

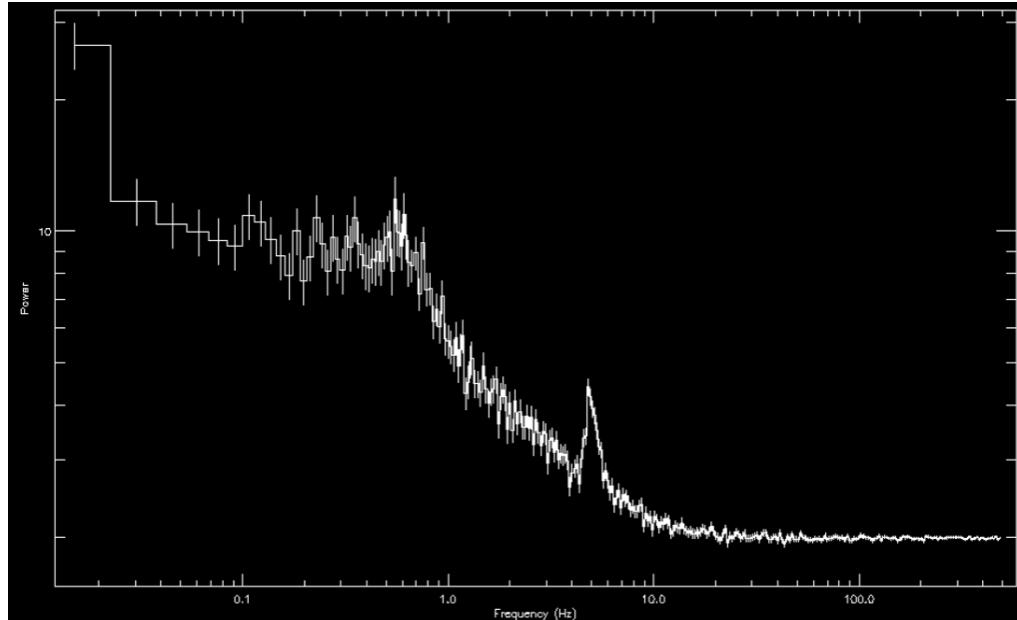
Fitting the power and cross spectra: Hidden Variability and the Link Between QPOs and the Jet

Mariano Méndez

Kapteyn Astronomical Institute, University of Groningen, The Netherlands

Based on several papers with: Federico García, Pei Jin, Candela Bellavita, Pengcheng Yang, Sandeep Rout, Federico Vincentelli, Diego Altamirano, Kevin Alabarta, Federico Fogantini, Maïmouna Brigitte, Ole König, Dave Russell, Valentina Peirano and Tomaso Belloni

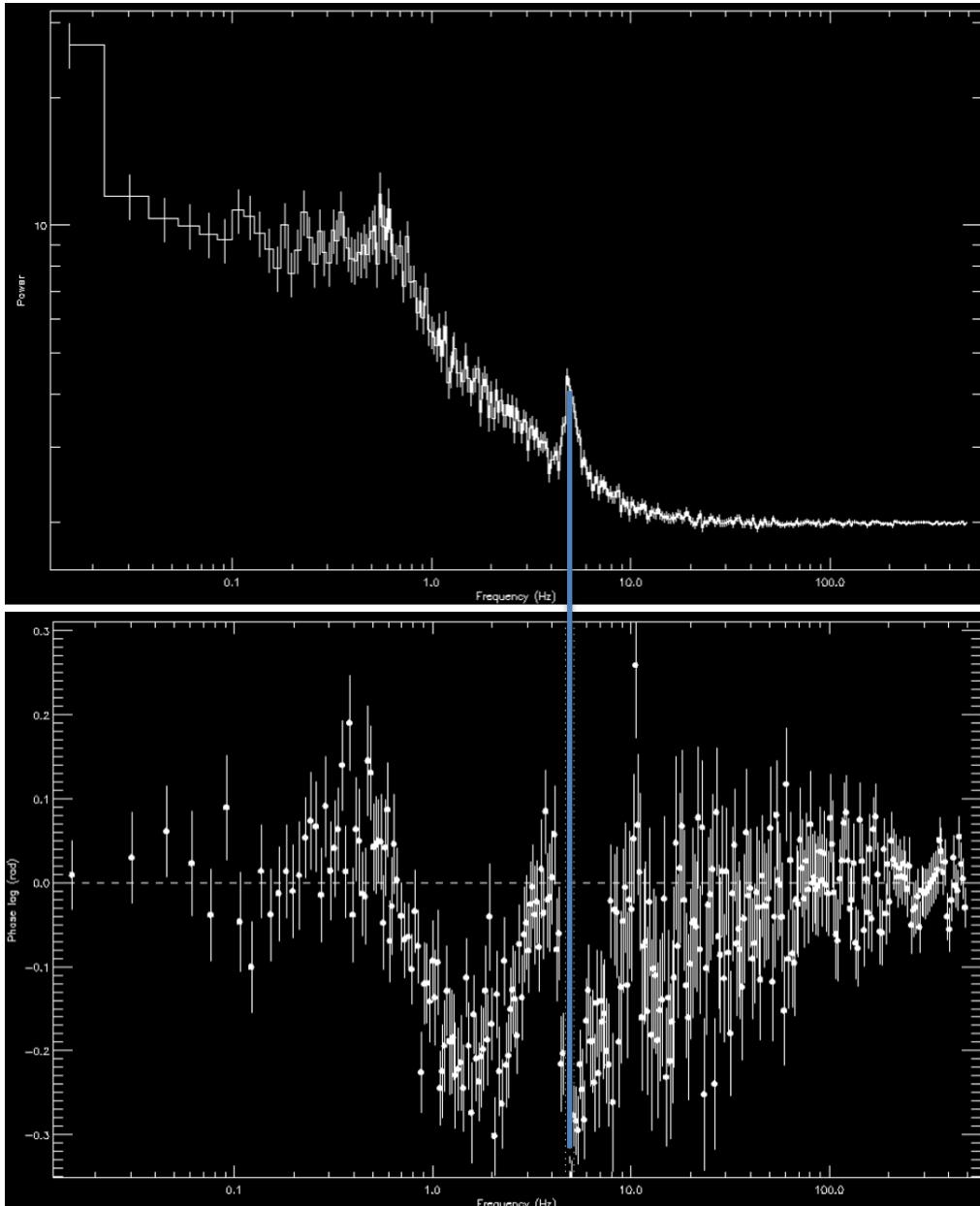
Take Tomaso's example of yesterday^(*)



Power spectrum (PDS) of
GRS 1915+105 with AstroSat.

()This slide is from a talk at ISSI in 2022*

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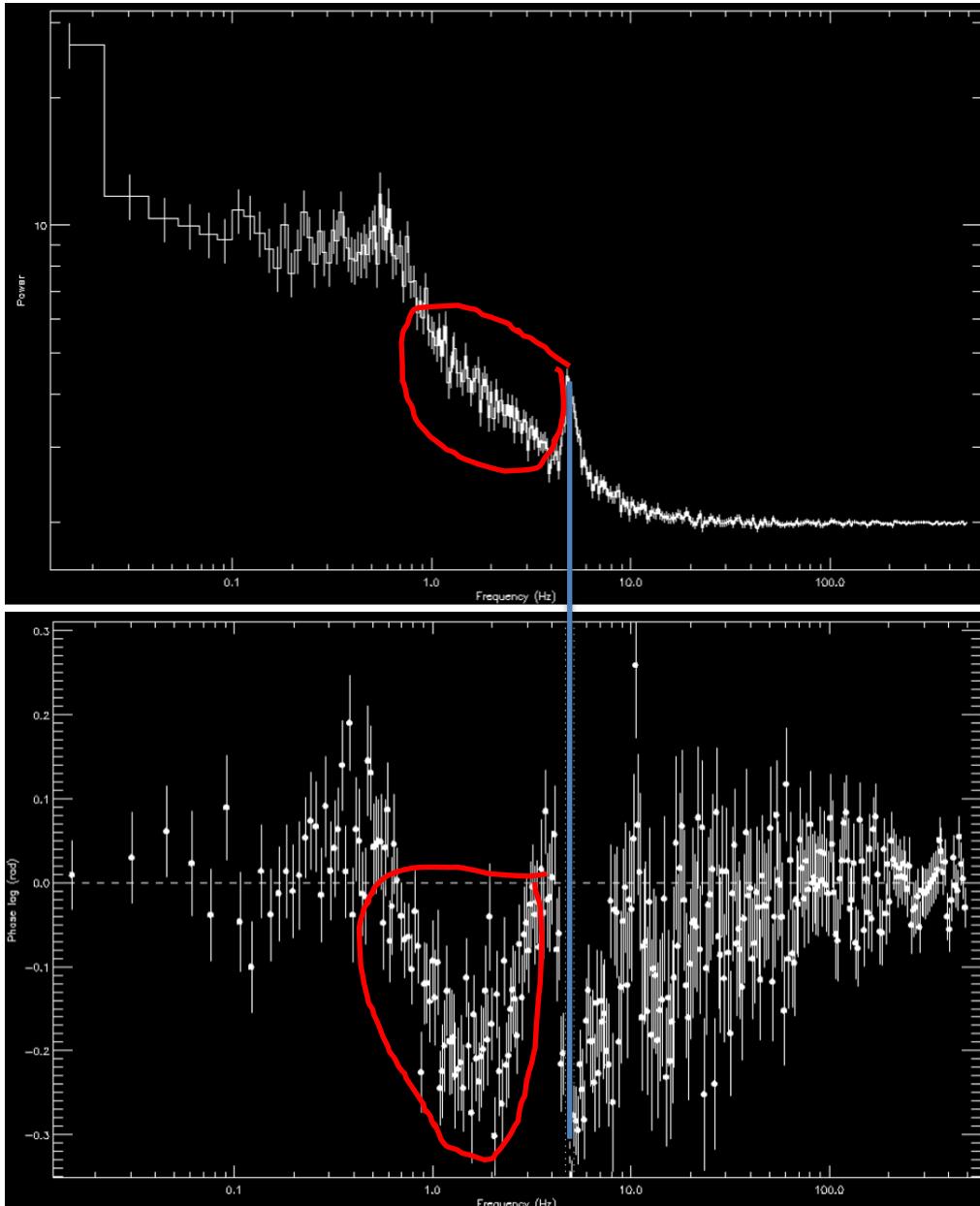


Power spectrum (PDS) of
GRS 1915+105 with AstroSat.

No clear feature in the PDS at ~ 1.5 Hz
but a clear feature in the lag vs.
frequency spectrum at that frequency.

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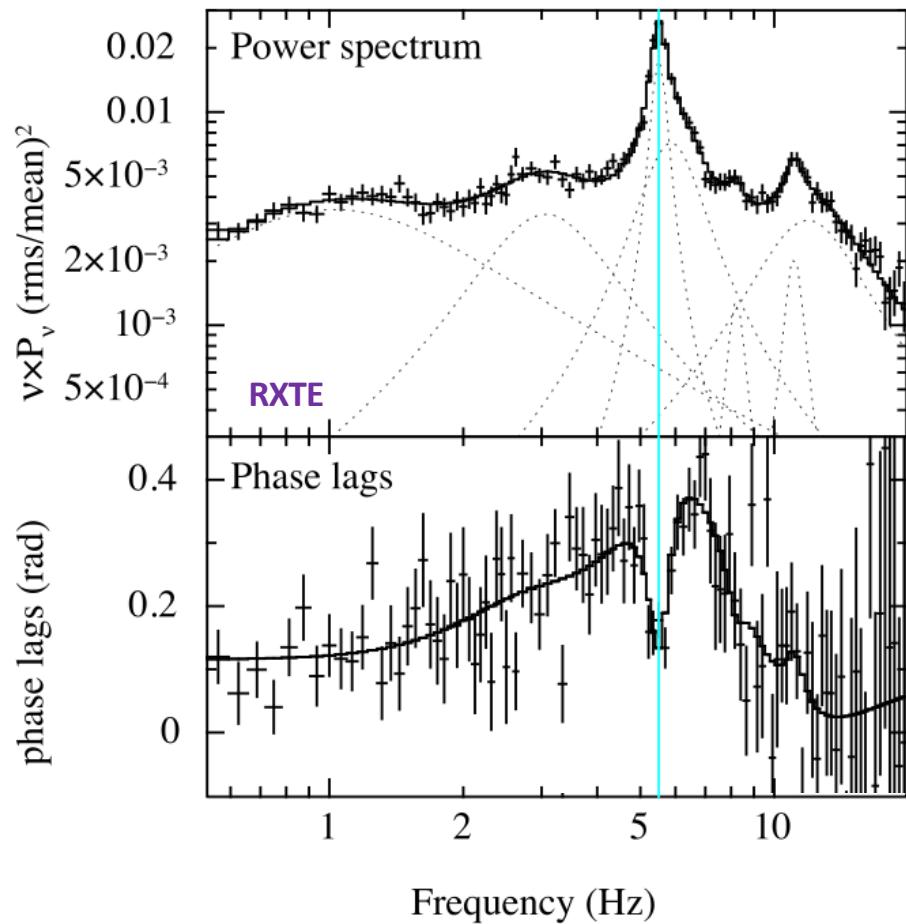


Power spectrum (PDS) of
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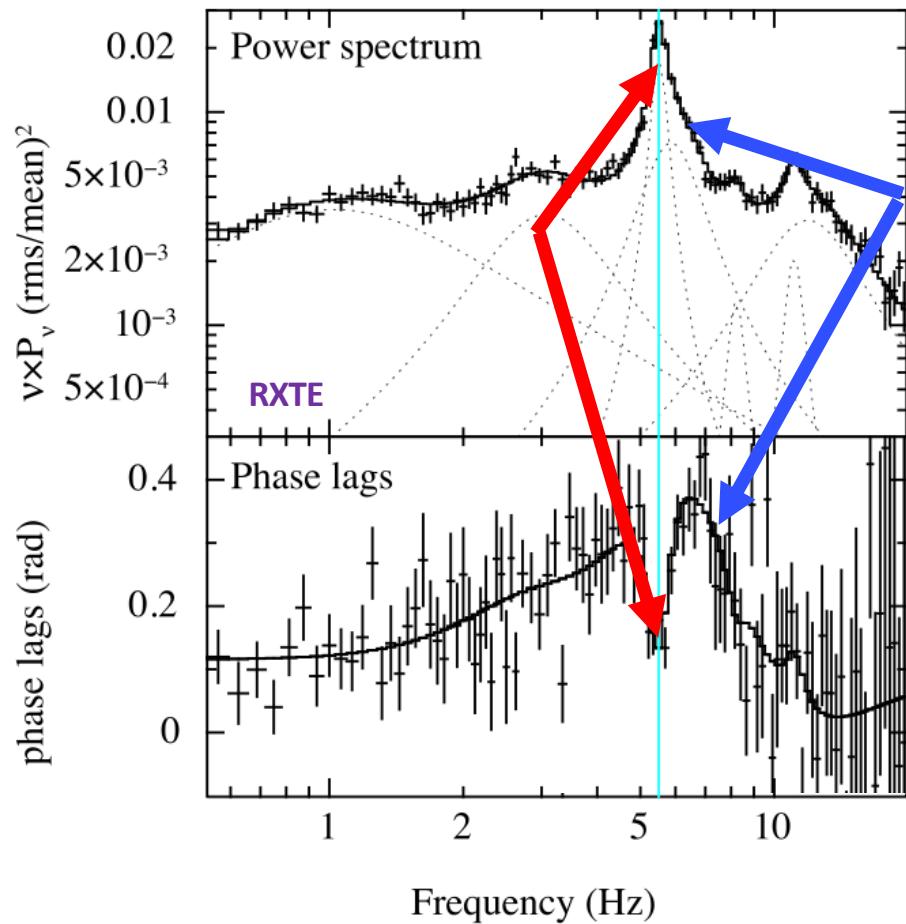
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GX 339–4: Simultaneous type-C and -B QPO in the HIMS

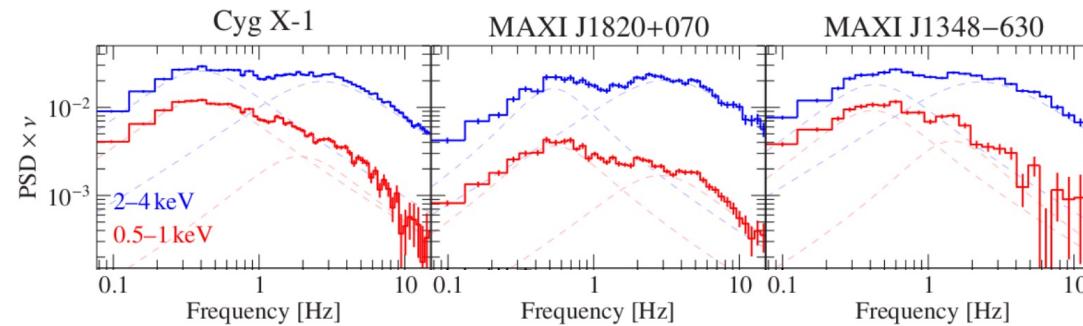


GX 339–4: Simultaneous type-C and -B QPO in the HIMS



Why “hidden variability” in the title?

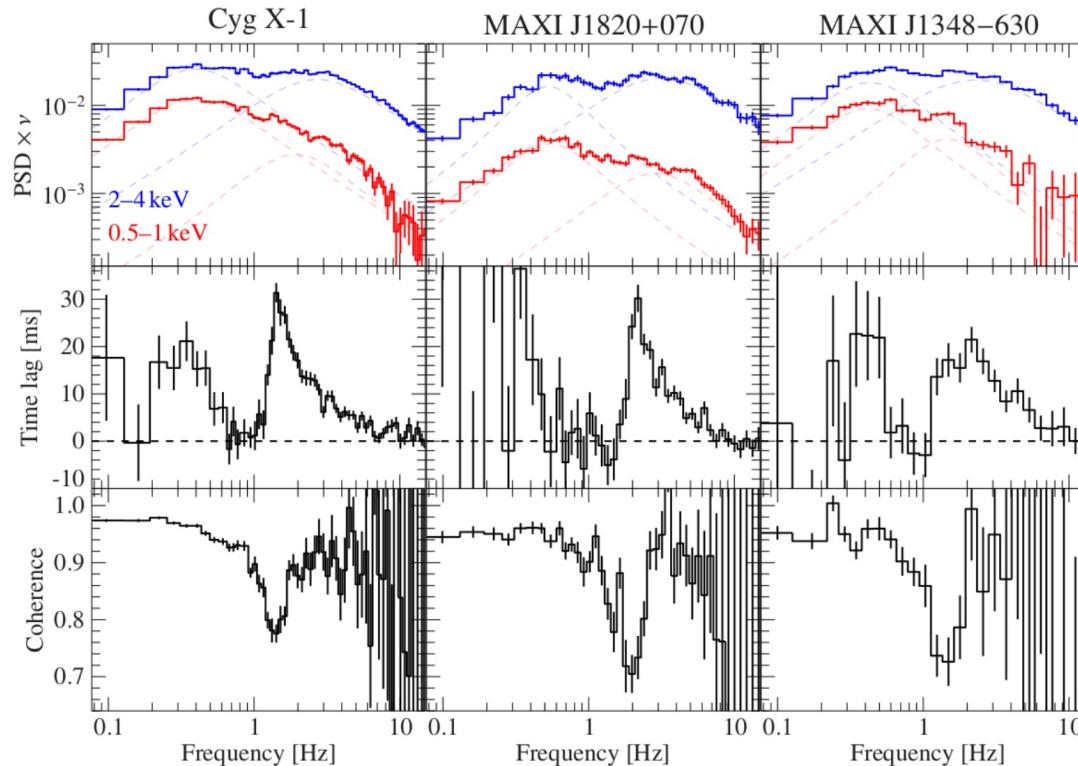
Featureless power spectra



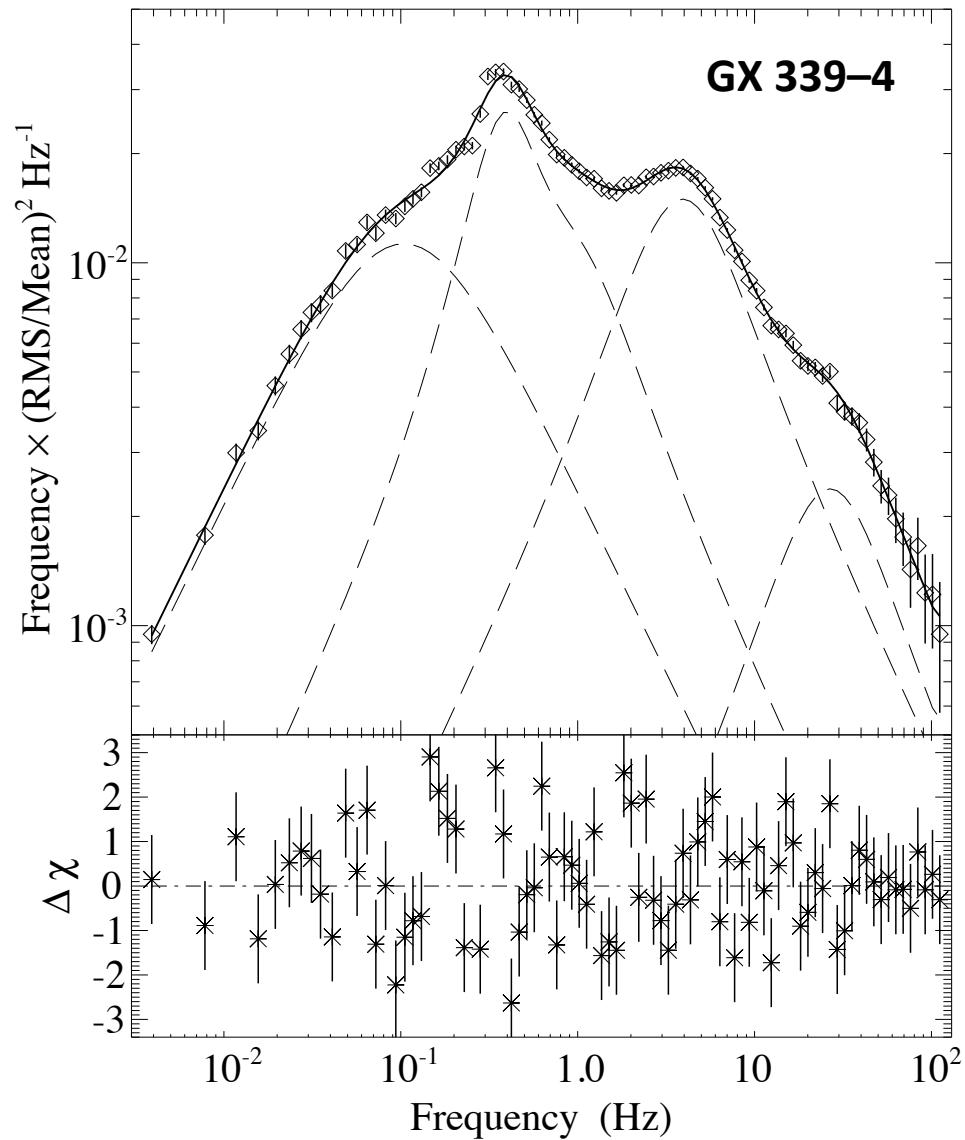
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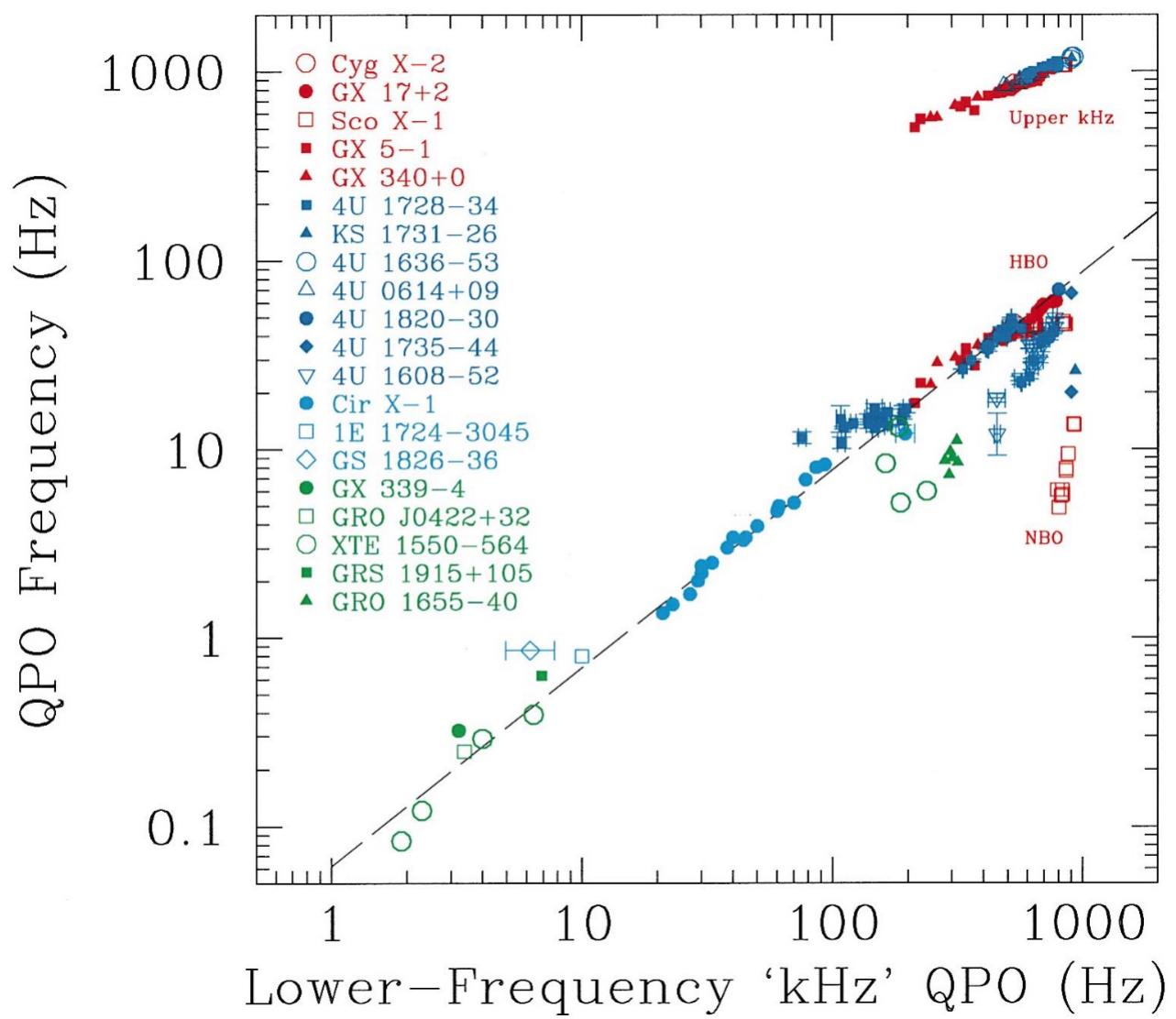
!!! But narrow features in the lags and coherence !!!



Lorentzian frequencies correlated with each other ...

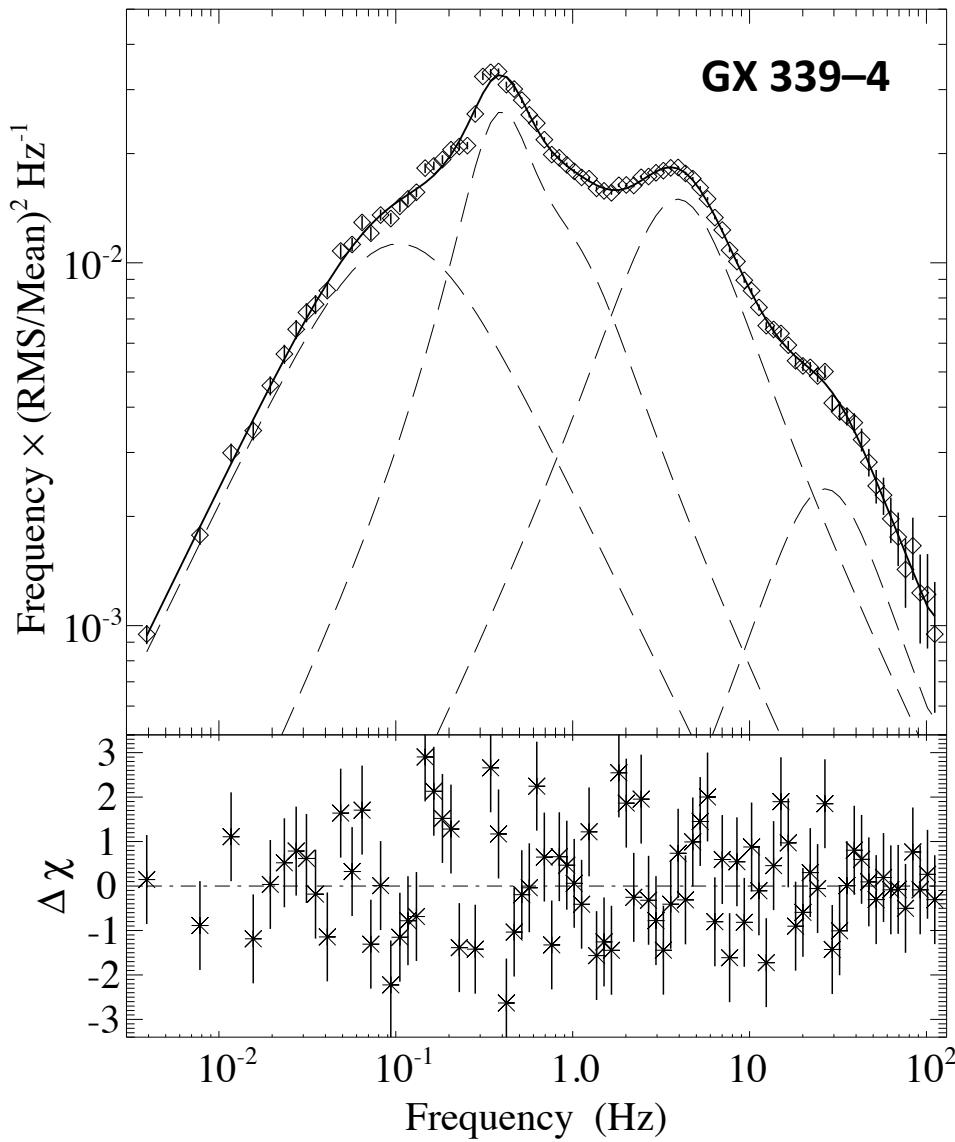


Nowak 2000



Psaltis et al. 1999

... and with (almost?) every other physical parameter



Nowak 2000

Not just frequency –frequency correlations, but:

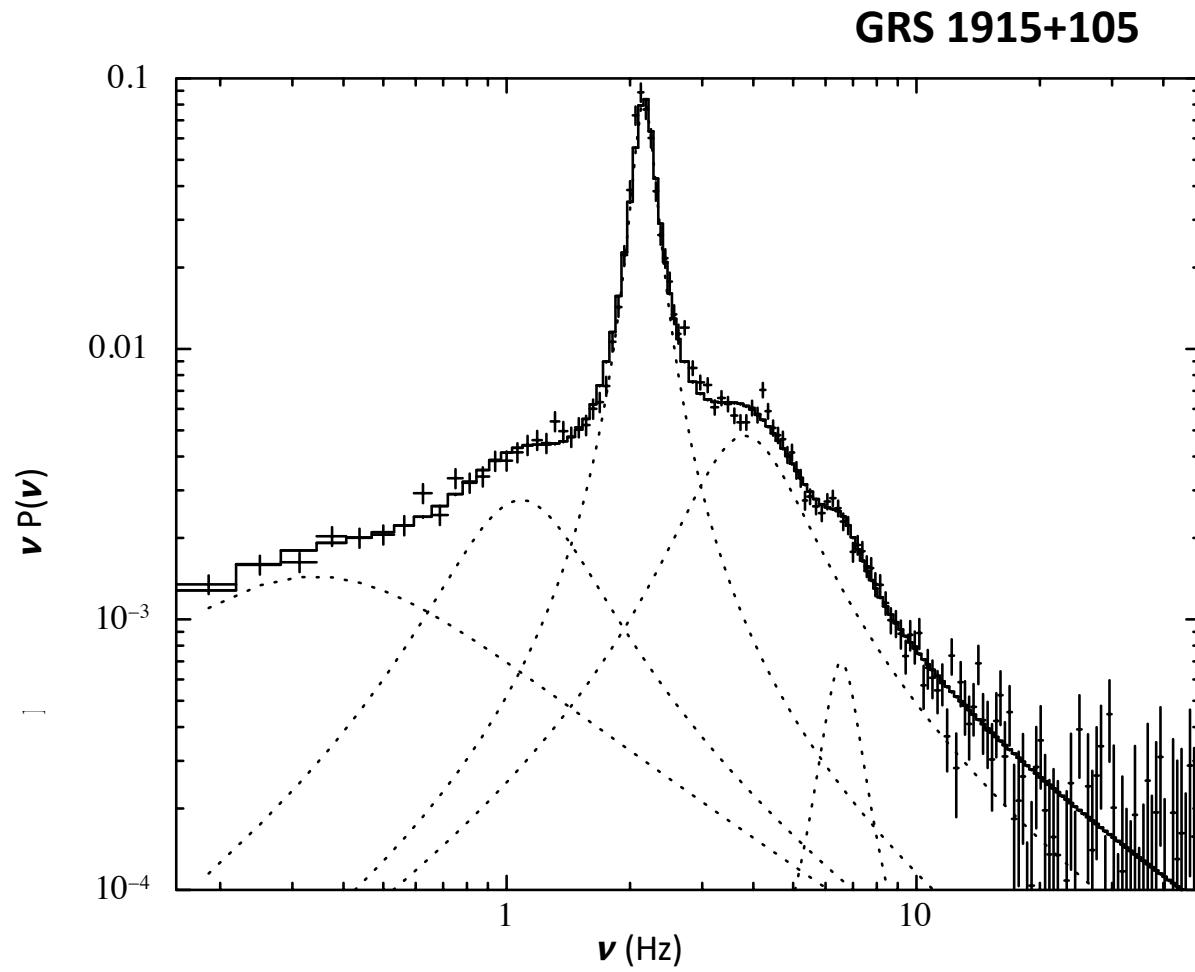
- *Lorentzian frequencies, rms amplitudes, FWHM, lags*
- *Broadband rms amplitude, lags*
- *Power colours (hue)*
- *Source intensity, hardness ratios*
- *Spectral parameters (kT_{in} , Γ , kT_{e} , etc.)*
- *Total flux, flux of individual spectral components (disc, corona, etc.)*
- *Radio flux and radio spectral index*
- ... etc...

e.g., Altamirano et al. 2005, 2008; Altamirano & Méndez 2015; Belloni et al. 2002; Casella et al. 2004; De Marco et al. 2015, 2017, 2021; Di Matteo & Psaltis 1999; Fender et al. 2004; Ford et al. 2000; García et al. 2022; Grinberg et al. 2014; Homan et al. 2001; Jonker et al. 2002; Kalamkar et al. 2015; Markwardt et al. 1999; Méndez et al. 1999, 2001, 2022; Motta et al. 2009, 2010, 2011; Munoz et al. 2001; Nowak 2000; Pottschmidt et al. 2003; Rao et al. 2010; Reig et al. 2013, 2018; Remillard et al. 2002; Shaposhnikov & Titarchuk 2009; Shidatsu et al. 2014; Sobczak et al. 2000; Stiele et al. 2013; van Straaten et al. 2002, 2003, 2005; Vignarca et al. 2003; Zdziarski et al. 2005 ...

(... and many, many, many more...)

Where can one measure the broadband lags?

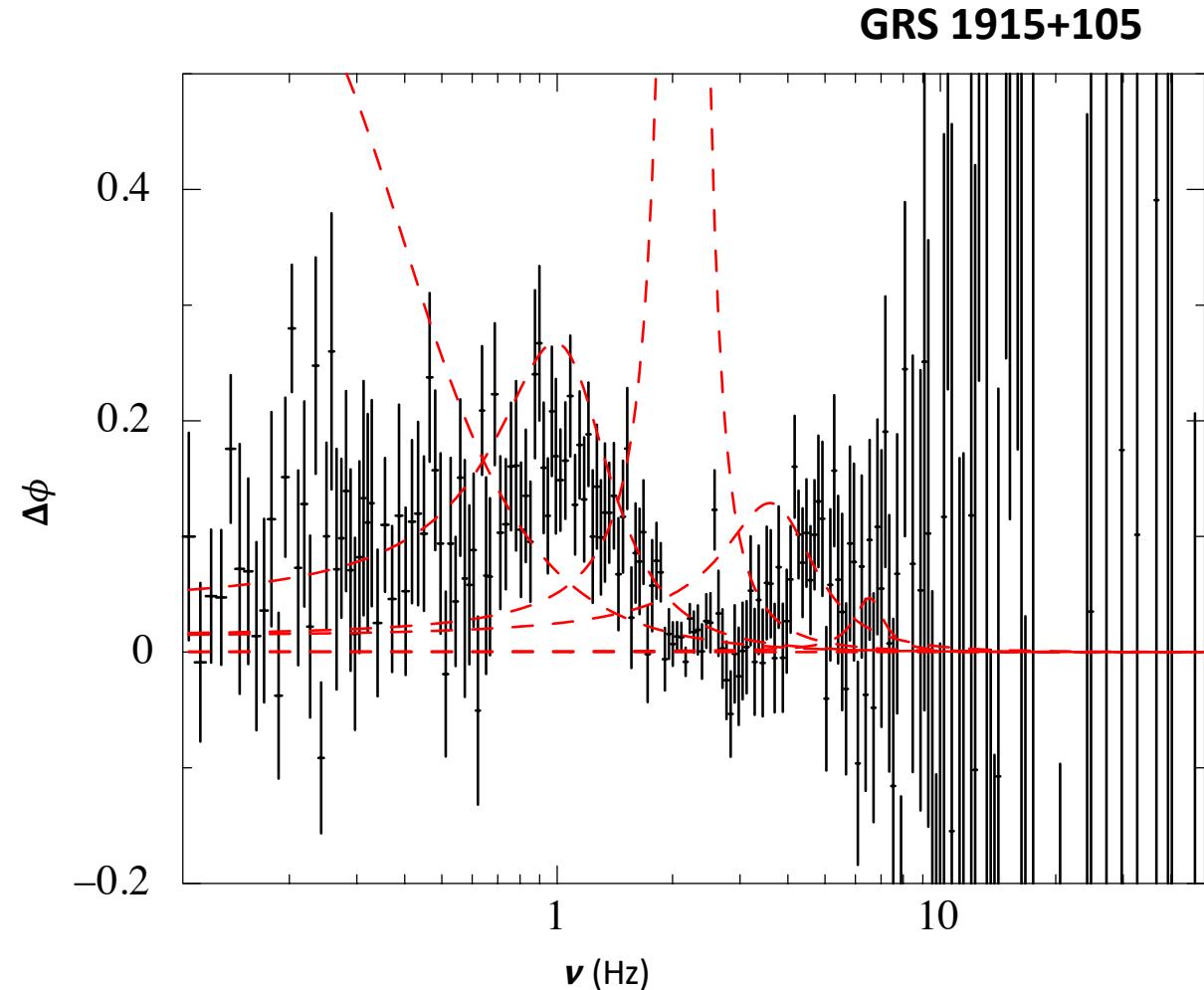
Power spectrum of an observation of GRS 1915+105 showing a fit with 5 Lorentzians for the fundamental of the QPO at ~ 2 Hz, the sub-harmonic, second and third harmonics plus a 0-centred Lorentzian



Where can one measure the broadband lags?

