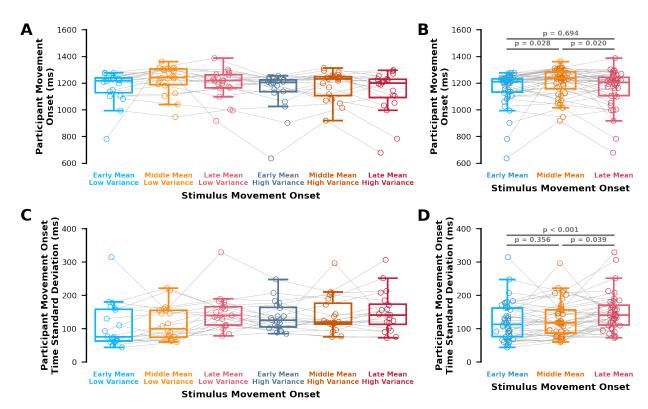
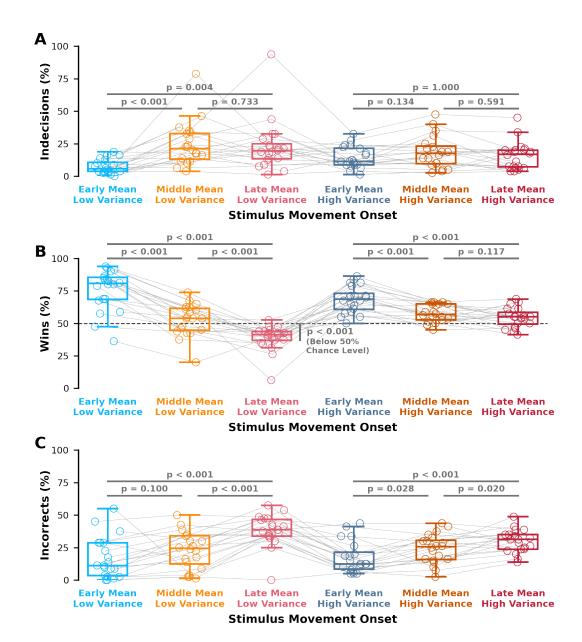
Supplementary Supplementary A: High Variance Results



Supplementary Figure 1: Timing Behaviour. A) Average participant movement onset (y-axis) for each experimental condition (x-axis). We found a significant main effect of stimulus movement onset mean (F[1.551,29.475] = 4.36, p = 0.030) and variance (F[1.000,19.000], p = 0.017)) on participant movement onset. There was no significant interaction (F[1.657,31.474], p = 0.565) B) Corresponding participant movement onset (y-axis) for the three different mean stimulus onsets (x-axis) when collapsed across variance. During the early mean conditions, participants had a high probability of reacting to the stimulus and reaching a target within the time constraint. For the middle mean conditions, participants often waited longer to react and had a later guess than the early mean conditions. Waiting longer to react and/or late guessing led to later movement onset. In the late mean condition, participants waited to react to the stimulus, but ended up guessing a majority of the time (see Supplementary Figure 6). The combination of waiting for the stimulus and eventually guessing, led to an earlier movement onset compared to the middle mean condition. For both the middle mean and late mean conditions, late guessing often led to indecisions (see Fig. 2). C) Participant movement onset standard deviation (y-axis) for each experimental condition (x-axis). We found a significant main effect of mean (F[1.383, 26.284], p = 0.018) and variance (F[1.000,19.000], p < 0.001) on participant movement onset standard deviation There was no significant interaction between stimulus movement onset mean and variance (F[1.978, 37.575], p = 0.097) D) Corresponding participant movement onset standard deviation (y-axis) for the three different mean onsets (x-axis), when collapsed across variance. In the late mean conditions, waiting to react and then guessing led to a larger movement onset standard deviation compared to the early mean and middle mean conditions. Box and whisker plots display the 25th, 50th, and 75th percentiles. Open gray circles and connecting lines represent individual data.



