Electric Circuit Analysis

Memorize scientific notation prefixes

$1.3E3 = 1.3 \cdot 10^3$

↖ "mantissa"

$5\,\mu m = 5E{-6}$ $10kV = 10E3$

$2E2\,g = 0.2\,E3 = 0.2\,kg$

$e = -1.602 \times 10^{-19}\,C$ $1C = 1/1.602E19 = 6.24E18$ electrons

$I = \frac{dq}{dt}$ or C/s $1A = 1C/s$

$10\,\mu A \cdot 2k\Omega = 10E10^{-6} \cdot 2E10^{3} = 20$

~
400
2.16
2.5
s
20

SI prefixes

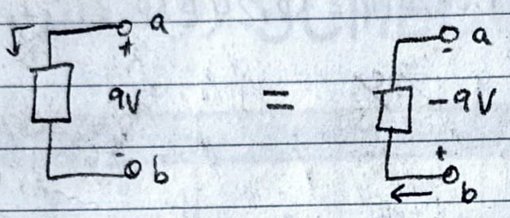
10^{18}	exa	E	my exa gf said I'm shaped like a snowman 18
10^{15}	peta	P	~~5~~ my pet goldfish?
10^{12}	tera	T	Twelve → Tera
10^{9}	giga	G	me singing at my first gig!
10^{6}	mega	M	Mega baymax's Big hero 6!
10^{3}	kilo	k	3E kiss marry kilo
10^{2}	hecto	h	only 2 places: heaven or hec
10	deka	da	≈ 50 cards deka cards
10^{-1}	deci	d	first decimal
10^{-2}	centi	c	centimeter = 100
10^{-3}	milli	m	m → 10^{-3} milli
10^{-6}	micro	μ	mew (mu) is pretty six, it would be too if I got a good million
10^{-9}	nano	n	nano → nine
10^{-12}	pico	p	1...2... pico few... pp 100!
10^{-15}	femto	f	femto → fifteen
10^{-18}	atto	a	1@ @ don't atto me right now!

Section 1.4 Voltage

emf - external electromotive force (battery)
↳ voltage / potential difference

$$1\ \text{volt} = 1\ J/C = 1\ \text{N-meter}/C = \frac{\Delta W}{\Delta J}$$
↳ or q

Alessandro Antonio Volta (1745-1827) Como, Italy



electric current → through an element
" voltage → across " " " or between 2 points

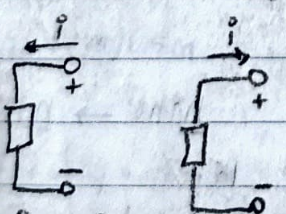
1.5 Power & Energy

Power is the time rate of expending or absorbing energy, watts (W)

$$P = \frac{dw}{dt} = J/s = J/C \cdot C/s = vi$$
↳ instantaneous power

+power, element is absorbing
−" , " " supplying

PS used here ★ Passive sign convention: when current enters through + terminal and $p = +vi$
Active is opposite)



$P = +vi$ Passive Sign

$p = -vi$ Active Sign

+Power absorbed = −Power supplied

$$\Sigma p = 0$$

Energy is the capacity to do work, measured in Joules (J)

Watt-hours (Wh)
$1\,Wh = 3{,}600\,J$

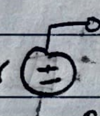
$$Q(t)\,(\text{charge}) = \int_{t_0}^{t} i(t)\,dt$$

$\boxed{\text{1.6 Circuit Elements}}$

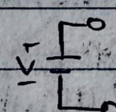
- Active element : able to generate energy
- Passive " : not "
  - generators, batteries, operational amplifiers
  - resistors, capacitors, inductors

ideal independent source: active element that provides a specified voltage / current that is independent from other elements

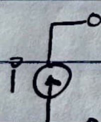
batteries / generators



constant or time-varying voltage



only constant voltage (DC)

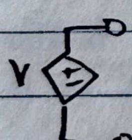


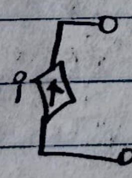
independent current source

transistors, operational amp, integrated circuits

ideal dependent (or controlled) source: source quantity is controlled by another voltage / current

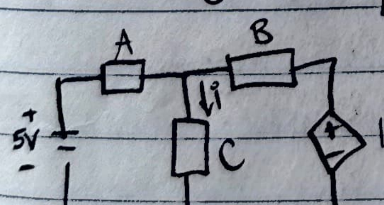
dependent V source





dependent i source

4 types:
- V-controlled v source (VCVS)
- current-controlled v source (CCVS)
- (VCCS)
- (CCCS)



← value is 10i V

★ ask about p_4 on example 1.7 !
- why is it -0.2??
- where are the terminals??
- why is voltage positive??

Voltage: "how much energy was expected to move a unit of charge from a to b

- sep pos + neg creates voltage

Energy = P·t   kW-hours (kw per hour)

1.) 6KW charger , 240 V

$P = Vi$

$6KW = 6{,}000\ W = 240V \cdot i$

$i = 25$ amps

2.) $6Kw \cdot 4h = 24Kwh$

3.) \$0.112 per kWh → \$2.68

4.) \$2.68/70 = 4¢ / mile

5.) \$4.00/25 = 16¢ / mile

Current going into positive → power consumed

Basic Electr. Hw

$p1 = -10V \cdot 1.0A = -10W$ $\quad$ $P4 = 3V \cdot 0.5A = 1.5W$

$p2 = 2V \cdot 1.0A = 2W$ $\quad$ $P5 = 2.5W$

$p3 = 8V \cdot 0.5A = 4W$

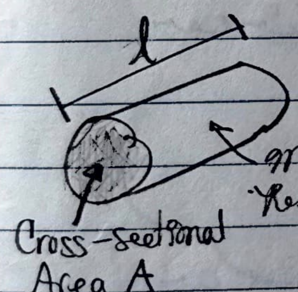
$1.2kW \cdot 2h = 2.4\ kWh \cdot .15 =$

$1kW \cdot \frac{1}{6}h = .167$

Chapter 2

$\boxed{\text{Section 2.1 Intro}}$: Remember to analyze stuff.

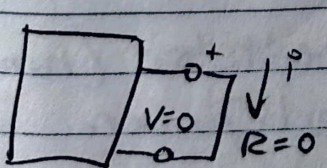
$\boxed{\text{Section 2.2 Ohm's Law}}$



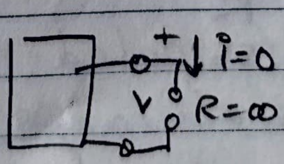
$R = \rho \frac{l}{A}$

Ohm's law : $V = IR$ $\quad$ $R = V/I$ $\quad$ $1\Omega = 1V/A$

Must be passive sign convention.
Current must flow from high to low for $V = IR$. If low to high, $V = -IR$.



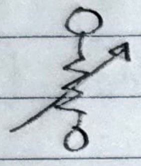
"Short circuit"



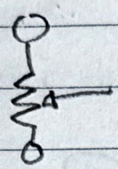
Open circuit ($R = \infty$)

Fixed resistor: remains constant
- wirewound
- composition - large resistance

Variable resistor: changes
- potentiometer
- also wirewound or composition



Variable resistor in General



potentiometer

Resistors that obey Ohm's law: <u>linear</u>
" " don't : nonlinear
↳ lightbulb, diode

Conductance: $G = \frac{1}{R} = \frac{i}{V}$
- how well conducts current

unit → mho or $\mho$
↳ siemens (S)
$1S = 1\mho = 1A/V$
$10\Omega = 0.1S$

$P = Vi = i^2R = \frac{V^2}{R}$
↳ power dissipated in a resistor (non-linear)
↳ Resistors always ab<u>sorb</u> power.

R_{eq} = "equivalent resistance"

$V = iR \rightarrow i = V/R$ $\quad P = \frac{V^2}{R}$

$P = iV$

$V = iR$ $\quad P = i^2 R$

1.) $V = 10\,V$, $R_{eq} = 100\,\Omega$

$i = 10/100 = .1\,A$

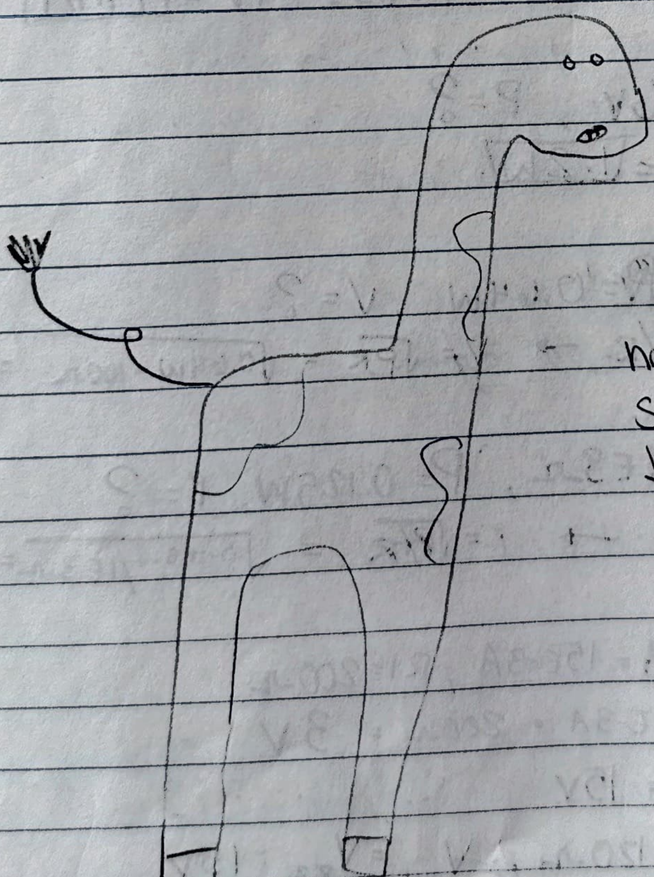
2.) $i = 250\,E{-6}$ $\quad R = 1E3$ $\quad V = .25\,V$

3.) $R = 2\,\Omega$ $\quad i = 100\,E{-3}\,A$ $\quad P = .2\,W$

4.) $R = R$ $\quad V = 3V$ $\quad i = ?$

$V = iR$ $\quad 3V = iR$ $\quad 5\,\Omega$ $\quad i =$ $\quad 1V$ $\quad i = 1/5\,A$ $\quad i = 3/5\,A$

increased by 300%



"I'm so grateful I'm not a giraffe. I'm tall and skinny, but at least I'm not tall, skinny and stupid.

Ohm's Law HW

Seth Ricks
Bro. Jack

1.) $V = 6V$, $R = 3.3\ K\Omega = 3300\ \Omega$, $i = ?$

$i = 6V / 3300\ \Omega = .0018\ A = \boxed{1.8\ mA}$

2.) $V = 8V$, $i = 3.3\ mA = 3.3E\text{-}3\ A$, $R = ?$

$R = 8V / 3.3E\text{-}3\ A = 2424\ \Omega = \boxed{2.4\ K\Omega}$

3.) $i = 320\ \mu A = 320E\text{-}6\ A$, $R_{eq} = 10\ K\Omega = 10E3\ \Omega$, $V = ?$

$V = 320E\text{-}\cancel{6}^{-3}A \cdot 10E\cancel{3}\ \Omega = \boxed{3.2V}$

4.) $V = 9V$, $G_{eq} = 1.0\ mS = 1.0E3S$, $i = ?$

$1/R = S = i/V \rightarrow i = 1.0E3S \cdot 9V = \boxed{9\ mA}$

5.) $i = 0.2A$, $V = 6V$, $P = ?$

$P = 0.2 \cdot 6 = \boxed{1.2W}$

6.) $R_{eq} = 100\ \Omega$, $P = 0.64W$, $V = ?$

$P = V^2/R \rightarrow V = \sqrt{PR} = \sqrt{0.64W \cdot 100\Omega} = \boxed{8V}$

7.) $R_{eq} = 1\ K\Omega = 1E3\ \Omega$, $P = 0.125W$, $i = ?$

$P = i^2 R \rightarrow i = \sqrt{P/R} = \sqrt{0.125W / 1E3\ \Omega} = 0.011\ A = \boxed{11\ mA}$

8.) $V_{R1} = ?$, $i = 15\ mA = 15E\text{-}3A$, $R1 = 200\ \Omega$

$V_{R1} = 15E\text{-}3A \cdot 200\ \Omega = 3V$

$V_{out} = 5(3V) = 15V$

$i_{R2} = ?$, $R2 = 120\ \Omega$, $V_{out} = V_{R2} = 15V$

$i_{R2} = 15V / 120\ \Omega = 0.125\ A = \boxed{12.5\ mA}$

?) $i_{R1} = ?$, $V = 6V$, $R1 = 300\Omega$

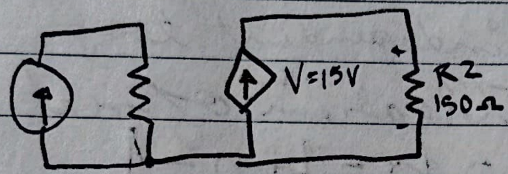
$i_{R1} = 6V / 300\Omega = 0.02A$

$i_{out} = (2.5)(0.02A) = 0.05A$

$V_{R2} = ?$, $i_{R2} = i_{out} = 0.05A$, $R2 = 150\Omega$

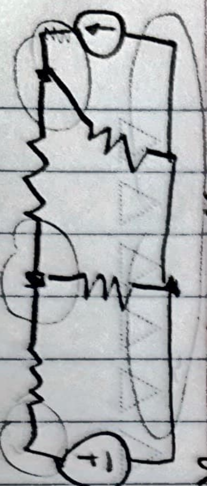
$V_{R2} = 0.05A \cdot 150\Omega = \boxed{7.5V}$

Question:



How can we assume that the voltage across R2 is 15V?

2.3 Nodes, Branches, and Loops



* Branch - a single element such as a voltage source or a resistor
* Node - point of connection between two or more branches
* loop - $\boxed{\text{any}}$ closed path in a circuit
  * Mesh - independent loop

Fundamental theorem of network topology: $b = l + n - 1$
branches = meshes + nodes + 1

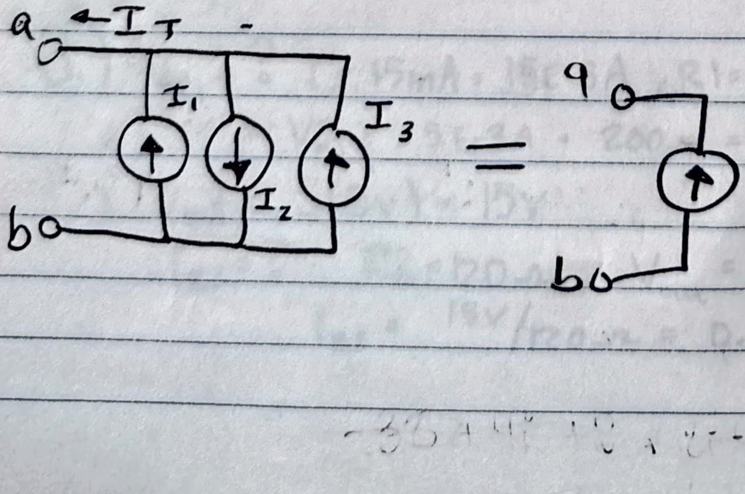
Difference between circuit + network?

2.4 Kirchhoff's Laws

Kirchoff's current law: (KLC)

$$\sum_{n=1}^{N} i_n = 0$$

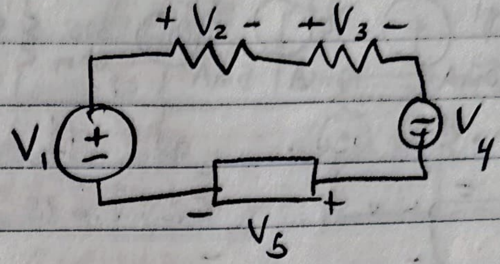
The sum of the currents entering a node is equal to the sum of the currents leaving the node.
- can be applied to a closed boundary



$I_T = I_1 - I_2 + I_3$

Kirchoff's voltage laws (KVL):

$$\sum_{m=1}^{M} V_m = 0$$

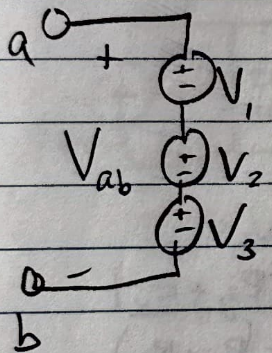


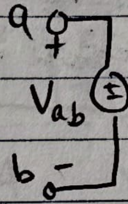
$-V_1+V_2+V_3-V_4+V_5=0$

$V_2+V_3+V_5=V_1+V_4$

Sum of voltage drops = Sum of voltage rise

* Why is this a negative voltage drop?



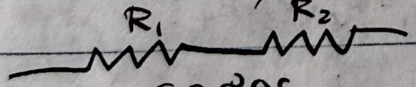


$V_s = V_1+V_2-V_3$

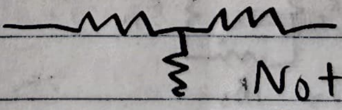
$V_{ab} = V_1+V_2-V_3$

* ALL points on a single node have the same voltage (not current)

Series: exclusively share a node → same i not v



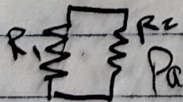
Series



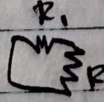
Not Series

Parallel: 2 nodes in common → same v not i

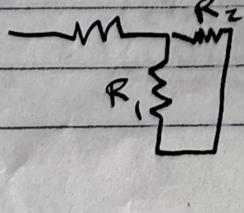
— Okay to have something jutting in



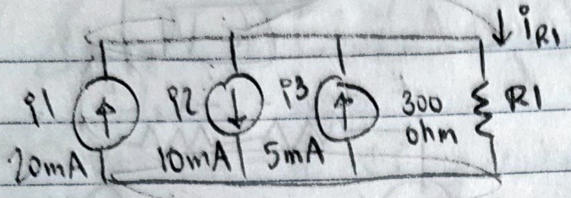
Parallel



Parallel



Paralel



1.) 2 nodes

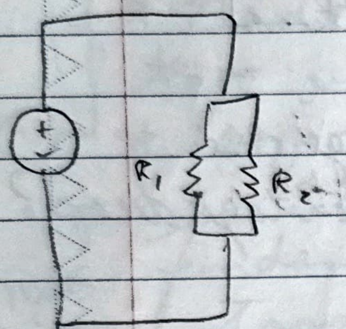
2.) $i_1 + i_3 = i_2 + i_{R1}$

3.) $20mA + 5mA = 10mA + i_{R1}$

$i_{R1} = 15mA$

4.) $V_{R1} = 4.5V$

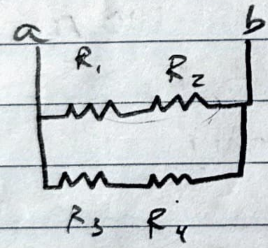
5.) $V_{i2} = 4.5V$



$$R_{eq} = R_1 \| R_2 = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

Lab 1:

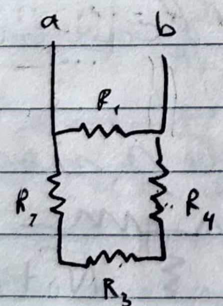
3.)



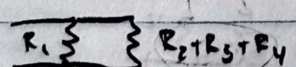
$$R_{eq} = \frac{(R_1 + R_2) \cdot (R_3 + R_4)}{(R_1 + R_2) + (R_3 + R_4)}$$

$$= \frac{(082 + 4.3) \cdot (22 + 6.2)}{0.82 + 4.3 + 22 + 6.2}$$

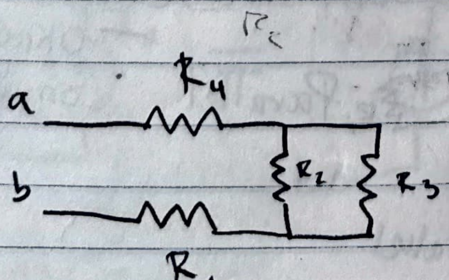
4.)



$$R_{eq} = \frac{R_1 (R_2 + R_3 + R_4)}{R_1 + R_2 + R_3 + R_4}$$



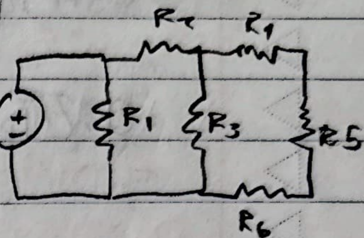
5.)



$$\frac{R_2 \cdot R_3}{(R_2 + R_3)} + R_4 + R_1$$

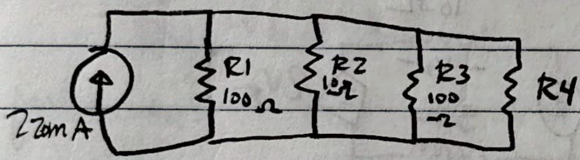
KCL HW

1. Number of branches: 7
Number of nodes: 5
Number of loops: 6
Number of meshes: 3



Components in parallel: VS & R1
Components in series: R4 & R5 & R6

2.



$I_{R1} = 60.0\text{mA}$ $\quad V_{R1} = 0.06\text{ A} \cdot 100\,\Omega = 6V_{R1}$

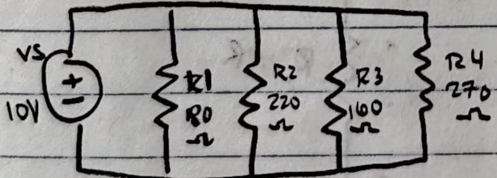
$I_T = 220\text{mA}$ $\quad \boxed{V_{R4} = 6V}$

$I_T = 220\text{mA} = 60.0 + 60.0 + 60.0 + R4$

$R4 = 40.0\text{ mA}$

$P_{R4} = VI = 6V \cdot .04A = \boxed{0.24W}$

3.



$i_{R1} = \frac{10V}{180\,\Omega} = 0.0556\text{ A} = \boxed{55.6\text{mA}}$

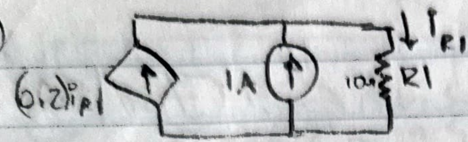
$i_{R2} = 10V/220\,\Omega = \boxed{45.5\text{mA}}$

$i_{R3} = 10V/160\,\Omega = \boxed{62.5\text{ mA}}$

$i_{R4} = 10V/270\,\Omega = \boxed{37.0\text{mA}}$

$i_s = \boxed{200.5\text{mA}}$

4.



$0.2\,i_{R1} + 1 = i_{R1}$

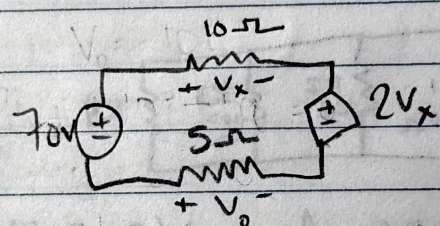
$1 = 0.8\,i_{R1}$

$i_{R1} = 1.25\,A$

$V_{R1} = i_{R1} \cdot R1 = \boxed{12.5V}$

$P_{R1} = 1.25A \cdot 12.5V = \boxed{15.6W}$

KVL practice; find V_x and V_o



why twice?

$-70 + V_x + 2V_x - V_o = 0$ $\qquad V_x = 10i \qquad V_o = -5i$

$-70 + 10i + 20i + 5i = 0$

$i = 2$

$V_x = 20V,\ V_o = -10V$

$i_S R_1$

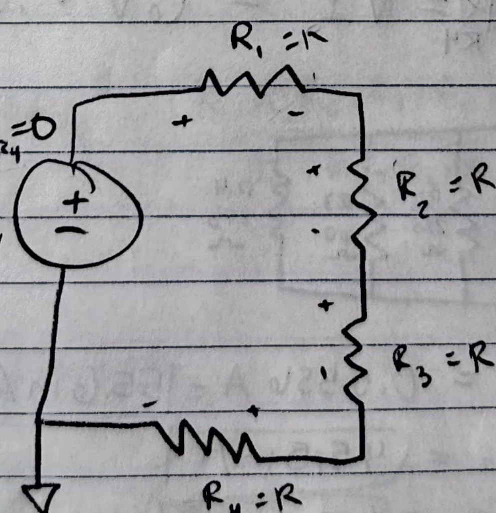
$-4V + V_{R_1} + V_{R_2} + V_{R_3} + V_{R_4} = 0$

$-4V + i_S R_1 + i_S R_2 + i_S R_3 + i_S R_4 = 0$

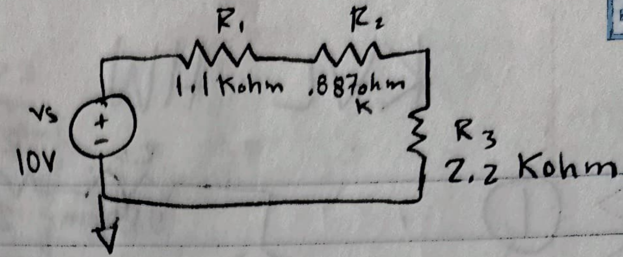
VS 4V

$-4V + 4i_S R = 0$

$-4V + 4V = 0$



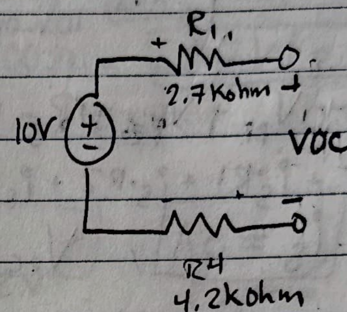
$$i_s = \frac{10V}{(1.1 + .887 + 2.2)\,\text{Kohm}} = 2.39\,mA$$



$V_{R1} = .00239^{A} \cdot 1100.^{\Omega} = 2.6V$

$V_{R2} = .00239^{A} \cdot 887\,\Omega = 2.12V$

$V_{R3} = .00239\,A \cdot 2200\,\Omega = 5.26V$



$R = \infty$

$V = \frac{I}{R} = \frac{I}{\infty}$

$V \to 0$

$-10V + V_{R1} + V_{oc} + V_{R4} = 0$

$\cancel{i_s R_1} + V_{oc} + \cancel{i_s R_4}$

$10v = V_{oc}$

$V_1 + V_2 = 15 \qquad V_1 - V_2 = -2$

$-2 + V_2 + V_2 = 15 \qquad V_1 = -2 + V_2$

$2V_2 - 2 = 15$

$2V_2 = 17$

$V_2 = 17/2$

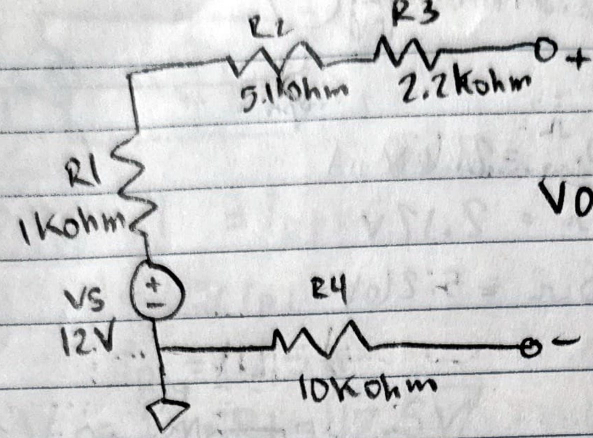
$V_2 \quad V_1$

$8.5, 6.5$

KVL HW

Seth Ricks
ECEN 150
1/20/2023

1.

