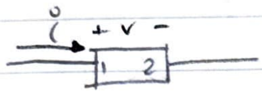
★ It's ♫ Review ♫ time! ★

1.4 Voltage & Current

Voltage $v = \frac{dw}{dq}$ v = voltage w = energy in J q = charge in coulumbs

Current $i = \frac{dq}{dt}$ i = current in amps q = charge t = time



← ideal basic circuit element

ex; No charge at left terminal of element for $t<0$. at $t=0$, 5A current begins to flow.

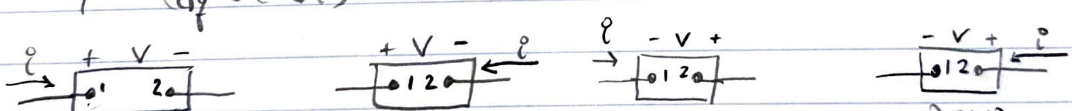
$$q(t) = \int_0^t i(x)dx = \int_0^t 5\,dx = 5x\Big|_0^t = 5t \text{ C for } t>0$$

Total charge in 10 seconds → 50 C

1.6 Power & Energy

Power $p = \frac{dw}{dt}$ p = power in watts t = time w = energy in joules

$$p = \left(\frac{dw}{dq}\right)\left(\frac{dq}{dt}\right) = vi$$

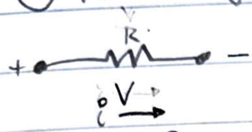


$p = vi$ $p = -vi$ $p = -vi$ $p = vi$

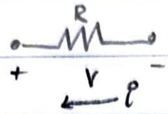
$p > 0$ → element being charged

$p < 0$ → power is extracted

2.2 Ohm's Law $V = iR$



$V = iR$



$V = -iR$

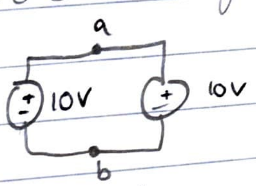
Conductance → $G = 1/R$

Power in terms of current for resistor → $p = i^2 R$

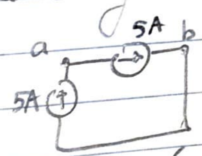
" " " voltage " " → $p = \frac{v^2}{R} = v^2 G$

2.1 Voltage & Current Sources

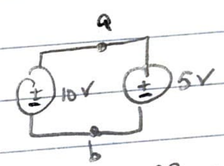
Are the following valid?



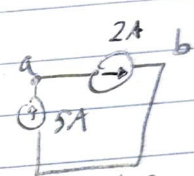
yes ✓



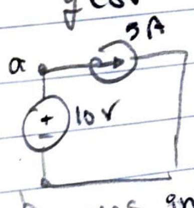
yes ✓



NO, different voltages for ab



No, dif currents for ab



yes, independent sources

(Same concept w/ combined dependent & independent sources)

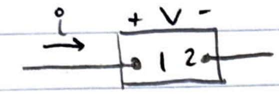
Seth Ricks

ECEN 250

1/10/2024

PS1 : 1.8, 1.14, 2.4, 2.8, 2.18, 2.32

$\boxed{1.8}$ There is no charge at the left termenal for $t<0$. At $t=0$ a current of $29e^{-1000t}$ mA enters left.



Part A

$$q(t) = \int_0^t i(x)\,dx = \int_0^t 0.029e^{-1000t}\,dt = 0.029\int e^{-1000t}\,dt$$

$u = -1000t$

$du/dt = -1000$

$dt = -\frac{du}{1000}$

$$= 0.029\int e^{u}\cdot -\frac{du}{1000} = -\frac{0.029}{1000}\int e^{u}\,du = -\frac{0.029}{1000}e^{u}$$

$$= -\frac{0.029}{1000}e^{-1000t}$$

$$\frac{0.029}{1000} - \frac{0.029}{1000}e^{-1000t}\ \mu C$$

$$= \boxed{q(t) = 29\left(1-e^{-1000t}\right)\mu C}$$

Part B

as $t \to \infty$, $q(t) \to \boxed{29\,\mu C}$

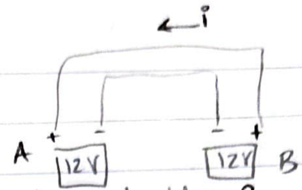
Part C If the current is stopped at $t = 0.4$ms, how much charge has accumulated at the left terminal.

as $t \to 0.4$ms, $q(t) = 29\left(1-e^{-1000(0.0004)}\right) = \boxed{9.56\,\mu C}$

$i_A = -25A$

Seth Ricks

$\boxed{1.14}$ Part A



Which car has the dead battery?

$\boxed{\text{Car A}}$ → This is because the current is entering the positive terminal of A

Part B Assume $i = 30A$

If the connection is maintained for 1 min, how much energy is transferred to the dead battery?

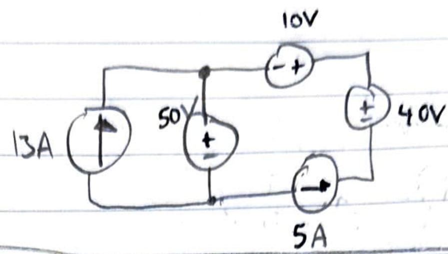
$1 \text{ min} = 60s \qquad P = \frac{dw}{dt}$

$$P = vi = 12v \cdot 30A = 360W = 360 \, J/s \cdot 60s$$
$$= 21600 \, J$$
$$= \boxed{21.6 \, kJ}$$

Seth Ricks

Problem 2.4

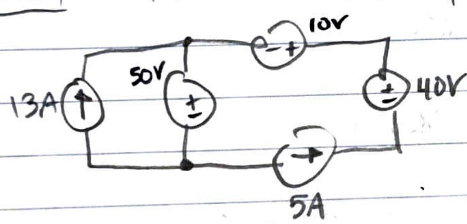
Assume $i = 13A$



Part A Is the interconnection in ↑ valid?