Supplementary Figure 2: Trial Outcomes. A) Indecisions (%) **B)** Wins (%), and **C)** Incorrects (%) for each condition. **A)** We found that the middle mean low variance and late mean low variance conditions had a significantly greater number of indecisions compared to the early mean low variance condition. That is, in the middle mean low variance and late mean low variance conditions, participants often waited to react to the stimulus but ended up guessing late. These late guesses often led to indecisions. **B)** Interestingly, we also found that the average win percentage (4X.X%) was significantly below the 50% chance level (p < 0.001) in the late mean low variance condition, clearly demonstrating suboptimal behaviour. Critically, participants would have earned more points if they had simply guessed early on all trials, rather than attempting to react to the stimulus. Collectively, these results show that participants were suboptimal decision makers that led to excessively indecisive behaviour.

Supplementary B: Optimal Models

We tested three different models to investigate the ability of participants to maximize reward during Experiment 1: i) No Switch Time Model, ii) Full Switch Time Model and iii) Partial Switch Time Model (Fig. 2). Each model represents a different hypothesis on how

humans time their decisions, which we address further below. Intuitively, participants should react to the stimulus on trials where it moved earlier in time and guess on trials where it moved later in time. That is, reacting to early stimulus movement onsets ensures they can select the correct target. Likewise, guessing on late stimulus movement onsets affords the participant a 50% chance of selecting the correct target. However, if participants wait too long to react to a late stimulus movement onset, then they might make too many indecisions and fail to maximize reward. We modelled the time to switch from reacting to guessing as an optimal stopping problem. This differs from past models that have used a similar Bayesian framework, which has been to determine reach aim (trommershauser) and reach timing (Hudson, Maloney, Landy 2008). Here a model represents a decision-maker that selects a stopping time, τ . This stopping time determines when to stop waiting to react to the stimulus and switch to guessing one of the two targets. The optimal stopping time is determined from a decision policy that maximizes reward, given task constraints (i.e. stimulus movement onset distribution and time constraint), and knowledge of sensorimotor delays and uncertainties.

Model Parameters

Here we define the parameters used for the three models. Note that not all the parameters are used in each model, which we specify further below and in Fig. 2. The Response Time parameters has both a mean (μ_{rt}) and uncertainty (σ_{rt}) that represents the delay between observing the stimulus movement onset to initiating a movement. Neuromechanical Delay is the mean (μ_{nmd}) and uncertainty (σ_{nmd}) of the time between a volitional decision to move and movement onset. Movement Time represents the mean (μ_{mt}) and uncertainty (σ_{mt}) of the delay between movement onset and reaching a target. Stimulus Movement onset is knowledge of the mean (μ_A) and uncertainty (σ_A) of the stimulus's movement onset distribution. Timing uncertainty (σ_{τ}) is the participant uncertainty around the intended stopping time, τ . Switch Time represents the additional delay (μ_{switch}) and uncertainty (σ_{switch}) of switching from reacting to guessing. All probability distributions are assumed to be normally distributed with a mean μ and standard deviation σ . Each of our three models has a different set of known and unknown parameters. A model decision policy has full knowledge of known parameters when determining the optimal decision time. A model decision policy has no or partial knowledge of unknown parameters when determining the optimal decision time.

No Switch Time Model

The No Switch Time Model had the following known parameters: response time mean and uncertainty, movement time mean and uncertainty, neuromechanical delay mean and uncertainty, timing uncertainty, and stimulus movement onset. This model had no unknown parameters. Importantly, the No Switch Time Model did not include the switch time mean and uncertainty in neither the known nor the unknown parameter sets. The No Switch Time Model reflects the hypothesis that there is no additional delay and uncertainty when switching from reacting to guessing.

Full Switch Time Model

The Full Switch Time Model had the following known parameters: response time mean and uncertainty, neuromechanical delay mean and uncertainty, movement time mean and uncertainty, stimulus movement onset, timing uncertainty, and switch time mean and uncertainty. This model did not have any unknown parameters. The Full Switch Time

Model reflects the hypothesis that participants fully account for a delay and uncertainty associated with switching from reacting to guessing.

Partial Switch Time Model

The Partial Switch Time Model had the following known parameters: response time mean and uncertainty, neuromechanical delay mean and uncertainty, movement time mean and uncertainty, and stimulus movement onset. This model had the following unknown parameters: switch time mean and uncertainty, and timing uncertainty. The Partial Switch Time Model reflects our hypothesis that participants do not account for the delay and uncertainties associated with guessing.

