A Computational Analysis of a Set Parameter Beta-Type Stirling Engine and Flywheel Design Optimization

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Abstract

The effectiveness of an energy conversion system is often defined by its losses: what causes them, how significant they are, and how they can be reduced to improve overall system performance. For Beta-type Stirling engines, we examine work generation in real versus ideal conditions, torque balance across the cycle, crankshaft speed fluctuation, and phase angle sensitivity. We also size a flywheel to meet a target coefficient of fluctuation given material and geometric constraints.

1 Introduction

Stirling engines provide a straightforward yet effective method of converting heat into mechanical work. To evaluate performance, we analyze each phase of the cycle versus crank angle θ, comparing pressure, volume, work, and torque distributions. Beyond the thermodynamic cycle, the flywheel must supply sufficient inertia to maintain near-constant angular velocity without overloading the system.

2 Given Parameters

Key parameters used throughout the analysis are summarized below.

Table 1: Initial engine and operating parameters (given).

Parameter	**Symbol**	**Value**	**Units**
Power piston crank length	r_p	0.025	m
Power piston connecting rod	ℓ_p	0.075	m
Displacer crank length	r_d	0.02	m
Displacer connecting rod	ℓ_d	0.14	m
Displacer volume	V_d	4.0×10^{-5}	m^3
Cylinder bore diameter	D	0.05	m
Phase shift	ϕ	$\pi/2$	rad
Compression ratio	CR	1.7	—
Hot temperature	T_h	900	K
Cold temperature	T_c	300	K
Gas pressure at BDC	P_{BDC}	500	kPa (abs)
Atmospheric pressure	P_{atm}	101.3	kPa (abs)
Regenerator dead volume	V_{reg}	2.0×10^{-5}	m^3
Flywheel width	w	0.025	m
Flywheel rim thickness	t	0.05	m
Flywheel material density	ρ	8000	kg/m^3
Coefficient of fluctuation	C_f	0.003	—
Average rotational speed	$\overline{\Omega}$	650	rpm

3 Stirling Engine Overview

Figure 1 shows the Beta-type Stirling engine schematic used in this study.

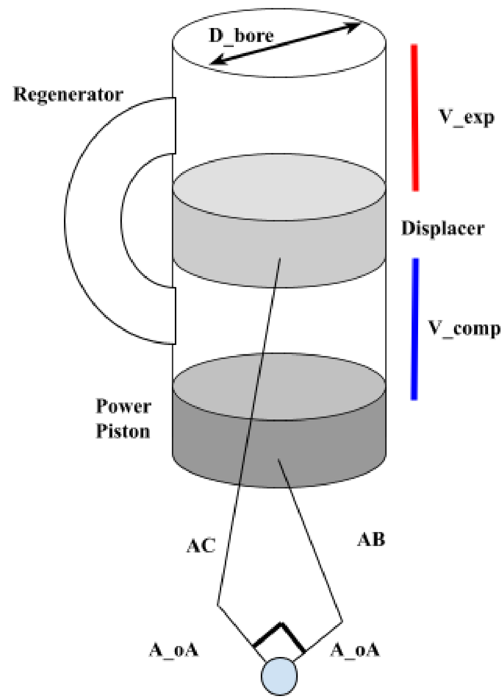

Figure 1: Stirling engine diagram.

4 Methodology

We discretize the crank angle as $\theta \in [0, 2\pi)$ with 360 evenly spaced points. With equal bore diameters, the cross-sectional area is $A = \pi D^2/4$.

4.1 Slider–Crank Kinematics

Piston/displacer positions follow the slider–crank with rod obliquity. For a crank radius r and rod length ℓ:

$$\beta(\theta) = \arcsin\left(\frac{r}{\ell}\sin\theta\right), \tag{1}$$

$$x(\theta) = \ell\cos\beta(\theta) - r\cos\theta. \tag{2}$$

We use $x_p(\theta)$ for the power piston and $x_d(\theta + \phi)$ for the displacer (phase shifted by ϕ).

4.2 Cold/Hot Volumes

Cold height is the separation between displacer and power piston minus fixed offsets; hot height is measured from the top space above the displacer:

$$h_c(\theta) = \big[x_d(\theta+\phi) - x_p(\theta)\big] - h_{\text{pin}} - \tfrac{1}{2}h_d, \tag{3}$$

$$h_h(\theta) = H_{\text{tot}} - \tfrac{1}{2}h_d - x_d(\theta+\phi), \tag{4}$$

with volumes

$$V_c(\theta) = A\,h_c(\theta), \qquad V_h(\theta) = A\,h_h(\theta), \qquad V_r = \text{const.} \tag{5}$$

