Note: This is an open-book and open-notes assignment. Please don't take help from the internet.

Instruction: You must choose one problem set from each week.

Week 1 Discussion Problem Sets

Problem Set 1:

Rewrite each of the following statements symbolically and in the if-then form in English.

(a) It is necessary to walk 8 miles to get to the top of Long's Peak.
(b) The audience will go to sleep if the chairperson gives the lecture.
(c) For hiking on the trail to be safe, it is necessary that berries not be ripe along the trail and for grizzly bears not to have been seen in the area.
(d) You can go to the Super Bowl unless you can't afford the ticket.
(e) Knowing the email address is a sufficient condition for sending an email.
(f) Either the Seahawks will win the Super Bowl, or they won't play in the Super Bowl.
(g) Unless you win the lottery, you won't be rich. (h) When John sings, my ears hurt.
(i) A sufficient condition for the warranty to be good is that you bought the computer less than a year ago.
(j) A necessary condition for a life of excellence is experiencing some choices in your life.

Now, write the negation of each statement of the above exercise in English.

Problem Set 2:

(a) Find a statement form involving the statement variables p, q, and r that is true when p and q are true and r is false, but is false otherwise. [Hint: Use a conjunction of each statement variable or its negation.]
(b) Write the converse, inverse and contrapositive of each of the following statements in the if-then form in English.
   I. $|4| < 3$ only if $-3 < 4 < 3$.
   II. A sufficient condition for Kate to take CS 325 is that she passes CS 225.
   III. Getting that job requires knowing someone who knows the boss.
   IV. The message is scanned for viruses whenever the message was sent from an unknown system.
   V. You will reach the summit unless you begin your climb too late.

Problem Set 3:

Show that each of the following statement forms is a tautology by using truth tables.

(a) $[p \wedge (p \rightarrow q)] \rightarrow q$
(b) $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$

Determine whether the statement forms in (c) – (d) are logically equivalent by using truth tables.

(c) $p \leftrightarrow q$ and $\sim p \leftrightarrow \sim q$
(d) $(p \rightarrow q) \rightarrow (r \rightarrow s)$ and $(p \rightarrow r) \rightarrow (q \rightarrow s)$

Problem Set 4:

Determine whether the statement forms in (a) – (d) are logically equivalent by using truth tables.

(a) $\sim (p \oplus q)$ and $p \leftrightarrow q$ ['exclusive or' is denoted by $\oplus$]
(b) $\sim p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$
(c) $p \leftrightarrow q$ and $(p \rightarrow q) \wedge (q \rightarrow p)$
(d) $(p \rightarrow q) \vee (p \rightarrow r)$ and $p \rightarrow (q \vee r)$

Problem Set 5:

Verify the following logical equivalences using laws. In each proof, supply a reason for each step. If they are not logically equivalent, provide a truth table showing that they are not equivalent.

(a) $(p \wedge q) \rightarrow (p \vee q) \equiv t$
(b) $\sim (p \leftrightarrow q) \equiv \sim p \leftrightarrow q$
(c) $(\sim q \wedge (p \rightarrow q)) \rightarrow \sim p \equiv t$

Problem Set 6:

Verify the following logical equivalences using laws. In each proof, supply a reason for each step. If they are not logically equivalent, provide a truth table showing that they are not equivalent.

(a) $(p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r) \equiv t$
(b) $(p \rightarrow q \vee r) \equiv (p \wedge \sim q \rightarrow r)$
(c) $((\sim p \wedge q) \vee (\sim p \wedge \sim q)) \vee (\sim p \wedge q) \equiv$
$\sim p$

Problem Set 7

For each of the following problems,

(a) Use the logical equivalences $p \rightarrow q \equiv \sim p \vee q$ and $p \leftrightarrow q \equiv (\sim p \vee q) \wedge (\sim q \vee p)$ to rewrite the statement form without using the symbol $\rightarrow$ or $\leftrightarrow$
(b) Use the logical equivalence $p \vee q \equiv \sim (\sim p \wedge \sim q)$ to rewrite the statement form using only $\wedge$ and $\sim$.

I. $\sim (p \rightarrow q) \leftrightarrow \sim q$
II. $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

Problem Set 8

For each of the following problems,

(a) Use the logical equivalences $p \rightarrow q \equiv \sim p \vee q$ and $p \leftrightarrow q \equiv (\sim p \vee q) \wedge (\sim q \vee p)$ to rewrite the statement form without using the symbol $\rightarrow$ or $\leftrightarrow$
(b) Use the logical equivalence $p \vee q \equiv \sim (\sim p \wedge \sim q)$ to rewrite the statement form using only $\wedge$ and $\sim$.

I. $(p \leftrightarrow q) \leftrightarrow ((p \rightarrow q) \wedge (q \rightarrow p))$
II. $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$

Problem Set 9

For each of the following problems,

(a) Use the logical equivalences $p \rightarrow q \equiv \sim p \vee q$ and $p \leftrightarrow q \equiv (\sim p \vee q) \wedge (\sim q \vee p)$ to rewrite the statement form without using the symbol $\rightarrow$ or $\leftrightarrow$
(b) Use the logical equivalence $p \vee q \equiv \sim(\sim p \wedge \sim q)$ to rewrite the statement form using only $\wedge$ and $\sim$.
I. $((p \rightarrow q) \vee (p \rightarrow r)) \leftrightarrow (q \vee r)$
II. $(p \vee q) \vee (p \wedge q) \rightarrow q$

Week 2 Discussion Problem Sets

Problem Set 10:

Express the negation of each of the following statements using quantifiers, variables and connectives and then in English.