Yes, the interconnection is valid. The independent power and current sources are properly aligned.

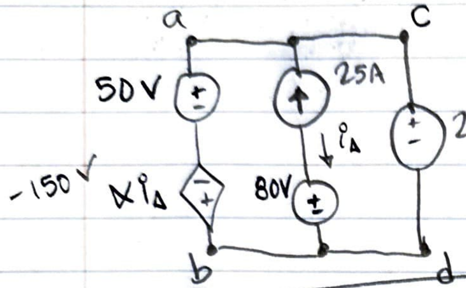
Part B Find the total power developed by the current sources.



$50V \cdot 13A + 20 \cdot 5A = 750W$

$\boxed{750\ W}$

Problem 2.8 Suppose that $\alpha = 6.0$



Part A Determine voltage drop from the top terminal to the bottom terminal, V_{ab}, in the left hand branch and, V_{cd} in the right hand branch of the circuit. Determine each voltage drop based on the elements in the corresponding branch and the middle branch.

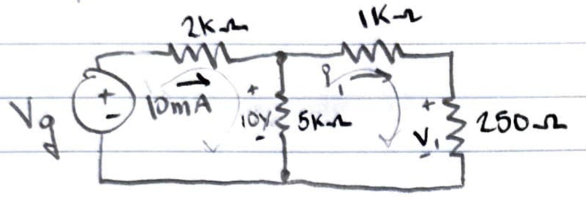
$i_A = -25A$ $\alpha = 6.0$ $\alpha i_A = -150$

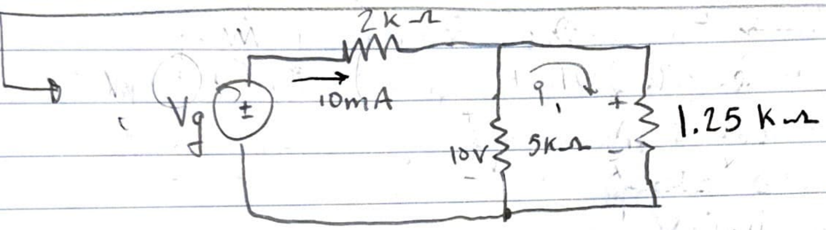
$V_{ab} = 200V$, $V_{cd} = 250V$

Part A

2.18 Find i_1

The current i_x in the circuit shown is 10 mA, and the voltage V_x is 10V





$$(1.25\times10^3)i_1 + (5\times10^3)(i_1 - i_x) = 0$$
$$(6.25\times10^3)i_1 - (5\times10^3)(10\times10^{-3}) = 0$$
$$i_1 = \frac{50}{6.25\times10^3} = 8\text{ mA} = \boxed{8.00\text{mA}}$$

Part B Find V_1

$$V_1 = i_1R_1 = 8.00\text{mA}\cdot250\Omega = 8\times10^3\cdot250 = 2V = \boxed{2.00V}$$

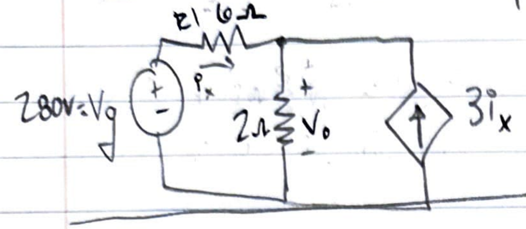
Part C Find V_g

$$-V_g + (2\times10^3)(10\times10^{-3}) + 10V = 0$$
$$-V_g + 20 + 10 = 0$$
$$\boxed{V_g = 30.0V}$$

Part D Find the magnitude of the power supplied by the voltage source

$$P = Vi = (30.0V)(10\times10^{-3}A) = \boxed{0.3W}$$

Problem 2.32 Suppose that $V_g = 280V$



Part A Find V_o

$V_o = i_o \cdot R_o = (4i_x)(2\Omega) = 8i_x V$

$V_{R1} = i_x \cdot 6\Omega = 6i_x V$

$280V = (6i_x + 8i_x)V = 14 i_x V = 20 = i_x$

$\boxed{V_o = 160V}$

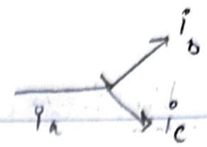
Part B Find the magnitude of the total power supplied in the circuit.

$P_{vg} = Vi = 280V \cdot 20A = 5600\ W = 5.60\ KW$

$P_{dep\ Source} = Vi = V_o \cdot 3i_x = 160 \cdot 60 = 9600W$
$= 9.60\ KW$

$\boxed{P_T = 15.2\ KW}$

Video Solution Problem 1



Part A A circuit node has current i_a entering and currents i_b and i_c exiting, $i_a = 5mA$ & $i_b = 9mA$.