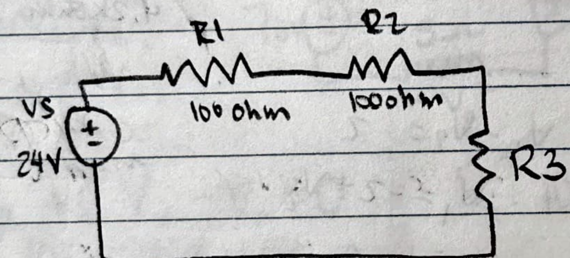


$-V_S + V_{R1} + V_{R2} + R_3 + V_{OC} + R_4 = 0$

$-12V + i_S R1 + i_S \cdot R2 + i_S \cdot R3 + V_{OC} + i_S \cdot R_4 = 0$

$\boxed{i_S = 0A}$, $\boxed{V_{oc} = 12V}$

2.



$V_{R1} = 4.80V \quad i_{R3} = ? \quad P_{R3} = ?$

$i_{R1} = 4.80V / 100\Omega = 0.048\ A \quad \boxed{i_{R3} = 48\ mA}$

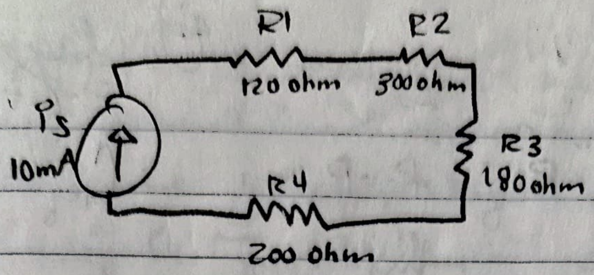
$-VS + i_S R1 + i_S R2 + i_S R3 = 0$

$-24 + 0.048(100 + 100 + R3) = 0$

$R3 = 24/0.048 - 100 - 100$

$\boxed{R3 = 300\Omega}$ $\boxed{P_{R3} = i_{R3}^2 \cdot R3 = 691\ mW}$

(3.)

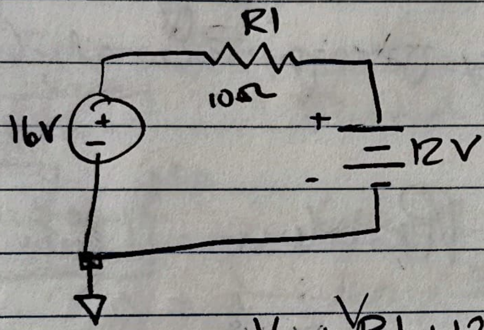


$$V_a = i_s(R1+R2+R3+R4)$$

$$= 0.01A(120+300+180+200) = \boxed{8V = V_a}$$

$$P_{R4} = i_s^2 \cdot R4 = (0.01A)^2 \cdot 200\Omega = \boxed{20mW = P_{R4}}$$

(4.)



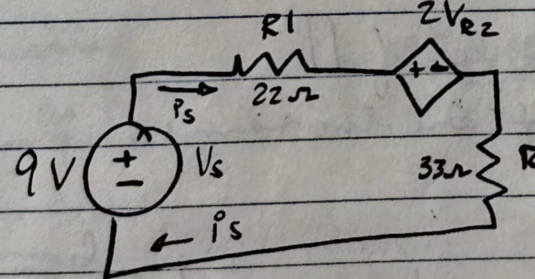
$i_{R1} = ?$

$$-V_s + V_{R1} + 12V = 0$$

$$-16V + i_{R1} \cdot 10\Omega + 12V = 0$$

$$\boxed{i_{R1} = 0.40A}$$

(5.)



$V_{R2} = ?$

$$\boxed{-V_s + V_{R1} + 2V_{R2} + V_{R2} = 0}$$

$$-9 + i_s \cdot 22\Omega + 3(i_s \cdot 33\Omega) = 0$$

$$i_s(22\Omega + 99\Omega) = 9$$

$$i_s = 9/(22+99) \approx 0.074$$

$$-V_s + i_s \cdot R1 + 3V_{R2} = 0$$

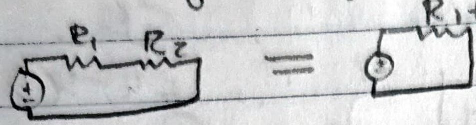
$$\boxed{V_{R2} = \frac{V_s - i_s \cdot R1}{3}}$$

$$\boxed{V_{R2} = 2.45V}$$

Same i
dif v

2.5 Series resistors & Voltage Division

$V = iR_{eq}$, $R_{eq} = R_1 + R_2 \ldots$ $G_{eq} = \frac{G_1 G_2}{G_1 + G_2}$

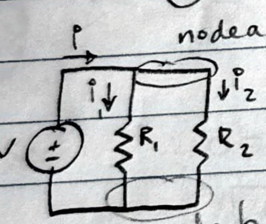


$V_1 = \frac{R_1}{R_1 + R_2} V$, $V_2 = \frac{R_2}{R_1 + R_2} V$, $V_n = \frac{R_n}{R_1 + R_2 + \ldots + R_N} v$

Voltage v is divided among the resistors and is directly proportional to their resistance.

Same v
dif i

2.6 Parallel Resistors and Current Division



$V = i_1 R_1 = i_2 R_2$

$i = i_1 + i_2$

$i = V/R_1 + V/R_2 = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{V}{R_{eq}}$

$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$

$R_{eq} = \frac{R_1 \cdot R_2}{R_1 + R_2}$ ← only if ||

If all the resistors are the same

$R_{eq} = R/N$

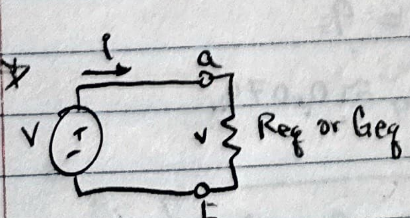
$i_1 = \frac{G_1}{G_1 + G_2} i$

Conductance:

$G_{eq} = G_1 + G_2 + G_3 + \ldots + G_N$

where $G_1 = 1/R_1$, $G_2 = 1/R_2$

$i_n = \frac{G_n}{G_1 + G_2 + \ldots + G_N} i$



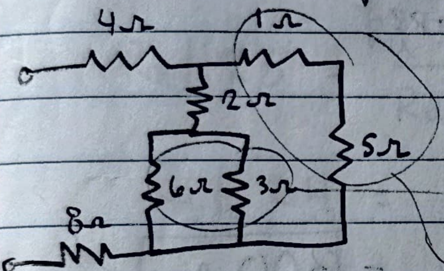
$V = iR_{eq} = \frac{iR_1 R_2}{R_1 + R_2}$

$i_1 = \frac{R_2 i}{R_1 + R_2}$ $i_2 = \frac{R_1 i}{R_1 + R_2}$

Total current i is shared by the resistors in inverse proportion to their resistance.
- principle of current division
- current travels through the path of least resistance.

$$i_1 = \frac{G_1}{G_1+G_2} i \; ; \; i_2 = \frac{G_2}{G_1+G_2} i \; ; \; i_n = \frac{G_n}{G_1+G_2+\dots+G_N} i$$

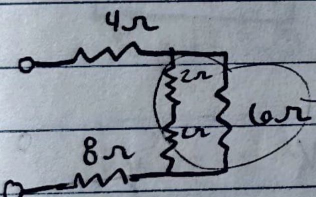
ex;



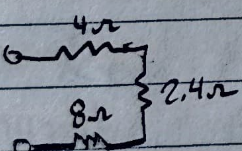
Find R_{eq}

$$6\Omega \| 3\Omega = \frac{6\cdot 3}{6+3} = 2\Omega$$

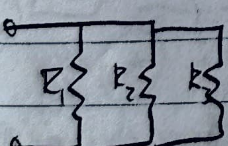
$$1\Omega + 5\Omega = 6\Omega$$



$$4\Omega \| 6\Omega = \frac{4\cdot 6}{4+6} = 2.4\Omega$$



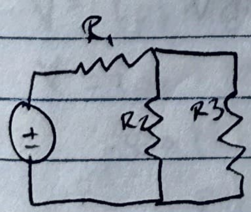
$$R_{eq} = 4\Omega + 2.4\Omega + 8\Omega = \boxed{14.4\Omega}$$



$R_1 = 2$ Kohm
$R_2 = 1.5$ Kohm
$R_3 = 1.2$ kohm

$$R_1 \| R_2 = \frac{2\cdot 1.5}{2+1.5} = \frac{6}{7}$$

$$R_{eq} = \; \| R_3 = \frac{\frac{6}{7}\cdot 1.2}{\frac{6}{7}+1.2} = 0.5 \text{ Kohm} = 500\,\Omega$$



$R_1 = 3.3\,k\Omega$
$R_2 = 4.7\,k\Omega$
$R_3 = 2.7\,k\Omega$

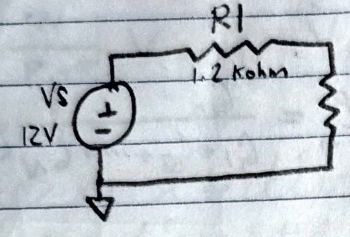
$$R_2 \| R_3 = \frac{4.7\cdot 2.7}{4.7+2.7} =$$

$$R_1 + \; = \frac{3711}{740}\,k\Omega \approx 5.01\,k\Omega$$

Voltage and Current Division HW

1. Once equivalent resistance is calculated, it has similar properties to a single resistor with the same value.

2. $V_{R2} = 2.4V$



$$V_{R2} = \frac{R2}{R1+R2} V$$

$$\frac{V_{R2}}{V} = \frac{R2}{R1+R2}$$

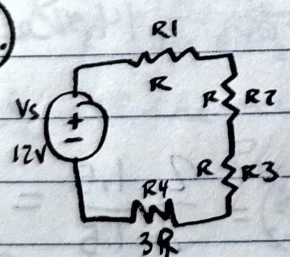
$$\frac{V_{R2}}{V} \cdot R1 = R2 - \frac{V_{R2}}{V} \cdot R2$$

$$R2\left(1 - \frac{V_{R2}}{V}\right) = \frac{V_{R2}}{V} \cdot R1$$

$$R2 = \frac{\left(\frac{V_{R2}}{V} \cdot R1\right)}{\left(1 - \frac{V_{R2}}{V}\right)} = \frac{\frac{2.4V}{12V} \cdot 1200\Omega}{1 - \frac{2.4V}{12V}} = \boxed{300\Omega}$$

Check: $2.4V = \frac{300\Omega}{1200\Omega + 300\Omega} \cdot 12V = 2.4V$

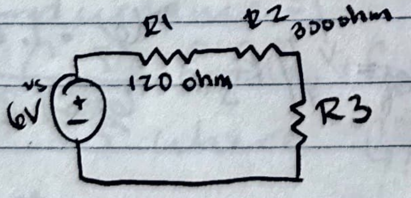
3.



$$R_{eq} = R + R + R + 3R = 6R$$

$$V_{3R} = \frac{R4}{R1+R2+R3+R4} V = \frac{3R}{6R} = \frac{1}{2} V = \boxed{6V}$$

4.



30% vs dropped across R3

$$V_{R3} = \frac{3}{10} \cdot 6V = 1.8V$$

$$V_{R3} = \frac{R3}{R1+R2+R3} V$$

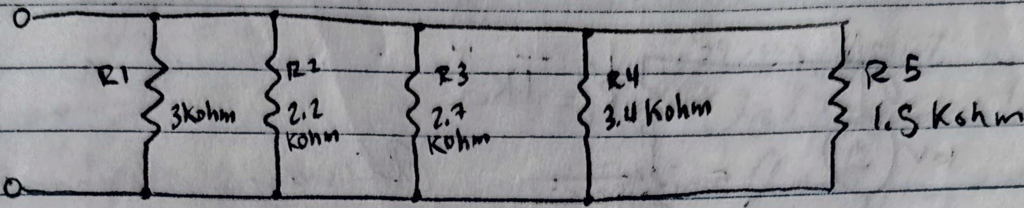
$$\frac{V_{R3}}{V}(R1+R2) = R3 - \frac{V_{R3}}{V} R3$$

$$\frac{\frac{V_{R3}}{V}(R1+R2)}{\left(1 - \frac{V_{R3}}{V}\right)} = R3$$

$$\boxed{R3 = 180\Omega}$$

Control Systems

(5.)



$$R1 \| R2 = \frac{3K\Omega \cdot 2.2K\Omega}{3K\Omega + 2.2K\Omega} = R_a$$

$$R_a \| R3 = \frac{R_a \cdot 2.7K\Omega}{R_a + 2.7K\Omega} = R_b$$

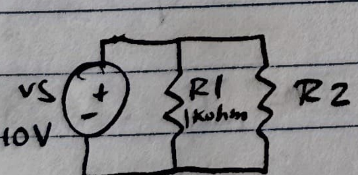
$$R_b \| R4 = \frac{R_b \cdot 3.4K\Omega}{R_b + 3.4K\Omega} = R_c$$

$$R_c \| R5 = \frac{R_c \cdot 1.5K\Omega}{R_c + 1.5K\Omega} = 0.472K\Omega = 472\Omega$$

$$\boxed{R_{eq} = 472\Omega}$$

(6.) Req is inversely proportional to the number of resistors

(7.)



$i_{R2} = 60\%$ of i_{Total} $\qquad i_T = \frac{3}{500}$

$R2 = ?$

$i_{R1} = 40\%$ of i_{Total}

$$V = i_{R1} \cdot R1 = 0.4 \cdot i_T \cdot R1 = 0.4 \cdot i_T \cdot 1000\Omega$$

$$V = i_{R2} \cdot R2 = 0.6 \cdot i_T \cdot R2$$

$$400\, i_T = 0.6\, i_T \cdot R2$$

$$R2 = \frac{400\, i_T}{0.6\, i_T} = \boxed{667\Omega}$$

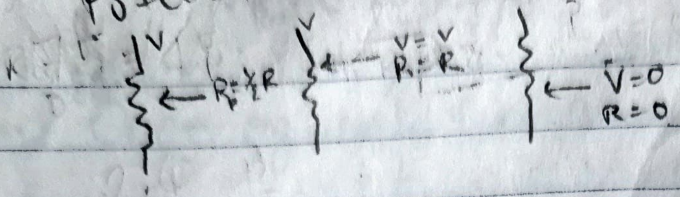
$$R_{eq} = \frac{1000 \cdot 667}{1000 + 667} \approx 400\Omega \qquad i_T = \frac{10V}{400\Omega} = 25mA$$

$i_{R2} = 15mA \quad i_{R1} = 10mA$

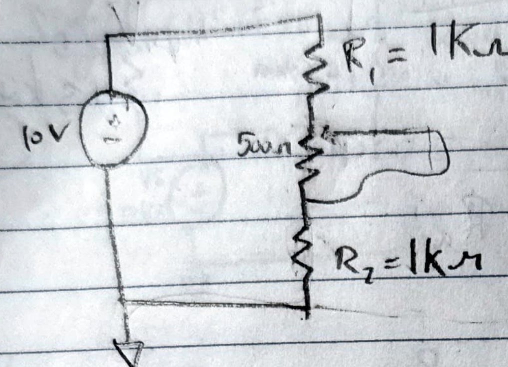
$V = 0.4 \cdot .015 \cdot 1000 = 10V$

$V = 0.6 \cdot .01 \cdot 1667 = 10$

Potentiometer



ex;



Always assume respect to ground if not explicitly said otherwise

3.2 Nodal Analysis
(non-voltage source)

Choose node voltages instead of element voltages → node-voltage method

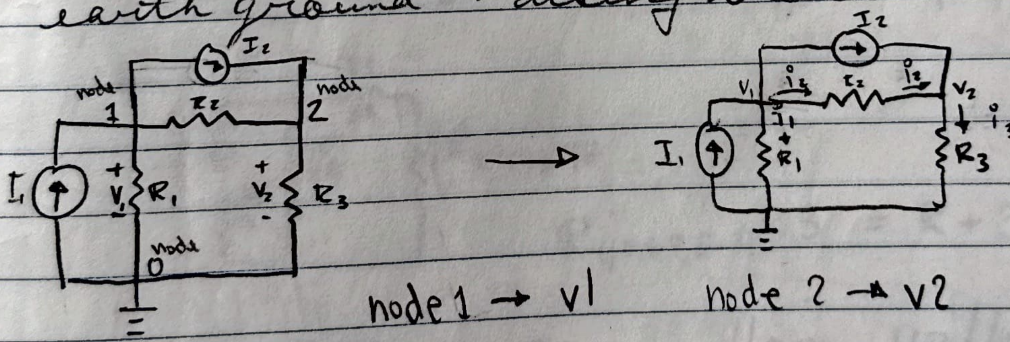
Steps to determine Node Voltages: (n nodes)

"datum node" or ground → 1. Select a reference node. Assign voltages $V_1, V_2, \ldots, V_{n-1}$ to remaining nodes. Voltages referenced with respect to reference node

2. Apply KCL to each n-1 nonreference nodes. Use Ohm's law to express the branch currents in terms of node voltages.

3. Solve the resulting simultaneous to obtain the unknown node voltages.

Chassis ground → case or enclosure is used as ground
earth ground → using the earth



node 1 → v1 node 2 → v2

Voltages are measured w/ respect to voltage node (ground). So here only 2 voltages v_1 & v_2.

(KCL) Node 1: $I_1 = I_2 + i_1 + i_2$ Node 2: $I_2 + i_2 = i_3$

★ Current flows from a higher potential to a lower potential in a resistor. ★

$$i = \frac{V_{higher} - V_{lower}}{R} = \frac{\text{Previous voltage} - \text{next voltage}}{\text{Resistor inbetween}}$$

$i_1 = \frac{V_1 - 0}{R_1}$ or $i_1 = G_1 V_1$ $i_2 = \frac{V_1 - V_2}{R_2}$ or $i_2 = G_2(V_1 - V_2)$

$i_3 = \frac{V_2 - 0}{R_3}$ or $i_3 = G_3 V_2$

$$\begin{cases} I_1 = I_2 + \frac{V_1}{R} + \frac{V_1 - V_2}{R_2} \\ I_2 + \frac{V_1 - V_2}{R_2} = \frac{V_2}{R_3} \end{cases}$$

or

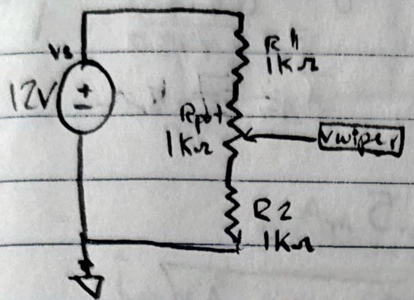
$$I_1 = I_2 + G_1 V_1 + G_2(V_1 - V_2)$$

$$I_2 + G_2(V_1 - V_2) = G_3 V_2$$

$\hookrightarrow$ Solve system of equations for V_1 & V_2

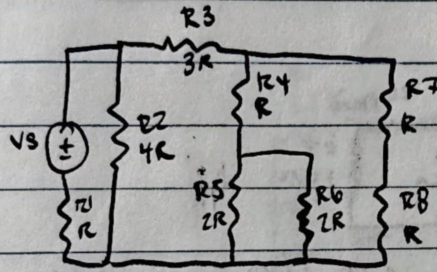
Series - Parallel Circuits HW

1.

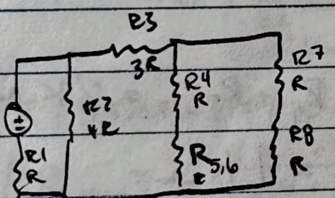


$$V_{wiper} = \frac{R2 \cdot 12V}{R1+R2+R_{pot}} \cdot \frac{1k\Omega}{3k\Omega} = \boxed{4V}$$

2.



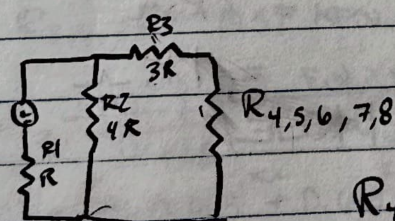
$$R5 \| R6 = 2R \| 2R = R$$



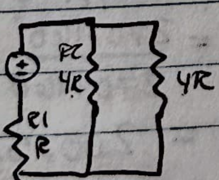
$$R4 + R_{5,6} = R+R = 2R$$

$$R7 + R8 = R+R = 2R$$

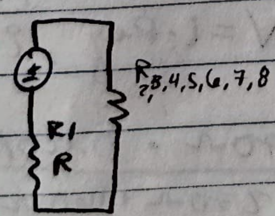
$$R_{4,5,6} \| R_{7,8} = 2R \| 2R = R$$



$$R_{4,5,6,7,8} + R3 = R + 3R = 4R$$

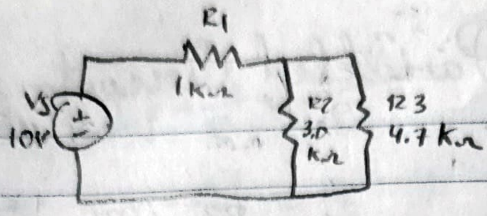


$$R_{3,4,5,6,7,8} \,//\, R2 = 4R // 4R \overset{16/8}{=} \underline{2R}$$



$$R1 // R_{2,3,4,5,6,7,8} = R + 2R = \boxed{3R}$$

3.

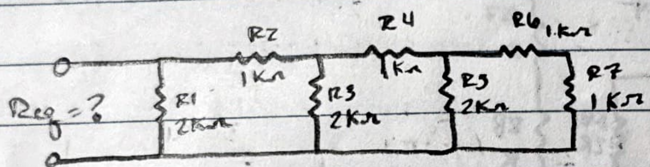


$$R_{eq} = (R2 // R3) + R1$$
$$= \frac{4.7 \cdot 3.0}{4.7 + 3.0} + 1$$
$$= 7\ k\Omega$$

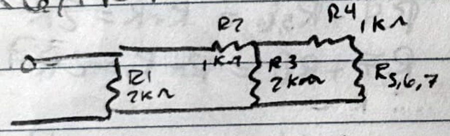
$$V = i R_{eq}$$
$$i = 10V / 7k\Omega = 3.5\ mA$$

$$i_2 = \frac{R_1 i}{R_1 + R_2} = \frac{4.7 \cdot 3.5}{3.0 + 4.7} = \boxed{2.14 mA}$$

4.



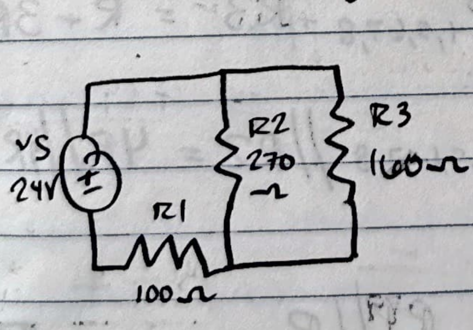
$$R6 // R7 = 0.5 k\Omega \qquad R5 // R_{6,7} = 1 k\Omega$$



$$R_4 + R_{5,6,7} = 2k\Omega$$

$$R3 // R_{4,5,6,7} = 1 k\Omega$$
$$R2 + R_{3,4,5,6,7} = 2 k\Omega$$
$$R1 // R_{2,3,4,5,6,7} = 1 k\Omega$$
$$\boxed{R_{eq} = 1 k\Omega}$$

5.

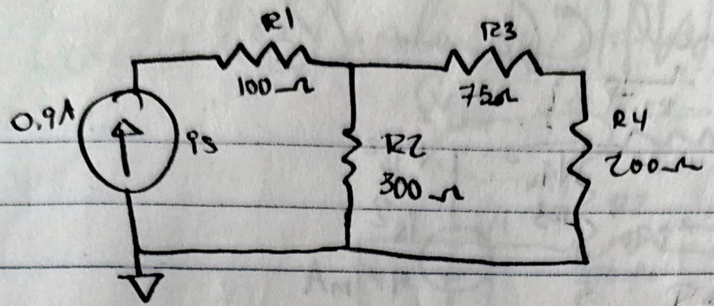


$$R_{eq} = R2 // R3 + R1$$
$$= \frac{270 \cdot 160}{270 + 160} + 100$$
$$= 200.5\ \Omega$$

$$V = i \cdot R_{eq}, \quad i = \frac{24V}{200.5\Omega} = 0.120 A$$

$$i_{R3} = \frac{R_2 \cdot i}{R_2 + R_3} = \frac{270\Omega \cdot 0.120A}{270\Omega + 160\Omega} = 0.075 A$$
$$= \boxed{75 mA}$$

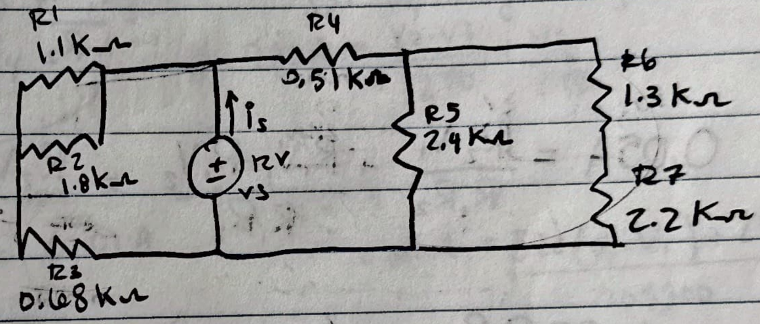
6.



$i_{R2} = ?$

$$i_{R2} = \frac{(R3+R4)\cdot i_s}{R2+R3+R4} = \boxed{430\,mA}$$

7.