(a) Every person is healthy if and only if the person run 10 laps daily and the person takes multivitamins.
(b) There is no one in this class who both knows French and who is learning Spanish.
(c) The squares of all real numbers greater than 2 are greater than 4.
(d) No polynomial functions have horizontal asymptotes.
(e) All that glitters is not gold.
(f) Some birds cannot fly.
(g) The square root of a natural number either has a decimal representation which terminates or has a non-terminating decimal representation and also a non-recurring decimal representation.
(h) Any integer with an even square is even.
(i) The product of any two fractions is a fraction.
(j) Everyone in your class enjoys either Thai or Chinese food.

Problem Set 11:

Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consist of the students in your class and second, let it consist of all people.

(a) Everyone in your class has not taken CS 161.
(b) There is at least one person in your class who hasn't taken CS 225 but has taken CS 161.
(c) No students in you class have taken CS 261.
(d) Some students in your class do not want to take CS 225.
(e) Any student in your class has either taken CS 161 or she/he has taken CS 225.

Problem Set 12:

Determine the truth value of each of the following statements. The domain of discourse is $\mathbb{R}$. Justify your answers.

(a) $\forall x(x^2 > x)$
(b) $\exists x(x^2 > x)$
(c) $\forall x(x > 1 \rightarrow x^2 > x)$
(d) $\forall x(x > 1 \rightarrow [\frac{x}{(x^2+1)} < \frac{1}{3}])$
(e) $\exists x(x > 1 \wedge [\frac{x}{(x^2+1)} < \frac{1}{3}])$

Now, write the negation of each statement of the above exercise symbolically and then in English.

Problem Set 13:

(a) The computer scientists Richard Conway and David Gries once wrote: The absence of error messages during translation of a computer program is only a necessary and not a sufficient condition for reasonable [program] correctness. Rewrite this statement in English without using the words necessary or sufficient.
(b) Whenever there is an active alert, all queued messages are transmitted. Rewrite this statement in English using a quantifier and without using the word whenever.
(c) The file system cannot be backed up unless there is not a user currently logged on. Rewrite this statement in English using a quantifier and without using the word unless.
(d) Answer the following question:
Taking the long view on your education, you go to the Prestige Corporation and ask what you should do in college to be hired when you graduate. The personnel director replies that you will be hired only if you major in mathematics or computer science, get a B average or better, and take accounting. You do, in fact, become a math major, get a B+ average, and take accounting. You return to Prestige Corporation, make a formal application, and are turned down. Did the personnel director lie to you?

Problem Set 14:

Let the following predicates be given-

F (x): "x is used frequently"

C (x): "x is in the correct place"

E (x): "x is in excellent condition"

Express each of the following English sentences in terms of F (x), C (x), E (x), quantifiers, and logical connectives. Assume that the domain for x consists of all the tools.

(a) No tools are used frequently and are in excellent condition.
(b) Some tools are neither in excellent condition nor are used frequently.
(c) All frequently used tools are in the correct places unless they are in excellent condition. (d) Every frequently used tool is in the correct place if they are in excellent condition.

Now write the negation of each statement of the above exercise symbolically and in English.

Problem Set 15:

Write the contrapositive, converse, and inverse for each of the following statements. Indicate as best as you can which of these statements (among the 4) are true and which are false. Give a counterexample for each that is false.

(a) For all integers d, d is a prime integer only if d is not a perfect square.
(b) For all real numbers a and b, if $a < b$ then $a^2 < b^2$. (c) For all integers n, if n is odd then $\frac{n-1}{2}$ is odd.
(d) For all integers n, n^2 is divisible by 4 unless n is not divisible by 4.
(e) For any positive integers m and n, if mn is a perfect square, then m and n are perfect squares.

Problem Set 16:

Compute the following summations. (Instruction: Showing your work is necessary. But you don't have to provide a final numerical value, an intermediate form will be sufficient.)

(a) $\sum_{i=23}^{899}(4i^2 + 6i + 3)$

(b) $\sum_{i=1}^{90}(i^3 + (-5)^{i+2} - 10)$

(c) $\sum_{i=12}^{100}(3^i + 2^{i+2} - 15i)$

(d) In a sequence a_1, a_2, a_3, ..., a_{200}, the kth term is defined by $a_k = \frac{1}{k} - \frac{1}{(k+1)}$ for all integers k from 1 through 200. What is sum of the 200 terms of this sequence?

Problem Set 17:

Compute the following summations.(Instruction: Showing your work is necessary. But you don't have to provide a final numerical value, an intermediate form will be sufficient.)

(a) $\sum_{i=10}^{77}(6i^2 + (-2)^i - 10)$

(b) $\sum_{i=1}^{20}(2 * 4^{i+2} - 2^{i+2})$

(c) $\sum_{i=12}^{50}\sum_{j=1}^{3}(7ij - 3)$

(d) $6\sum_{i=10}^{500}(10i^2 - 3) - 15\sum_{i=1}^{500}(i^2 + 2)$

Problem Set 18:

Find explicit formulas for sequences of the form a_1, a_2, a_3, . . . with the initial terms given below:

(a) 3, 6, 12, 24, 48, 96, 192, ...
(b) 15, 8, 1, −6, −13, −20, −27, ...
(c) 0, 2, 8, 26, 80, 242, 728, 2186, 6560, 19682, ...
(d) 1, 3, 15, 105, 945, 10395, 135135, 2027025, …
(e) 10, 2, 0.4, 0.08, 0.016, ...
(g) $90, -30, 10, -3\frac{1}{3}, 1\frac{1}{9}, ...$

(h) $-4, \frac{7}{2}, -\frac{5}{2}, \frac{13}{8}, -1, ...$