Discussion Weeks 1&2 Solutions

Contents

Week 1 Problems

Problem Set 1

Problem: Rewrite each of the following statements symbolically and in the if-then form in English

(a)

- Original:
  It is necessary to walk 8 miles to get to the top of Long's Peak.
- Symbolic
  Let p: you walk 8 miles
  Let q: you get to the top of Long's Peak
  So, we get, $\sim p \rightarrow \sim q$
  Alternatively, $q \rightarrow p$
- English
  If you do not walk 8 miles, then you cannot get to the top of Long's Peak.
  Alternatively: If you reach the top of Long's Peak, then you walked 8 miles.
- Negation
  You do not walk 8 miles and you get to the top of Long's Peak.

(b)

- Original:
  The audience will go to sleep if the chairperson gives the lecture.
- Symbolic
  Let p: the chairperson gives the lecture
  Let q: the audience goes to sleep
  So, we get, $p \rightarrow q$
- English
  If the chairperson gives the lecture, then the audience will go to sleep.
- Negation
  The chairperson gives the lecture and the audience does not go to sleep.

(c)

- Original:
  For hiking on the trail to be safe, it is necessary that berries not be ripe along the trail and for grizzly bears not to have been seen in the area.
- Symbolic
  Let p: hiking on the trail is safe
  Let q: berries are ripe along the trail
  Let r: grizzly bears have been seen in the area
  So, we get, $\sim(\sim q \wedge \sim r) \rightarrow \sim p$
  simplified: $q \vee r \rightarrow \sim p$
  or $p \rightarrow (\sim q \wedge \sim r)$
- English

If berries are ripe along the trail or grizzly bears have been seen in the area, then hiking on the trail is not safe.
or: If hiking on the trail is safe, then berries are not ripe along the trail and grizzly bears have not been seen in the area.

- Negation
  Hiking on the trail is safe and berries are ripe along the trail or grizzly bears have been seen in the area.

(d)

- Original:
  You can go to the Super Bowl unless you can’t afford the ticket.
- Symbolic
  Let p: you can afford the ticket
  Let q: you can go to the Super Bowl
  So, we get, $p \rightarrow q$
- English
  If you can afford the ticket, then you can go to the Super Bowl.
- Negation
  You can afford the ticket and you cannot go to the Super Bowl.

(e)

- Original:
  Knowing the email address is a sufficient condition for sending an email.
- Symbolic
  Let p: you know the email address
  Let q: you can send an e-mail message
  So, we get, $p \rightarrow q$
- English
  If you know the email address, then you can send an e-mail message.
- Negation
  You know the email address and you cannot send an e-mail message.

(f)

- Original:
  Either the Seahawks will win the Super Bowl, or they will not play in the Super Bowl.
- Symbolic
  Let p: the Seahawks win the Super Bowl
  Let q: the Seahawks play in the Super Bowl
  $p \vee \sim q$ or $\sim p \rightarrow \sim q$
- English
  If the Seahawks do not win the Super Bowl, then the Seahawks do not play in the Super Bowl.
- Negation

The Seahawks do not win the Super Bowl and the Seahawks do play in the Super Bowl.

(g)

- Original:
  Unless you win the lottery, you won't be rich.
- Symbolic
  Let p: you win the lottery
  Let q: you will be rich
  So, we have, $\sim p \to \sim q$
- English
  If you do not win the lottery, you will not be rich.
- Negation
  You do not win the lottery and you become rich.

(h)

- Original:
  When John sings, my ears hurt.
- Symbolic
  Let p: John sings
  Let q: my ears hurt
  So, we have, $p \to q$
- English
  If John sings, then my ears hurt.
- Negation
  John sings and my ears do not hurt.

(i)

- Original:
  A sufficient condition for the warranty to be good is that you bought the computer less than a year ago.
- Symbolic
  Let p: you bought the computer less than a year ago
  Let q: the warranty is good
  So, we have, $p \to q$
- English
  If you bought the computer less than a year ago, then the warranty is good.
- Negation
  You bought the computer less than a year ago and the warranty is not good.

(j)

- Original:

  A necessary condition for a life of excellence is experiencing some choices in your life.
- Symbolic

  Let p: you experience some choices in your life

  Let q: you have a life of excellence

  So, we have, $\sim p \rightarrow \sim q$
- English

  If you are have a life of excellence, then you experience some choices in your life. Equivalently, if you do not experience some choices in your life, then you will not have a life of excellence.
- Negation

  You experience some choices in your life and you don't have a life of excellence.

Problem Set 2

Part a

Problem: Find a statement form involving the statement variables p, q, and r that is true when p and q are true and r is false, but is false otherwise.

$(p \wedge q) \wedge \sim r$

Part b

Problem: Write the converse, inverse and contrapositive of each of the statements in the if-then form in English

I. $|4| < 3$ only if $-3 < 4 < 3$.

- Rewritten:
  If $|4| < 3$ then $-3 < 4 < 3$.
- Converse:
  if $-3 < 4 < 3$ then $|4| < 3$.
- Inverse:
  if $|4| \geq 3$ then $4 \geq 3$ or $4 \leq -3$.
- Contrapositive:
  if $4 \geq 3$ or $4 \leq -3$ then $|4| \geq 3$.

II. A sufficient condition for Kate to take CS 325 is that she passes CS 225.

- Rewritten:
  if Kate passes CS 225 then she can take CS 325.
- Converse:
  If she can take CS 325, then Kate passes CS 225.
- Inverse:
  If she does not pass CS 225, then she cannot take CS 325.
- Contrapositive:
  If she cannot take CS 325 then she did not pass CS 225.

III. Getting that job requires knowing someone who knows the boss.

- Rewritten:
  if you don't know someone who knows the boss then you won't get the job.
- Converse:
  If you don't get that job then you didn't know someone who knows the boss,
- Inverse:
  If you know someone who knows the boss then you will get that job.
- Contrapositive:

If you get that job, then you knew someone who knew the boss.

IV. The message is scanned for viruses whenever the message was sent from an unknown system.

- Rewritten:
  if the message was sent from an unknown system, then it is scanned for viruses.
- Converse:
  If the message was scanned for viruses, then the message was sent from an unknown system.
- Inverse:
  If the message was not sent from an unknown system, then the message was not sent scanned for viruses.
- Contrapositive:
  If the message was not scanned for viruses, then the message was not sent from an unknown system.

