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University of Jos
Department of Mathematics
B.Sc./B.Ed. Second Semester Examinations 2014/2015 Session
MTH201: Mathematical Methods 1. Time allowed: 2 hours
Instructions: Answer any FOUR questions.

1. (a) Determine the equations of the tangent and normal to the curve $x\sin xy = x^2 + y^2 - 1$ at the point $(1,0)$.
(b) Given that $f(x,y) = xy^2 - x^2 y$, where $x = r - 2s$ and $y = 3r + s$, find (i) $\frac{\partial f}{\partial r}$ (ii) $\frac{\partial f}{\partial s}$ (iii) $\frac{\partial^2 f}{\partial s^2}$ (iv) $\frac{\partial(x,y)}{\partial(r,s)}$ at the point $(r,s) = (2,1)$.

2. (a) Investigate the limiting behaviour of the function
$$f(x,y) = \frac{x^2 y^2}{x^4 + y^2} \text{ as } (x,y) \to (0,0).$$
(b) If $u(x,y) = x^3 y - 4xy^3 + 2y^3$, find du when $x = 3, y = 2, \Delta x = 0.3$ and $\Delta y = 0.02$.
(c) Given that $u = xyz$, $v = xy + xz + yz$ and $w = x + y + z$, show that
$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = (x-y)(y-z)(z-x).$$
$\frac{\partial u}{\partial x} = yz$ $\frac{\partial v}{\partial y} = x+z$ $\frac{\partial w}{\partial z} = 1$.

3. (a) Show that the function $u(x,y) = 2x^3 - 3xy^2 + y^3$ satisfies the partial differential equation $u_{xx} + 2u_{xy} + 2u_{yy} = 0$.
(b) Find and classify all the critical points of the function $f(x,y) = 6xy - 3x^2 y - 2xy^2$.

4. (a) The acceleration of a particle at any time t is given by $\vec{a} = 12\cos 2t\mathbf{i} - 8\sin 2t\mathbf{j} + 24t\mathbf{k}$. If the velocity $\vec{V}$ and displacement $\vec{r}$ are zero at $t = 0$, find $\vec{V}$ and $\vec{r}$ at any time t.
(b) Classify the critical points $(0,0,0)$, $(2,-2,-2)$ and $(-2,-2,2)$ of the function $f(x,y,z) = xyz - x^2 - y^2 - z^2$.

5. (a) If $f(x,y) = xye^{-xy}$, find (i) $f_{xx}(x,y)$ (ii) $f_{xy}(x,y)$ (i) $f_{yy}(x,y)$.
(b) Given $\vec{F} = x^2 y\mathbf{i} - xz\mathbf{j} + 5yz\mathbf{k}$, evaluate $\int_C \vec{F}.d\vec{r}$, C is the straight line joining $(-1,0,1)$ to $(1,2,-3)$.

6. (a) Define a harmonic function in R^2, hence show that the function $u(x,y) = x^2 - y^2 - 2xy - x + y - 3$ is harmonic in R^2.
(b) If $V = xy$, evaluate $\int_\tau V d\tau$, where τ is the volume bounded by the planes $x = y = z = 0$ and $x + y + z = 1$.

$\frac{\partial u}{\partial x} = f'(x+y+z) \cdot yz$

$\frac{\partial v}{\partial y} = f'(xy+xz+yz) \cdot x+z$

$\frac{\partial w}{\partial z} = f'(x+y+z) \cdot 1$

$(x-y)(y-z)$

$(xy - xz - y^2 + yz)(z - x)$

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UNIVERSITY OF JOS, DEPARTMENT OF MATHEMATICS,
B.Sc./B.Ed. FIRST SEMESTER EXAMINATION 2017/2018 SESSION
MTH202: ELEMENTARY DIFFERENTIAL EQUATIONS I – 3 CREDIT UNITS

Answer any FOUR Questions, Time Allowed: 2 Hours

1. (a) Use the method of variable separable to solve the differential equation $(1+x^2)\frac{dy}{dx} = xy(1+y)$. (b) Use the method of variation of parameters to solve the differential equation $y'' - y' - 2y = 2e^{-x}$.

2. (a) Solve the first order linear differential equation $2(y-4x^2)dx + xdy = 0$.
(b) Obtain the general solution of the Bernoulli's equation $xdy - \left[y + xy^3(1+\ln x)\right]dx = 0$.

3. (a) Show that the differential equation $(ye^{xy} - 2y^3)dx + (xe^{xy} - 6xy^2 - 2y)dy = 0$ is exact and solve it. (b) Use Laplace transform to solve the initial value problem $y'' + y' - 2y = 2t, y(0) = 0, y'(0) = 1$.

4. (a) Show that the differential equation $(x^2 - xy + y^2)dx - xydy = 0$ is homogeneous and solve it. (b) Obtain the solution of the differential equation $y'' + 4y = x^2 + 3e^x, y(0) = 0, y'(0) = 2$ using the method of undetermined coefficients.

5. (a) Find the general solution of the system $X' = \begin{pmatrix} 4 & -1 \\ -4 & 4 \end{pmatrix} X$. (b) If $L\{f(t)\} = F(s)$ and $L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}(F(s))$, then find $L\{t^2 \cos 2t\}$.