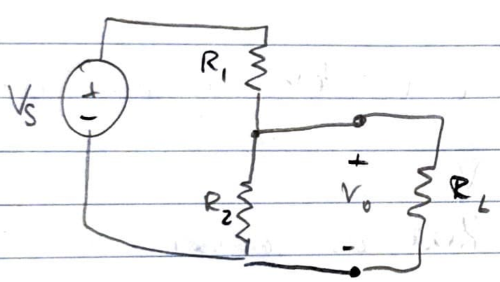
$i_c = ?$

$$i_a = i_b + i_c$$

$$5mA = 9mA - 4mA$$

$$\boxed{i_c = -4mA}$$

The Voltage Divider



when no load attached

$$V_o = \frac{R_2}{R_1 + R_2} V_s$$

when load,

$$V_o = \frac{R_2}{R_1\left(1 + \frac{R_2}{R_L}\right) + R_2} V_s$$

Part A

find R_2; $V_s = 18V$, $R_1 = 50K\Omega$, $V_o = 6V$
(No load)

$$V_o = \frac{R_2}{R_1 + R_2} V_s \longrightarrow \frac{V_o}{V_s} = \frac{R_2}{R_1 + R_2}$$

$$\frac{V_o}{V_s} R_1 + \frac{V_o}{V_s} R_2 = R_2$$

$$\frac{V_o}{V_s} R_1 = R_2 - \frac{V_o}{V_s} R_2$$

$$= R_2\left(1 - \frac{V_o}{V_s}\right)$$

$$R_2 = \frac{V_o}{V_s} R_1 \cdot \frac{1}{1 - \frac{V_o}{V_s}} = \frac{6}{18}(50 \times 10^3) \cdot \frac{1}{1 - \frac{6}{18}}$$

$$= 25000\Omega = \boxed{25K\Omega}$$

The Voltage Divider, Part B

Out of $R_L = 300k\Omega$, $200k\Omega$, $100k\Omega$
which is the most different output voltage from $V_o = 6V$?

$$V_o = \frac{R_2}{R_1\left(1+\frac{R_2}{R_L}\right)+R_2} V_s \qquad R_2 = 25\,k\Omega$$

$V_s = 18V$ $R_1 = 50k\Omega$

$R_L = 300k\Omega \longrightarrow V_o = 5.684$
$R_L = 200k\Omega \longrightarrow V_o = 5.538$
$R_L = 100k\Omega \longrightarrow V_o = 5.143$

Part C

Change R_1 and R_2 so that $V_o = 6V$ when the load resistance is $R_L = 200k\Omega$. The actual output voltage must not drop below 5.4V when $R_L = 100k\Omega$. What is the smallest resistor value that can be used for R_1?

$V_o = 6V$; $R_L = 200k\Omega$ $V_o \geq 5.4V$ when $R_L = 100k\Omega$

$$V_o = \frac{R_2}{R_1\left(1+R_2/R_L\right)+R_2} V_s \qquad R_2 = \frac{V_o}{V_s} R_1 \cdot \frac{1}{1-\frac{V_o}{V_s}}$$

$$R_2 = \frac{R_1 R_L V_o}{R_L(V_s - V_o) - R_1 V_o} = \frac{R_1 (200\cdot 10^3)\,6}{(200\cdot 10^3)(18-6) - R_1(6)} = \frac{R_1 \cdot 1.2\cdot 10^6}{(2.4\cdot 10^6) - 6R_1}$$

$$R_2 = \frac{R_1 R_L V_o}{R_L(V_s - V_o) - R_1 V_o} = \frac{R_1(100\cdot 10^3)\,5.4}{(100\cdot 10^3)(18-5.4) - R_1(5.4)} =$$

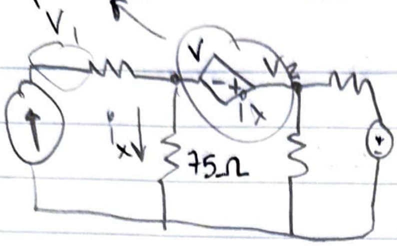
$$= \frac{R_1 \cdot 0.54 \cdot 10^6}{1.26 \cdot 10^6 - 5.4 R_1}$$

$$\boxed{R_1 = 66.7\,k\Omega}$$

Extra credit problem

Supernode = two nodes connected by a "floating" voltage source

{ ungrounded



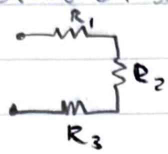
$V_2 - V = i_x$

Supernode gets its own equation

$i_x = \frac{V_1 - 0}{75}$

Chapter 3

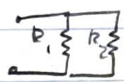
3.1 Resistors in Series



$$R_{eq} = R_1 + R_2 + R_3$$

3.2 Resistors in Parallel

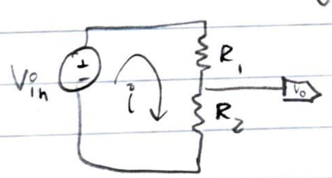
$$R_{eq} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$



$$R_{eq} = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

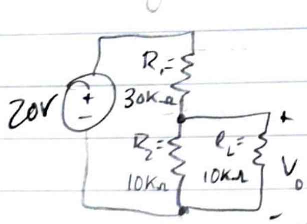
$$G_{eq} = \sum_{i=1}^{k} G_i = G_1 + G_2 + \dots + G_k$$