V. You will reach the summit unless you begin your climb too late.

- Rewritten:
  If you did not begin your climb too late, then you will reach the summit.
- Converse:
  If you reach the summit, then you didn't begin your climb too late.
- Inverse:
  If you began your climb too late, then you will not reach the summit.
- Contrapositive:
  If you do not reach the summit, then you began your climb too late.

Problem Set 3

Problem: Show that the statements are tautologies by using truth tables

(a) $[p \wedge (p \rightarrow q)] \rightarrow q$

p	q	p→q	p∧(p→q)	[p∧(p→q)]→q
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

The truth values of the last column are all T's. So, $[p \wedge (p \rightarrow q)] \rightarrow q$ is a tautology.

(b) $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$

p	q	r	p∨q	p→r	q→r	(p∨q)∧(p→r)	(p∨q)∧(p→r)∧(q→r)	[(p∨q)∧(p→r)∧(q→r)]→r
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	T	T	T	T	T	T
T	F	F	T	F	T	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	F	T	F	T
F	F	T	F	T	T	F	F	T
F	F	F	F	T	T	F	F	T

The truth values of the last column are all T's. So, $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$ is a tautology.

Problem: Determine whether the statements are logically equivalent using truth tables

(c) $p \leftrightarrow q$ and $\sim p \leftrightarrow \sim q$

p	q	$p \rightarrow q$	$q \rightarrow p$	$p \leftrightarrow q$	$\sim p \rightarrow \sim q$	$\sim p \rightarrow \sim q$	$\sim p \leftrightarrow \sim q$
T	T	T	T	T	T	T	T
T	F	F	T	F	T	F	F
F	T	T	F	F	F	T	F
F	F	T	T	T	T	T	T

The values in the highlighted columns match so the statements are logically equivalent

(d) $(p \to q) \to (r \to s)$ and $(p \to r) \to (q \to s)$

Let $w \equiv (p \to q)$, $x \equiv (r \to s)$, $y \equiv (p \to r)$, and $z \equiv (q \to s)$

p	q	r	s	w	x	y	z	$w \to x$	$y \to z$
T	T	T	T	T	T	T	T	T	T
T	T	T	F	T	F	T	F	F	F
T	T	F	T	T	T	F	T	T	T
T	T	F	F	T	T	F	F	T	T
T	F	T	T	F	T	T	T	T	T
T	F	T	F	F	F	T	T	T	T
T	F	F	T	F	T	F	T	T	T
T	F	F	F	F	T	F	T	T	T
F	T	T	T	T	T	T	T	T	T
F	T	T	F	T	F	T	F	F	F
F	T	F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	F	T	F
F	F	T	T	T	T	T	T	T	T
F	F	T	F	T	F	T	T	F	T
F	F	F	T	T	T	T	T	T	T
F	F	F	F	T	T	T	T	T	T

The highlighted cells are different so the statements are not logically equivalent.

Problem Set 4

Problem: Determine whether the statement pairs are logically equivalent using truth tables

(a) $\sim (p \oplus q)$ and $p \leftrightarrow q$

p	q	$p \oplus q$	$\sim(p \oplus q)$	$p \to q$	$q \to p$	$p \leftrightarrow q$
T	T	F	T	T	T	T
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

Since the highlighted cells match the statements are logically equivalent

(b) $\sim p \to (q \to r)$ and $q \to (p \vee r)$

p	q	r	$q \to r$	$\sim p \to (q \to r)$	$q \to (p \vee r)$
T	T	T	T	T	T
T	T	F	F	T	T
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	T	T	T
F	T	F	F	F	F
F	F	T	T	T	T
F	F	F	T	T	T

Since the highlighted cells match the statements are logically equivalent

(c) $p \leftrightarrow q$ and $(p \to q) \wedge (q \to p)$

p	q	$p \leftrightarrow q$	$(p \to q)$	$(q \to p)$	$(p \to q) \wedge (q \to p)$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

Since the highlighted cells match the statements are logically equivalent

(d) $(p \to q) \vee (p \to r)$ and $p \to (q \vee r)$

p	q	r	$p \to q$	$p \to r$	$(p \to q) \vee (p \to r)$	$p \to (q \vee r)$
T	T	T	T	T	T	T
T	T	F	T	F	T	T
T	F	T	F	T	T	T
T	F	F	F	F	F	F
F	T	T	T	T	T	T
F	T	F	T	T	T	T

F	F	T	T	T	T	T
F	F	F	T	T	T	T

Since the highlighted cells match the statements are logically equivalent

Problem Set 5

Problem: Verify the logical equivalences with laws.

(a) $\sim(p \vee (\sim p \wedge q)) \equiv \sim p \wedge \sim(\sim p \wedge q)$ by the De Morgan's Law

$\equiv \sim p \wedge (\sim(\sim p) \vee \sim q)$ by the De Morgan's law

$\equiv \sim p \wedge (p \vee \sim q)$ by the double negative law

$\equiv (\sim p \wedge p) \vee (\sim p \wedge \sim q)$ by the distributive law

$\equiv (p \wedge \sim p) \vee (\sim p \wedge \sim q)$ by the commutative law

$\equiv C \vee (\sim p \wedge \sim q)$ by the negation law

$\equiv (\sim p \wedge \sim q) \vee C$ by the commutative law

$\equiv (\sim p \wedge \sim q)$ by the identity law

Therefore, both statements are logically equivalent.

(b) $(p \wedge q) \rightarrow (p \vee q) \equiv \sim(p \wedge q) \vee (p \vee q)$ by logical equivalence of conditional statements

$\equiv (\sim p \vee \sim q) \vee (p \vee q)$ by the De Morgan's law

$\equiv (p \vee \sim p) \vee (q \vee \sim q)$ by the associative and commutative laws

$\equiv t \vee t$ by the negation law

$\equiv t$ by the universal bound law

Therefore, both statements are logically equivalent.

