

Optimal Timing Analytical Solution (Agent)

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Constants

$$\begin{aligned} P_{50} &= 1/2, P_{100} = 1, P_0 = 0 \\ \mu_1 &= \text{Player Mean Reach Time}; \mu_2 = \text{Agent Mean Reach Time} \\ \sigma_1 &= \text{Player SD Reach Time}; \sigma_2 = \text{Agent SD Reach Time} \\ \mu_d &= \frac{\mu_1 - \mu_2}{\sigma_d} \\ \sigma_d &= \sqrt{\sigma_1^2 + \sigma_2^2} \\ DT_1 &= \text{Player Decision Time}; DT_2 = \text{Agent Decision Time} \\ RT_1 &= \text{Player Reaction Time}; \\ RMT_1 &= \text{Player Reaction+Movement Time}; RMT_2 = \text{Movement Time} \end{aligned}$$

Probabilites

$$\begin{aligned} P_1 &= P(\text{win} | \mu_d \leq RT_1) = P(\mu_d \leq RT) * P(X_1 < 1500) * P(X_2 < 1500) * P_{50} \\ P_2 &= P(\text{win} | \mu_d > RT_1) = P(\mu_d > RT) * P(X_1 < 1500) * P(X_2 < 1500) * P_{100} \\ P_3 &= P(\text{win} | X_1 < 1500 \cap X_2 > 1500) = P(X_1 < 1500) * P(X_2 > 1500) * P_{100} \\ P_4 &= P(\text{win} | X_1 > 1500 \cap X_2 < 1500) = P(X_1 < 1500) * P(X_2 < 1500) * P_0 \end{aligned}$$

Win Rate

$$\text{Win Rate} = P_1 + P_2 + P_3 + P_4$$