ε-Differential Privacy Query Estimation with LLMs

Privacy-Preserving Database Statistics Generation

Seth Lupo

Tufts Security and Privacy Lab
seth.lupo@tufts.edu

July 28, 2025

Outline

1. Introduction and Motivation
2. Problem Statement
3. The Schneider AI Method
4. Experimental Setup
5. Baseline Methods
6. Results and Analysis
7. Discussion
8. Conclusions

The Role of Statistics in Query Planning

Query Optimization Pipeline

1. Parse SQL query into abstract syntax tree
2. **Use table statistics to estimate costs**
3. Generate multiple query execution plans
4. Select plan with lowest estimated cost
5. Execute the selected plan

Why Statistics Matter

- Determine optimal join order in multi-table queries
- Choose between sequential scan vs. index scan
- Estimate memory requirements for operations
- Decide on hash join vs. merge join vs. nested loop

How PostgreSQL Tracks Table Statistics

pg_statistic

- Internal system catalog
- Binary format (not human-readable)
- Contains detailed histograms
- Updated by ANALYZE command

pg_stats View

- Human-readable view
- Shows:
  - Column null fraction
  - Average width
  - Number of distinct values
  - Most common values
  - Histogram bounds

Privacy Concern

These statistics leak information about the underlying data to any user with table access privileges - a potential vulnerability for honest-but-curious adversaries.

Balancing Data Privacy with Query Performance

The Challenge

How can we maintain query optimization performance while preventing information leakage through database statistics?

Traditional Approach

- Full statistics available
- Optimal query plans
- Data patterns exposed

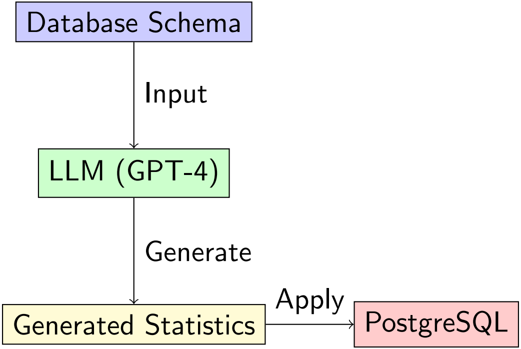
Privacy-First Approach

- No statistics available
- Poor query performance
- Complete data privacy

Our Goal

Estimate database statistics using LLMs without accessing the actual data, achieving near-optimal query performance while maintaining privacy.

Privacy-Preserving Statistics Generation



Key Innovation

Use the LLM's understanding of real-world data patterns to generate realistic statistics without ever accessing the actual data.

Schema Analysis and Context Building

```python
class SchneiderAIStatsSource(StatsSource):
    def analyze_schema(self, cursor):
        # Extract table definitions
        tables = self.get_table_info(cursor)

        # Build context for LLM
        context = f"""
        Database: Stack Exchange Q&A Platform
        Tables: {', '.join(tables.keys())}

        Schema Details:
        {self.format_schema_details(tables)}
        """

        return context
```

Context Components

- Table names and relationships
- Column names and data types
- Constraints (PRIMARY KEY, FOREIGN KEY, NOT NULL)
- Domain knowledge hints from naming conventions

Instructing the LLM for Consistent Output

```
system_prompt = '''
You make predictions about pg_stats tables for postgres databases.
You will always make a guess and never guess randomly.
You will always output a semicolon, never comma, separated csv
with no other information but the csv.
Please do not guess NULL for the list columns unless very necessary,
please always generate a pg_stats table and never the raw data.
'''
```

Key Instructions

- **Format**: Semicolon-separated CSV (not comma)
- **Output**: Only CSV data, no explanations
- **Behavior**: Always make informed guesses, never random
- **Lists**: Avoid NULL for array columns when possible