$R6 + R7 = 1.3K\Omega + 2.2K\Omega = 3.5\,K\Omega$

$R5 // R_{6,7} = \frac{2.4\,K\Omega \cdot 3.5\,K\Omega}{2.4+3.5\,K\Omega} = 1.4\,K\Omega$

$R4 + R_{5,6,7} = 0.51\,K\Omega + 1.4\,K\Omega = 1.93\,K\Omega$

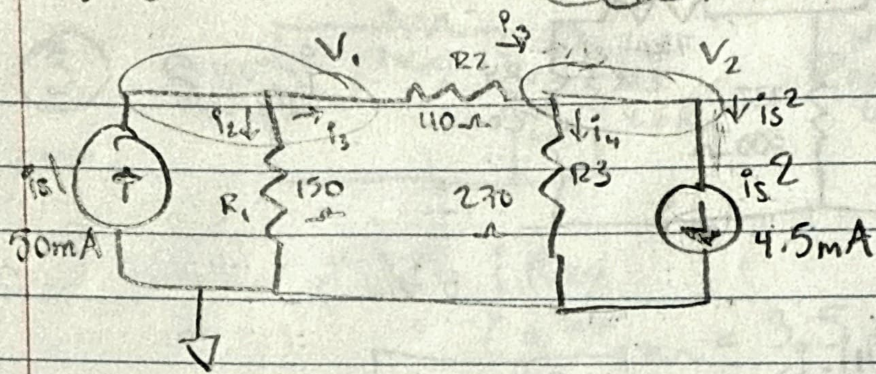
$R1 // R2 = \frac{1.1 \cdot 1.8}{1.1+1.8} = 0.68\,K\Omega$

$R_{1,2} + R_3 = 0.68\,K\Omega + 0.68\,K\Omega = 1.36\,K\Omega$

$R_{1,2,3} // R_{4,5,6,7} = \frac{1.36\,K\Omega \cdot 1.91\,K\Omega}{1.36\,K\Omega + 1.91\,K\Omega} = \boxed{799\,\Omega = R_{eq}}$

$V = i \cdot R_{eq}$ $\quad i_s = \frac{12V}{799\,\Omega} = \boxed{15\,mA = i_s}$

Nodal Anal. Class 1/30



$V_1 : i_{s1} = i_2 + i_3$

$V_2 : i_3 = i_4 + i_{s2}$

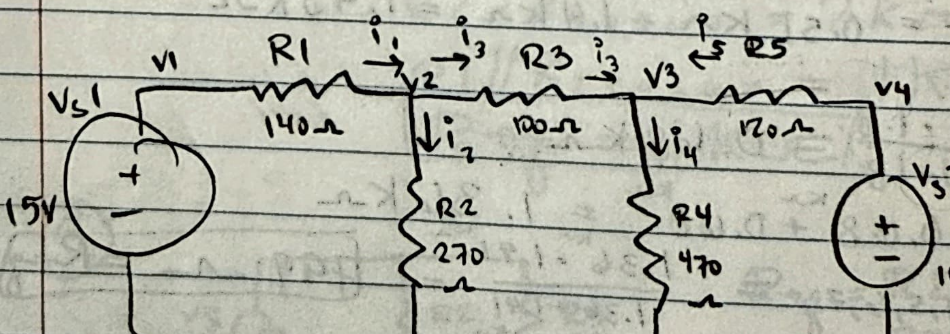
$$0.05A = \frac{V_1}{R1} + \frac{V_1 - V_2}{R2}$$

$$\frac{V_1 - V_2}{R_2} = \frac{V_2}{R_3} + 0.0045A$$

$$0.05A = \frac{R_2 V_1}{R_1 R_2} + \frac{R_1 V_1 - R_1 V_2}{R_1 R_2} = \frac{R_2 V_1 + R_1 V_1 - R_1 V_2}{R_1 R_2}$$

$$0.05 R_1 R_2 = V_1 (R_1 + R_2) - V_2 R_1$$

★Do for other one★

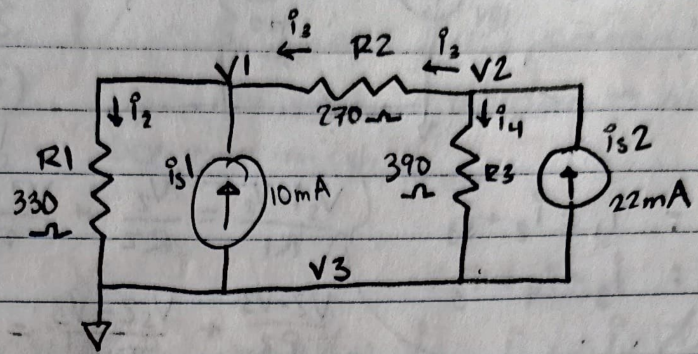


$V_2 : i_1 = i_2 + i_3$, $\frac{15V - V_2}{R_1} = \frac{V2}{R2} + \frac{V2 - V3}{R3}$

$V_3 : i_3 + i_5 = i_4$, $\frac{V2 - V3}{R3} + \frac{10V - V_3}{R5} = \frac{V_3}{R4}$

Nodal Analysis HW

1.



A.) V1: $i_2 = i_{s1} + i_3$ $\qquad \frac{V1}{330} = .01A + \frac{V2-V1}{270}$

V2: $i_{s2} = i_3 + i_4$ $\qquad .022A = \frac{V2-V1}{270} + \frac{V2}{390}$

$$\frac{V1}{330} - \left(\frac{V2-V1}{270}\right) = 0.01A$$

$$\frac{270V1 - 330V2 + 330V1}{89100} = 0.01$$

$$\boxed{600V1 - 330V2 = 891}$$

$$0.022A = \frac{390(V_2 - V_1) + 270V_2}{105300}$$

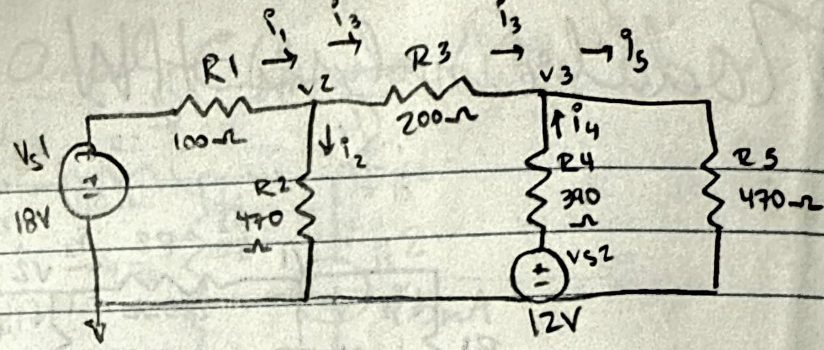
$$2316.6 = 390V_2 - 390V_1 + 270V_2$$

$$\boxed{2316.6 = 660V_2 - 390V_1}$$

MATLAB

B.) $V_1 = 5.06\,V \qquad V_2 = 6.5\,V$

(2)



$V2: \; i_1 = i_2 + i_3$

$V3: \; i_3 + i_4 = i_5$

$$\frac{V_s1 - V_2}{R1} = \frac{V_2}{R2} + \frac{V2 - V3}{R3}$$

$$\frac{V2 - V3}{R3} + \frac{V_s2 - V3}{R4} = \frac{V3}{R5}$$

$$\frac{18 - V_2}{100} = \frac{V_2}{470} + \frac{V_2 - V_3}{200}$$

$$\frac{18}{100} = \frac{V_2}{470} + \frac{V_2}{100} + \frac{V_2 - V_3}{200}$$

$$\frac{18}{10} = \frac{V_2}{47} + \frac{V_2}{10} + \frac{V_2 - V_3}{20}$$

$$= \frac{20V_2}{940} + \frac{94V_2}{940} + \frac{47(V_2 - V_3)}{940}$$

$$= \frac{20V_2}{940} + \frac{94V_2}{940} + \frac{47V_2 - 47V_3}{940}$$

$$\frac{18}{10} = \frac{161V_2 - 47V_3}{940}$$

$$1692 = 161V_2 - 47V_3$$

$$\frac{V2 - V3}{200} + \frac{12 - V3}{390} = \frac{V3}{470}$$

$$\frac{V2 - V3}{20\cancel{0}} - \frac{V3}{39\cancel{0}} - \frac{V3}{47\cancel{0}} = \frac{-12}{39\cancel{0}}$$

$$\frac{1833(V2 - V3) - 940V3 - 780V3}{36660} = \frac{-12}{39}$$

$$1833V2 - 1833V3 - 940V3 - 780V3 = -11280$$

$$1833V2 - 3553V3 = -11280$$

☆ MATLAB ☆

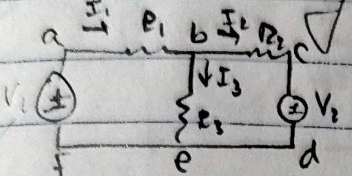
$$\boxed{\begin{aligned} V_2 &= 13.5V \\ V_3 &= 10.1V \end{aligned}}$$

Mesh - loop w/ no other loops

3.4 Mesh Analysis (loop or mesh-current method)

applies KVL to find unknown currents

— Only if planar (no crossing branches)



abef → mesh ; abcdefa → not mesh

bcde → mesh

Steps:

1.) Assign mesh currents $i_1 \ldots i_2 \ldots i_n$ to n meshes

2.) Apply KVL to each of the n meshes. Use ohm's law to express the voltages in terms of mesh currents

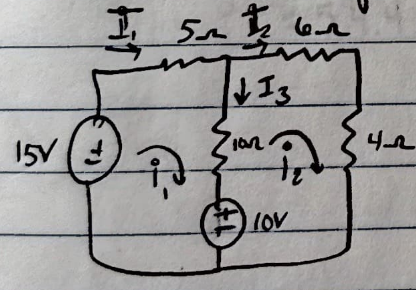
3.) Solve resulting n equations to get mesh currents

Mesh 1: $-V_1 + R_1 i_1 + R_3(i_1 - i_2) = 0$ or $V_1 = (R_1 + R_3)i_1 - R_3 i_2$

Mesh 2: $R_2 i_2 + V_2 + R_3(i_2 - i_1) = 0$ or $-V_2 = -R_3 i_1 + (R_2 + R_3) i_2$

* Solve for i_1 and i_2

Ex:



mesh 1:

$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$

$3i_1 - 2i_2 = 1$

mesh 2:

$6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0$

$i_1 = 2i_2 - 1$

$6i_2 - 3 - 2i_2 = 1$

$i_2 = 1A$

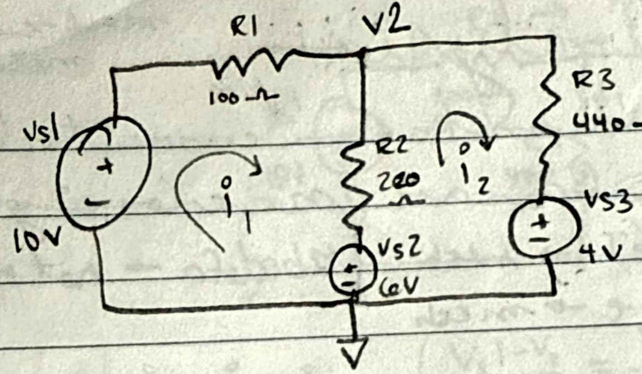
$I_1 = i_1 = 1A$

$I_2 = i_2 = 1A$

$I_3 = i_1 - i_2 = 0$

Nodal	Analysis	Mesh
parallel		series
nodes < meshes		meshes < nodes
op amp circuit		transistors
non-planar		planar

ex:



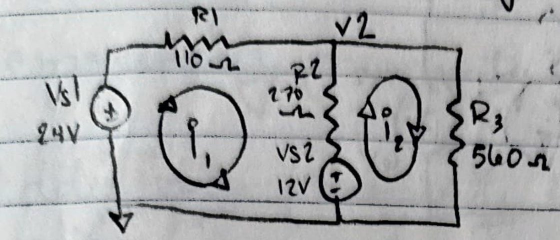
$$-10V + i_1R_1 + (i_1 - i_2)R_2 + 6V = 0$$
$$4V - 6V + (i_2 - i_1)R_2 + i_2R_3 = 0$$

$$4V = i_1(R_1 + R_2) - i_2(R_2)$$
$$2V = i_2(R_2 + R_3) - i_1(R_2)$$

$$4V = 320i_1 - 220i_2$$
$$2V = 660i_2 - 220i_1$$

Mesh Analysis HW

1.



mesh 1: $-24V + i_1R_1 + (i_1 - i_2)R_2 + 12V = 0$

$(R_1 + R_2)i_1 - R_2 i_2 = -12V$

$380i_1 - 270i_2 = -12V$

mesh 2: $-12V + (i_2 - i_1)R_2 + i_2R_3 = 0$

$-12V + (R_2 + R_3)i_2 - R_2 i_1 = 0$

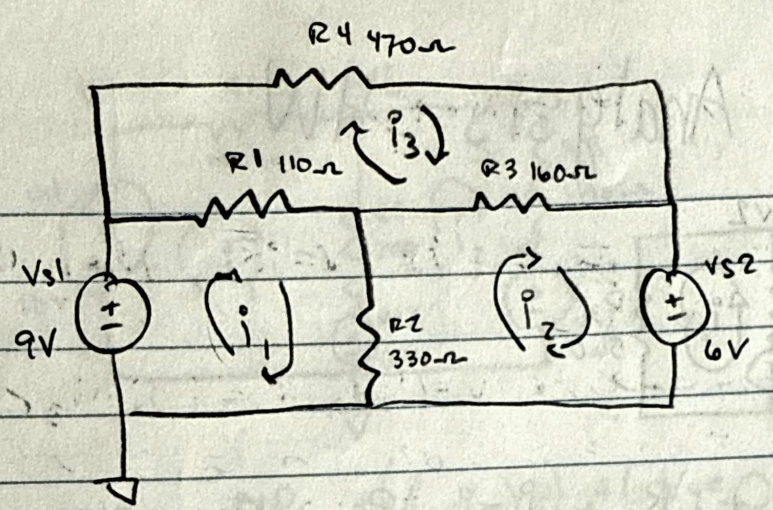
$-12V + 830i_2 - 270i_1 = 0$

$830i_2 - 270i_1 = 12V$

W/ MATLAB

$i_1 = 54.4\ mA$ $i_2 = 32.2\ mA$

2.



mesh 1 : $-9V + (i_1 - i_3)R_1 + (i_1 - i_2)R_2 = 0$

$$-9V + R_1 i_1 - R_1 i_3 + i_1 R_2 - i_2 R_2 = 0$$

$$-9V + (R_1 + R_2) i_1 - R_2 i_2 - R_1 i_3 = 0$$

$$-9V + 440 i_1 - 330 i_2 - 110 i_3 = 0$$

$$440 i_1 - 330 i - 110 i_3 = 9V$$

Mesh 2: $6V + (i_2 - i_1)R_2 + (i_2 - i_3)R_3 = 0$

$$6V + R_2 i_2 - R_2 i_1 + R_3 i_2 - R_3 i_3 = 0$$

$$6V + -R_2 i_1 + (R_2 + R_3) i_2 - R_3 i_3 = 0$$

$$6V - 330 i_1 + 490 i_2 - 160 i_3 = 0$$

$$-330 i_1 + 490 i_2 - 160 i_3 = -6V$$

mesh3 : $(i_3 - i_2) R_3 + (i_3 - i_1) R_1 + i_3 R4 = 0$

$$R_3 i_3 - R_3 i_2 + R_1 i_3 - R_1 i_1 + R_4 i_3 = 0$$

$$-R_1 i_1 - R_3 i_2 + (R_1 + R_3 + R_4) i_3 = 0$$

$$-110 i_1 - 160 i_2 + 740 i_3 = 0$$

MATLAB

i_1 = 29.2 mA

i_2 = 9.5 mA

i_3 = 6.4 mA

4.2 Linearity Property

Homogeneity property : if $V=iR$, then $KiR=KV$

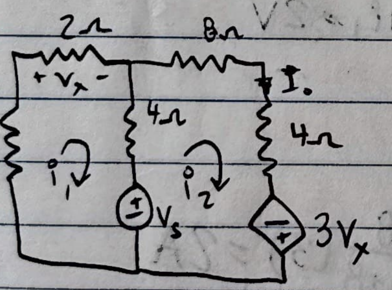
Additivity property : $V=(i_1+i_2)R = i_1R + i_2R = V_1+V_2$
- Response to a sum of inputs is the sum of the response to each input separately.

Circuit is linear if both are satisfied.
— Output is linearly related to its input (power relation not linear)

↳ if $P_1 = Ri_1$, $P_2 = Ri_2$

$P_3 = R(i_1+i_2)^2 \neq P_1+P_2$

Ex;



Find I_0 when $V_S = 12V$ and $V_S = 24V$

$12i_1 - 4i_2 + V_S = 0$ (KVL)

$-4i_1 + 16i_2 - 3v_x - V_S = 0$; $V_x = 2i_1$

$-10i_1 + 16i_2 - V_S = 0$

$2i_1 + 12i_2 = 0 \Rightarrow i_1 = -6i_2$

$-76i_2 + V_S = 0 \Rightarrow i_2 = \frac{V_S}{76}$

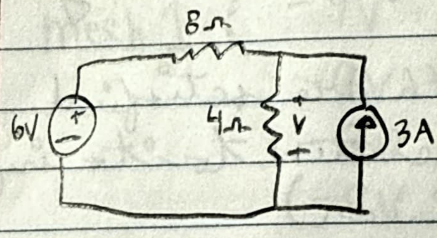
When $V_S = 12V$, $I_0 = i_2 = \frac{12}{76}$ A

$= 24V$, $= \frac{27}{76}$ A

4.3 Superposition

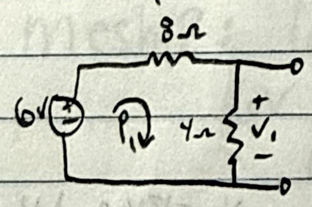
Superposition : Voltage (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or current through) that element due to each independent source acting alone.

ex;



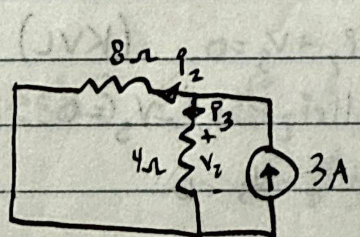
Let $V = V_1 + V_2$

split



$12i_1 - 6 = 0 \Rightarrow i_1 = 0.5A$

$V_1 = 4i_1 = 2V$

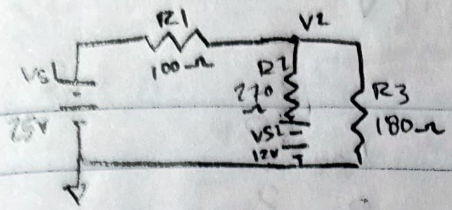


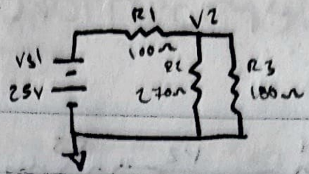
$i_3 = \frac{8}{4+8}(3) = 2A$

$V_2 = 4i_3 = 8V$

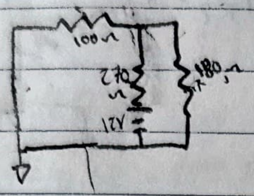
$V = V_1 + V_2 = 2 + 8 = \underline{10V}$

* To turn off voltage → short circuit (0V)
* To turn off current → open circuit (0A)





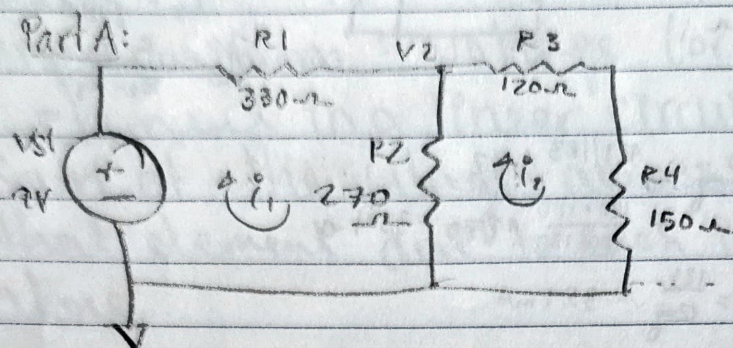
$$R_{eq} = R2 \| R3 + R1$$
$$= \frac{180 \cdot 270}{180+270} + 100 = 208\,\Omega$$
$$i_T = \frac{25V}{208\,\Omega} = 0.12\,A$$
$$i_{R3} = \frac{R_2 \cdot i_T}{R_2+R_3} = 72\,mA$$



$$R_{eq} = R1 \| R3 + R2$$
$$= \frac{100 \cdot 180}{100+180} + 270 = 334\,\Omega$$
$$i_T = \frac{12V}{R_{eq}} = 35.9\,mA$$

Superposition HW

1. Part A:



$i_{R2} = ?$

$P_{R2} = ?$

$R_{eq} = R1 + R2 // (R3+R4)$

$R3 + R4 = 270\Omega$

$R2 // R_{3,4} = 135$

$R1 + R_{2,3,4} = 465\Omega = R_{eq}$

$i_{TA} = \frac{9V}{465\Omega} = 19.4mA$

$-9V + i_1R_1 + (i_1 - i_2)R_2 = 0$

$i_2(R4+R3) + (i_2 - i_1)R2 = 0$

$V_{R2A} = i_{TA} \cdot R2 + (R3 // R4) = 2.619V$

$-9V + i_1R_1 + i_1R_2 - i_2R_2 = 0$

$-9V + (R_1 + R_2)i_1 - i_2R_2 = 0$

$-9 + 600i_1 - 270i_2 = 0$

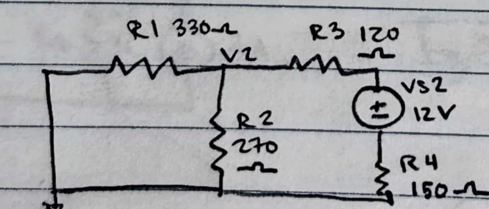
$(R4 + R3 + R2)i_2 - R2i_1 = 0$

$540i_2 - 270i_1 = 0$

$i_1 = 19.4mA \quad i_2 = 9.7mA$

$i_{R2A} = i_1 - i_2 = 9.7mA$

Part B:



$R_{eq} = R4 + R1 // R2 + R3$

$R1 // R2 = 148.5\Omega$

$R_{eq} = 418.5\Omega$

$i_{TB} = \frac{12V}{418.5\Omega} = 28.7mA$

$V_{R2} = i_{TB} \cdot (R1 // R2)$

$= 4.26V$

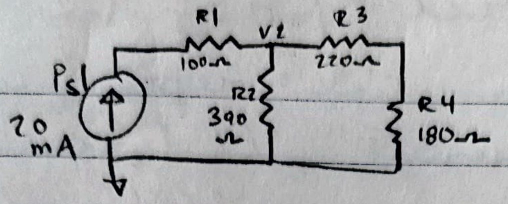
$i_{R2B} = V_{R2} / R2 = 15.8mA$

$i_{R2T} = i_{R2A} + i_{R2B} = \boxed{25.5mA}$

$V_{R2} = V_{R2A} + V_{R2B} = 6.879$

$P_{R2} = i_{R2T} \cdot V_{R2} = \boxed{175\ mW}$

(2) Part A:



$i_{R2} = ?$

$P_{R2} = ?$

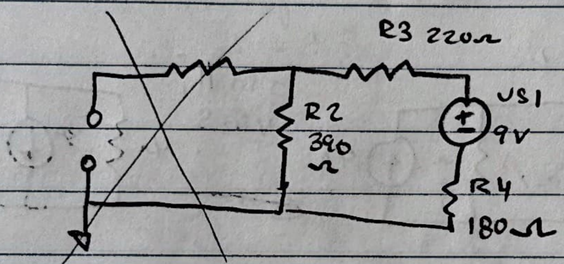
$R_{eq} = R1 + (R2 \,//\, (R3 + R4))$ $R2 // R_{3,4} = 197.5\,\Omega$

$= 297.5\,\Omega$

$V_{R2A} = 0.02 \cdot 197.5 = \underline{3.95}\,V$

$i_{R2A} = 3.95V / 390\,\Omega = 10\,mA$

Part B:



$R_{eq} = R3 + R2 + R4 = 790\,\Omega$

$i_{TB} = 9V / 790\,\Omega = 11.4\,mA = i_{R2B}$

$V_{R2B} = i_{R2B} \cdot R2$

$= 4.44\,V$

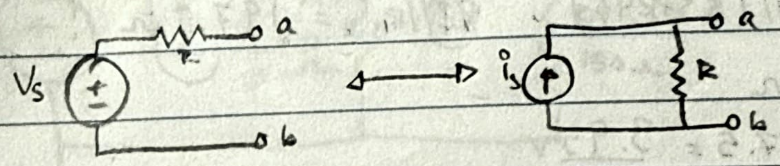
$i_{R2T} = i_{R2A} + i_{R2B} = \boxed{21.4\,mA}$

$V_{R2} = V_{R2A} + V_{R2B} = 8.39\,V$

$P_{R2} = i_{R2T} \cdot V_{R2} = \boxed{180\,mW}$

4.4 Source Transformation

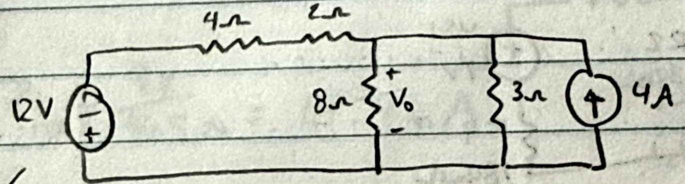
Source transformation: replacing a voltage source in series with a resistor R by a current source in parallel with R, or vice versa



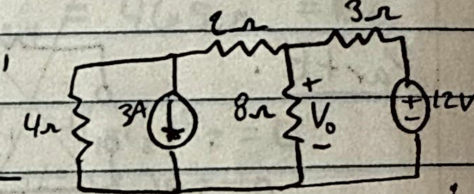
(Or dependent sources)

Note: arrow of current => + of voltage
: Not possible if R = 0

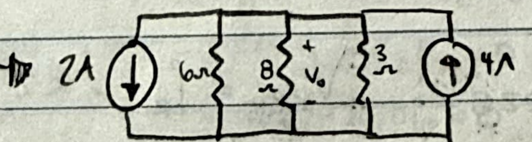
ex;



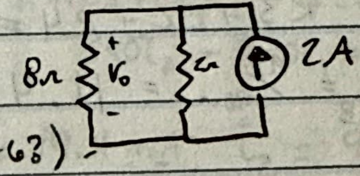
I in ||, V in S

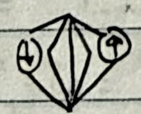


V in S, I in ||



combine I (why 2 not 6?)