Lag-frequency spectrum (lags vs. Fourier frequency; **black**) of the same observation of GRS 1915+105

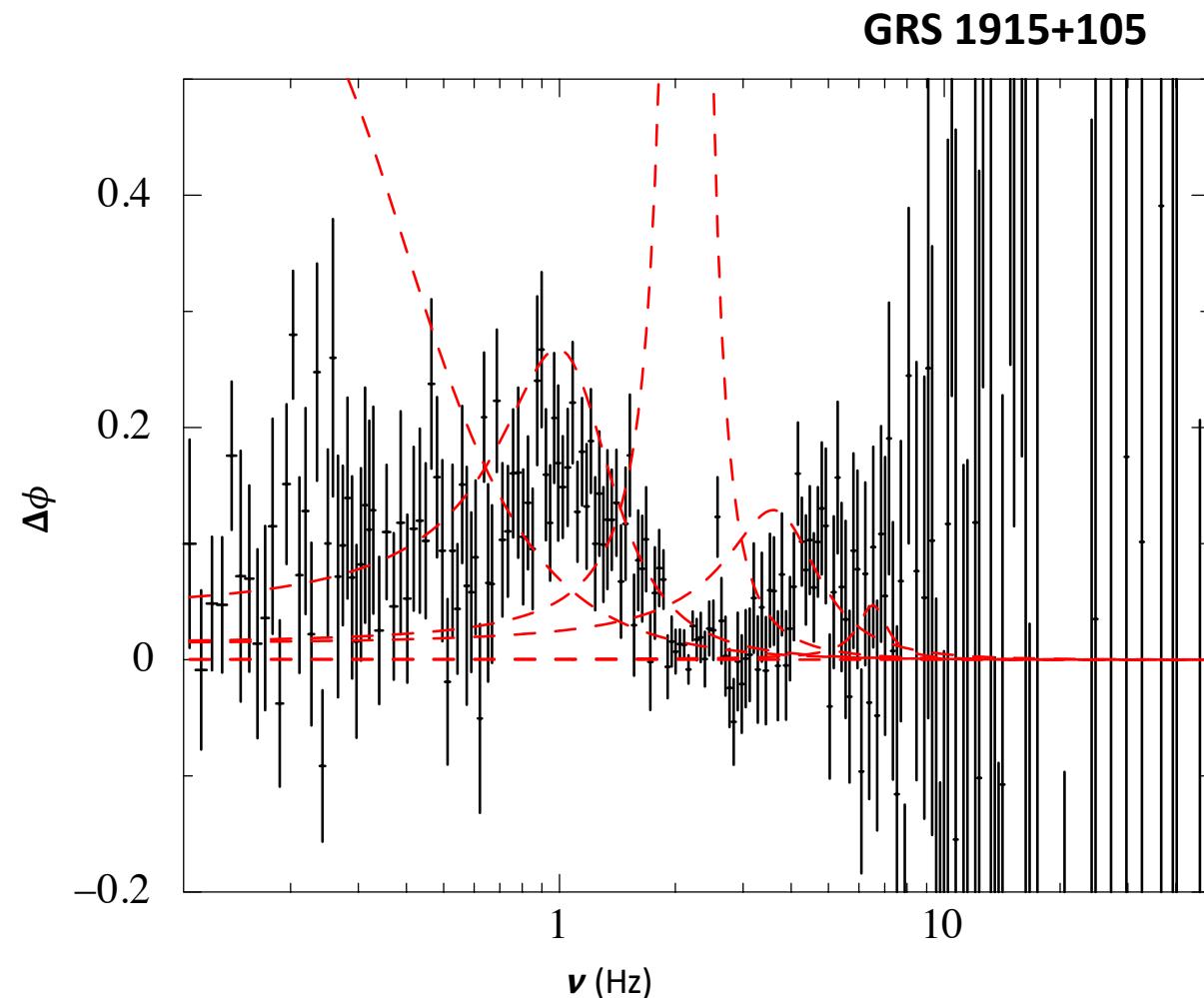
The dashed (**red**) lines show the Lorentzians (rescaled arbitrarily) used to fit the power spectrum



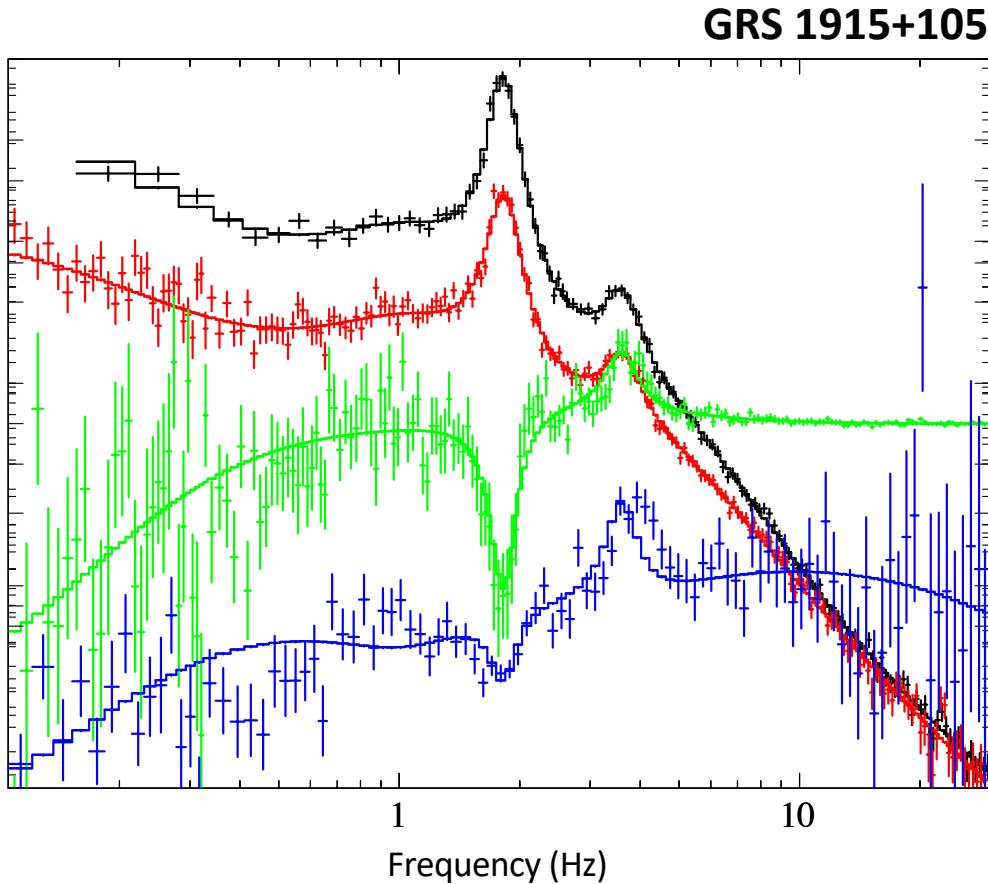
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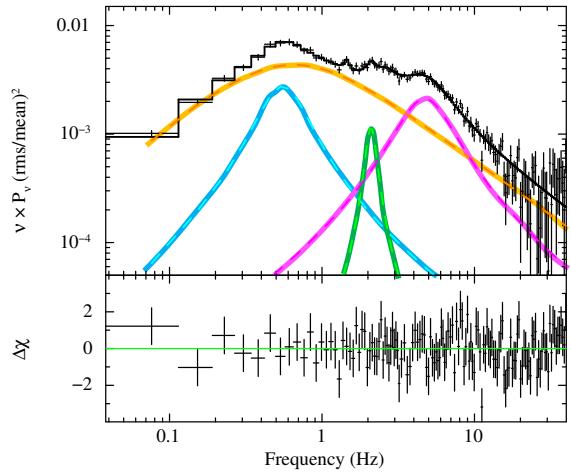
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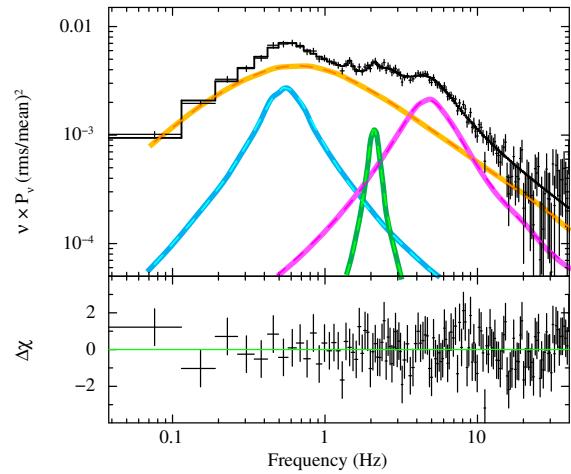
Black: Full-band Power Spectrum (PDS)
Red: Real part of the Cross Spectrum (CS)
Green: Imaginary part of the Cross Spectrum
Blue: Phase lags
(all of them arbitrarily scaled)

All four fitted with the same number of Lorentzians, with central frequencies and FWHM fixed to the best-fitting values of the PDS

Power spectrum (PDS)

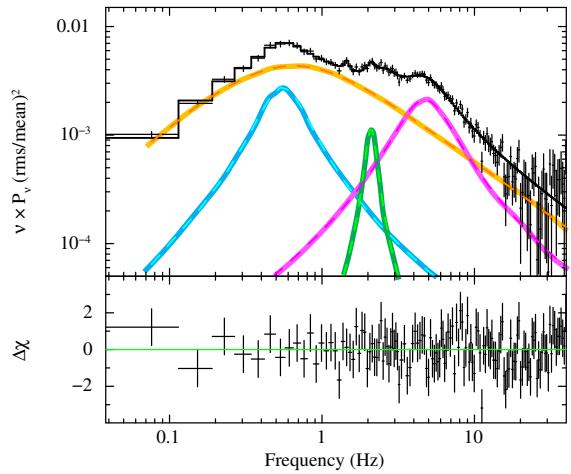


Power spectrum



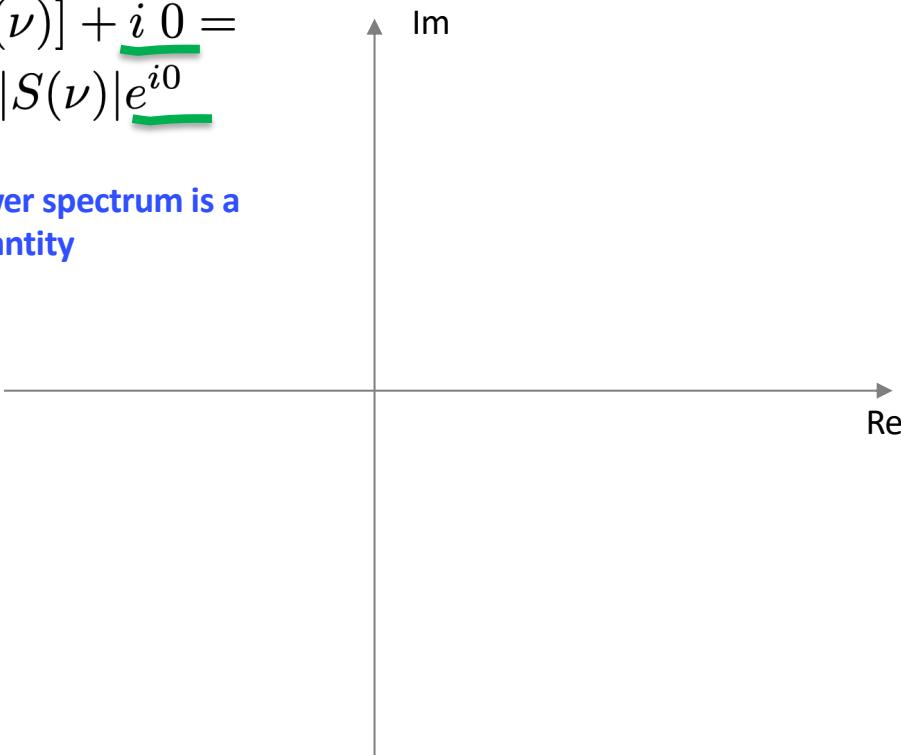
$$\begin{aligned}P(\nu) &= \langle S^*(\nu)S(\nu) \rangle = \\Re[P(\nu)] + i 0 &= \\|S(\nu)| |S(\nu)| e^{i0}\end{aligned}$$

Power spectrum

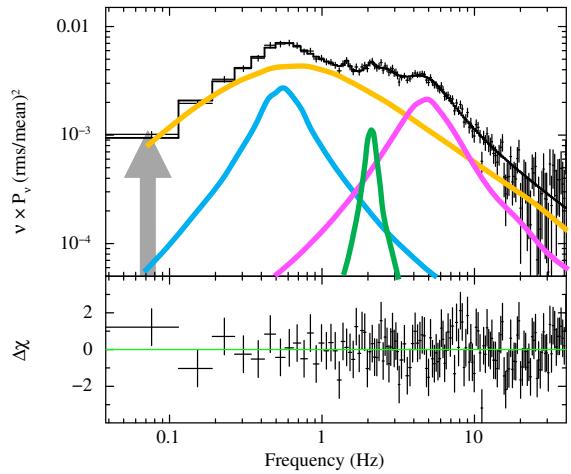


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The power spectrum is a
Real quantity

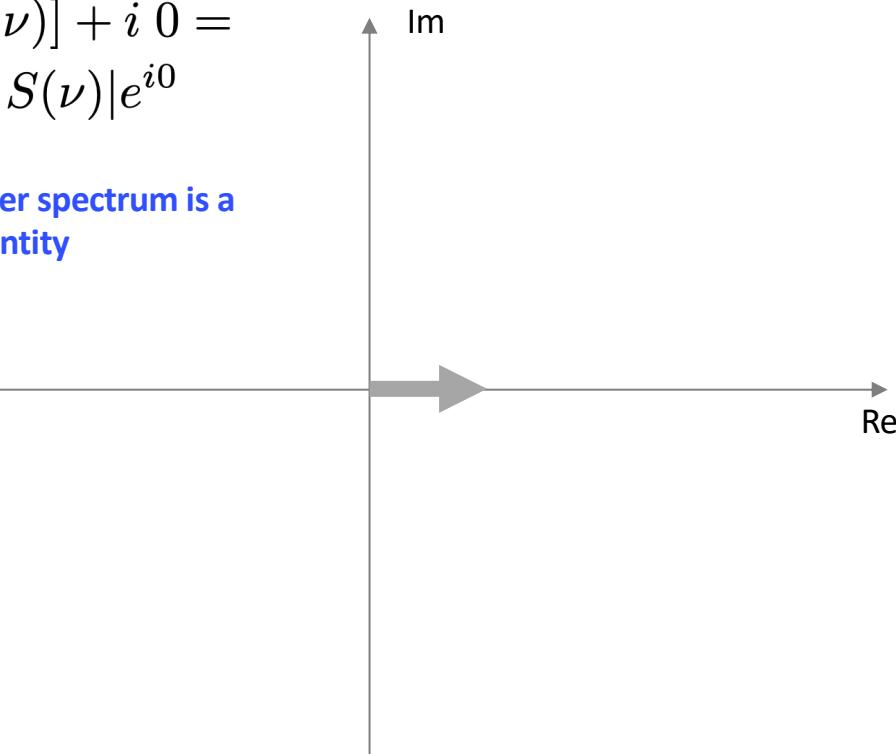


Power spectrum

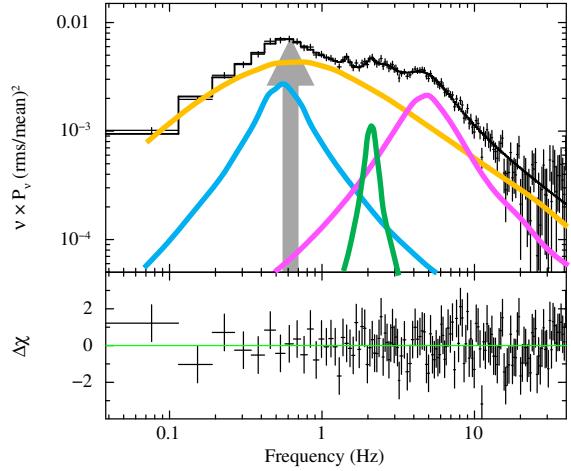


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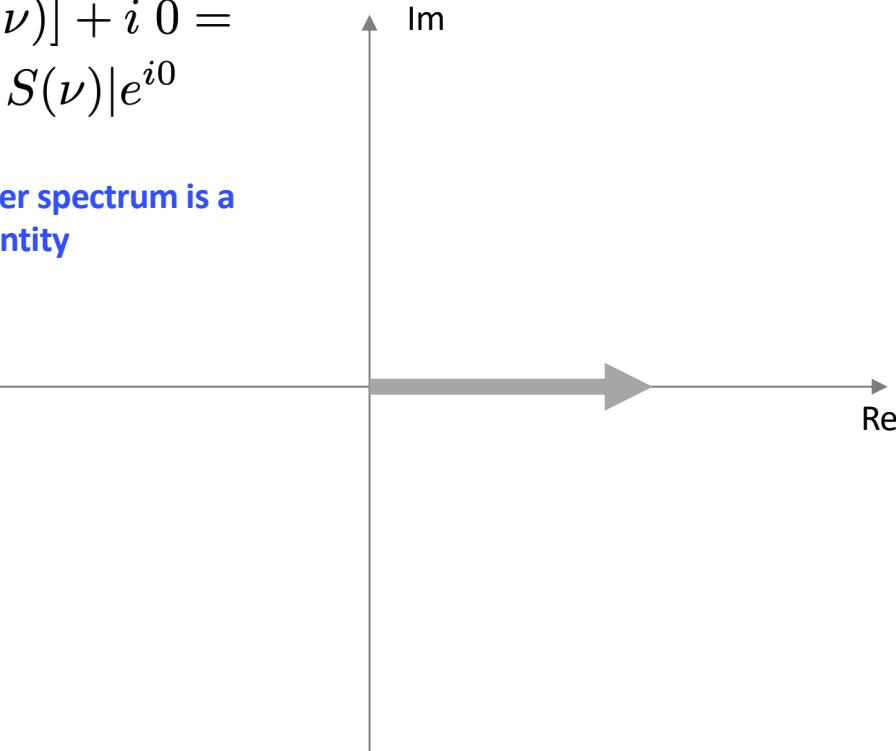


Power spectrum

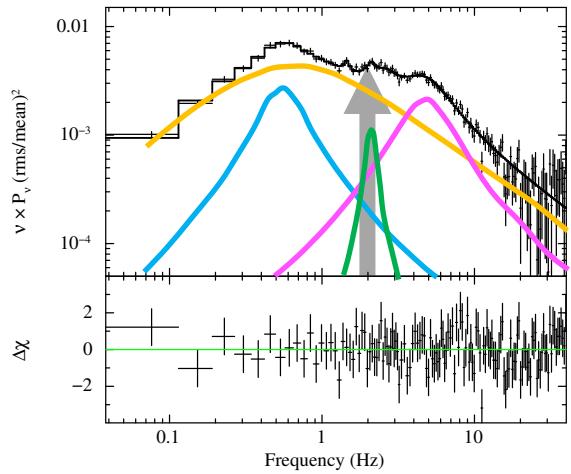


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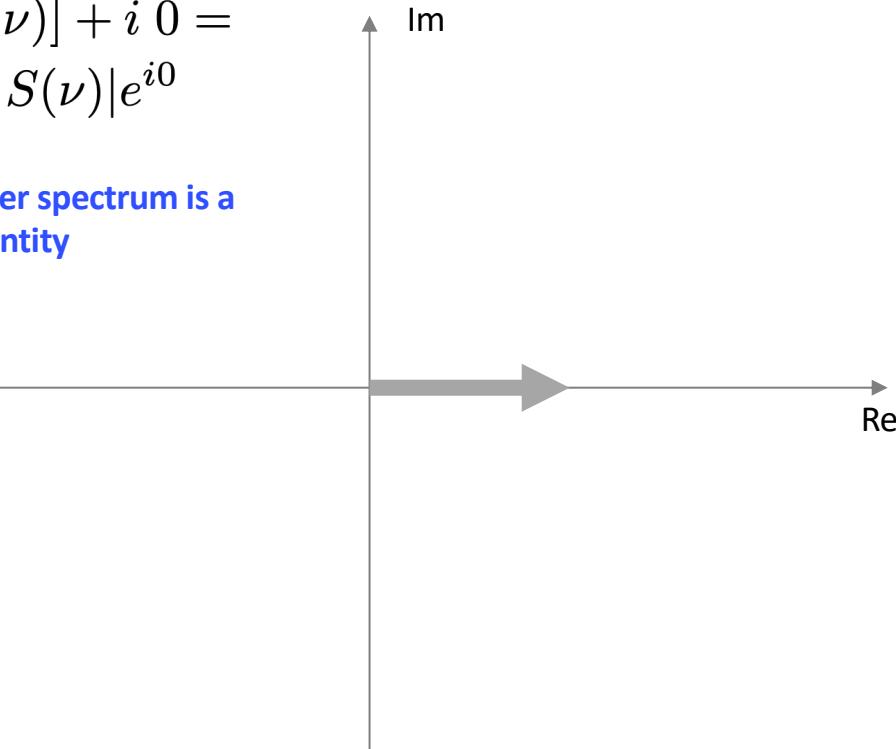


Power spectrum

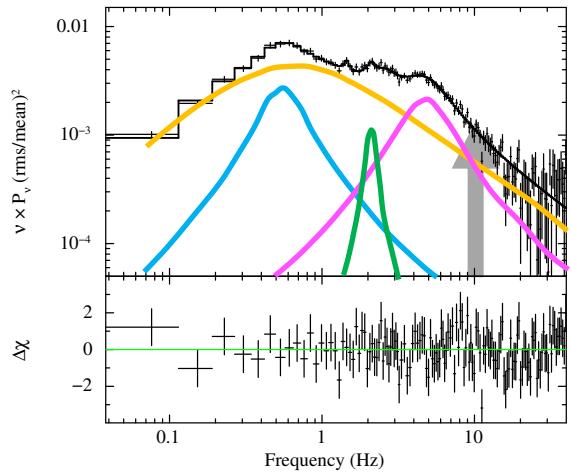


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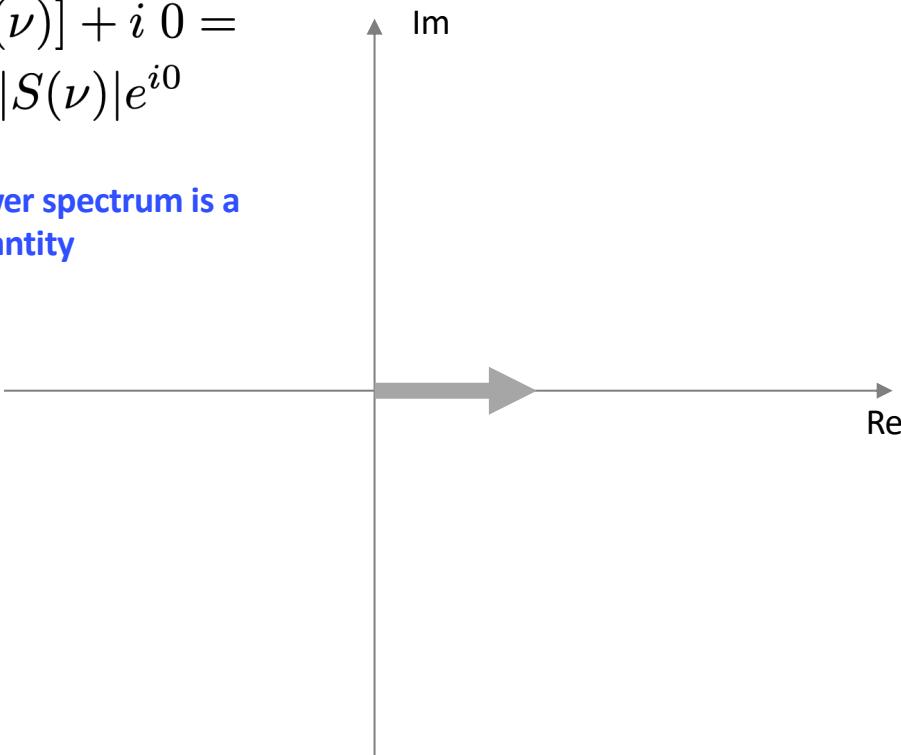


Power spectrum

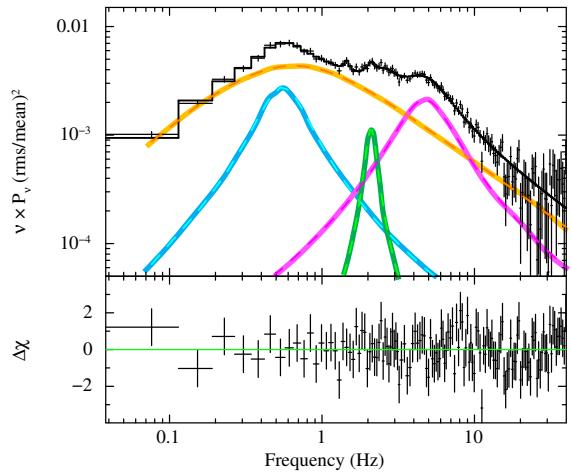


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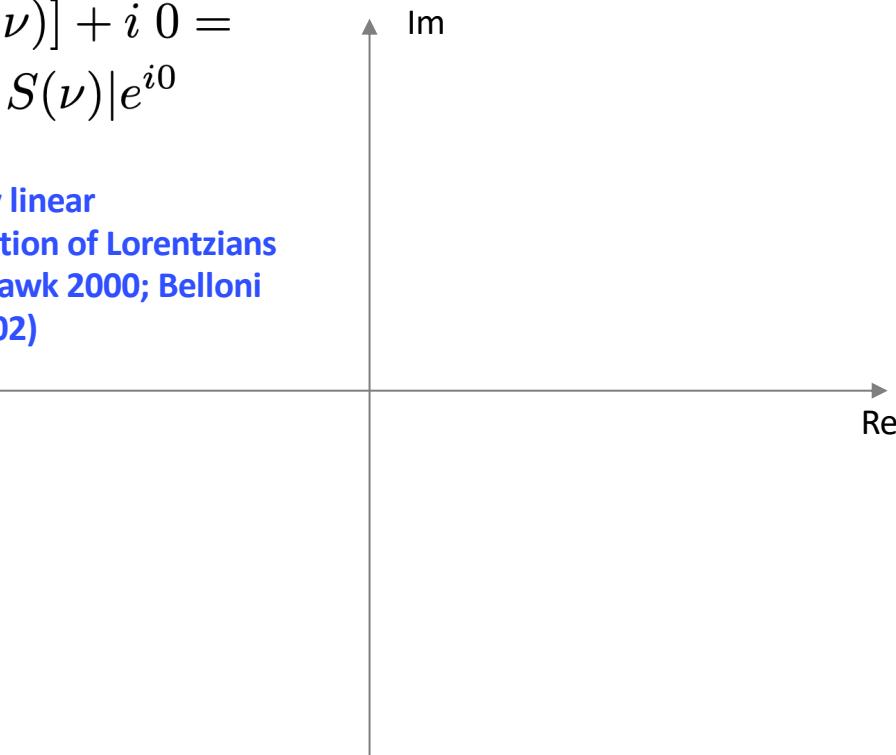


Power spectrum

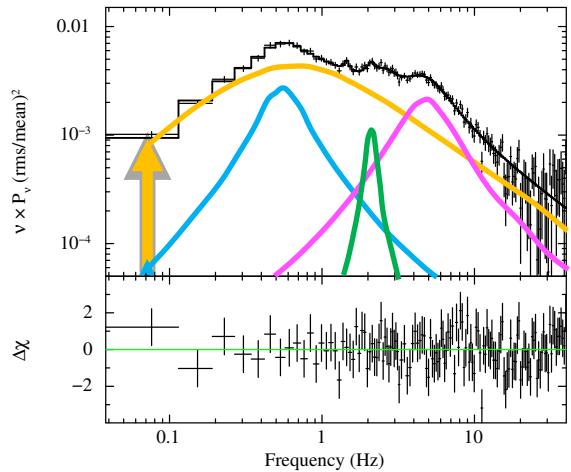


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Fitted by linear
combination of Lorentzians
(e.g., Noawk 2000; Belloni
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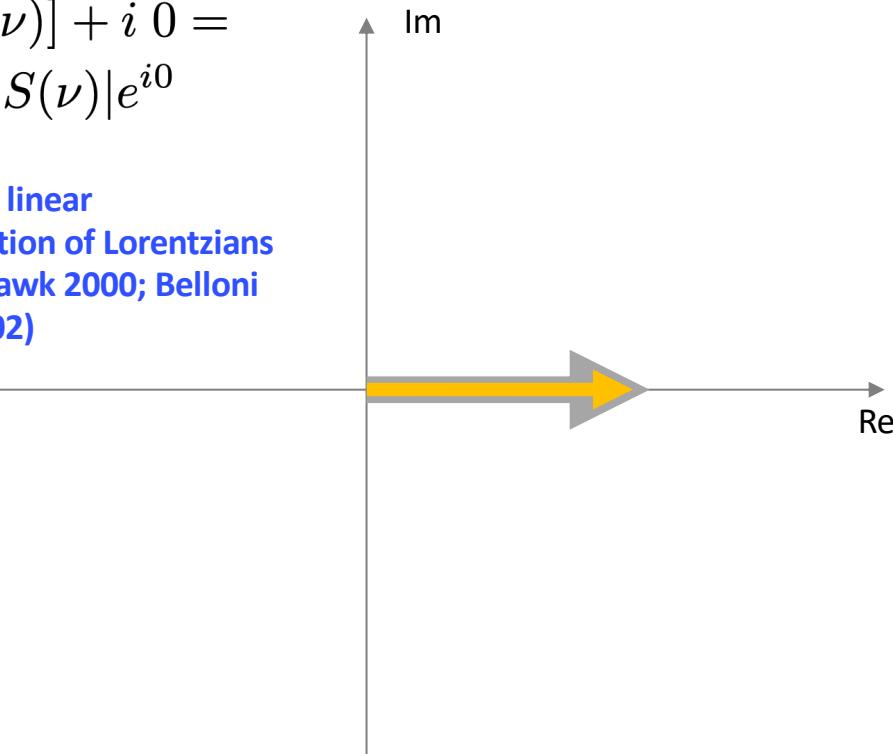


Power spectrum

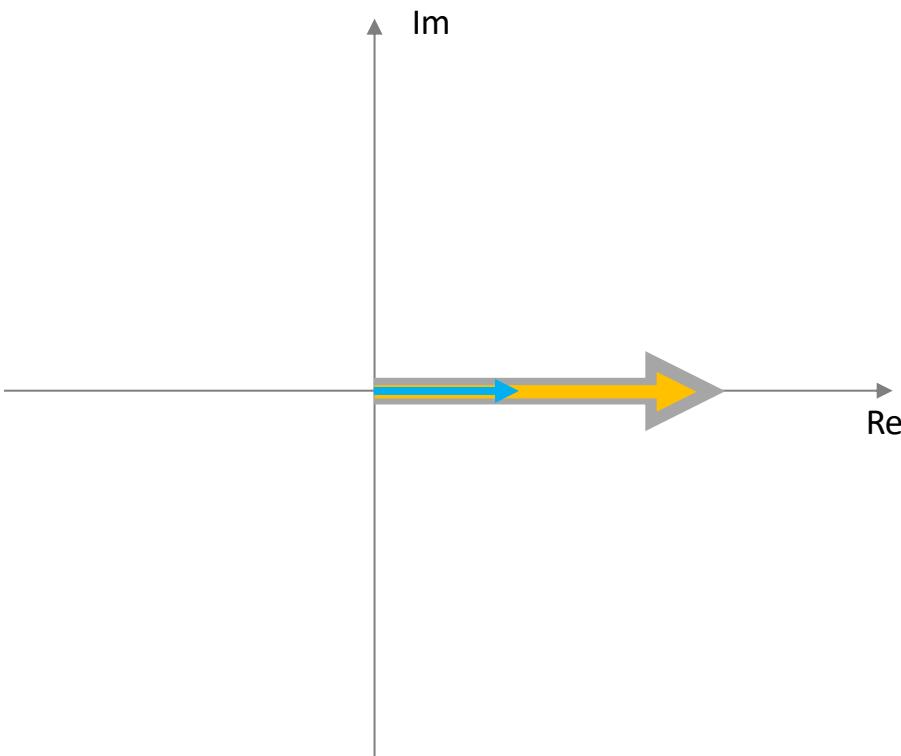
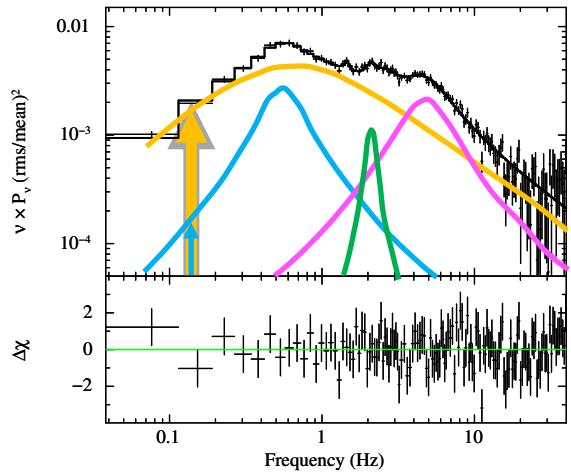


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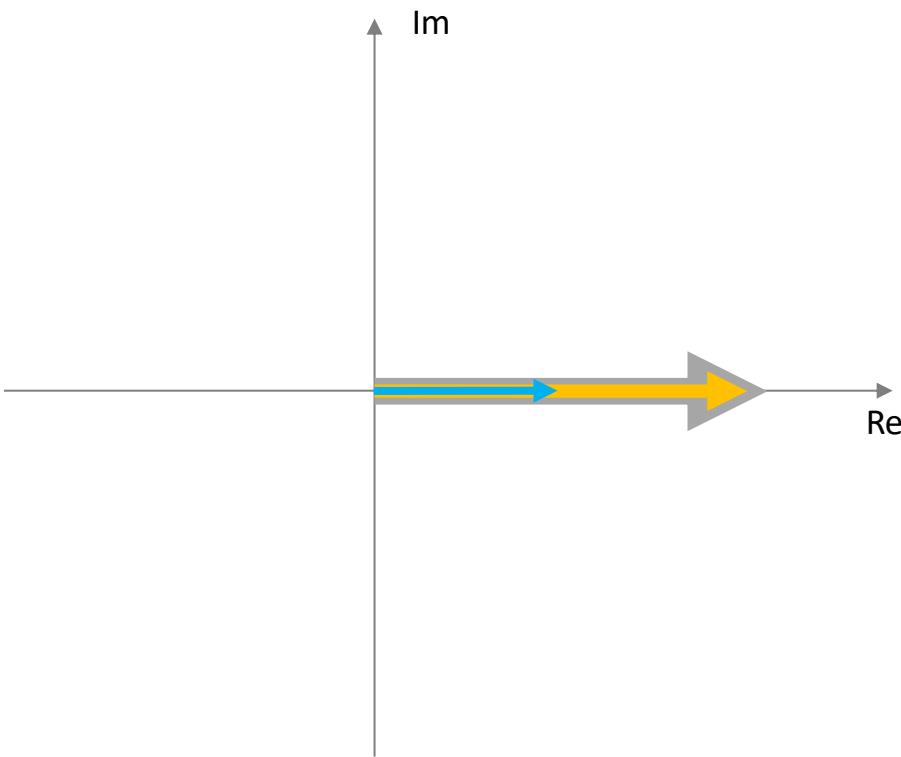
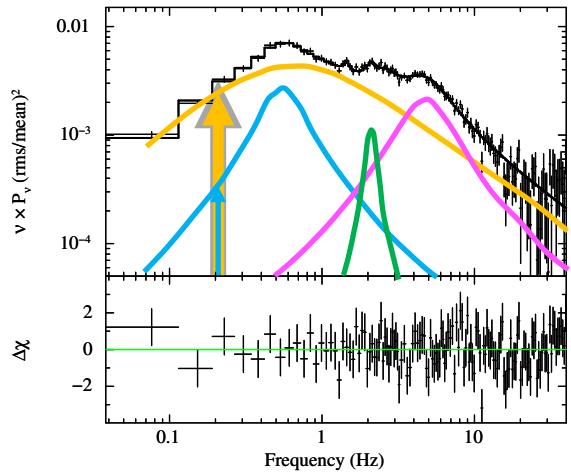
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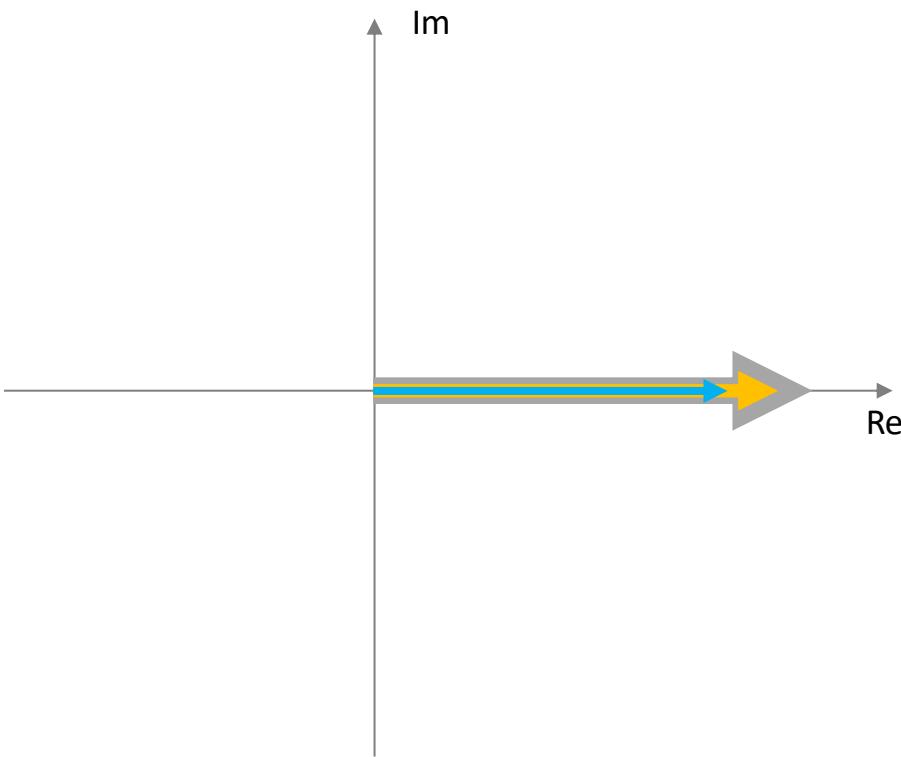
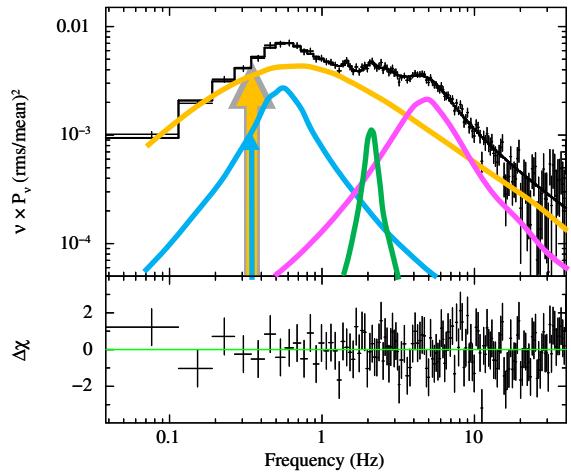
Power spectrum



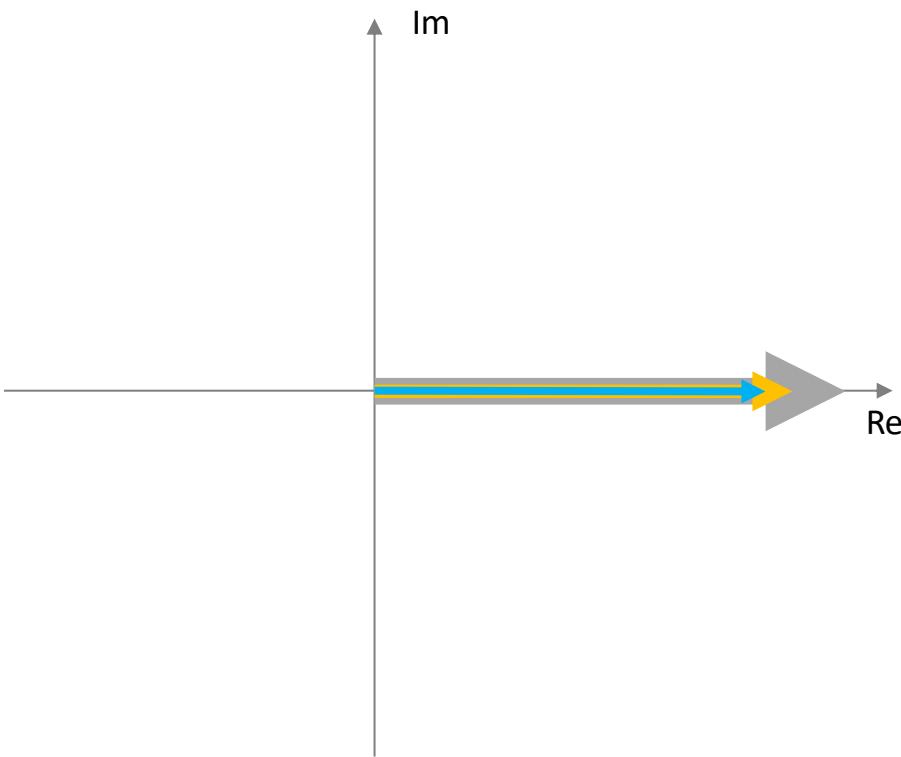
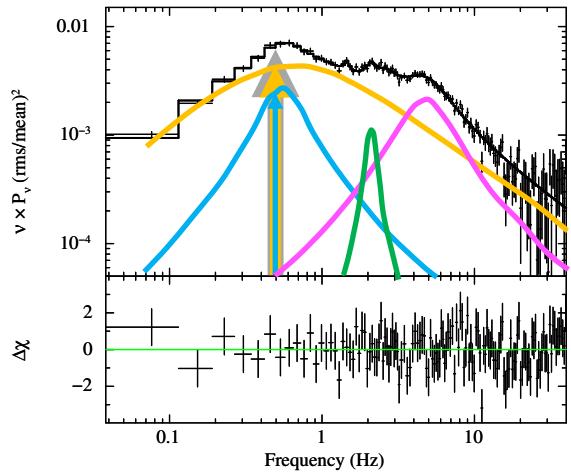
Power spectrum



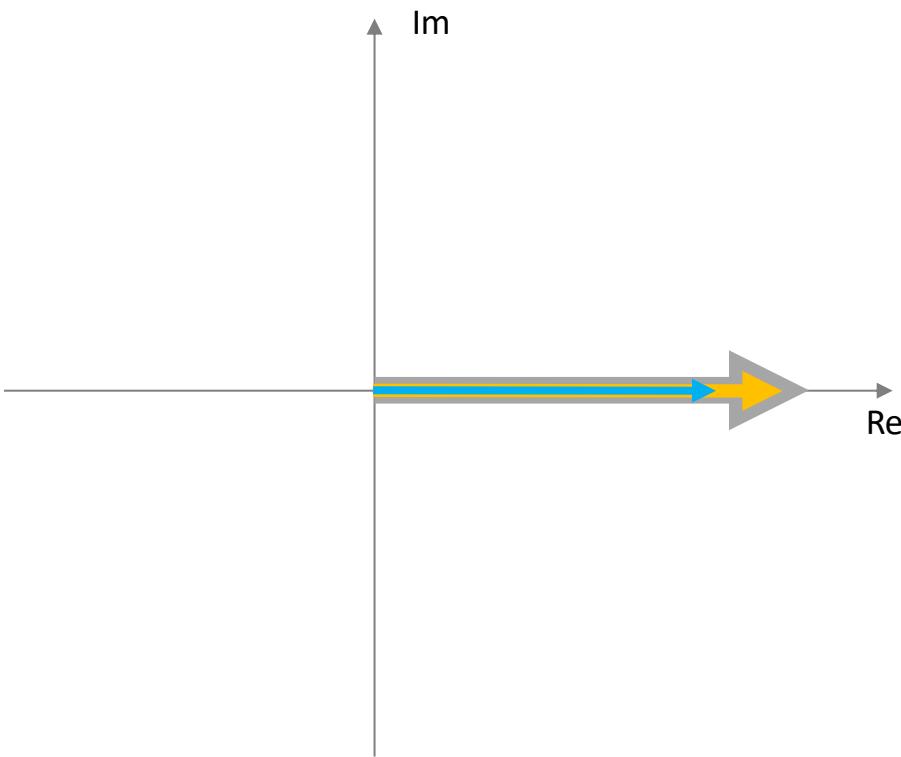
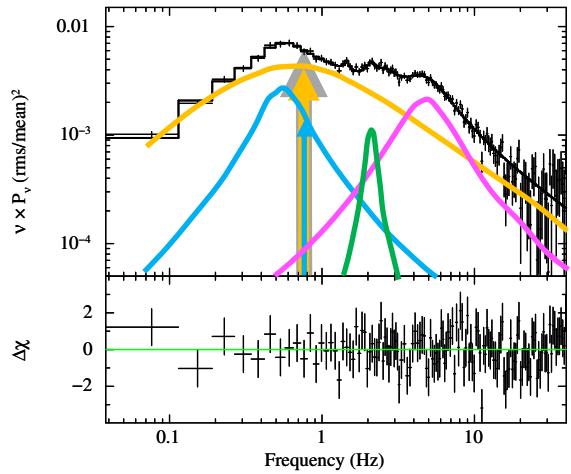
Power spectrum