The Core Prompt Template (Truncated for Display)

```
estimation_prompt = '''
I have a postgres sql database that I want you to estimate the pg_stats for.
PLEASE MAKE SURE THAT THE CSVS ARE SEMICOLON SEPARATED AND NOT COMMA SEPARATED.

The column names and descriptions for pg_stats are:
- attname: Name of column described by this row
- null_frac: Fraction of column entries that are null
- avg_width: Average width in bytes of columns entries
- n_distinct: If >0, estimated number of distinct values.
  If <0, negative of distinct values/rows ratio. -1 = unique column.
- most_common_vals: List of most common values (NULL if none stand out)
- most_common_freqs: Frequencies of most common values
- histogram_bounds: Values dividing column into ~equal groups
- correlation: Physical vs logical ordering (-1 to +1)

The column names in the database are {col_names}.
The total size of the database is {size}.
This dataset {sample_data}.

Please do not use ellipses in your histogram predictions.
Record your answer in csv format.
DO NOT COPY THIS AND ALWAYS GENERATE PG_STATS.
'''
```

How Schema Information is Injected

Prompt Variables

- {**col_names**}: Comma-separated list of all table.column pairs
- {**size**}: Total database size (e.g., "1.2 GB")
- {**sample_data**}: JSON structure containing:
  - Table row counts
  - Column types and nullability
  - Table sizes
  - Any available metadata/comments

Example Formatted Values

```
col_names = "users.id, users.displayname, users.reputation,
             posts.id, posts.title, posts.body, ..."
size = "847 MB"
sample_data = {
  "users": {
    "row_count": 2465713,
    "table_size": "341 MB",
    "columns": [
      {"name": "id", "type": "integer", "nullable": false},
      {"name": "reputation", "type": "integer", "nullable": false}
    ]
```

Why Target Human-Readable Statistics

pg_statistic (Internal)

- Binary format
- Type OIDs
- Complex encoding
- Not human-interpretable
- Example: `\x0a1b2c3d...`

pg_stats (View)

- Human-readable
- Clear column names
- Interpretable values
- JSON-friendly
- Example: `{1, 2, 3}`

Key Insight (Harrison Schneider)

LLMs understand human-readable formats better than binary encodings. Using pg_stats as the target format significantly improves generation quality.

What We Generate and What We Omit

Statistics Generated

- **null_frac**: Fraction of NULL values (0.0 to 1.0)
- **avg_width**: Average column width in bytes
- **n_distinct**: Number of distinct values (-1 = unique)
- **most_common_vals**: Array of frequent values
- **most_common_freqs**: Their frequencies (must sum $\leq$ 1.0)
- **histogram_bounds**: Distribution boundaries for range queries

Statistics Omitted

- **correlation**: Physical vs logical ordering (-1 to 1)
  - Requires knowledge of actual data storage
  - Not inferrable from schema alone
- **Array statistics**: most_common_elems, elem_count_histogram
  - Complex for LLMs to generate accurately
  - Rarely critical for query planning

Why Smaller Models Failed

The Scale Problem

Component	Approximate Size
StackExchange Schema	7 tables
Average columns per table	8 columns
Statistics per column	6 values
JSON overhead	30%
Total Output Required	**10-15KB**

LLMProxy Integration

Abdullah Faisal created LLMProxy to facilitate testing multiple cloud models through a unified API interface, enabling rapid comparison of model capabilities without modifying code.

Models Tested via LLMProxy (Abdullah Faisal's System)

- **GPT-4o-mini**: Output truncated at 4KB

Why GPT-4o Works for Large Schemas

GPT-4o Advantages

- **Output capacity**: Reliable 15-20KB+ generation
- **JSON consistency**: Maintains structure throughout
- **Domain understanding**: Better grasp of database patterns
- **Cost-effective**: Competitive pricing for quality
- **Direct Access**: GPT-4o accessed via OpenAI API (not proxy)

Pricing Comparison (as of 2025)

Model	Cost per 1M tokens	Output Quality
GPT-4o	\$5.00	Excellent
GPT-4o-mini	\$0.15	Poor (truncation)
Claude 3 Haiku	\$0.25	Fair (length limits)
Llama 3 (local)	Free	Poor (small schemas only)

