia

[6] A.J. Forsyth, S.V. Mollov, "Modelling and Control of DC-DC Converters", Power Engineering Journal, vol. 12, no. 5, 1998, pp. 229-236.

[7] W. Fischer, A. Lindemann,"Circuit Simulation in a Research Oriented Education of Power Electronics", COMPEL 11th Workshop on Control and Modeling of Power Electronics, 2008, pp. 1-5