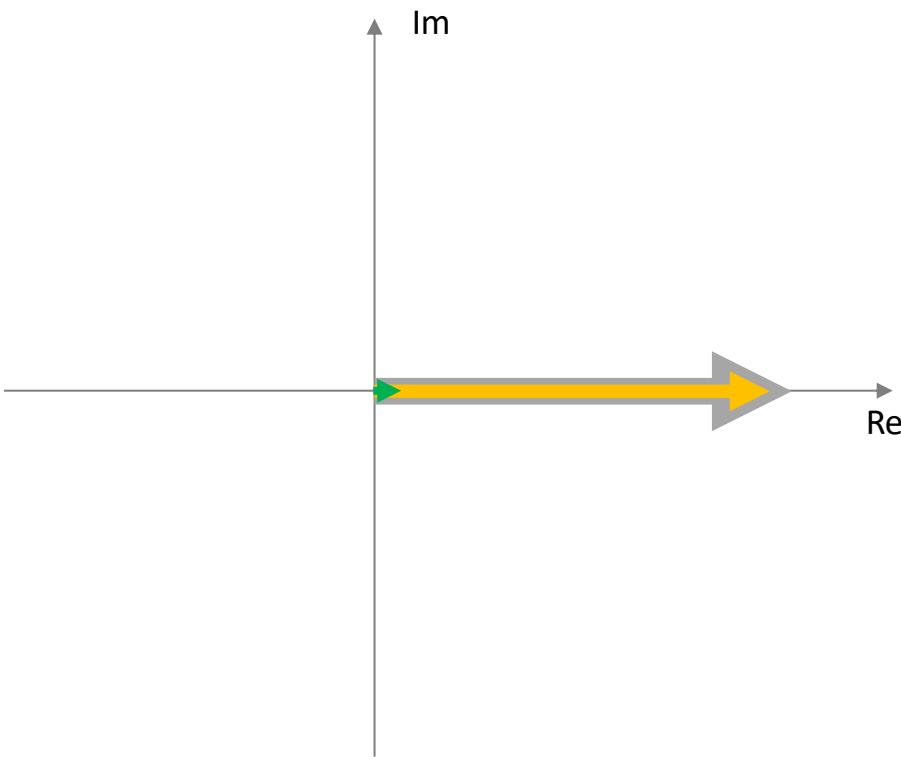
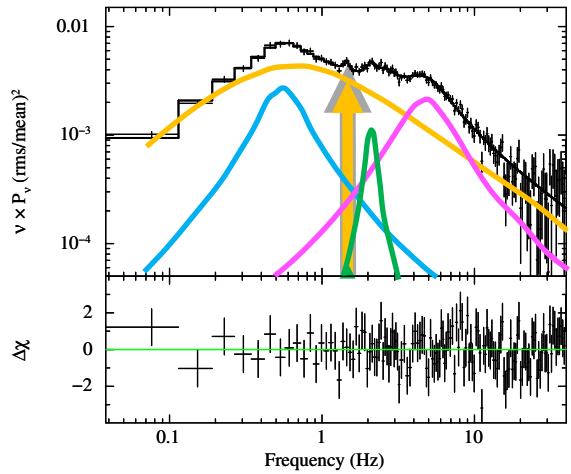
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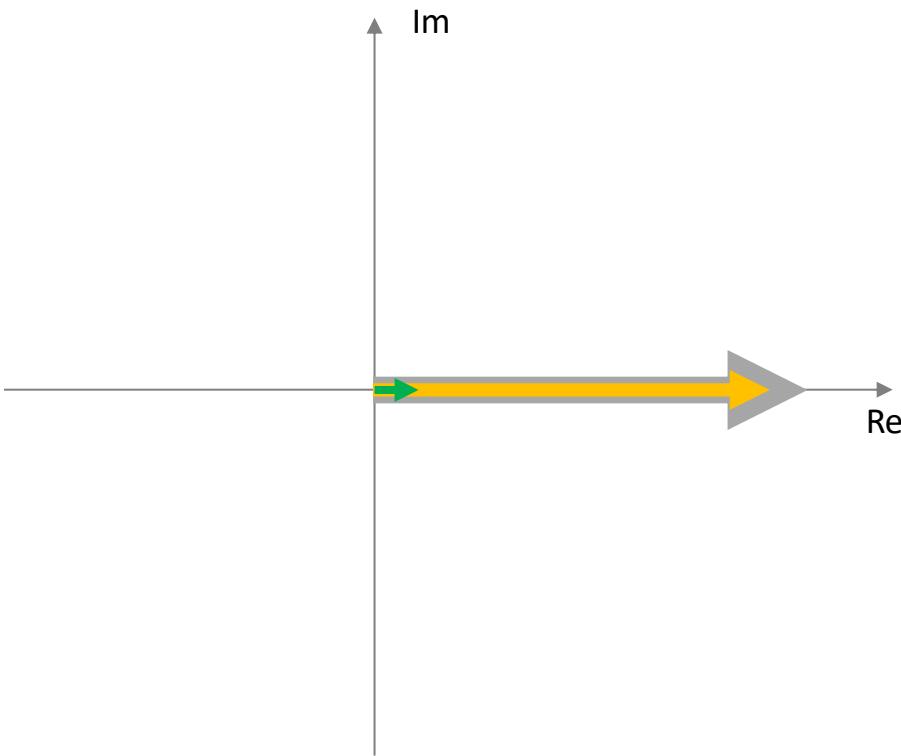
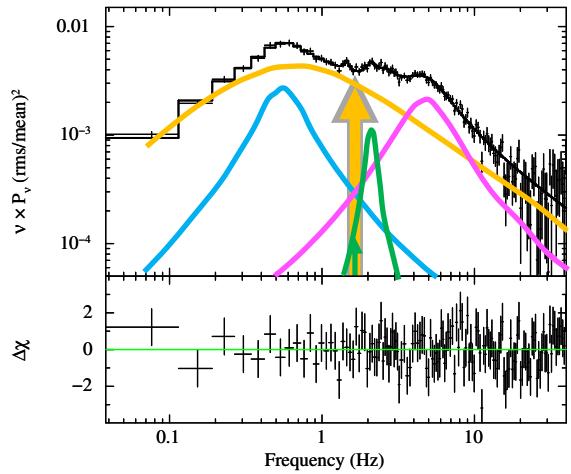
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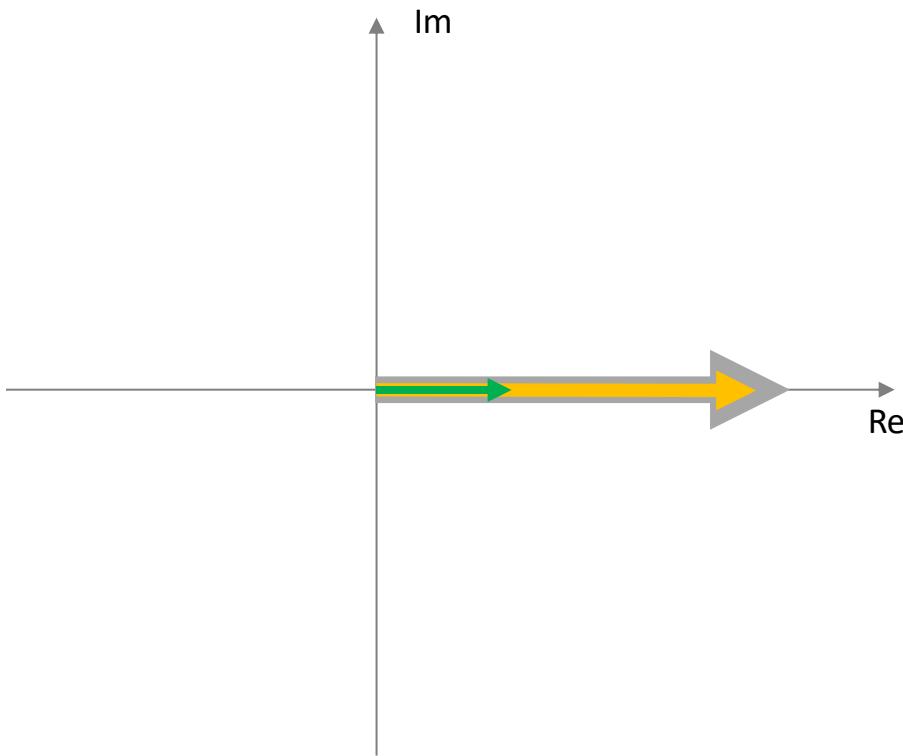
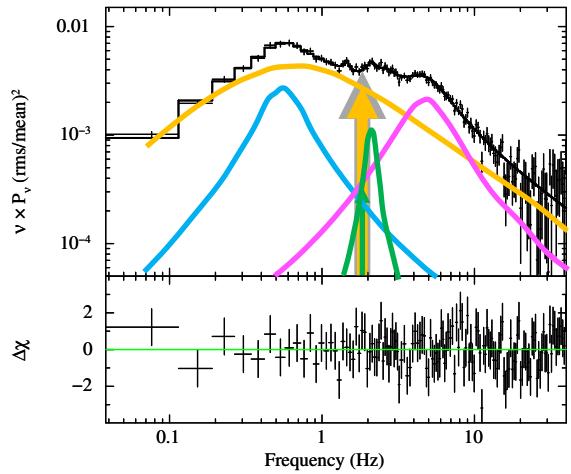
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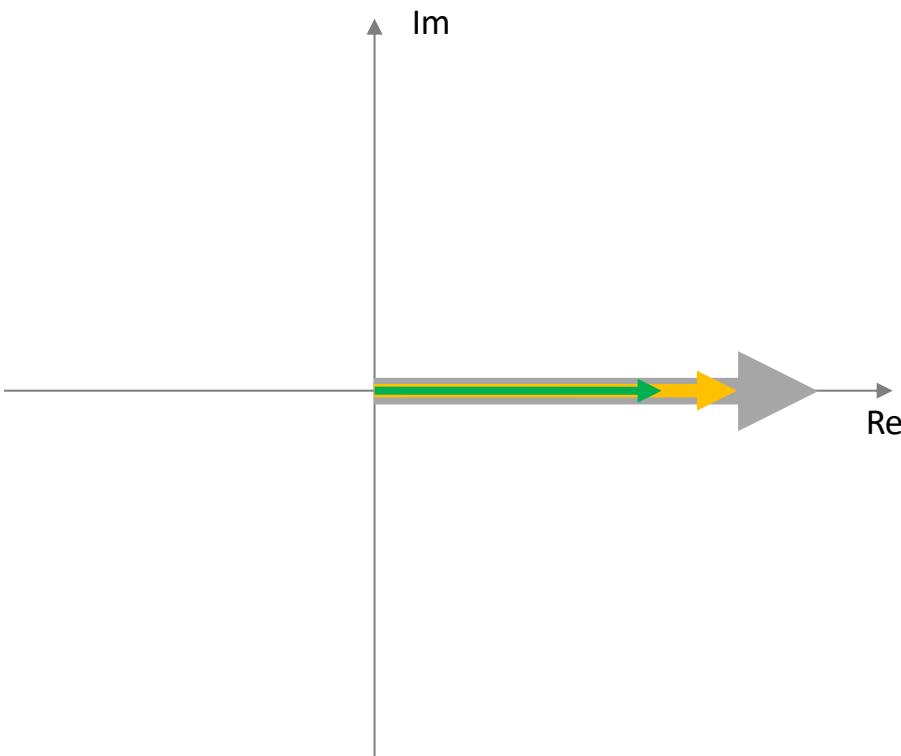
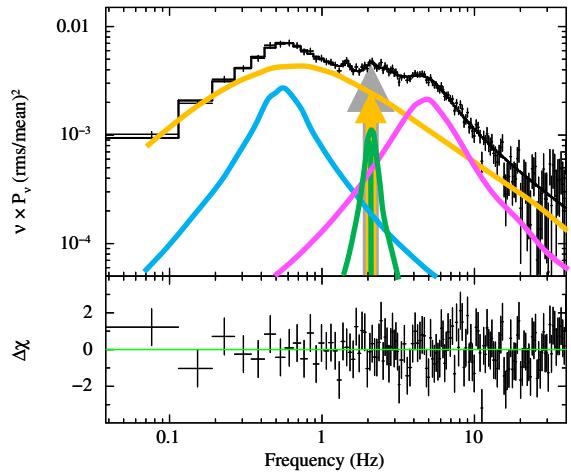
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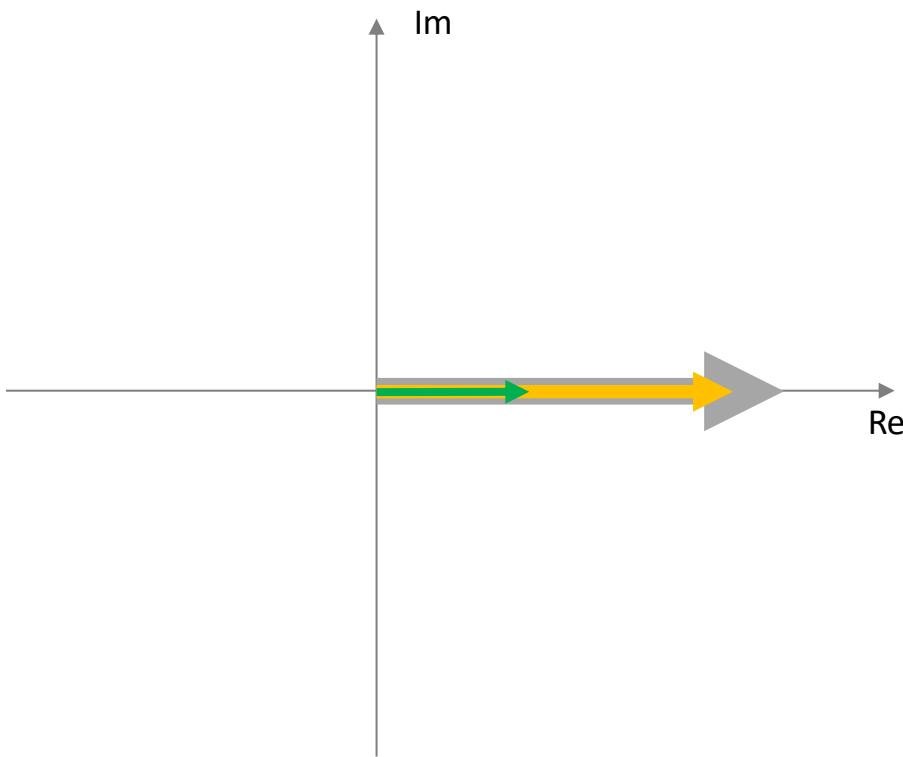
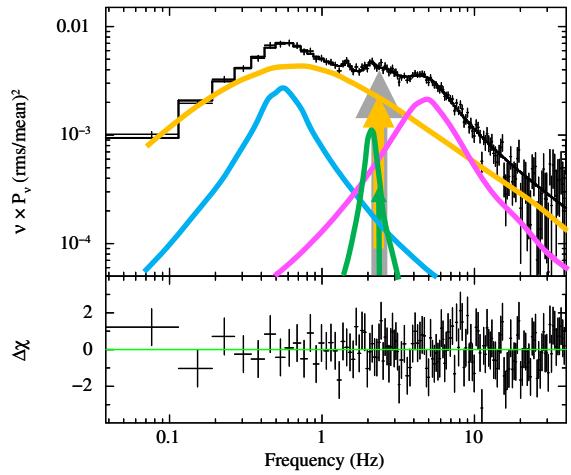
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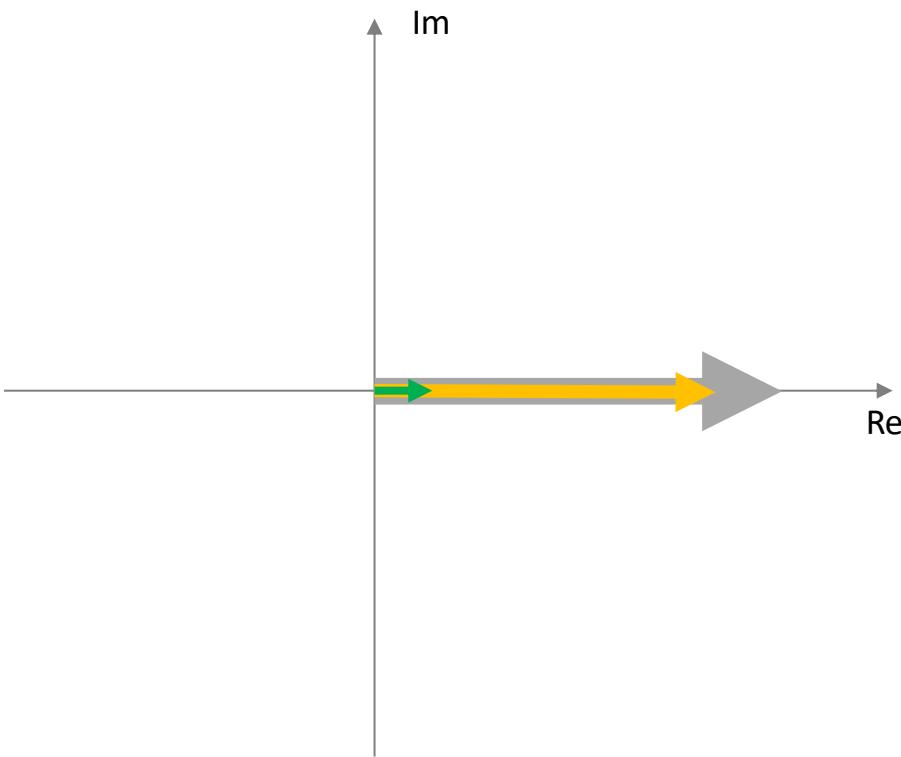
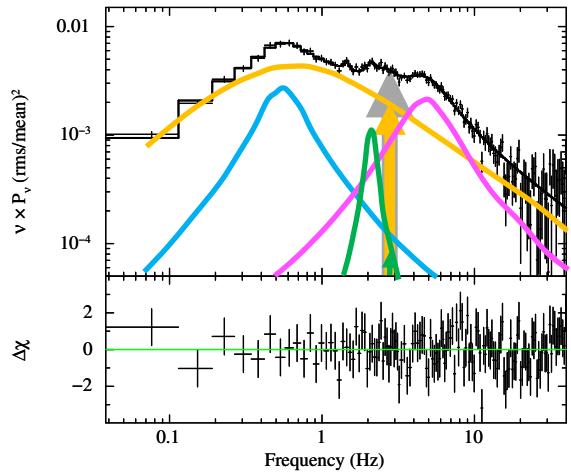
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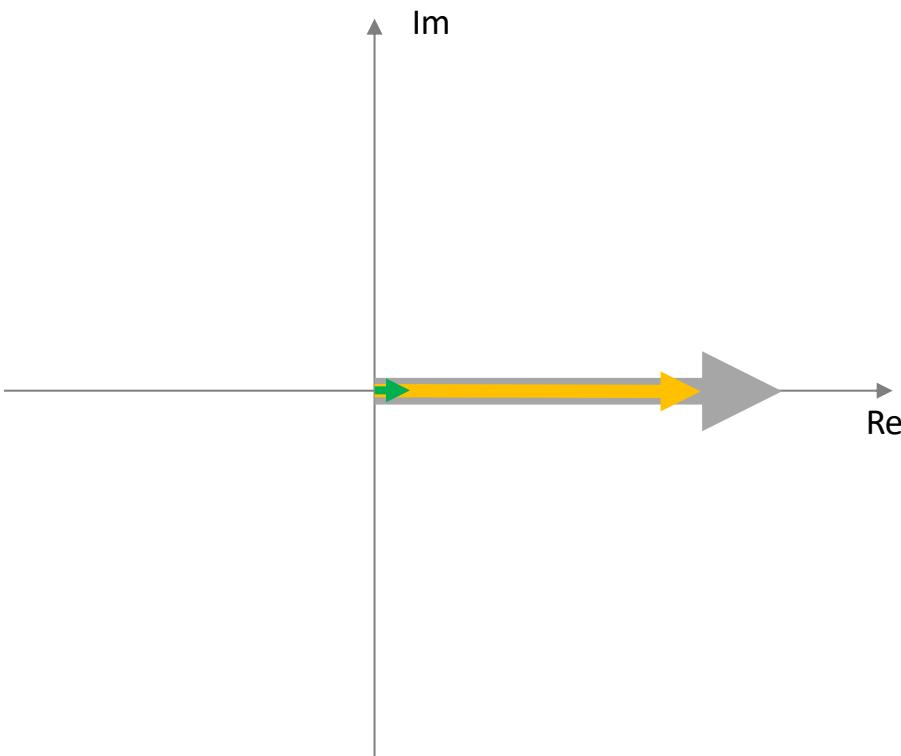
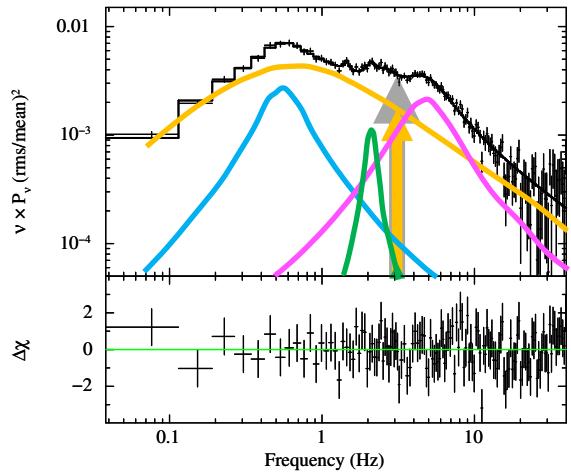
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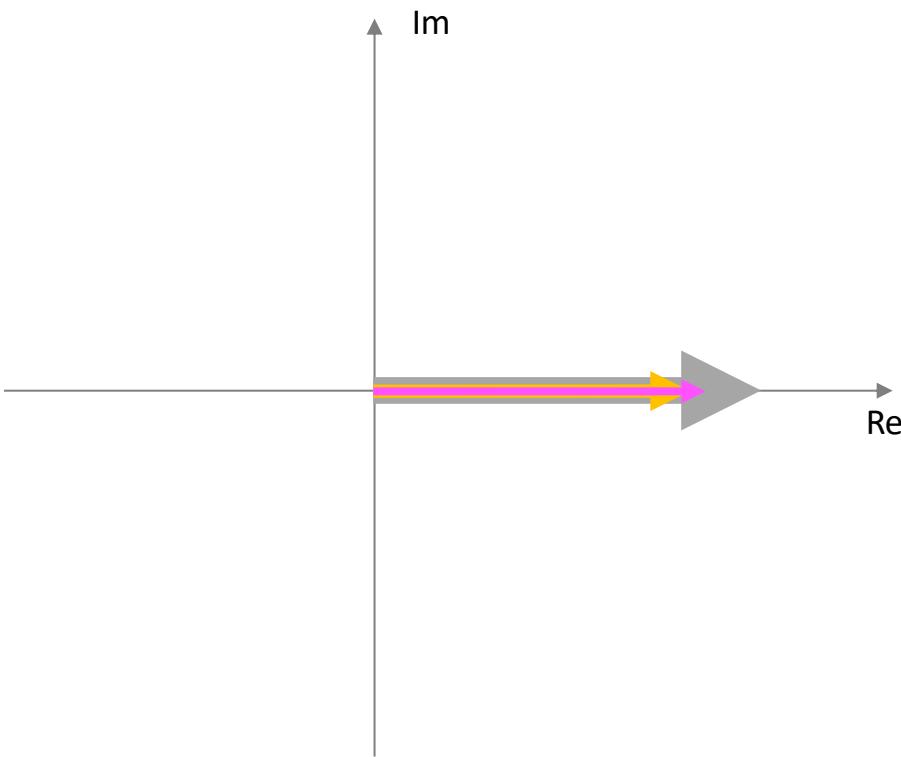
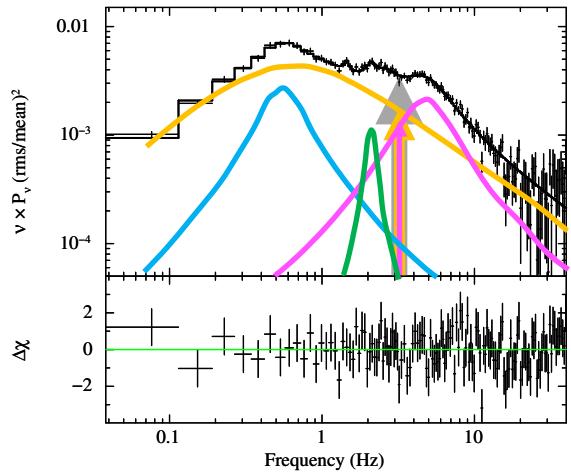
Power spectrum



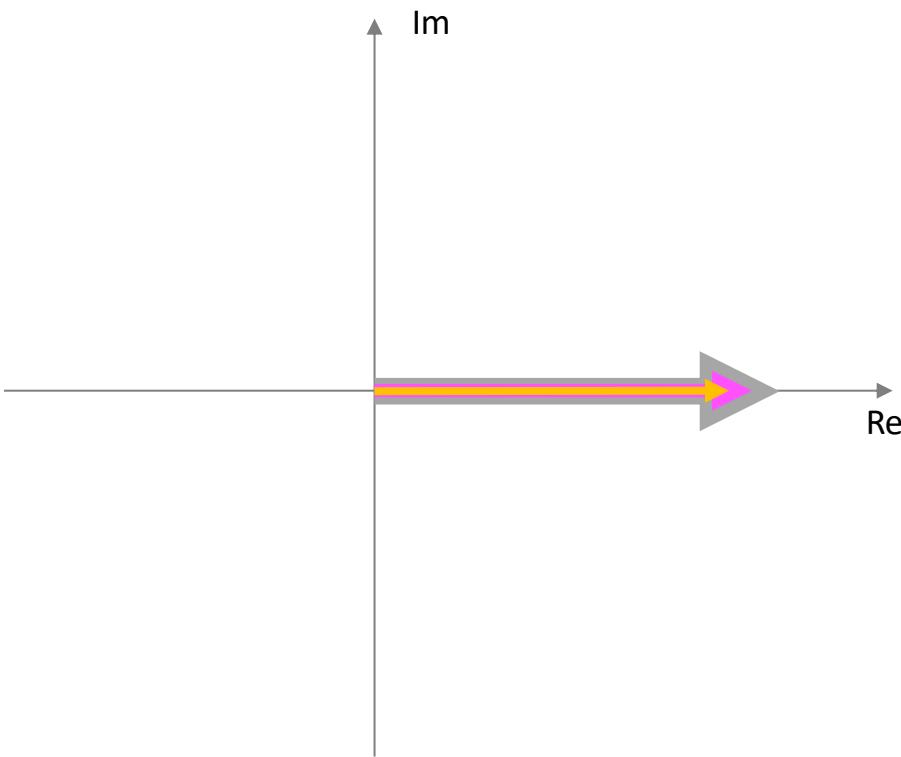
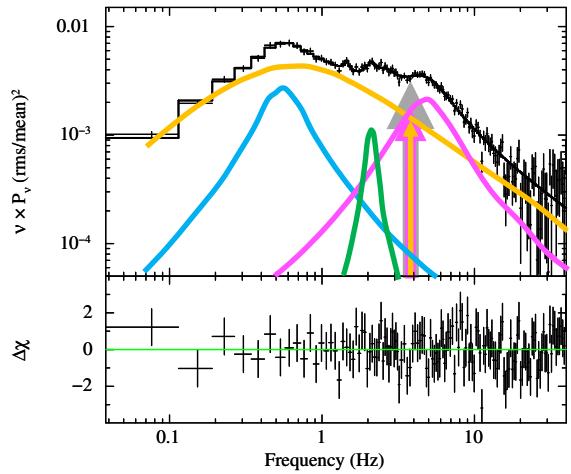
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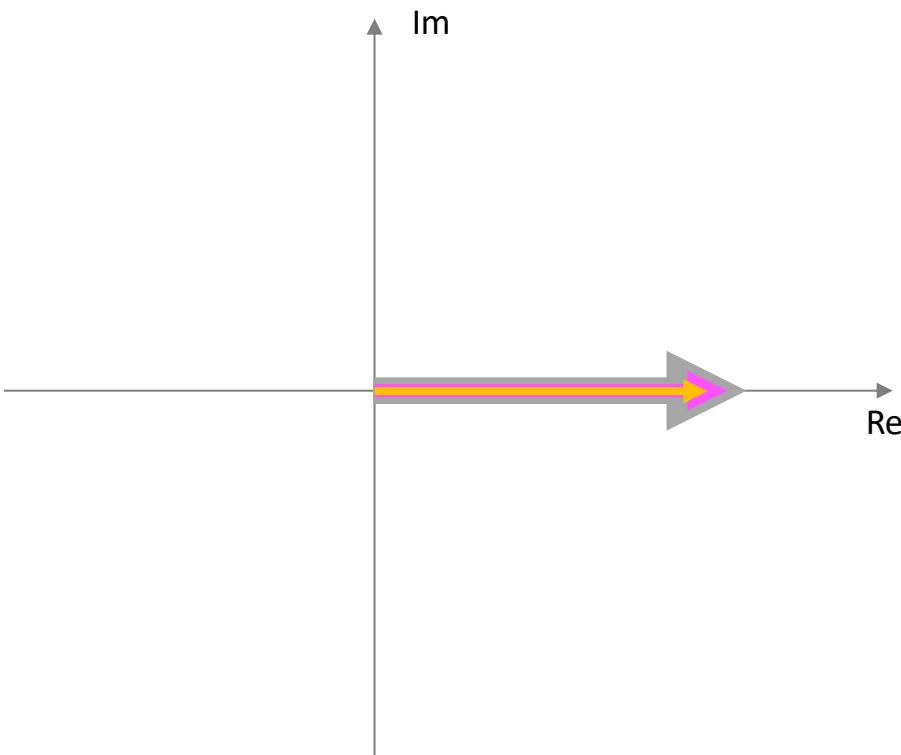
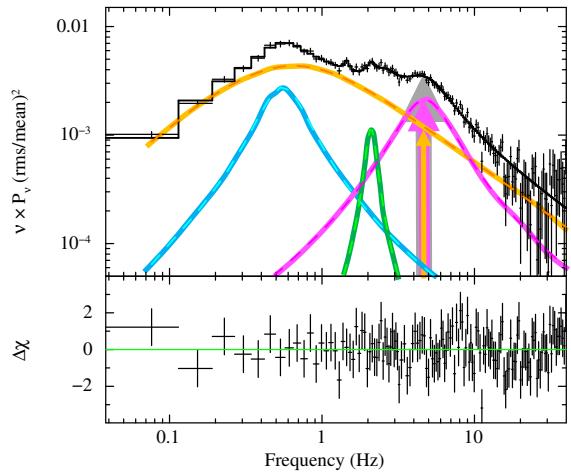
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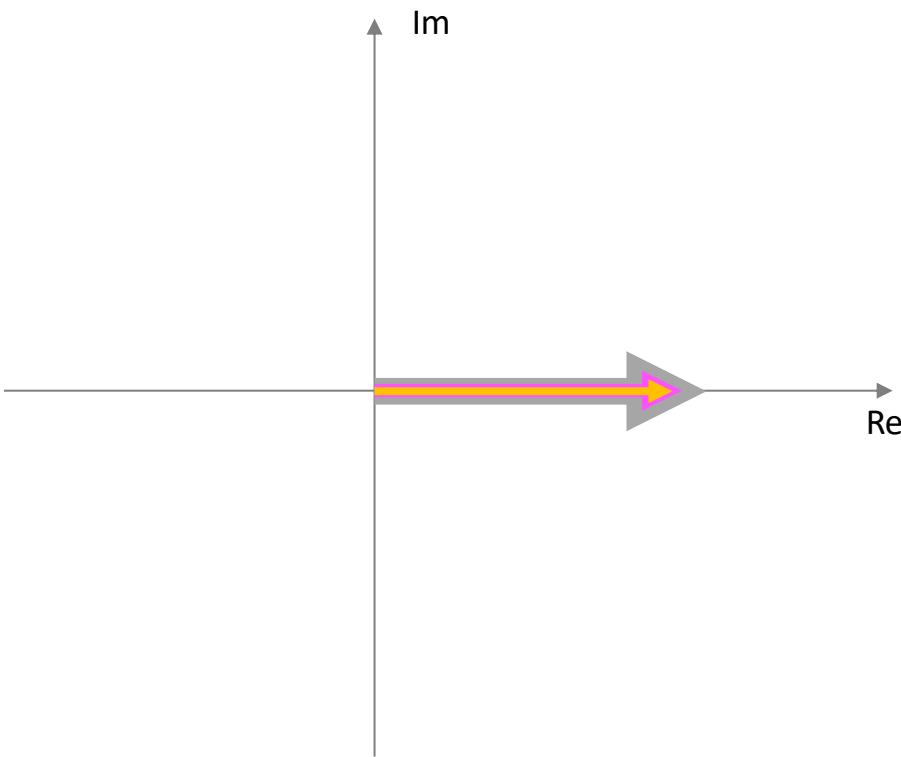
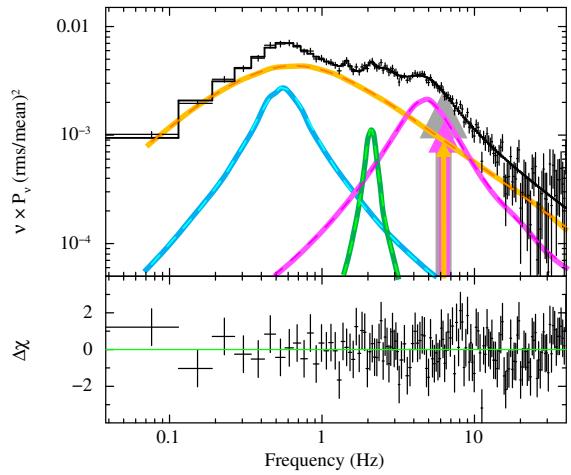
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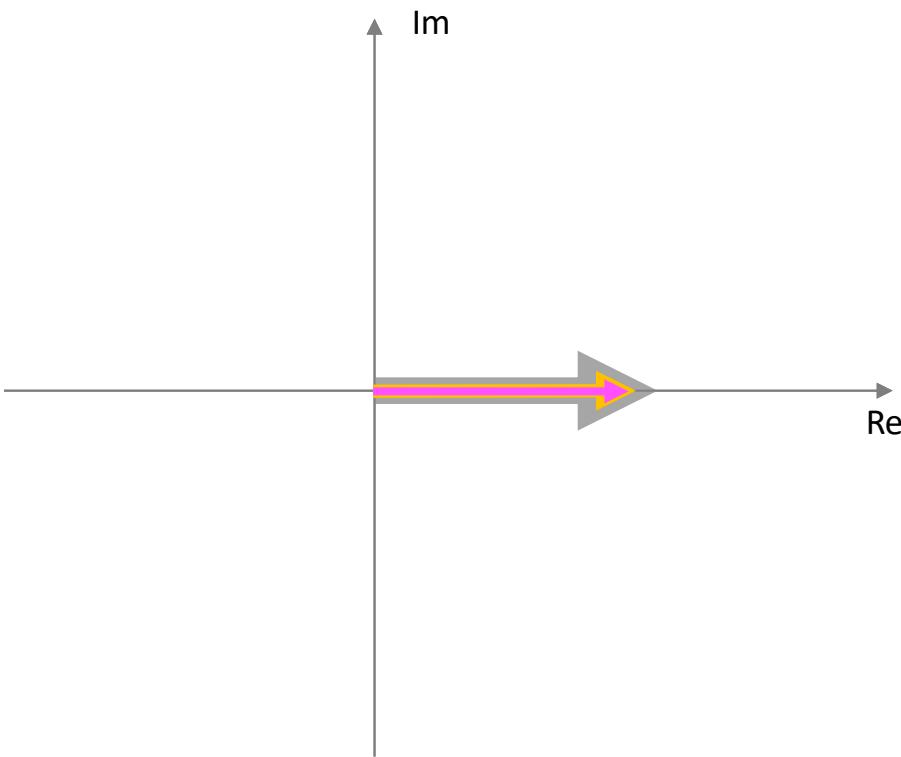
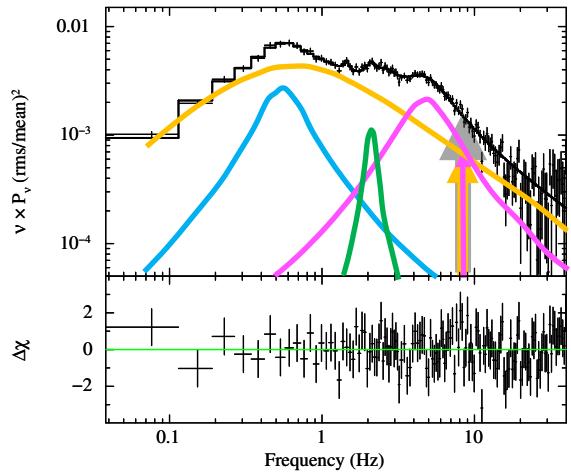
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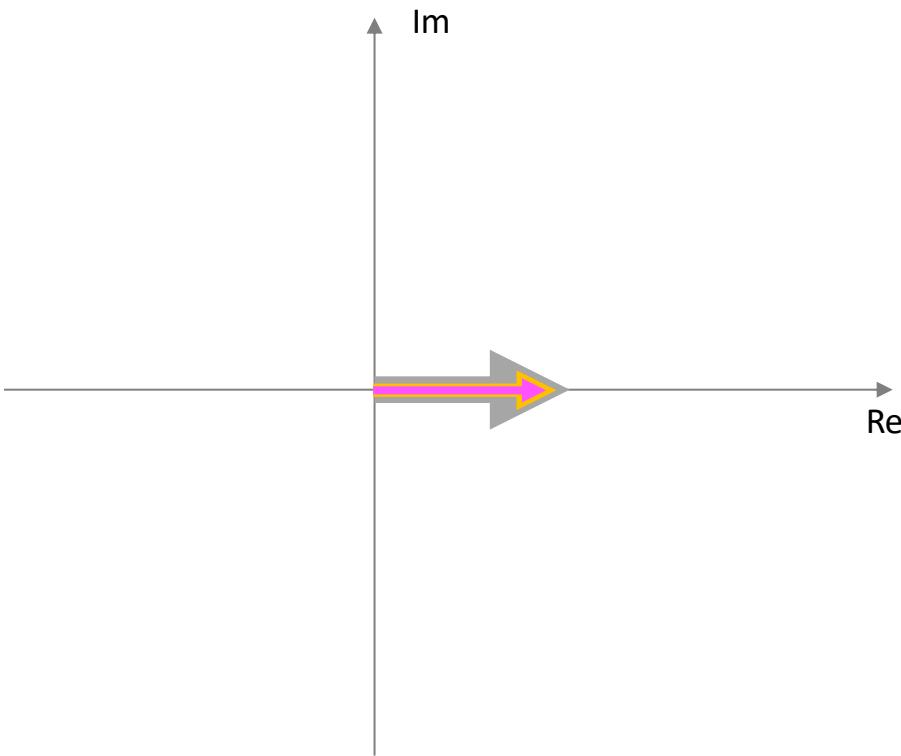
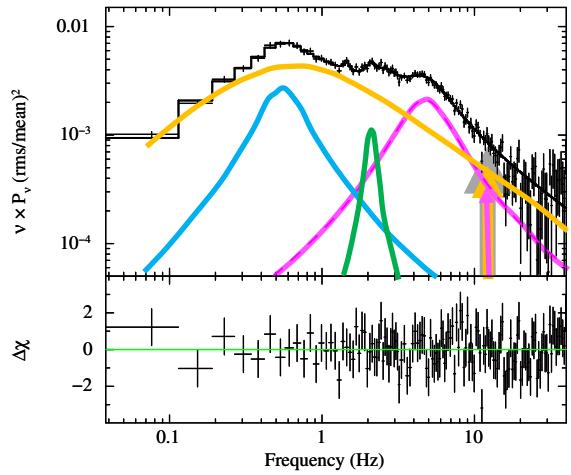
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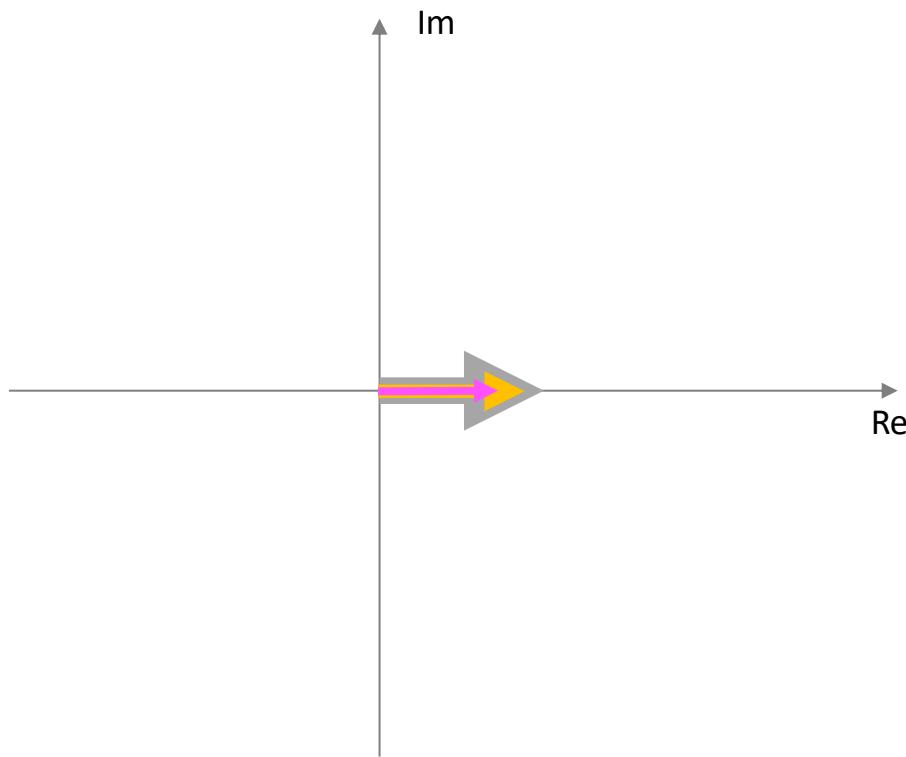
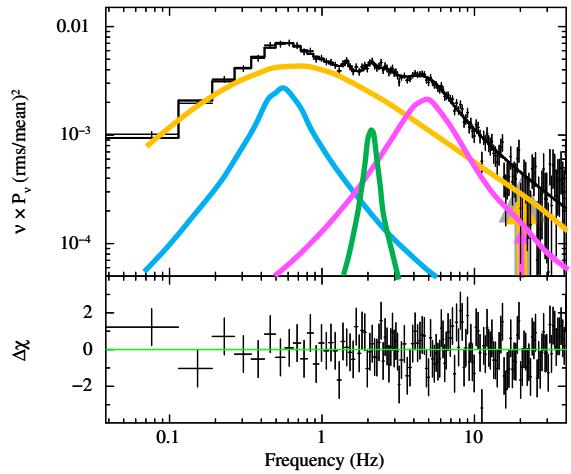
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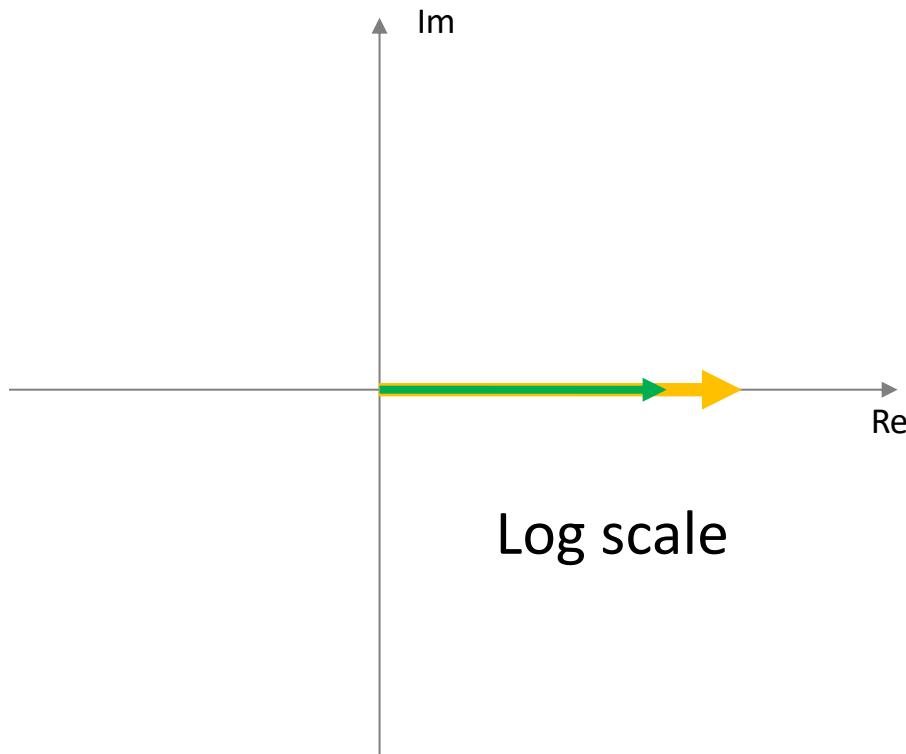
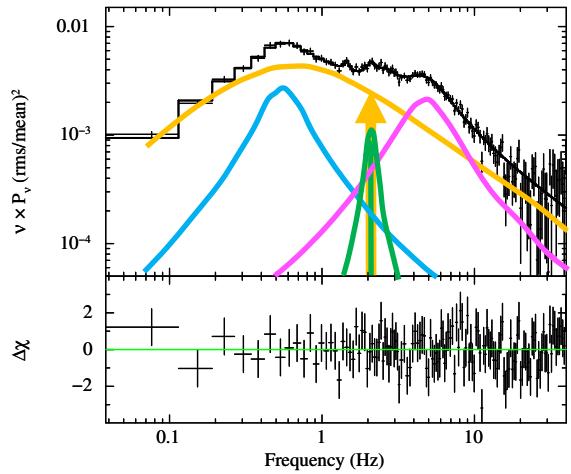
Power spectrum



Power spectrum

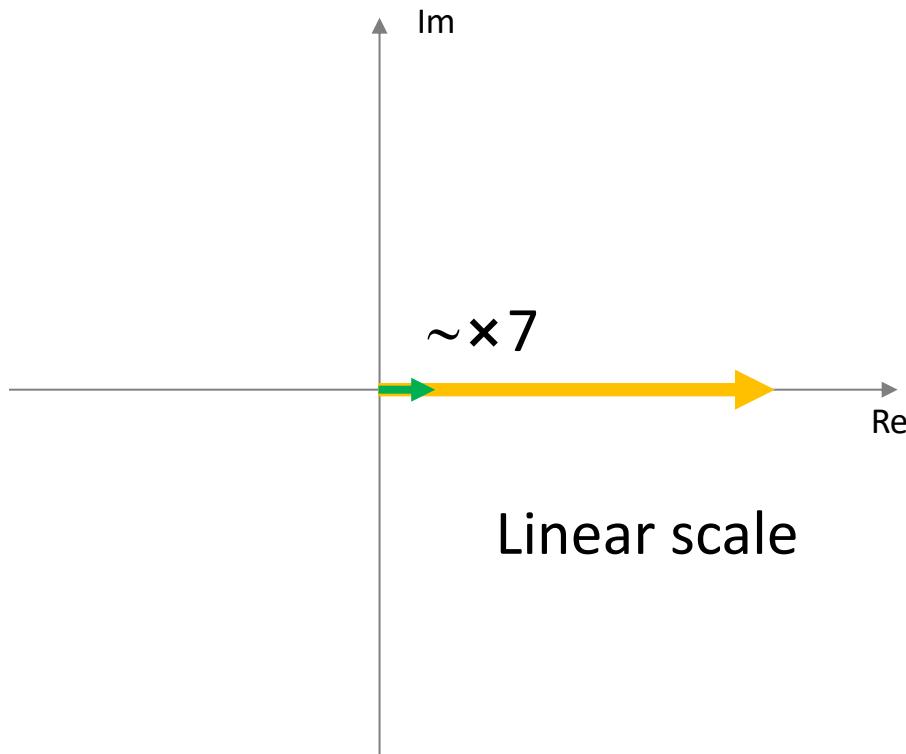
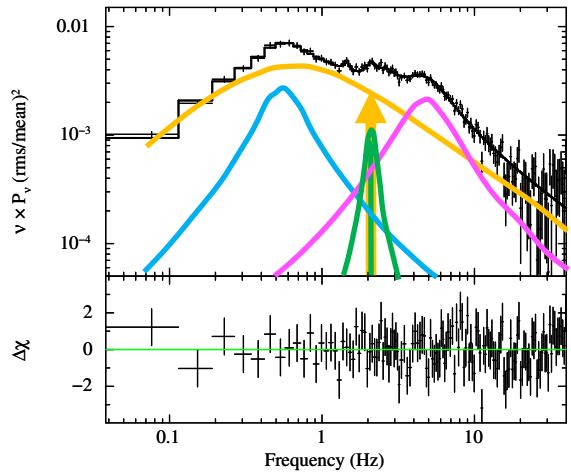