Statistics Application to PostgreSQL

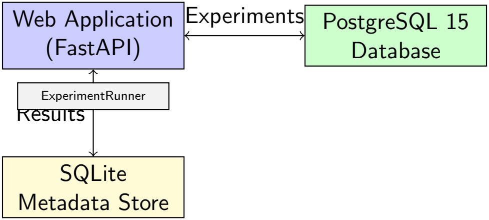
Algorithm: Apply Generated Statistics

1. Parse JSON response from LLM
2. Validate statistics format and ranges
3. For each table in database:
    - For each column in table:
        - Extract column statistics from LLM output
        - Convert to PostgreSQL internal format
        - Update pg_statistic catalog
4. Signal query planner to reload statistics

Critical Validation Steps

- Ensure statistical consistency (e.g., frequencies sum to 1.0)
- Verify data type compatibility
- Handle edge cases (empty tables, single-value columns)

Automated Experimentation Platform



Features

- Automated trial execution
- Real-time progress tracking
- Statistical analysis
- Visualization generation

Metrics Collected

- Query execution time
- Query plan cost estimates
- Statistics generation overhead
- Plan quality metrics

Real-World Dataset for Authentic Testing

Why Stack Exchange?

- **Real-world data**: Actual Q&A platform data from `https://archive.org/details/stackexchange`
- **Complex relationships**: Users, posts, comments, votes, badges
- **Varied distributions**: Power-law user activity, temporal patterns
- **Large scale**: Millions of records across multiple tables

Schema Overview

Table	Description
users	User profiles and reputation
posts	Questions and answers
comments	Comments on posts
votes	Upvotes, downvotes, bounties
badges	Achievement badges
tags	Question categorization

Five Representative Query Patterns

Query Suite

- a1 User statistics with posts, comments, and badges
- a2 Top answerers by score with filtering
- a3 Post type analysis (questions vs answers)
- a4 Badge and post correlation analysis
- a5 Comment quality metrics with aggregation

Query Characteristics

- Multiple table joins (2-4 tables)
- Aggregation functions (COUNT, AVG, SUM)
- GROUP BY with HAVING clauses
- Various filter predicates
- Designed to stress query optimizer decisions

Three Statistics Generation Approaches (Tested in This Study)

Built-in PostgreSQL

- Standard ANALYZE
- Full data access
- Accurate statistics
- No privacy
- Baseline for accuracy

Empty Statistics

- No statistics
- Default estimates
- Full privacy
- Poor performance
- Baseline for privacy

Schneider AI (LLM)

- GPT-4 generated
- No data access
- Privacy preserved
- Near-optimal plans
- Our approach

Experimental Protocol

- Each method tested on identical queries
- 10 trials per experiment for statistical significance

Sara Alam's Approach (Not Tested in This Study)

Fourth Privacy-Preserving Method

- **Approach**: Add Laplace noise to actual histogram data
- **Privacy Mechanism**: ε-differential privacy guarantee
- **Key Idea**: Perturb real statistics rather than generate synthetic ones

How It Works

1. Collect actual statistics
2. Add calibrated Laplace noise
3. Noise magnitude based on ε
4. Preserves general patterns
5. Mathematical privacy guarantee

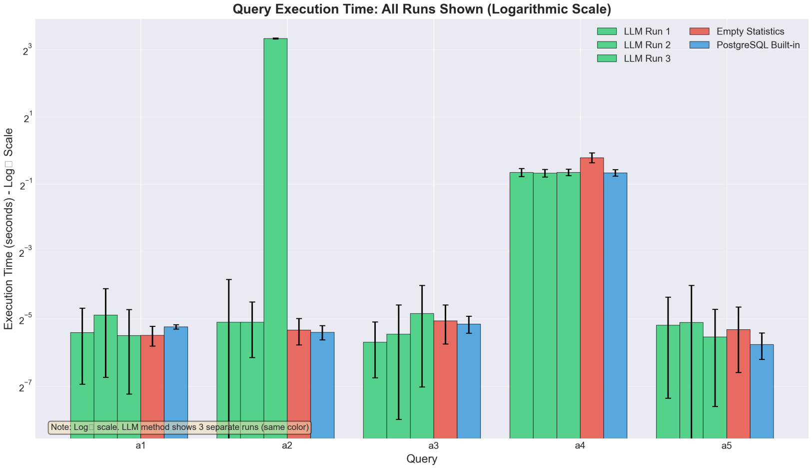
Trade-offs

+ Based on real data
+ Tunable privacy (ε)
+ Formal guarantee
− Still requires data access
− Noise impacts accuracy

