A Computational Analysis of a Set Parameter Beta-Type Stirling Engine and Flywheel Design Optimization

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Abstract

The effectiveness of an energy conversion system is often defined by its losses: what causes them, how significant they are, and how they can be reduced to improve overall system performance. For Beta-type Stirling engines, we examine work generation in real versus ideal conditions, torque balance across the cycle, crankshaft speed fluctuation, and phase angle sensitivity. We also size a flywheel to meet a target coefficient of fluctuation given material and geometric constraints.

1 Introduction

Stirling engines provide a straightforward yet effective method of converting heat into mechanical work. To evaluate performance, we analyze each phase of the cycle versus crank angle θ, comparing pressure, volume, work, and torque distributions. Beyond the thermodynamic cycle, the flywheel must supply sufficient inertia to maintain near-constant angular velocity without overloading the system.

2 Given Parameters

Key parameters used throughout the analysis are summarized below.

Table 1: Initial engine and operating parameters (given).

Parameter	**Symbol**	**Value**	**Units**
Power piston crank length	r_p	0.025	m
Power piston connecting rod	ℓ_p	0.075	m
Displacer crank length	r_d	0.02	m
Displacer connecting rod	ℓ_d	0.14	m
Displacer volume	V_d	4.0×10^{-5}	m^3
Cylinder bore diameter	D	0.05	m
Phase shift	ϕ	$\pi/2$	rad
Compression ratio	CR	1.7	—
Hot temperature	T_h	900	K
Cold temperature	T_c	300	K
Gas pressure at BDC	P_{BDC}	500	kPa (abs)
Atmospheric pressure	P_{atm}	101.3	kPa (abs)
Regenerator dead volume	V_{reg}	2.0×10^{-5}	m^3
Flywheel width	w	0.025	m
Flywheel rim thickness	t	0.05	m
Flywheel material density	ρ	8000	kg/m^3
Coefficient of fluctuation	C_f	0.003	—
Average rotational speed	$\overline{\Omega}$	650	rpm

3 Stirling Engine Overview

Figure 1 shows the Beta-type Stirling engine schematic used in this study.

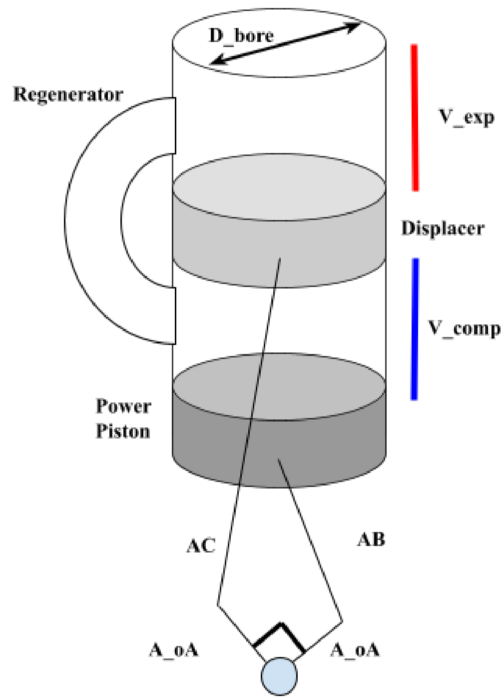

Figure 1: Stirling engine diagram.

4 Methodology

We begin by discretizing the crank angle as $\theta \in [0, 2\pi)$ with 360 evenly spaced points. With equal bore diameters, the cross-sectional area is $A = \pi D^2/4$.