Power spectrum

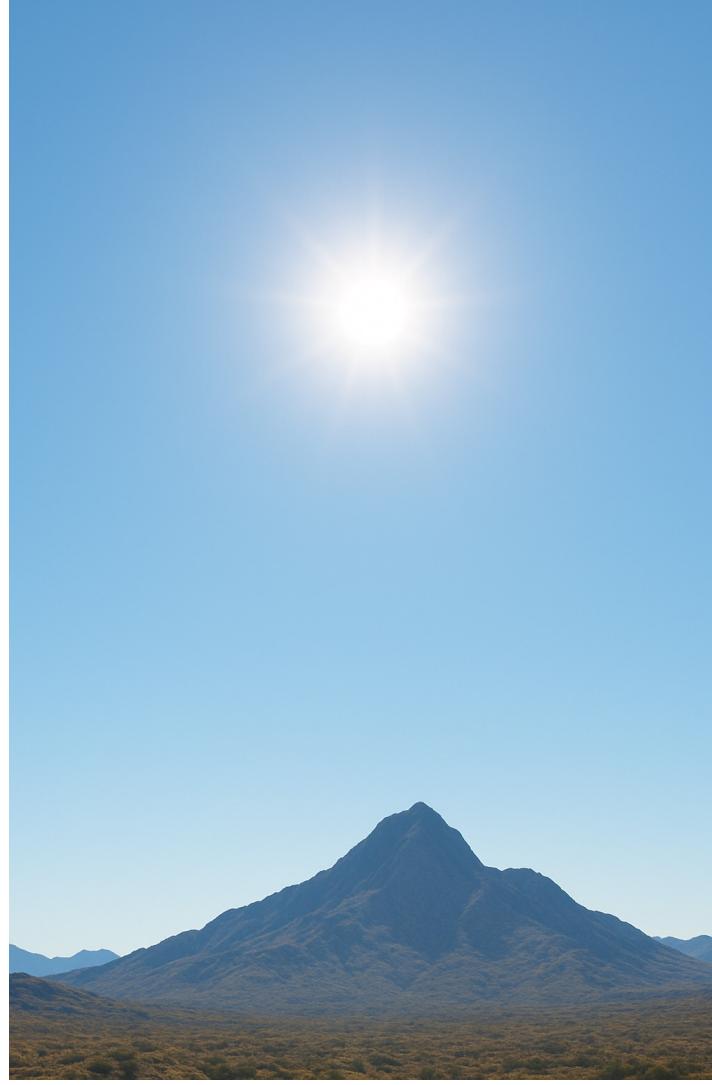


Log scale

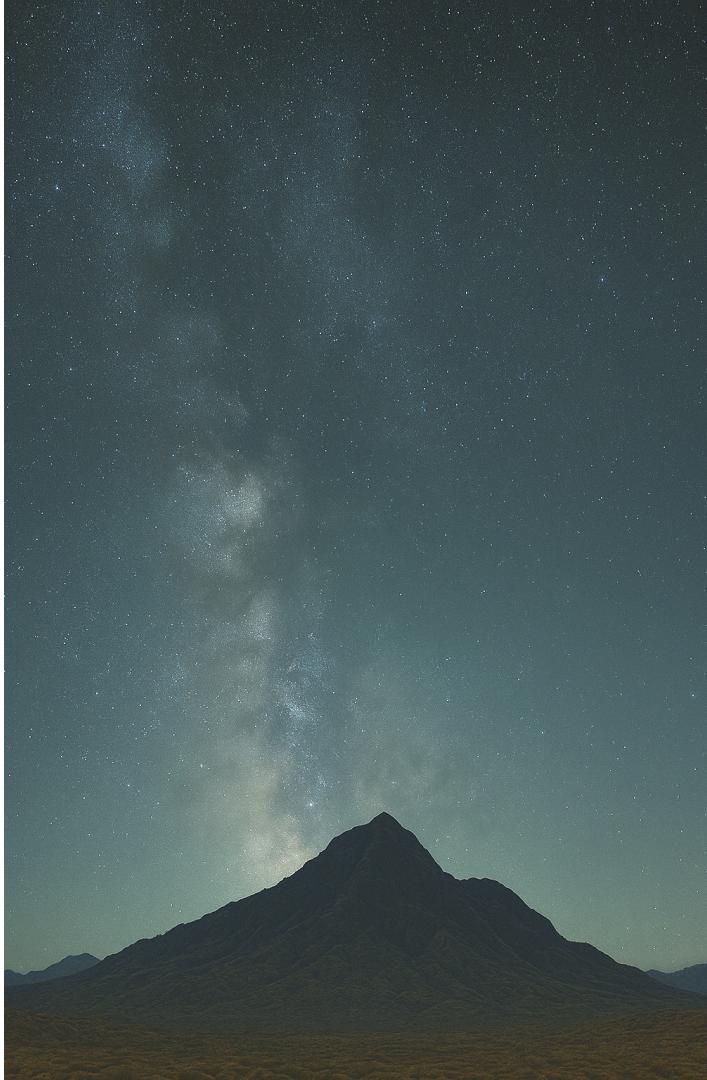
Power spectrum



Linear scale



Sirius is $\sim \times 10$
weaker than blue sky



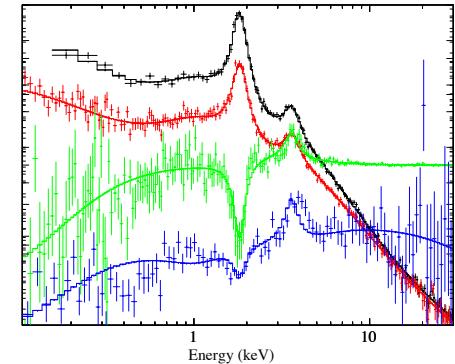
Lorentzians in the power spectrum → Lorentzians in the cross spectrum

Assumptions:

1. PDS in each band is a linear combination of Lorentzians
2. Each Lorentzian has the same centroid frequency and FWHM in all energy bands:

$$G_{xx} := \sum_{i=1}^n (G_{xx})_i, \text{ with } (G_{xx})_i(\nu) = A_i^2 L(\nu; \nu_{0,i}, \Delta_i)$$

$$G_{yy} := \sum_{i=1}^n (G_{yy})_i, \text{ with } (G_{yy})_i(\nu) = B_i^2 L(\nu; \nu_{0,i}, \Delta_i)$$



Lorentzians in the power spectrum → Lorentzians in the cross spectrum

Assumptions:

3. Each Lorentzians is perfectly coherent in two energy bands:

$$(\gamma_{xy}^2)_i(\nu) = \frac{|(G_{xy})_i(\nu)|^2}{(G_{xx})_i(\nu)(G_{yy})_i(\nu)} = 1$$

4. Any two Lorentzian that overlap in frequency are incoherent to one another:

If $X(\nu) = X_1(\nu) + X_2(\nu)$ then $\langle X_1(\nu)X_2^*(\nu) \rangle = 0$

(and the same for the cross spectra)

Lorentzians in the power spectrum → Lorentzians in the cross spectrum

$$\text{Re}[G_{xy}(\nu)] = \sum_{i=1}^n C_i^2 L(\nu; \nu_{0,i}, \Delta_i) \cos [\Delta\phi_i(\nu)]$$

$$\text{Im}[G_{xy}(\nu)] = \sum_{i=1}^n C_i^2 L(\nu; \nu_{0,i}, \Delta_i) \sin [\Delta\phi_i(\nu)]$$

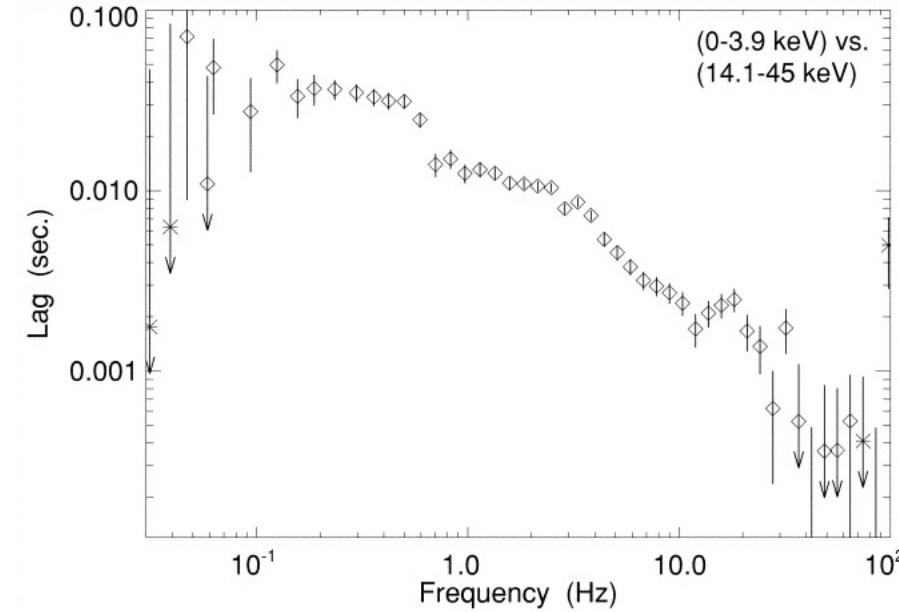
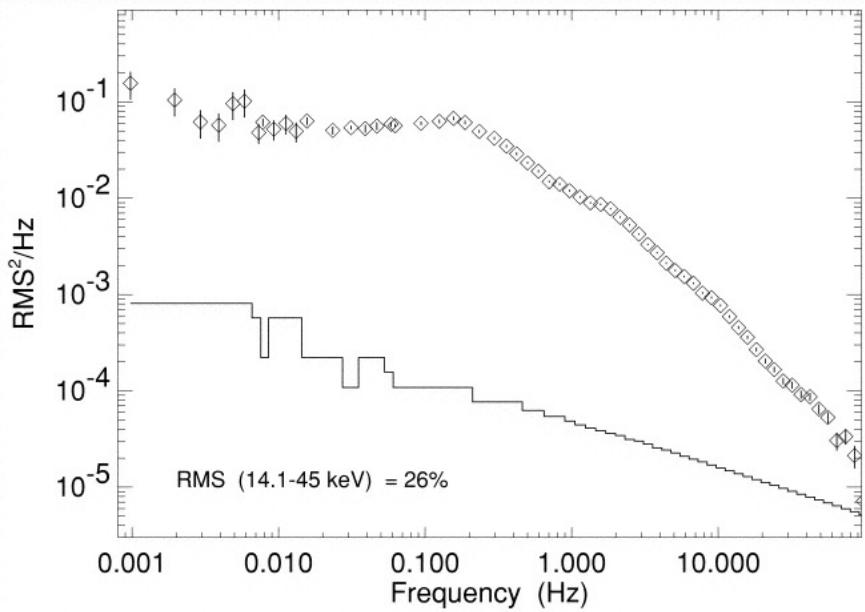
We need to assume a functional form for $\Delta\phi_i(\nu)$

Out of the infinite possibilities, the two simplest cases:

$$\Delta\phi_i(\nu) = \begin{cases} 2\pi k_i & \text{constant phase lags (time lags } \propto \nu^{-1}\text{)} \\ 2\pi k_i \nu & \text{constant time lags (phase lags } \propto \nu\text{)} \end{cases}$$

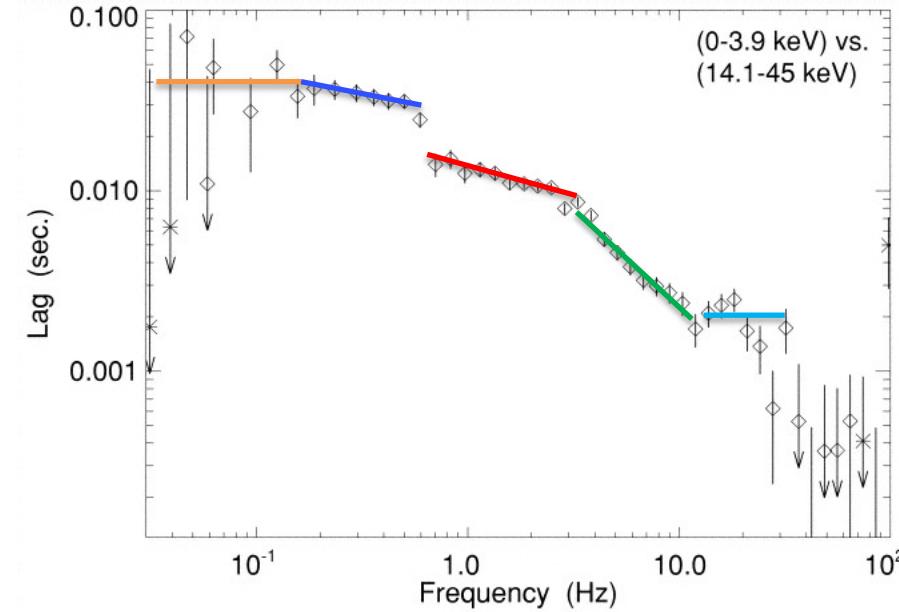
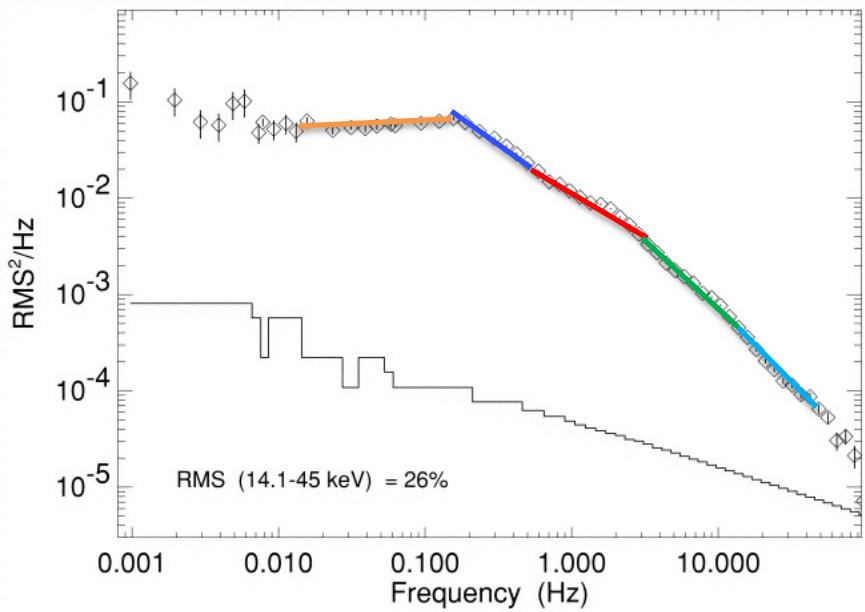
Phase-lag shelves match changes in the PDS

Cyg X-1



Phase-lag shelves match changes in the PDS

Cyg X-1



!!!!!! THIS IS JUST A MODEL !!!!!

AND THEREFORE, IT IS WRONG (AS ALL MODELS!!!)

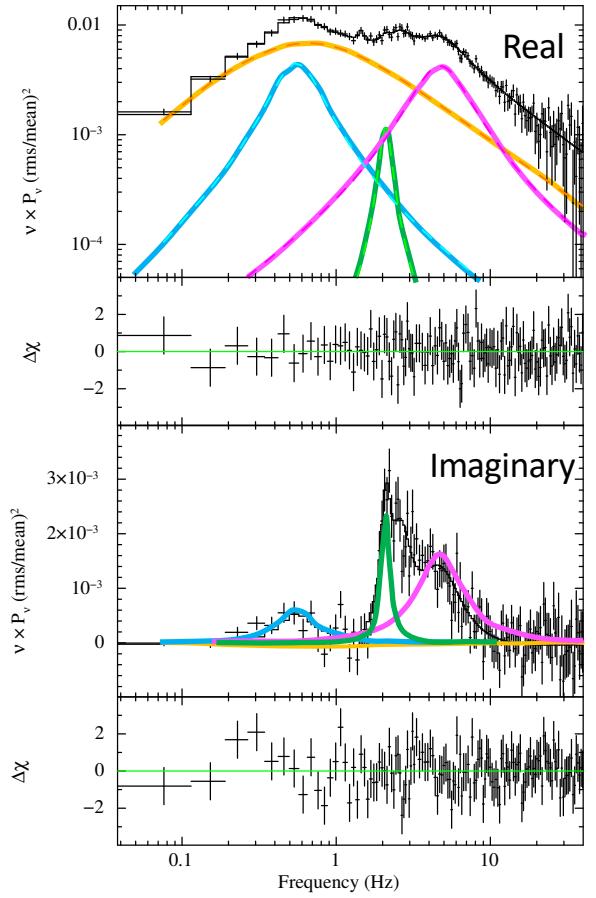
YET IT CAN BE USEFUL^(*)

^(*) see quote by George Box

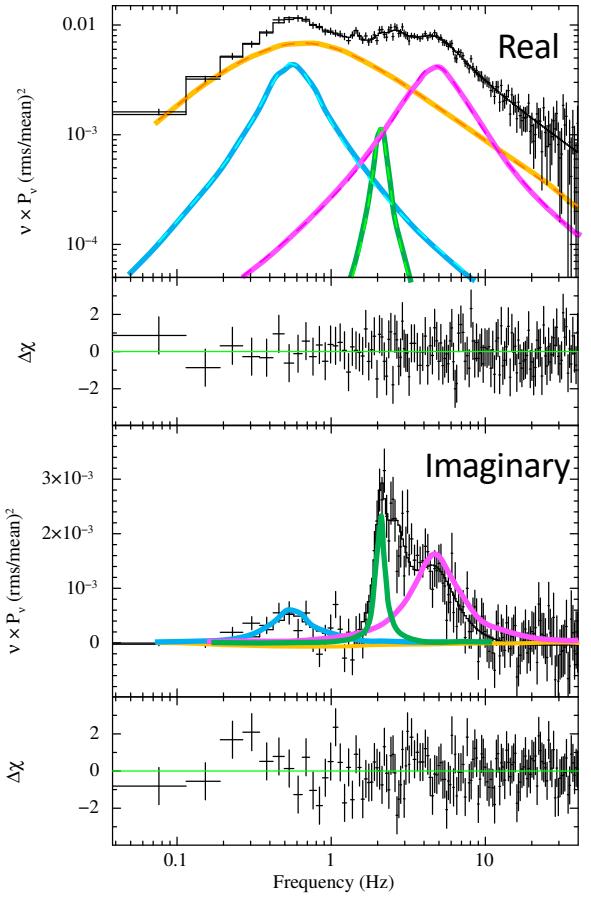
As any model, it should be judged on the basis of

1. How good it reproduces the data
2. What predictions it makes (how many; how good they reproduce the data; potential for new discoveries)
3. Does it do better/worse than other models?

Cross spectrum (CS)

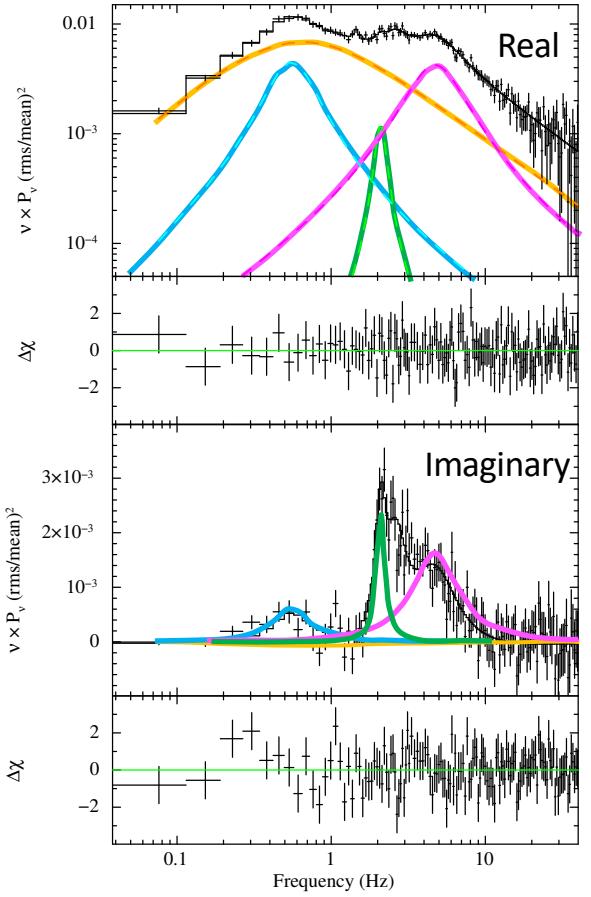


Cross spectrum

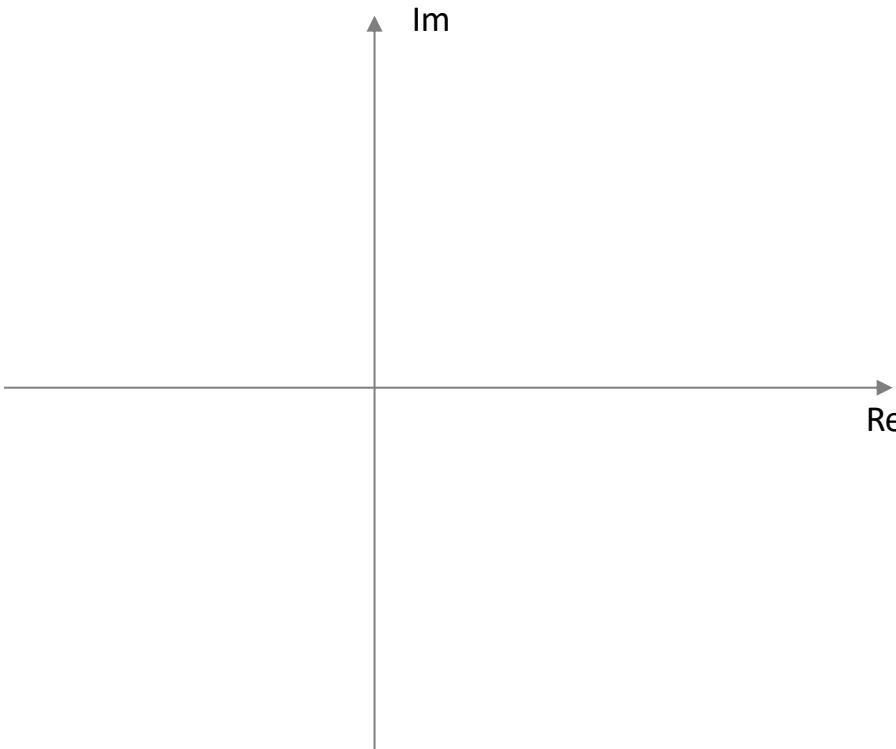


$$C(\nu) = \langle S^*(\nu)H(\nu) \rangle = \\ Re[C(\nu)] + i Im[C(\nu)] =$$

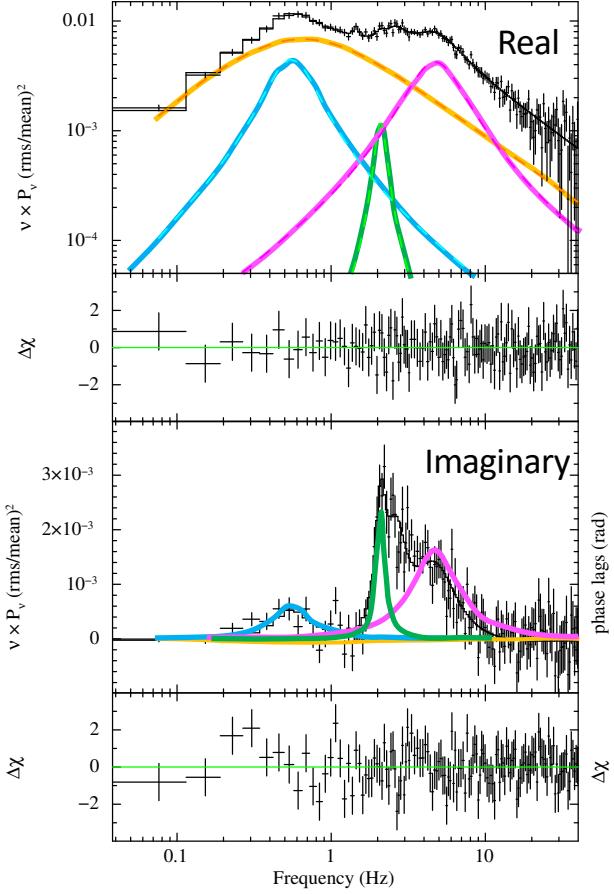
Cross spectrum



$$C(\nu) = \langle S^*(\nu)H(\nu) \rangle = \\ Re[C(\nu)] + i Im[C(\nu)] =$$

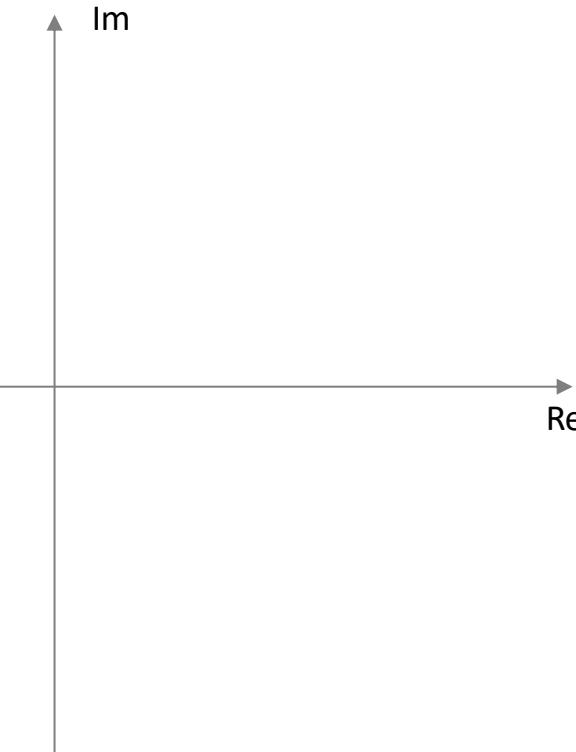
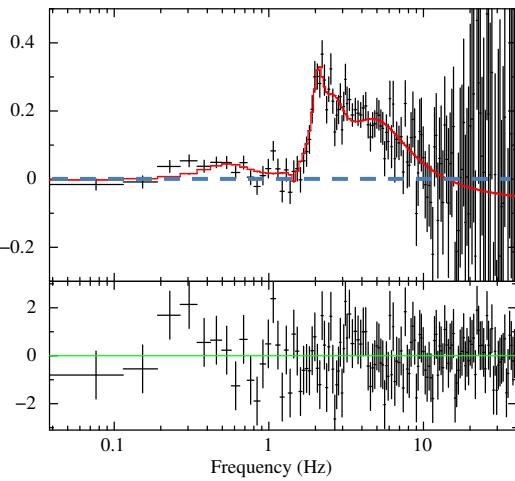


Cross spectrum

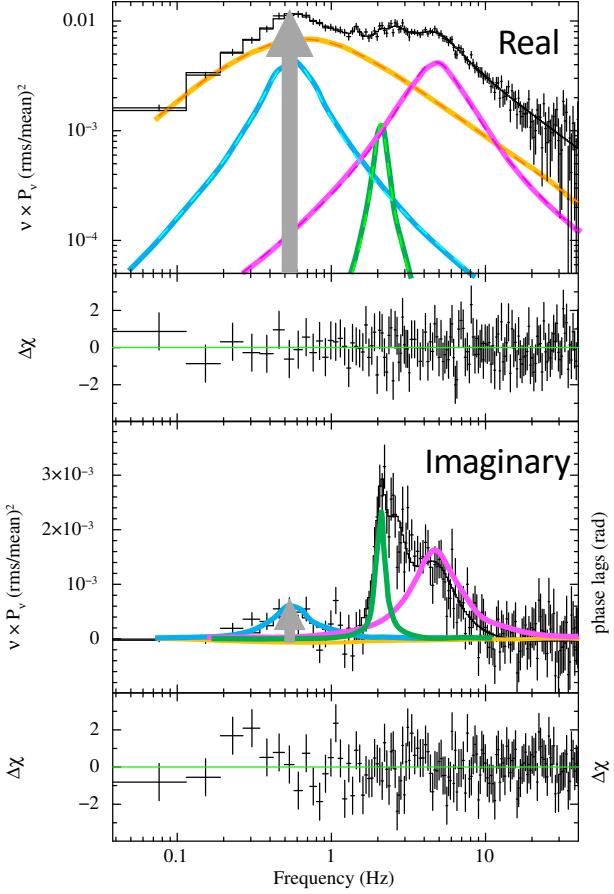


$$C(\nu) = \langle S^*(\nu)H(\nu) \rangle = \\ \text{Re}[C(\nu)] + i \text{Im}[C(\nu)] =$$

The cross spectrum is a
Complex quantity (Re/Im)

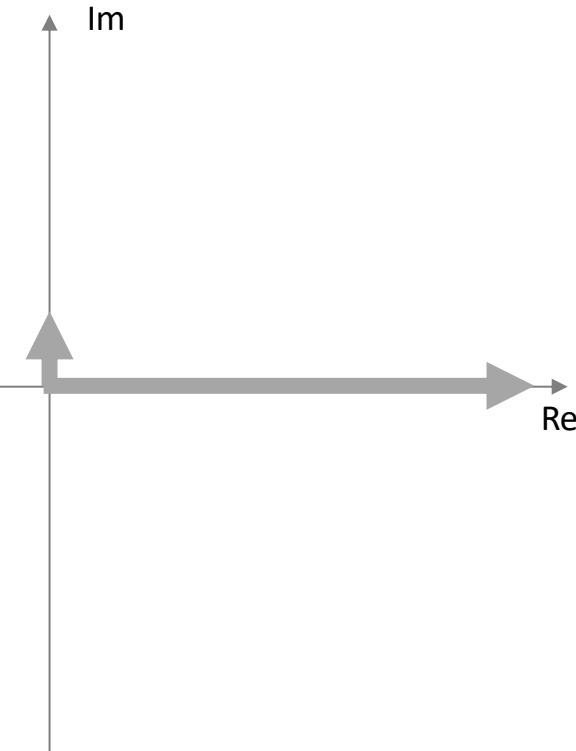
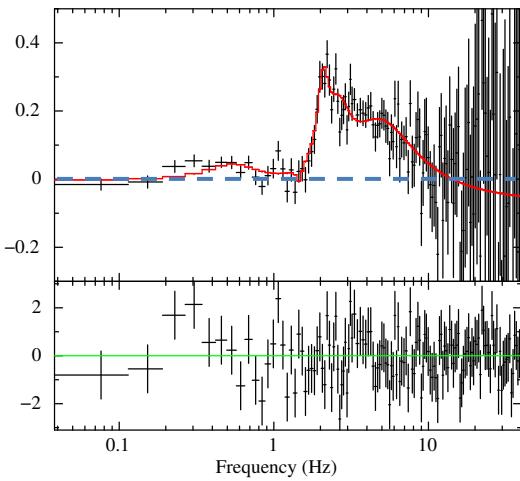


Cross spectrum

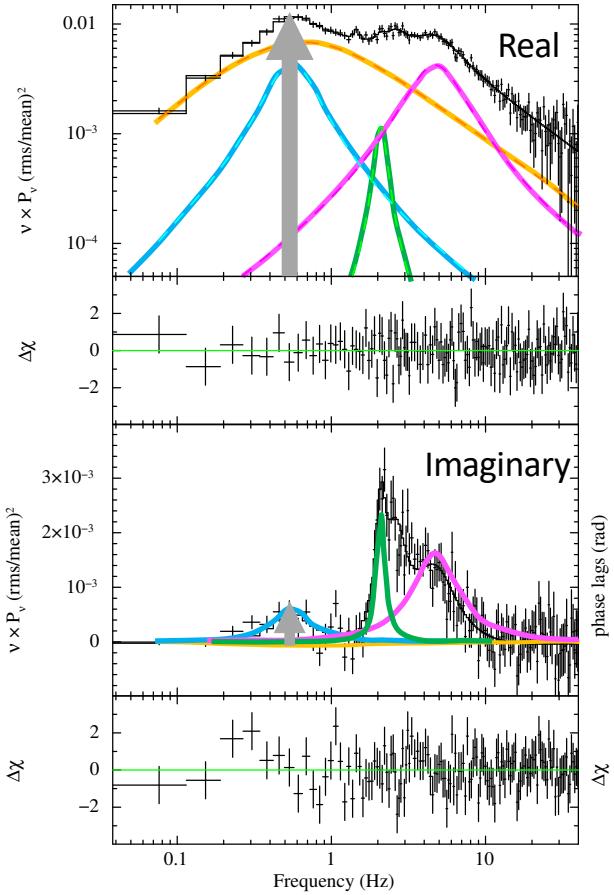


$$C(\nu) = \langle S^*(\nu) H(\nu) \rangle = \\ \text{Re}[C(\nu)] + i \text{Im}[C(\nu)] =$$

The cross spectrum is a
Complex quantity (Re/Im)

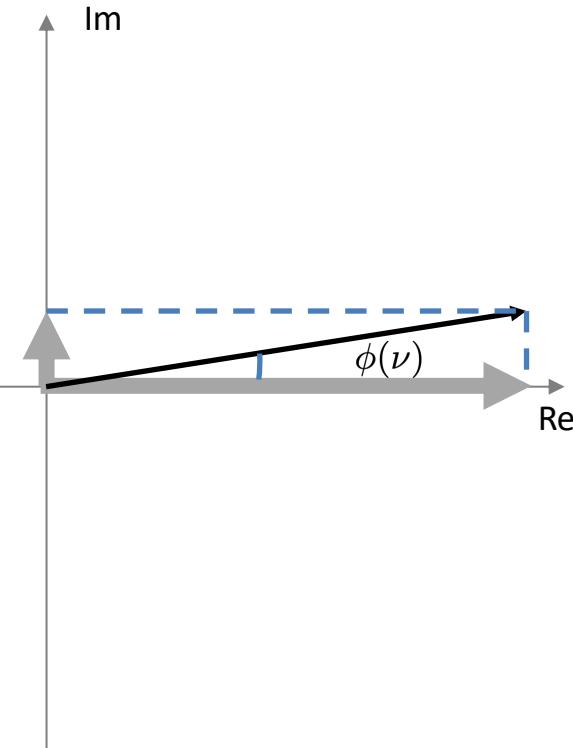
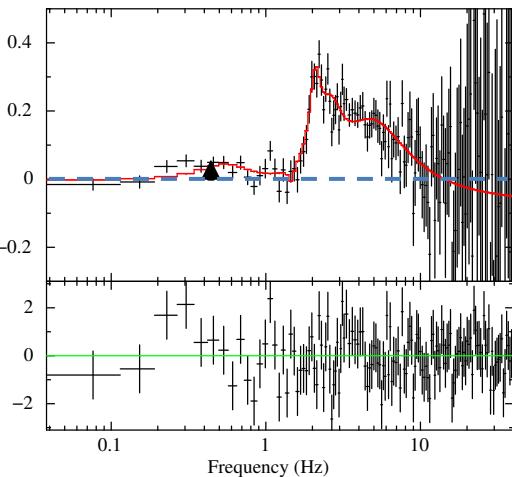


Cross and lag spectrum

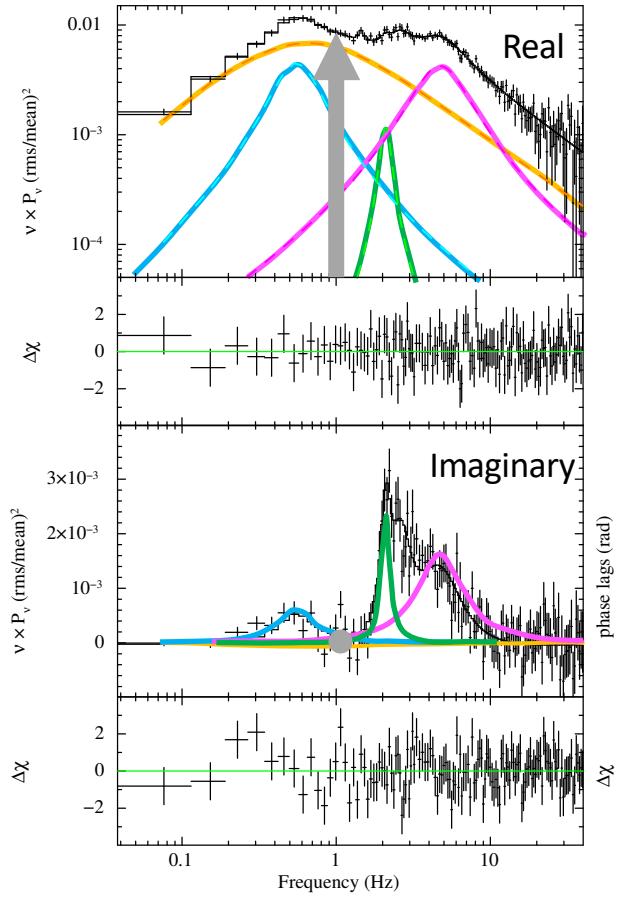


$$\begin{aligned} C(\nu) &= \langle S^*(\nu) H(\nu) \rangle = \\ Re[C(\nu)] + i Im[C(\nu)] &= \\ |S(\nu)| |H(\nu)| e^{i\phi(\nu)} \end{aligned}$$

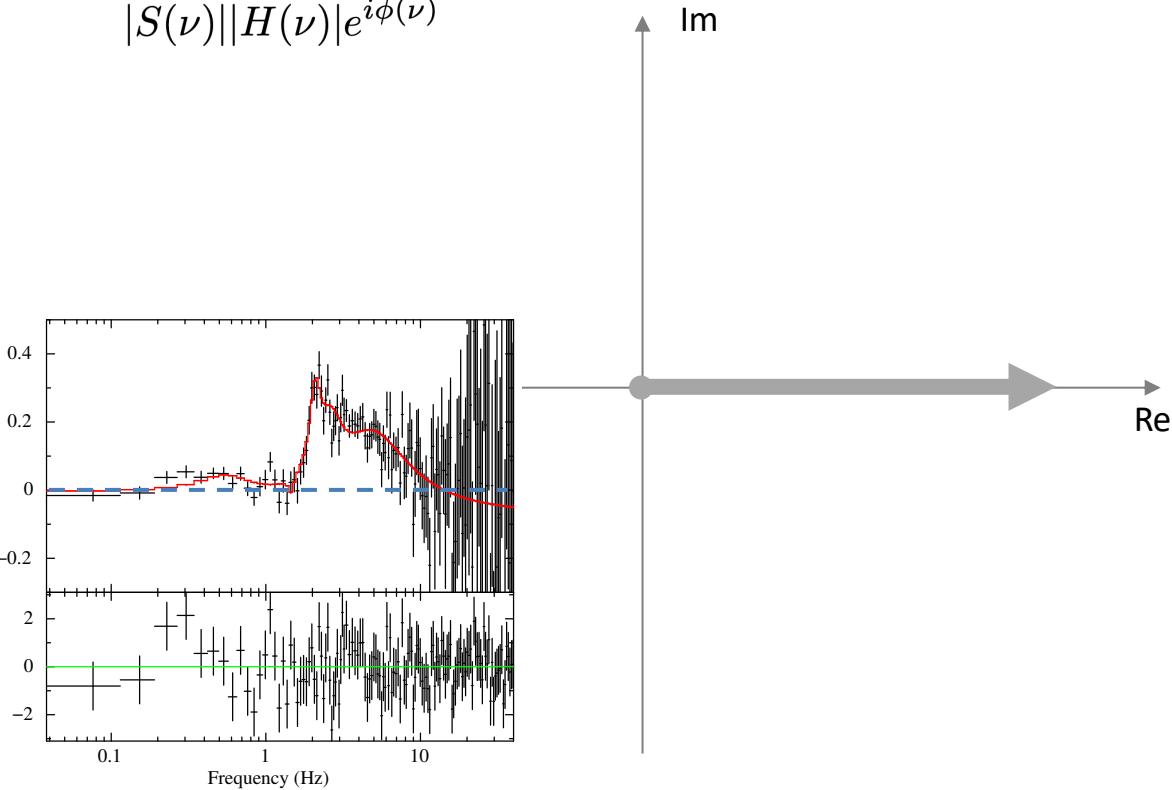
The cross spectrum is a
Complex quantity
(Modulus/Phase)



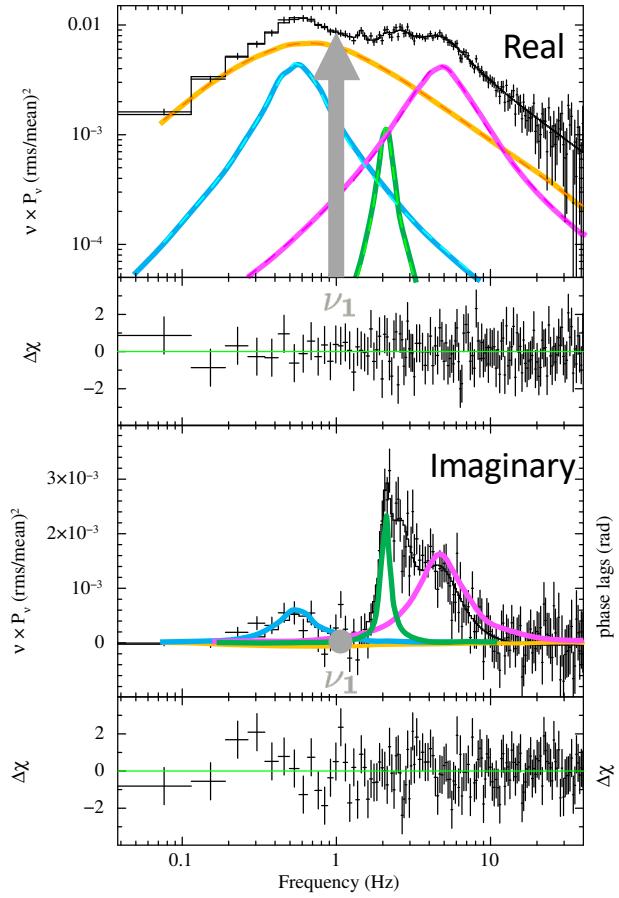
Cross and lag spectrum



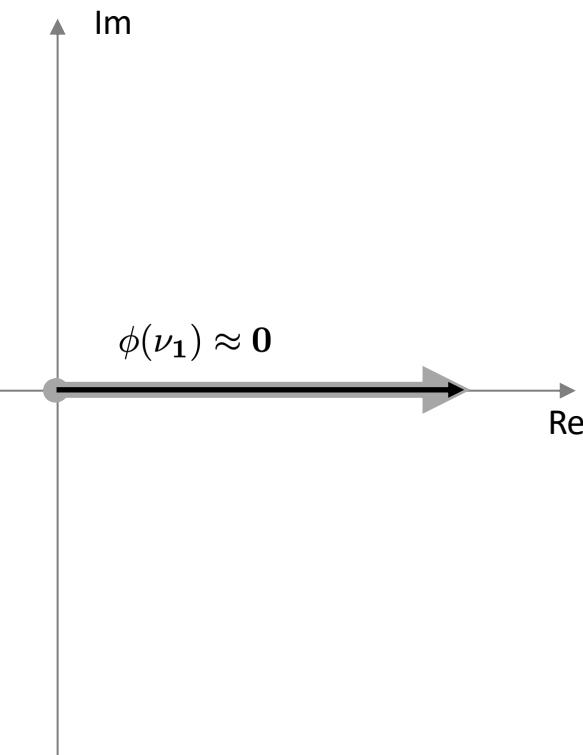
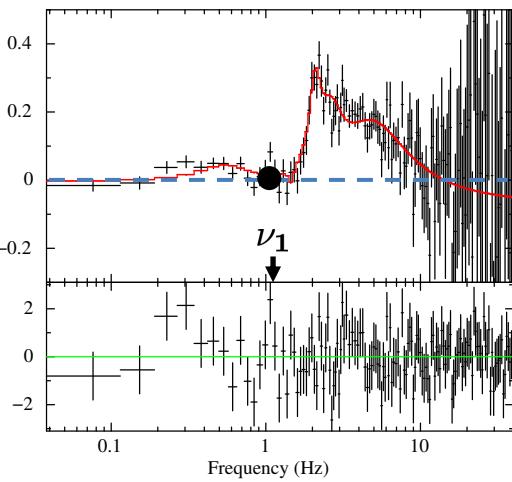
$$\begin{aligned} C(\nu) &= \langle S^*(\nu) H(\nu) \rangle = \\ Re[C(\nu)] + i Im[C(\nu)] &= \\ |S(\nu)| |H(\nu)| e^{i\phi(\nu)} \end{aligned}$$



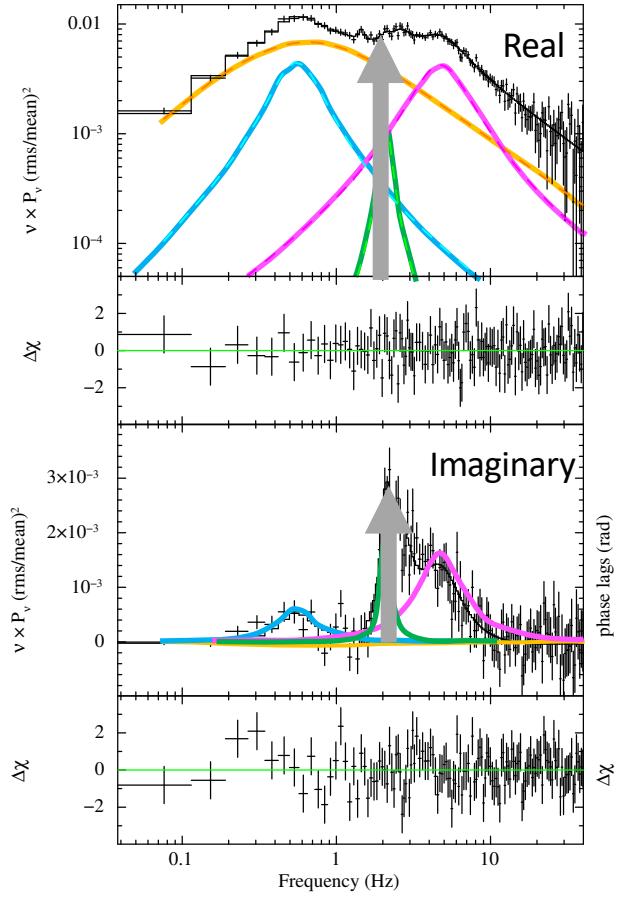
Cross and lag spectrum



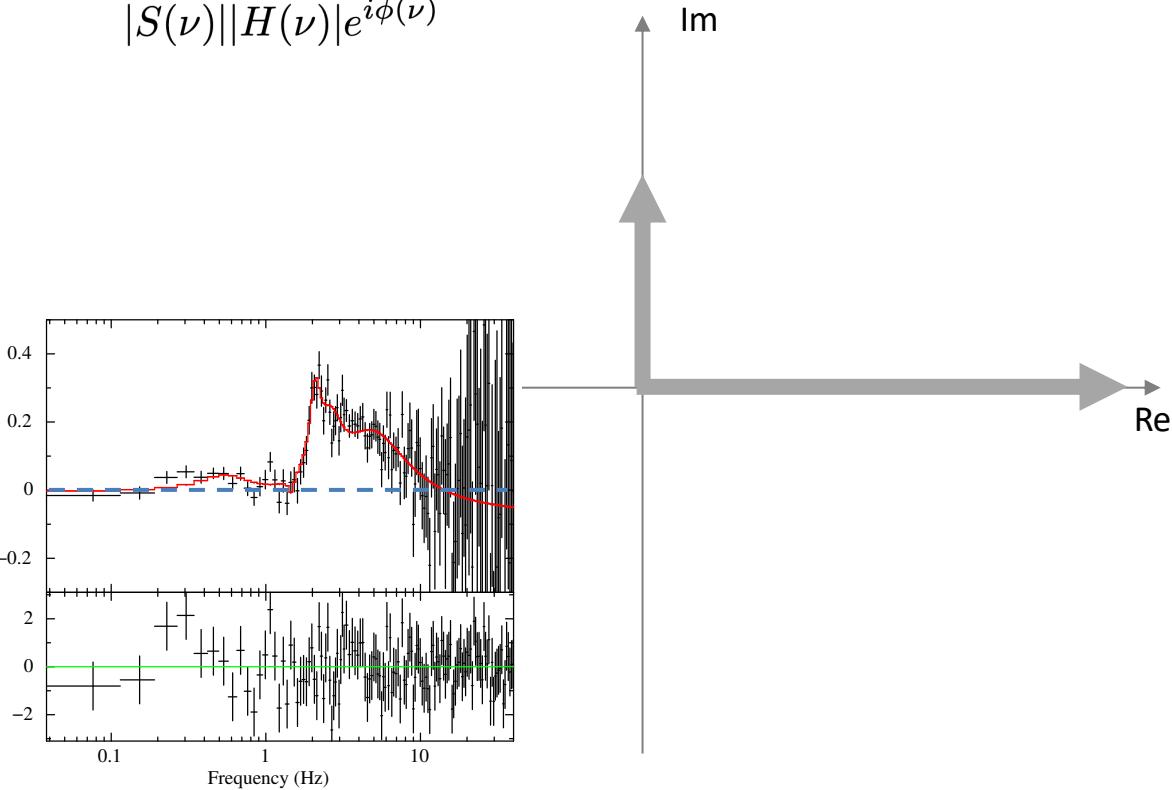
$$C(\nu) = \langle S^*(\nu)H(\nu) \rangle = \\ Re[C(\nu)] + i Im[C(\nu)] = \\ |S(\nu)||H(\nu)|e^{i\phi(\nu)}$$