(c) $\sim(p \leftrightarrow q) \equiv \sim((p \rightarrow q) \wedge (q \rightarrow p))$ by logical equivalence of biconditional statements

$\equiv \sim((\sim p \vee q) \wedge (\sim q \vee p))$ by logical equivalence of conditional statements

$\equiv \sim(\sim p \vee q) \vee \sim(\sim q \vee p)$ by the De Morgan's law

$\equiv (\sim(\sim p) \wedge \sim q) \vee (\sim(\sim q) \wedge \sim p)$ by the De Morgan's law

$\equiv (p \wedge \sim q) \vee (q \wedge \sim p)$ by the double negative law

$\equiv ((p \wedge \sim q) \vee q) \wedge ((p \wedge \sim q) \vee \sim p)$ by the distributive law

$\equiv (q \vee (p \wedge \sim q)) \wedge (\sim p \vee (p \wedge \sim q))$ by the commutative law

$\equiv ((q \vee p) \wedge (q \vee \sim q)) \wedge ((\sim p \vee p) \wedge (\sim p \vee \sim q))$ by the distributive law

$\equiv ((q \vee p) \wedge (q \vee \sim q)) \wedge ((p \vee \sim p) \wedge (\sim p \vee \sim q))$ by the commutative law

$\equiv ((q \vee p) \wedge t) \wedge (t \wedge (\sim p \vee \sim q))$ by the negation law

$\equiv ((q \vee p) \wedge t) \wedge ((\sim p \vee \sim q) \wedge t)$ by the commutative law

$\equiv (q \vee p) \wedge (\sim p \vee \sim q)$ by the identity law

$\equiv (p \vee q) \wedge (\sim q \vee \sim p)$ by the commutative law

$\equiv (\sim(\sim p) \vee q) \wedge (\sim q \vee \sim p)$ by the double negative law

$\equiv (\sim p \rightarrow q) \wedge (q \rightarrow \sim p)$ by logical equivalence of conditional statements

$\equiv \sim p \leftrightarrow q$ by the logical equivalence of biconditional statements

Therefore, both statements are logically equivalent.

(d) $\left(\sim q \wedge (p \rightarrow q)\right) \rightarrow \sim p$

$\equiv \sim\left(\sim q \wedge (\sim p \vee q)\right) \vee \sim p$, by the logical equivalence of conditional statements
$\equiv \left(q \vee \sim(\sim p \vee q)\right) \vee \sim p$, by De Morgan's and double negative laws
$\equiv \left(q \vee (p \wedge \sim q)\right) \vee \sim p$, by De Morgan's and double negative laws
$\equiv \left((q \vee \sim q) \wedge (q \vee p)\right) \vee \sim p$, by distributive law
$\equiv \left(t \wedge (q \vee p)\right) \vee \sim p$, by negation law
$\equiv q \vee (p \vee \sim p)$, by identity and associative laws
$\equiv q \vee t$, by negation law
$\equiv t$, by universal bound law

Therefore, both statements are logically equivalent.

Problem Set 6

Problem: Verify the logical equivalences with laws.

(a) $\sim(q \rightarrow p) \vee (p \wedge q) \equiv \sim(\sim q \vee p) \vee (p \wedge q)$ by logical equivalence of conditional statements

$\equiv (\sim(\sim q) \wedge \sim p) \vee (p \wedge q)$ by the De Morgan's law

$\equiv (q \wedge \sim p) \vee (p \wedge q)$ by the double negative law

$\equiv q \wedge (\sim p \vee p)$ by the distributive law

$\equiv q \wedge t$ by the commutative and negation law

$\equiv q$ by the identity law

Therefore, both statements are logically equivalent.

(b) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

$\equiv \sim[(\sim p \vee q) \wedge (\sim q \vee r)] \vee (\sim p \vee r)$ by logical equivalence of conditional statements

$\equiv [\sim(\sim p \vee q) \vee \sim(\sim q \vee r)] \vee (\sim p \vee r)$ by the De Morgan law

$\equiv (\sim(\sim p) \wedge \sim q) \vee (\sim(\sim q) \wedge \sim r) \vee \sim p \vee r$ by the De Morgan law

$\equiv (p \wedge \sim q) \vee (q \wedge \sim r) \vee \sim p \vee r$ by the double negative law

$\equiv \sim p \vee (p \wedge \sim q) \vee (q \wedge \sim r) \vee r$ by the commutative law

$\equiv ((\sim p \vee p) \wedge (\sim p \vee \sim q)) \vee ((q \vee r) \wedge (\sim r \vee r))$ by the distributive law

$\equiv (t \wedge (\sim p \vee \sim q)) \vee ((q \vee r) \wedge t)$ by the negation law

$\equiv (\sim p \vee \sim q) \vee (q \vee r)$ by the identity law

$\equiv \sim p \vee (\sim q \vee q) \vee r$ by the associative law

$\equiv \sim p \vee t \vee r$ by the negation law

$\equiv t$ by the universal bound law

Therefore, the statement is a tautology.

(c) $p \rightarrow (q \vee r) \equiv \sim p \vee (q \vee r)$ by logical equivalence of conditional statements

$\equiv (\sim p \vee q) \vee r$ by the associative law

$\equiv \sim(p \wedge \sim q) \vee r$ by the De Morgan's law

$\equiv (p \wedge \sim q) \to r$ by the logical equivalence of conditional statements

Therefore, both statements are logically equivalent.

(d) $\big((\sim p \wedge q) \vee (\sim p \wedge \sim q)\big) \vee (\sim p \wedge q)$

$\equiv \big((\sim p \wedge \sim q) \vee (\sim p \wedge q)\big) \vee (\sim p \wedge q)$, Commutative law

$\equiv (\sim p \wedge \sim q) \vee \big((\sim p \wedge q) \vee (\sim p \wedge q)\big)$, Associative law

$\equiv (\sim p \wedge \sim q) \vee (\sim p \wedge q)$, Idempotent law

$\equiv \big((\sim p \wedge \sim q) \vee \sim p\big) \wedge \big((\sim p \wedge \sim q) \vee q\big)$, Distributive law

$\equiv (\sim p) \wedge \big((\sim p \wedge \sim q) \vee q\big)$, Absorption law

$\equiv \sim p \wedge \big((\sim p \vee q) \wedge (\sim q \vee q)\big)$, Distributive law