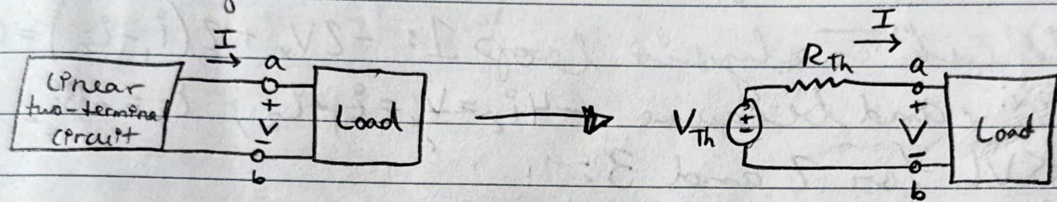




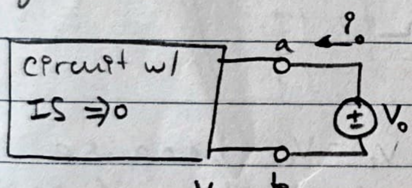
RS → Source resistance

4.5 Thevenin's Theorem

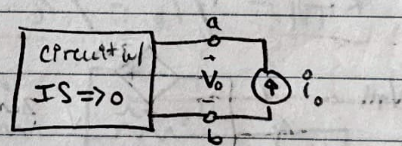
A linear, two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series w/ R_{Th}. $V_{Th} \to$ open circuit voltage
$R_{Th} \to R_{eq}$



Turn off all independent sources. Not dependent.
Apply V_o to determine i_o, or apply i_o to find V_o. Then $R_{Th} = V_o / i_o$



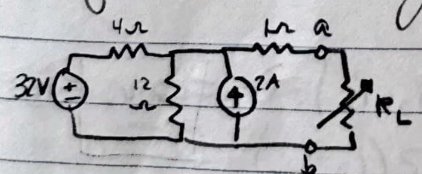
$R_{Th} = \frac{V_o}{i_o}$

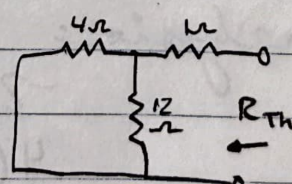


$R_{Th} = V_o / i_o$

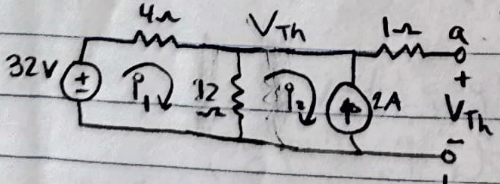
★If R_{Th} is negative, the circuit is supplying power.

ex;



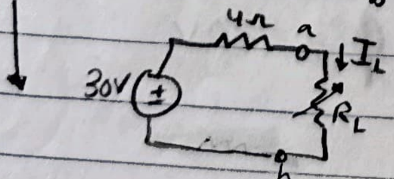


$R_{Th} = 4 \| 12 + 1 = 4\Omega$



(Nodal Analysis)

$$\frac{32 - V_{Th}}{4} + 2 = \frac{V_{Th}}{12} \qquad V_{Th} = 30V$$

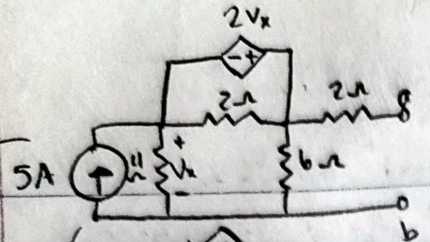


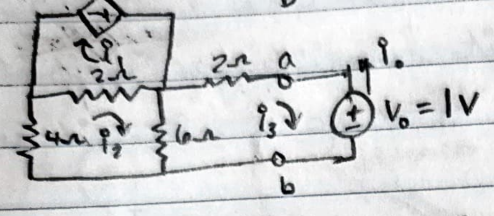
$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{30}{4 + R_L}$$

If $R_L = 6$, $I_L = \frac{30}{10} = 3A$
$= 16$, $I_L = \frac{30}{20} = 1.5A$
$= 36$, $I_L = \frac{30}{40} = 0.75A$

★ w/ Dependent Source!! ★

Find Thevenin Equivalent





Add simple Voltage Source

Mesh analysis loop 1: $-2V_x + 2(i_1 - i_2) = 0$ or $V_x = i_1 - i_2$

and because $-4i_2 = V_x = i_1 - i_2$, $i_1 = -3i_2$

KVL for 2 and 3:

$$4i_2 + 2(i_2 - i_1) + 6(i_2 - i_3) = 0$$

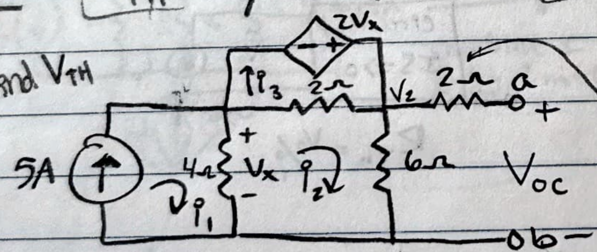
$$6(i_3 - i_2) + 2i_3 + 1 = 0$$

Solve : $i_3 = \frac{1}{6}$ A

and $i_0 = -i_3 = \frac{1}{6}$ A

So $\boxed{R_{TH}} = 1/i_0 = 1/\tfrac{1}{6} = \boxed{6\Omega}$

Use og circuit to find V_{TH}



$V_2 = V_{oc}$ because current through is 0

Mesh analysis: $i_1 = 5$

$$-2V_x + 2(i_3 - i_2) = 0 \Rightarrow V_x = i_3 - i_2$$

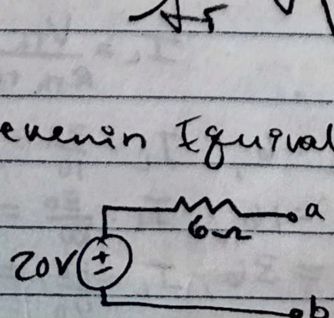
$$4(i_2 - i_1) + 2(i_2 - i_3) + 6i_2 = 0$$

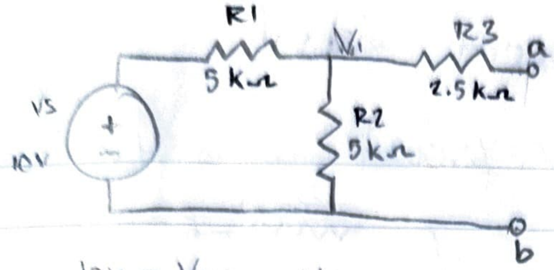
$12i_2 - 4i_1 - 2i_3 = 0$ and $4(i_1 - i_2) = V_x$

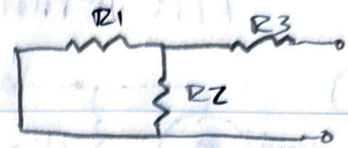
Solving leads to $i_2 = 10/3$,

So $V_{TH} = V_{oc} = 6i_2 = 20V$

Thevenin Equivalent:







$$\frac{10V - V_{TH}}{5k\Omega} + \frac{V_{TH}}{5k\Omega} = 0$$

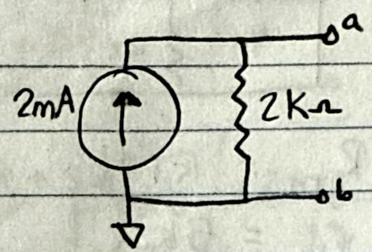
$V_{TH} = 5V$

$R_{TH} = R1 \parallel R2 + R3$

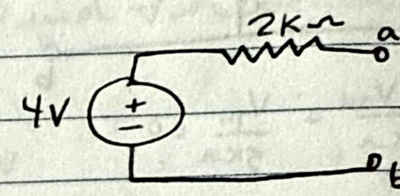
$= 5k\Omega$

Thevenin's Theorem HW

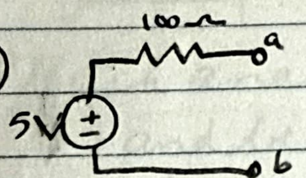
1.



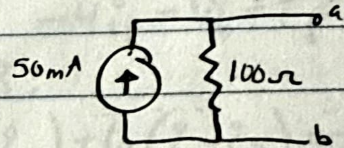
$V = 0.002\,A \cdot 2000\,K\Omega = 4V$



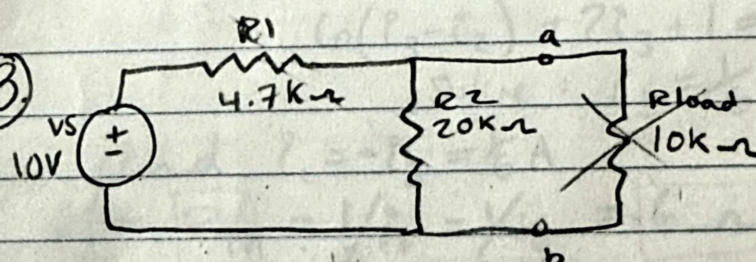
2.

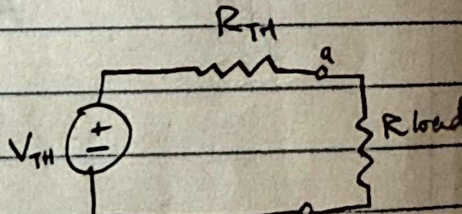


$I = V/R = \frac{5V}{100\Omega} = 50\,mA$



3.





$R_{TH} = R1 \parallel R2 = 3.8\,K\Omega$

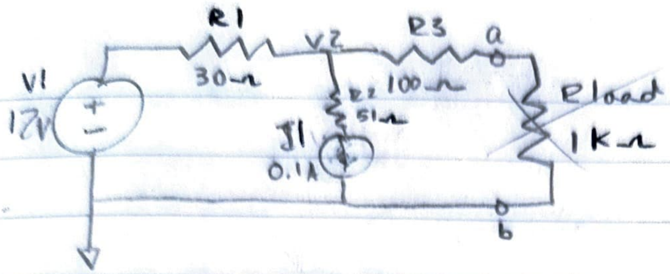
$V_{TH} = V_{R2} = V_S\left(\frac{R_2}{R_1+R_2}\right)$

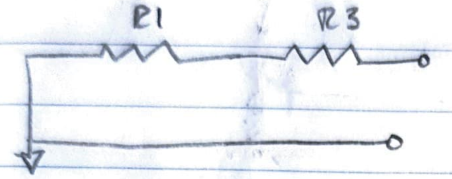
$= 10 \cdot \left(\frac{20}{24.7}\right) = 8.10\,V$

$V_{TH} = (R_{TH} + R_{load})\,I_T$

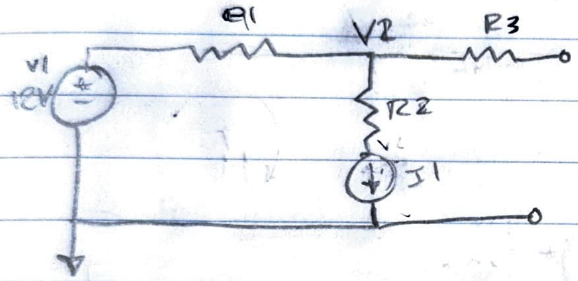
$I_T = V_{TH} / (R_{TH} + R_{load}) = 586\,\mu A$

4.





$R_{TH} = 130\Omega$



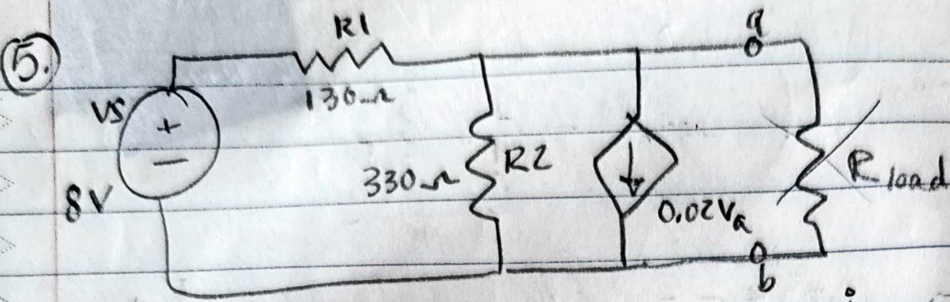
$$V2 = V1 - V_{R1}$$
$$= 12 - (0.1)(30) = 9V$$

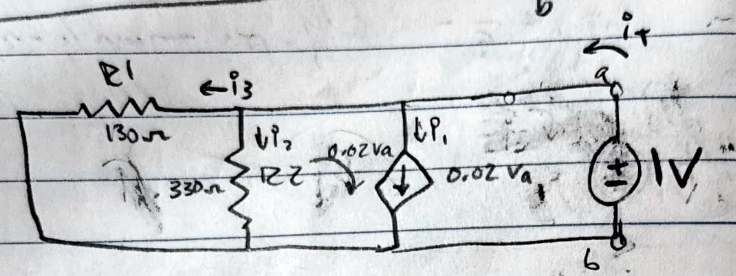
$$V_{TH} = 9V$$

$$i_{Rload} = \frac{9V}{(130\Omega + 1k\Omega)} = 7.9mA$$

$$P_{Rload} = (7.9mA)^2 \cdot 1k\Omega = 63.4\ W$$

5.





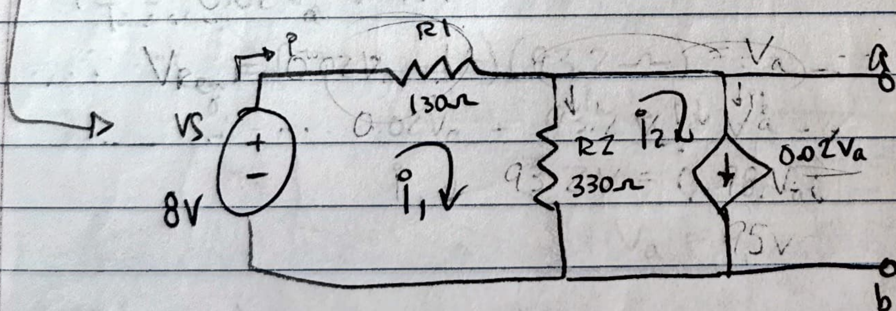
$V_a = 1V$

$i_T = i_1 + i_2 + i_3$

$= 0.02(1) + \frac{1V}{330\Omega} + \frac{1V}{130\Omega}$

$= 0.031A$

$R_{TH} = \frac{1V}{0.031A} = 32.5\,\Omega$



Mesh 1: $-8V + 130i_1 + 330(i_1 - 0.02V_a) = 0$

Mesh 2: $i_2 = 0.02V_a$

$i_1 = \frac{8V - V_a}{130}$

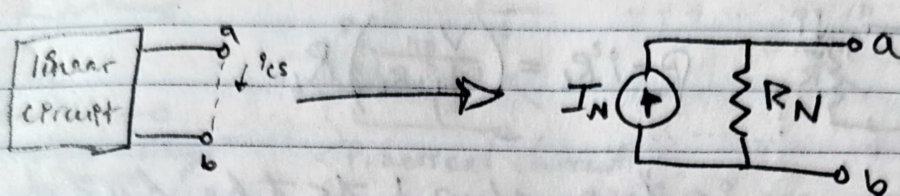
$-8V + 130\left(\frac{8V - V_a}{130}\right) + 330\left(\frac{8V - V_a}{130} - 0.02V_a\right) = 0$

☆ Mathematica ☆

$V_a = 2.0V$

♪ ☆ FANFARE MUSIC ☆ ♪

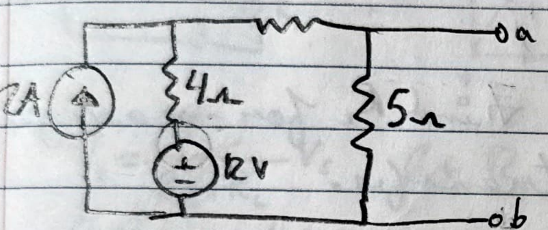
4.6 Norton's Theorem

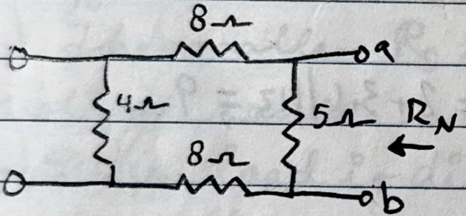


$R_N = R_{TH}$ $\quad I_N = \frac{V_{TH}}{R_{TH}}$ $\quad$ If you short a + b, $i_{sc} = I_N$

$V_{oc} = V_{TH}$ $\quad R_{TH} = \frac{V_{oc}}{i_{sc}} = R_N$

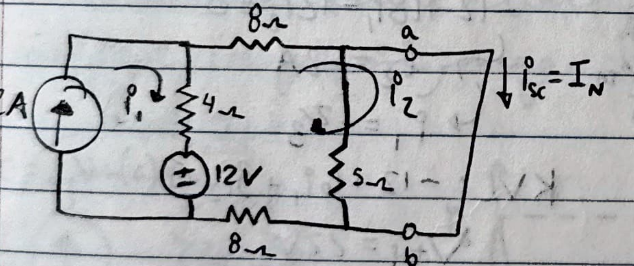
ex:





$R_N = 5 \| (8+4+8) = 5 \| 20 = \frac{20 \times 5}{25} = 4\Omega$

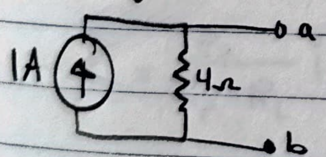
(5 is short circuited)



$i_1 = 2A \quad , \quad 20 i_2 - 4 i_1 - 12 = 0$

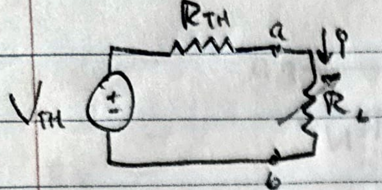
$i_2 = 1A = i_{sc} = I_N$

(Or, you could find V_{TH} and divide by R_{TH})



Norton equivalent

4.8 Maximum power transfer

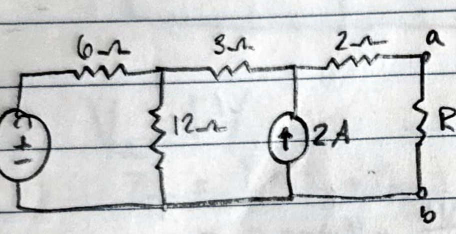


$$p = i^2 R_L = \left(\frac{V_{TH}}{R_{TH}+R_L}\right)^2 R_L$$

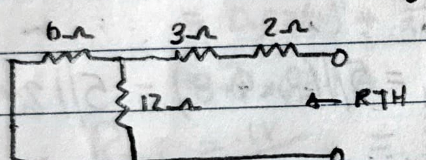
Max power is transfered to the load when the load resistance = thevenin resistance.

$$P_{max} = \frac{V_{TH}^2}{4R_{TH}}$$

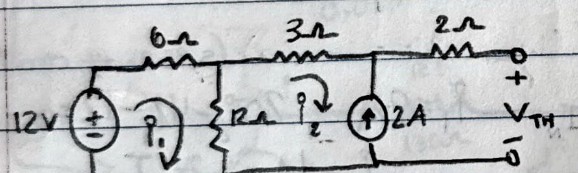
ex;



Find R_L for max power transfer.



$R_{TH} = 2 + 3 + 6 \| 12 = 9\Omega$



$-12 + 18i_1 - 12i_2 = 0$

$i_2 = -2A$

$i_1 = -2/3$

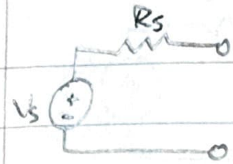
KVL: $-12 + 6i_1 + 3i_2 + 2(0) + V_{TH} = 0$

$V_{TH} = 22V$

$R_L = R_{TH} = 9\Omega$

$$P_{max} = \frac{V_{TH}^2}{4R_L} = \frac{22^2}{(4)(9)} = 13.44\,W$$

4.16.1 Source Modeling

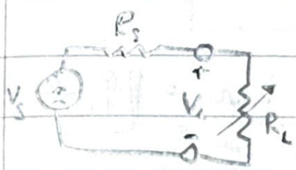


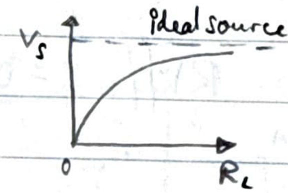
Practical voltage source

Ideal: $R_S \to 0$

Practical current source

Ideal: $R_p \to \infty$

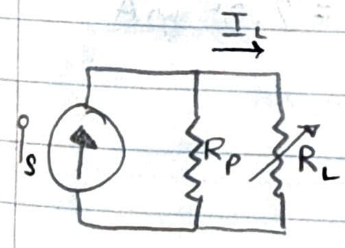


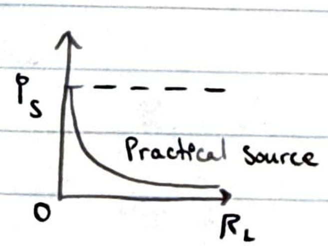


$V_L = \frac{R_L}{R_S + R_L} V_S$ As R_L increases, the load voltage approaches a source voltage V_S.

1.) The smaller R_S compared to R_L, closer the voltage source is to being ideal.

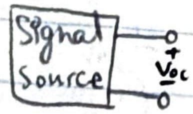
2.) When load is disconnected, $V_{oc} = V_S$. V_S is unloaded source voltage. Connecting load, terminal voltage drops in mag. → loading effect

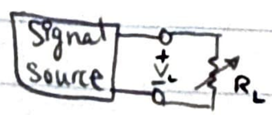




$i_L = \frac{R_p}{R_p + R_L} i_S$ Load current is constant when when internal resistance is very large.

To find unloaded sourve voltage V_S, and internal resistance R_S.



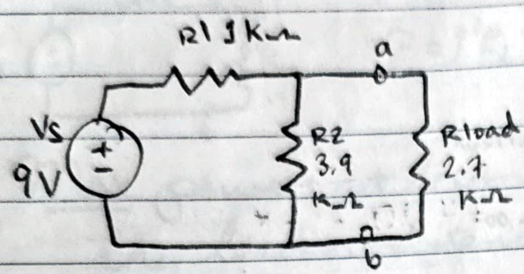


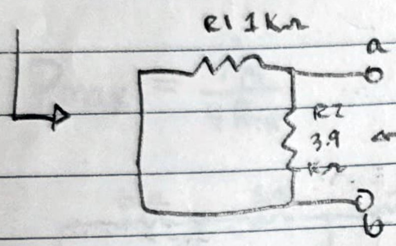
Measure V_{oc}, then adjust R_L until $V_L = V_{oc}/2$.

$R_L = R_{TH} = R_S$

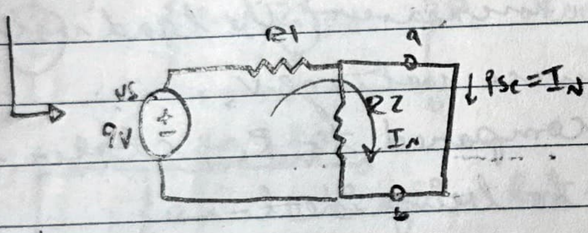
Norton's Theorem HW

(1.)



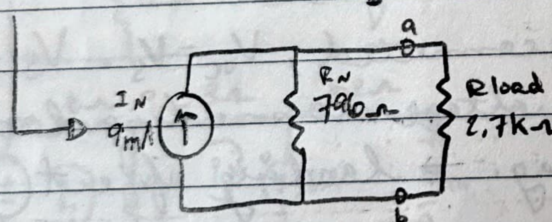


$R1 \parallel R2 = 0.796\ k\Omega = \boxed{796\ \Omega = R_N}$



$-9V + (R1)(I_N) = 0$

$\boxed{I_N = 9\ mA}$

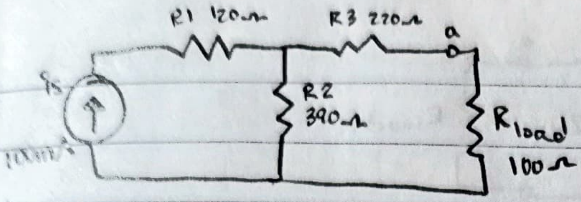


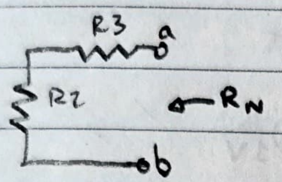
$i_{Rload} = I_N \left(\frac{R_N}{R_N + R_{Load}} \right)$

$= 0.009 \left(\frac{796}{796 + 2700} \right)$

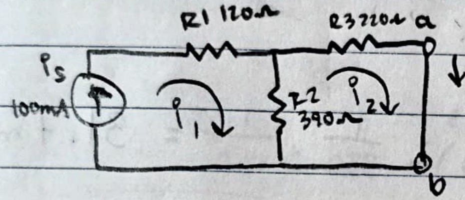
$\boxed{i_{Rlod} = 2.05\ mA}$

2.





$\leftarrow R_N$ $\quad R2 + R3 = \boxed{610\Omega = R_N}$



$i_{sc} = I_N = i_2$

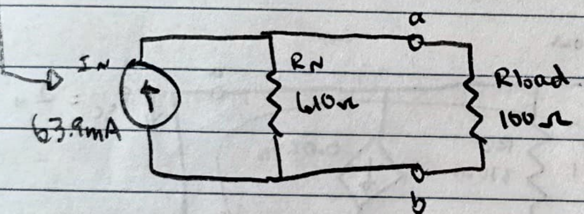
$i_1 = 100mA$

$(R2)(I_N - i_1) + R3(I_N) = 0$

$(R2+R3)(I_N) - (R2)(i_1) = 0$

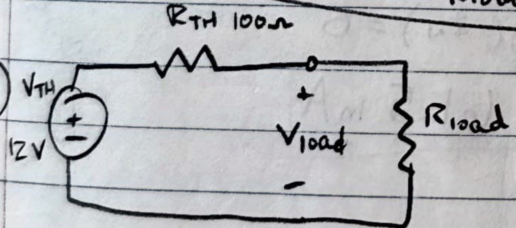
$I_N = \dfrac{(R2)(i_1)}{(R2+R3)} = \dfrac{(390)(0.1)}{(390+220)} =$

$\boxed{I_N = 63.9 \text{ mA}}$



$i_{Rload} = I_N \left(\dfrac{R_N}{R_N + R_{load}}\right) = \boxed{54.9 mA}$

3.

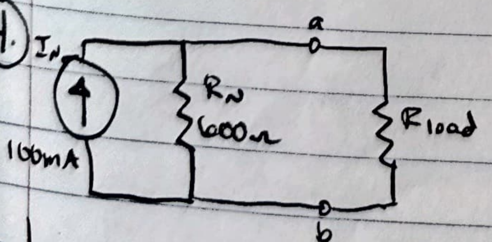


$R_L = R_{TH} = 100\Omega$

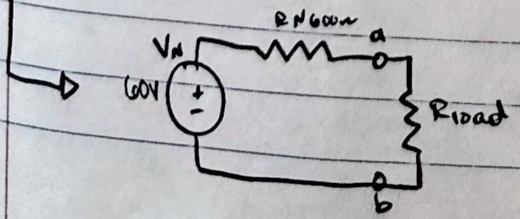
$P_{max} \left(\dfrac{V_{TH}}{R_{TH} + R_L}\right)^2 R_L = \left(\dfrac{12}{100 + 100}\right)^2 (100)$

$\boxed{P_{max} = 0.36 \text{ W}}$

4.



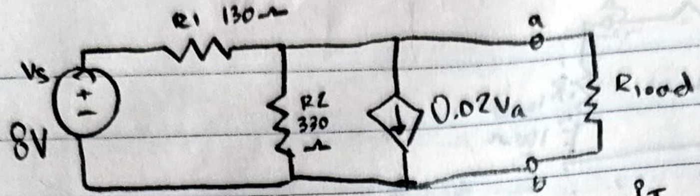
$R_L = R_N = 600\Omega$

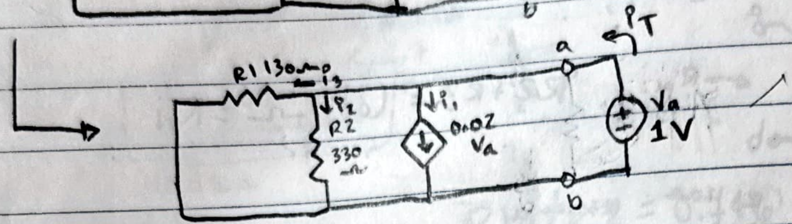


$P_{max} = \left(\dfrac{V_N}{R_N + R_L}\right)^2 R_L = \left(\dfrac{60}{1200}\right)^2 (600)$

$\boxed{P_{max} = 1.5 \text{ W}}$

5.



