CS 225: Discrete Structures in CS

Homework 2, Part 1- Solution Key

Exercise Set 3.1 of the required textbook: Question#32(b, d)

Exercise Set 3.2 of the required textbook: Question 4(b, d), #12, #29, #33, #46, #48

Extra problem on Canvas

Exercise Set 3.1:

#32(b, d)

b. $x > 2 \Rightarrow x^2 > 4$

The above statement is true.

$\forall$ real number x, if x is greater than 2, then square of the x is greater than 4.

d. $x^2 > 4 \Leftrightarrow |x| > 2$

The above statement is true.

$\forall$ real number x, the square of x is greater than 4 if and only if the absolute value of x is greater than 2.

Exercise Set 3.2:

#4(b, d)

b. Some graphs are not connected. / There is at least one graph that is not connected.

d. No estimate is accurate. / All estimates are not accurate.

#12.

The negation proposed is incorrect. The correct negation should be “There exists some irrational number and rational number such that the product is rational.”

#29

Given Statement: $\forall n \in Z$, if n is prime then n is odd or n= 2.

Contrapositive: $\forall n \in Z$, if n is even and n≠2 then n is not prime. This statement is true.

Converse: $\forall n \in Z$, if n is odd or n=2, then n is prime. This statement is false.

As a counterexample, consider n=1, which is odd but is not prime.

Inverse: $\forall n \in Z$, if n is not prime then n is even and n≠2.This statement is false.

As a counterexample, consider n=9, which is not prime but is not even.

#33

Given Statement: $\forall$ function f, if f is differentiable, then f is continuous.

Contrapositive: $\forall$ function f, if f is not continuous, then f is not differentiable. This statement is true.

Converse: $\forall$ function f, if f is continuous, then f is differentiable. This statement is false.

As a counterexample, consider a function with a vertical tangent which may be continuous, but fails to be differentiable at the location of the anomaly (https://en.wikipedia.org/wiki/Vertical_tangent).

Inverse: $\forall$ function f, if f is not differentiable, then f is not continuous. This statement is false.

As a counterexample, consider the absolute value function which is not differentiable at x=0, but is actually continuous at x=0.

#46

Given Statement: Having a large income is not a necessary condition for a person to be happy.
If-then form: ~($\forall$ person x, if x is happy then x has a large income).
The negation is, "$\exists$ a person x such that x is happy and x doesn't have a large income".

#48

Given Statement: Being a polynomial is not a sufficient condition for a function to have a real root.
If-then form: ~($\forall$ function x, if x is a polynomial then x has a real root).
The negation is, "$\exists$ a function x such that x is a polynomial and x doesn't have a real root".

Extra problem on Canvas:

Let B(x), W(x), and S(x) be the predicates
B(x): x is a female
W(x): x is a good athlete
S(x): x is young
Express each of the following English sentences in terms of B(x), W(x), S(x), quantifiers, and logical connectives. Assume the domain is the set of all people.

a) All young females are not good athletes.
Answer: $\forall x \in D, (S(x) \wedge B(x)) \rightarrow \sim W(x)$

b) A person is a good athlete if it is the case that both she is a female and she is young.
Answer: $\forall x \in D, (B(x) \wedge S(x)) \rightarrow W(x)$

c) Some good athletes are not female.
Answer: $\exists x \in D$ such that $W(x) \wedge \sim B(x)$

d) All good athletes are neither young nor they are female.

Answer: $\forall x \in D, W(x) \rightarrow \sim S(x) \wedge \sim B(x)$

e) Any person is a good athlete unless he/she is not young.

Answer: $\forall x \in D, S(x) \rightarrow W(x)$ or $\forall x \in D, ((B(x) \vee \sim B(x)) \wedge S(x)) \rightarrow W(x)$