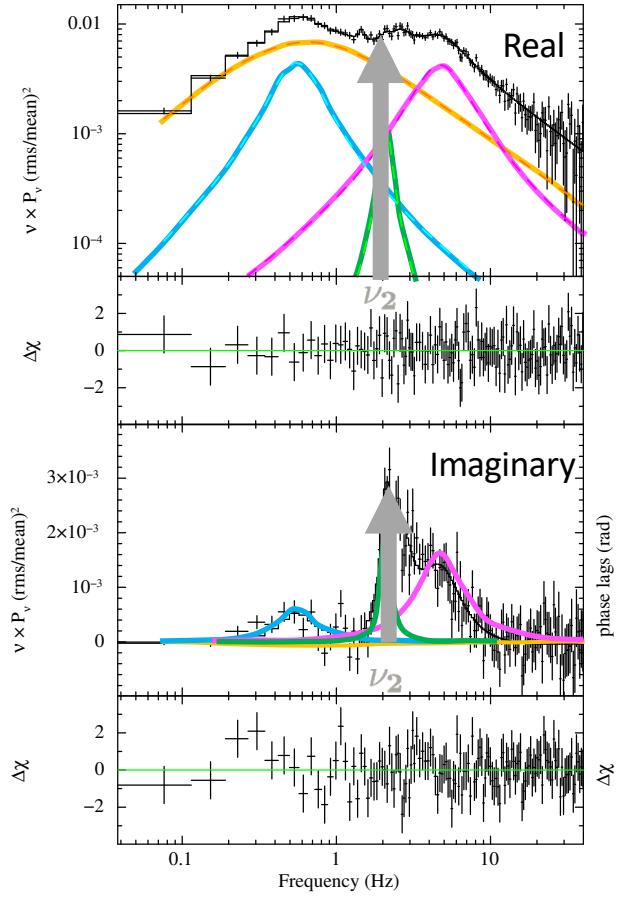
Cross and lag spectrum



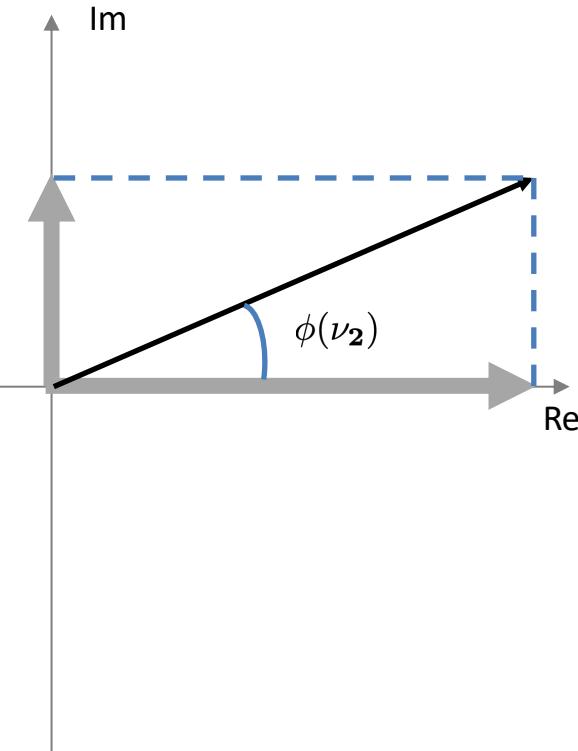
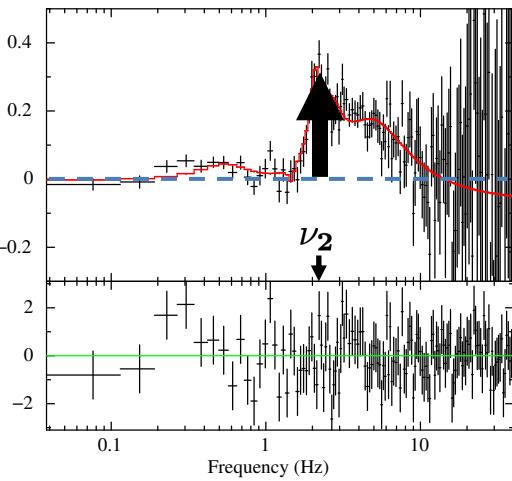
$$C(\nu) = \langle S^*(\nu)H(\nu) \rangle = \\ Re[C(\nu)] + i Im[C(\nu)] = \\ |S(\nu)||H(\nu)|e^{i\phi(\nu)}$$



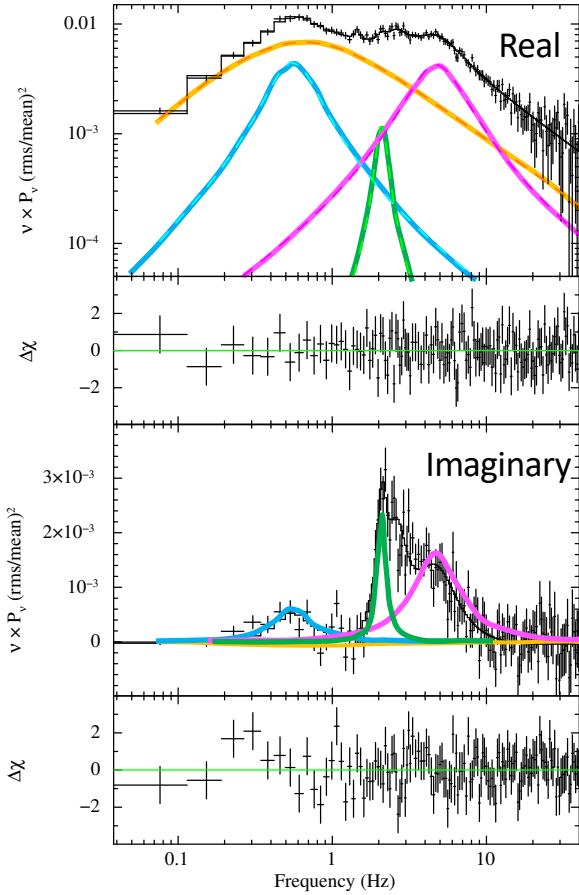
Cross and lag spectrum



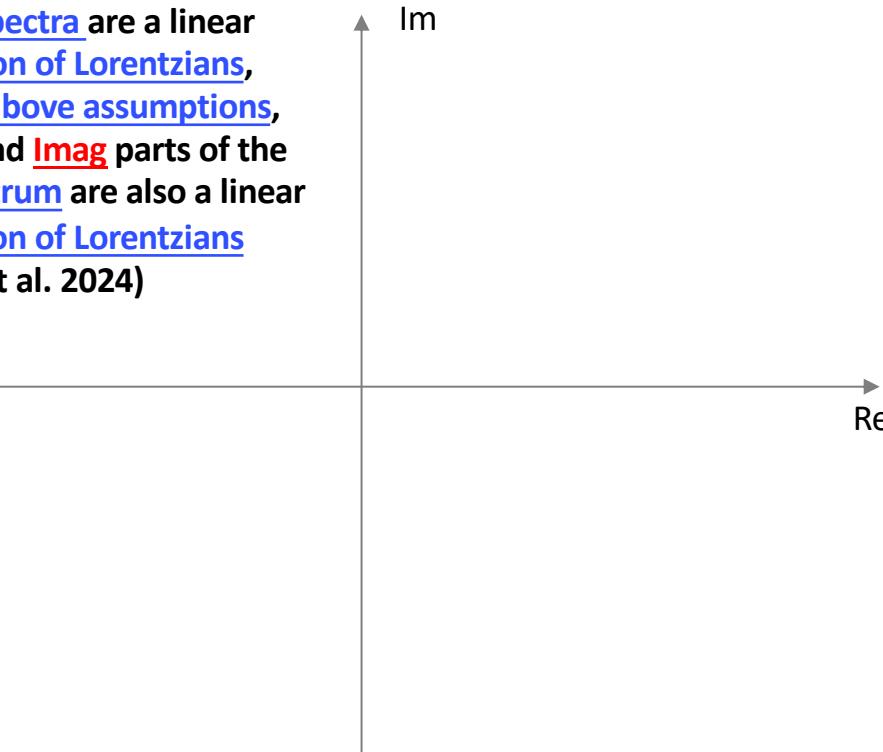
$$C(\nu) = \langle S^*(\nu)H(\nu) \rangle = \\ Re[C(\nu)] + i Im[C(\nu)] = \\ |S(\nu)||H(\nu)|e^{i\phi(\nu)}$$



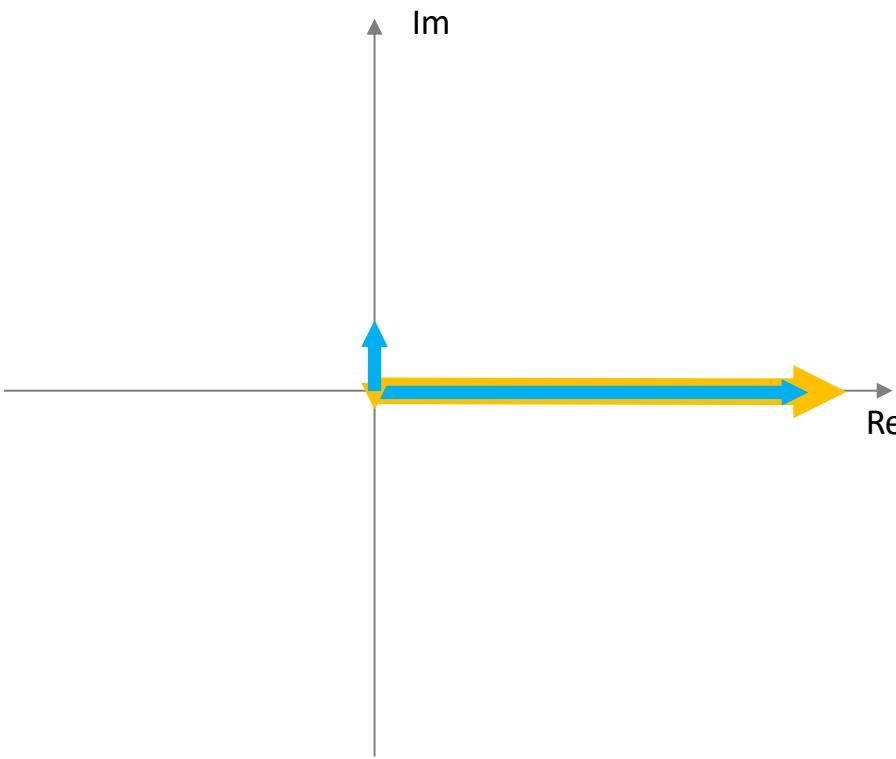
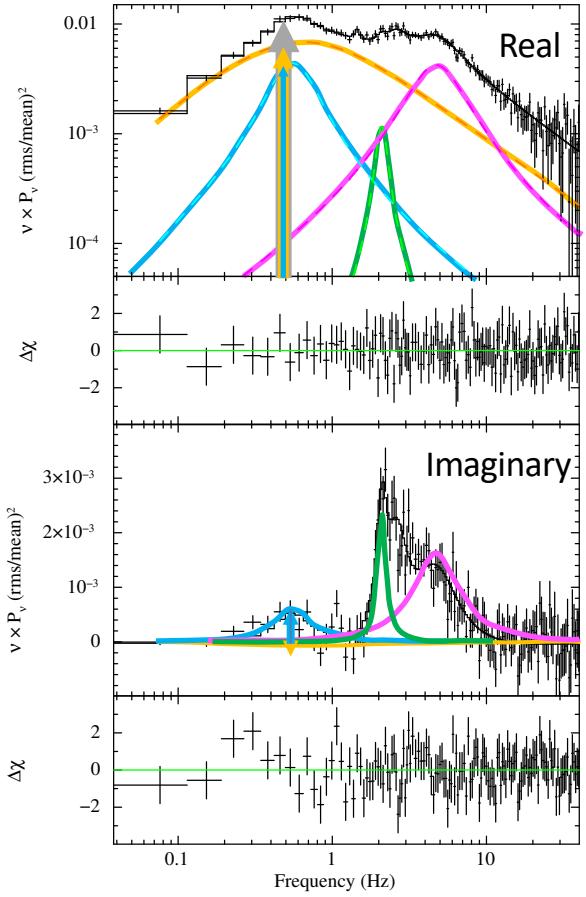
Cross spectrum



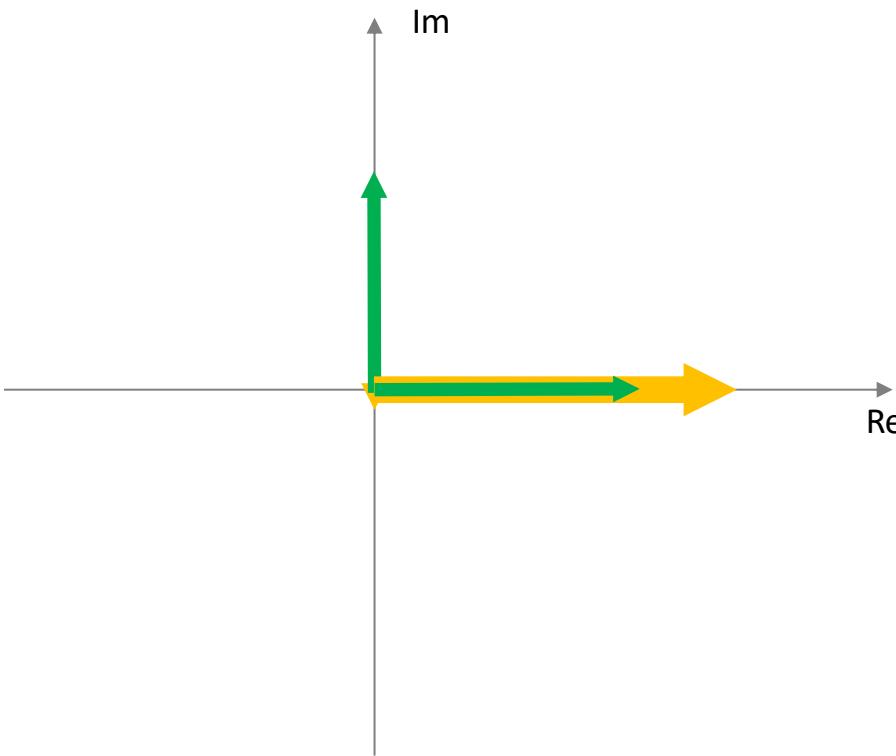
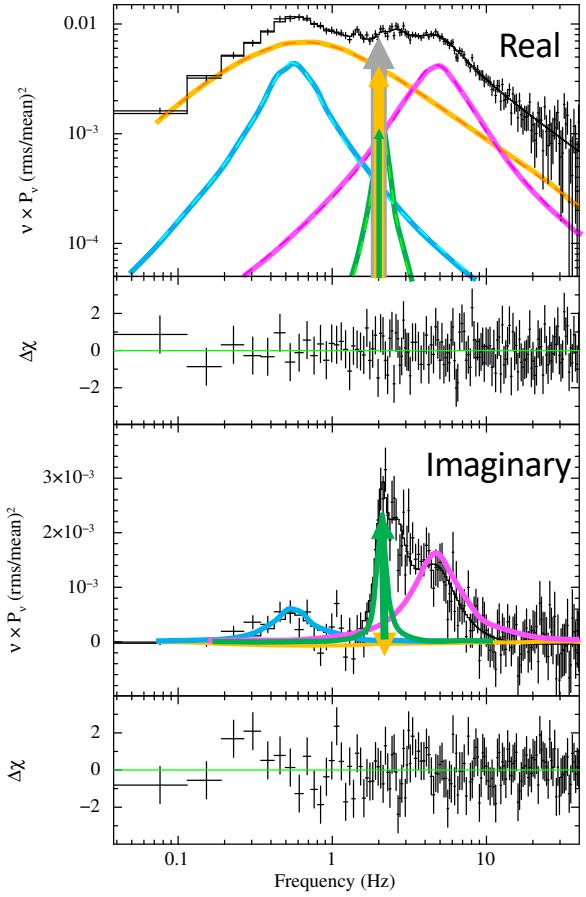
If [power spectra](#) are a linear combination of Lorentzians, given the [above assumptions](#), the **Real** and **Imag** parts of the [cross spectrum](#) are also a linear combination of Lorentzians
(Méndez et al. 2024)



Cross spectrum



Cross spectrum



Summary of discoveries from fitting the cross spectrum

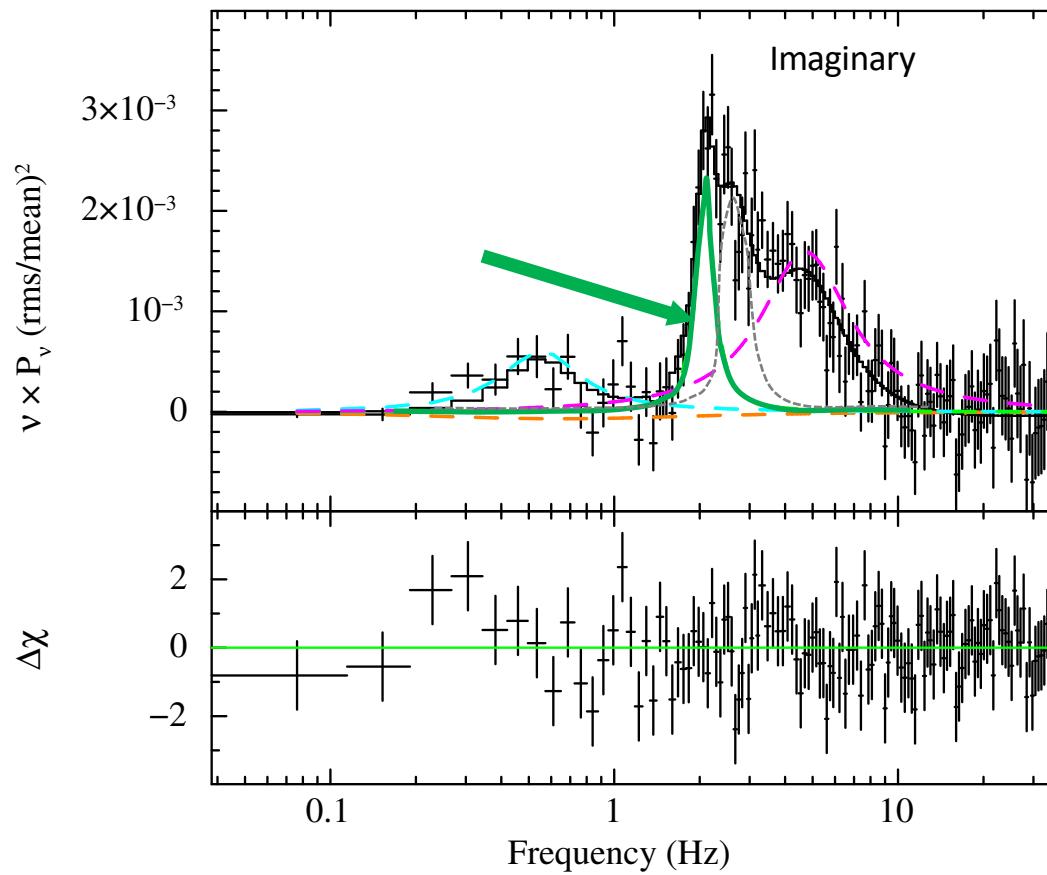
1. New (previously undetected) “imaginary” QPOs (**iQPOs**) in MAXI J1820+070, Cyg X-1, MAXI J1348–630, AT2019wey and Swift J1727.8–1613.
2. A mHz QPO in intermediate states of Cyg X-1 that is not detected in the PDS.
3. MAXI J1820+070, MAXI J1348–630, AT2019wey: The properties of the **iQPO** are consistent with those of the type-C QPO.
4. MAXI J1820+070, MAXI J1348–630, Swift J1727.8–1613 (transient sources): The **iQPO** appears in the soft-to-hard transition.

It remains to be tested whether in Cyg X-1, AT2019wey (“persistent” sources) the **iQPOs** appear when the source evolves from soft to hard, from hard to soft or both.

Summary of discoveries from fitting the cross spectrum

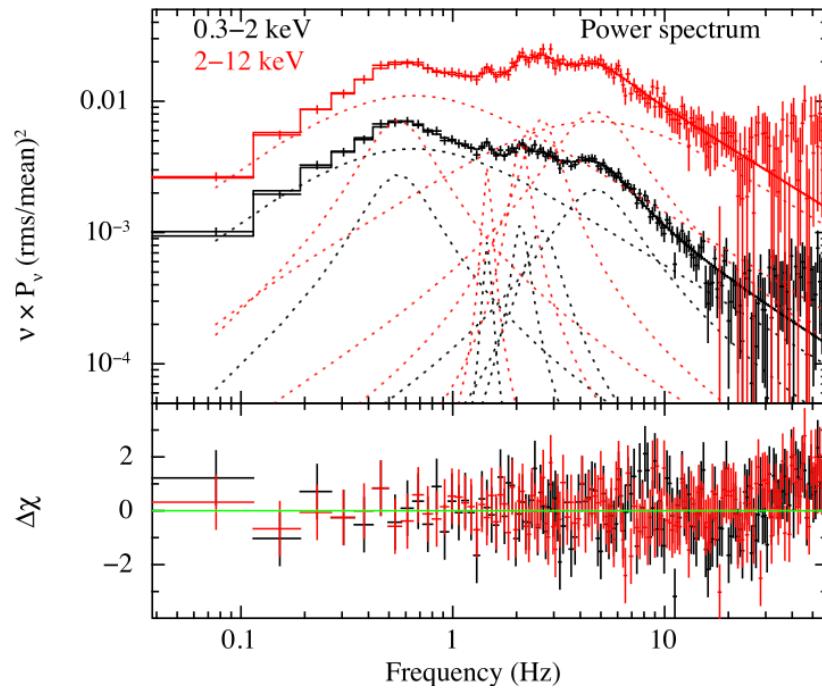
5. Swift J1727.8–1613, MAXI J1820+070, GX 339–4 : The type-B QPO is present in the HIMS, together with the type-C QPO and the strong broadband noise, before the discrete jet is ejected.
 - ⇒ The type-B QPO cannot be produced by the jet that appears at the HIMS-to-SIMS transition.
6. The apparent energy dependency of the QPO frequency in some sources is due to a shoulder of the QPO with a steeper rms spectrum than that of the main QPO.
7. In Swift J1727.8–1613 and GX 339–4 the type-B QPO is a shoulder of the type-C QPO. Shoulders of the type-C QPO have been seen in GRS 1915+105 in the HIMS.
Are those also type-B QPOs?
8. A mHz QPO in the Be/X-ray binary 1A0535+262.

“Imaginary” QPO in MAXI J1820+070

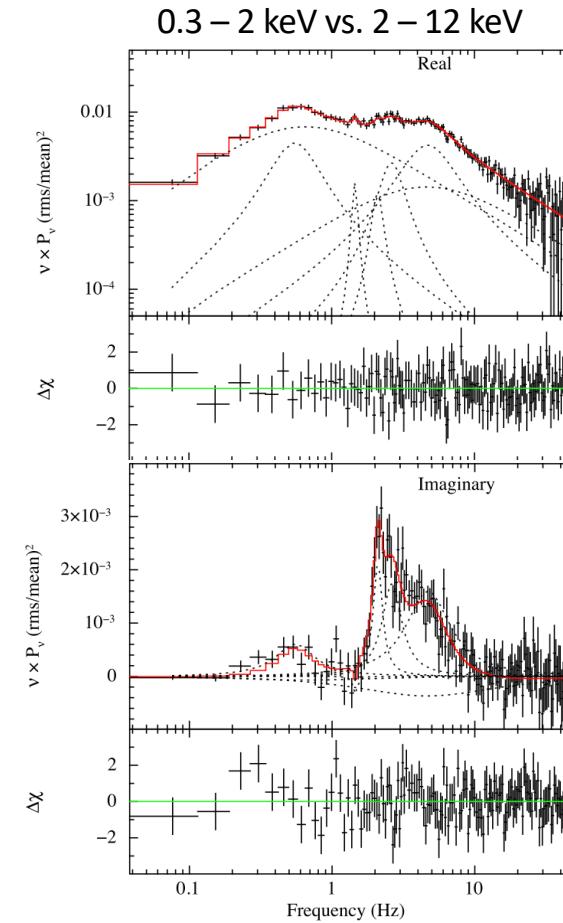


Bellavita et al. 2025

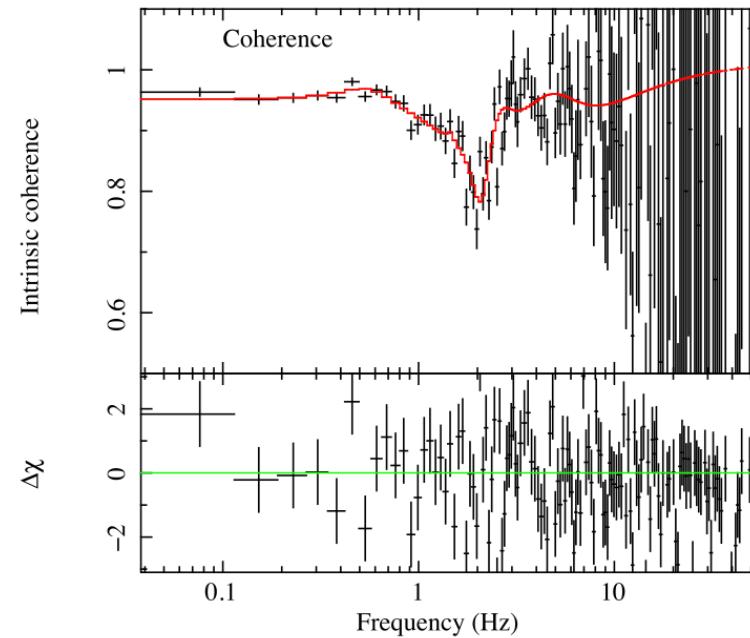
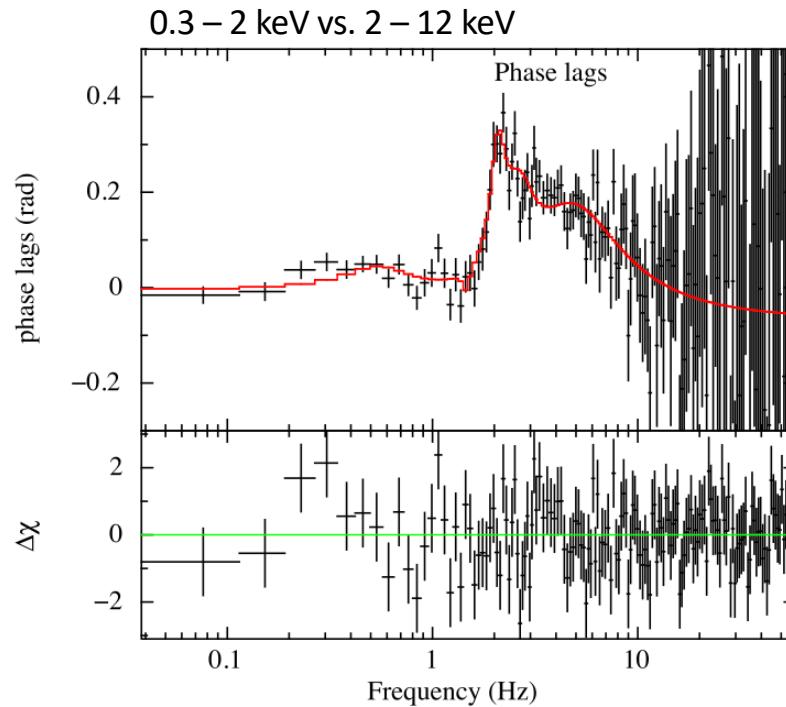
MAXI J1820: Fit power and cross spectra ...



Méndez et al. 2024; Bellavita et al. 2025

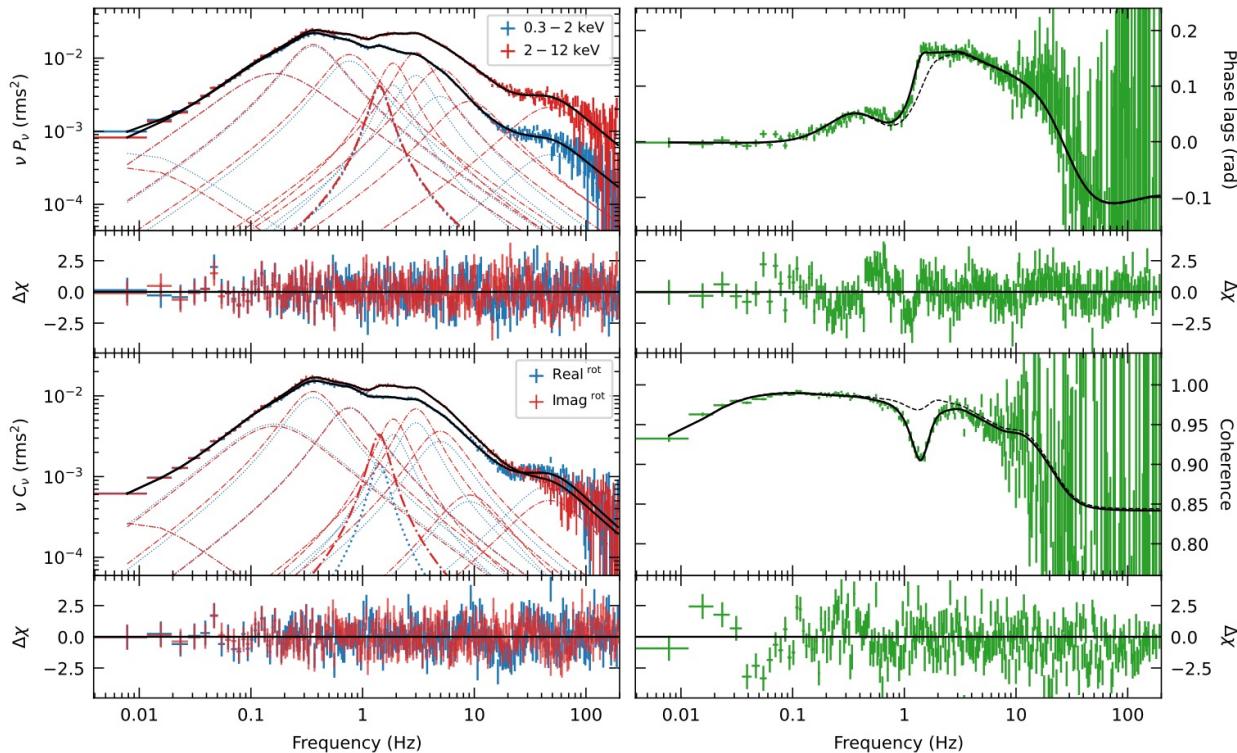


MAXI J1820: ... [predict](#) phase lags and coherence



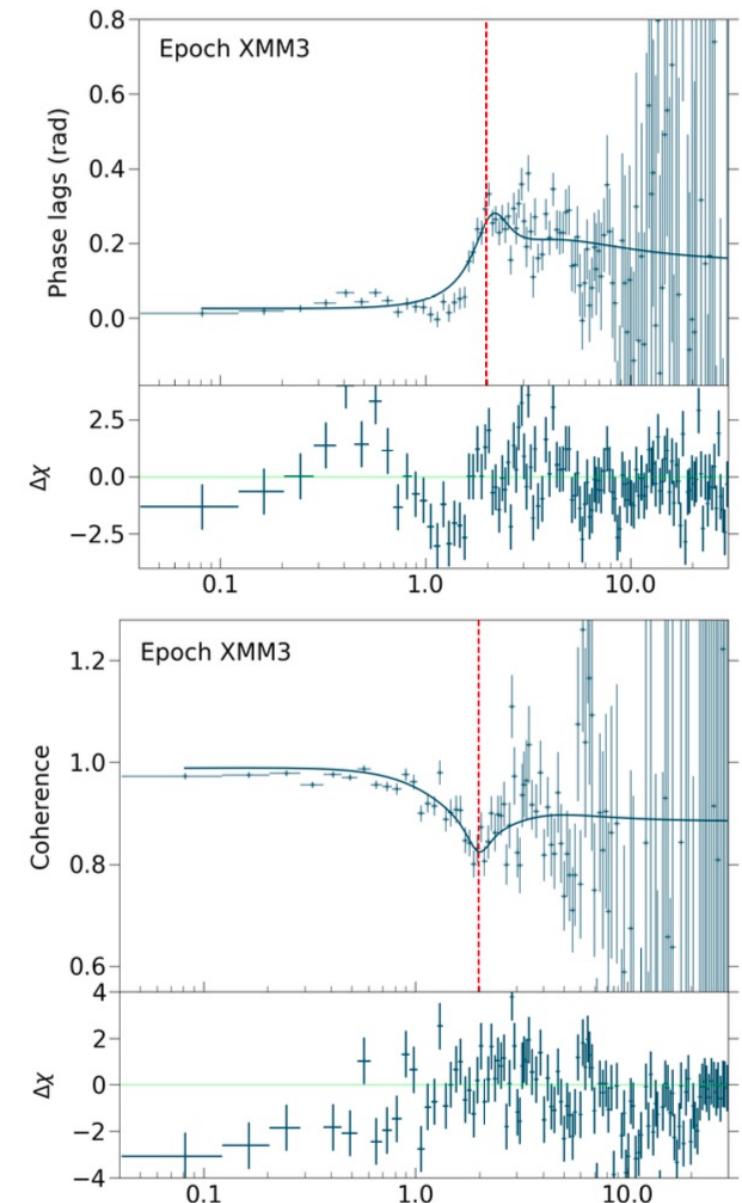
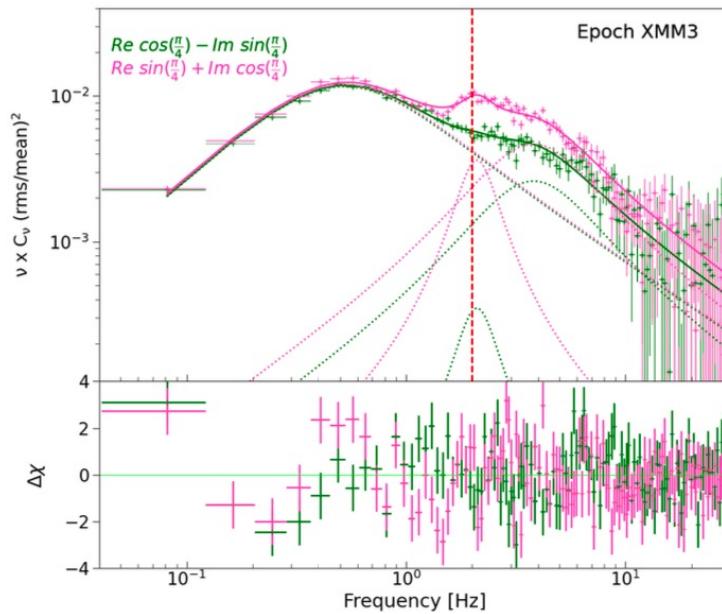
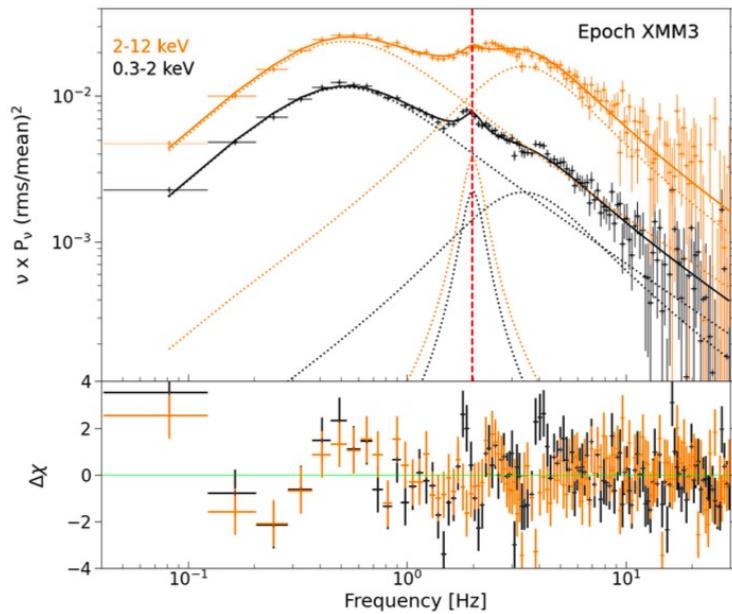
Imaginary QPOs