4.1 Step 0: Assisting Calculations

Before analyzing the cycle, we first compute several derived geometric parameters from the given inputs. We calculate the displacer height as $h_d = V_d/A$ to determine its physical dimensions. Next, we find the power piston positions at bottom and top dead center using the slider-crank equations, which gives us the swept volume $V_{\text{swept}} = A(x_{\text{TDC}} - x_{\text{BDC}})$. We then determine the total cylinder volume at BDC using the compression ratio: $V_{\text{total,BDC}} = V_r - V_d + \text{CR} \cdot V_{\text{swept}}/(\text{CR} - 1)$. From this, we compute the total cylinder height as:

$$H_{\text{tot}} = \frac{V_{\text{total,BDC}}}{A} + h_d + h_{\text{pin}} + \ell_p - r_p, \tag{1}$$

where the terms account for the cold/hot space height, displacer height, pin offset, and connecting rod geometry. Finally, we set the regenerator temperature $T_r = (T_h + T_c)/2$. These calculations establish the complete engine geometry needed for the kinematic and thermodynamic analysis that follows.

4.2 Step 1: Slider–Crank Kinematics

Then, to begin the analysis, we calculate piston and displacer positions using slider–crank kinematics with rod obliquity. For a crank radius r and rod length ℓ:

$$\beta(\theta) = \arcsin\!\left(\frac{r}{\ell}\sin\theta\right), \tag{2}$$

$$x(\theta) = \ell\cos\beta(\theta) - r\cos\theta. \tag{3}$$

We then apply this to get $x_p(\theta)$ for the power piston and $x_d(\theta + \phi)$ for the displacer (phase shifted by ϕ).

4.3 Step 2: Cold/Hot Volumes

Next, we compute the cold and hot space volumes from the piston positions. Cold height is the separation between displacer and power piston; hot height is measured from the top space above the displacer:

$$h_c(\theta) = \big[x_d(\theta + \phi) - x_p(\theta)\big] - h_{\text{pin}} - \tfrac{1}{2}h_d, \tag{4}$$

$$h_h(\theta) = H_{\text{tot}} - \tfrac{1}{2}h_d - x_d(\theta + \phi), \tag{5}$$

which then gives us volumes

$$V_c(\theta) = A\,h_c(\theta), \qquad V_h(\theta) = A\,h_h(\theta), \qquad V_r = \text{const.} \tag{6}$$

4.4 Step 3: Schmidt Analysis and Mass from BDC

Now we determine the total gas mass and pressure distribution. We start by setting the total gas mass at a known reference point—bottom dead center (BDC, $\theta = 0$)—using the known absolute pressure P_{BDC}:

$$m = \frac{P_{\text{BDC}}}{R}\left(\frac{V_c(0)}{T_c} + \frac{V_r}{T_r} + \frac{V_h(0)}{T_h}\right). \tag{7}$$

With this fixed mass, we then calculate the instantaneous absolute pressure at any angle using the Schmidt relation:

$$P(\theta) = \frac{m\,R}{\dfrac{V_c(\theta)}{T_c} + \dfrac{V_r}{T_r} + \dfrac{V_h(\theta)}{T_h}}. \tag{8}$$

Finally, we evaluate cycle work numerically as

$$W = \oint P\,\mathrm{d}V \approx \sum_k P(\theta_k)\,\Delta V(\theta_k) \quad \text{(trapezoidal rule)}. \tag{9}$$

4.5 Step 4: Torque with Rod Obliquity

We then convert pressure to torque. The net axial force on the power piston is $F_p(\theta) = \big(P(\theta) - P_{\mathrm{atm}}\big)\,A$. Using β from Eq. (1) and crank radius r_p, the torque becomes:

$$\tau(\theta) = -\,F_p(\theta)\,\frac{r_p\,\sin\theta}{\cos\beta(\theta)}. \tag{10}$$

4.6 Step 5: Flywheel Sizing

Next, we size the flywheel to smooth out torque variations. We start by computing the torque deviation about its mean:

$$T_{\mathrm{dev}}(\theta) = \tau(\theta) - \overline{\tau}, \quad \text{where } \overline{\tau} = \frac{1}{2\pi}\int_0^{2\pi} \tau(\theta)\,\mathrm{d}\theta. \tag{11}$$