3.3 The Voltage-Divider



$$V_{out} = V_{in}\left(\frac{R_2}{R_1 + R_2}\right)$$

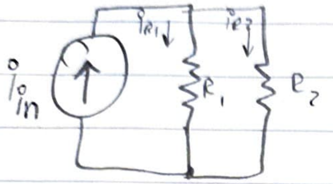
Voltage Divider w/ R_L



$$V_o = \frac{5000}{30{,}000 + 5000}(20) = 2.86V$$

$R_2 // R_L = 5K\Omega$

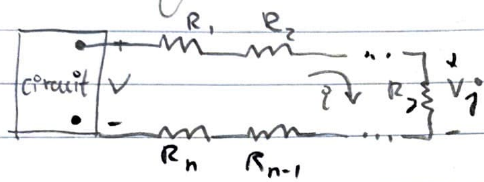
3.3 Current Divider circuit



$$i_{R_1} = i_{in}\left(\frac{R_2}{R_1+R_2}\right)$$

$$i_{R_2} = i_{n}\left(\frac{R_1}{R_1+R_2}\right)$$

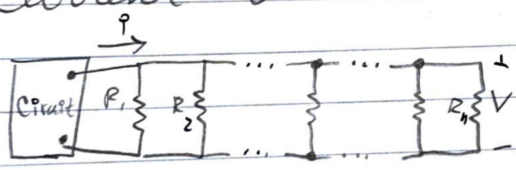
3.4 Voltage Division



% R of total

$$V_j = iR_j = \frac{R_j}{R_{eq}} V$$

Current Division

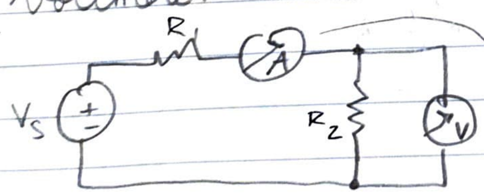


Same but $R_{eq} < R_j$

$$i_j = \frac{V}{R_j} = \frac{R_{eq}}{R_j} i$$

3.5 Measuring Voltage and Current

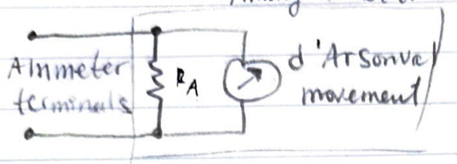
ammeter → measures current. Put in series → $\approx R_{eq} = 0$
voltmeter → measures volts. Put in || → ideal $R_{eq} = \infty$

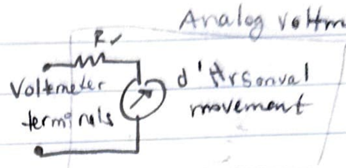


→ acts like short circuit

→ acts like open circuit

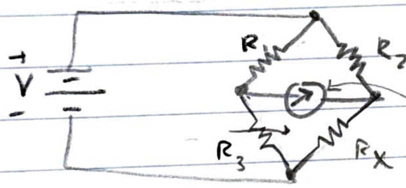
Analog Meters → based on coil in d'Arsonval movement





Digital Meters → measure from descrete points in time

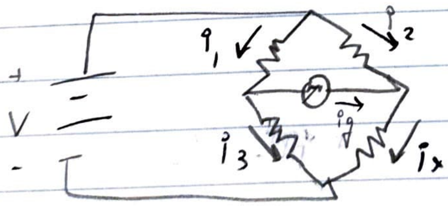
3.6 Measuring Resistance - The Wheatstone Bridge



microamp detector called a galvanometer

To find R_x, we adjust R_3 until there is no current in the galvanometer. Then,

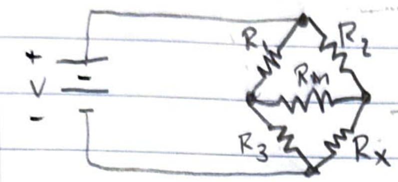
$$\boxed{R_x = \frac{R_2}{R_1} R_3}$$



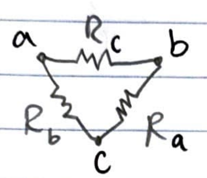
If $i_g = 0$, then $i_2 = i_x$ and $i_1 = i_3$

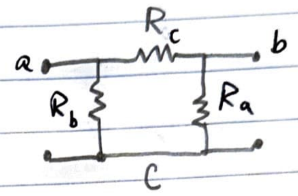
Δ-to-Y   π-to-T

$\boxed{3.7}$ Delta-to-Wye (Pi-to-Tee) Equivalent Circuits



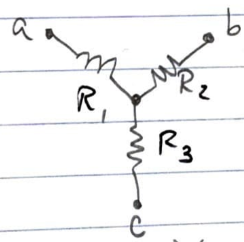
$\left.\begin{matrix} R_1, R_2, R_m \\ R_3, R_m, R_x \end{matrix}\right\} \longrightarrow$ delta interconnection or pi ''

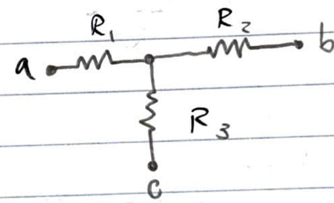




Δ   {or}   π

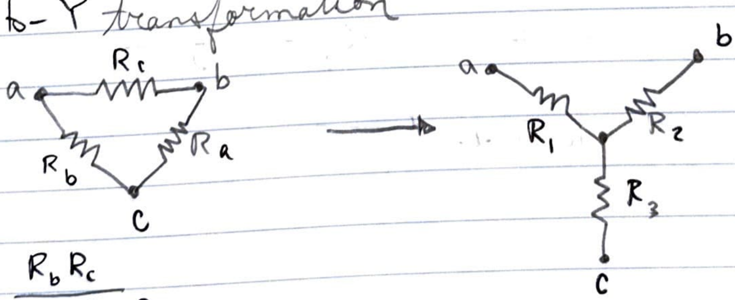
$\left.\begin{matrix} R_1, R_m, R_3 \\ R_2, R_m, R_x \end{matrix}\right\} \longrightarrow$ wye (Y) interconnection or tee (T) interconnection





