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Order of growth

1) Prove $32n^2 + 17n + 1$ is $O(n^2)$

   Proof: consider c=33 and k=200.
   Then for any $n \geq k$, $n^2 \geq 200n \geq 17n+1$. Since n is positive, we also have $0 \leq 32n^2+17n+1$
   Therefore for any $n \geq k$, $0 \leq 32n^2+17n+1 \leq 32n^2 + n^2 = 33n^2 = cn^2$
   So $32n^2 + 17n+1$ is $O(n^2)$

   $32n^2 + 17n+1$ is not $O(n)$ nor $O(n\log n)$

   Proof by contradiction: suppose $32n^2 + 17n+1$ is $O(n)$
   Then there are c and k such that $0 \leq 32n^2+17n+1 \leq cn$ for every $n \geq k$ but $32n^2+17n+1 \leq cn$ implies that $n \leq 32n+17+(1/n) \leq c$ But this fails for any n that is greater than $32n+17+(1/n)$ and as n approaches infinity there won't be a c that is greater than $32n+17+(1/n)$ So we have a contradiction and $32n^2 + 17n+1$ is not $O(n)$

   Proof by contradiction: suppose $32n^2 + 17n+1$ is $O(n\log n)$
   Then there are a c and k such that $0 \leq 32n^2 + 17n+1 \leq C(n\log n)$ for every $n \geq k$
   However, there is no c that will satisfy $32n^2 + 17n+1$ being greater than $C(n\log n)$ as n gets closer to infinity.
   So $32n^2 + 17n+1$ must not be $O(n\log n)$

2) Prove $32n^2 + 17n+1$ is both $\Omega(n^2)$ and $\Omega(n)$

   To prove $32n^2 + 17n+1$ is $\Omega(n^2)$, prove there exists a c such that $c(n^2) < 32n^2 + 17n+1$
   For any $c \leq 32$ this inequality holds true.
   Therefore $32n^2 + 17n+1$ is $\Omega(n^2)$

   To prove $32n^2 + 17n+1$ is $\Omega(n)$, prove there exists a c such that $c(n) < 32n^2 + 17n+1$
   For every $n>0$, if c is less than $(32n^2 + 17n+1)/n$ then this inequality holds true.
   For example, if n=20 and c<81 then $(81)(20) < 32((20)^2)+17(20)+1$
   Therefore, there exists a c such that $c(n) < 32n^2 + 17n+1$ and $32n^2 + 17n+1$ is $\Omega(n)$.

   Prove $32n^2 + 17n+1$ is not $\Omega(n^3)$
   If $32n^2 + 17n+1$ was $\Omega(n^3)$ then there would exist a c such that $c(n^3) < 32n^2 + 17n+1$

For any n>0. This implies that every c≤(32/n)+(17/ n^2)+(n^3) as you approach infinity the right side will get very small and it won't be possible for there to be a c that fits the inequality

3) Prove that $32n^2$ +17n+1 is ø(n^2)

f(n) is ø(n^2) if f(n) is bounded both above and below by n^2 therefore it is both O(n^2) and Ω(n^2),
To prove $32n^2$ +17n+1 is Ω(n^2) and bounded below, prove there exists a c such that c(n^2)< $32n^2$ +17n+1
For any c≤32 this inequality holds true.
Therefore $32n^2$ +17n+1 is Ω(n^2)
To prove $32n^2$ +17n+1 is O(n^2) and bounded above consider c=33 and k=200.
Then for any n ≥ k, n^2≥200n ≥17n+1. Since n is positive, we also have 0≤$32n^2$+17n+1
Therefore for any n ≥ k, 0≤$32n^2$+17n+1≤32 n^2 + n^2 =$33n^2$=cn^2
So $32n^2$ +17n+1 is O(n^2)
Since it is bounded both above and below by n^2, $32n^2$ +17n+1 is ø(n^2)

Prove that $32n^2$ +17n+1 is neither ø(n) nor ø(n^3)

in order for $32n^2$ +17n+1 to be ø(n) it needs to be bounded above and below i.e. both O(n) and Ω(n)
It is not bounded above and therefore not O(n) as proven by contradiction: suppose $32n^2$ +17n+1 is O(n)
Then there are c and k such that 0≤$32n^2$+17n+1≤cn for every n ≥ k but $32n^2$+17n+1≤cn implies that n ≤32n+17+(1/n) But this fails for any n that is greater than 32n+17+(1/n) So we have a contradiction and $32n^2$ +17n+1 is not O(n)and there it is not bounded from above by n and cannot be ø(n)

In order for $32n^2$ +17n+1 to be ø(n^3) in needs to be both O(n^3) and Ω(n^3)
If $32n^2$ +17n+1 was Ω(n^3) then there would exist a c such that c(n^3)< $32n^2$ +17n+1
For any n>0. This implies that every c≤(32/n)+(17/ n^2)+(n^3) as you approach infinity the right side will get very small and it won't be possible for there to be a c that fits the inequality. Therefore, it is not bounded from below by n^3 so $32n^2$ +17n+1 is not ø(n^3)
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