General Formulation for the Models

We begin by defining the general normal probability density function and cumulative density function:

$$X \sim \mathcal{N}(\mu, \sigma)$$
 (1)

$$f_X(x;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$$
 (2)

$$F_X(b) = P(X \le b) = \int_{-\infty}^b f_X(x; \mu, \sigma) dx \tag{3}$$

X is the random variable drawn from a normal distribution $\mathcal N$ with some mean and standard deviation. $f_X(x;\mu,\sigma)$ is the normal probability density function over the variable x, with a mean (μ) and standard deviation (σ) . $F_X(b)$ is the normal cumulative density, which is the integral of the probability density function $f_X(x;\mu,\sigma)$ from $-\infty$ to b. Throughout, capitalized variables denote normally distributed random variables. Additionally, lowercase variables such as x refer to the realization of the random variable X. The lowercase notation simply indicates that the value is known and is not a random variable.

Next we define the mean and standard deviation of the participant movement onset separately for reaction decisions and guess decisions. We begin by defining A as the random variable drawn from the stimulus's movement onset distribution (Eq. 4) and T as the random variable drawn from the stopping time distribution (Eq. 5).

$$S \sim \mathcal{N}(\mu_S, \sigma_S)$$
 (4)

$$T \sim \mathcal{N}(\tau, \sigma_{\tau})$$
 (5)

Consider a and t as the realizations of random variables A and T for a single trial. On any specific trial, a decision-maker will react if the stimulus movement onset is before the stopping time (i.e., a < t). Conversely, the decision-maker will guess at the stopping time if the stimulus has not moved by the stopping time (i.e., a > t). Thus, the probability that a participant will react or guess depends on the participants choice of a stopping time τ . Specifically, the probability that the participant will react is the probability that the random variable A is less than T.

$$P(React|\tau) = P(\tau > \mu_S; \sigma_\tau, \sigma_S)$$
 (6)

Since the participants can only react or guess, the probability of guessing is simply one minus the probability of reacting.

$$P(Guess|\tau) = P(\tau < \mu_S; \sigma_\tau, \sigma_S) \tag{7}$$

As a result, a decision-maker only reacts to the portion of the stimulus's distribution that is prior to their stopping time. Thus, the participant reacts to a truncated distribution of the stimulus movement onset. The truncated stimulus movement onset distribution is generated from only taking the random variable A if it is less than the random variable A. We can then define an indicator function ($\mathbb{1}_{a < t}$), which is equal to one if the realized value A is less than A and zero otherwise:

$$\mathbb{1}_{s < t} = \begin{cases} 1 & \text{if } s < t, \\ 0 & \text{otherwise.} \end{cases}$$
 (8)

The mean of the truncated stimulus movement onset distribution ($\mu_{A_{react}}$) can be calculated by combining the indicator function and the method of moments. We start by calculating the expected value of the random variable A, which is the integral from $-\infty$ to ∞ of the value a multiplied by the probability density function $f_A(a)$.

$$\mu_S = \mathbb{E}[A] = \int_{-\infty}^{\infty} a \cdot f_S(s) da \tag{9}$$

Note that the expected value of the full stimulus movement onset distribution defined here is equivalent to the mean of the distribution. For the truncated stimulus movement onset distribution, we only take values a if they are less than t. Since the inclusion of a depends on t, we calculate the mean of the truncated stimulus movement onset distribution by integrating over every possible combination of a and t. Inside the double integral, we multiply the realized value a by its probability density function (i.e., $f_A(a)$). We also multiply the probability density function for t (i.e., $f_T(t)$) and the indicator function ($\mathbb{1}_{a < t}$). Since this double integral multiplies two gaussian probability density functions and only sums values if a is less than t, we must normalize by dividing by the probability that A is less than T. The mean (first moment) of the truncated stimulus movement onset distribution is

$$\mu_{S_{react}} = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} s \cdot f_S(s) \cdot f_T(t) \cdot \mathbb{1}_{s \in S} \ dadt}{P(S < T)}$$
(10)

