Graph Theory

Varying Applications (examples)

- Computer networks
- Distinguish between two chemical compounds with the same molecular formula but different structures
- Solve shortest path problems between cities
- Scheduling exams and assign channels to television stations

Topics Covered

- Definitions
- Types
- Terminology
- Representation
- Sub-graphs
- Connectivity
- Hamilton and Euler definitions
- Shortest Path
- Planar Graphs
- Graph Coloring

Definitions - Graph

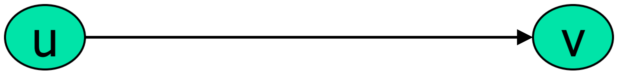
A generalization of the simple concept of a set of dots, links, edges or arcs.

Representation: *Graph G =(V, E) consists set of vertices denoted by V, or by V(G) and set of edges E, or E(G)*

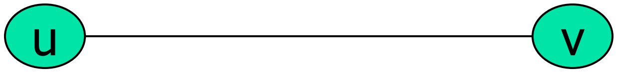
- A graph with infinite number of vertices or edges is called infinite graph.
- A graph with finite number of vertices as well as finite number of edges is called finite graph.

Definitions – Edge Type

Directed: Ordered pair of vertices. Represented as (u, v) directed from vertex u to v.

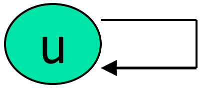


Undirected: Unordered pair of vertices. Represented as {u, v}. Disregards any sense of direction and treats both end vertices interchangeably.



Definitions – Edge Type

- **Loop (self loop):** A loop is an edge whose endpoints are equal i.e., an edge joining a vertex to it self is called a loop. Represented as {u, u} = {u}

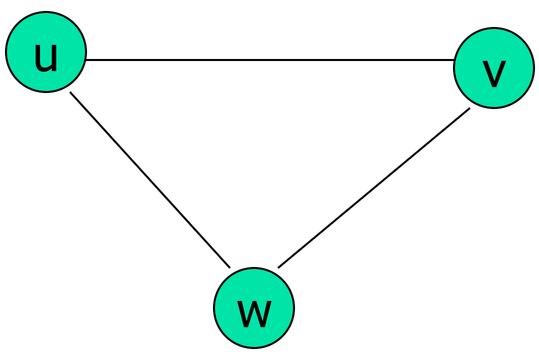


- **Multiple Edges:** Two or more edges joining the same pair of vertices.

Definitions – Graph Type

Simple (Undirected) Graph: consists of V, a nonempty set of vertices, and E, a set of unordered pairs of distinct elements of V called edges (undirected)

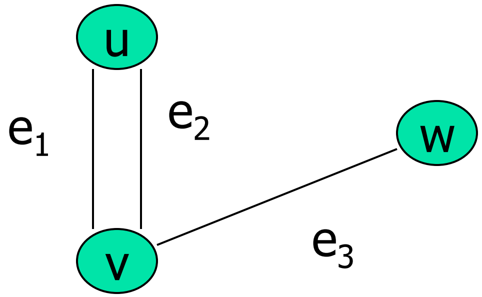
Representation Example: G(V, E), V = {u, v, w}, E = {{u, v}, {v, w}, {u, w}}



Definitions – Graph Type

Multigraph: G(V,E), consists of set of vertices V, and set of Edges E. The edges e1 and e2 are called multiple or parallel edges.

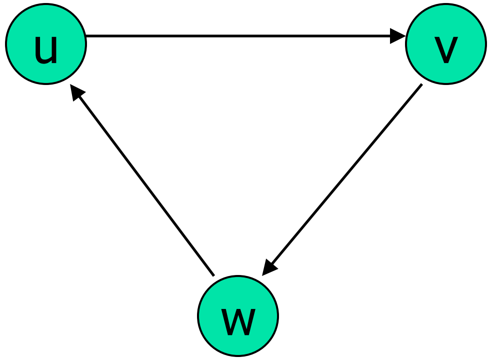
Representation Example: $V = \{u, v, w\}$, $E = \{e_1, e_2, e_3\}$



Definitions – Graph Type

Directed Graph (Digraph): G(V, E), set of vertices V, and set of Edges E, that are ordered pair of elements of V (directed edges)