Y   {or}   T

Δ-to-Y transformation

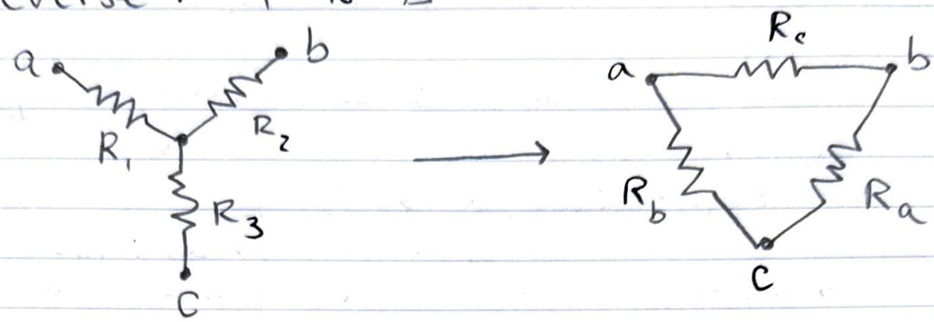


$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

Reverse : Y-to-Δ

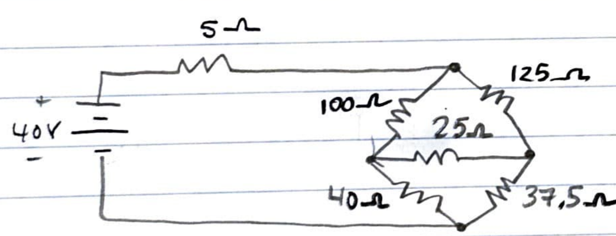


$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

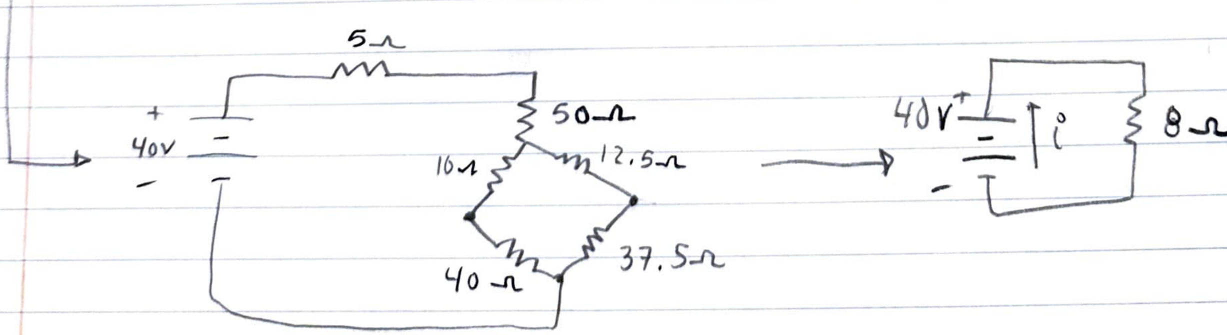
$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

ex;



$$R_1 = \frac{100 \cdot 125}{250} = 50\,\Omega \qquad R_3 = \frac{100 \cdot 25}{250} = 10\,\Omega$$

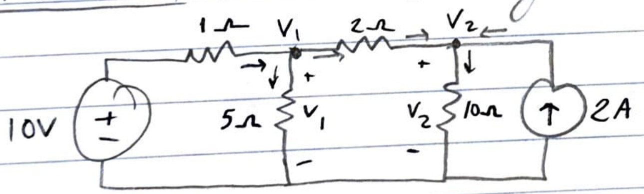
$$R_2 = \frac{125 \cdot 25}{250} = 12.5\,\Omega$$



4.1 Terminology

Planar circuit → a circuit that can be drawn with no overlapping branches
node → where two elements combine
essential node → where three or more elements join
path → a trace of elements, none included twice
branch → path that connects two nodes
essential branch → a path that connects two essential nodes
loop → path whose last node is same as starting
mesh → loop that doesn't include another loop

4.2 - 4.4 Node-Voltage Method



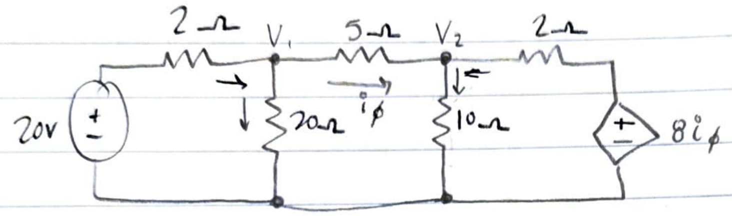
node 1: $\frac{V_1 - 10}{1} + \frac{V_1}{5} + \frac{V_1 - V_2}{2} = 0$ node 2: $\frac{V_2 - V_1}{2} + \frac{V_2}{10} - 2 = 0$

100/11 120/11

Solve → $V_1 = 9.09V$, $V_2 = 10.91V$

$\frac{10 - V_1}{1} = \frac{V_1}{5} + \frac{V_1 - V_2}{2}$ $\frac{V_1 - V_2}{2} + 2 = \frac{V_2}{10}$ Same answer!

The Node-Voltage Method and Dependent Sources



Node 1: $\frac{20-V_1}{2} = \frac{V_1}{20} + i_\phi$ Node 2: $\frac{V_2}{10} = i_\phi + \frac{8i_\phi - V_2}{2}$

$i_\phi = \frac{V_1 - V_2}{5}$

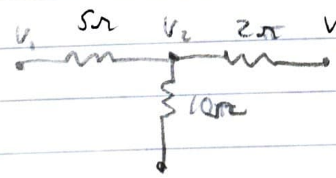
★ What the book has (dumber) → $\frac{V_1 - 20}{2} + \frac{V_1}{20} + \frac{V_1 - V_2}{5} = 0$

$\frac{V_2 - V_1}{5} + \frac{V_2}{10} + \frac{V_2 - 8i_\phi}{2} = 0$

Solve → $V_1 = 16V$, $V_2 = 10V$, $i_\phi = 1.2A$ $i_\phi = \frac{V_1 - V_2}{5}$

Basically, start at the node and minus next voltage.

