If g,f are surjections then (g o f) is a surjection.

Let y be an element of C. Then, since g from B to C is a surjection, there exists $u \in B$ such that $g(u) = y$ and $g(u) \in C$. Since f from A to B is a surjection and $u \in B$, there exists $v \in A$ such that $f(v) = u$ and $f(v) \in B$. We would like to find $x \in A$ s.t. $g(f(x)) = y$ and $g(f(x)) \in C$.

If f is a surjection then $f(A)^c \subset f(A^c)$

Let x be an element of $(f(A))^c$. Then $x \notin f(A)$. Then it is not that case that $x \in f(A)$. We would like to find $y \in (A)^c$ s.t. $f(y) = x$. But $y \in (A)^c$ if and only if $y \notin A$. But $y \notin A$ if and only if it is not that case that $y \in A$.

Prove that $(A \cap B)^c \subset A^c \cup B^c$

Let x be an element of $(A \cap B)^c$. Then $x \notin A \cap B$. Then it is not that case that $x \in A \cap B$. We would like to show that $x \in (A)^c \cup (B)^c$, i.e. that $x \in (A)^c$ or $x \in (B)^c$. We would like to show that $x \in (A)^c$, i.e. that $x \notin A$. We would like to show that $x \notin A$, i.e. that it is not that case that $x \in A$. We would like to show that $x \in (B)^c$, i.e. that $x \notin B$. We would like to show that $x \notin B$, i.e. that it is not that case that $x \in B$.

Prove that $A^c \cup B^c \subset (A \cap B)^c$

Let x be an element of $(A)^c \cup (B)^c$. Then $x \in (A)^c$ or $x \in (B)^c$. Since $x \in (A)^c$, we have that $x \notin A$. Then it is not that case that $x \in A$. Since $x \in (B)^c$, we have that $x \notin B$. Then it is not that case that $x \in B$. We would like to show that $x \in (A \cap B)^c$, i.e. that $x \notin A \cap B$. We would like to show that $x \notin A \cap B$, i.e. that it is not that case that $x \in A \cap B$. We would like to show that $x \in (A \cap B)^c$, i.e. that $x \notin A \cap B$. We would like to show that $x \notin A \cap B$, i.e. that it is not that case that $x \in A \cap B$.

Prove that $(A \cup B)^c = A^c \cap B^c$

We would like to show that $(A \cup B)^c \subset (A)^c \cap (B)^c$, i.e. that $(A \cup B)^c \subset (A)^c$ and $(A \cup B)^c \subset (B)^c$.

Prove that $A^c \cap B^c \subseteq (A \cup B)^c$.

Let x be an element of $(A)^c \cap (B)^c$. Then $x \in (A)^c$ and $x \in (B)^c$. Then $x \notin A \cup B$ and $x \notin A$. Then it is not that case that $x \in A$. Since $x \in (B)^c$, we have that $x \notin B$. Then it is not that case that $x \in B$. Since $x \notin A \cup B$, it is not that case that $x \in A \cup B$. We would like to show that $x \in (A \cup B)^c$, i.e. that $x \notin A \cup B$. We would like to show that $x \notin A \cup B$, i.e. that it is not that case that $x \in A \cup B$. But this is clearly the case, so we are done.

If A, B,and C are open sets, then $A \cap (B \cap C)$ is also open.

Let x be an element of $A \cap B \cap C$. Then $x \in A$ and $x \in B \cap C$. Therefore, since A is open, there exists $\eta > 0$ such that $u \in A$ whenever $d(x, u) < \eta$ and $x \in B$ and $x \in C$. Therefore, since B is open, there exists $\theta > 0$ such that $v \in B$ whenever $d(x, v) < \theta$ and since C is open, there exists $\alpha > 0$ such that $w \in C$ whenever $d(x, w) < \alpha$. We would like to find $\delta > 0$ s.t. $y \in A \cap B \cap C$ whenever $d(x, y) < \delta$. But $y \in A \cap B \cap C$ if and only if $y \in A$ and $y \in B \cap C$. We know that $y \in A$ whenever $d(x, y) < \eta$. But $y \in B \cap C$ if and only if $y \in B$ and $y \in C$. We know that $y \in B$ whenever $d(x, y) < \theta$ and that $y \in C$ whenever $d(x, y) < \alpha$. Assume now that $d(x, y) < \delta$. Then $d(x, y) < \eta$ if $\delta \leqslant \eta$, $d(x, y) < \theta$ if $\delta \leqslant \theta$ and $d(x, y) < \alpha$ if $\delta \leqslant \alpha$.

If A and B are closed sets, then $A \cup B$ is also closed.

Let (a_n) and a be such that (a_n) is a sequence in $A \cup B$ and $a_n \to a$. We would like to show that $a \in A \cup B$, i.e. that $a \in A$ or $a \in B$. Since A is closed and $a_n \to a$, $a \in A$ if (a_n) is a sequence in A. Since B is closed and $a_n \to a$, $a \in B$ if (a_n) is a sequence in B. Take n. Take n'.