6. (a) Convert the differential equation $(x^4 + y^4)dx + xy^3dy = 0$ into an exact equation using an appropriate integrating factor and hence find a function, $u(x,y)$ that satisfies such an equation.
(b) Solve the system of equations
$$X_1' = X_1 + X_2 + 2e^t$$
$$X_2' = 4X_1 + X_2 - e^t.$$

UNIVERSITY OF JOS
DEPARTMENT OF MATHEMATICS
FIRST SEMESTER EXAMINATION 2016/2017 SESSION
B.SC./B.ED: ELEMENTARY DIFFERENTIAL EQUATIONS 1 (3 CREDITS)
ANSWER **ANY FOUR** QUESTIONS, TIME ALLOWED: 2 ½ HOURS

1. What is a Differential equation? (a) Solve the First Order Differential equation $xdy + ydx = \sin x dx$. (b) Show that the equation $(xe^{\frac{y}{x}} + y)dx - xdy = 0$ is homogeneous and solve it.
2. (a) Show that the equation $(2xy - \tan y)dx + (x^2 - x\sec^2 y)dy = 0$ is exact and solve it. (b) A certain radioactive material is known to decay at a rate proportional to the amount present. A block of this material originally having a mass of 100g is observed after 20 years to have a mass of only 80g. (i) Find an expression for the mass of the material at any time t. (ii) Find the half-life of the material.
3. (a) Solve the Differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$. (b)(i) Verify that e^x and x are solutions of the homogeneous equation corresponding to $(1-x)y'' + xy' - y = 2(x-1)^2 e^x,\ 0 < x < 1$ (ii) Find the general solution using variation of parameters.
4. Define a First Order System (a) Solve the system $\begin{matrix} (2D+1)X_1 + DX_2 = t \\ (D-1)X_1 + DX_2 = 2 \end{matrix}$ (b) Find the general solution of the system $X' = \begin{pmatrix} 2 & 9 \\ 1 & 2 \end{pmatrix} X$.
5. Given a function $f(t)$, what is the Laplace Transform of $f(t)$? (a) Prove that $L\{\sin 3t\} = \frac{3}{s^2+9}$.
(b) Hence find $L\{t^2 \sin 3t\}$
6. (a) Show that $L\{e^{kt}\} = \frac{1}{s-k}$. (b) Find the inverse Laplace Transform of $F(s) = \frac{2s+1}{s^2+s-6}$.

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2. (a) Solve the first order linear differential equation $2\left(y-4x^2\right)dx + xdy = 0$.
(b) Obtain the general solution of the Bernoulli's equation $xdy - \left[y + xy^3\left(1+\ln x\right)\right]dx = 0$.

3. (a) Show that the differential equation $\left(ye^{xy} - 2y^3\right)dx + \left(xe^{xy} - 6xy^2 - 2y\right)dy = 0$ is exact and solve it. (b) Use Laplace transform to solve the initial value problem $y'' + y' - 2y = 2t, y(0) = 0, y'(0) = 1$.

4. (a) Show that the differential equation $\left(x^2 - xy + y^2\right)dx - xydy = 0$ is homogeneous and solve it. (b) Obtain the solution of the differential equation $y'' + 4y = x^2 + 3e^x, y(0) = 0, y'(0) = 2$ using the method of undetermined coefficients.

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$$X_1' = X_1 + X_2 + 2e^t$$
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ANSWER ANY FOUR QUESTIONS. TIME ALLOWED: 2 ½ HOURS

can be solved by the use of an integrating factor. Give the integrating factor. (b) Hence solve the equations (i) $x\frac{dy}{dx}+2y=e^x$. (ii) $(1+x)\frac{dy}{dx}+xy=(1+x)^2$.

2. (a) Solve the first order differential equation $\frac{dy}{dx}=xy-y$.

   (b) Show that the equation $(x^2-xy+y^2)dx-xydy=0$ is homogeneous and solve it.

   (c) Find the general solution of the Bernoulli equation $y^2\frac{dy}{dx}+\frac{y^3}{x}=x^3+4$.

3. (a) Determine the most general form of M or N, if the following equations are exact;

   (i) $M(x,y)dx+(2x^2y^3+x^4y)dy=0$. (ii) $(2xy+y+1)dx+N(x,y)dy=0$.

   (b) Show that the equation $[\cos(x+y)-y\sin(xy)]dx+[cox(x+y)-x\sin(xy)]dy=0$ is exact and solve it.

4. (a) Show that the Laplace transformation of y'' is given by $s^2Y(s)-sy(0)-y'(0)$.

   (b) Hence solve the initial value problem $y''+3y'+2y=0, y(0)=1, y'(0)=0$ by Laplace transform.

   (c) Evaluate $L\{t^3 \sinh at\}$.

5. (a) Solve the differential equation $\left(\frac{d^2y}{dx^2}+4\frac{dy}{dx}+8y=0.\right)$

   (b) Use the method of undetermined coefficients to obtain the solution of the initial value problem $\frac{d^2y}{dx^2}+4y=x^2+3e^x, y(0)=0, y'(0)=2$. (c) Use variation of parameters to solve the equation $\frac{d^2y}{dx^2}+y=\sin x$.

6. (a) Given a matrix $A=\begin{pmatrix}-2 & 1\\ -5 & 4\end{pmatrix}$, obtain the eigen values of A and its corresponding eigenvectors. (b) Solve the system of equations $\begin{pmatrix}X_1' = X_1+X_2\\ X_2' = 4X_1+X_2\end{pmatrix}$