$\equiv \sim p \wedge \big((\sim p \vee q) \wedge t\big)$, Negation law

$\equiv \sim p \wedge (\sim p \vee q)$, Identity law

$\equiv \sim p$, Absorption law

Therefore, both statements are logically equivalent

Problem Set 7

Problem:

(a) Use the logical equivalences p →q ≡~ p ∨ q and p ↔ q ≡ (~p ∨ q) ∧ (~q ∨ p) to rewrite thestatement form without using the symbol → or ↔

(b) Use the logical equivalence p ∨ q ≡~(~p ∧~q) to rewrite the statement form using only ∧ and

I. ~(p → q) ↔ ~q

(a)~(p → q) ↔ ~q

$\equiv \sim(\sim p \vee q) \leftrightarrow \sim q$ by logical equivalence of conditional statement

$\boldsymbol{\equiv ((\sim p \vee q) \vee \sim q) \wedge (\sim(\sim p \vee q) \vee q)}$ by logical equivalence of biconditional statement

(b)Result from I. $((\sim p \vee q) \vee \sim q) \wedge (\sim(\sim p \vee q) \vee q)$

$\equiv \sim\big(\sim(\sim p \vee q) \wedge \sim(\sim q)\big) \wedge ((p \wedge \sim q) \vee q)$ By De Morgan's law

$\boldsymbol{\equiv \sim\big((p \wedge \sim q) \wedge q\big) \wedge \sim(\sim (p \wedge \sim q) \wedge \sim q)}$ By De Morgan's law

II. $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

(a) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

$\equiv [(\sim p \vee q) \wedge (\sim q \vee r)] \rightarrow (\sim p \vee r)$, by logical equivalence of conditional statement (→)

$\equiv \sim[(\sim p \vee q) \wedge (\sim q \vee r)] \vee (\sim p \vee r)$, by logical equivalence of conditional statement

(b)Result from I. $\equiv \sim[(\sim p \vee q) \wedge (\sim q \vee r)] \vee (\sim p \vee r)$

$\equiv \sim[\sim(p \wedge \sim q) \wedge \sim(q \wedge \sim r)] \vee \sim(p \wedge \sim r)$, by De Morgan's Law

$\equiv \boldsymbol{\sim\{[\sim(p \wedge \sim q) \wedge \sim(q \wedge \sim r)] \wedge (p \wedge \sim r)\}}$, by De Morgan's Law

Problem Set 8

Problem:

(a) Use the logical equivalences p →q ≡~ p ∨ q and p ↔ q ≡ (~p ∨ q) ∧ (~q ∨ p) to rewrite thestatement form without using the symbol → or ↔

(b) Use the logical equivalence p ∨ q ≡~(~p ∧~q) to rewrite the statement form using only ∧ and ~

I. $(p \leftrightarrow q) \leftrightarrow \big((p \rightarrow q) \wedge (q \rightarrow p)\big)$

(a) $(p \leftrightarrow q) \leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p)$

$\equiv (p \leftrightarrow q) \leftrightarrow (\sim p \vee q) \wedge (\sim q \vee p)$, by logical equivalence of conditional statement

$\equiv \big((\sim p \vee q) \wedge (\sim q \vee p)\big) \leftrightarrow (\sim p \vee q) \wedge (\sim q \vee p)$, by logical equivalence of biconditional statement

$\equiv \Big(\big((\sim p \vee q) \wedge (\sim q \vee p)\big) \vee \sim\big((\sim p \vee q) \wedge (\sim q \vee p)\big)\Big) \wedge \Big(\sim\big((\sim p \vee q) \wedge (\sim q \vee p)\big) \vee \big((\sim p \vee q) \wedge (\sim q \vee p)\big)\Big)$

(b) $\Big(\big((\sim p \vee q) \wedge (\sim q \vee p)\big) \vee \sim\big((\sim p \vee q) \wedge (\sim q \vee p)\big)\Big) \wedge \Big(\sim\big((\sim p \vee q) \wedge (\sim q \vee p)\big) \vee \big((\sim p \vee q) \wedge (\sim q \vee p)\big)\Big)$

$\equiv \sim\Big(\sim\big(\sim(p \wedge \sim q) \wedge \sim(q \wedge \sim p)\big) \wedge \big(\sim(p \wedge \sim q) \wedge \sim(q \wedge \sim p)\big)\Big) \wedge \sim\Big(\big(\sim(p \wedge \sim q) \wedge \sim(p \wedge \sim p)\big) \wedge \sim\big(\sim(p \wedge \sim q) \wedge \sim(q \wedge \sim p)\big)\Big)$, by De Morgan's law

II. [(p ∨ q) ∧ (p → r) ∧ (q → r)] → r

(a) [(p ∨ q) ∧ (p → r) ∧ (q → r)] → r

≡ [(p ∨ q) ∧ (~p ∨ r)] ∧ (~q ∨ r)] → r by logical equivalence of conditional statement (→)

≡~[(p ∨ q) ∧ (~p ∨ r)] ∧ (~q ∨ r)] ∨ r by logical equivalence of conditional statement (→)

(b) Result from I. ≡~[(p ∨ q) ∧ (~p ∨ r)] ∧ (~q ∨ r)] ∨ r

≡~(~(~[(p v q) ∧ (~p v r) ∧ (~q v r)]) ∧ ~ r) By De Morgan's law

≡~([~(~p ∧ ~q) ∧ ~(~(~p) ∧ ~r) ∧ ~(~(~q) ∧ ~r)]) ∧ ~ r) By De Morgan's law

Problem Set 9

Problem:

(a) Use the logical equivalences p →q ≡~ p ∨ q and p ↔ q ≡ (~p ∨ q) ∧ (~q ∨ p) to rewrite thestatement form without using the symbol → or ↔

(b) Use the logical equivalence p ∨ q ≡~(~p ∧~q) to rewrite the statement form using only ∧ and

I. $\big((p \rightarrow q) \vee (p \rightarrow r)\big) \leftrightarrow (q \vee r)$

(a) $\big((p \rightarrow q) \vee (p \rightarrow r)\big) \leftrightarrow (q \vee r)$

$\equiv \big((\sim p \vee q) \vee (\sim p \vee r)\big) \leftrightarrow (q \vee r)$, by logical equivalence of conditional statement