Similarly we can find the standard deviation of the truncated stimulus movement onset distribution, where variance is the second moment, by:

$$\sigma_{S_{react}}^2 = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (s - \mu_{S_{react}})^2 \cdot f_S(s) \cdot f_T(t) \cdot \mathbb{1}_{s < t} \, da \, dt}{P(S < T)} \tag{11}$$

The player's movement onset distribution consists of a mix between reacting to the truncated stimulus movement onset and guessing. Importantly, reacting and guessing have unique delays and uncertainties. Thus, reaction decisions and guess decisions will

lead to unique means and uncertainties of the participant movement onset distribution. The participant mean reaction movement onset ($\mu_{mo_{react}}$; Eq. 10) is the truncated stimulus movement onset mean ($\mu_{A_{react}}$) plus the participant mean response time (μ_{rt}). Likewise, the standard deviation of participant reaction movement onset (($\sigma_{mo_{react}}$; Eq. 11) is the square root of the variance of the truncated stimulus movement onset distribution ($\sigma_{A_{react}}$) plus the participant response time variance (σ_{rt}^2).

$$\mu_{mot_{react}} = \mu_{S_{react}} + \mu_{rt} \tag{12}$$

$$\sigma_{mot_{react}} = \sqrt{\sigma_{S_{react}}^2 + \sigma_{rt}^2} \tag{13}$$

Here, μ_{rt} represents the time to process the stimulus movement plus the neuromechanical delay (μ_{nmd}). For the no switch time model, the participant mean guess movement onset ($\mu_{mot_{quess}}$, Eq. 12) is the sum of the participant stopping time τ and the mean neuromechanical delay μ_{nmd} . The participant standard deviation guess movement onset $(\sigma_{mot_{guess}})$ is the square root of the sum of the timing variance $\sigma_{ au}^2$ and neuromechanical delay variance σ_{nmd}^2 (Eq. 13).

$$\mu_{mot_{guess}} = \tau + \mu_{nmd} \tag{14}$$

$$\sigma_{mot_{guess}} = \sqrt{\sigma_{\tau}^2 + \sigma_{nmd}^2} \tag{15}$$

Crucially, for the known switch time model and unknown switch time model, the mean and standard deviation of the guess movement onset includes the switch time mean and uncertainty,

$$\mu_{mot_{guess}} = \tau + \mu_{nmd} + \mu_{switch} \tag{16}$$

$$\sigma_{mot_{guess}} = \sqrt{\sigma_{\tau}^2 + \sigma_{nmd}^2 + \sigma_{switch}^2} \tag{17}$$

For both the known switch time model and unknown switch time models, the switch time mean and uncertainty influence the model outputs. However, the decision policy of the Full Switch Time Model has perfect knowledge of the switch time, while the decision policy of the Partial Switch Time Model has imperfect knowledge of the switch time. Further below (see Model Fitting), we address how we estimate the amount of imperfect knowledge for the Partial Switch Time Model.

We define the react target reach time mean ($\mu_{reach_{react}}$) and standard deviation ($\sigma_{reach_{react}}$), and the guess target reach time mean ($\mu_{reach}guess$) and standard deviation ($\sigma_{reach}guess$) as

$$\mu_{reach_{react}} = \mu_{mot_{react}} + \mu_{mt} \tag{18}$$

$$\sigma_{reach_{react}} = \sqrt{\sigma_{mot_{react}}^2 + \sigma_{mt}^2} \tag{19}$$

$$\mu_{reach_{guess}} = \mu_{mot_{guess}} + \mu_{mt} \tag{20}$$

$$\mu_{reach_{guess}} = \mu_{mot_{guess}} + \mu_{mt}$$

$$\sigma_{reach_{guess}} = \sqrt{\sigma_{mot_{guess}}^2 + \sigma_{mt}^2}$$
(20)

To obtain the probability of reaching the target given the participant has either reacted ($X_{reach_{react}}$) or guessed ($X_{reach_{guess}}$), we need to define a random variable for the distribution of target reach times according to

$$X_{reach_{react}} \sim \mathcal{N}(\mu_{reach_{react}}, \sigma_{reach_{react}})$$
 (22)

$$X_{reach_{quess}} \sim \mathcal{N}(\mu_{reach_{quess}}, \sigma_{reach_{quess}})$$
 (23)

$$\hat{X}_{reach_{quess}} \sim \mathcal{N}(\hat{\mu}_{reach_{quess}}, \hat{\sigma}_{reach_{quess}})$$
 (24)