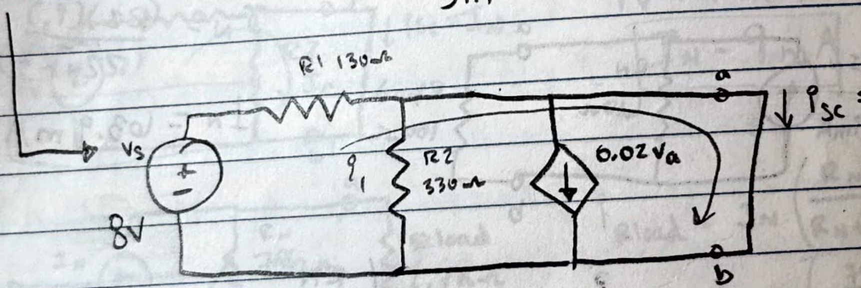


$V_a = 1V$

$i_T = i_1 + i_2 + i_3$

$= 0.02(1) + \frac{1}{330} + \frac{1}{130} = 31.7\,mA$

$R_{th} = \frac{1V}{31.7mA} = \boxed{32.5\,\Omega}$

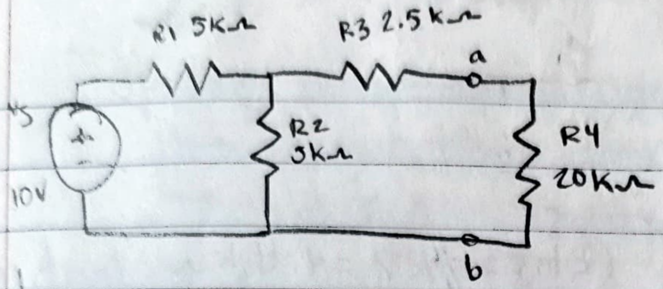


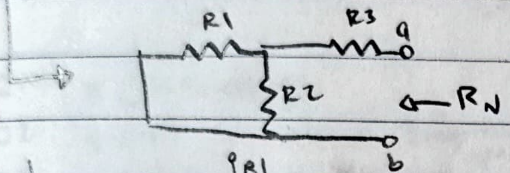
$-8V + (R1)(I_N) = 0$

$\boxed{I_N = 61.5\,mA}$

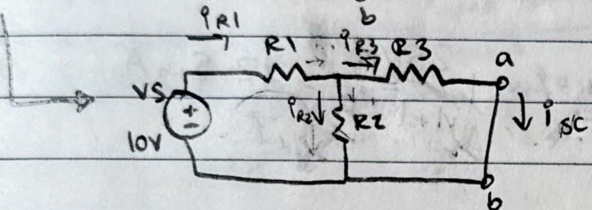
You're the best! -Izzy ♡

Class 2/14/2023





$R_N = R1 || R2 + R3 = 5 \text{ k}\Omega$



$i_{R1} = i_{R2} + i_{R3}$

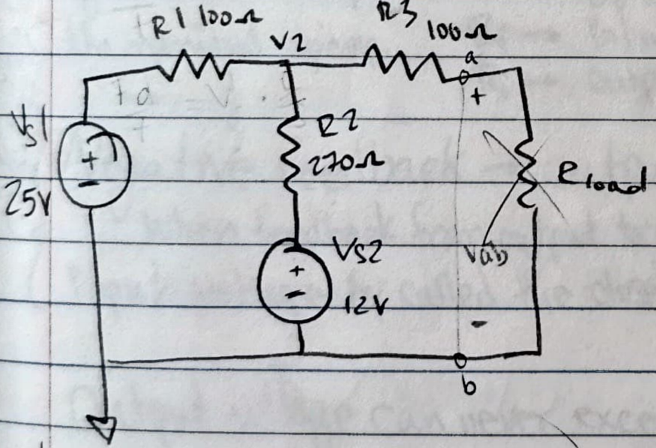
$\frac{10 - V_1}{5K} = \frac{V_1}{R_2} + \frac{V}{R_3}$

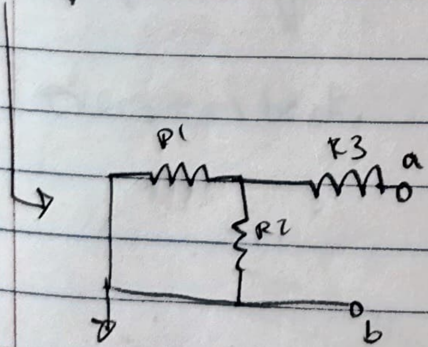
$\frac{10}{5k} = V_1\left(\frac{1}{5k} + \frac{1}{5k} + \frac{1}{2.5k}\right)$

$V_1 = 2.5V$

$i_{SC} = \frac{2.5}{R_3} = 1mA$

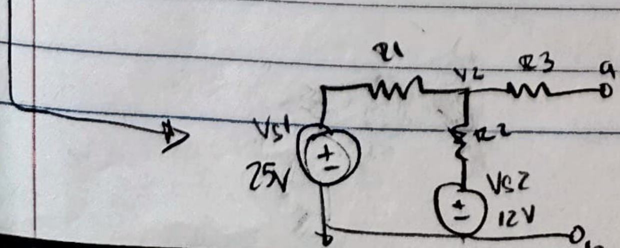
Lab 2/17/2023





$R_{TH} = R1 || R2 + R3$

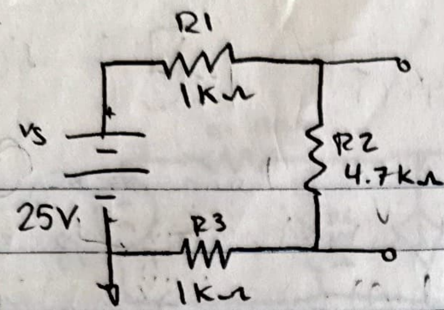
$= 173 \Omega$



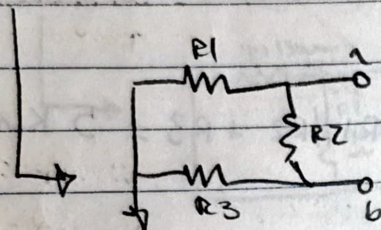
$-25V + iR1 + iR2 + 12V = 0$

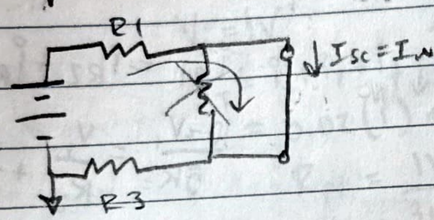
$i = \frac{13V}{(R1 + R2)} = 0.035A$

$V_{ab} = 21.49V$



$$(R1+R3)\,//\,R2 = 1.4\,k\Omega$$



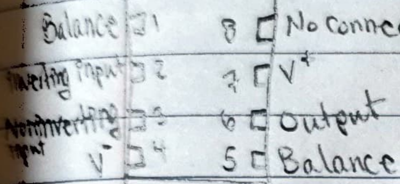


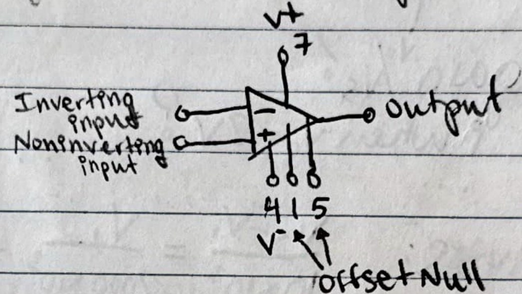
$$\frac{25V}{1+1\,k\Omega} = 12.5\,mA$$

5.1 Introduction / 5.2 Op amp

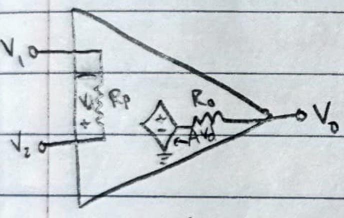
op amp → operational amplifier
- → behaves like a voltage-controlled voltage source
- → able to sum, amplify, integrate, and differentiate signals

8-pin dual in-line package (DIP) → type of op amp





power supply is needed through 4 and 7



equivalent circuit of the nonideal opamp.

$V_d = V_2 - V_1$

$V_0 = AV_d = A(V_2 - V_1)$

A → open loop voltage gain

R_i → input resistance

R_0 → output resistance

?{ Negative feedback → output is fed back to the inverting terminal
When feedback from output to input, ratio of output voltage to input voltage is called the closed-loop gain.

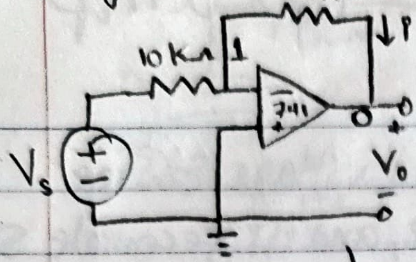
Output voltage can never exceed power supply voltage.

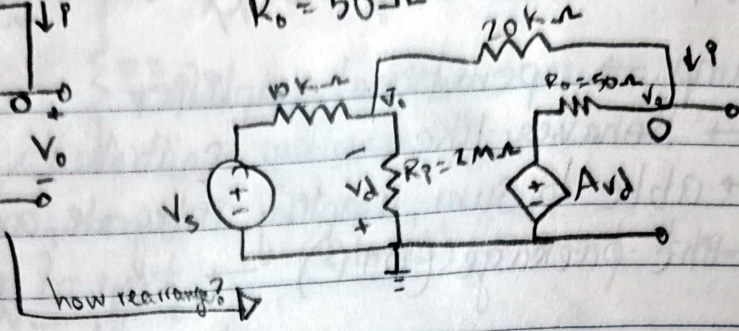
example on back.

given the circuit: And $A = 2\times10^5$

$R_i = 2M\Omega$

$R_o = 50\Omega$

ex: 



Closed-loop gain V_o/V_s?

current i when $V_s = 2V$?

Nodal Analysis: $\frac{V_s - V_1}{10\times10^3} = \frac{V_1}{2000\times10^3} + \frac{V_1 - V_o}{20\times10^3}$

$$V_1 = \frac{2V_s + V_o}{3}$$

At node O, $\frac{V_1 - V_o}{20\times10^3} = \frac{V_o - Av_d}{50}$

$v_d = -V_1$, $A = 200{,}000$

$$V_1 - V_o = 400(V_o + 200{,}000V_1)$$

$\rightsquigarrow \frac{V_o}{V_s} = -1.999$

When $V_s = 2V$, $V_o = -3.999\,V$, $V_1 = 20.0666\,\mu V$

$\therefore i = \frac{V_1 - V_o}{20\times10^3} = 0.1999\,mA$

What does Neg Voltage mean??

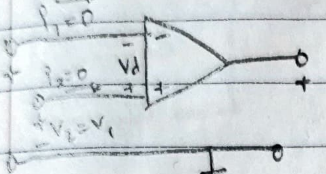
5.3 Ideal Op Amp

Ideal opamp:

1.) Infinite loop gain, $A \simeq \infty$

2.) Infinite input resistance, $R_i \simeq \infty$

3.) zero output resistance, $R_o \simeq 0$



Properties of ideal opamp:

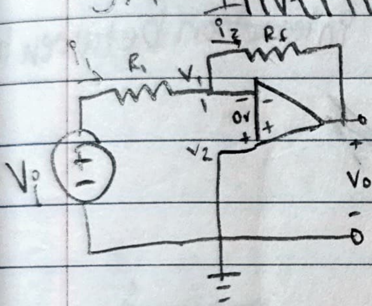
$i_1 = 0$, $i_2 = 0$ (because of ∞ resistance)

$V_d = V_2 - V_1 = 0$

$V_1 = V_2$

5.4 Inverting Amplifier

+ con. to ⏚, − has voltage

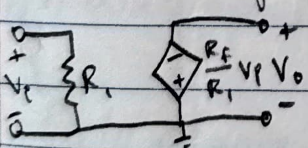


$$i_1 = i_2 \Rightarrow \frac{V_i - V_1}{R_1} = \frac{V_1 - V_o}{R_f}$$

$$\frac{V_i}{R_1} = \frac{-V_o}{R_f} \qquad V_o = \frac{-R_f}{R_1} V_i$$

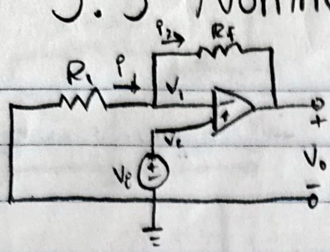
Voltage gain: $A_v = V_o / V_i = \frac{-R_f}{R_1}$

An inverting amplifier reverses polarity of input signal while amplifying it.



Equivalent circuit for Inverting Amplifier

5.5 Noninverting Amplifier



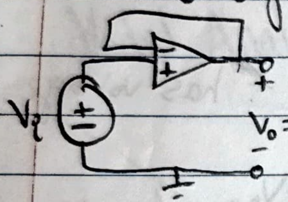
$$i_1 = i_2 \Rightarrow \frac{0-V_1}{R_1} = \frac{V_1-V_o}{R_f}$$

$$\frac{-V_i}{R_1} = \frac{V_i-V_o}{R_f}$$

$$V_o = \left(1+\frac{R_f}{R_1}\right)V_i$$

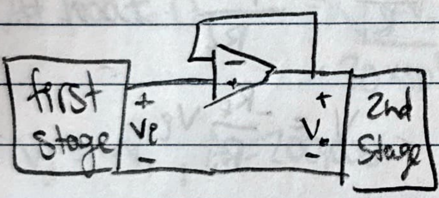
Voltage gain: $A_v = V_o/V_i = 1+\frac{R_f}{R_1}$

designed to provide positive voltage gain



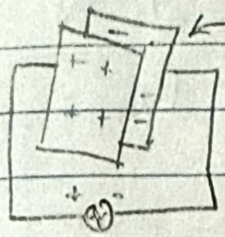
$V_o = V_i$ Voltage follower.

Can minimize interaction between two stages.



6.1 Intro / 6.2 Capacitors

Capacitor → store energy
↳ two conducting plates separated by an insulator. (dielectric)



capacitor.

charge stored → $q = Cv$

C → constant of proportionality (capacitance)

↳ unit farad (F) $1 \text{ farad} = \frac{1C}{1V}$

capacitance → ratio of the charge on one plate to the voltage between the two plates.

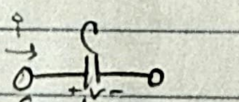
↳ doesn't depend of q or v, rather, the dimensions of the plate.

$$C = \frac{\epsilon A}{d}$$

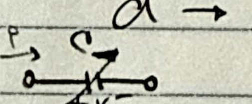
ϵ → permittivity of material between plates

A → surface area of each plate

d → distance between plates



fixed capacitor



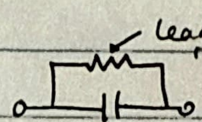
variable capacitor

$$i = \frac{dq}{dt} = C\frac{dv}{dt}$$

$$W = \frac{1}{2}Cv^2 = \frac{q^2}{2C}$$

* a capacitor is DC current
* voltage can't change abruptly

non-ideal capacitor:



ex; Calc charge stored on a 3-pF capacitor w/ 20V across it

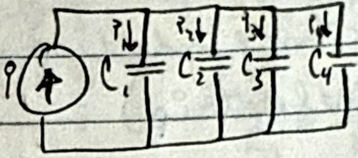
$$q = 3 \cdot 10^{-12} \cdot 20 = 60\,pC$$

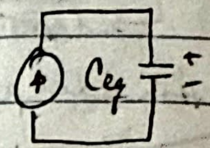
energy stored:

$$w = \frac{1}{2}Cv^2 = \frac{1}{2} \cdot 3 \cdot 10^{-12} \cdot 400 = 600\,pJ$$

6.3 series and parallel capacitors

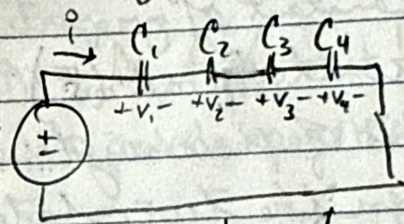
Parallel:





$$C_{eq} = C_1 + C_2 + C_3 + C_4 + \dots + C_N$$

Series:



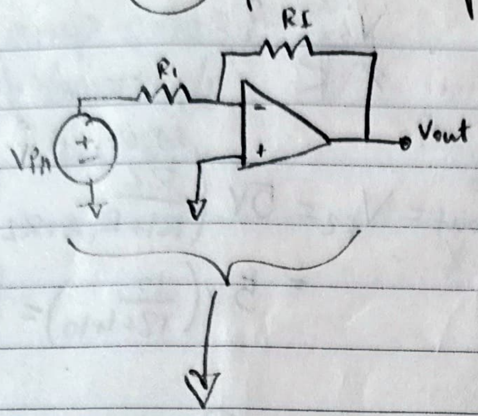
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} + \dots + \frac{1}{C_N}$$

two capacitors in series: $C_{eq} = \dfrac{C_1 C_2}{C_1 + C_2}$

Op-Amp HW

Seth Ricks

1.



$R_1 = 2K\Omega \quad R_f = 10K\Omega \quad V_{in} = 2V$

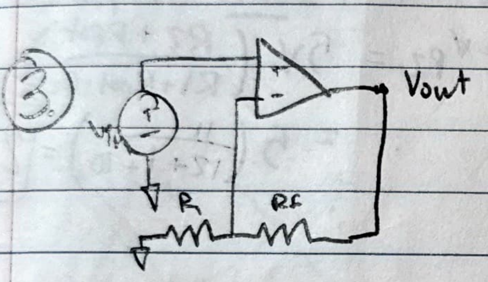
$$V_{out} = V_{in}\left(-\frac{R_f}{R_1}\right)$$
$$= (2V)\left(-\frac{10}{2}\right) = \boxed{-10V}$$

2.

$R_1 = 2K\Omega \quad V_{out} = 8V \quad V_{in} = -1V$

$$V_{out} = V_{in}\left(-\frac{R_f}{R_1}\right)$$
$$-\left(\frac{V_{out}}{V_{in}}\right)(R_1) = R_f = \boxed{16\,K\Omega}$$

3.

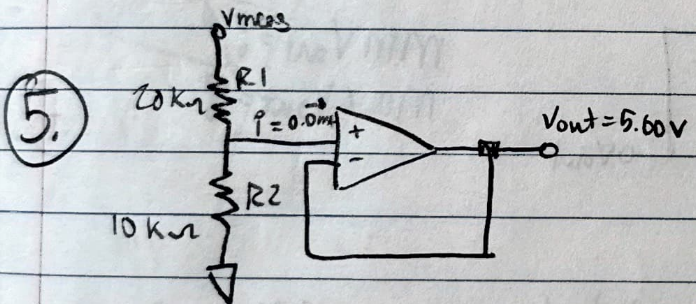


$$\frac{V_{out}}{V_p} = \left(1 + \frac{R_f}{R_1}\right)$$

$R_1 = 2K\Omega$
$R_F = 10K\Omega$
$V_{in} = 2V$

$$V_{out} = \left(1 + \frac{10}{2}\right)(2) = \boxed{12V}$$

5.

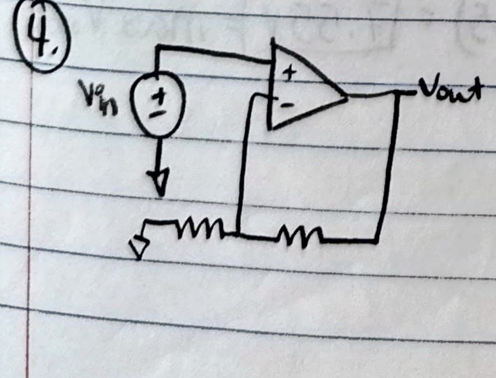


$V_{R2} = V_{out} = 5.00\,V$

$$V_{out} = V_{meas}\left(\frac{R_2}{R_1 + R_2}\right)$$

$$V_{meas} = V_{out}\left(\frac{R_1 + R_2}{R_2}\right) = 15V$$

4.

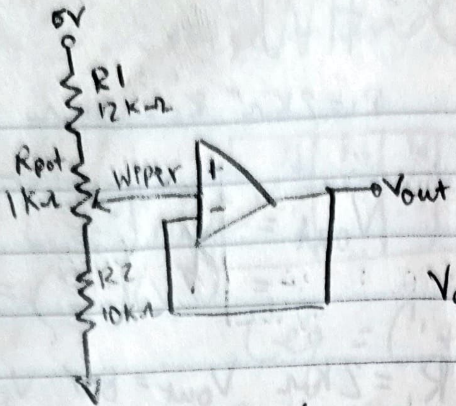


$$\frac{V_{out}}{V_p} = \left(1 + \frac{R_F}{R_1}\right)$$

$V_{out} = 8V$
$R_1 = 2K\Omega$
$V_{in} = 1V$

$$\left(\frac{V_{out}}{V_p} - 1\right)(R_1) = R_F$$
$$R_F = \boxed{14K\Omega}$$

⑥



$V_{out} = ?$

* 1 R above wiper → 1KΩ

$$V_{out} = V_{R2} = 5V\left(\frac{R2}{R1 + R_{pot} + R2}\right)$$

$$= 5V\left(\frac{10}{12+1+10}\right) = \boxed{2.17V}$$

⑦.

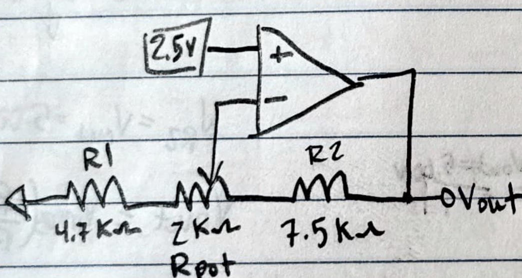
$V_{out} = ?$

* 1 R below wiper → 1KΩ

$$V_{out} = V_{R2} = 5V\left(\frac{R2 + R_{pot}}{R1 + R_{pot} + R_2}\right)$$

$$= 5\left(\frac{11}{12+1+10}\right) = \boxed{2.39V}$$

⑧.



min $V_{out} = ?$

max $V_{out} = ?$

If resistance to the right of wiper is 2KΩ:

$$V_{out} = \left(1 + \frac{7.5}{6.7}\right)(2.5) = \boxed{5.30V} = \text{min } V_{out}$$

If resistance to the left of wiper is 2KΩ:

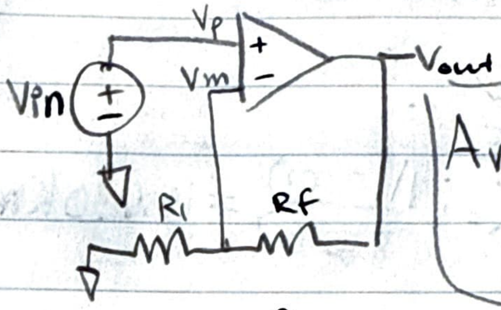
$$V_{out} = \left(1 + \frac{9.5}{4.7}\right)(2.5) = \boxed{7.55V} = \text{max } V_{out}$$

OP Amp Summary

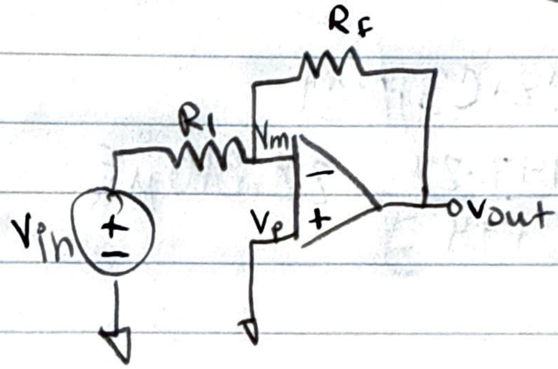
Op amp rules:

1) $V_m = V_p$
2) $I_{in} = 0A$

non-inverting:



$$\boxed{A_{v,noninverting} = \left(1 + \frac{R_F}{R_1}\right)}$$



$$\boxed{A_{v,inverting} = \frac{-R_F}{R_1}}$$

Capacitor HW

(1) $C = \frac{\epsilon A}{d}$

$A = Cd/\epsilon$

$l = \sqrt{\frac{Cd}{\epsilon}} = \frac{1(0.001)}{8.85\times10^{-12}} = 10.630\,Km$

(2) $q = CV$

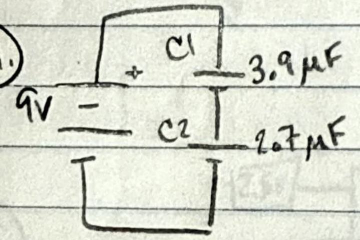
$= 47\times10^{-6}\cdot 12$

$= 564\,\mu C$

(3) $C_{eq} = (C2+C3)//C1$

$= \frac{147\cdot 27}{174} = 22.810\,\mu F$

(4)



$V_{c1} = ?$

$V_{c2} = ?$

$q = CV$

$q = C_{eq}V = \left(\frac{3.9\cdot 2.7}{3.9+2.7}\right)(9)$

$= 14.359\,\mu C$

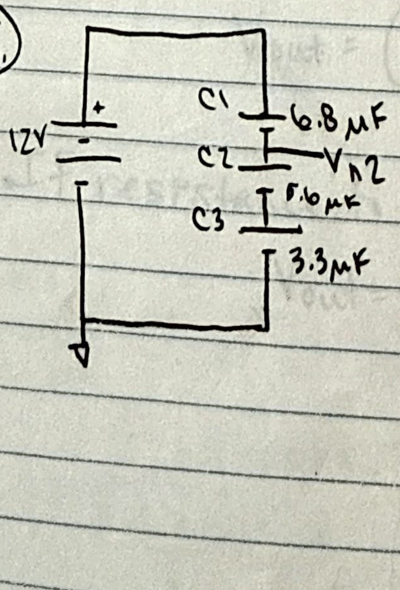
Series capacitors have the same separated charge

$V_{c1} = \frac{q_{eq}}{C_1} = 3.682\,V$

$V_{c2} = \frac{q_{eq}}{C_2} = 5.318\,V$

$V_{c1} + V_{c2} = 9V$ ✓

(5)



$V_n2 = ?$

$C1//C2 = \left(\frac{6.8\cdot 5.6}{6.8+5.6}\right) = 3.07$

$//C3 = 1.591\,\mu F$

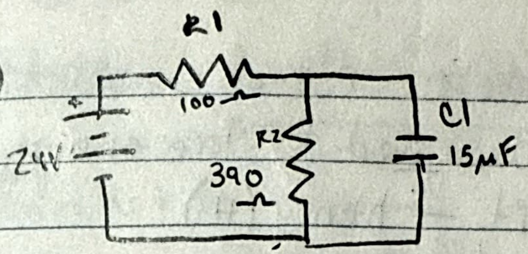
$q = C_{eq}V = 19.088\,\mu C$

$V_n2 = 12 - V_{c1}$

$V_{c1} = q/C1 = 2.867\,V$

$V_n2 = 9.193\,V$

(6)



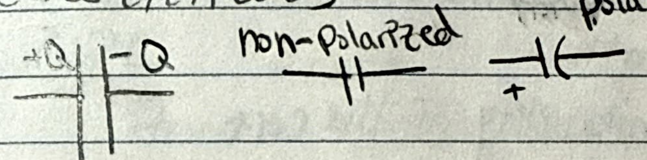
$W = \frac{1}{2} C V^2 = \frac{q^2}{2C}$

$V_{C1} = V_{R2} = 24V\left(\frac{R2}{R1+R2}\right) = 19.102\,V$

$W_{C1} = \frac{1}{2} C_1 V_{C1}^2 = \frac{1}{2}(15\mu F)(19.102V)^2$

$W_{C1} = 2.7\ mJ$

Class 2/27/2023



$\epsilon = \epsilon_r \epsilon_0$

=

ϵ_0 = air vaccum = 8.85×10^{-12} F/m

air : 1 barium titnetrate : 1,000

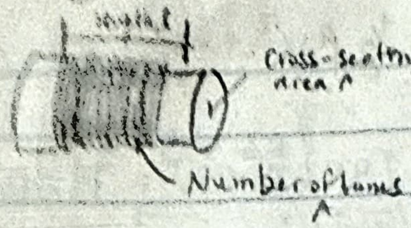
glass : 4-10

$i_c(t) = C \frac{dv_c}{dt}$

$V_c(t) = \frac{1}{C} \int_0^t i_c(\tau)\,d\tau + v(t_0)$

$W = \frac{1}{2} C v^2$ (Joules) Energy stored in capacitor

6.4 Inductors



inductor – coil of conducting wire designed to store energy

unit – henry (H)

↳ inductance, which is opposition to the change of current flow ↳ $\frac{V/s}{A}$

$$L = \frac{N^2 \mu A}{l}$$

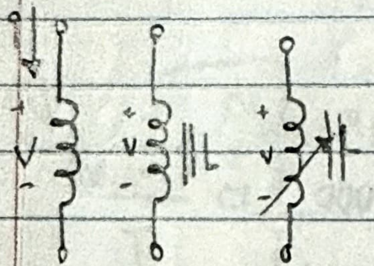
$N \rightarrow$ # of turns

$A \rightarrow$ cross-sectional area

$l \rightarrow$ length

$\mu \rightarrow$ permeability of the core

$$v(t) = L\frac{di}{dt}$$

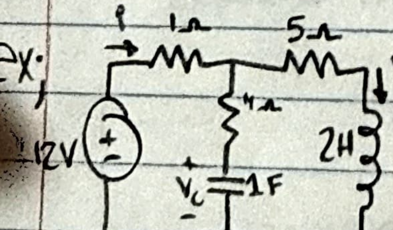


air core   iron-core   variable iron-core

Energy stored:

$$W = \frac{1}{2}Li^2$$

ex;

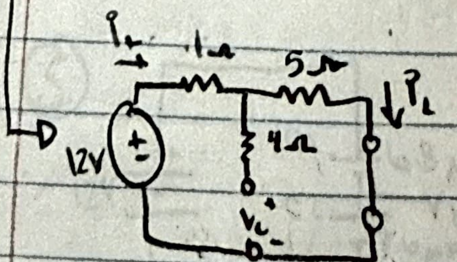


$i = ?$

$i_L = ?$

$V_c = ?$

energy stored by capacitor and inductor?



$$i = i_L = \frac{12}{1+5} = 2A$$

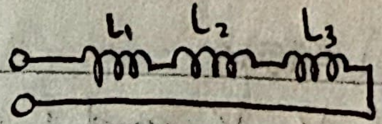
$$V_c = 5i = 10V$$

$$W_c = \frac{1}{2}CV_c^2 = \frac{1}{2}(1)(10^2) = 50J$$

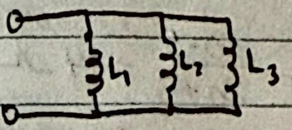
$$W_L = \frac{1}{2}Li_L^2 = \frac{1}{2}(2)(2^2) = 4J$$

Inductors resist fast changes in current

6.5 Series and Parallel Inductors

Series: 

$L_{eq} = L_1 + L_2 + L_3 + \cdots + L_N$

parallel: 

$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}$

So, basically same as resistors.