$\equiv \Big(\big((\sim p \vee q) \vee (\sim p \vee r)\big) \vee \sim(q \vee r)\Big) \wedge \Big(\sim\big((\sim p \vee q) \vee (\sim p \vee r)\big) \vee (q \vee r)\Big)$, by logical equivalence of biconditional statement

(b) $\Big(\big((\sim p \vee q) \vee (\sim p \vee r)\big) \vee \sim(q \vee r)\Big) \wedge \Big(\sim\big((\sim p \vee q) \vee (\sim p \vee r)\big) \vee (q \vee r)\Big)$

$\equiv \Big(\big(\sim(p \wedge \sim q) \vee \sim(p \wedge \sim r)\big) \vee (\sim q \wedge \sim r)\Big) \wedge \Big(\big(\sim(\sim p \vee q) \wedge \sim(\sim p \vee r)\big) \vee (q \vee r)\Big)$, by De Morgan's law

$\equiv \Big(\sim\big((p \wedge \sim q) \wedge (p \wedge \sim r)\big) \vee (\sim q \wedge \sim r)\Big) \wedge \Big(\big((p \wedge \sim q) \wedge (p \wedge \sim r)\big) \vee (q \vee r)\Big)$, by De Morgan's law

$\equiv \sim\Big(\big((p \wedge \sim q) \wedge (p \wedge \sim r)\big) \wedge \sim(\sim q \wedge \sim r)\Big) \wedge \sim\Big(\sim\big((p \wedge \sim q) \wedge (p \wedge \sim r)\big) \wedge (\sim q \wedge \sim r)\Big)$, by De Morgan's law

II. $(p \vee q) \vee (p \wedge q) \rightarrow q$

(a) $(p \vee q) \vee (p \wedge q) \rightarrow q$

$\equiv \sim[(p \vee q) \vee (p \wedge q)] \vee q$, by logical equivalence of conditional statement (→)

(b) Result from I. $\sim[(p \vee q) \vee (p \wedge q)] \vee q$

$\equiv \sim([(p \vee q) \vee (p \wedge q)] \wedge \sim q)$, by De Morgan's Law

$\equiv \sim(\sim[\sim(p \vee q) \wedge \sim(p \wedge q)] \wedge \sim q)$, by De Morgan's Law

$\equiv \sim(\sim[(\sim p \wedge \sim q) \wedge \sim(p \wedge q)] \wedge \sim q)$, by De Morgan's Law

Week 2 Problems

Problem Set 10

(a) Every person is healthy if and only if the person run 10 laps daily and the person takes multi-vitamins.

- Statement: ∀x, P(x) [healthy] ↔ (Q(x) [run 10 laps daily] ∧ R(x) [takes multi-vitamins])
- Negation: ∃x such that [P(x) ∧ (~Q(x) ∨ ~R(x)) ∨ (Q(x) ∧ R(x)) ∧ ~P(x)]
- (There is at least one person who is healthy, but neither run 10 laps daily nor takes multi-vitamins) or (There is at least one person who runs 10 laps daily and takes multi-vitamins, but is not healthy).

(b) There is no one in this class who both knows French and who is learning Spanish.

- Statement: ∀x, P(x) [people in class] → ~(Q(x) [knows French] ∧ R(x) [learning Spanish]) Equivalent Statement: ∀x, P(x) → ~Q(x) ∨ ~R(x)
- Negation: ∃x such that P(x) ∧ ~(~Q(x) ∨ ~R(x)) ≡ ∃x, such that P(x) ∧ (Q(x) ∧ R(x))
- There is at least one person in class, who both knows French and is learning Spanish.

(c) The squares of all real numbers greater than 2 are greater than 4.

- Statement: ∀x ∈ R {x | x > 2}, x^2 > 4
- Negation: ∃x ∈ R {x | x > 2}, such that x^2 ≤ 4
- There is at least one real number greater than 2 that when squared is no more than 4.

(d) No polynomial functions have horizontal asymptotes.

- Statement: ∀x, P(x) [polynomial functions] → ~Q(x) [have horizontal asymptotes] Negation: ∃x such that P(x) ∧ ~(~Q(x))
- Equivalent Statement: ∃x, such that P(x) ∧ Q(x))
- There exists a function that has horizontal asymptotes.

(e) All that glitters is not gold.

- Statement: ∀x, P(x) [glitters] → ~Q(x) [gold]
- Negation: ∃x such that P(x) ∧ ~(~Q(x))
- Equivalent Statement: ∃x, such that P(x) ∧ Q(x))
- Some that glitters and is gold.

(f) Some birds cannot fly.

- Statement: $\forall x$ such that $P(x)$ [birds] such that $\sim Q(x)$ [fly]
- Negation: $\exists x, P(x) \rightarrow Q(x)$
- All birds can fly.

(g) The square root of a natural number either has a decimal representation which terminates or has a non-terminating decimal representation and also a non-recurring decimal representation.

- Statement: $\forall x$, $P(x)$ [square root of natural number] $\rightarrow$ ($Q(x)$ [decimal representation that terminates]) $\vee$ ($R(x)$ [non-terminating decimal representation] $\wedge$ $T(x)$ [non-recurring decimal representation])
- Negation: $\exists x$ such that $P(x) \wedge \sim(Q(x) \vee (R(x) \wedge T(x)))$
- There is at least one square root of a natural number that does not have a decimal representation that terminates and does not have both a non-terminating decimal representation and a non-recurring decimal representation.

(h) Any integer with an even square is even.

- Statement: $\forall x$, $P(x)$ [has an even square] $\rightarrow$ ($Q(x)$ [is even]
- Negation: $\exists x$ such that $P(x) \wedge \sim(Q(x)$
- There exists an integer which has an even square and is odd.

(i) The product of any two fractions is a fraction.

- Statement: $\forall x \in Q$, $\forall y \in Q$, if $P(x)$[is a fraction] $\wedge$ $P(y)$[is a fraction] $\rightarrow$ $P(x*y)$ [is a product of two fractions]
- Negation: $\exists x \in Q, \exists y \in Q, P(x) \wedge P(y) \wedge \sim P(x*y)$
- There exist two fractions such that their product is not a fraction.