We then integrate this deviation to get cumulative energy variation. The energy fluctuation ΔE is defined as the peak-to-peak of that signal. This gives us the required inertia:

$$I_{\mathrm{req}} = \frac{\Delta E}{C_f\,\omega_{\mathrm{avg}}^2}. \tag{12}$$

We then model the flywheel as a thick ring (like a washer) with width w, thickness t, density ρ, and outer radius R. Most of the mass is concentrated near the outer edge, giving moment of inertia:

$$I_{\mathrm{rim}}(R) = \tfrac{1}{2}\,M(R)\,\big(R^2 + R_{\mathrm{in}}^2\big), \quad M(R) = \rho\,\pi w\,\big(R^2 - R_{\mathrm{in}}^2\big), \quad R_{\mathrm{in}} = R - t. \tag{13}$$

To solve $I_{\mathrm{rim}}(R) = I_{\mathrm{req}}$, we use a fixed-point iteration. We start from a ring-based guess $R^{(0)} = \sqrt{I_{\mathrm{req}}/(\pi\,\rho\,w\,t)} + t/2$. At each step, we evaluate $I_{\mathrm{act}} = I_{\mathrm{rim}}\big(R^{(k)}\big)$, form $\eta = I_{\mathrm{req}}/I_{\mathrm{act}}$, and rescale:

$$R^{(k+1)} = R^{(k)}\,\eta^{1/3}, \qquad R_{\text{in}} = R^{(k+1)} - t. \tag{14}$$

We stop when the relative error $|I_{\text{act}} - I_{\text{req}}|/I_{\text{req}}$ drops below a tolerance. Why cube root? For a rim-dominant geometry, inertia scales approximately like R^3, so scaling R by $\eta^{1/3}$ moves directly toward the required inertia with stable, fast convergence. Finally, we set diameters $D_{\text{out}} = 2R$, $D_{\text{in}} = 2R_{\text{in}}$ and mass $M(R)$.

4.7 Step 6: Energy-Based Dynamics

Now we simulate how the flywheel affects engine speed. We set the load torque equal to mean engine torque for steady-state operation. The net torque then drives angular acceleration $\alpha = T_{\text{net}}/I$. Using the work–energy theorem over angle increments with cumulative work $W_{\text{net}}(\theta)$, we update speed via:

$$\Omega^2(\theta) = \Omega_{\text{avg}}^2 + \frac{2\,W_{\text{net}}(\theta)}{I}, \qquad \Omega(\theta) = \sqrt{\Omega^2(\theta)}. \tag{15}$$

We then normalize Ω to recover the target average. This normalization is necessary because the energy-based calculation can drift slightly from the desired mean speed due to numerical integration, so we rescale the entire speed profile to maintain the correct average while preserving the fluctuation pattern.

4.8 Step 7: Phase Optimization

Finally, we optimize the phase shift to maximize power output. We perform a three-stage search over ϕ: first, a coarse scan from 30° to 150° in 2° steps to find a good neighborhood. Then, we zoom in with a medium scan within ±6° of the coarse best at 0.1° resolution. Finally, we pinpoint the optimum with a fine scan within ±0.5° at 0.01° resolution. At each phase, we compute mean torque integrated over θ, calculate power as $\bar{\tau}\,\omega_{\text{avg}}$, and select the ϕ that maximizes power.

5 Results

Results include p–V work, torque versus crank angle, phase sensitivity, and flywheel size meeting the specified coefficient of fluctuation. Plots are omitted here.

6 Discussion

The analysis highlights sensitivity to phase shift and clearance volumes. Idealized assumptions (Schmidt analysis, isothermal zones) tend to overpredict work; incorporating losses would reduce predicted performance.

7 Conclusion

We reproduced a codified analysis of a Beta-type Stirling engine, sized a flywheel to achieve a target fluctuation coefficient, and identified an optimal phase shift to maximize cycle work. Future work should incorporate non-ideal heat transfer and frictional losses.