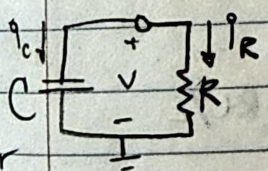
7.1 Introduction

"First order circuits" $\begin{cases} \text{RC circuit} \rightarrow \text{resistor and capacitor} \\ \text{RL } \text{''} \rightarrow \text{''} \quad \text{''} \text{ inductor} \end{cases}$

Two way to excite → C & L are charged, or
→ Dependent sources

7.2 The source-free RC circuit

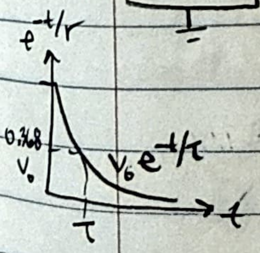
Current both the ways?? — dc source disconnected



initial voltage: $v(0) = V_0$

$w(0) = \frac{1}{2} C V_0^2$

$i_c + i_R = 0$



$C\frac{dv}{dt} + \frac{v}{R} = 0 \longrightarrow \frac{dv}{dt} + \frac{v}{RC} = 0$

$\rightarrow \boxed{v(t) = V_0 e^{-t/RC}}$

Time constant → time required for the response to decay to a factor of $1/e$ or 36.8%.

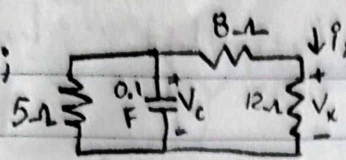
$v(t+\tau) = 0.368 v(t)$

$\tau = RC$

So, $v(t) = V_0 e^{-t/\tau}$

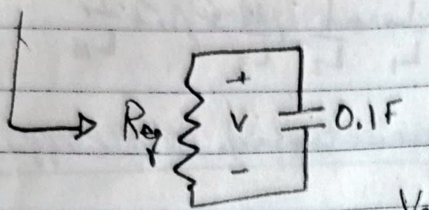
A circuit 5τ to charge.

ex;



$V_c(0) = 15V$

Find V_c, V_x, and i_x for $t>0$



$$R_{eq} = \frac{20 \cdot 5}{20+5} = 4\Omega$$

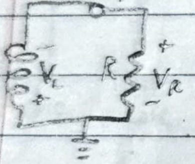
$$\tau = R_{eq}C = 4(0.1) = 0.4s$$

$$V_c = V = V(0)e^{-t/\tau} = 15e^{-t/0.4}V = 15e^{-2.5t}V$$

• Voltage div. $V_x = \frac{12}{12+8}V = 0.6(15e^{-2.5t}) = 9e^{-2.5t}V$

• Ohm's law $i_x = \frac{V_x}{12} = 0.75e^{-2.5t}A$

7.3 The Source-Free RL circuit:



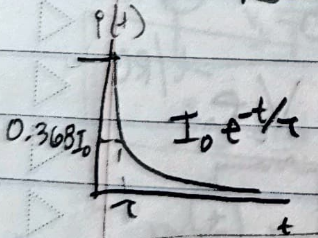
$$i(0) = I_0$$

$$w(0) = \frac{1}{2}LI_0^2$$

$$V_L + V_R = 0 \qquad L\frac{di}{dt} + Ri = 0$$

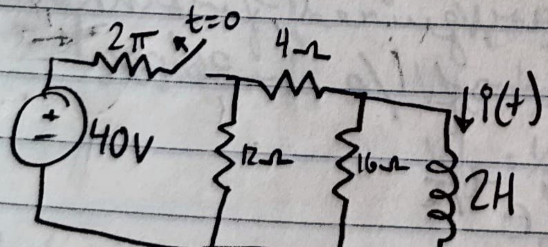
$$\rightarrow i(t) = I_0e^{-Rt/L}$$

$$\tau = \frac{L}{R} \rightarrow i(t) = I_0e^{-t/\tau}$$

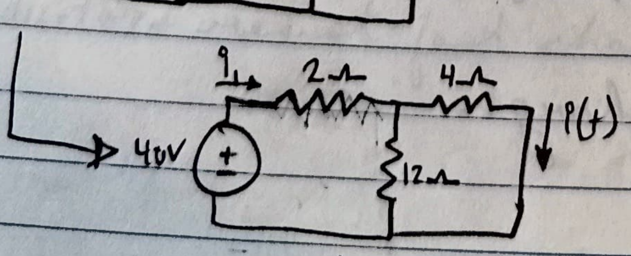


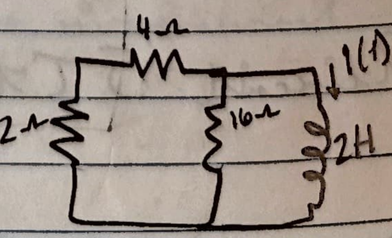
$$i(t+\tau) = 0.368i(t)$$

ex;



Find $i(t)$ for $t>0$





$t<0$ $\qquad$ $t>0$

(t<0 circuit) combine 12Ω & 4Ω → 3Ω

$i_1 = \frac{40}{2+3} = 8A$

$i(0) = i(t) = \frac{12}{12+4} i_1 = 6\,A,\ t<0$

$t>0 \rightarrow R_{eq} = 8\Omega$ $\tau = \frac{L}{R_{eq}} = \frac{1}{4}s$

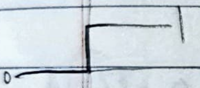
$i(t) = i(0)e^{-t/\tau} = \boxed{6e^{-4t}\,A}$

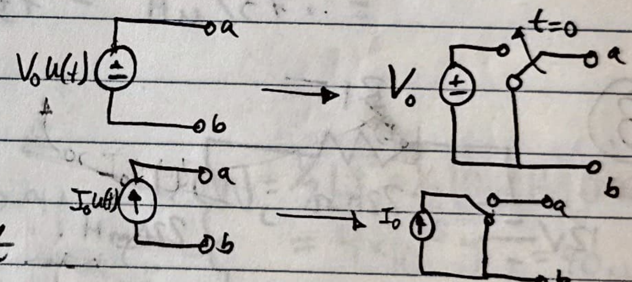
7.4 Singularity functions: (or switching functions)

↳ functions that are either discontinuous or have discontinuous derivatives

unit step function: $u(t)$ is 0 for $t<0$, and 1 for $t>0$

$$u(t) = \begin{cases} 0, & t<0 \\ 1, & t>0 \end{cases}$$

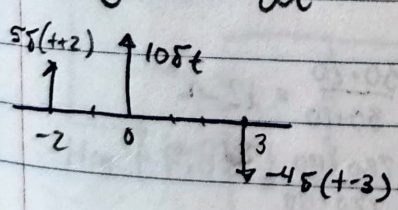




derivative of unit step function → unit impulse function $\delta(t)$

$$\delta(t) = \frac{d}{dt}u(t) = \begin{cases} 0, & t<0 \\ \text{undefined}, & t=0 \\ 0, & t>0 \end{cases}$$

$$\int_{0^-}^{0^+} \delta(t)\,dt = 1$$

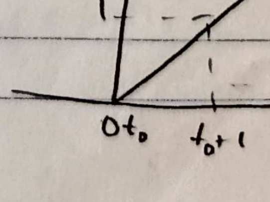


$$\int_a^b f(t)\delta(t-t_0)\,dt = f(t_0)$$

↖ function at point where impulse occurs

Integrating unit step function → unit ramp function

$$r(t) = \begin{cases} 0, & t\leq 0 \\ 1, & t\geq 0 \end{cases}$$



Inductors HW Seth Ricks

1. ~~Four inductors in series.~~

$L1 = 120\mu H,\ L2 = 47\mu H,\ L3 = 750nH,\ L4 = 560\mu H$

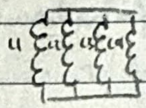
$L_{eq} = 120\times10^{-6} + 47\times10^{-6} + 750\times10^{-9} + 560\times10^{-6}$

$= 7.27\times10^{-4} = 727\ \mu H$

2. ~~Four inductors in parallel.~~

$L1 = 120\mu H,\ L2 = 47\mu H,\ L3 = 750nH,\ L4 = 560\mu H$

$= .750\mu H$



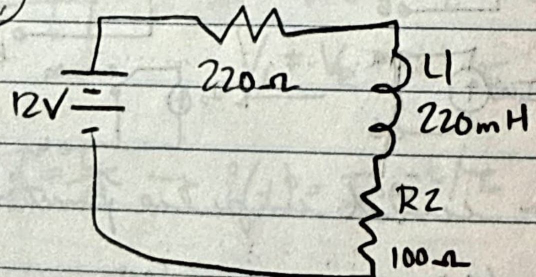
$L1//L2 = \frac{120\cdot47}{120+47} = 33.77$

$L3//L4 = \frac{.75\cdot560}{.75+560} = 0.749$

$(L1//L2)//(L3//L4)$

$= .732\mu H = 732 nH$

3.



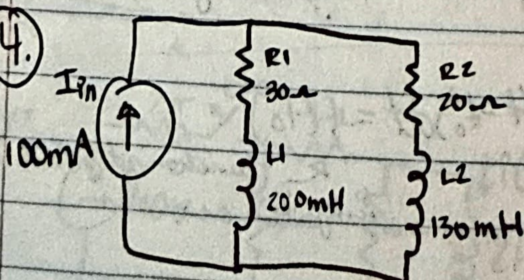
$i = \frac{12V}{330\Omega} A = 0.0364 A$

$W = \frac{1}{2}Li^2 = \frac{1}{2}(.22H)(0.0364)^2$

$= 1.54\times10^{-4} J$

$= 154\ \mu J$

4.



$R_{eq} = \frac{30\cdot20}{30+20} = 12\Omega$

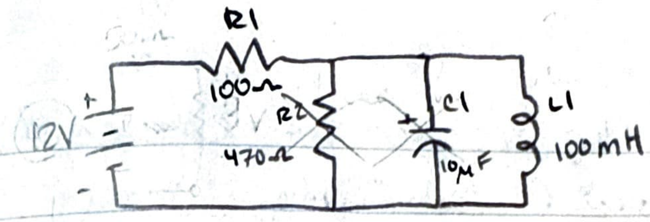
$L_{eq} = \frac{200\cdot130}{220+130} = 78.79\ mH$

$W = \frac{1}{2}Li^2 = \frac{1}{2}(0.07879)(.1^2)$

$= 3.940\times10^{-4} J$

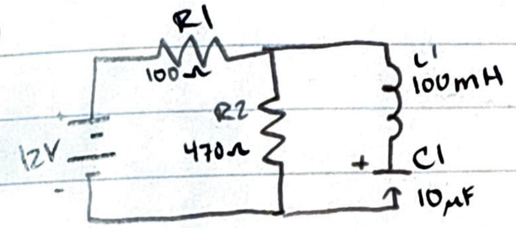
$= 394\ \mu J$

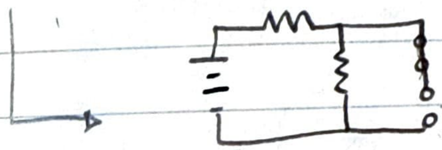
5.



$$W = \tfrac{1}{2} L i^2 = \tfrac{1}{2}(0.1)\left(\tfrac{12}{100}\right)^2 = 7.2\times10^{-4}\,J = 720\,\mu J$$

6.





$$V_{R2} = V_{C1}$$

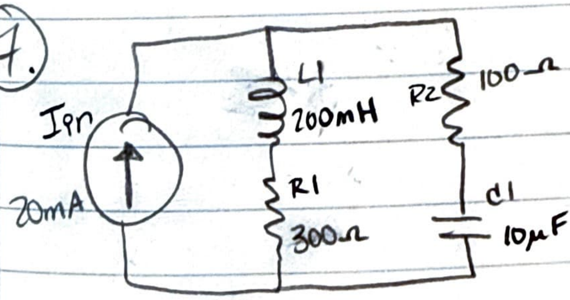
$$V_{R2} = 12V\left(\frac{R2}{R1+R2}\right) = 12V\left(\frac{470}{570}\right)$$

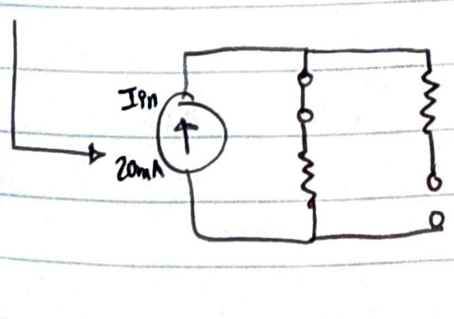
$$= 9.895\,V$$

$$W = \tfrac{1}{2} C_1 V_{C1}^2 = \tfrac{1}{2}(10\,\mu F)(9.895^2)$$

$$= 490.0\,\mu J$$

7.





$$W_L = \tfrac{1}{2} L i^2 = \tfrac{1}{2}(0.2H)(0.02A)^2$$

$$= 4\times10^{-5}\,J = 40\,\mu J$$

$$W = \tfrac{1}{2} C_1 V_{C1}^2$$

$$V_{C1} = V_{R1} = (300\Omega \cdot 0.02\,A) = 6\,V$$

$$W_C = \tfrac{1}{2}(10\times10^{-6}\,H)(6V)^2$$

$$= 180\,\mu J$$

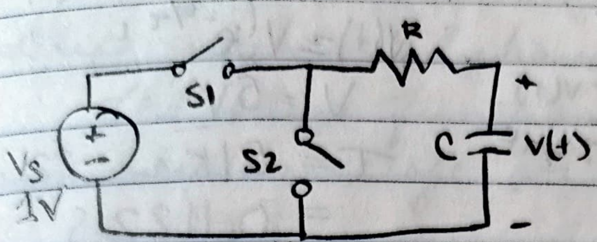
$$W_T = W_L + W_C = 220\,\mu J$$

Question: Voltage across capacitor in this circuit is independent of the resistor???!

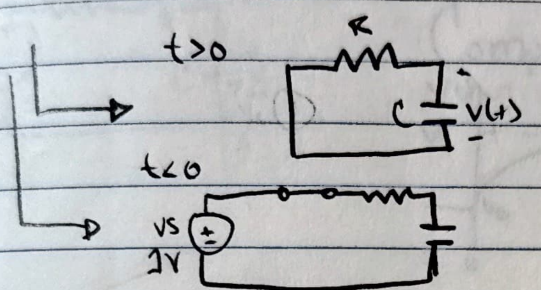
$e^{-t/} \quad e^{t/5}$

Source Free RC and RL Circuits HW

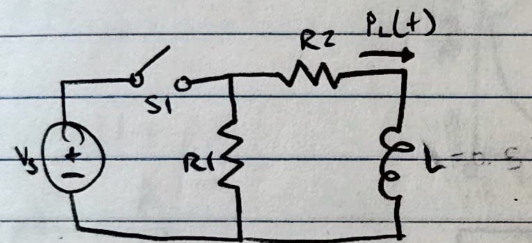
1.



$$V(t) = V_0 e^{-t/\tau}$$
$$V(0) = 1V$$
$$V(\tau) = e^{-1}$$
$$V(3\tau) = e^{-3}$$
$$V(5\tau) = e^{-5}$$
$$V(10\tau) = e^{-10}$$

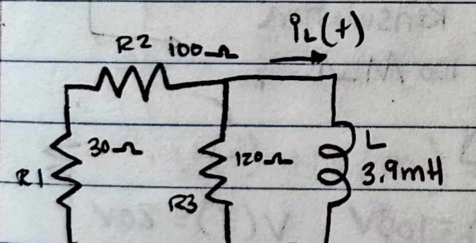


2.



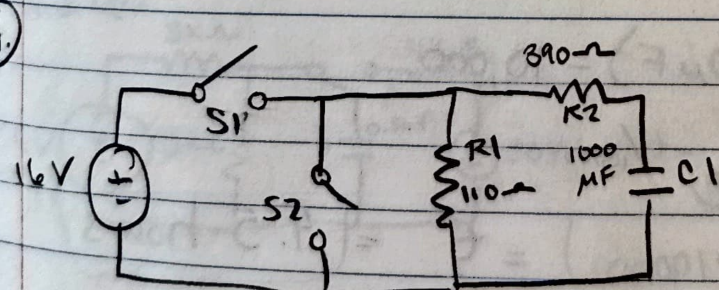
$$i_L(0) = 0.5A$$
$$i(t) = I_0 e^{-t/\tau}$$
$$i(\tau) = 0.5e^{-1}$$
$$i(3\tau) = 0.5e^{-3}$$
$$i(5\tau) = 0.5e^{-5} \qquad i(10\tau) = 0.5e^{-10}$$

3.

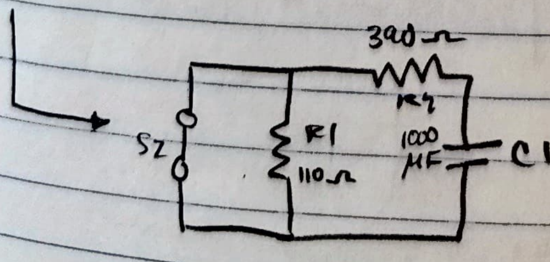


$$\tau = \frac{L}{R} \qquad R_{eq} = \frac{130 \cdot 120}{130 + 120} = 62.4\Omega$$
$$= \frac{3.9mH}{62.4\Omega} = \boxed{62.5\mu s}$$

4.

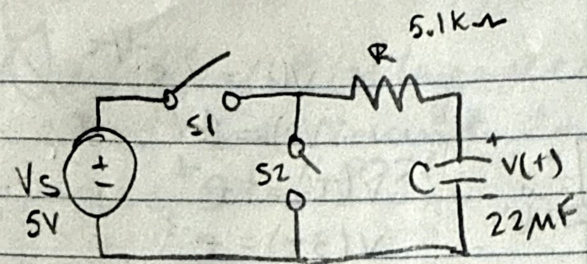


$$v(t) = V_0 e^{-t/\tau}$$
$$R_{eq} = \frac{390 \cdot 110}{390 + 110} = 78\Omega$$
$$\tau = R_{eq} \cdot C = 0.078 s$$
$$V(0) = 16V\left(\frac{R1}{R1 + R2}\right) = 3.52V$$



$$V(t) = 3.52e^{-t/0.078}$$

5.



$V(?) = 2.2V$

$V(t) = V_0 e^{-t/\tau}$

$V_0 = 5V$

$\tau = 5.1k\Omega \cdot 22\mu F$

$= 0.1122\,s$

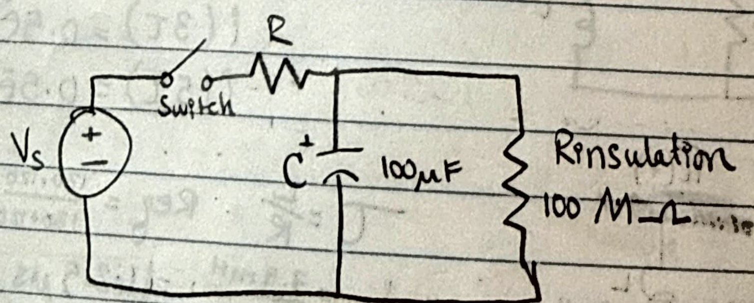
$2.2V = 5V e^{-t/0.1122}$

$t = -\ln\left(\frac{2.2}{5}\right) \cdot 0.1122$

$= 0.092\,s$

$= 92\,ms$

6.



time (hrs) for Vs: 100V → 20V

$V(0) = 100V$ $V(?) = 20V$

$\tau = RC = (100M\Omega)(100\mu F) = 10{,}000$

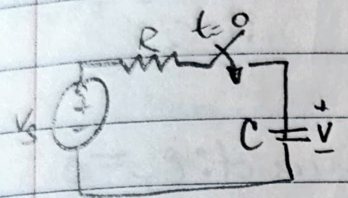
$20V = 100V \cdot e^{-t/10000}$

$-\left(\ln\left(\frac{1}{5}\right) \cdot 10000\right) = t = \boxed{4.5 \text{ hours}}$

7.5 Step response of an RC circuit

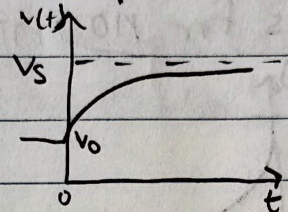
Step response - behavior when the excitation is the step function, which may be voltage or current

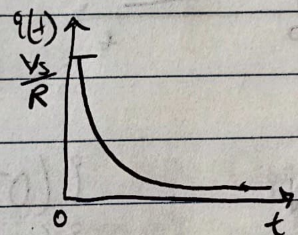
sudden voltage or current



Complete response: $v(t) = \begin{cases} V_0 & , t<0 \\ V_s + (V_0 - V_s)e^{-t/\tau} & , t>0 \end{cases}$

This is if it is charged initially. Otherwise $V_0 = 0$.





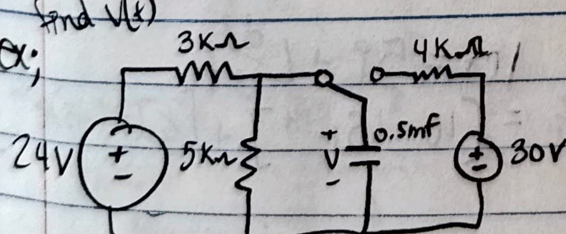
Complete response = natural response + forced response

	natural response	forced response
	~~store energy~~	~~independent source~~
	temporary part	permanent part
	transient response	steady-rate response

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$$

final V_c — initial V_c — time constant

Ex; find $v(t)$

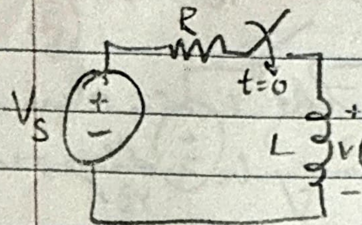


$$v(0^-) = \frac{5}{5+3}(24) = 15V = v(0)$$

$$\tau = R_{TH}C = 2s$$

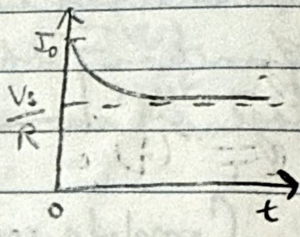
$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$$
$$= 30 + (15-30)e^{-t/2} = 30 - 15e^{-0.5t} \text{ V}$$

7.6 Step response of an RL circuit

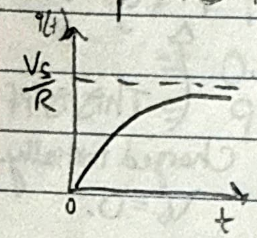


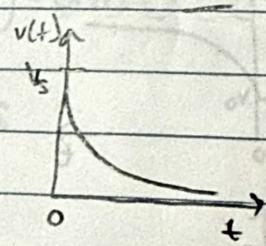
$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$$

initial i_I final i_I

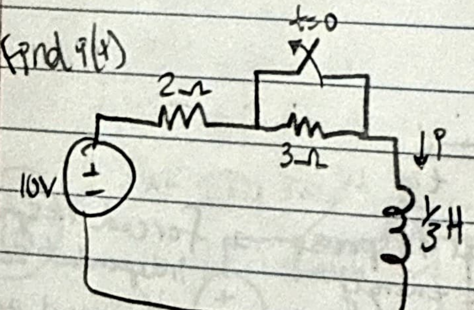


Step RL reponses for no initial current: $i_I = 0$





ex: Find i(t)



$$i(0^-) = \frac{10}{2} = 5A = i(0)$$

$$i(\infty) = \frac{10}{2+3} = 2A$$

$$R_{TH} = 2+3 = 5\,\Omega$$

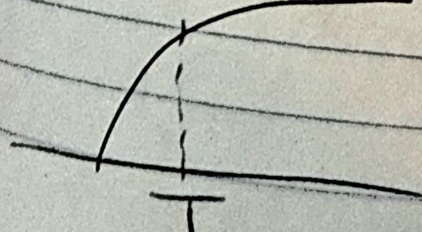
$$\tau = L/R_{TH} = \frac{1/3}{5} = \frac{1}{15}\,s$$

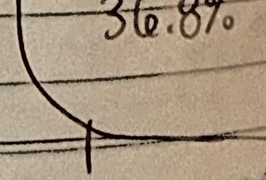
$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$$
$$= 2 + (5-2)e^{-15t} = 2 + 3e^{-15t}\ A,\ t>0$$

Check: for $t>0$, KVL: $10 = 5i + L\frac{di}{dt}$

$$5i + L\frac{di}{dt} = [10 + 15e^{-15t}] + [\tfrac{1}{3}(3)(-15)e^{-15t}]$$

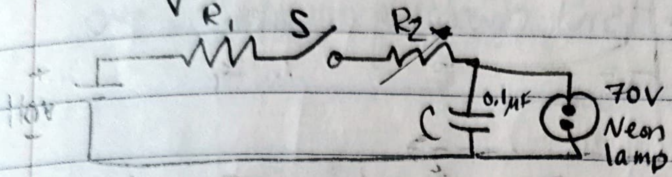
Instantaneous value = Final + [Initial − Final] $e^{-(t-t_0)/\tau}$





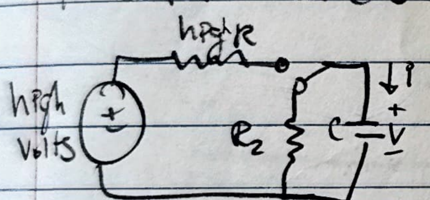
7.9 Applications

Delay Circuits



when S is closed, C charges until it reaches 70V, which then fires through lamp quickly and restarts. Can make the light flash several times a second.