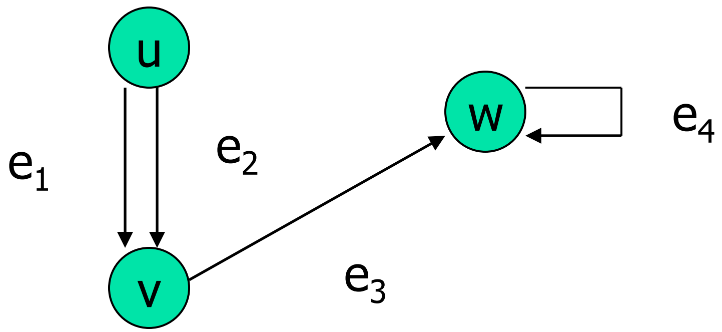
Representation Example: G(V, E), V = {u, v, w}, E = {(u, v), (v, w), (w, u)}



Definitions – Graph Type

Directed Multigraph: G(V,E), consists of set of vertices V, set of Edges E. The edges e1 and e2 are multiple edges.

Representation Example: $V = \{u, v, w\}$, $E = \{e_1, e_2, e_3, e_4\}$



Definitions – Graph Type

Type	Edges	Multiple Edges Allowed ?	Loops Allowed ?
Simple Graph	undirected	No	No
Multigraph	undirected	Yes	No
Directed Graph	directed	No	Yes
Directed Multigraph	directed	Yes	Yes

Terminology – Undirected graphs

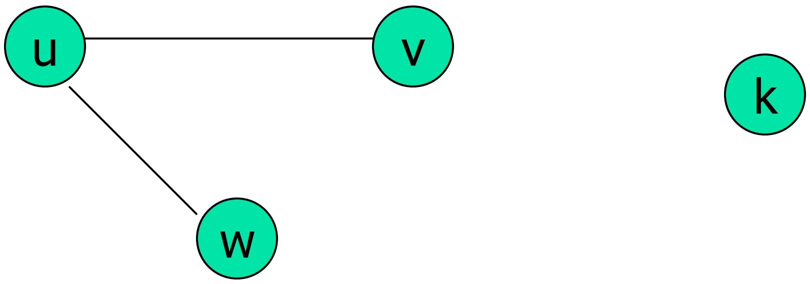
- u and v are **adjacent** if {u, v} is an edge, e is called **incident** with u and v. u and v are called **endpoints** of {u, v}

- **Degree of Vertex (deg (v)):** the number of edges incident on a vertex. A loop contributes twice to the degree.

- **Pendant Vertex:** deg (v) =1

- **Isolated Vertex:** deg (k) = 0

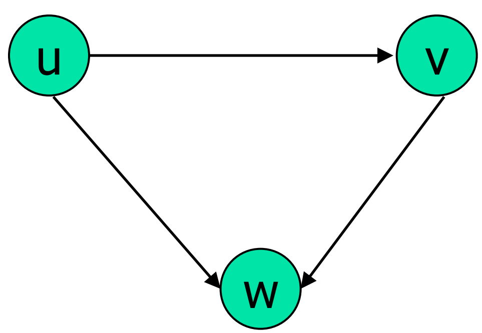
Representation Example: For V = {u, v, w} , E = { {u, w}, {u, v} }, deg (u) = 2, deg (v) = 1, deg (w) = 1, deg (k) = 0, w and v are pendant , k is isolated



Terminology – Directed graphs

- For the edge (u, v), u is **adjacent to** v OR v is **adjacent from** u, u – **Initial vertex**, v – **Terminal vertex**

- **Null graph:** If E = Ø, in a graph G = (V, E), then such a graph without any edges is called a null graph



Theorems: Undirected Graphs

Theorem 1

The Handshaking theorem:

$$2e = \sum_{v \in V} \deg(v)$$

Every edge connects 2 vertices

Theorems: Undirected Graphs

Theorem 2:

An undirected graph has even number of vertices with odd degree.

Odd and even vertices: A vertex of a graph is called odd or even depending on whether its degree is odd or even.

Null graph: If $E = \emptyset$, in a graph $G = (V, E)$, then such a graph without any edges is called a null graph