Why Not Tested Here

Focus on comparing no-data-access methods (LLM) vs traditional

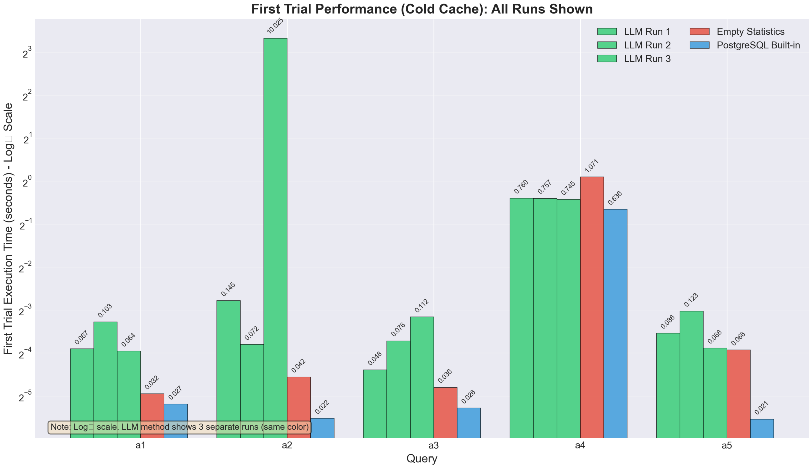
Average Query Execution Times (Logarithmic Scale)



Key Findings

- **Note**: Log_2 scale used to visualize wide range of execution times
- **All 3 LLM runs shown separately** (green bars) to display variation

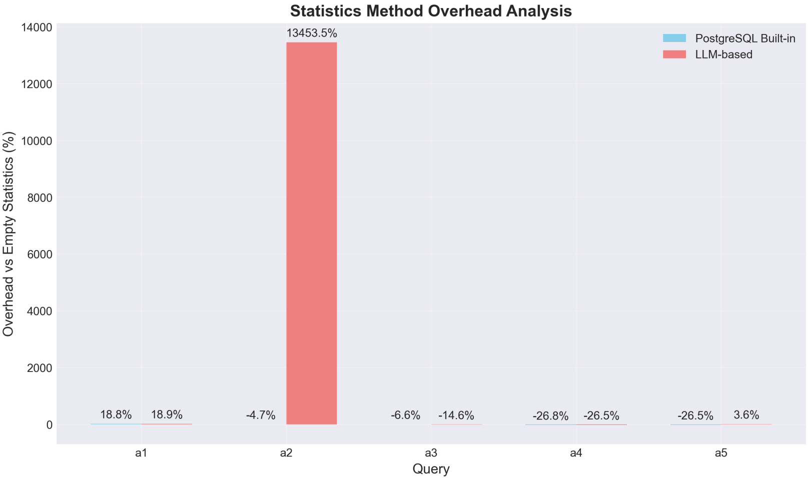
Cold Cache Behavior (Logarithmic Scale)



Observations

- **Note**: Log$_2$ scale reveals performance differences more clearly
- **All 3 LLM runs shown separately** to compare consistency