We can then define the probability that the participant will reach one of the two targets before the time constraint of 1500ms given they react (P(Reach|React)) or guess (P(Reach|Guess)):

$$P(Reach|React) = P(X_{reach_{react}} < 1500)$$
 (25)

$$P(Reach|Guess) = P(X_{reach_{quess}} < 1500)$$
 (26)

We can now define the conditional probabilities for three outcome metrics: indecision, win, and incorrect. The conditional probability of an indecision given the participant has reacted (P(Indecision|React)) or guessed (P(Indecision|Guess)) follows immediately from Eqs. 20-21. An indecision is simply the probability that the participant reaches the target after the time constraint:

$$P(Indecision|React) = 1 - P(Reach|React)$$
 (27)

$$P(Indecision|Guess) = 1 - P(Reach|Guess)$$
(28)

To define a win or incorrect trial, we need the conditional probability of selecting the correct target given the participant has either reacted (P(Correct|React)) or guessed (P(Correct|Guess)). The probability of selecting the correct target when the participant reacts is 100% (Eq. 24). The probability of selecting the correct target when the participant guesses is 50% (Eq. 25).

$$P(Correct|React) = 1.0 (29)$$

$$P(Correct|Guess) = 0.5 (30)$$

The conditional probability the participant wins given they react (P(Win|React)) or guess (P(Win|Guess)) is the probability that they reach the target within the time constraint and they select the correct target:

$$P(Win|React) = P(Reach|React) \cdot P(Correct|React)$$
(31)

$$P(Win|Guess) = P(Reach|Guess) \cdot P(Correct|Guess)$$
 (32)

The conditional probability the participant is incorrect given they react (P(Incorrect|React)) or guess (P(Incorrect|Guess)) is the probability that they reach the target within the time constraint and they select the wrong target:

$$P(Incorrect|React) = P(Reach|React) \cdot (1 - P(Correct|React))$$
(33)

$$P(Incorrect|Guess) = P(Reach|Guess) \cdot (1 - P(Correct|Guess))$$
(34)

Finally we define the final probability of wins $(P(Win|\tau))$, indecisions $(P(Indecision|\tau))$, and incorrects $(P(Incorrect|\tau))$, by considering their and their associated conditional probabilities (Eq. x-x) and the probability of reaching and guessing given a stopping time (Eq. x-x), as

$$P(Win|\tau) = P(React|\tau) \cdot P(Win|React)$$

$$+ P(Guess|\tau) \cdot P(Win|Guess)$$

$$P(Indecision|\tau) = P(React|\tau) \cdot P(Indecision|React)$$

$$+ P(Guess|\tau) \cdot P(Indecision|Guess)$$

$$P(Incorrect|\tau) = P(React|\tau) \cdot P(Incorrect|React)$$

$$+ P(Guess|\tau) \cdot P(Incorrect|Guess)$$

$$(35)$$

Decision Policy

For each model, the goal of the decision policy is to find the optimal stopping time that maximizes expected reward. The reward on a particular trial for wins (R_{win}) , indecisions $(R_{indecision})$, and incorrects $(R_{incorrect})$ is,

$$R_{win} = 1 (38)$$

$$R_{indecision} = 0 (39)$$

$$R_{incorrect} = 0 (40)$$

The expected reward is defined as the reward of a trial outcome multiplied by the probability of that outcome, which is

$$\mathbb{E}[R|\tau] = P(Win|\tau) \cdot R_{Win} + P(Incorrect|\tau) \cdot R_{Incorrect} + P(Indecision|\tau) \cdot R_{Indecision}$$
(41)