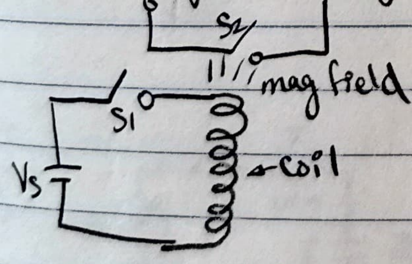
Photoflash Unit



Cap. charges slowly through high R, ($5R_1C$) and then discharges quickly throug low R_2 ($5R_2C$)

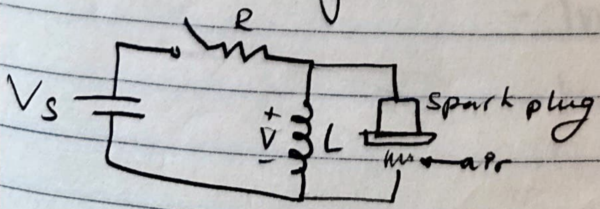
Relay Circuits

- magnetically controlled switch

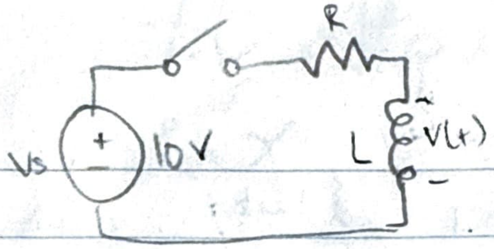


time to switch → relay delay time

Automobile Ignition circuit



Each cylinder is ignited at proper times w/ inductor. When switch is opened suddenly, large V created through a lot of Amps. Causes air to ignite. (gas)



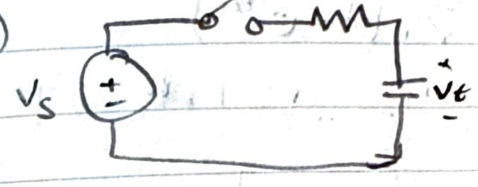
$V(0) = 10V$

- Current $= 0$ because A can't change instantaneously

$V(\infty) = 0$, because $V(t) = L\frac{di}{dt} \to \frac{di}{dt}$ isn't changing anymore so $\to 0$

Step Response of RC and RL Circuits

1.

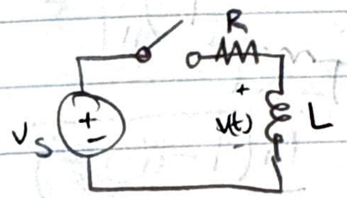


$V(t=\tau) = V_s(1-e^{-1})\,V$

$V(t=3\tau) = V_s(1-e^{-3})\,V$

$V(t=5\tau) = V_s(1-e^{-5})\,V$

2.



$R = 100\,\Omega$

$V_s = 1V$

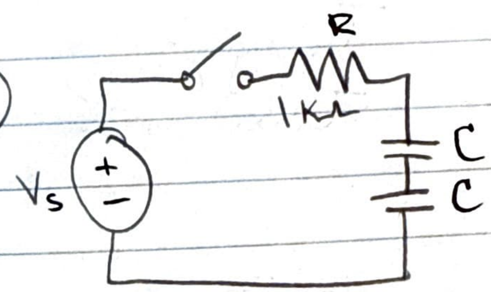
$i(t) = \frac{V_s}{R} + \left(0 - \frac{V_s}{R}\right)e^{-t/\tau}\,A$

$i(t=\tau) = \frac{V_s}{100}\left(1 - e^{-1}\right)A$

$i(t=3\tau) = \frac{V_s}{100}\left(1-e^{-3}\right)A$

$i(t=5\tau) = \frac{V_s}{100}\left(1-e^{-5}\right)A$

3.

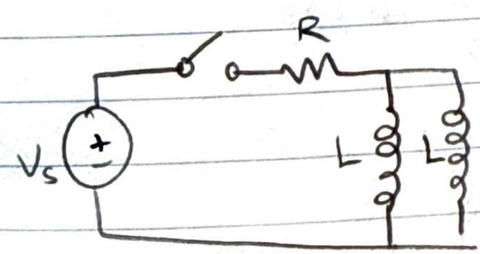


$RC = \tau = 1ms \quad C_{eq} = C/2$

$C = ?$

$C_T = \frac{1ms}{1K\Omega} \cdot 2 = 2\mu F$

4.

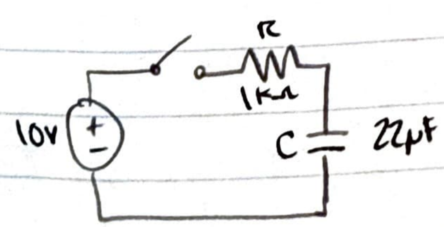


$L = 10mH \quad L_{eq} = 5mH$

$R = ? \quad L/R = 50\mu s$

$R = 5mH / 50\mu s = 100\,\Omega$

5.



C is discharged at $t = 0^-$

t to 5V?

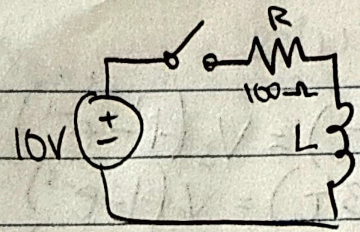
$5V = 10V\left(1 - e^{-t/\tau}\right)$

$-\ln\left[-\left(\frac{1}{2}V - 1\right)\right] \cdot \tau = t$

$-\ln\left(1 - \frac{1}{2}V\right) \cdot (22\mu F)(1K\Omega) = 0.015\,s$

$t = 15mS$

6.



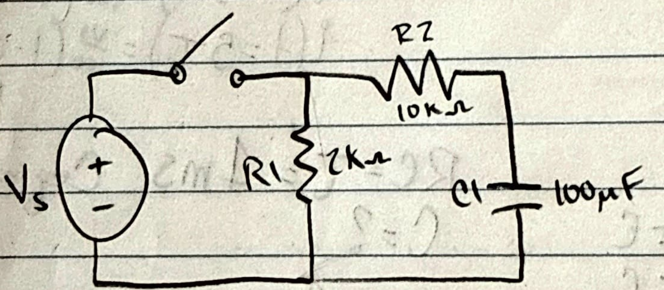
L=? for 50mA after 10ms

$50mA = i(t) = \frac{R}{V_s}\left(1-e^{-Rt/L}\right)$

$$\ln\left[-\left(i(t)\cdot\frac{V_s}{R} - 1\right)\right] = -\frac{Rt}{L}$$

$$L = \frac{-Rt}{\ln\left(1-i(t)\frac{V_s}{R}\right)} = 199.5H$$

7.



$\tau_{charging} = ?$

$\tau_{discharging} = ?$

While charging, the only resistor in question is R2?

$\tau = R2\,C1 = 1.0s$

while discharging, resistors in series

$R_{eq} = 12K\Omega$

$\tau = 1.2s$

9.1/9.2 Sinusoids

Sinusoid – a signal that has the form of the sine or cosine function

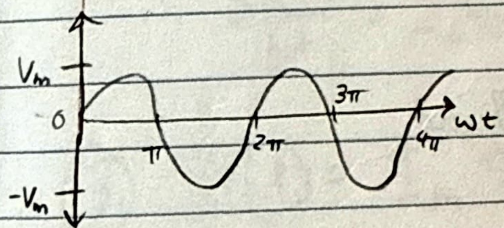
↳ alternating current!!

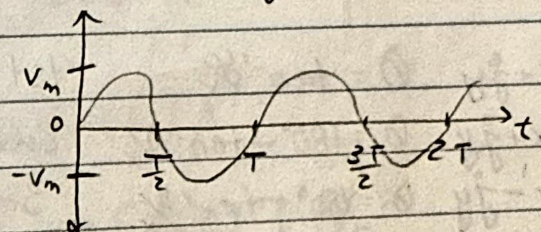
$v(t) = V_m \sin \omega t$

V_m = amplitude of sinusoid

ω = angular frequency in rads.

ωt = "argument" of sinusoid





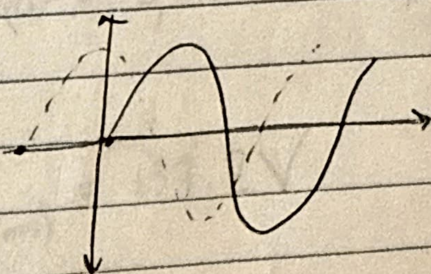
$T = \frac{2\pi}{\omega}$

$v(t)$ repeats itself every T seconds, so $v(t+T) = v(t)$

(it's a periodic function) frequency → $f = \frac{1}{T}$

$\omega = 2\pi f$

$v(t) = V_m \sin(\omega t + \phi)$ → The "phase"



$\sin(\omega t \pm 180°) = -\sin \omega t$

$\cos(\omega t \pm 180°) = -\cos \omega t$

$\sin(\omega t \pm 90°) = \pm \cos \omega t$

$\cos(\omega t \pm 90°) = \mp \sin \omega t$

$A \cos \omega t + B \sin \omega t = C \cos(\omega t - \theta)$, where $C = \sqrt{A^2 + B^2}$, $\theta = \tan^{-1} \frac{B}{A}$

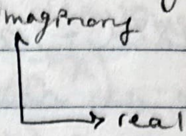
ex; $3 \cos \omega t - 4 \sin \omega t = 5 \cos(\omega t + 53.1°)$

Imaginary part → phase

real part → magnitude

B.1 Representation of Complex numbers

$z = \underbrace{x}_{\text{real}} + \underbrace{jy}_{\text{imaginary}}$, where $j = \sqrt{-1}$



polar $z = \underbrace{r}_{\text{magnitude}} \underline{/\theta} \leftarrow$ angle w/ real axis , where $r = \sqrt{x^2+y^2}$

$\theta = \tan^{-1} y/x$

$x = r\cos\theta$, $y = r\sin\theta$

$$z = x + jy = r\underline{/\theta} = r\cos\theta + jr\sin\theta$$

$z = x + jy \quad \theta = \tan^{-1} y/x$ 1st Quad

$z = -x + jy \quad \theta = 180° - \tan^{-1} y/x$ 2nd

$z = -x - jy \quad \theta = 180° + \tan^{-1} y/x$ 3rd

$z = x - jy \quad \theta = 360° - \tan^{-1} y/x$ 4th

exponential Or $z = re^{j\theta}$, $r = \sqrt{x^2+y^2}$, $\theta = \tan^{-1} y/x$

Sinusoids & Complex Numbers HW

(1.) $v(t) = 170\sin(120\pi t)V$

a.) Peak-to-peak voltage $= 350$

b.) frequency $= \frac{120\pi}{2\pi} = 60\ Hz$

c.) Period $T = \frac{2\pi}{120\pi} = \frac{1}{60}$

(2.) $v(t) = V_m\cos(2\pi ft)V$

$$\frac{dv(t)}{dt} = -2\pi f V_m \sin(2\pi ft)\ V/s$$

(3.) $i(t) = I_m \sin[2\pi ft]A$

$$\frac{dA}{dt} = 2\pi f I_m \cos[2\pi ft]\ A/s$$

(4.) $v(t) = V_m\cos(\omega t)V$; $V_m = ?$, $f = 60Hz$, $v(2ms) = 100V$

$$f = \frac{\omega}{2\pi} = 60\ Hz$$

$$\omega = 120\pi\ Hz$$

$$V_m = \frac{v(t)}{\cos\omega t} = \frac{100V}{\cos(120\pi\ Hz \cdot 2ms)} = 137.2V$$

$r = \sqrt{a^2+b^2}$

$\theta = \tan^{-1}\left(\frac{imag}{real}\right)$

(5.) $60 - j30$

a.) $r = \sqrt{60^2+30^2} = \sqrt{4500} = 10\sqrt{45}$

$\theta = 360° - \tan^{-1}\left(\frac{30}{60}\right) = 333.4°$

$Z = 10\sqrt{45}\angle 333.4°$

b.) $24 + j15$

$r = \sqrt{24^2+15^2} = \sqrt{801}$

$\theta = \tan^{-1}\left(\frac{15}{24}\right) = 32.01°$

$Z = \sqrt{801}\angle 32.01°$

6. $a = r\cos\theta$
$b = r\sin\theta$

a.) $120\angle 32^\circ$

$a = 120\cos 32^\circ = 101.77$
$b = 120\sin 32^\circ = 63.59$

$z = 101.77 + j63.59$

b.) $100\angle 0.5\pi$

$a = 100\cos\frac{\pi}{2} = 0$
$b = 100\sin\frac{\pi}{2} = 100$

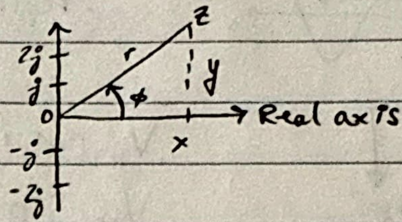
$z = j100$

7. $\frac{3+j6}{3-j2} \cdot \frac{3+j2}{3+j2} = \frac{9+j18+j6+12(-1)}{9-(j2)^2}$

$= \frac{-3+j24}{13} = \frac{-3}{13} + j\frac{24}{13}$

9.3 Phasors

phasor : complex number that represents amplitude and phase of a sinusoid



Imaginary numbers!

$z = x + jy = r\angle\phi = r(\cos\phi + j\sin\phi)$

Operations:

addition : $z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$

subtraction : $z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$

Mult : $z_1 z_2 = r_1 r_2 \angle\phi_1 + \phi_2$

Div : $z_1/z_2 = r_1/r_2 \angle\phi_1 + \phi_2$

Reciprocal : $1/z = \frac{1}{r}\angle -\phi$

Square root : $\sqrt{z} = \sqrt{r}\angle\phi/2$

Complex Conjugate : $z^* = x - jy = r\angle -\phi = re^{-j\phi}$

$\longrightarrow 1/j = -j$

Euler's identity for phasor representation

$$e^{\pm j\phi} = \cos\phi \pm j\sin\phi$$ ← Proof in appendix B

$\cos\phi = Re(e^{j\phi})$ → "real" part of

$\sin\phi = Im(e^{j\phi})$ → "imaginary" part of

given $v(t) = V_m \cos(\omega t + \phi)$

$v(t) = Re(Ve^{j\omega t})$, where $V = V_m e^{j\phi} = V_m \angle\phi$

V is the phasor representation of the sinusoid v(t). It's a complex representation of the magnitude and phase of a sinusoid.

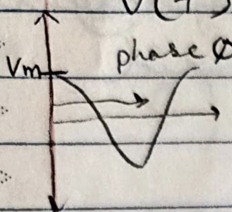
- Mathematical equivalent of a sinusoid with the time dependence dropped.

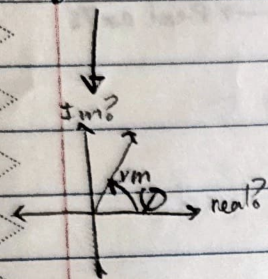
Sinusoid $\qquad\qquad\qquad\qquad$ vector

amplitude $\quad$ angle added inside?

$$v(t) = V_m \cos(\omega t + \phi) \longleftrightarrow V = V_m \angle \phi$$

Time-domain representation $\qquad\qquad$ Phasor-domain representation





$V_m \cos(\omega t + \phi)$	$V_m \angle \phi$
$V_m \sin(\omega t + \phi)$	$V_m \angle \phi - 90°$
$I_m \cos(\omega t + \theta)$	$I_m \angle \theta$
$I_m \sin(\omega t + \theta)$	$I_m \angle \theta - 90°$

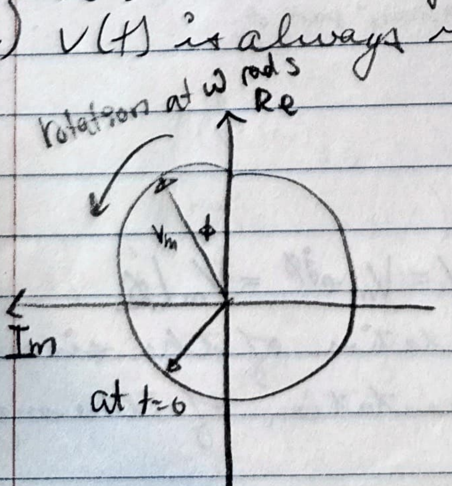
Differentiating a sinusoid is equivalent to multiplying its corresponding phasor by $j\omega$

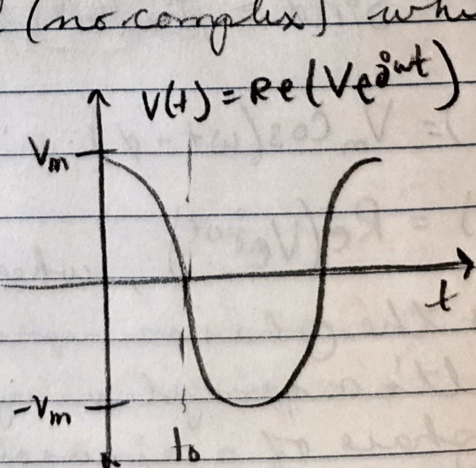
$$\frac{dv}{dt} \longleftrightarrow j\omega V$$

Integrating a sinusoid is the same as dividing its phasor by $j\omega$

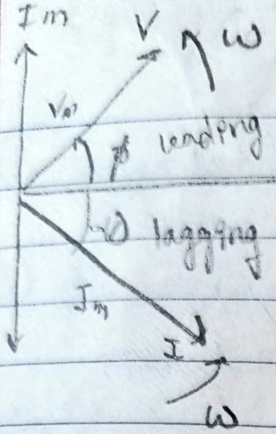
$$\int v\,dt \longleftrightarrow \frac{V}{j\omega}$$

1.) $v(t)$ is instantaneous or time domain representation, while V is frequency or phasor domain representation
2.) $v(t)$ is time dependent, while V is not
3.) $v(t)$ is always real (no complex) while V is complex





$V = V_m \angle \phi$



starts at θ and goes ω

$I = I_m \angle -\theta$

ex, $i = 6\cos(50t - 40°)$ A

$I = 6 \angle -40°$ A

$v = -4\sin(30t + 50°)$ V

$-\sin A = \cos(A + 90°)$

$= 4\cos(30t + 140°)$ V

$V = 4 \angle 140°$ V

ex, $v(t) = v_1(t) + v_2(t)$

$v_1(t) = 10\cos(\omega t - 25°)$ V

$v_2(t) = 4\sin(\omega t + 35°)$

$10 \angle -25 + 4 \angle -55$

$V_1 = 10\cos(-25) - j10\sin(-25)$
$= 9.06 - j4.23$

$V_2 = 4\cos(-55) - j4\sin(-55)$
$= 2.29 - j3.28$

$V_1 + V_2 = 11.35 - j7.51$

$V = 13.6 \angle -33.5°$ V

Seth Ricks

Phasor HW

$r=\sqrt{a^2+b^2}$

$\theta=\tan^{-1}\left(\frac{im}{real}\right)$

1.)

	original	$j(1+j1)$	$j^2(1+j1)$	$j^3(1+j1)$	$j^4(1+j1)$
Regular	$1+j1$	$-1+j1$	$-1-j$	$1-j1$	$1+j1$
Polar	$1\angle\pi/4$	$1\angle\frac{3\pi}{4}$	$1\angle-\frac{3\pi}{4}$	$1\angle-\pi/4$	$1\angle\pi/4$
Quadrant	1st	2nd	3rd	4th	1st

$-j-1$

2.)

a.) $v(t)=12\sin(\omega t+45°)\,V$

$\mathbf{V}=12\angle 45°-90° = 12\angle-45°$

b.) $v(t)=9\sin(\omega t-\pi/3)\,V$

$\mathbf{V} = 9\angle-\pi/3-\pi/2 = 9\angle-60°-90° = 9\angle-150°$

c.) $i(t)=i_1(t)+i_2(t)$

where $i_1(t)=4\sin(\omega t+\pi/3)$ and $i_2(t)=3\cos(\omega t-\pi/6)A$

$I_1 = 4\angle\pi/3-\pi/2 = 4\angle-\pi/6 = 4\angle-30°$

$I_2 = 3\angle-\pi/6 = 3\angle-30°$

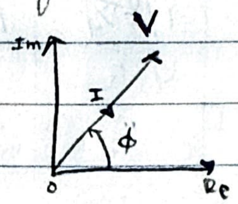
$I_1=3.46-j2 \qquad I_2=2.60-j1.5$

$I_1+I_2 = 6.06-j3.5$

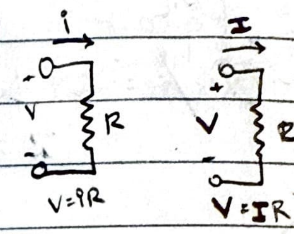
$= 7\angle-30°$

9.4 Phasor relationships for Circuit elements

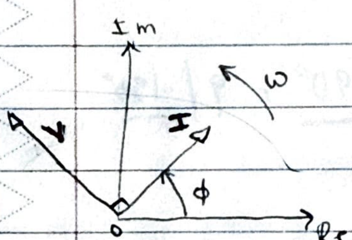
- If current through resistor R is $i = I_m \cos(\omega t + \phi)$



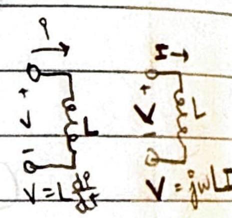
$$V = iR = RI_m \cos(\omega t + \phi)$$
$$\mathbf{V} = RI_m \angle \phi$$
$$\mathbf{V} = R\mathbf{I}$$



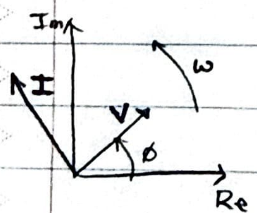
- For the inductor L, assume current $i = I_m \cos(\omega t + \phi)$



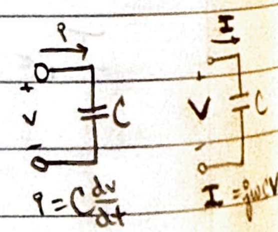
$$v = L\frac{di}{dt} = -\omega L I_m \sin(\omega t + \phi)$$
$$= \omega L I_m \cos(\omega t + \phi + 90°)$$
$$= \omega L I_m \angle \phi + 90°$$
$$\mathbf{V} = j\omega L \mathbf{I}$$



- For the capacitor C, assume voltage $v = V_m \cos(\omega t + \phi)$



$$i = C\frac{dv}{dt}$$
$$\mathbf{I} = j\omega C \mathbf{V} \rightarrow \mathbf{V} = \frac{\mathbf{I}}{j\omega C}$$



Element	Time Domain	Frequency Domain
R	$v = Ri$	$\mathbf{V} = R\mathbf{I}$
L	$v = L\frac{di}{dt}$	$\mathbf{V} = j\omega L\mathbf{I}$
C	$i = C\frac{dv}{dt}$	$\mathbf{V} = \frac{\mathbf{I}}{j\omega C}$

9.5 Impedance and Admittance

$V = RI \quad V = j\omega L I \quad V = \frac{I}{j\omega C}$

can be written as

$\frac{V}{I} = R \quad \frac{V}{I} = j\omega L \quad \frac{V}{I} = \frac{1}{j\omega C}$

So, ohm's law in phasor form for any element

$Z = \frac{V}{I}$ or $V = ZI$

Z is a frequency-dependent quantity → impedance
↳ measured in ohms.

$Z = R \pm jX = |Z|\angle\theta$, where $|Z| = \sqrt{R^2+X^2}$, $\theta = \tan^{-1}\left(\frac{\pm X}{R}\right)$

$R = |Z|\cos\theta$, $X = |Z|\sin\theta$

Impedance: $R = \mathrm{Re}\,Z$ → resistance
$X = \mathrm{Im}\,Z$ → reactance

Reciprocal of impedance → admittance, (siemens (S) or mhos)

$Y = \frac{1}{Z} = \frac{I}{V}$

$Y = G + jB$ → susceptance
↳ conductance

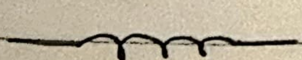
$G + jB = \frac{1}{R + jX}$

⇝ or $G = \frac{R}{R^2+X^2}$ and $B = \frac{-X}{R^2+X^2}$

Impedance and admittance of Passive elements

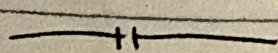
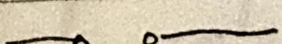
Element	Impedance	Admittance
R	$Z = R$	$Y = \frac{1}{R}$
L	$Z = j\omega L$	$Y = \frac{1}{j\omega L}$
C	$Z = \frac{1}{j\omega C}$	$Y = j\omega C$

Always $X_L \angle 90°$

 → 
short at DC
$\omega = 0$


open at high frequencies
$\omega = \infty$

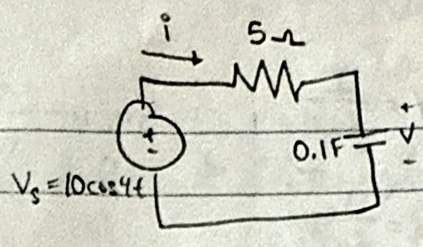
Always $X_C \angle -90°$

 → 
open at DC
$\omega = 0$


short at high frequencies
$\omega = \infty$

W/out j → reactance
reactance is mag of impedance

ex;



$$V_s = 10\angle 0^\circ \text{ V}$$

$$Z = 5 + \frac{1}{j\omega C} = 5 + \frac{1}{j4\times 0.1} = 5 - j2.5\,\Omega$$

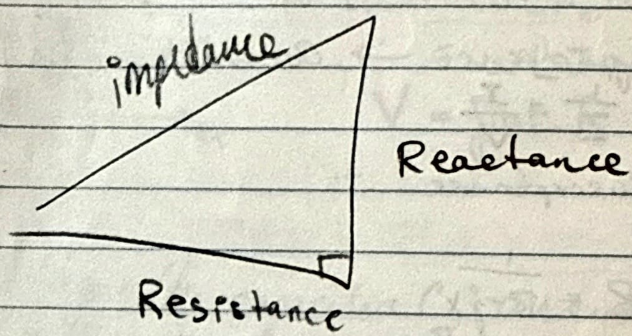
$$I = \frac{V_s}{Z} = \frac{10\angle 0^\circ}{5 - j2.5} = \frac{10(5 + j2.5)}{5^2 + 2.5^2}$$

$$= 1.6 + j0.8 = 1.789\angle 26.57^\circ \text{ A}$$

$$V = IZ_c = \frac{I}{j\omega C} = \frac{1.789\angle 26.57^\circ}{j4\times 0.1}$$

$$= \frac{1.789\angle 26.57^\circ}{0.4\angle 90^\circ} = 4.47\angle -63.43^\circ \text{ V}$$

$$\left.\begin{aligned} i(t) &= 1.789\cos(4t + 26.57^\circ)\text{ A} \\ v(t) &= 4.47\cos(4t - 63.43^\circ)\text{ V} \end{aligned}\right\}$$ (notice i(t) leads v(t) by 90°, as expected)



Impedance HW

① Reactance X_c of a 2.7 μF capacitor at frequency 150Hz.