(j) Everyone in your class enjoys either Thai or Chinese food.

- Statement: $\forall x$, $P(x)$[enjoys Thai food] $\vee$ $Q(x)$[enjoys Chinese food]
- Negation: $\exists x, \sim(P(x) \vee Q(x)) \equiv \exists x, \sim P(x) \wedge \sim Q(x)$
- There exists a person in class who does not enjoy Thai food and does not enjoy Chinese food.

Problem Set 11

Problem: Translate each statement 2 ways. First with domain as students in your class then as all people

(a) Everyone in your class has not taken CS 161.

- ≡ All students in my class have not taken CS 161.
  Let P(x): x is in your class and Q(x): x has taken CS 161
  - $\forall x \in D, \sim Q(x)$
  - $\forall x \in D, P(x) \rightarrow \sim Q(x)$

(b) There is at least one person in your class who have not taken CS 225 but have taken CS 161

- Let P(x): x is in your class and R(x): x has taken CS 225 and Q(x): x has taken CS 161
  - $\exists\, x \in D$ such that $\sim R(x) \wedge Q(x)$
  - $\exists\, x \in D$ such that $P(x) \wedge (\sim R(x) \wedge Q(x))$

(c) No students in you class have taken CS 261.

- ≡ All students in my class have not taken CS 261.
  Let P(x): x is in your class and G(x): x has taken CS 261
  - $\forall x \in D, \sim G(x)$
  - $\forall x \in D, P(x) \rightarrow \sim G(x)$

(d) Some students in your class do not want to take CS 225.

- Let P(x): x is in your class and R(x): x wants to take CS 225
  - $\exists\, x \in D$ such that $\sim R(x)$
  - $\exists\, x \in D$ such that $P(x) \wedge \sim R(x)$

(e) Any student in your class has either taken CS 161 or she/he has taken CS 225.

- Let P(x): x is in your class and R(x): x has taken CS 225 and Q(x): x has taken CS 161
  - $\forall x \in D, Q(x) \vee R(x)$
  - $\forall x \in D, P(x) \rightarrow (Q(x) \vee R(x))$

Problem Set 12

Problem: Determine the truth for the statements with the domain as $\mathbb{R}$ and justify. Then write the negation

(a) $\forall x(x^2 > x)$ – False because if x = 1, 1^2 is not >1. This should be true for ALL real numbers in domain R.

Negation: $\exists x(x^2 \leq x)$; There is a real number x, such that its square is less than or equal to itself.

(b) $\exists x(x^2 > x)$ –True because for some numbers in domain R this is true. Ex) if x > 1 than its square is greater than itself.

Negation: $\forall x(x^2 \leq x)$; All real numbers have squares less than or equal to themselves.

(c) $\forall x(x > 1 \rightarrow x^2 > x)$ –True because if a real number is >1, then its square is greater than itself.

Negation: $\exists x(x > 1 \wedge x^2 \leq x)$; There is a real number x, such that it is greater than 1 and its square is also less than or equal to itself.

(d) $\forall x(x > 1 \rightarrow [\frac{x}{x^2+1} < \frac{1}{3})$]) – False because when x >1, the sum of itself divided by the sum of itself squared plus one is not less than $\frac{1}{3}$. if x=2, then $\frac{2}{5}$ is not less than $\frac{1}{3}$.

Negation: $\exists x(x > 1 \wedge [\frac{x}{x^2+1} \geq \frac{1}{3}])$; There is a real number x such that x is greater than 1 and the expression $\frac{x}{x^2+1}$ is also greater than or equal to $\frac{1}{3}$.

(e) $\exists x(x > 1 \wedge [\frac{x}{(x^2+1)} < \frac{1}{3}])$ – True because for some real numbers this is the case. When x = 4, then 4>1 and $\frac{4}{17} < \frac{1}{3}$.

Negation: $\forall x(x > 1 \rightarrow [\frac{x}{x^2+1} \geq \frac{1}{3}])$; All real numbers x that are greater than 1 result in a value of $\frac{x}{x^2+1}$ that is greater than or equal to $\frac{1}{3}$.

Problem Set 13

Problem: Answer the questions

(a) The computer scientists Richard Conway and David Gries once wrote: The absence of error messages during translation of a computer program is only a necessary and not a sufficient condition for reasonable [program] correctness. Rewrite this statement in English without using the words necessary or sufficient.
   - If a computer program has error messages during translation, then the computer program is not correct and there also exists a computer program that has no error messages during translation and that is not correct.

(b) Whenever there is an active alert, all queued messages are transmitted. Rewrite this statement in English using a quantifier and without using the word whenever.
   - For all alerts, if there is an active alert, then all queued messages are transmitted.

(c) The file system cannot be backed up unless there is not a user currently logged on. Rewrite this statement in English using a quantifier and without using the word unless.
   - For every file system, if a user is logged on then the file system cannot be backed up.

(d) Taking the long view on your education, you go to the Prestige Corporation and ask what you should do in college to be hired when you graduate. The personnel director replies that you will be hired only if you major in mathematics or computer science, get a B average or better, and take accounting. You do, in fact, become a math major, get a B+ average, and take accounting. You return to Prestige Corporation, make a formal application, and are turned down. Did the personnel director lie to you?
   - No, the director did not lie.

     Let p: "Hired by Prestige Corporation" and q: "Major in mathematics or computer science" and "B average grade or better" and "Take accounting"

     The personnel director implies that getting hired (denoted as p) "only if" a person majored in math/CS, got a B average or better and took accounting. (denoted by q). However, "p only if q" does not mean "p if q." it is possible for "p only if q" to be true at the same time that "p if q" is false. For instance, to say that a person would get hired if they accomplished major, grade, and accounting requirements does not mean that if they accomplished major, grade and accounting requirements that they would get hired.

     Simplified truth table would look like this (with applicable situation highlighted):

p	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

As a result, the personnel director did not lie.

1Problem Set 14

Problem: Express each of the following English sentences in terms of F (x), C (x), E (x), quantifiers, and logical connectives. Assume that the domain for x consists of all the tools. Then write the negation of each statement of the above exercise symbolically and in English.