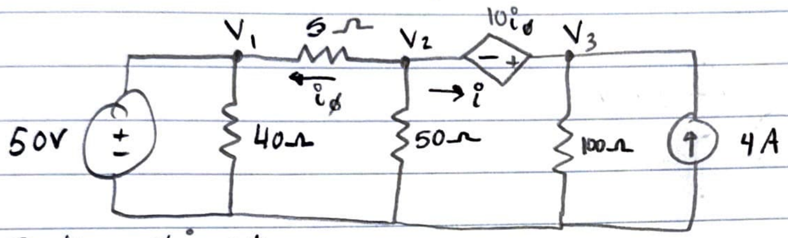
Then $= 0$.



For V_2,

$\frac{V_2 - V_1}{5} + \frac{V_2}{10} + \frac{V_2 - V_3}{2} = 0$

$\boxed{4.4}$ - Dependent voltage source between two nodes



* Book equations *

Node 2: $\frac{V_2 - 50}{2} + \frac{V_2}{50} + i = 0$

Node 3: $\frac{V_3}{100} - i - 4 = 0$

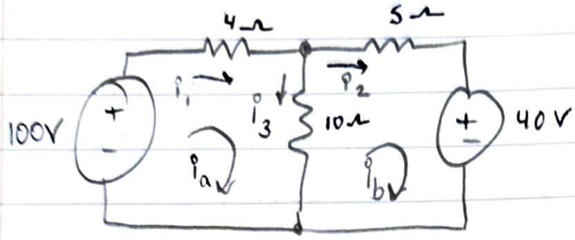
* add two equations * $\frac{V_2 - 50}{2} + \frac{V_2}{50} + \frac{V_3}{100} - 4 = 0$

Voltage source between two nodes → Supernode!

Supernode equation: $V_3 - V_2 = 10 i_\phi$ → $i_\phi = \frac{V_2 - 50}{5}$

Solve → $V_2 = 60V$, $V_3 = 80V$, $i_\phi = 2A$

4.5 Mesh-Current Method Intro



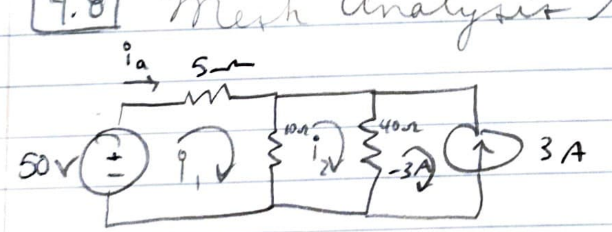
Mesh 1 : $-100 + 4i_a + 10(i_a - i_b) = 0$

Mesh 2 : $5i_b + 40 + 10(i_b - i_a) = 0$

Solve → $i_a = 10A$, $i_b = 4A$

Solve again → $i_1 = i_a = 10\,A$
$i_2 = i_b = 4A$
$i_3 = i_a - i_b = 6A$

4.8 Mesh Analysis Special cases



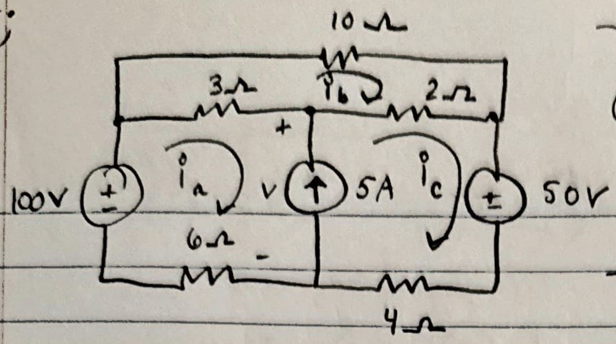
→ current source in a singular mesh

Equations : $-50 + 5i_1 + 10(i_1 - i_2) = 0$ &
$10(i_2 - i_1) + 40(i_2 - (-3)) = 0$

→ $i_1 = 2A$ & $i_2 = -2A$

→ $i_a = i_1 = 2A$
$i_b = i_1 - i_2 = 4A$
$i_c = i_2 + 3 = 1A$

ex;



Current source in two meshes

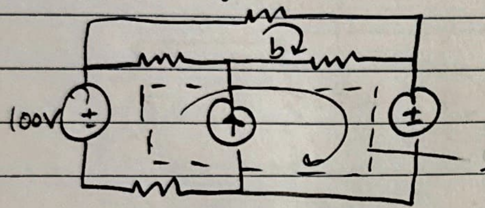
voltage drop across current source

Mesh a : $-100 + 3(i_a - i_b) + v + 6i_a = 0$

Mesh C : $50 + 4i_c - v + 2(i_c - i_b) = 0$

* add equations * ⟶ $-50 + 9i_a - 5i_b + 6i_c = 0$

the same



Supermesh

Supermesh : $-100 + 3(i_a - i_b) + 2(i_c - i_b) + 50 + 4i_c + 6i_a = 0$

⟶ simplify ⟶ $-50 + 9i_a - 5i_b + 6i_c = 0$

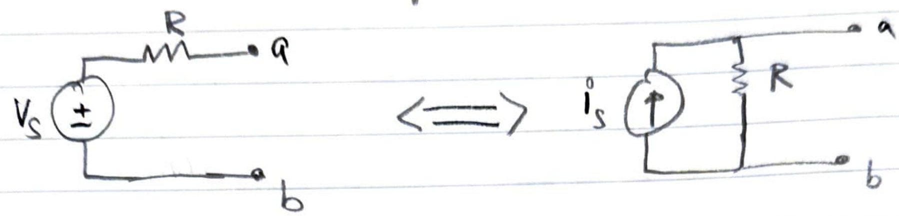
Mesh b : $10i_b + 2(i_b - i_c) + 3(i_b - i_a) = 0$