The decision policy uses known parameters to select an optimal stopping time τ^* that maximizes expected reward:

$$\tau^* = \underset{\tau}{argmax}[\mathbb{E}(R|\tau)] \tag{42}$$

Model Parameter Estimation and Fitting Procedure

As a reminder, we had three models: No Switch Time Model, Full Switch Time Model, and Partial Switch Time Model. Each of these models had full knowledge of the reaction time mean and standard deviation, neuromechanical delay mean and standard deviation,

movement time mean and standard deviation, and the stimulus movement onset distribution. These parameter values were estimated from experimental data by bootstrapping the meansfrom the response time experiment (μ_{rt} , σ_{rt}), timing experiment (σ_{τ}), and Experiment 1 (μ_{mt} , σ_{mt}). We describe the bootstrap procedure below. Neuromechanical delay (μ_{nmd}) and uncertainty (σ_{nmd}) were estimated from prior literature (Norman and Komi 1979, Bruce et al. 1985, Rossini and Rossi 1998).

We used both a warm-start initialization and bootstrap procedure (see below) to determine the switch time mean (μ_{switch}) and uncertainty (σ_{switch}), as well as the decision policy's knowledge of the switch time mean ($\hat{\mu}_{switch}$, switch time uncertainty ($\hat{\sigma}_{switch}$), and timing uncertainty ($\hat{\sigma}_{\tau}$).

Warm-Start Initialization

The fitting procedure for the Full Switch Time Model and Partial Switch Time Model began with a "warm-start" to find an initial set of μ_{switch} , σ_{switch} , $\hat{\sigma}_{switch}$, and $\hat{\sigma}_{\tau}$ (Roth 2023r). Here the remaining parameters estimated from experimental data were set as the group level means. We found the best-fit parameters that led to the lowest loss between model outputs ($Model_{i,j}$) and group data means($Data_{i,j}$) according to

$$\mathcal{L} = \sum_{i=1}^{6} \sum_{j=1}^{5} \frac{|Data_{i,j} - Model_{i,j}|}{Data_{i,j}},$$
(43)

where i corresponds to experimental condition and j corresponds to each dependent measure (i.e., movement onset, standard deviation of participant movement onset, wins, indecisions, and incorrects) over the target metrics. This fitting procedure was repeated 1,000 times to avoid local minimums. From these 1,000 optimizations, we used the set of parameters that resulted in the lowest loss as the initial guess for our bootstrap procedure. Model fitting was performed using the Powell algorithm in the Minimize function from the Scipy Python library. The parameter set with the lowest loss was used as the initial guess for the bootstrap procedure.

Warm-Start Initialization

In the bootstrap procedure we randomly sampled participants with replacement 10,000 times (Cashaback 2017, Cashback 2019, Roth 2024, Roth 2023). We used the mean of the bootstrapped participant data for each of the parameter values that were estimated from data. For each bootstrap iteration, the optimization process selected the free parameters for the Full Switch Time Model (μ_{switch} , σ_{switch}) and Partial Switch Time Model (μ_{switch} , σ_{switch} , $\hat{\mu}_{switch}$, $\hat{\sigma}_{\tau}$, $\hat{\sigma}_{\tau}$) that minimized the loss function (Eq. 44).