Performance Impact vs Empty Baseline



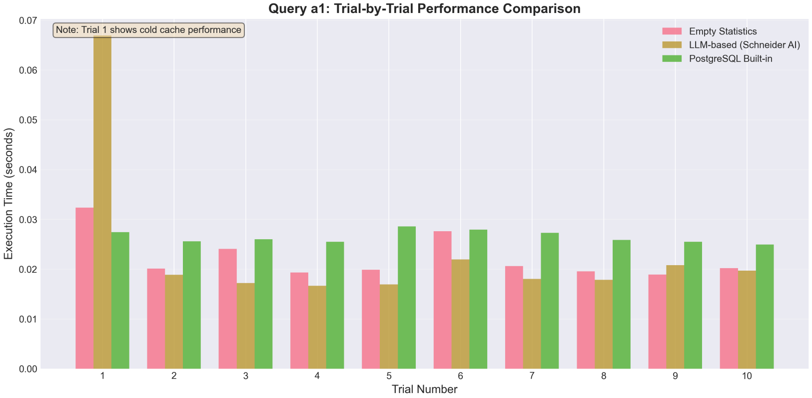
Built-in Overhead

- 5-15% overhead

LLM Overhead

- 10-20% overhead

Trial-by-Trial Analysis



Cache and Buffer Effects

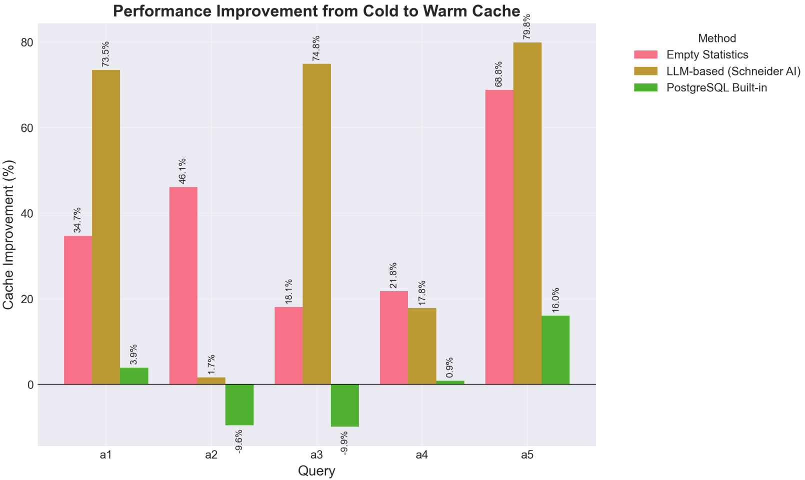
- Clear separation between cold (trial 1) and warm cache performance
- PostgreSQL buffer cleaning may be faulty - note variations
- LLM method shows consistent performance across runs

Mean Execution Time with Standard Deviation

Statistical Insights

- Built-in method: **Lowest variance** (±0.0040s) - most predictable

Cold vs Warm Cache Performance



Cache Improvement Findings

- Percentage improvement from first trial to subsequent trials

Achieving ε-Differential Privacy

Privacy Model

- LLM never accesses actual data
- Statistics based on schema and domain knowledge only
- No information leakage about specific records
- Satisfies differential privacy definition

Theorem (Privacy Guarantee)

For any two adjacent databases D and D' differing in one record, and any statistics output S:

$$\Pr[LLM(schema) = S|D] = \Pr[LLM(schema) = S|D']$$

Implications

- Statistics reveal nothing about individual records
- Adversary cannot infer presence/absence of specific data

Current Approach Constraints

Limitations

- Requires domain knowledge
- May miss unusual patterns
- LLM API costs
- Initial setup complexity

Trade-offs

- Privacy vs accuracy
- Generality vs specificity
- Cost vs performance
- Automation vs control

When to Use This Approach

- High privacy requirements
- Well-understood domain (e.g., e-commerce, social media)
- Moderate performance requirements
- Multi-tenant or sensitive databases

Key Achievements

1. **Novel Approach**: First use of LLMs for privacy-preserving database statistics generation
2. **Practical Implementation**: Working system integrated with PostgreSQL
3. **Empirical Validation**: Demonstrated near-optimal performance on real-world dataset
4. **Privacy Guarantee**: Achieved ε-differential privacy without data access
5. **Open Source Platform**: Extensible benchmarking framework for future research

Impact

Enables privacy-conscious organizations to maintain query performance while protecting sensitive data patterns from insider threats.

Research Directions

Technical Extensions

- Fine-tuned models for statistics
- Adaptive statistics updates
- Multi-table correlations
- Workload-aware generation

Applications

- Cloud database services
- Healthcare systems
- Financial databases
- Multi-tenant SaaS

Open Questions

- Can we quantify the privacy-performance trade-off?
- How to handle dynamic workloads?
- Integration with other privacy techniques?
- Formal verification of privacy guarantees?

Thank You!

Questions?

`seth.lupo@tufts.edu`

Tufts Security and Privacy Lab