F(x): "x is used frequently"
C(x): "x is in the correct place"
E(x): "x is in excellent condition"

(a) No tools are used frequently and are in excellent condition.

Symbolic: $\sim (\exists x$ such that $(F(x) \wedge E(x))) \equiv \forall x, (F(x) \rightarrow \sim E(x))$
Symbolic Negation: $\exists x$ such that $(F(x) \wedge E(x))$
English Negation: There exists a tool that is used frequently and is in excellent condition.

(b) Some tools are neither in excellent condition nor are used frequently.

Symbolic: $\exists x$ such that $(\sim E(x) \wedge \sim F(x)) \equiv \exists x$ such that $\sim(E(x) \vee F(x)) \equiv \sim \forall x, (\sim E(x) \rightarrow F(x))$
Symbolic Negation: $\forall x, (\sim E(x) \rightarrow F(x))$
English Negation: All not excellent tools are used frequently.

(c) All frequently used tools are in the correct places unless they are in excellent condition.

Symbolic: $\forall x, \sim E(x) \rightarrow (F(x) \rightarrow C(x)) \equiv \forall x, (E(x) \vee (\sim F(x) \vee C(x))) \equiv \forall x, \sim(\sim E(x) \wedge (F(x) \wedge \sim C(x))) \equiv \sim \exists x$ such that $(\sim E(x) \wedge (F(x) \wedge \sim C(x)))$
Symbolic Negation: $\exists x$ such that $((\sim E(x) \wedge F(x) \wedge \sim C(x))$
English Negation: There exists a frequently used tool that is not in excellent condition and not in the correct place.

(d) Every frequently used tool is in the correct place if they are in excellent condition

Symbolic: $\forall x, E(x) \rightarrow (F(x) \rightarrow C(x)) \equiv \forall x, (\sim E(x) \vee (\sim F(x) \vee C(x))) \equiv \forall x, \sim(E(x) \wedge (F(x) \wedge \sim C(x))) \equiv \sim \exists x$ such that $(E(x) \wedge (F(x) \wedge \sim C(x)))$
Symbolic Negation: $\exists x$ such that $(E(x) \wedge (F(x) \wedge \sim C(x))$
English Negation: There is some tool in excellent condition and frequently used and not in the correct place.

Problem Set 15

Problem: Write the contrapositive, converse, and inverse. Indicate as best as you can which of these statements are true and which are false. Give a counterexample for each that is false

(a) **For all integers *d*, *d* is a prime integer only if *d* is not a perfect square.**
- Rewritten as a universal conditional statement:
- **For all integers *d*, if *d* is a prime integer, then *d* is not a perfect square.**
- This is true.
- **Contrapositive:** For all integers *d*, if *d* is a perfect square, then *d* is not a prime integer.
- **Converse**: For all integers *d*, if *d* is not a perfect square, then *d* is a prime integer. False. 15 is not a perfect square and 15 is not a prime integer.
- **Inverse**: For all integers *d*, if *d* is not a prime integer, then *d* is a perfect square. False. False. 15 is not a prime integer and 15 is not a perfect square.

(b) **For all real numbers *a* and *b*, if $a < b$ then $a^2 < b^2$.**
- This is **False**. If $a = -1$ and $b = 0$ then $a < b$ but $a^2 > b^2$
- **Contrapositive**: For all real numbers *a* and *b, if $a^2 \geq b^2$ then $a \geq b$.* False $\mathrm{a} = -1$ and $\mathrm{b} = 0$ results in $\mathrm{a}^2 \geq \mathrm{b}^2$ and $\mathrm{a} < \mathrm{b}$
- **Converse**: For all real numbers *a* and *b, if $a^2 < b^2$ then $a < b$.* False $\mathrm{a} = 1$ and $\mathrm{b} = -2$ results in $\mathrm{a}^2 < \mathrm{b}^2$ and $\mathrm{a} > \mathrm{b}$
- **Inverse**: For all real numbers *a* and *b, if $a \geq b$ then $a^2 \geq b^2$.* False $\mathrm{a} = 1$ and $\mathrm{b} = -2$ is a situation in which this is false

(c) **For all integers *n*, if *n* is odd, then $\frac{n-1}{2}$ is odd.**
- This is **false**. If n = 1, then $\frac{n-1}{2}$, and 0 is even.
- **Contrapositive**: For all integers *n*, if $\frac{n-1}{2}$ is even, then *n* is even. False $n = 1$
- **Converse**: For all integers *n*, if $\frac{n-1}{2}$ is odd, then *n* is odd. True
- **Inverse**: For all integers *n*, if *n* is even, then $\frac{n-1}{2}$ is even. True (since, the parity of $\frac{n-1}{2}$ can't be determined when *n* is even)

(d) **For all integers *n*, n^2 is divisible by 4 unless *n* is not divisible by 4.**
- As only if: For all integers *n*, n^2 is not divisible by 4 only if *n* is not divisible by 4.
- Rewritten as universal conditional:
- **For all integers *n*, if n^2 is not divisible by 4, then *n* is not divisible by 4.**
- This is true.
- **Contrapositive**: For all integers *n*, if *n* is divisible by 4, then n^2 is divisible by 4. True
- **Converse**: For all integers *n*, if *n* is not divisible by 4, then n^2 is not divisible by 4. False $n = 2$ and $4|n^2$
- **Inverse**: For all integers *n*, if n^2 is divisible by 4, then *n* is divisible by 4. False $n = 2$ $4|n^2$ but 2 is not divisible by 4

(e) **For any positive integers m and n, if mn is a perfect square, then m and n are perfect squares.**

- This is **false**. If $m = 3$ and $n = 27$ then $mn = 81$, which is a perfect square, but neither m nor n are perfect squares.
- **Contrapositive**: For all integers *m* and *n*, if *m* and *n* are not both perfect squares then *mn* is not a perfect square. False $m = 3$ and $n = 27$
- **Converse**: For all integers *m* and *n*, if *m* and *n* are perfect squares then *mn* is a perfect square. True
- **Inverse**: For all integers *m* and *n*, if *mn* is not a perfect square then *m* and *n* are not both perfect squares. True

Problem Set 16

Problem: Computer the summations (must show work)