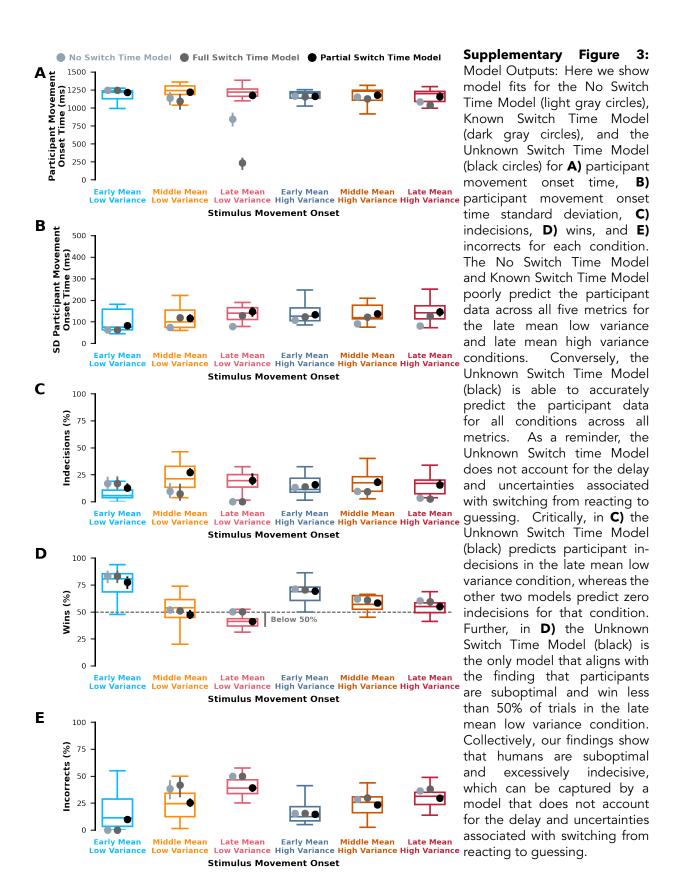
Model Parameters

Here we report the mean and 95% confidence intervals for the bootstrapped model parameters. For each parameter, each model had a value that determined the model outputs and a value that the decision policy used to select τ . The No Switch Time Model and Full Switch Time Model were both optimal in the sense that they had perfect knowledge of all parameters. That is, the value that determined model outputs was equal to the value used by the decision policy. Parameter values for reaction time mean, reaction time uncertainty, movement time mean, movement time uncertainty, and timing uncertainty were all calculated from participant data on each bootstrap iteration: reaction time mean (247.4 ms; [239.2, 256.35]), reaction time uncertainty (38.5 ms; [35.7, 41.4]), move-

ment time mean (171.7 ms; [157.6, 197.2]), movement time uncertainty (25.4 ms; [22.2, 28.7]), and timing uncertainty (77.8 ms; [69.4, 86.6]). The No Switch Time Model had no parameters to fit, and the decision policy had full knowledge of all of these parameters. The Full Switch Time Model fit the switch time mean and switch time uncertainty and the decision policy had full knowledge of all parameters. The bootstrapped mean of the switch time delay parameter was 0.95 ms with a 95% confidence interval of [0.21, 2.20]. The bootstrapped mean of the switch time uncertainty parameter was 103.20 ms with a 95% confidence interval of [97.77, 115.31].

The Partial Switch Time Model fit the timing uncertainty, switch time mean, switch time uncertainty. Critically, this model did not have full knowledge of these parameters, so the value that the decision policy used to select the optimal time was fit separately from the value that was used to determine the model outputs. Note that the timing uncertainty value used for model outputs is from experimental data, but this model allowed the knowledge of the timing uncertainty to be a fit parameter. The values that were used for model outputs were: timing uncertainty (77.83 ms; [69.4, 86.8]), switch time mean (21.10 ms; [1.46, 63.71]), switch time uncertainty (134.73 ms; [111.22, 155.06]. The values that were used by the decision policy were: timing uncertainty (1.65 ms; [0.08, 4.63]), switch time mean (9.0 ms; [0.00, 16.65]), switch time uncertainty (40.4 ms; [34.54, 48.02]). Critically, the fitting procedure for the Partial Switch Time Model selected the decision policy's values for the switch time mean, switch time uncertainty, and stopping time certainty to be less than the values that impact the model outputs, indicating that humans have imperfect knowledge of these parameters.

Supplementary C: Model Results



Supplementary D: Response Time Task

Response Time Experiment

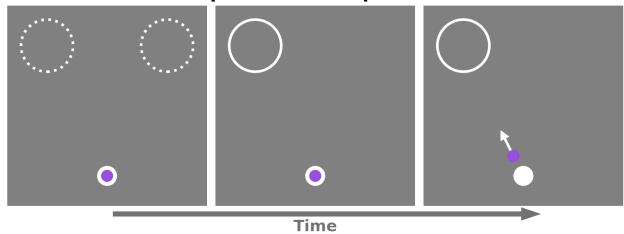


Figure 4: Response Time Experiment: One of two potential targets (dashed white rings; not visible to participants) would appear on the screen. Once a target appeared (solid white ring), participants were instructed to rapidly move their cursor to hit the displayed target. We used the mean and standard deviation of response times as model parameters in the optimal stopping models.

Supplementary E: Timing Uncertainty Task Supplementary F: Guess Decisions