Cyg X-1

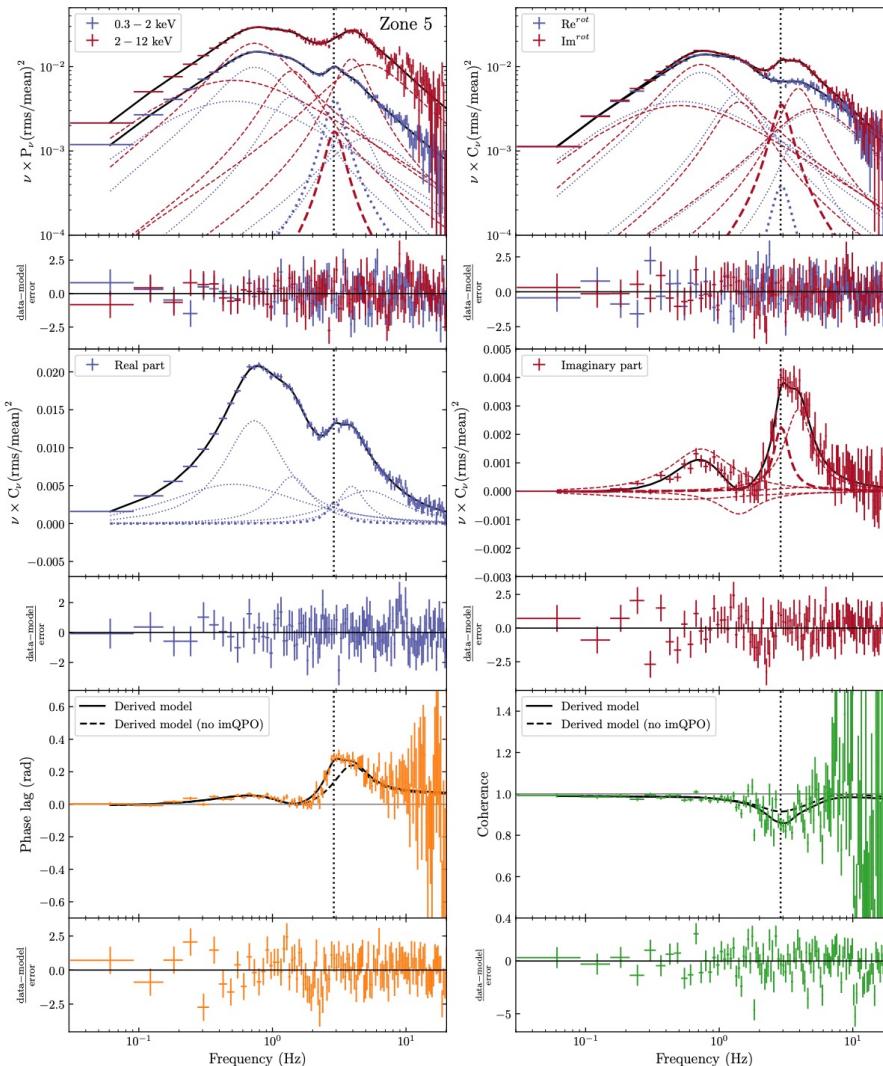


Imaginary QPOs

Swift J1727

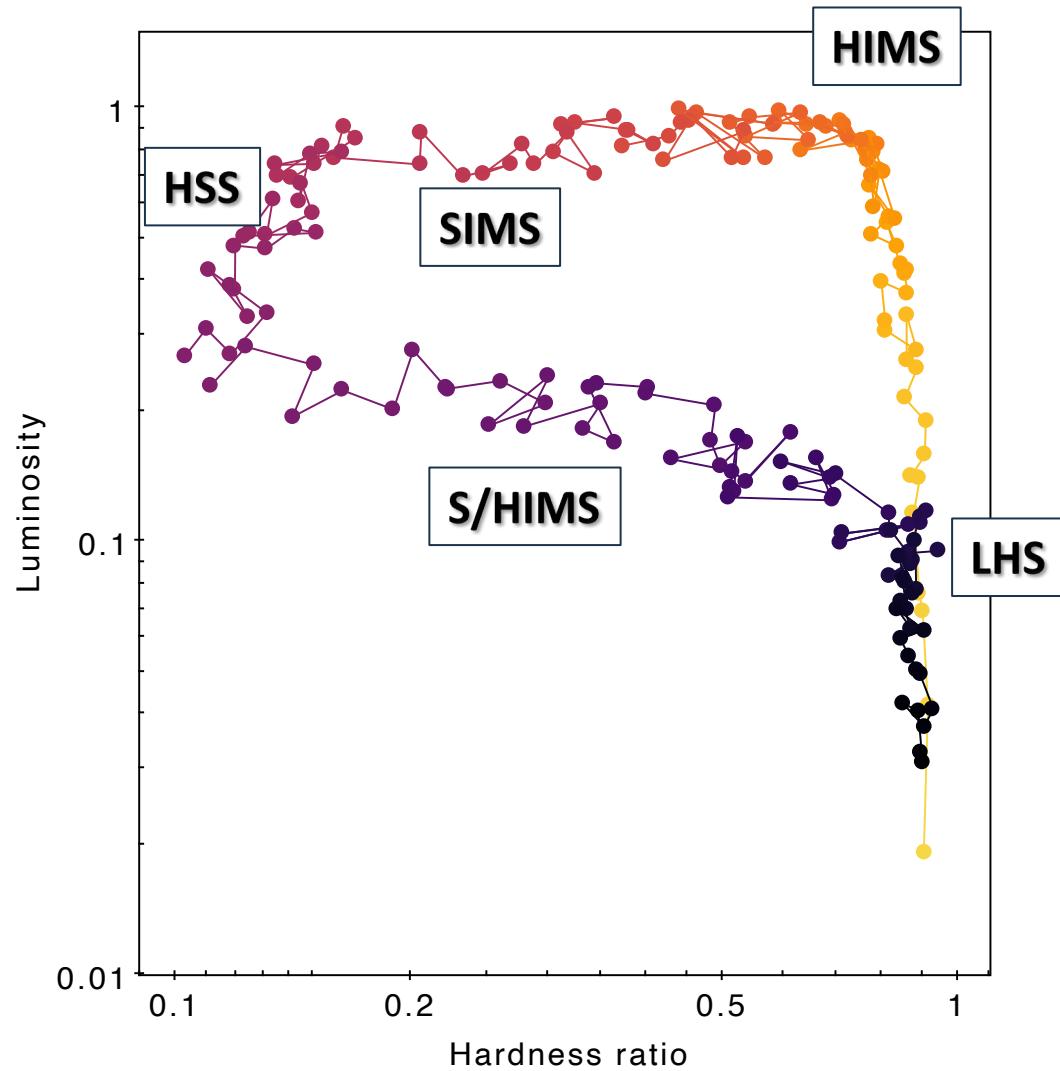


Imaginary QPOs

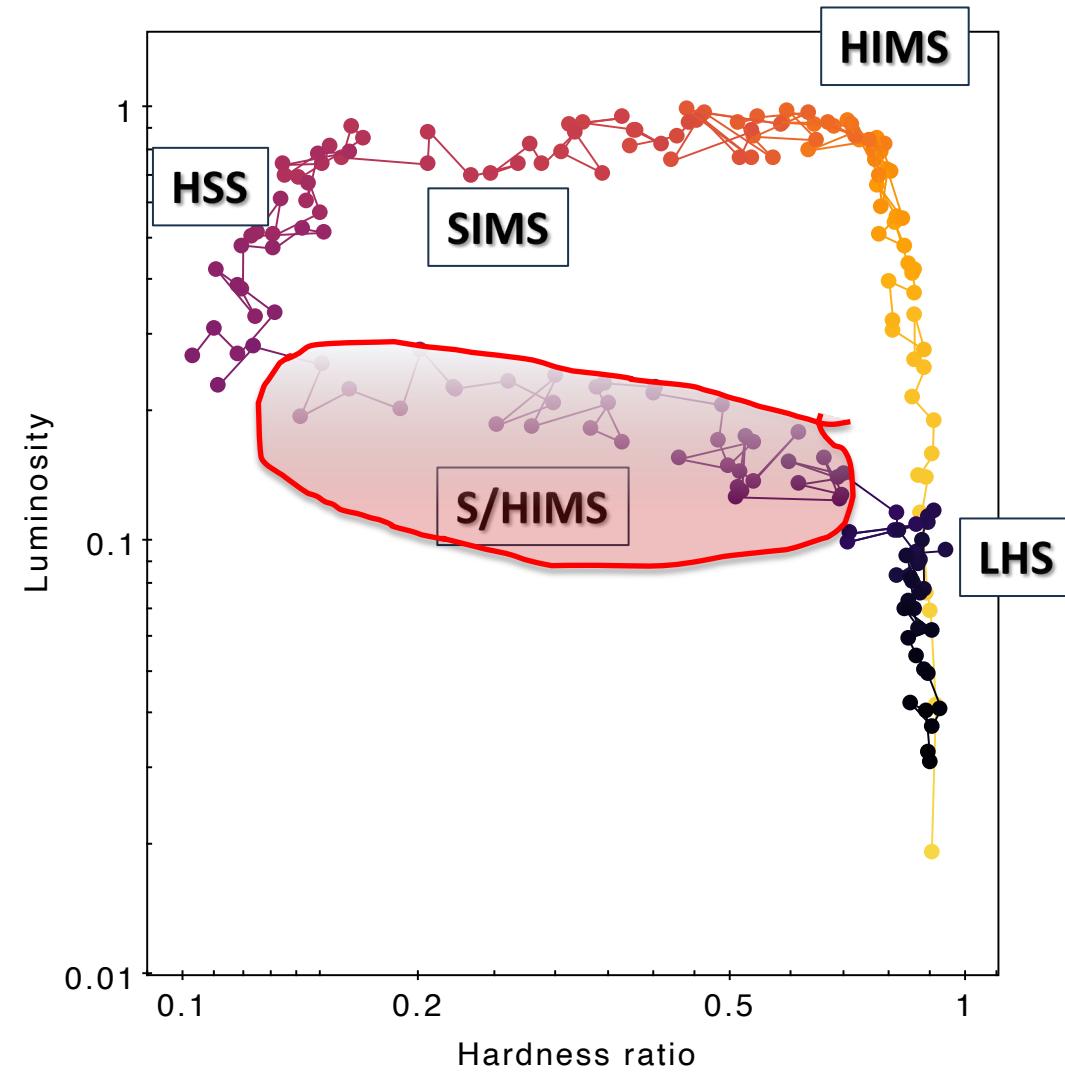


AT2019wey

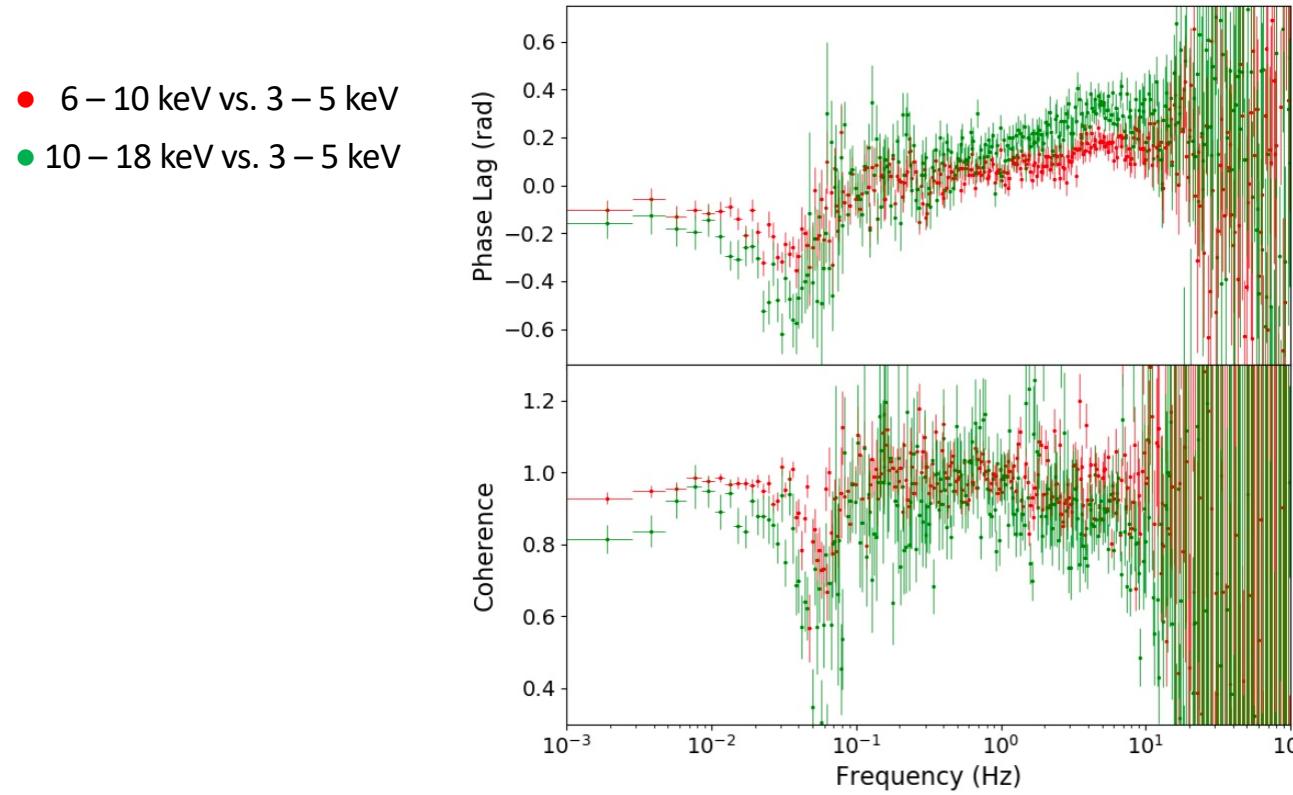
Imaginary QPOs



Imaginary QPOs

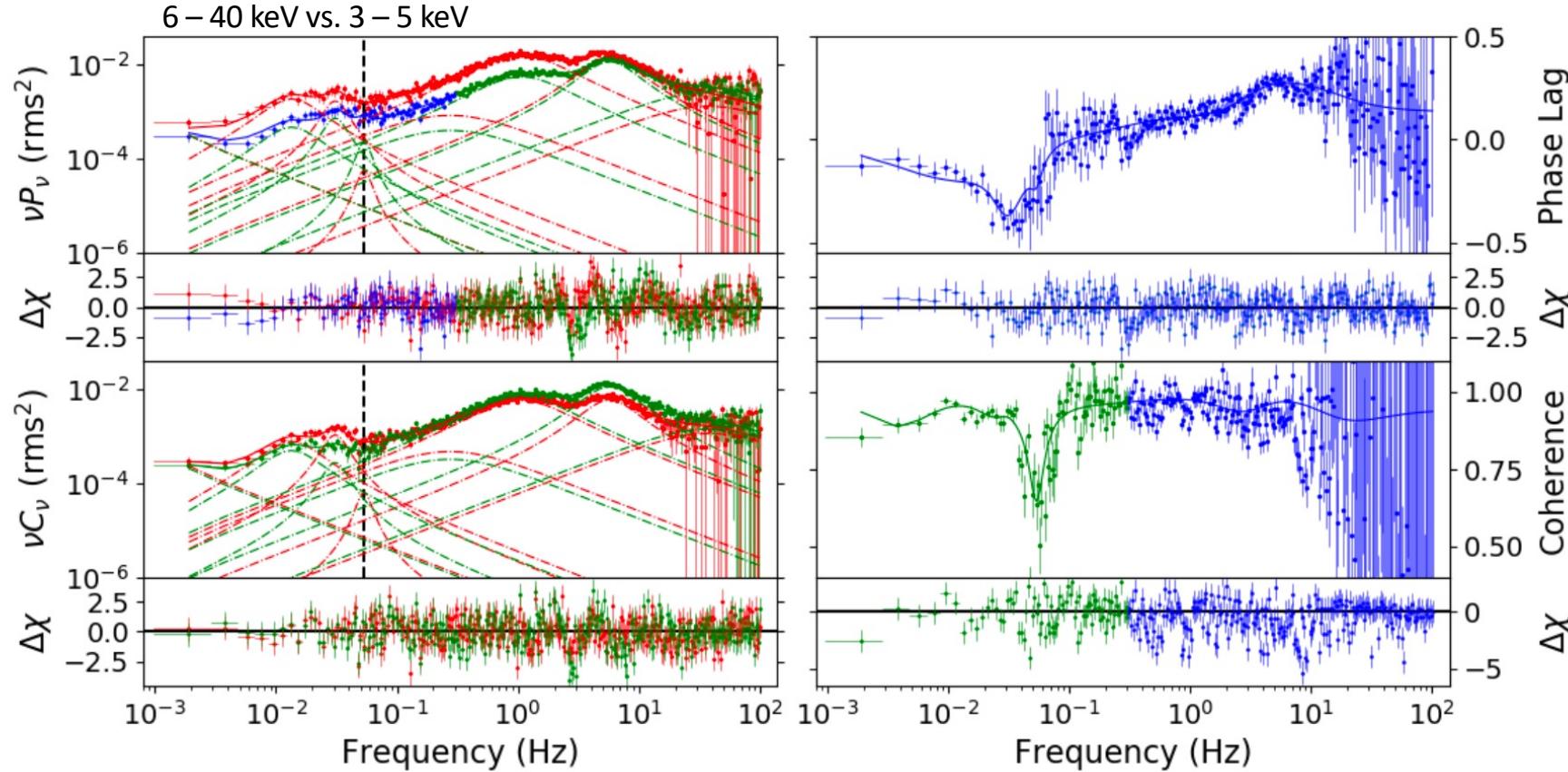


Cyg X-1 (AstroSat): phase lags and coherence



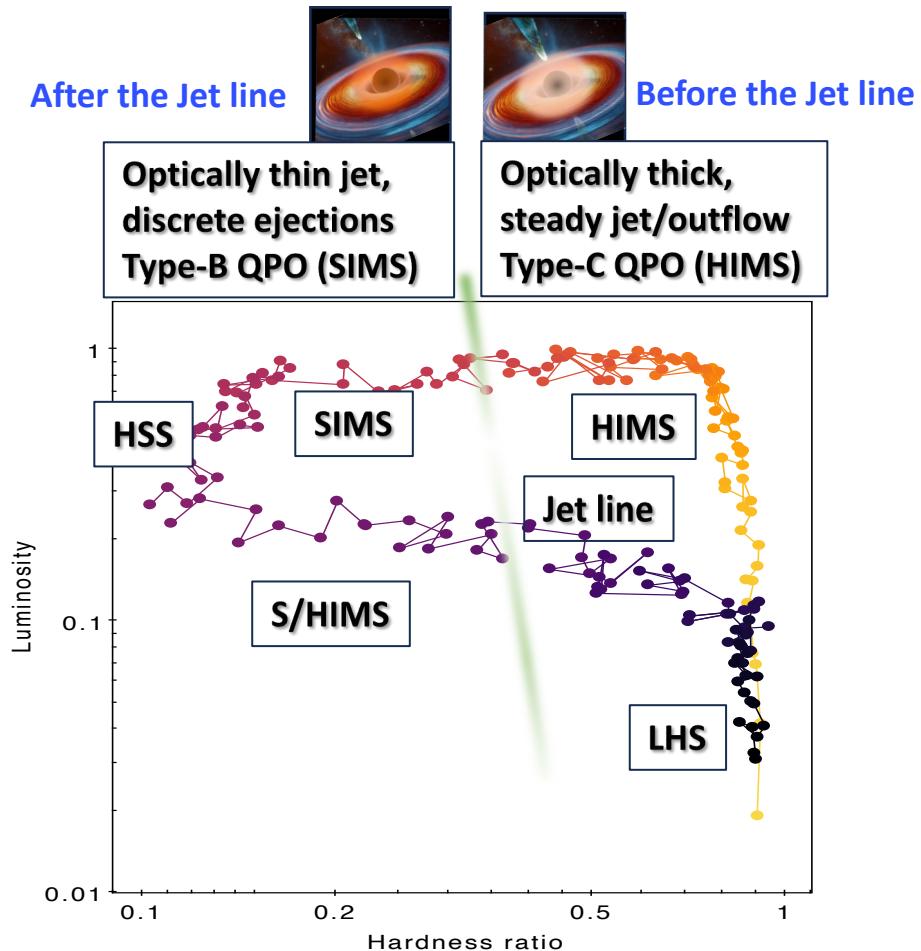
Rout et al. 2025

Cyg X-1 (AstroSat): phase lags and coherence

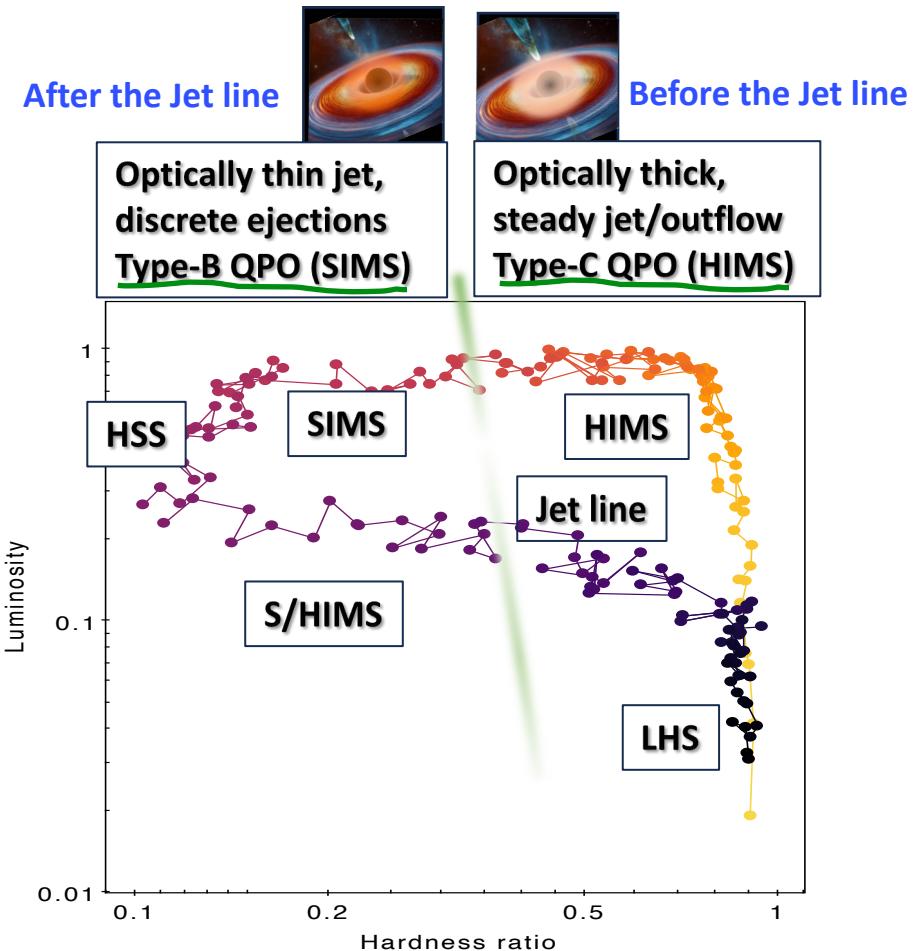


Rout et al. 2025 (see also Fogantini et al. 2025)

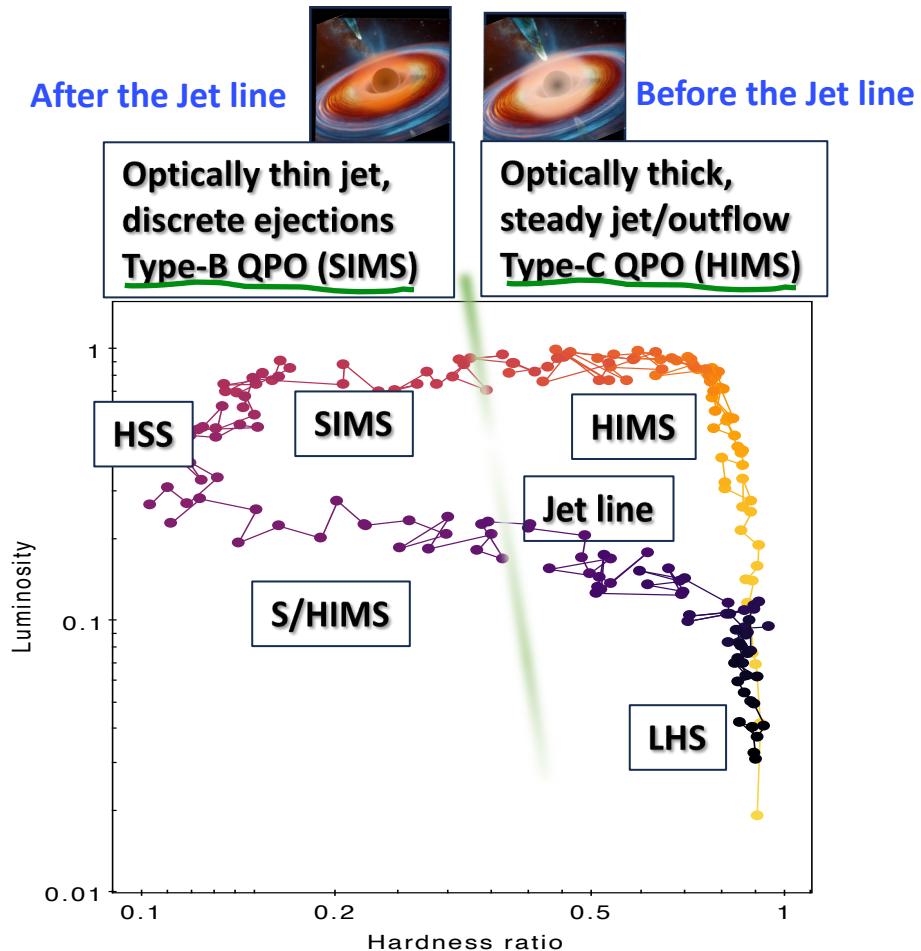
Black-hole states



Black-hole states



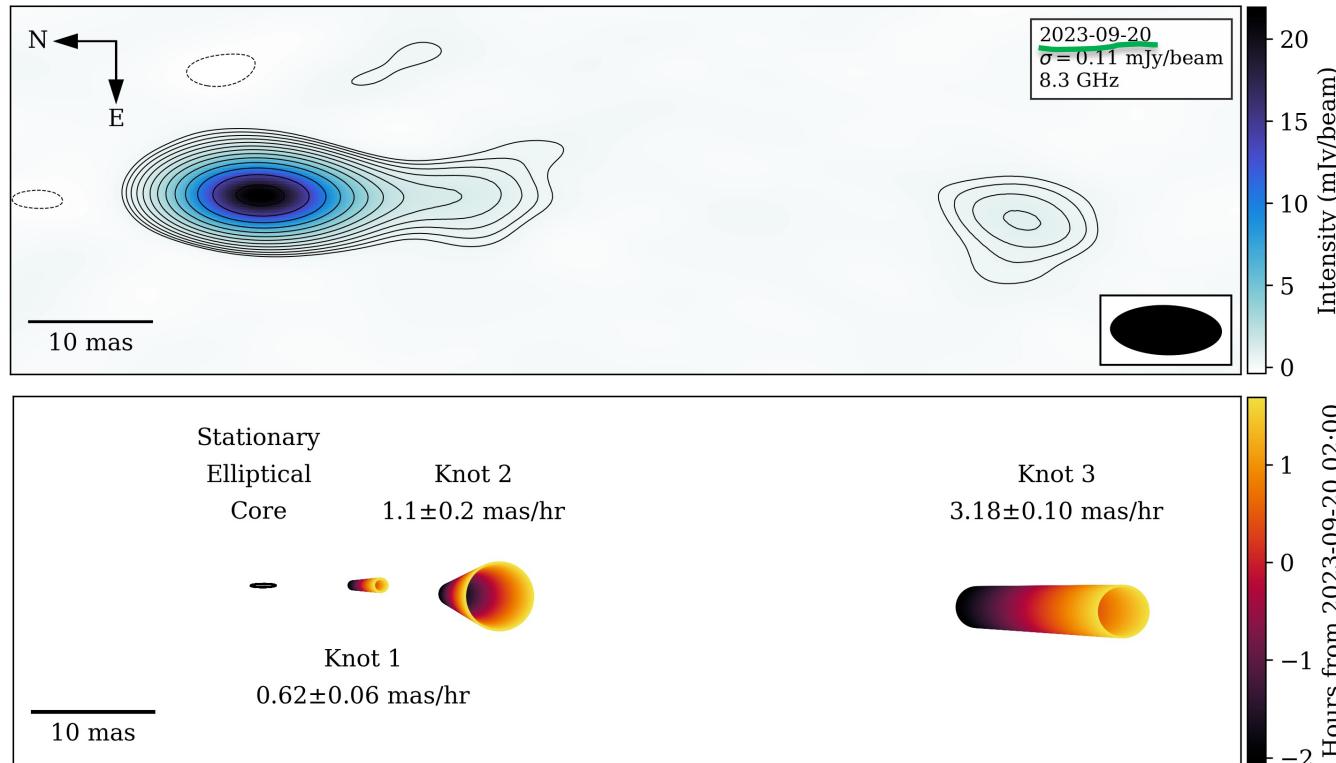
Black-hole states



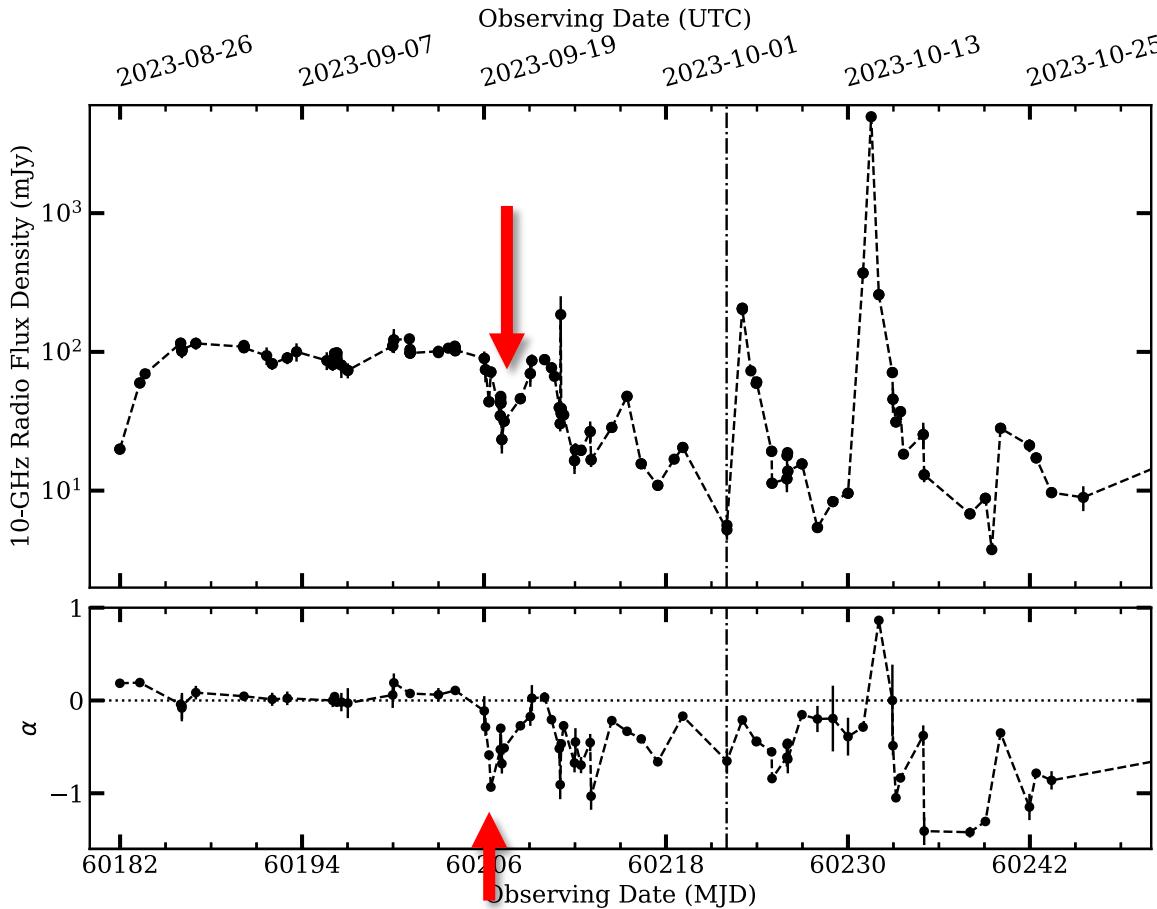
"Type-B QPOs are associated (produced) by the (discrete) jet ejections"

E.g., Uttley & Stevens 2016 (precession), Kylafis, Reig & Papadakis 2020, and others
(idea first proposed by Motta et al. 2016)

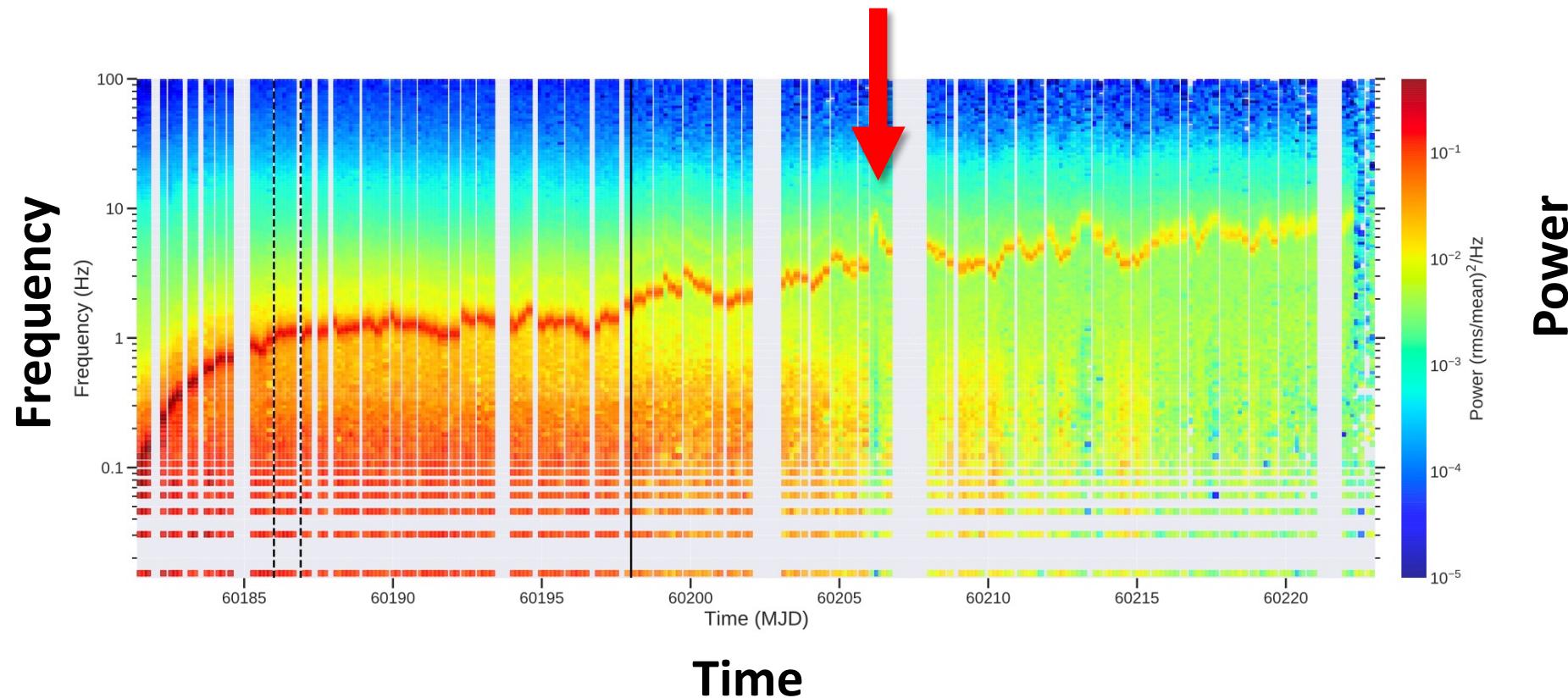
Swift J1727: Discrete jet ejections on 2023-09-20



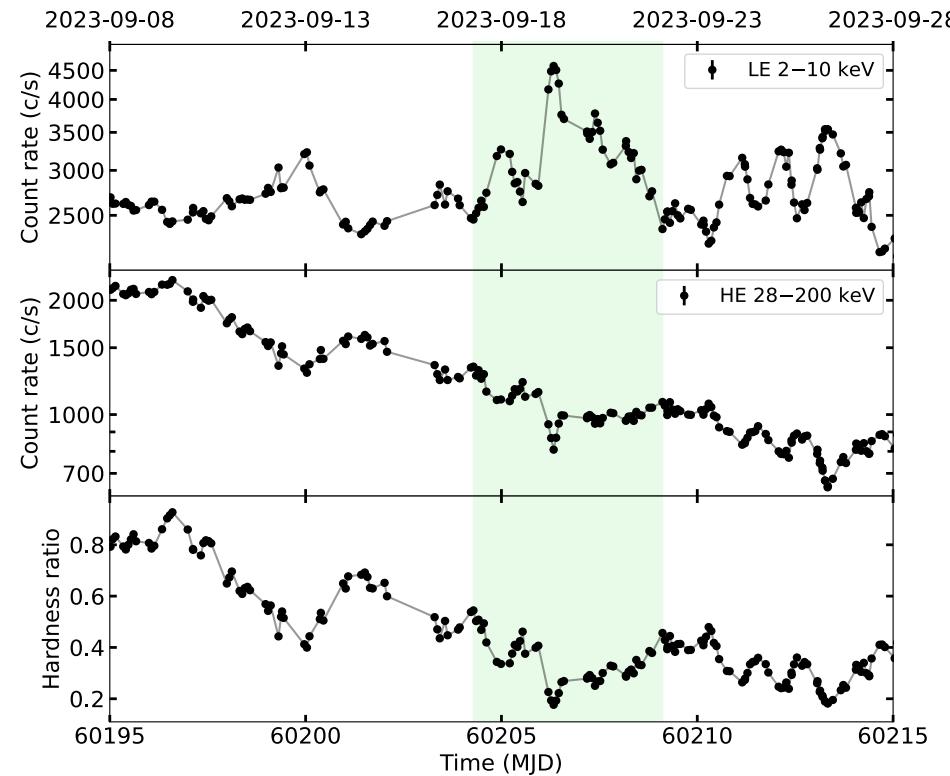
Swift J1727: Radio flux and spectral index



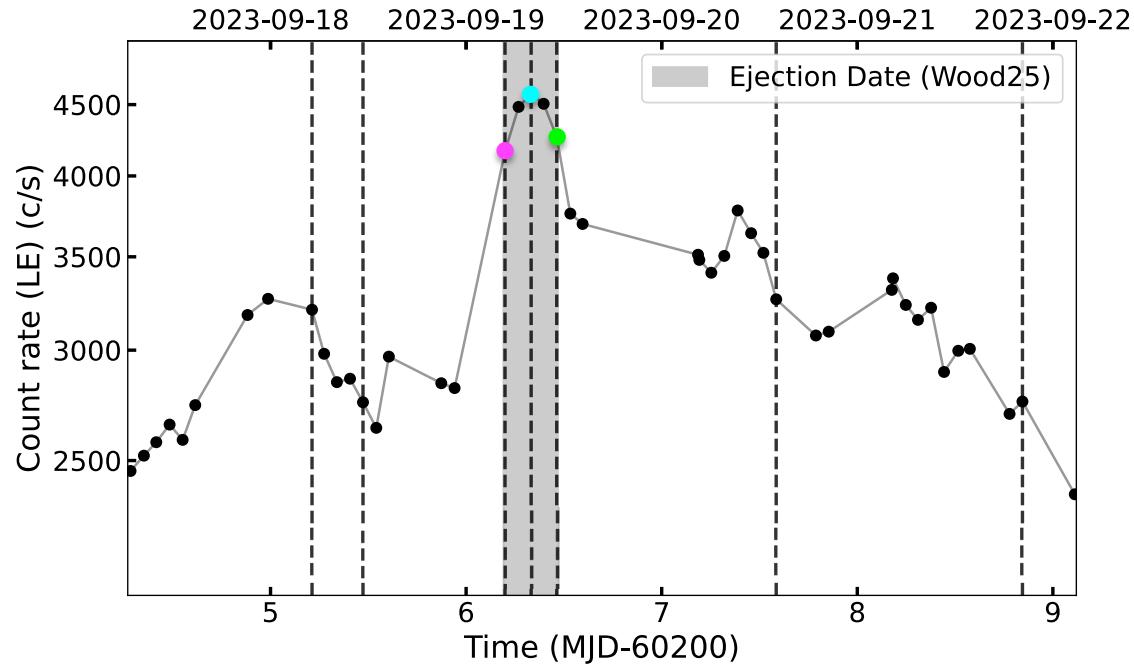
Swift J1727: Dynamical power spectrum



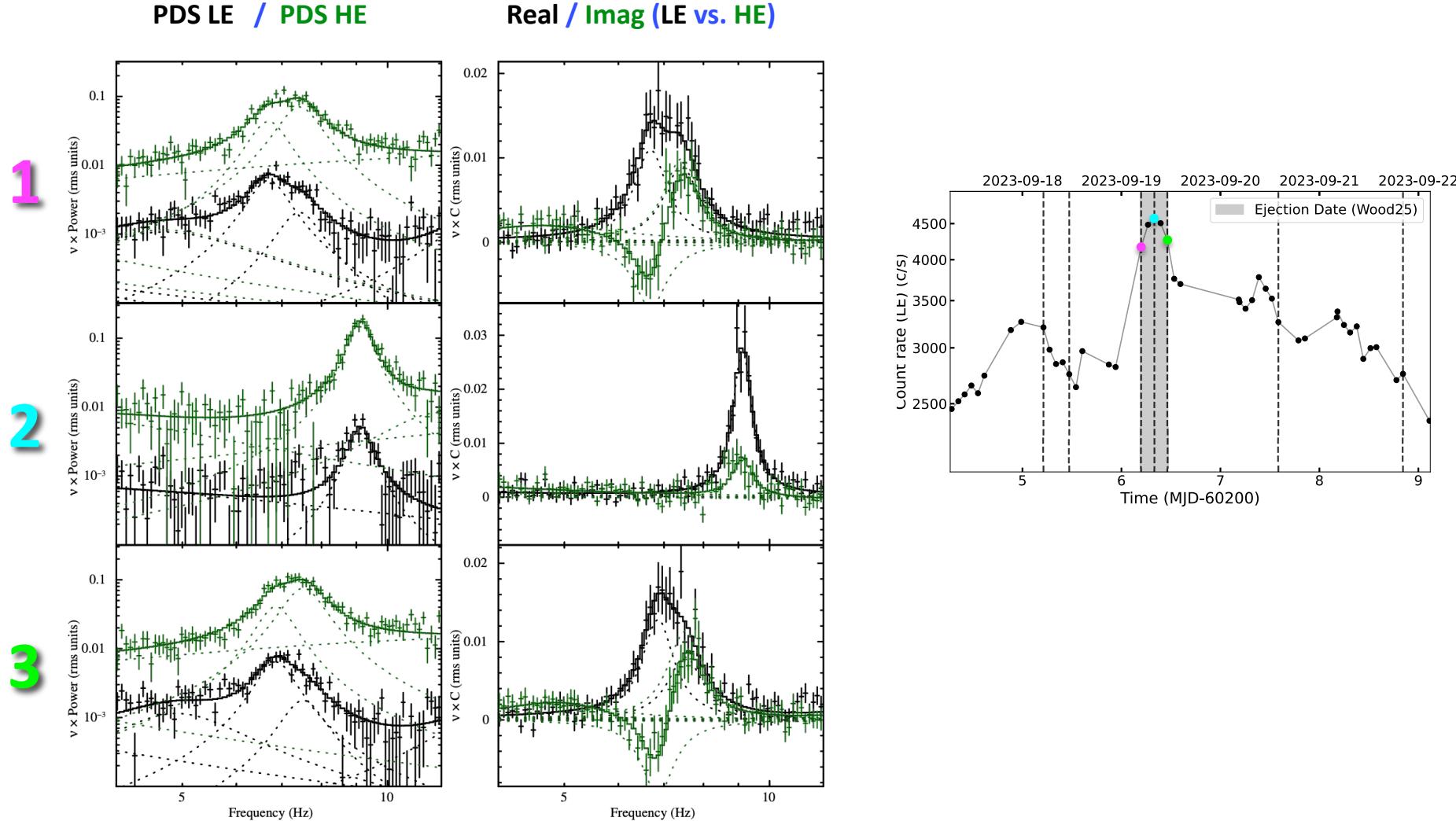
Swift J1727: X-ray flux



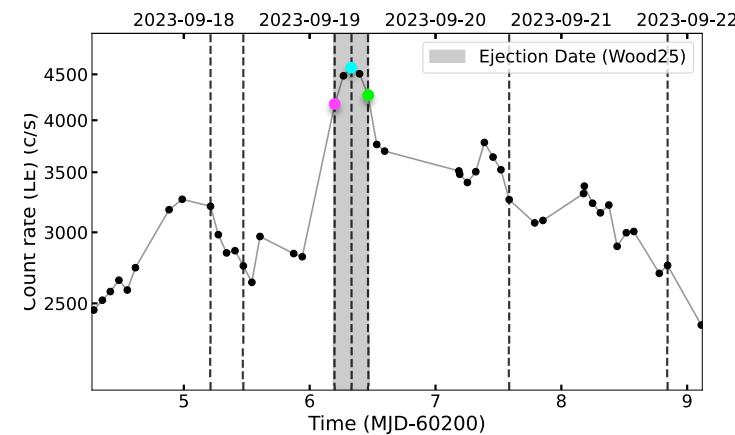
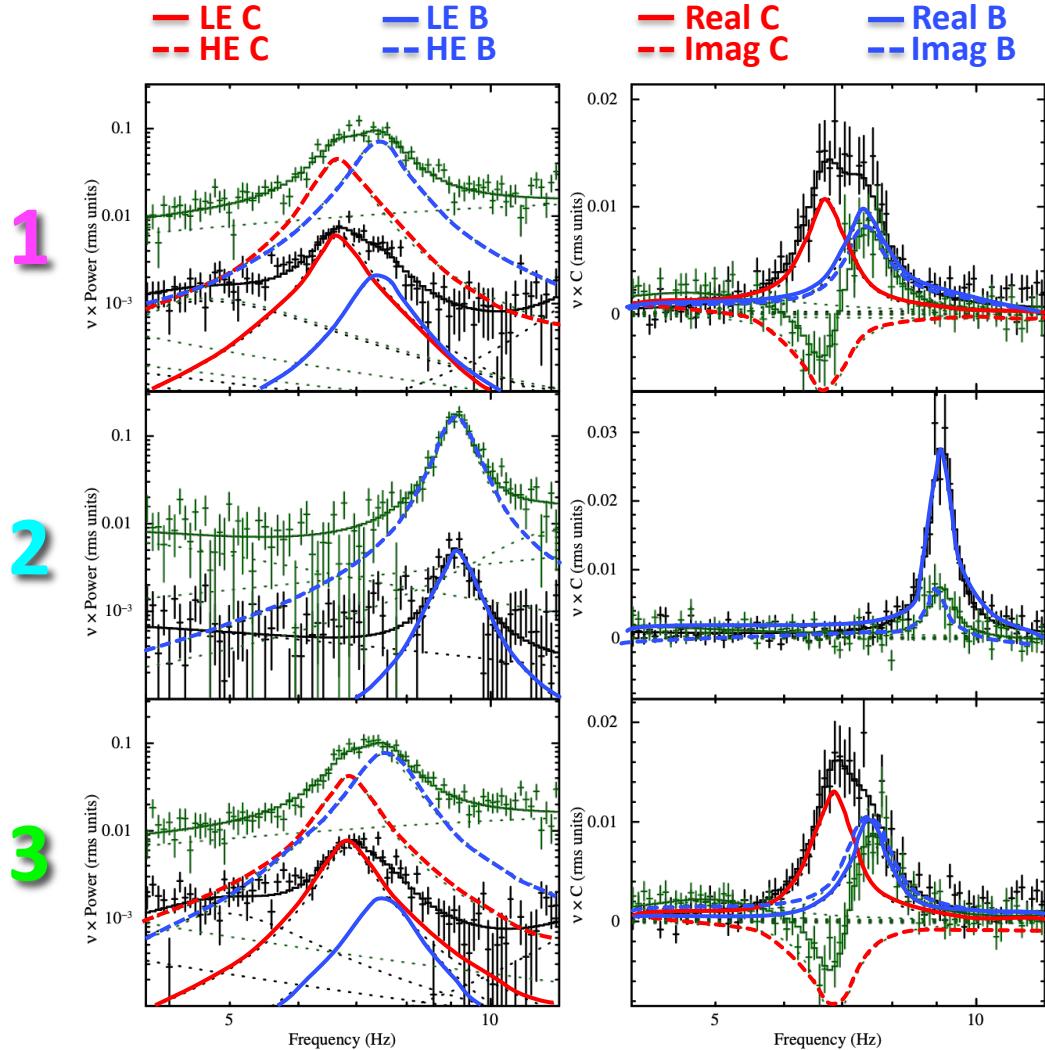
Swift J1727: X-ray flux