(a) $\sum_{i=23}^{899}(4i^2+6i+3)$

$$= \sum_{i=1}^{899}(4i^2+6i) - \sum_{i=1}^{22}(4i^2+6i) + \sum_{i=23}^{899}(3)$$
$$= 4\sum_{i=1}^{899}(i^2) + 6\sum_{i=1}^{899}(i) - 4\sum_{i=1}^{22}(i^2) - 6\sum_{i=1}^{22}(i) + \sum_{i=23}^{899}(3)$$
$$= 4\left(\frac{899*900*1799-22*23*45}{6}\right) + 6\left(\frac{899*900-22*23}{2}\right) + 3(899-23+1)$$
$$= 4(899*150*1799-11*23*15) + 6(899*450-11*23) + 3*877$$

(b) $\sum_{i=1}^{90}\left(i^3+(-5)^{i+2}-10\right)$

$$= \sum_{i=1}^{90}(i^3) + \sum_{i=1}^{90}\left((-5)^{i+2}\right) - \sum_{i=1}^{90}(10)$$
$$= \frac{90^2 91^2}{4} + 25\left(\frac{-5^{91}-5}{-6}\right) - 900$$

(c) $\sum_{i=12}^{100}\left(3^i+2^{i+2}-15i\right)$

$$= \sum_{i=12}^{100}\left(3^i\right) + \sum_{i=12}^{100}\left(2^{i+2}\right) - \sum_{i=12}^{100}(15i)$$
$$= \frac{3^{101}-3^{12}}{2} + 4\left(\frac{2^{101}-2^{12}}{1}\right) - 15\left(\frac{100*101-11*12}{2}\right)$$
$$= \frac{3^{101}-3^{12}}{2} + 4(2^{101}-2^{12}) - 15\left(\frac{100*101-11*12}{2}\right)$$

(d) In a sequence a1, a2, a3, ..., a200, the kth term is defined by ak=1/k−1/(k+1)for all integers k from 1 through 200. What is sum of the 200 terms of this sequence?

Since when you add any 2 consecutive terms like so:

$$a_k + a_{k+1} = \left(\frac{1}{k} - \frac{1}{k+1}\right) + \left(\frac{1}{k+1} - \frac{1}{k+2}\right)$$

the red portions will cancel each other out we will be left with only the positive portion of the first term and the negative portion of the last term giving us:

$$\sum_{i=1}^{200}\left(\frac{1}{i} - \frac{1}{i+1}\right) = 1 - \frac{1}{201}$$

Problem Set 17

Problem: Computer the summations (must show work)

(a) $\sum_{i=10}^{77}\left(6i^2+(-2)^i-10\right)$

$$= \sum_{i=10}^{77}(6i^2)+\sum_{i=10}^{77}\left((-2)^i\right)-\sum_{i=10}^{77}(10)$$
$$= 6\left(\sum_{i=1}^{77}(i^2)-\sum_{i=1}^{9}(i^2)\right)+\left(\frac{(-2)^{78}+(-2)^{10}}{-3}\right)-\left(10*(77-10+1)\right)$$
$$= 6\left(\left(\frac{77*78*155}{6}\right)-\left(\frac{9*10*19}{6}\right)\right)+\left(\frac{(-2)^{78}+(-2)^{10}}{-3}\right)-(10*68)$$

(b) $\sum_{i=1}^{20}\left(2*4^{i+2}-2^{i+2}\right)$

$$= \sum_{i=1}^{20}\left(32*4^i-4*2^i\right)$$
$$= 32\sum_{i=1}^{20}\left(4^i\right)-4\sum_{i=1}^{20}\left(2^i\right)$$
$$= 32\left(\frac{4^{21}-4}{3}\right)-4(2^{21}-2)$$

(c) $\sum_{i=12}^{50}\left(\sum_{j=1}^{3}(7ij-3)\right)$

$$= \sum_{i=12}^{50}\left(\left(7i\left(\frac{12}{2}\right)-3*3\right)\right)$$
$$= 3\sum_{i=12}^{50}(28i-3)$$
$$= 3\left(14\sum_{i=12}^{50}(i)-\sum_{i=12}^{50}(3)\right)$$
$$= 3\left(14\left(\sum_{i=1}^{50}(i)-\sum_{i=1}^{11}(i)\right)-\sum_{i=12}^{50}(3)\right)$$
$$= 3\left(14\left(\frac{50*51}{2}-\frac{11*12}{2}\right)-3(50-12+1)\right)$$
$$= 42(25*51-11*6)-117$$

(d) $6\sum_{i=10}^{500}(10i^2-3)-15\sum_{i=1}^{500}(i^2+2)$

$$= 6\left(\sum_{i=10}^{500}(10i^2)-\sum_{i=10}^{500}(3)\right)-\left(15\sum_{i=1}^{500}(i^2)+15\sum_{i=1}^{500}(2)\right)$$
$$= 60\sum_{i=10}^{500}(i^2)-60\sum_{i=1}^{9}(i^2)-6\sum_{i=10}^{500}(3)-\left(15\sum_{i=1}^{500}(i^2)+15\sum_{i=1}^{500}(2)\right)$$
$$= 60\left(\frac{500*501*1001}{6}-\frac{9*10*19}{6}\right)-18(500-10+1)-15\left(\frac{500*501*1001}{6}+2(500)\right)$$
$$= 60(250*167*1001-15*19)-18*491-15(250*167*1001+1000)$$

Problem Set 18

<u>Problem:</u> Find explicit formulas

(Domain for all given solutions is that $n \geq 1$)

(a) $3 * 2^{n-1}$
(b) $15 - 7(n-1)$
(c) $3^{n-1} - 1$
(d) $\frac{(2n)!}{2^n*(n!)}$ or $\frac{(2n-1)!}{2^{n-1}*((n-1)!)}$ or $(2n-1)!!$
(e) $\frac{10}{5^{n-1}}$
(f) $90\left(-\frac{1}{3}\right)^{n-1}$
(g) $\frac{(-1)^n(3n+1)}{2^{n-1}}$
or $1 - \left(\frac{1+(-1)^{n+1}}{2}\right)\left(\frac{23-3n}{4}\right) + \left(\frac{1+(-1)^n}{2}\right)\frac{5}{2^{n-1}}$