☆ $i_c - i_a = 5$ ☆ ⟵ Supermesh "constraint" equation

Solve 3 equations (Supermesh, Mesh b, constraint)

⟶ $i_a = 1.75A$, $i_b = 1.25A$, $i_c = 6.75A$

4.9 Source Transformations

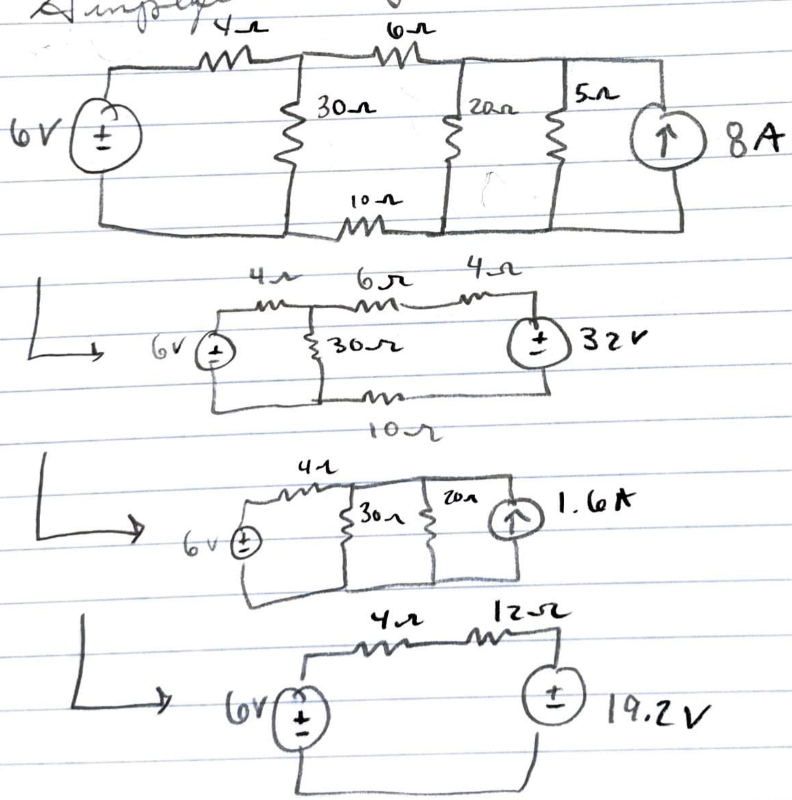


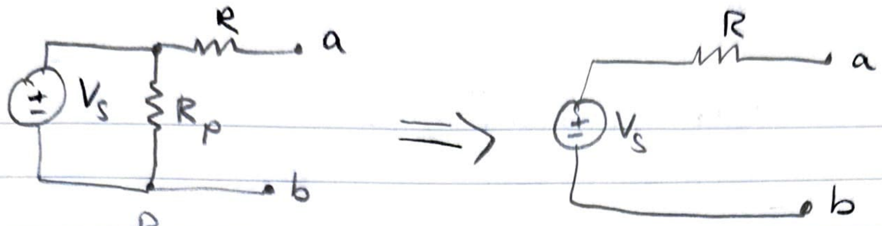
A resistor in series with a voltage source

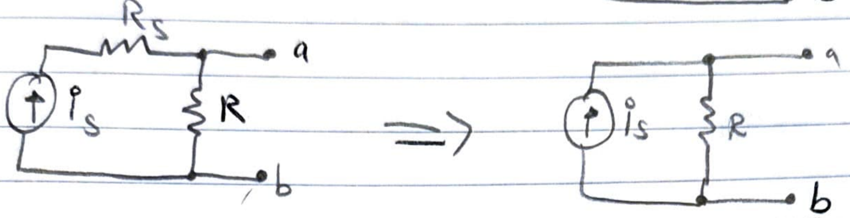
A resistor in // with a current source

☆ $i_s = \dfrac{V_s}{R}$

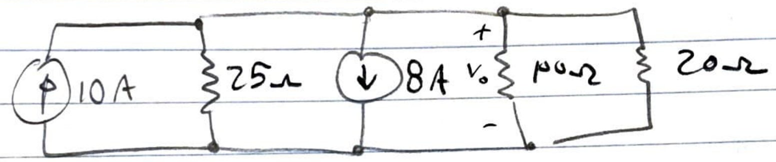
ex; Simplification process

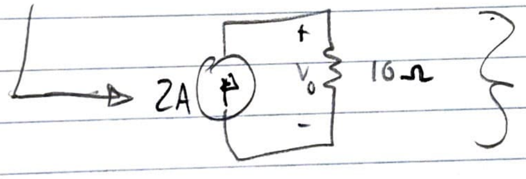






★ How is this possible?? ★

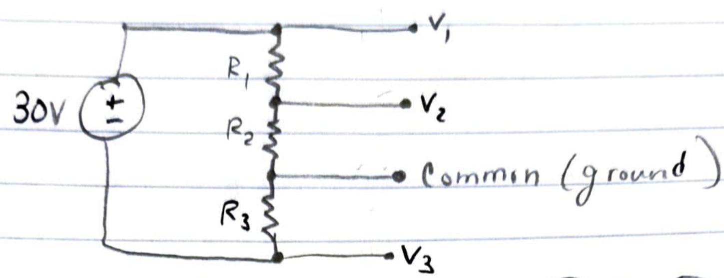




why 2A and not 18A?