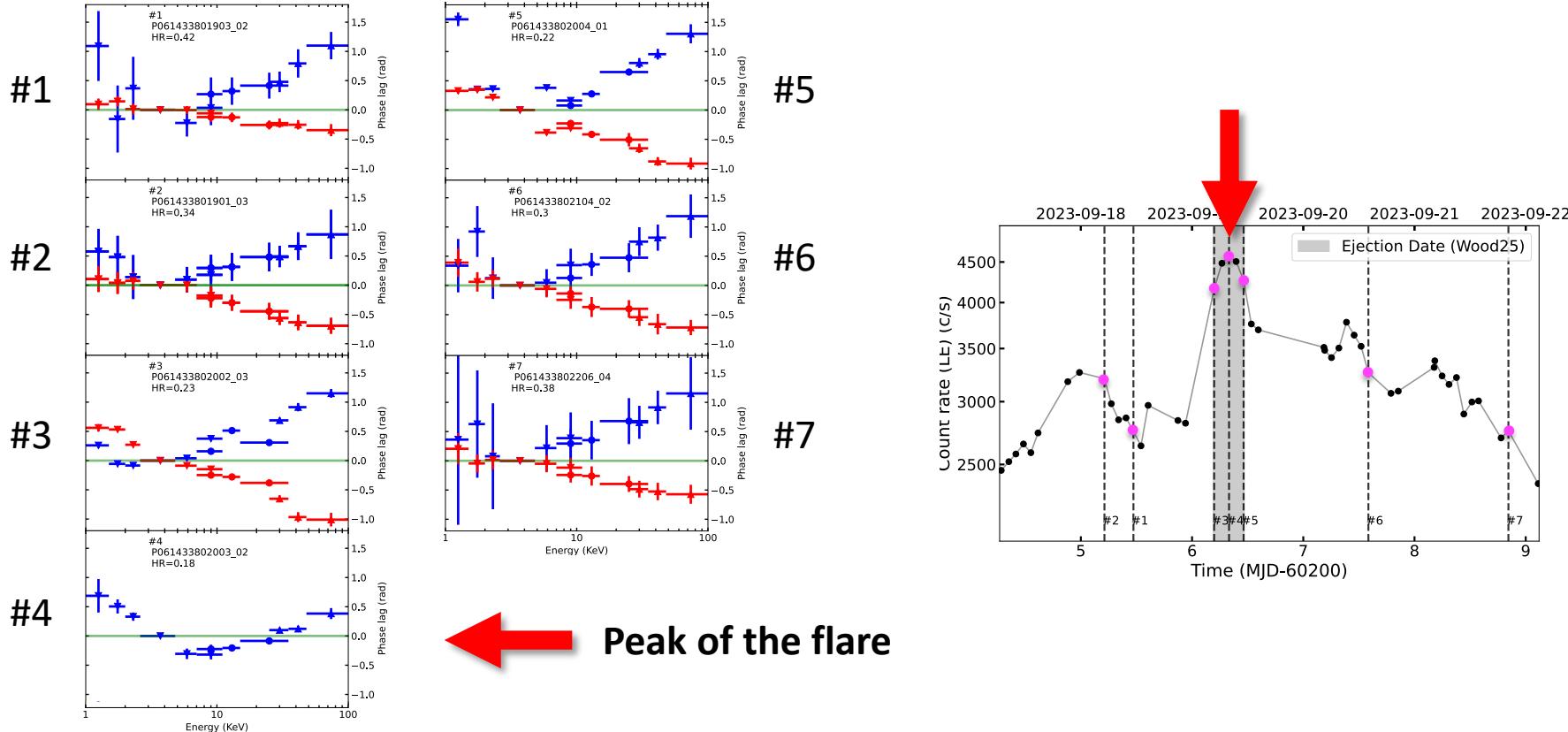
Swift J1727: X-ray flux



Swift J1727: X-ray flux

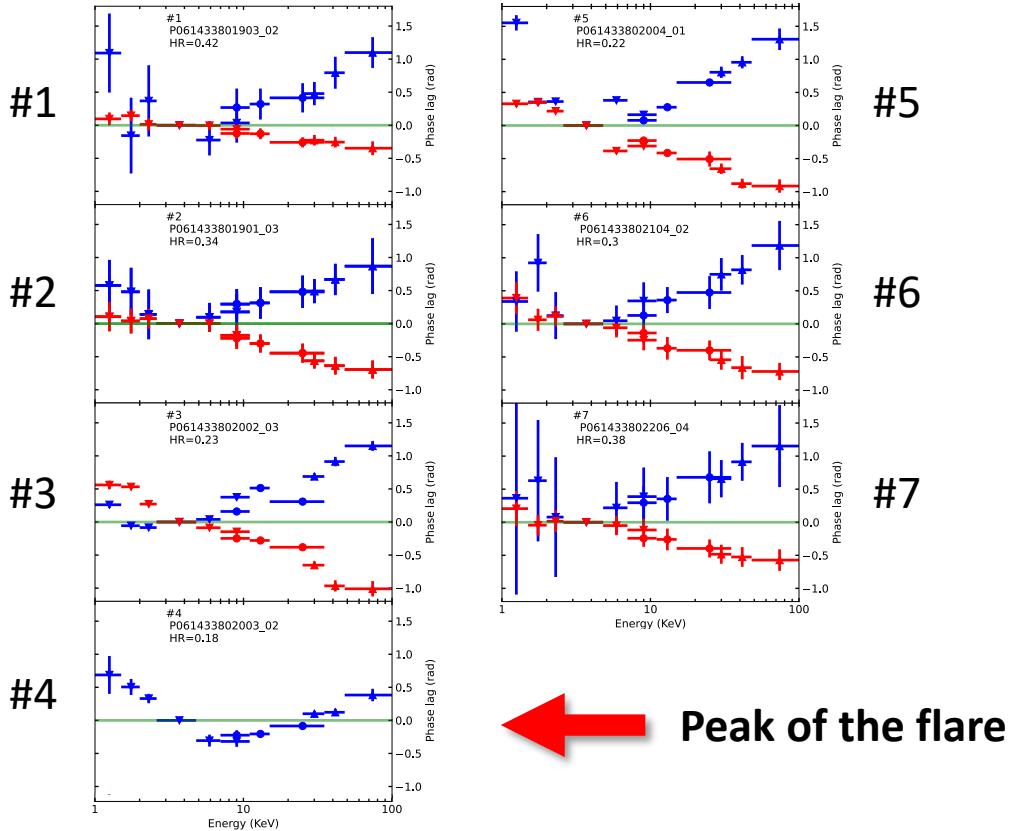


Swift J1727: lag-energy spectra

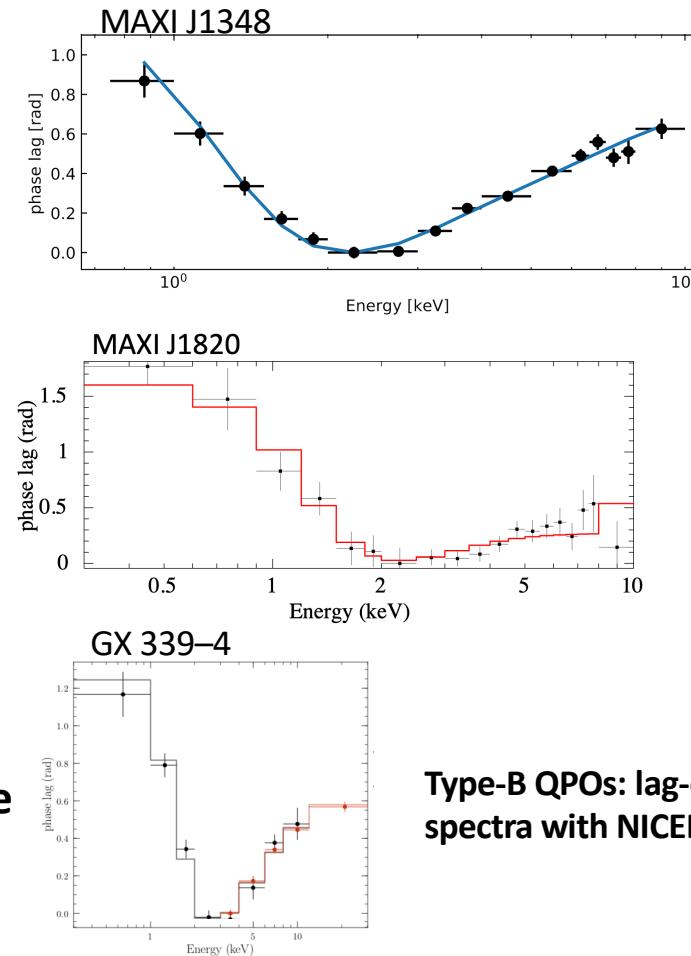


Red is type-C QPO

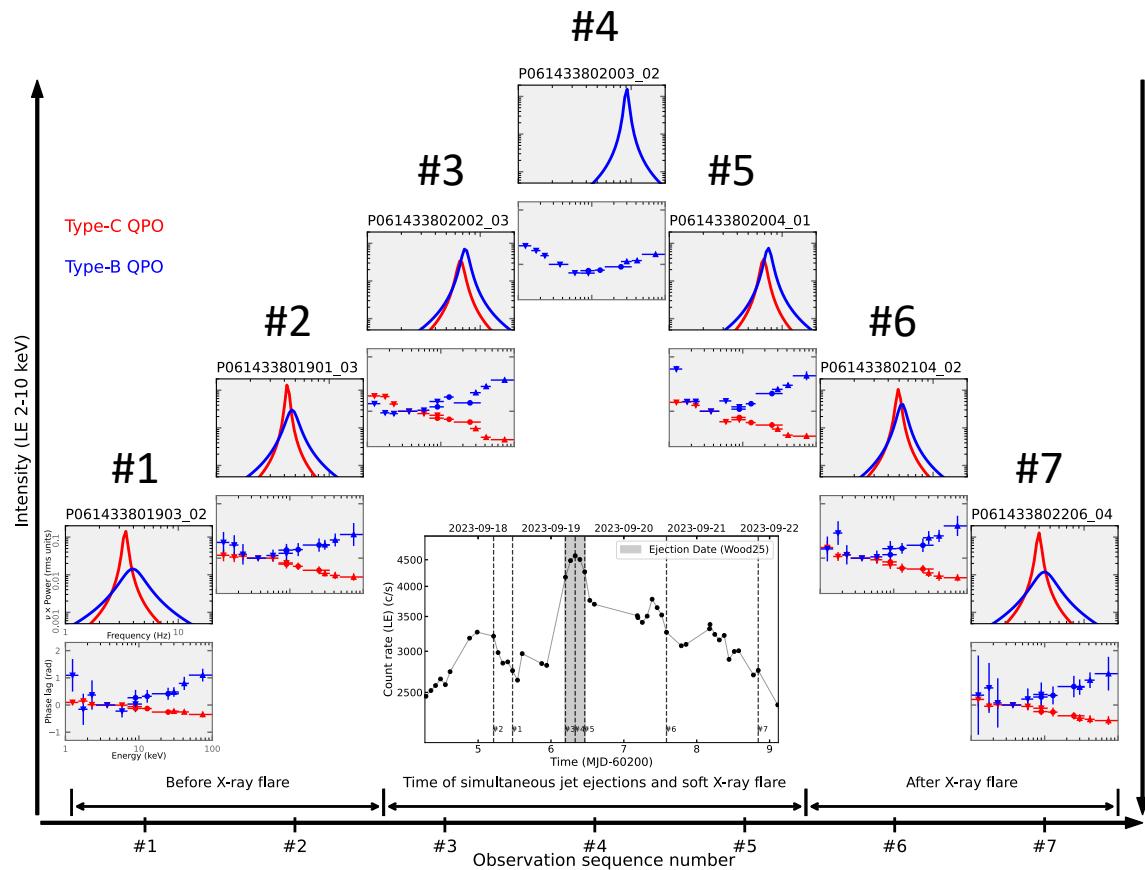
Swift J1727: lag-energy spectra



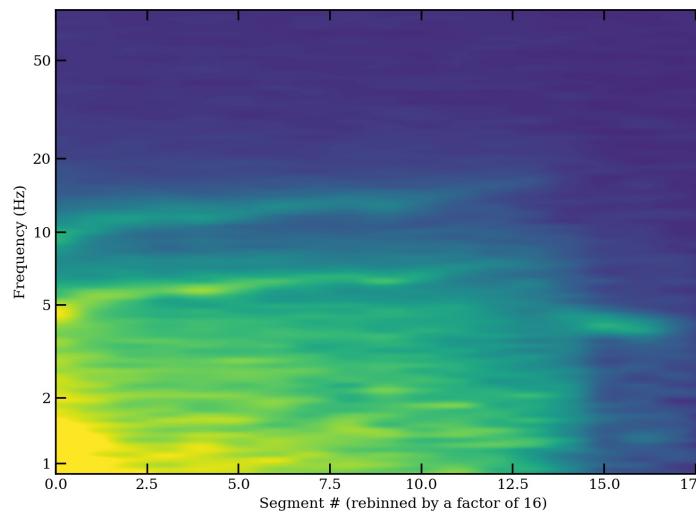
Red is type-C QPO
Blue is type-B QPO



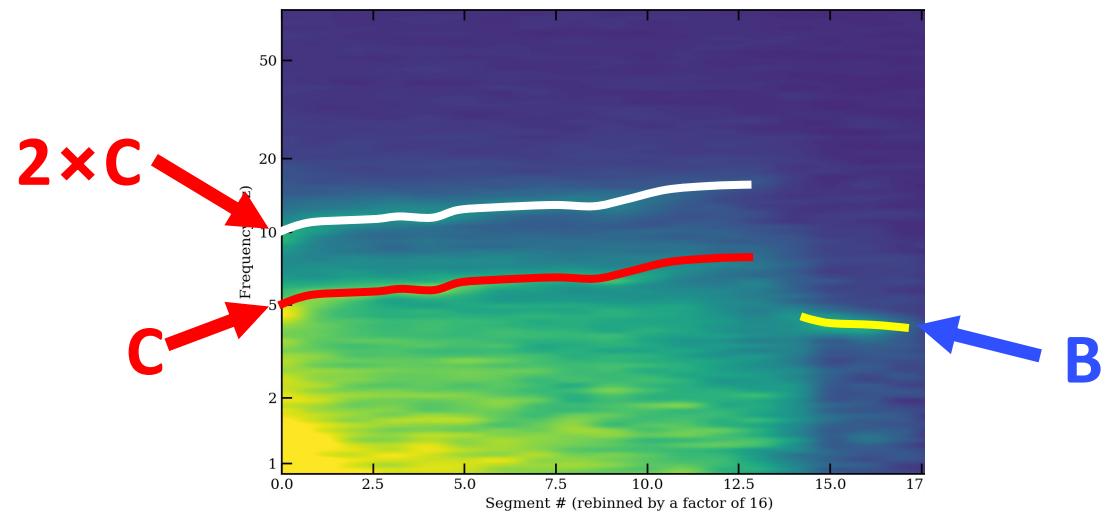
Type-B QPOs: lag-energy spectra with NICER



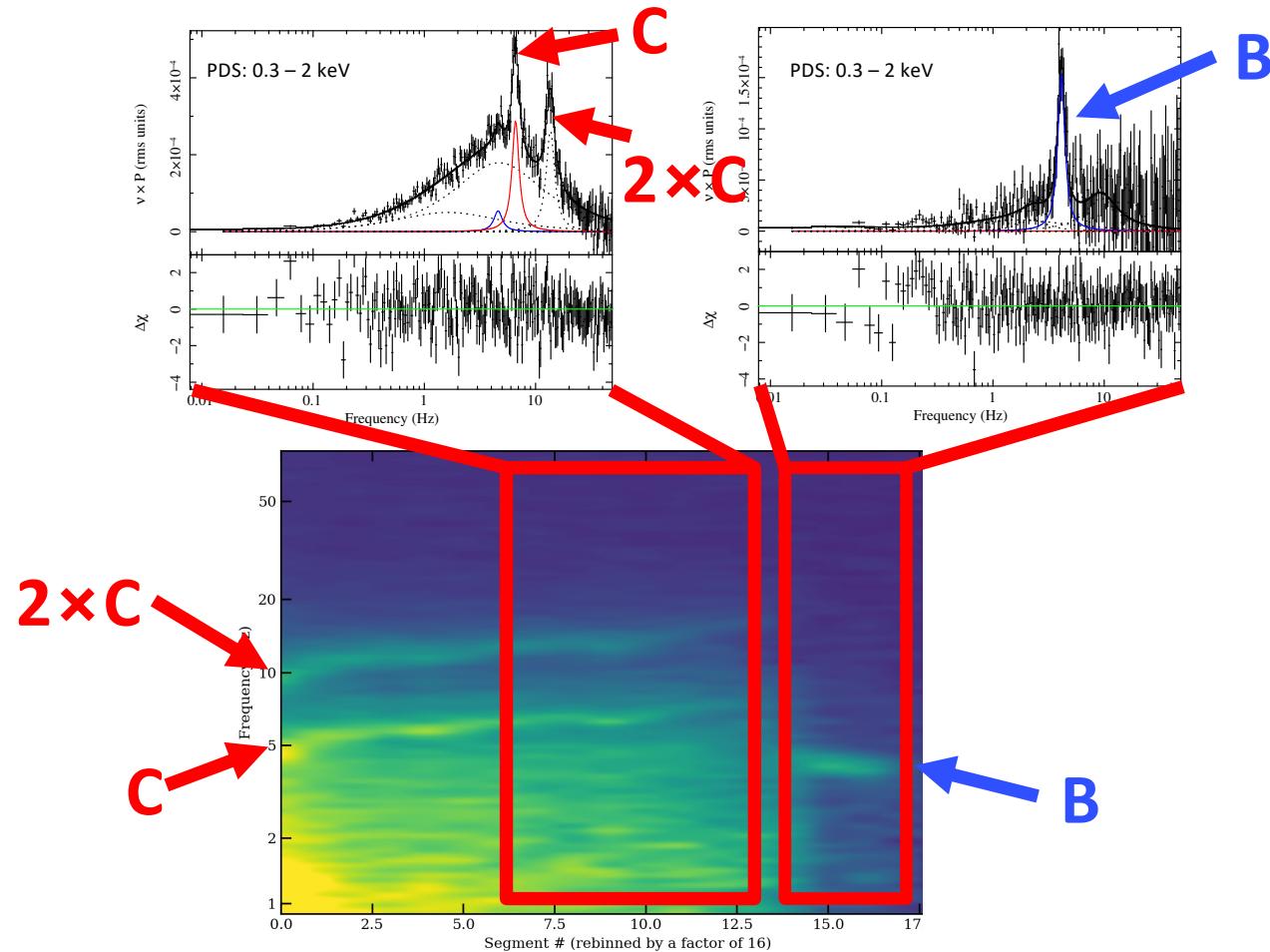
MAXI J1820+070



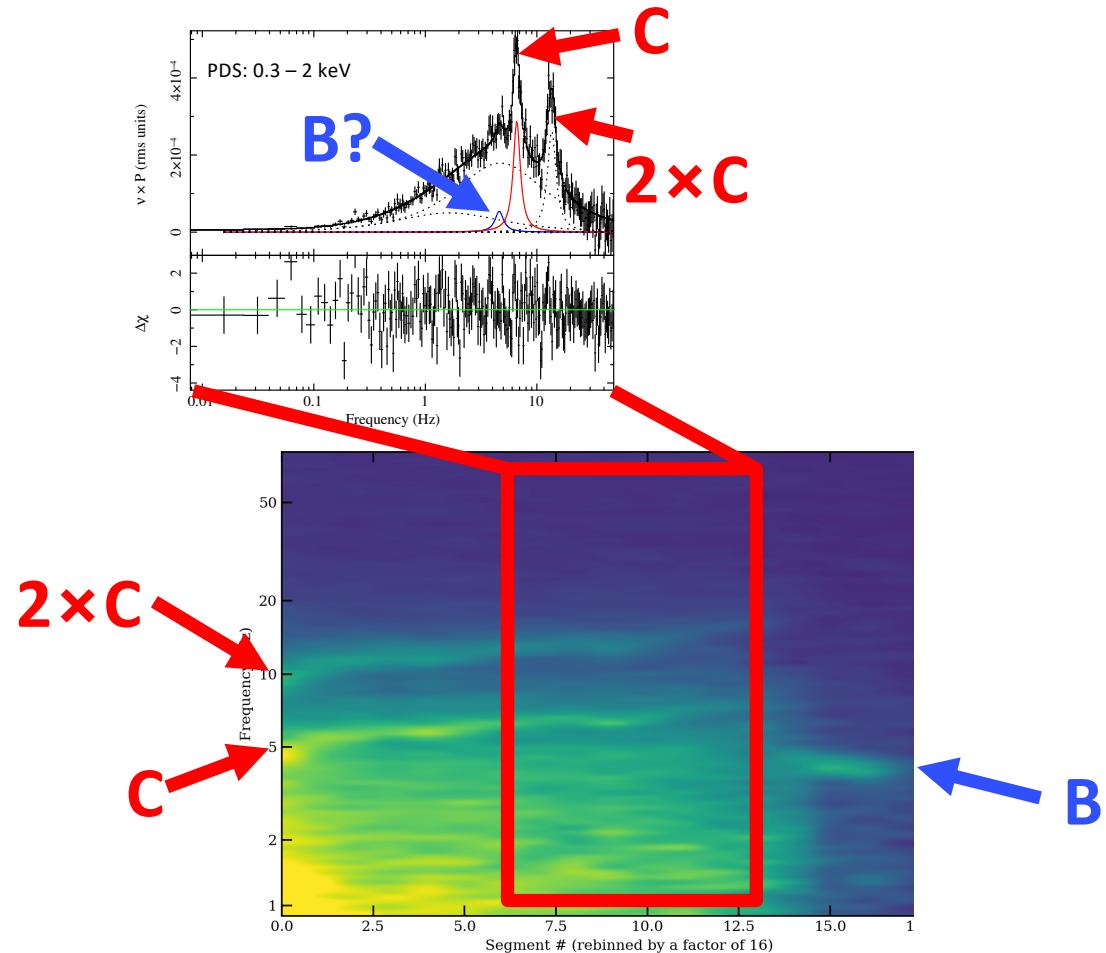
MAXI J1820+070



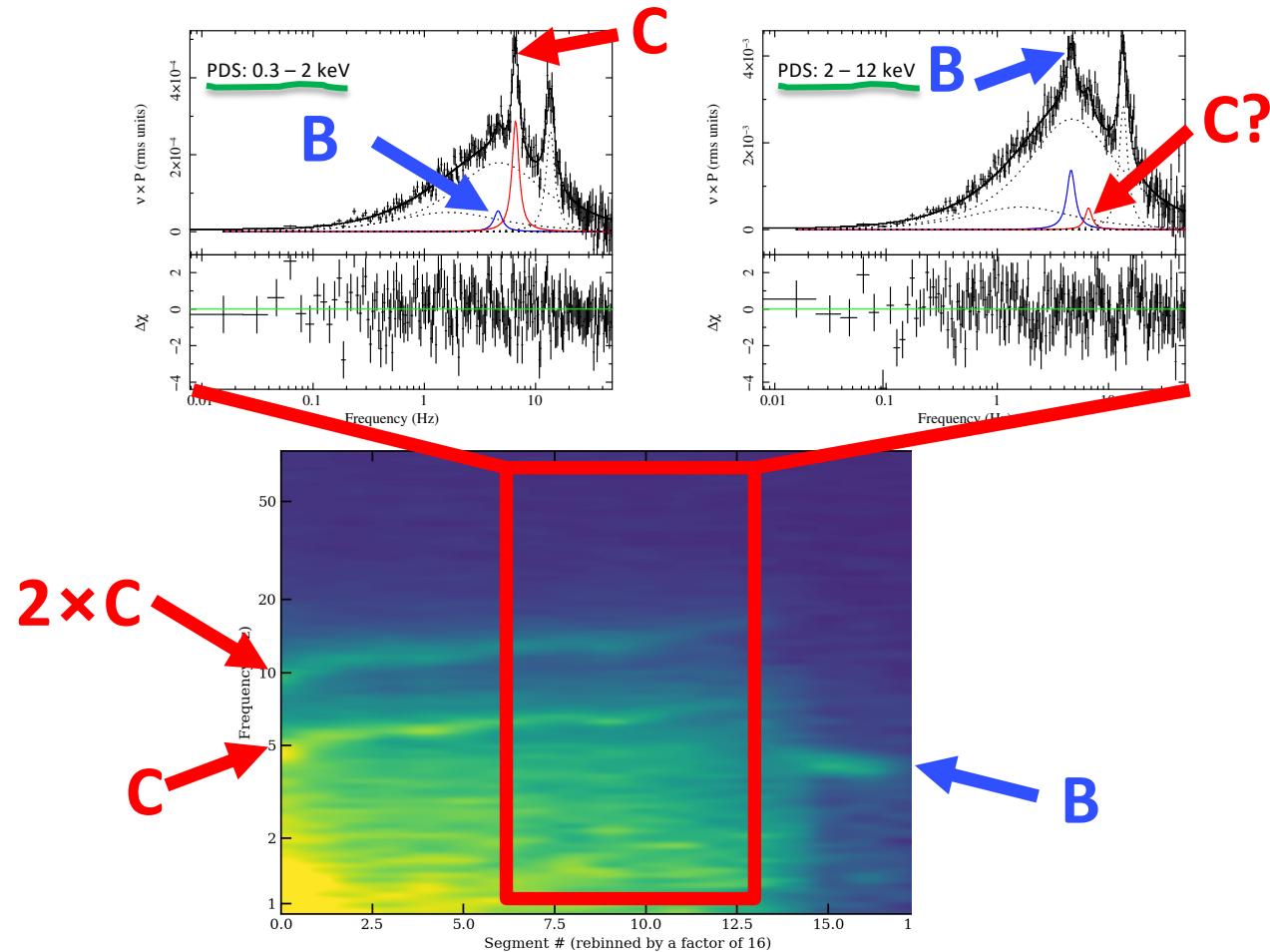
MAXI J1820+070



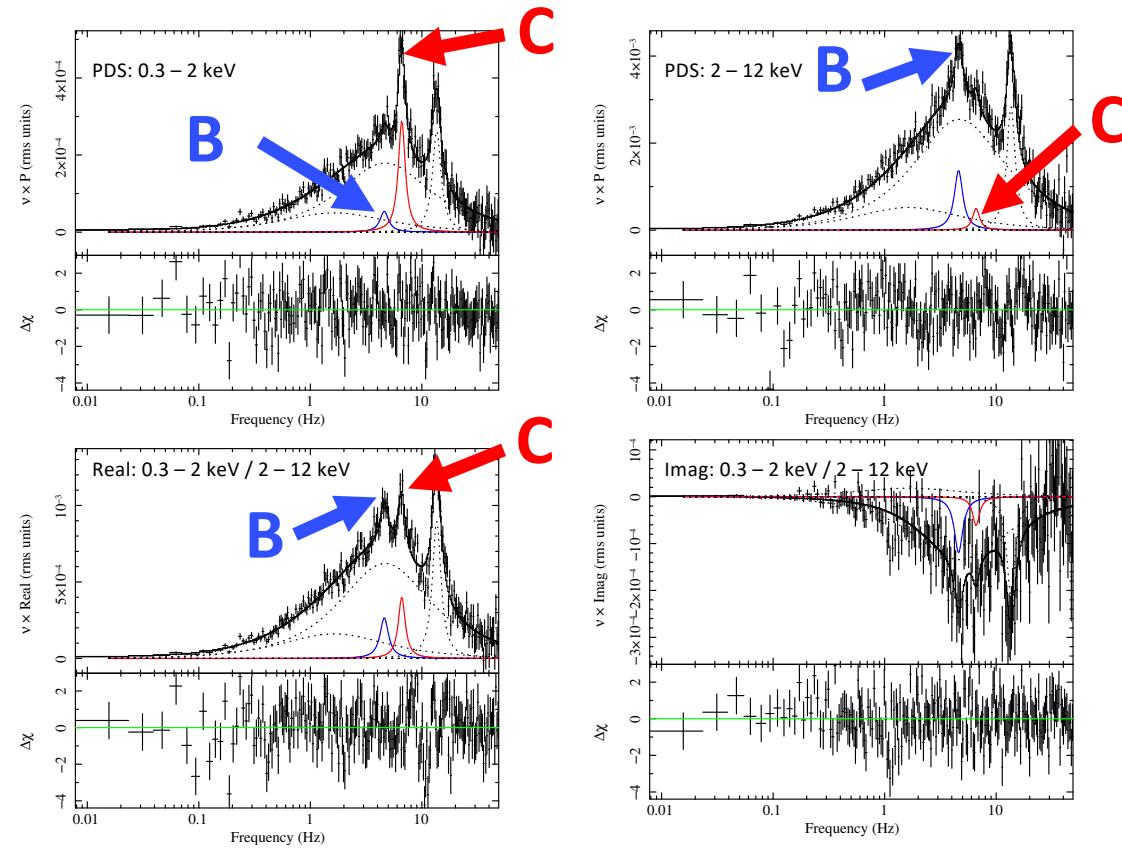
MAXI J1820+070



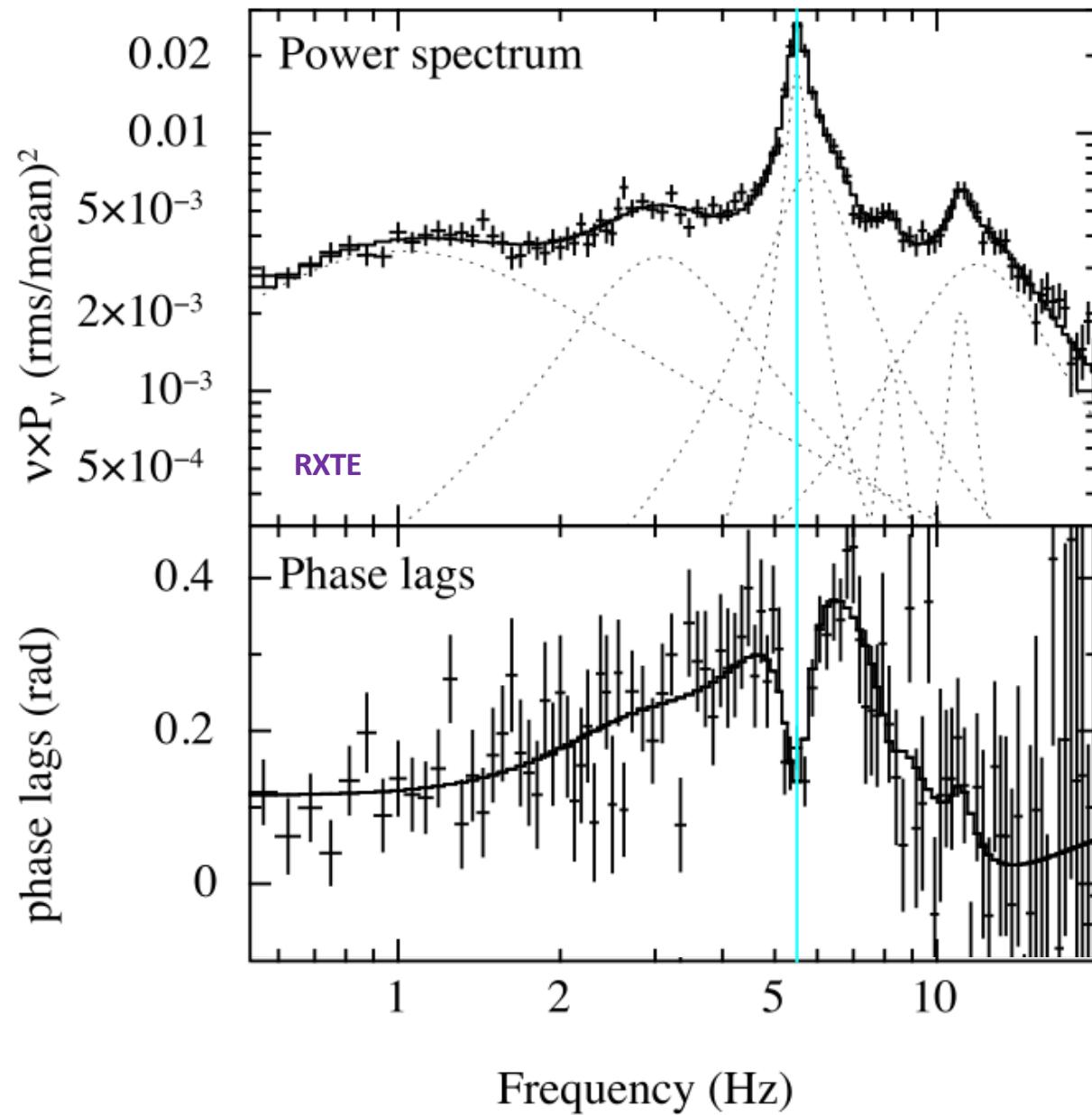
MAXI J1820+070



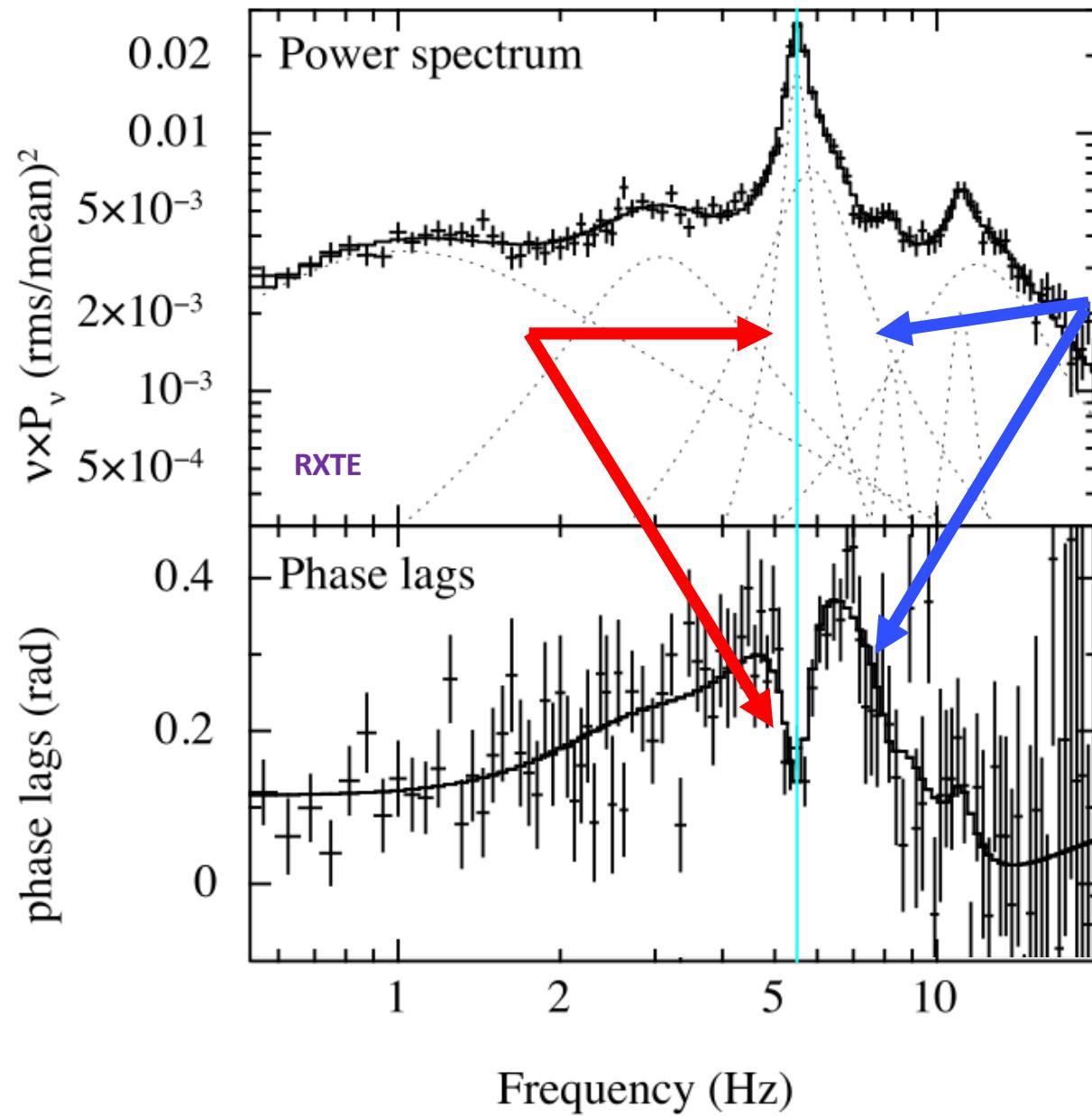
MAXI J1820+070



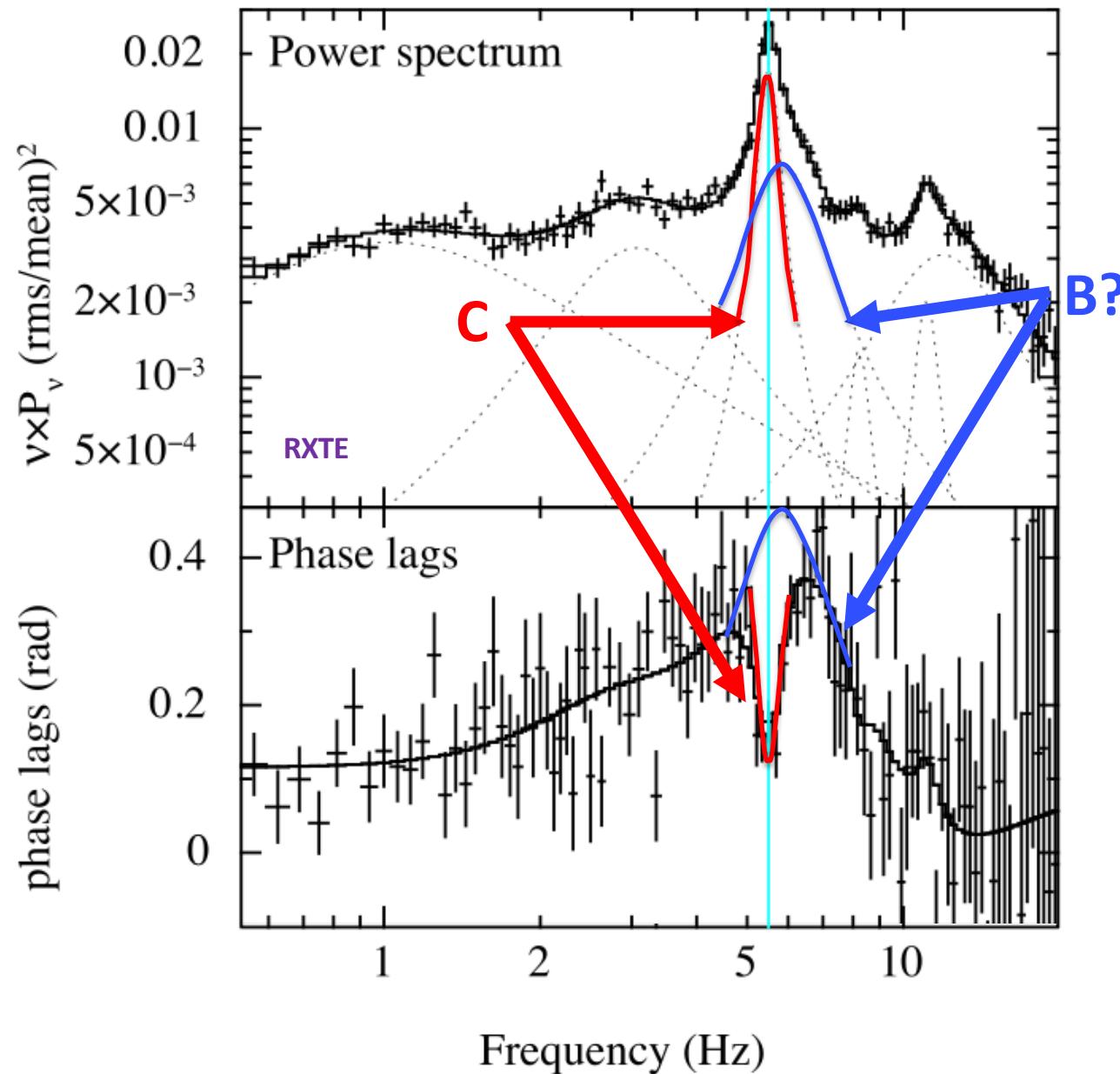
A local phase-lag dip at the QPO frequency in GX 399–4



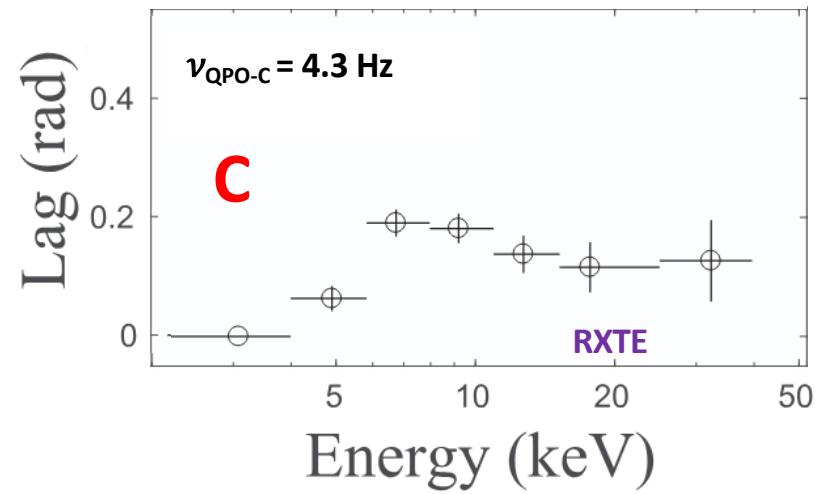
A local phase-lag dip at the QPO frequency in GX 399–4



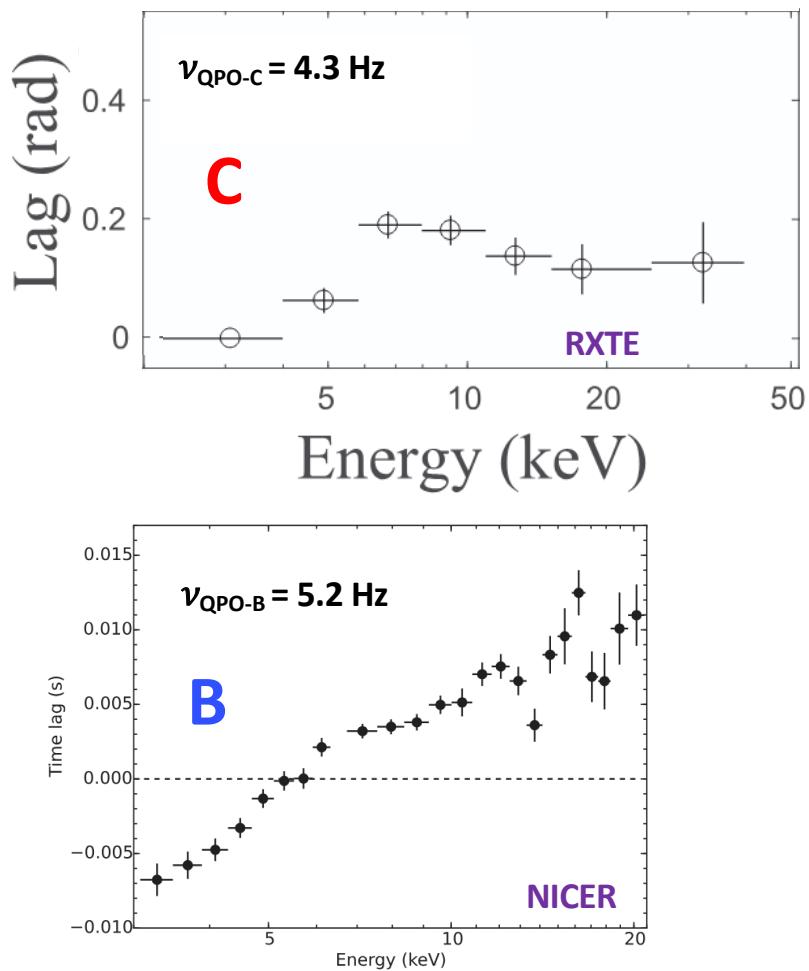
A local phase-lag dip at the QPO frequency in GX 399–4



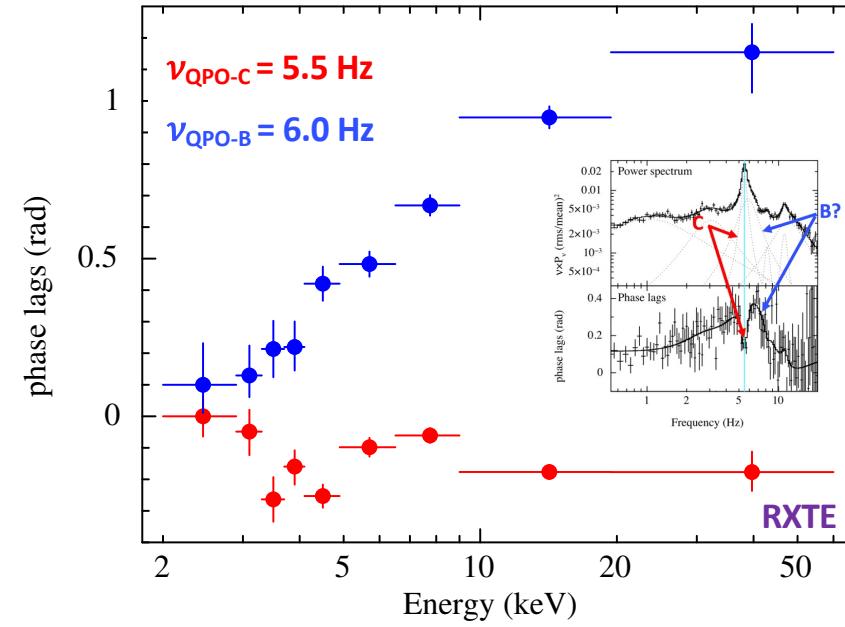
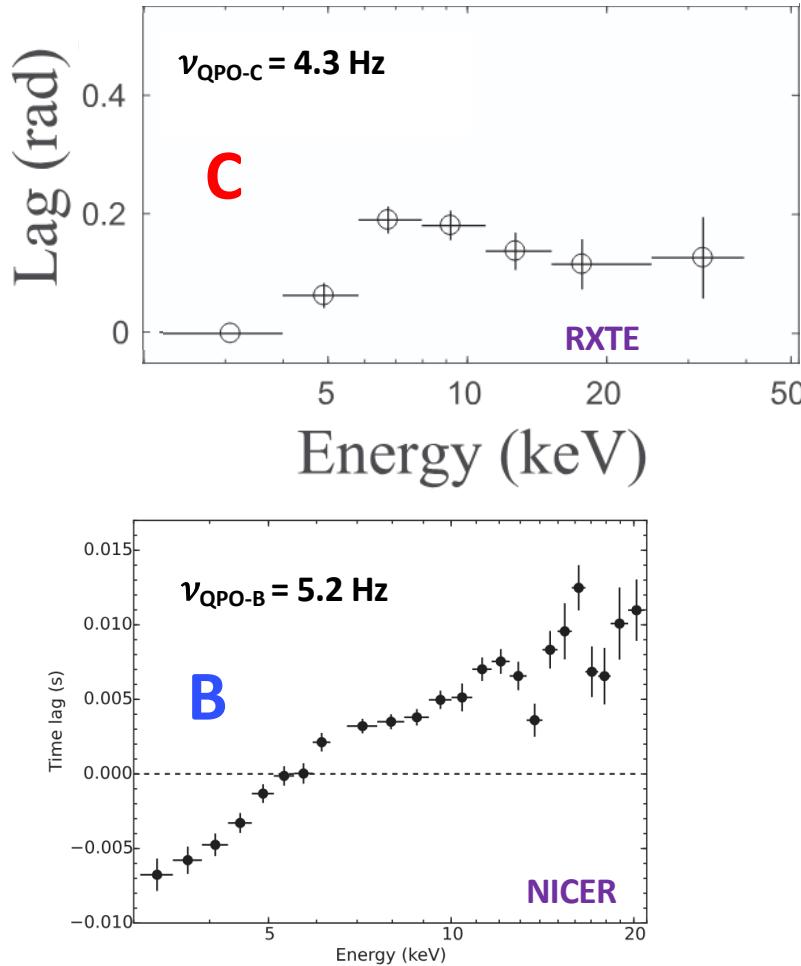
Type-C and -B QPOs in GX 339–4 in the HIMS



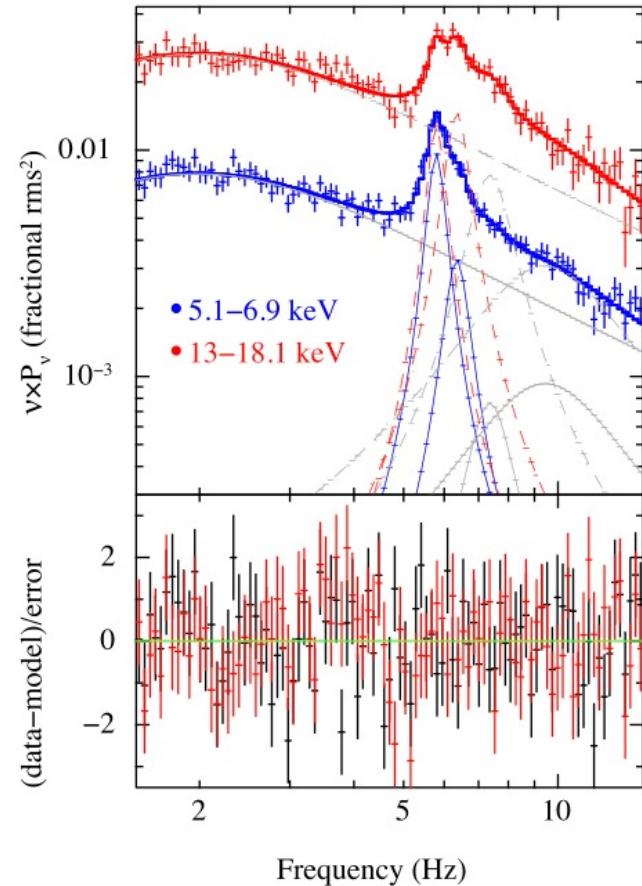
Type-C and -B QPOs in GX 339–4 in the HIMS