4.3 Schmidt Analysis and Mass from BDC

Let T_c, T_h, T_r be cold, hot, and regenerator temperatures and R the specific gas constant. Total mass is fixed and determined at BDC ($\theta = 0$) using the known absolute pressure P_{BDC}:

$$m = \frac{P_{\text{BDC}}}{R}\left(\frac{V_c(0)}{T_c} + \frac{V_r}{T_r} + \frac{V_h(0)}{T_h}\right). \tag{6}$$

Instantaneous absolute pressure then follows the Schmidt relation

$$P(\theta) = \frac{m\,R}{\dfrac{V_c(\theta)}{T_c} + \dfrac{V_r}{T_r} + \dfrac{V_h(\theta)}{T_h}}. \tag{7}$$

Cycle work is evaluated numerically as

$$W = \oint P\,\mathrm{d}V \approx \sum_k P(\theta_k)\,\Delta V(\theta_k) \quad \text{(trapezoidal rule).} \tag{8}$$

4.4 Torque with Rod Obliquity

Net axial force on the power piston is $F_p(\theta) = \big(P(\theta) - P_{\text{atm}}\big)\,A$. With β from Eq. (1) and crank radius r_p, the torque is

$$\tau(\theta) = -\,F_p(\theta)\,\frac{r_p\,\sin\theta}{\cos\beta(\theta)}. \tag{9}$$

The displacer contributes zero torque (equal pressure both sides, zero rod area).

4.5 Flywheel Sizing

Let ω_{avg} be the average angular speed and C_f the target coefficient of fluctuation. From the torque deviation about its mean, compute the cumulative energy variation and define the energy fluctuation ΔE as the peak-to-peak of that signal. The required inertia is

$$I_{\text{req}} = \frac{\Delta E}{C_f\,\omega_{\text{avg}}^2}. \tag{10}$$

Using a rim-dominant ring of width w, thickness t, density ρ, and outer radius R, the moment of inertia is

$$I_{\text{rim}}(R) = \tfrac{1}{2}\,M(R)\left(R^2 + R_{\text{in}}^2\right), \quad M(R) = \rho\,\pi w\left(R^2 - R_{\text{in}}^2\right), \quad R_{\text{in}} = R - t. \tag{11}$$

We solve $I_{\text{rim}}(R) = I_{\text{req}}$ with a short fixed-point update. Start from a ring-based guess $R^{(0)} = \sqrt{I_{\text{req}}/(\pi\,\rho\,w\,t)} + t/2$. At each step, evaluate $I_{\text{act}} = I_{\text{rim}}\big(R^{(k)}\big)$, form $\eta = I_{\text{req}}/I_{\text{act}}$, and rescale

$$R^{(k+1)} = R^{(k)}\,\eta^{1/3}, \qquad R_{\text{in}} = R^{(k+1)} - t. \tag{12}$$

Stop when the relative error $|I_{\text{act}} - I_{\text{req}}|/I_{\text{req}}$ drops below a tolerance (or after a max iteration count). Why cube root? For a rim-dominant geometry, inertia scales approximately like R^3, so scaling R by $\eta^{1/3}$ moves directly toward the required inertia with stable, fast convergence. Finally, set diameters $D_{\text{out}} = 2R$, $D_{\text{in}} = 2R_{\text{in}}$ and mass $M(R)$, checking against any maximum diameter constraint.

4.6 Energy-Based Dynamics

With load torque equal to mean engine torque, the net torque drives angular acceleration $\alpha = T_{\text{net}}/I$. Using the work–energy theorem over angle increments with cumulative work $W_{\text{net}}(\theta)$, we update speed via

$$\Omega^2(\theta) = \Omega_{\text{avg}}^2 + \frac{2\,W_{\text{net}}(\theta)}{I}, \qquad \Omega(\theta) = \max\!\Big(0.1\,\Omega_{\text{avg}},\ \sqrt{\Omega^2(\theta)}\Big), \tag{13}$$

then normalizes Ω to recover the target average.

4.7 Phase Optimization

We perform a three-stage search over ϕ: a coarse scan from 30° to 150° in 2° steps; a medium scan within ±6° of the coarse best at 0.1° resolution; and a fine scan within ±0.5° at 0.01°

resolution. Mean torque is integrated over θ, power is $\bar{\tau}\,\omega_{\text{avg}}$, and the best ϕ maximizes power.

5 Results

Results include p–V work, torque versus crank angle, phase sensitivity, and flywheel size meeting the specified coefficient of fluctuation. Plots are omitted here.

6 Discussion

The analysis highlights sensitivity to phase shift and clearance volumes. Idealized assumptions (Schmidt analysis, isothermal zones) tend to overpredict work; incorporating losses would reduce predicted performance.

7 Conclusion

We reproduced a codified analysis of a Beta-type Stirling engine, sized a flywheel to achieve a target fluctuation coefficient, and identified an optimal phase shift to maximize cycle work. Future work should incorporate non-ideal heat transfer and frictional losses.