$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi \cdot 150 \cdot 2.7\times10^{-6}} = 392.98\,\Omega$$

② Capacitance of Capacitor w/ 200Ω reactance at 20KHz

$$X_c = \frac{1}{\omega C} \quad C = \frac{1}{\omega X_c} = \frac{1}{2\pi \cdot 20\times10^3 \cdot 200} = 39.79\ nF$$

③ frequency at which 0.1 μF capacitor has a reactance 100Ω

$$X_c = \frac{1}{2\pi f C} \quad f = \frac{1}{2\pi \cdot C \cdot X_c} = \frac{1}{2\pi \cdot 100\Omega \cdot 0.1\times10^{-6}F} = 15.92\ KHz$$

④ reactance X_L of 270 μH inductor at frequency of 1 MHz

$$X_L = \omega L = 2\pi \cdot 10^6 Hz \cdot 270\times10^{-6} H = 1.697\ K\Omega$$

⑤ Inductance of an inductor that has a reactance 250Ω at 20 kHz

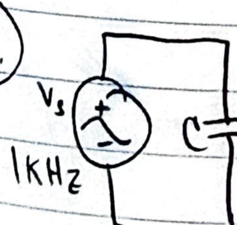
$$X_L = \omega L \quad L = \frac{X_L}{2\pi f} = \frac{250\Omega}{2\pi \cdot 20\times10^3} = 1.99\ mH$$

⑥ frequency at which a 1.5 mH inductor that has a reactance 100Ω

$$X_L = \omega L = 2\pi f L \quad f = \frac{X_L}{2\pi L} = \frac{100\Omega}{2\pi \cdot 1.5\times10^{-3}}$$

$$= 10.61\ kHz$$

⑦

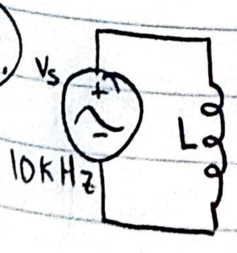


Impedance Z_c = ?

$$Z_c = \frac{1}{j\omega C} = \frac{1}{j2\pi f C} = -j \cdot \frac{1}{2\pi \cdot 10^3 Hz \cdot 10^{-6} F}$$

$$= -j159.15\,\Omega$$

⑧

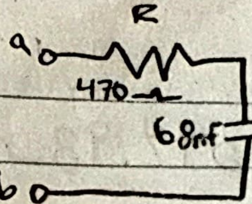


impedance Z_L = ?

$$Z_L = j\omega L = j2\pi f L = j\,2\pi \cdot 10^4 Hz \cdot 4.7\times10^{-3} H$$

$$= j\,295.31\,\Omega$$

9.



impedance = ? frequency = 5 kHz

$$Z = 470 + \frac{1}{jwC} = 470 - \frac{1}{j2\pi f C}$$

$$= 470 + \frac{1}{j2\pi \cdot 5 \cdot 10^3 \cdot 68 \cdot 10^{-9}}$$

$$= 470 - j468.10$$

$$|Z| = \sqrt{R^2 + X^2} = \sqrt{470^2 + 468.10^2}$$

$$= 663\ \Omega$$

$$\theta = \tan^{-1}\left(\frac{-468.10}{470}\right) = -44.88^\circ$$

$$Z = 663 \angle{-44.88^\circ}$$

9.6 Kirchoff's Laws in the Frequency Domain

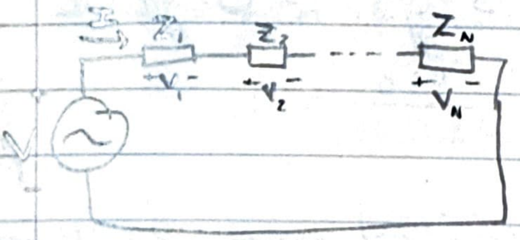
KVL: $v_1 + v_2 + \dots + v_n = 0$

translates to $\mathbf{V}_1 + \mathbf{V}_2 + \dots \mathbf{V}_n = 0$

KCL: $i_1 + i_2 + \dots + i_n = 0$

translates to $\mathbf{I}_1 + \mathbf{I}_2 + \dots \mathbf{I}_n = 0$

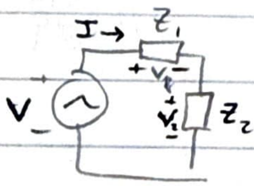
9.7 Impedance Combinations



KVL: $\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2 + \dots + \mathbf{V}_N = \mathbf{I}(\mathbf{Z}_1 + \mathbf{Z}_2 + \dots + \mathbf{Z}_N)$

$\mathbf{Z}_{eq} = \frac{\mathbf{V}}{\mathbf{I}} = \mathbf{Z}_1 + \mathbf{Z}_1 + \dots \mathbf{Z}_N$

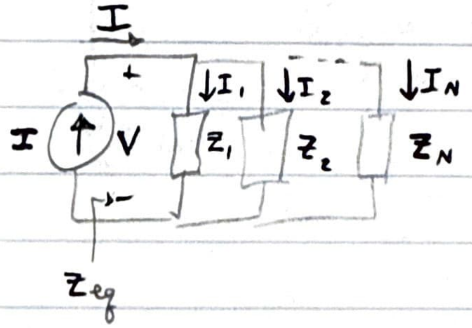
If $N = 2$,



(Voltage division)

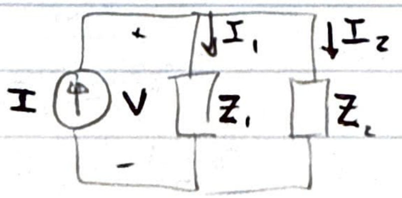
$\mathbf{V}_1 = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{V}, \quad \mathbf{V}_2 = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{V}$

If in //:



$\mathbf{Y}_{eq} = \mathbf{Y}_1 + \mathbf{Y}_2 + \dots + \mathbf{Y}_N$

If $N = 2$,



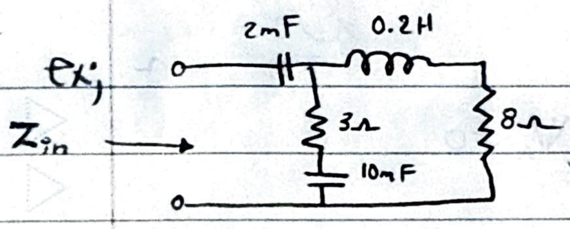
(current division)

$\mathbf{I}_1 = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{I}, \quad \mathbf{I}_2 = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{I}$

$Y_{eq} = X \angle \theta°$

$$\frac{1}{Y_{eq}} = \frac{1 \angle 0°}{X \angle \theta°} = \frac{1}{X} \angle 0 - \theta°$$

Find input impedance. Assume circuit operates w/ $\omega = 50$ rad/s



Z_1 = Impedance of 2-mF cap.
Z_2 = Impedance of 3Ω resistor w/ 10mF-cap
Z_3 = Impedance of the 0.2 H inductor in series with 8Ω-resistor

$$Z_1 = \frac{1}{j\omega C} = -j10\,\Omega; \quad Z_2 = 3 + \frac{1}{j\omega C} = (3-j2)\,\Omega$$

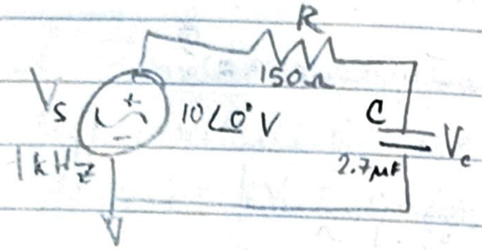
$$Z_3 = 8 + j\omega L = (8 + j10)\,\Omega$$

$$Z_{in} = Z_1 + Z_2 || Z_3 = -j10 + \frac{(3-j2)(8+j10)}{11+j8} = 3.22 - j11.09\,\Omega$$

Impedance HW 2

Seth Ricks

1.



$$I = \frac{V_s}{Z_{eq}} \qquad \omega = 1\text{kHz}$$

$$Z_{eq} = 150 + -j \cdot \frac{1}{\omega C} = 150 - j \cdot \frac{1}{2\pi(10^3) \cdot 2.7 \cdot 10^{-6}}$$

$$= 150 - j\,58.946\,\Omega$$

$$|Z| = \sqrt{150^2 + (58.946)^2} = 161.166\,\Omega$$

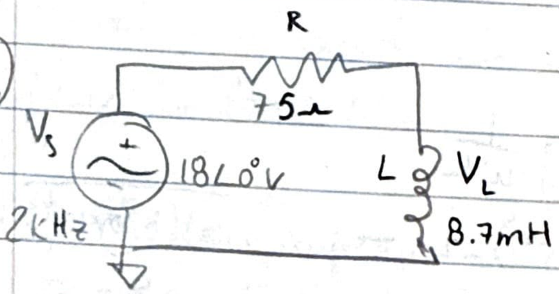
$$\theta = \tan^{-1}\left(-\frac{58.946}{150}\right) = -21.45°$$

$$I = \frac{10\angle 0°}{161.166\angle -21.45°\,\Omega} = 62\,\text{mA}\angle 21.45°$$

$$V_c = IZ_c = \frac{I}{j\omega C} = \frac{62\,\text{mA}\angle 21.45°}{j(2\pi)(10^3\,\text{Hz})(2.7\times10^{-6}\,\text{F})} = \frac{62\,\text{mA}\angle 21.45°}{j\,0.01696}$$

$$V_c = \frac{62\,\text{mA}\angle 21.45°}{0.01696\angle -90} = 3.669\angle 111.45°$$

2.



$$Z_{eq} = 75 + j\omega L$$

$$= 75 + j(2\pi)(2\times10^3)(8.7\times10^{-3})$$

$$= 75 + j\,109.327$$

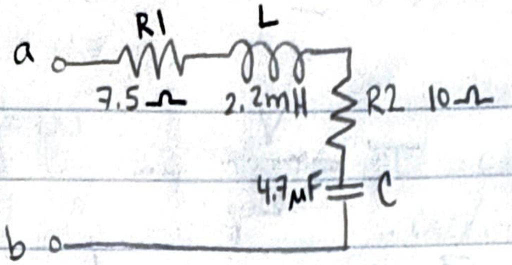
$$|Z| = \sqrt{75^2 + (109.327)^2} = 132.58$$

$$\theta = \tan^{-1}\left(\frac{109.327}{75}\right) = 55.55°$$

$$I = \frac{V_s}{Z_{eq}} = \frac{18\angle 0°\,\text{V}}{132.58\angle 55.55°} = 135.77\,\text{mA}\angle -55.55°$$

$$V_L = IZ = 135.77\,\text{mA}\angle -55.55° \cdot 109.327\angle 90° = 14.84\angle 34.45°\ \text{V}$$

(3.)



$f = 1\,kHz$

Impedance = ?

$$Z_{eq} = 7.5\Omega + 10\Omega + -j\frac{1}{\omega C}\Omega + j\omega L\Omega$$

$$= 17.5 - j\frac{1}{(2\pi)(10^3)(4.7\times10^{-6})} + j(2\pi)(10^3)(2.2\times10^{-3})$$

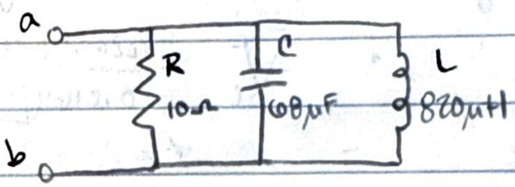
$$= 17.5 - j33.863 + j13.823 = 17.5 - j20.040$$

$$|Z| = \sqrt{17.5^2 + (-20.040)^2} = 26.605$$

$$\theta = \tan^{-1}\left(\frac{-20.040}{17.5}\right) = -48.871^\circ$$

$$Z_{eq} = 26.605 \angle -48.871^\circ\ \Omega$$

(4.)



$f = 1\,kHz$

Admittance = ?

$$Y_{eq} = \frac{1}{10\Omega} + j\omega C\Omega + -j\frac{1}{\omega L}$$

$$= \frac{1}{10\Omega} + j(2\pi)(10^3)(68\times10^{-6})\Omega - j\frac{1}{(2\pi)(10^3)(820\times10^{-6})\Omega}$$

$$= \frac{1}{10}\Omega + j0.427\Omega - j0.194\Omega$$

$$= 0.1 + j0.233\Omega$$

$$|Y| = \sqrt{0.1^2 + 0.233^2} = 0.253 \text{ Siemens}$$

$$\theta = \tan^{-1}\left(\frac{0.233}{0.1}\right) = 66.772^\circ$$

$$Y_{eq} = 0.253 \angle 66.772^\circ\ S$$

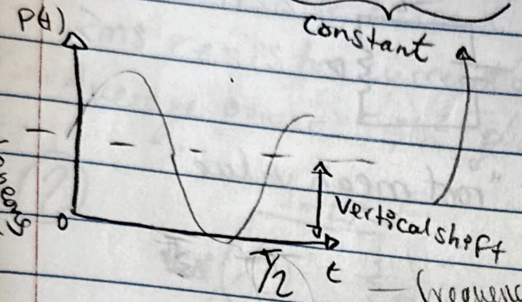
11.1/11.2 Instantaneous and Average Power

$\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$

Instantaneous power - power at any instant in time

$p(t) = v(t)i(t)$; $v(t) = V_m \cos(\omega t + \theta_v)$; $i(t) = I_m \cos(\omega t + \theta_i)$

$$p(t) = \underbrace{\tfrac{1}{2} V_m I_m \cos(\theta_v - \theta_i)}_{\text{constant}} + \underbrace{\tfrac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)}_{\text{sinusoidal}}$$



frequency increases high

Average Power: $P = \frac{1}{2}\mathrm{Re}[\mathbf{V}\mathbf{I}^*] = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$

$P_{avg} = \frac{1}{2} V_m I_m$ → Resistive load (R) - absorbs power at all times ($\theta_v = \theta_i$)

"Capacitive or Inductive load" Reactive load (L or C) - absorbs zero average power ($\theta_v - \theta_i = \pm 90°$)

ex; Calc average power absorbed by an impedance $Z = 30 - j70\,\Omega$ when $V = 120\angle 0°$ applied across it.

$$I = \frac{V}{Z} = \frac{120\angle 0°}{30 - j70} = \frac{120\angle 0°}{76.16\angle -66.8°} = 1.576\angle 66.8°\text{ A}$$

$$P = \tfrac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \tfrac{1}{2}(120)(1.576)\cos(0 - 66.8°)$$
$$= 37.24\text{ W}$$

Resistor Power dissapated:

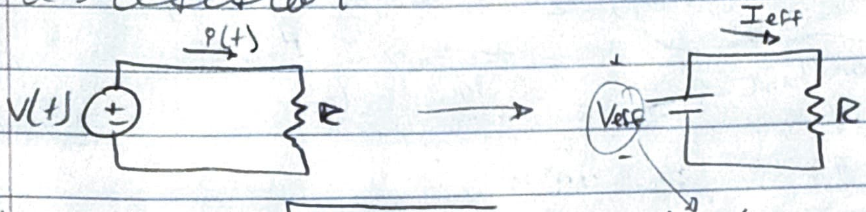
$\frac{1}{2} V_m I_m$

Capacitor / Inductor

0

11.4 Effective or RMS value

Effective value of a periodic current is the dc current that delivers the same average power to a resistor



$X_{eff} = X_{rms} = \sqrt{\frac{1}{T}\int_0^T x^2 dt}$ ← "root mean value"

For $i(t) = I_m \cos\omega t$

$I_{rms} = \sqrt{\frac{1}{T}\int_0^T I_m^2 \cos^2\omega t\, dt} = \frac{I_m}{\sqrt{2}}$

$V_m \cos\omega t$: $V_{rms} = \frac{V_m}{\sqrt{2}}$

$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(\theta_v - \theta_i) = V_{rms} V_{Irms} \cos(\theta_v - \theta_i)$

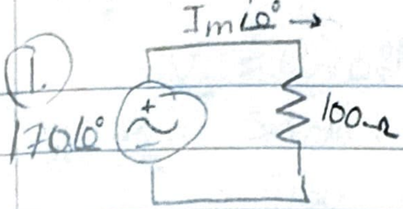
Avg. Power by resistor : $P = I_{rms}^2 R = \frac{V_{rms}^2}{R}$

RMS is NOT Average Voltage or Current.

It is used to find average power!

AC Power HW

(1.)



$f = 120\,Hz$ Peak voltage $= 170\,V$

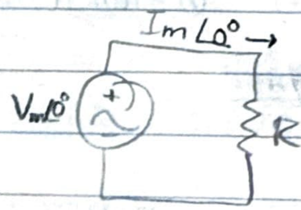
average resistor current $= 0$

Peak resistor current $= V_m/R = 1.7\,A$

Rms resistor current $= \frac{I_m}{\sqrt{2}} = \frac{1.7}{\sqrt{2}}A = 1.202\,A$

Average power $= I_{rms}^2 R = 144.5\,W$

(2.)



$v(t) = 8\cos(2\pi f t)\,V$

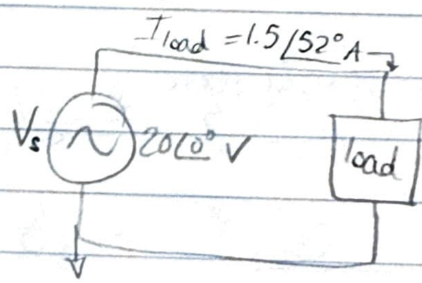
$I_{rms} = 1.2\,A$

$I_m = 1.2 \cdot \sqrt{2}\,A = 1.697\,A$

$R = V_m/I_m = 8/1.697\,\Omega = 4.714\,\Omega$

$P_{avg} = I_{rms}^2 R = (1.2)^2(4.714) = 6.788\,W$

(3.)

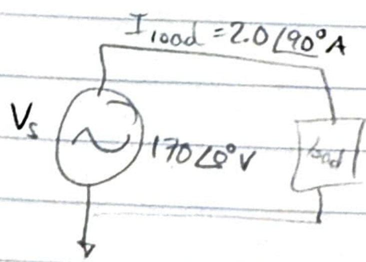


$Z_{load} = \frac{20\angle 0°}{1.5\angle 52°} = 13.33\angle -52°\,\Omega$

$I_{rms} = \frac{1.5}{\sqrt{2}}$

$P_{avg} = I_{rms}^2 R \cdot \cos(0-52°) = \left(\frac{1.5}{\sqrt{2}}\right)^2(13.33)\cos -52°$
$= 9.235\,W$

(4.)

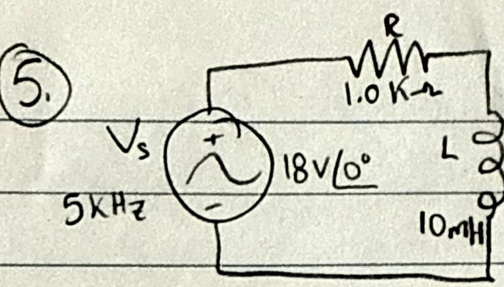


$Z_{load} = \frac{170\angle 0°\,V}{2.0\angle 90°\,A} = 85\angle -90\,\Omega$

$I_{rms} = \frac{2.0}{\sqrt{2}}$

$P_{avg} = I_{rms}^2 R(\cos(-90)) = 0\,W$

5.



$P_{source} = ?$ $P_R = ?$

$$P_{source} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$Z_{eq} = Z_R + Z_L = 1.0k\Omega + j(2\pi)(5\times10^3)(10\times10^{-3}) = 1000 + j100\pi\,\Omega$$

$$|Z_{eq}| = \sqrt{1000^2 + 100\pi^2} = 1048.187\,\Omega \qquad \theta = \tan^{-1}\left(\frac{100\pi\,\Omega}{1000}\right) = 17.441^\circ$$

$$I = V/R = \frac{18\angle 0^\circ\ V}{1048.187\angle 17.441^\circ} = 0.0172\angle -17.441^\circ$$

$$P_{source} = \frac{1}{2}(18)(0.0172)\cos(0 + +17.441^\circ) = 0.1477\ W$$

$$P_R = \cancel{I_{rms}^2 R \cos(\theta_v - \theta_i) = \left(\frac{0.0172}{\sqrt{2}}\right)^2 \cos(17.441^\circ) = 141\ \mu W}$$

$$= 0.1477\ W$$

Question: Difference between these two?

lab

$V_s = 10\angle 0° \, V$

$Z_{eq} = 5.1\times10^3 k\Omega - j/(2\pi)(300)(0.1\times10^{-6}))$

$= 5100 - j5305.16 \, \Omega$

$|Z| = \sqrt{5100^2 + 5305.16^2} = 7359 \, \Omega$

$\theta = \tan^{-1}\left(\frac{-5305.16}{5100}\right) = 46.13°$

$I = V_s / Z = \frac{10\angle 0° \, V}{7359\angle 46.13° \, \Omega} = 1.36 \, mA \angle +46.13°$

$Z_c = -j5305.16 \, \Omega$

$= 5305.16 \angle -90° \, \Omega$

$1.36 \, mA \angle +46.13° * 5305.16 \angle -90° \, \Omega$

$= 7.21 \angle -43.87°$

$V_s = X\angle 0° \, V$

$Z_L = Y\angle \theta° \, \Omega$

$\frac{V_s}{Z_L} = \frac{X\angle 0° \, V}{Y\angle \theta°}$ $0 - \theta = -\theta$

$I = -\theta°$

$j\frac{xx}{\quad}$

14.1 / 14.2 Transfer Function

Frequency response – the variation in a circuit's behavior with change in signal frequency

Transfer function $H(\omega)$ → (network function)

Frequency response → graph $H(\omega)$ vs ω

$X(\omega)$ → [linear network $H(\omega)$] → $Y(\omega)$

Input ... output

$$H(\omega) = \frac{Y(\omega)}{X(\omega)}$$

all these should be bolded

$H(\omega)$ = Voltage gain $= \dfrac{V_o(\omega)}{V_i(\omega)}$ $\qquad H(\omega)$ = Transfer impedance $= \dfrac{V_o(\omega)}{I_i(\omega)}$

$H(\omega)$ = Current gain $= \dfrac{I_o(\omega)}{I_i(\omega)}$ $\qquad H(\omega)$ = Transfer admittance $= \dfrac{I_o(\omega)}{V_i(\omega)}$

$H(\omega) = H(\omega)\angle\phi$

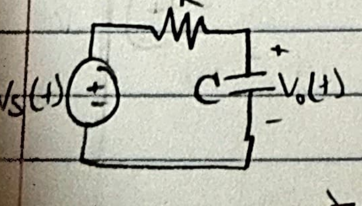
$$H(\omega) = \frac{N(\omega)}{D(\omega)}$$

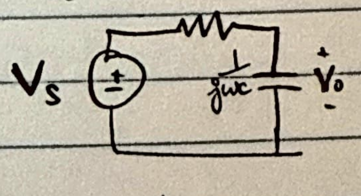
num. polynomial

denom. polynomial

A zero is a root of num. poly, and results in zero value of the function. A pole is a root of the denom. poly, is a value for which the function is infinite.

ex; Obtain V_o/V_s and frequency response. Let $V_s = V_m \cos \omega t$





$$H(\omega) = \frac{V_o}{V_s} = \frac{I \cdot 1/j\omega C}{I \cdot R + 1/j\omega C} = \frac{1}{1 + j\omega RC} \qquad H = \frac{1}{\sqrt{1+(\omega/\omega_0)^2}} \qquad \theta = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

14.3 The Decibel Scale

G = Number of bels $= \log_{10} \frac{P_2}{P_1}$ "Gain" has no units

1/10th of a bel → $G_{dB} = 10\log_{10}\frac{P_2}{P_1}$ } used for Power

If $P_2 = 2P_1$ → $G_{dB} = 10\log_{10} 2 \approx 3\,dB$

$G_{dB} = 20\log_{10}\frac{V_2}{V_1}$, $G_{dB} = 20\log_{10}\frac{I_2}{I_1}$ * for $R_{out} = R_{in}$

used for voltage / current

* $R_{out} \neq R_{in}$: $A_{v_{dB}} = 20\log_{10}\left(\frac{R_{in}}{R_{out}}\right) - 10\log_{10}\left(\frac{R_{in}}{R_{out}}\right)$

14.4 Bode Plots:

I ♥ Izzy Moomey

Bode plots - semilog plots of the magnitude (in decibels) and phase (in degrees) of a transfer function versus frequency.

$$H = H\angle\phi = He^{j\phi}$$

$$\ln H = \ln H + \ln e^{j\phi} = \ln H + j\phi$$

$$H_{dB} = 20\log_{10} H$$

Transfer Function HW

1. 30mW → 10W

$$G_{dB} = 10\log_{10}\left(\frac{10}{30e-3}\right) = 25\text{ dB}$$

2. 10μV → 10mV

$$G_{dB} = 20\log_{10}\left(\frac{10e-3}{10e-6}\right) = 60\text{ dB}$$

3. 1V 1MHz → 1mV 1MHz

$$G_{dB} = 20\log_{10}\left(\frac{10e-3}{1}\right) = -40\text{ dB}$$

4. Determine peak sinusoidal voltage

$$V_{im} \longrightarrow \boxed{40\text{dB}} \longrightarrow V_{om} = 1V$$

$$G_{dB} = 20\log_{10}\left(\frac{1V}{V_{im}}\right)$$

$$10^{\frac{G_{dB}}{20}} = \frac{1}{V_{im}} \longrightarrow V_{im} = 10^{-\frac{G_{dB}}{20}} = 10mV$$

5. Determine peak output voltage

$$\underset{1mV}{V_{im}} \longrightarrow \boxed{42\text{dB}} \longrightarrow \boxed{38\text{dB}} \longrightarrow V_{om}$$

$$G_{dB} = 20\log_{10}\left(\frac{V_{out}}{1e-3}\right)$$

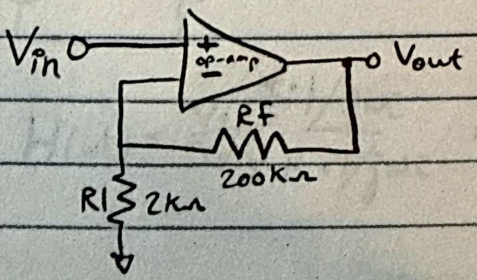
$$V_{out} = 10^{\frac{G_{dB}}{20}} \cdot 1e-3$$

$$= 0.1259V$$

$$V_{om} = 10^{\frac{G_{dB}}{20}} \cdot 0.1259V$$

$$= \boxed{10V}$$

6. Determine voltage gain



$$A = \left(1 + \frac{R_f}{R_1}\right)$$

$$G_{dB} = 20\log_{10}\left(1 + \frac{R_f}{R_1}\right)$$

$$= 40.086\text{ dB}$$

* Gain if input is 120Hz 1V, output 120Hz 3mV

$A_{v_{dB}} = 20\log_{10}\left(\frac{3mV}{V}\right) = -50.5\ dB$

$dB < 1 \rightarrow$ attenuation

$V_{in1} \rightarrow \boxed{A_{v1}} \xrightarrow{V_{in2}} \boxed{A_{v2}} \rightarrow V_{out}$

36 dB   32 dB $36^{dB} + 32^{dB} = 68\ dB$

40 dB = ?

$\rightarrow \boxed{A_{v3}} \rightarrow$

$40\ dB = 20\log_{10}\left(\frac{V_{out}}{V_{in}}\right)$

$10^{4/2} = 100$

40 dB = 100 linear multiple

* $V_{in1} \rightarrow \boxed{A_{v1}} \xrightarrow{V_{in2}} \boxed{A_{v2}} \rightarrow V_{out}$

2000

$A_{v_{total}} = 20\log_{10}(2000) = 66\ dB$

$-25\ dB$

$= A_{v2_{dB}} = 41\ dB$

$A_{v2} = 10^{\frac{41}{20}} = 112$