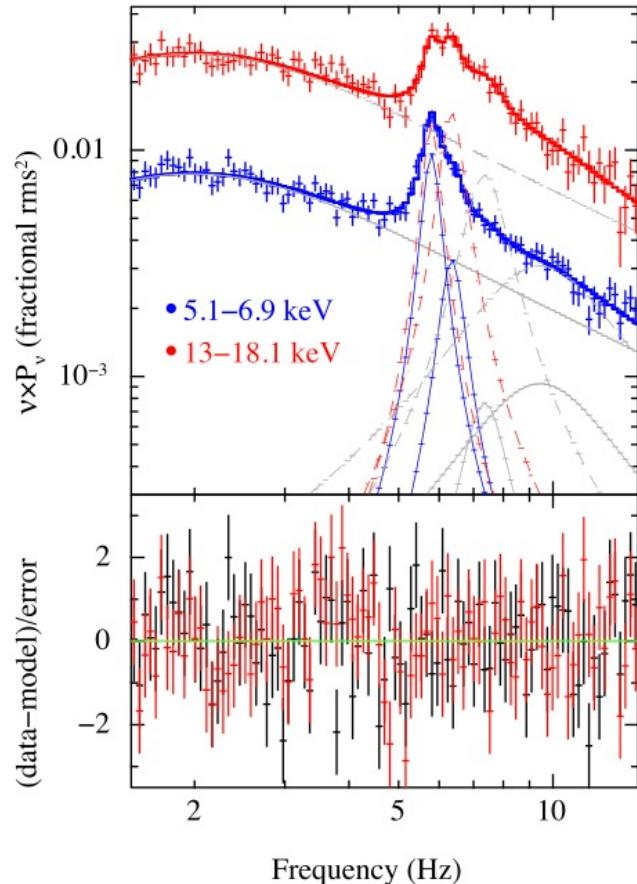
Type-C and -B QPOs in GX 339–4 in the HIMS



GRS 1915+105: no energy dependence of QPO frequency



GRS 1915+105: no energy dependence of QPO frequency



Model 1: 1 QPO, energy-dependent centroid frequency, FWHM and amplitude

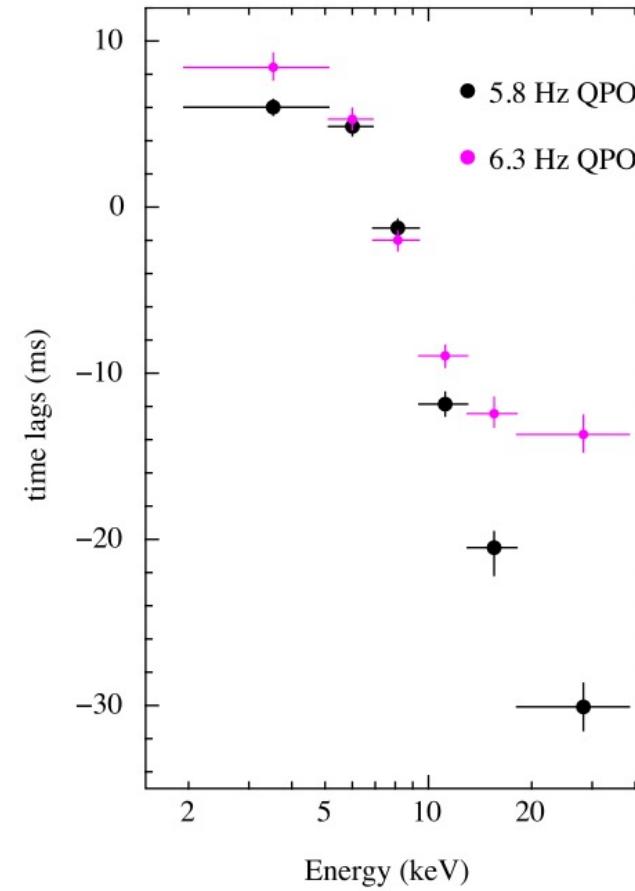
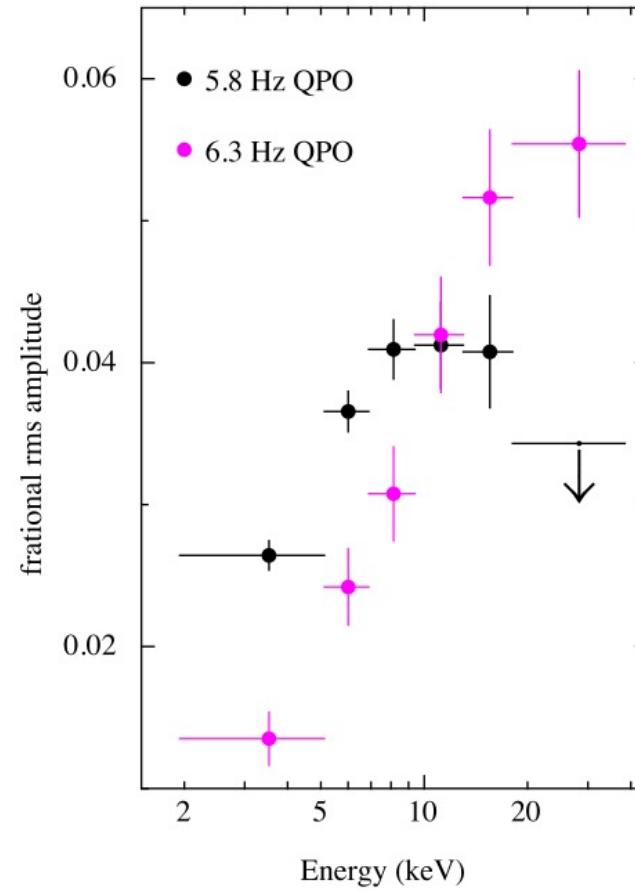
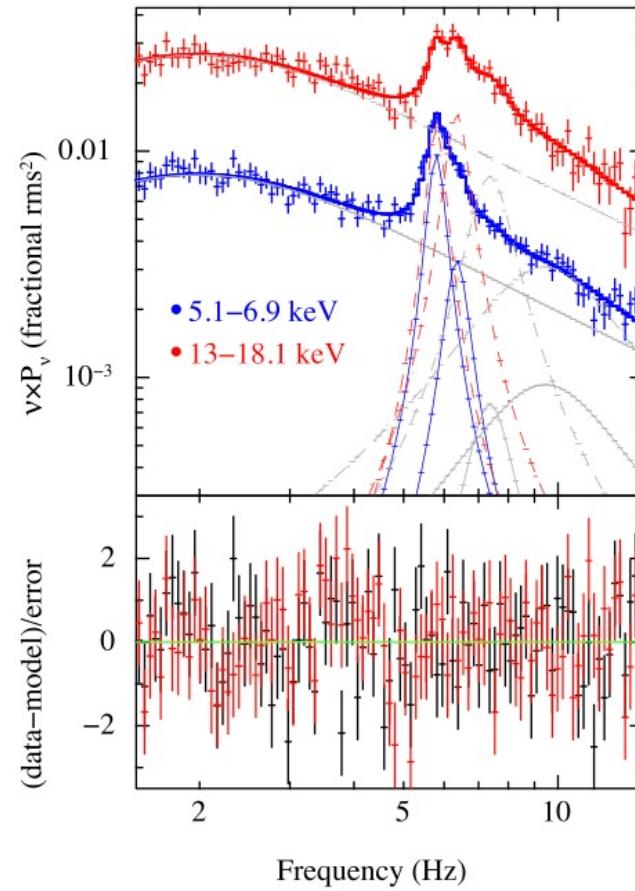
Model 2: 2 QPOs, each with the same frequency and FWHM in all bands, energy-dependent amplitudes

For $N=6$ energy bands:

Model 1: $\chi^2/\text{dof} = 601.9/528$

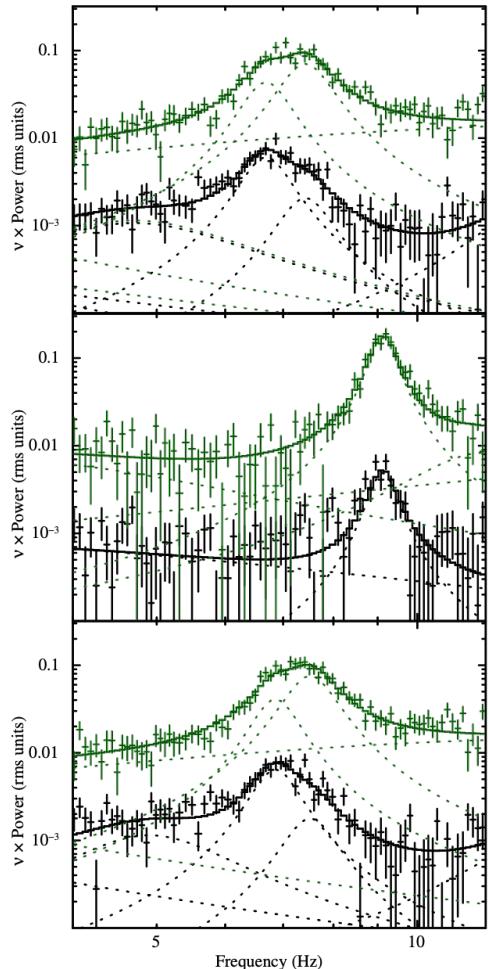
Model 2: $\chi^2/\text{dof} = 566.1/530$

GRS 1915+105: no energy dependence of QPO frequency

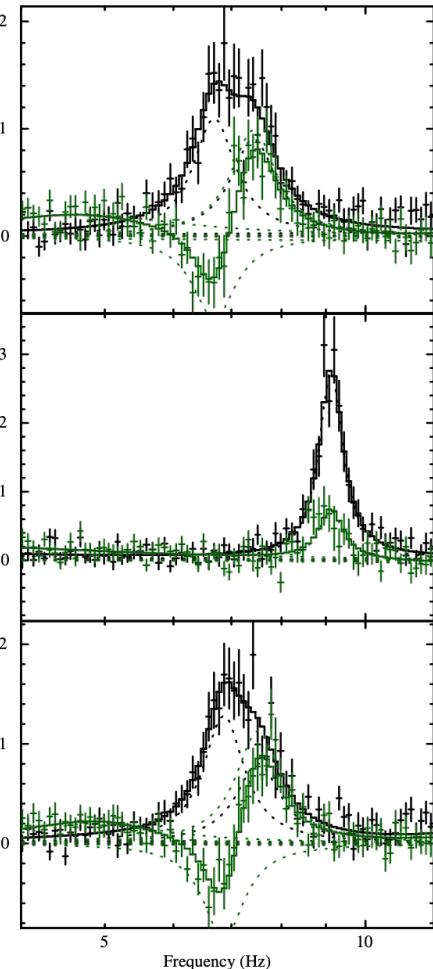


Swift J1727

PDS LE / PDS HE



Real / Imag (LE vs. HE)



For $N=12$ energy bands:

Model 1: $\chi^2/\text{dof} = 1504.8 / 1354$

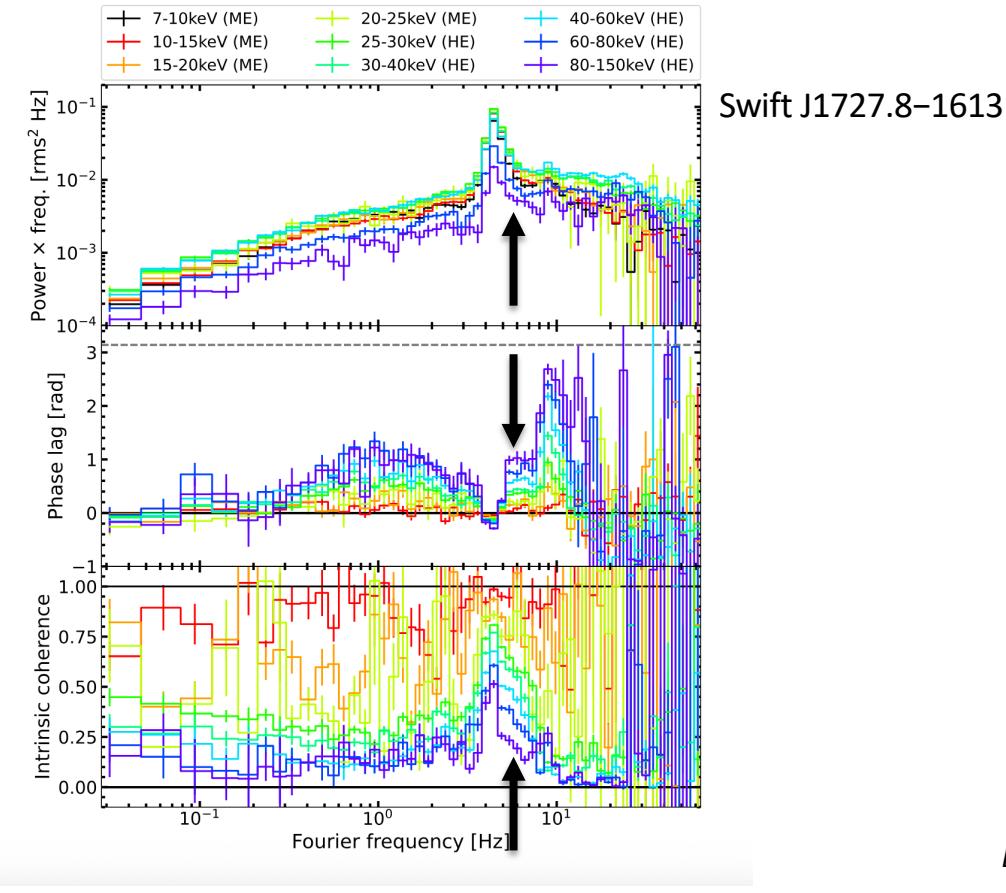
Model 2: $\chi^2/\text{dof} = 1495.3 / 1362$

dof Model 1 = $3N$

dof Model 2 = $4 + 2N$

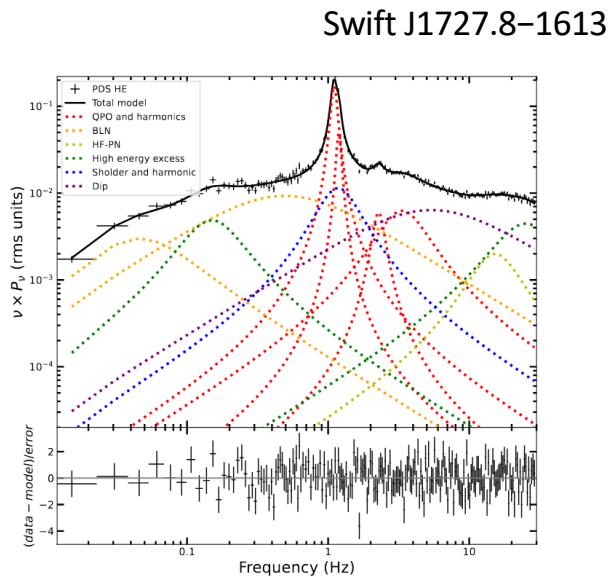
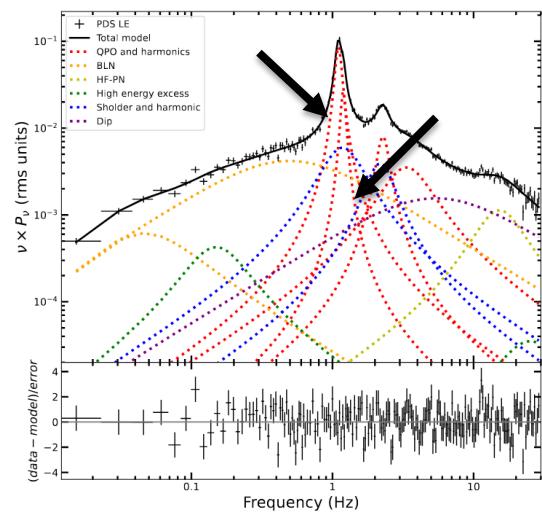
Why not seen before?

QPO asymmetry – Previous evidence



Bollemeijer et al. 2025

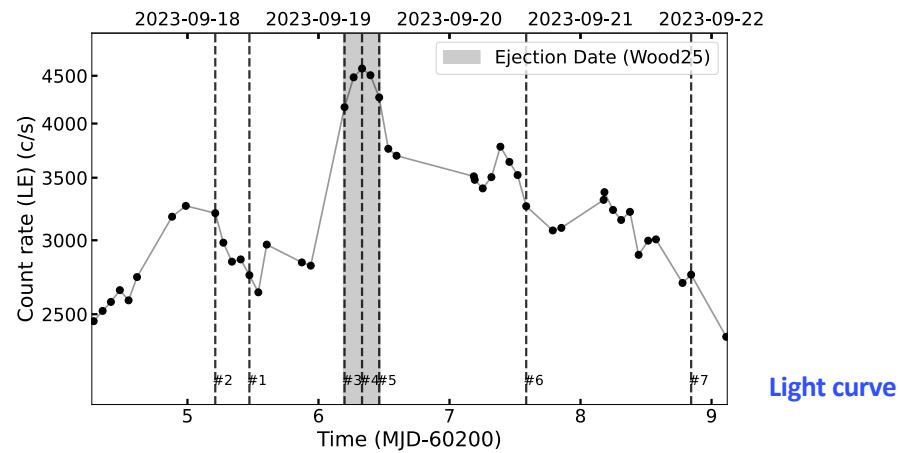
QPO asymmetry – Previous evidence

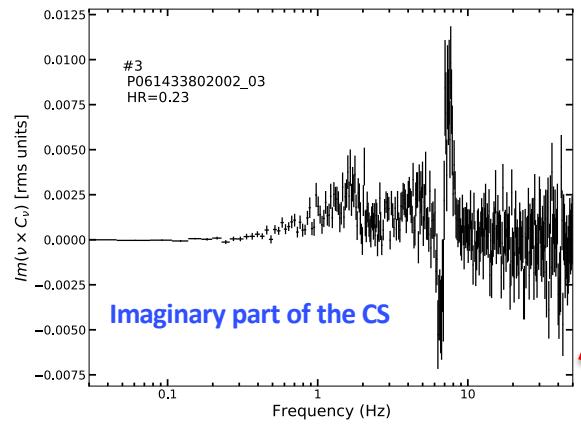


Jin et al. 2025

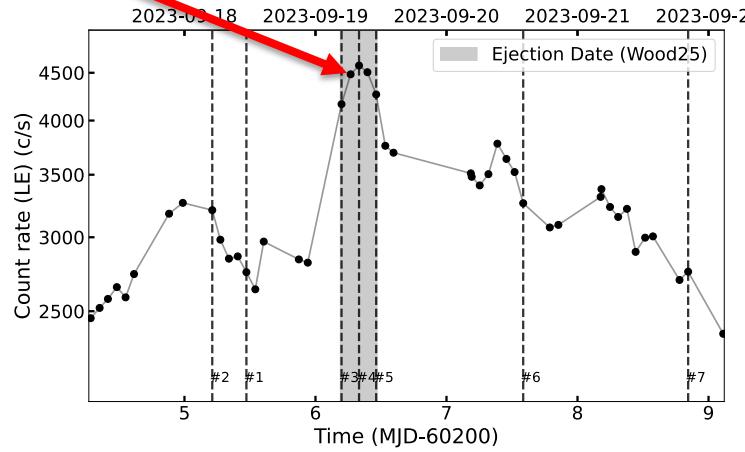
Evidence from the data, not the model

Evidence from the data (independent of the model assumptions)

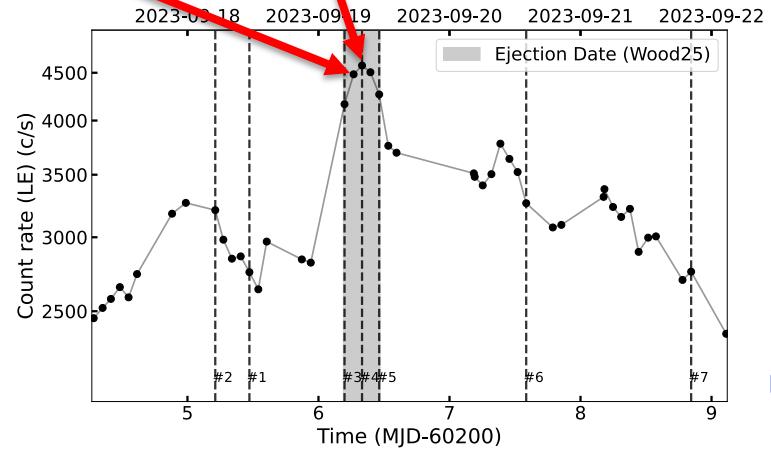
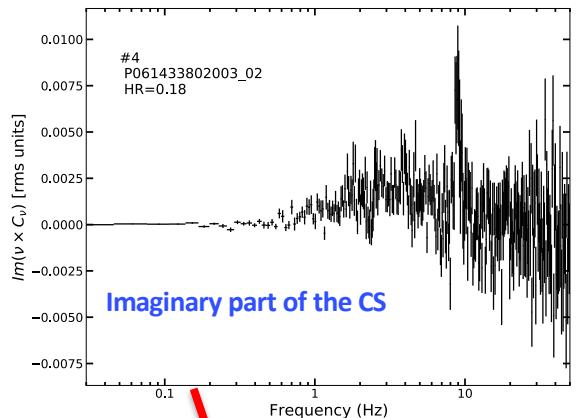
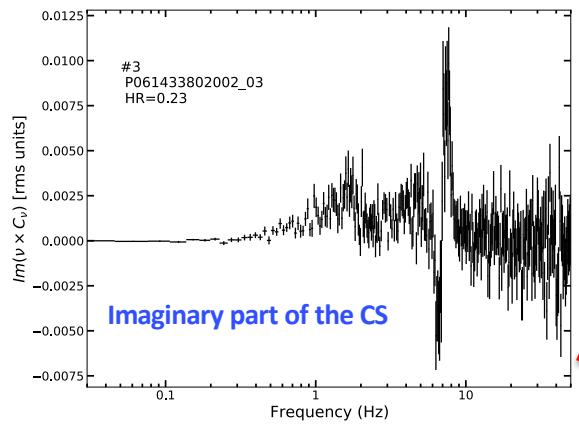




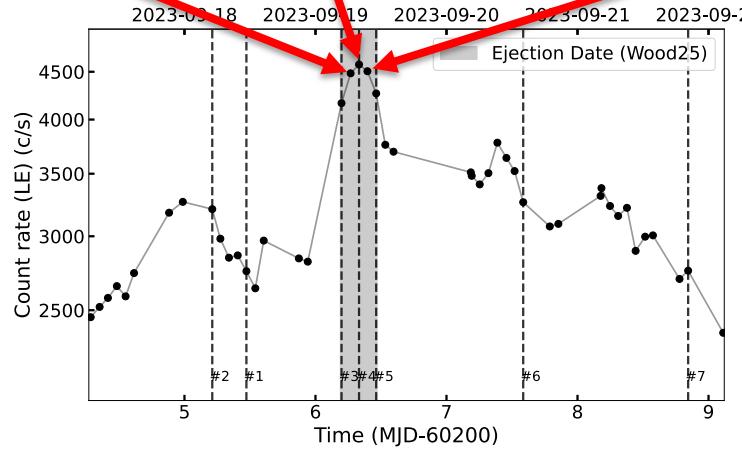
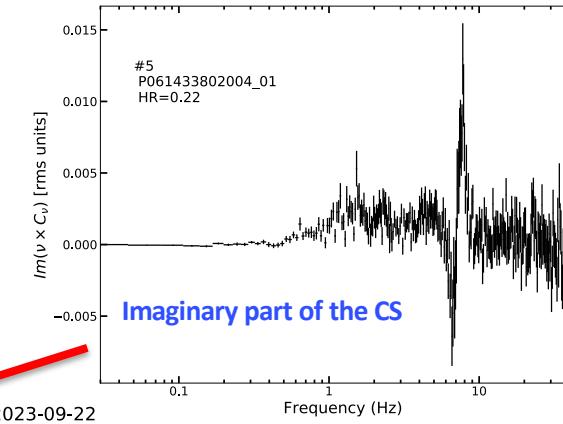
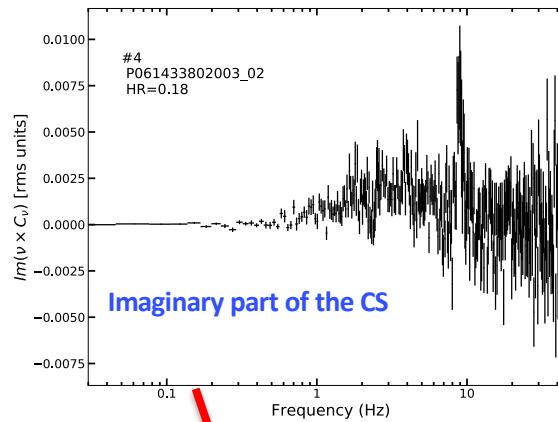
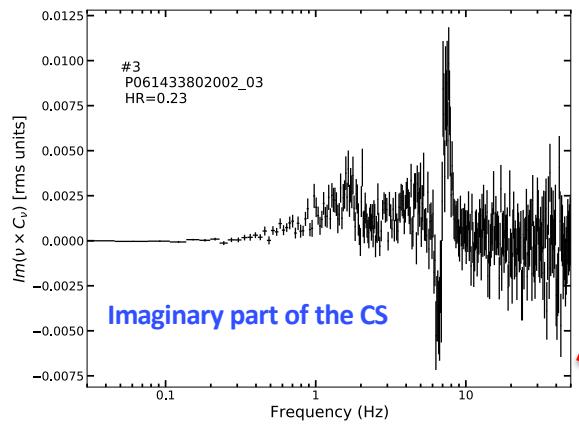
Imaginary part of the CS



Light curve

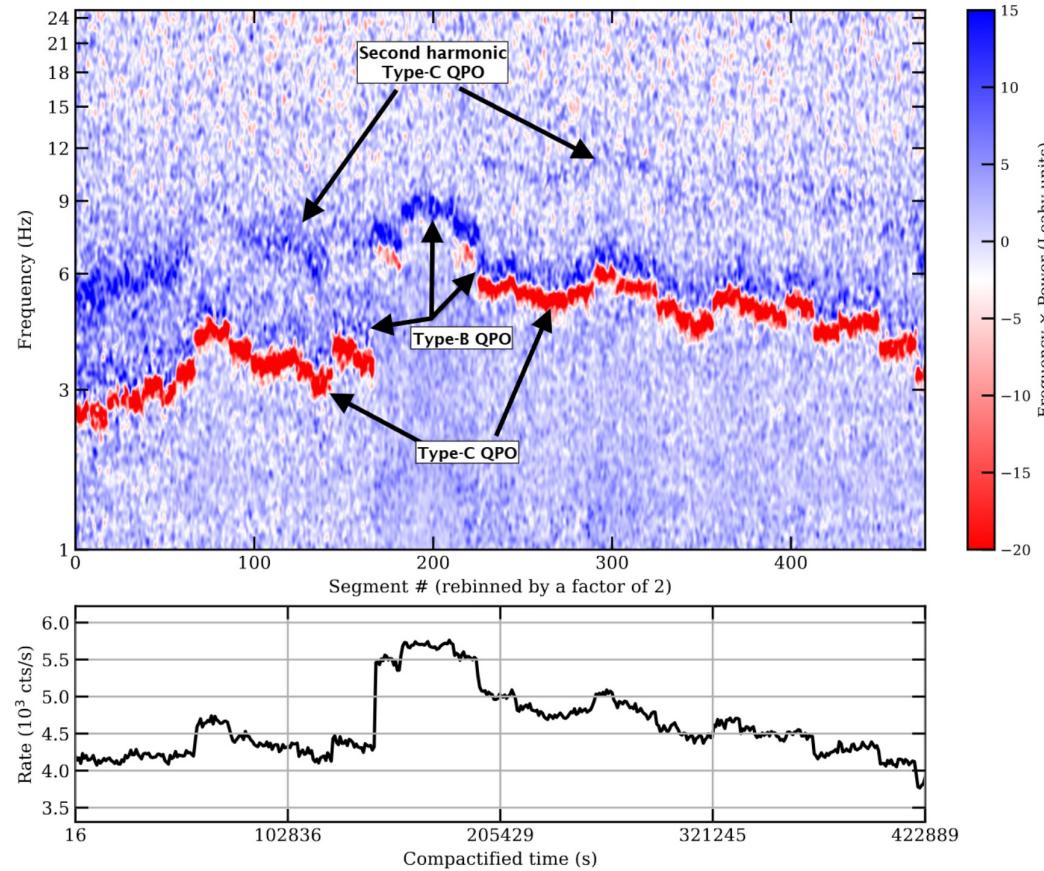


Light curve



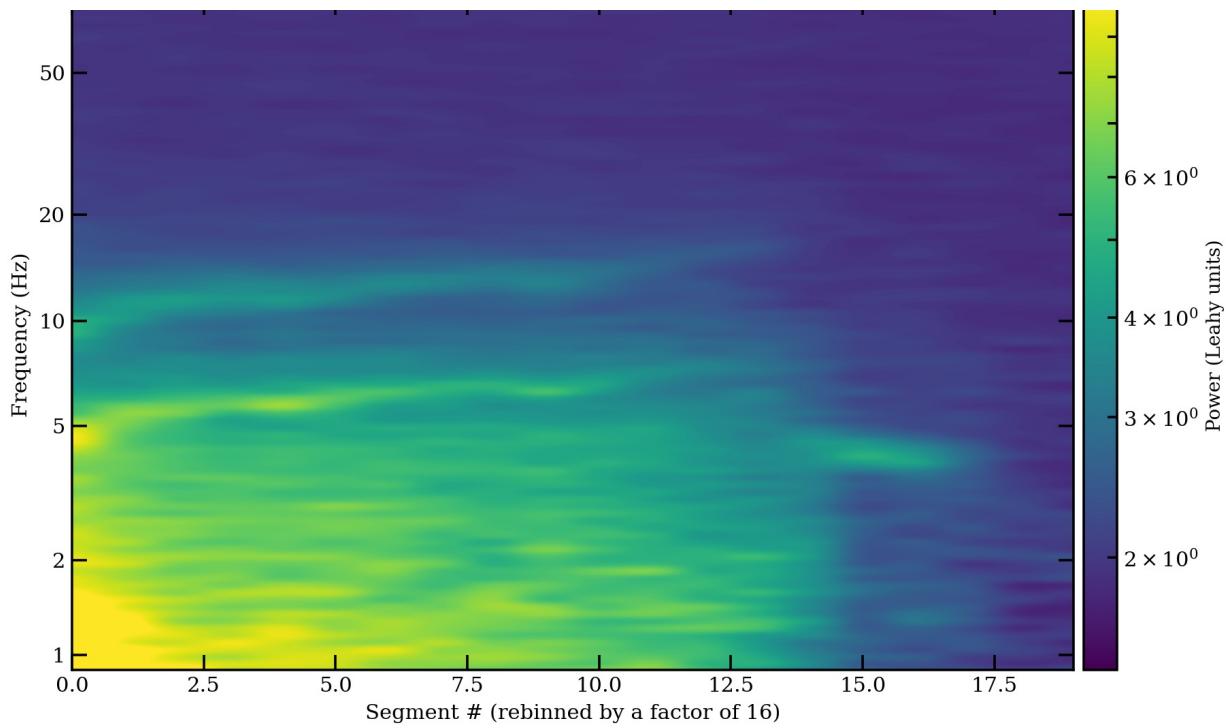
Light curve

Imaginary spectrogram (only data, no model!)



MAXI J1820+070

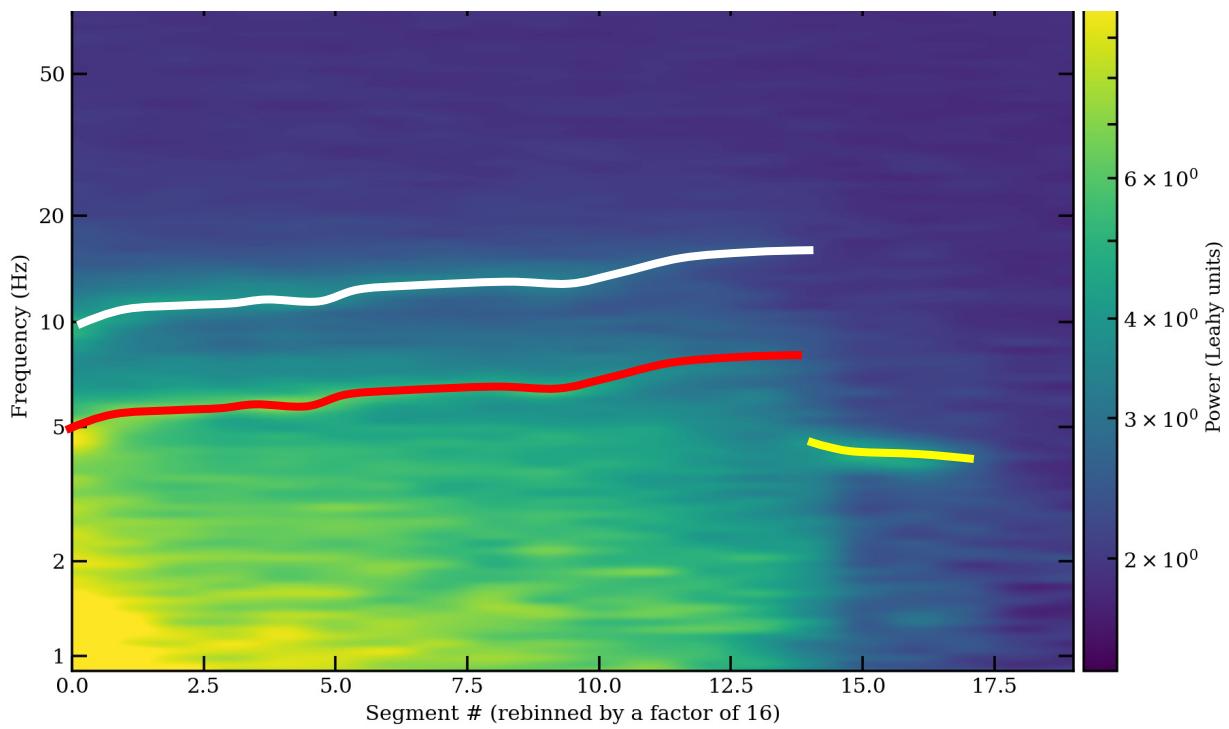
Spectrogram (a.k.a. dynamical PDS)
0.3 – 12 keV



Homan et al. 2020

MAXI J1820+070

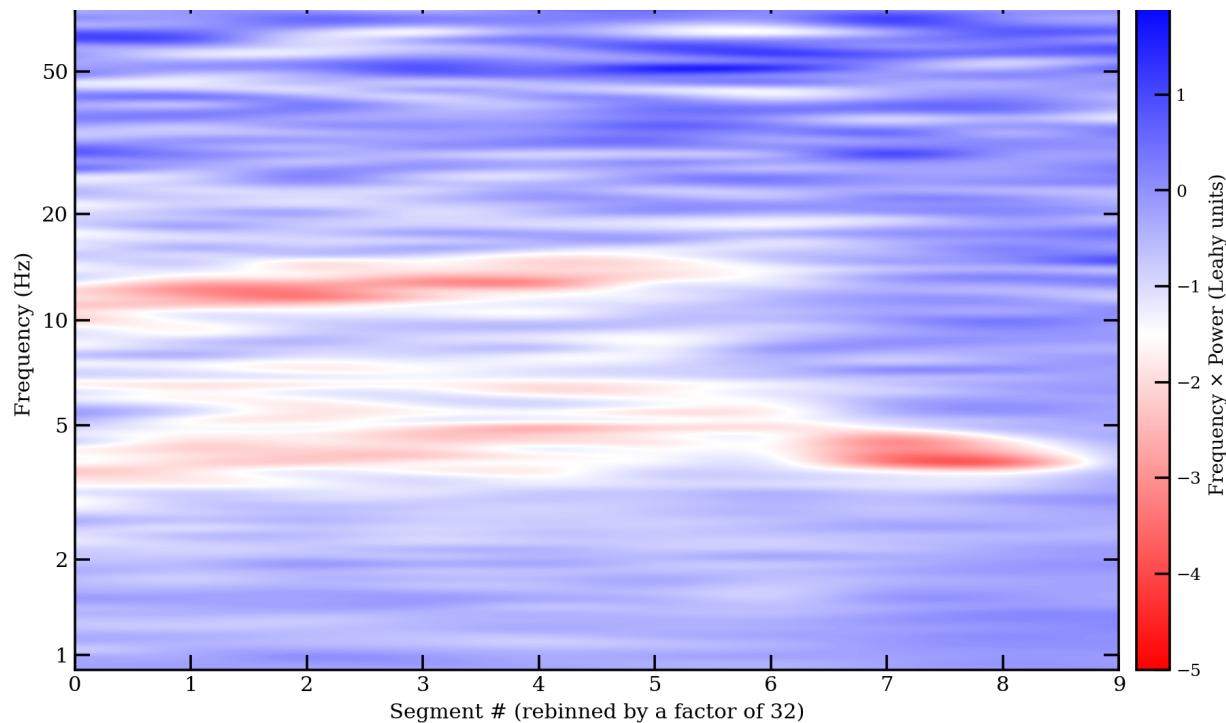
Spectrogram (a.k.a. dynamical PDS)
0.3 – 12 keV



Homan et al. 2020

MAXI J1820+070

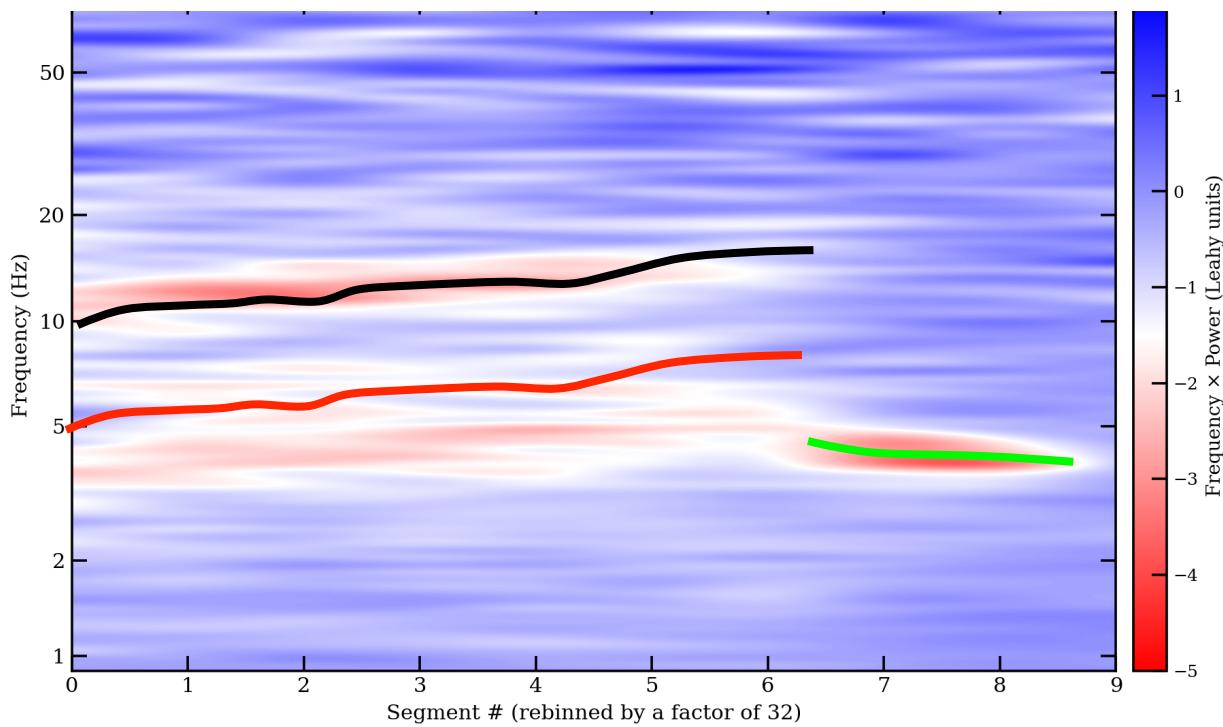
Imaginary spectrogram 0.3 – 2 keV / 2 – 12 keV



Jin et al. (in prep.)

MAXI J1820+070

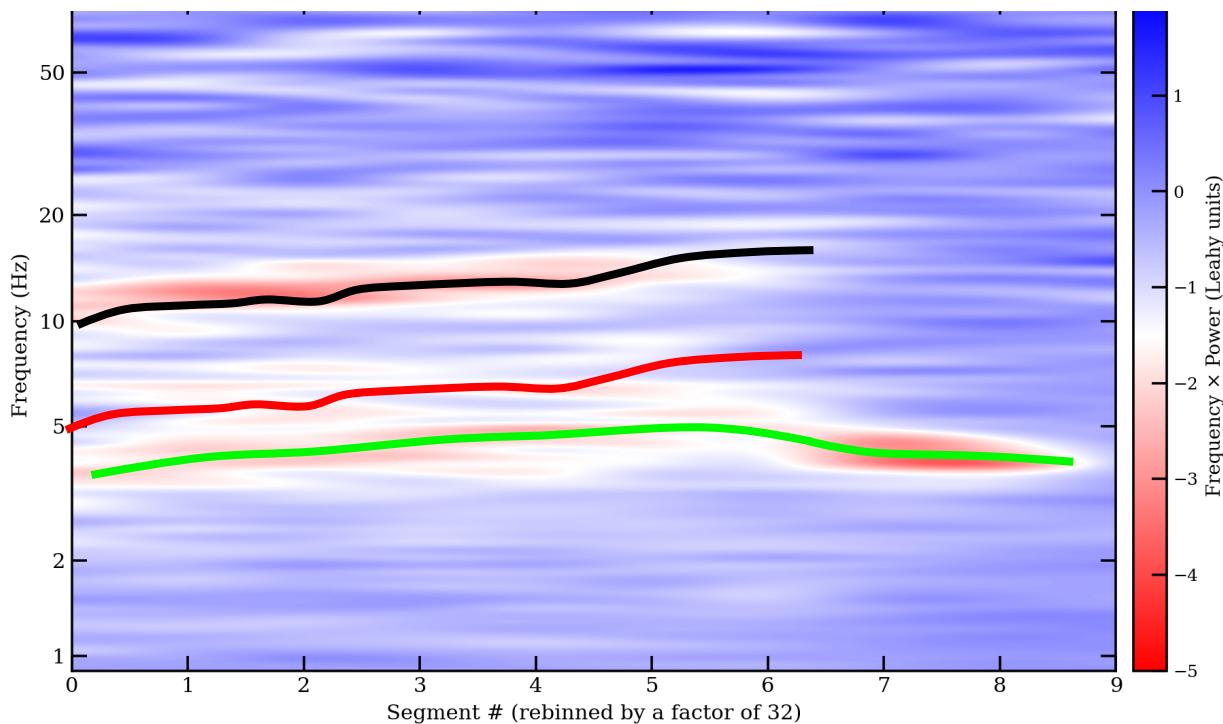
Imaginary spectrogram 0.3 – 2 keV / 2 – 12 keV



Jin et al. (in prep.)

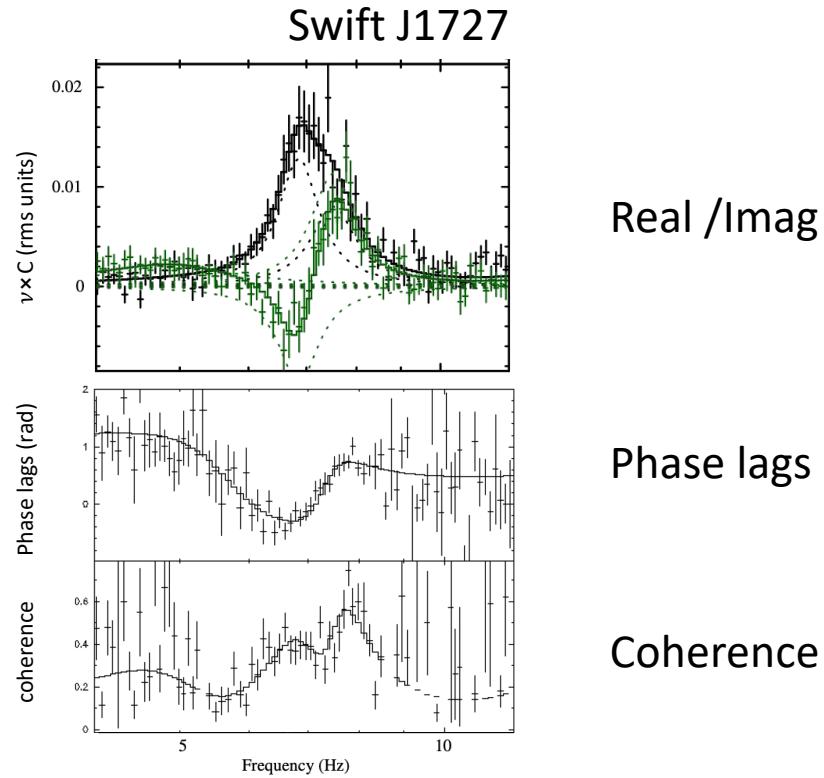
MAXI J1820+070

Imaginary spectrogram 0.3 – 2 keV / 2 – 12 keV



Jin et al. (in prep.)

**Fit power (not shown) and cross spectra
predict phase lags and coherence**



Real / Imag

Phase lags

Coherence

Swift J1727: Power and cross spectra