Seth Ricks | ECEN 250 1/16/2024 | PS 2 : 3.18, 3.21, 3.62, 4.12, 4.21, 4.37

$\boxed{3.18}$



Part A

Select values R_1, R_2, R_3

→ total power when divider unloaded = 30 W

→ $V_1 = 15V$; $V_2 = 5V$; $V_3 = -15V$

$V_1 - \text{Common} = 15$ $\quad V_3 - \text{Common} = -15V$

$V_2 - \text{Common} = 5$

$V_{R1} = V_1 - V_2 = 10V \quad V_{R2} = V_2 = 5V \quad V_{R3} = |V_3| = 15V$

$P = 30W$; $V = 30V \rightarrow i = 1A$

$i_{R1} = i_{R2} = i_{R3} = 1A$

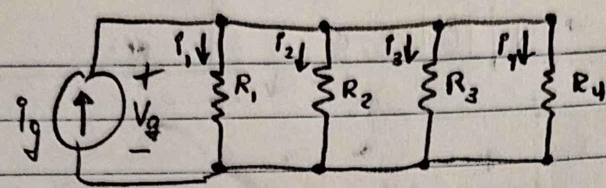
$$R_1 = \frac{V_{R1}}{i_{R1}} = \frac{10V}{1A} = 10\Omega$$

$$R_2 = \frac{V_{R2}}{i_{R2}} = \frac{5V}{1A} = 5\Omega$$

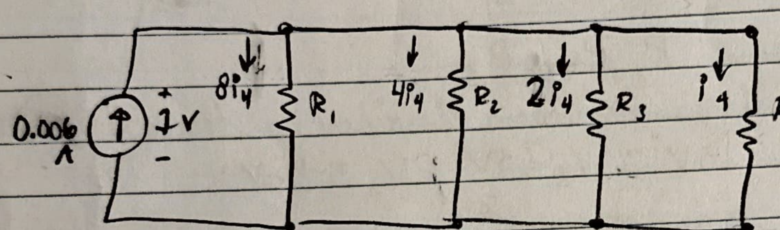
$$R_3 = \frac{V_{R3}}{i_{R3}} = \frac{15V}{1A} = 15\Omega$$

$\boxed{R_1 = 10\Omega \ ; \ R_2 = 5\Omega \ ; \ R_3 = 15\Omega}$

$\boxed{3.21}$



$i_g = 6mA$, $V_g = 1V$, $i_1 = 2i_2$, $i_2 = 2i_3$, $i_3 = 2i_4$



$i_4 = i_4$

$i_3 = 2i_4$

$i_2 = 2i_3 = 2(2i_4) = 4i_4$

$i_1 = 2i_2 = 2(2i_3) = 4i_3 = 4(2i_4) = 8i_4$

$8\cancel{i_4}(R_1) = 4\cancel{i_4}(R_2) = 2\cancel{i_4}(R_3) = \cancel{i_4}(R_4)$

$8R_1 = 4R_2 = 2R_3 = R_4$

$8i_4 + 4i_4 + 2i_4 + i_4 = 0.006\ A$

$15i_4 = 0.006A$

$i_4 = 0.4mA = 0.0004\ A$

$R_4 = V/I = \frac{1V}{0.0004A} = 2500\ \Omega = \boxed{2.50\ k\Omega}$

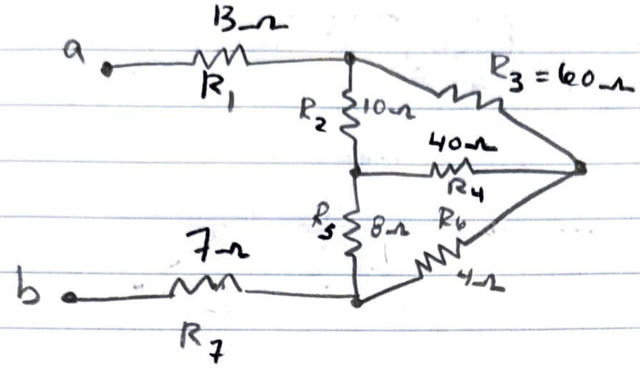
$2R_3 = R_4 = 2500\ \Omega$

$R_3 = 1250\ \Omega = \boxed{1.25\ k\Omega}$

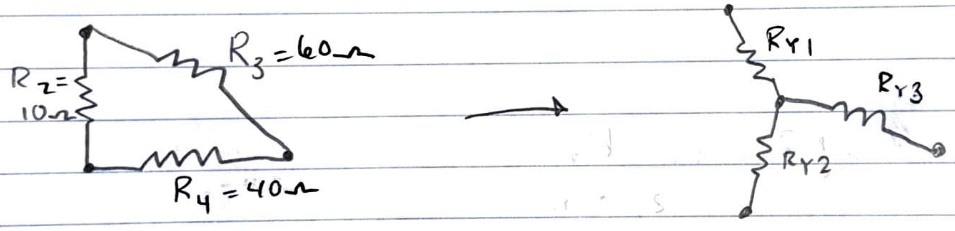
$4R_2 = 2500\ \Omega \rightarrow R_2 = 625\ \Omega = \boxed{0.625\ k\Omega}$

$8R_1 = 2500\ \Omega \rightarrow R_1 = 312.5\ \Omega = \boxed{0.313\ k\Omega}$

3.62



Part A Simplify R_2, R_3, and R_4 by using Δ-to-Y transformation

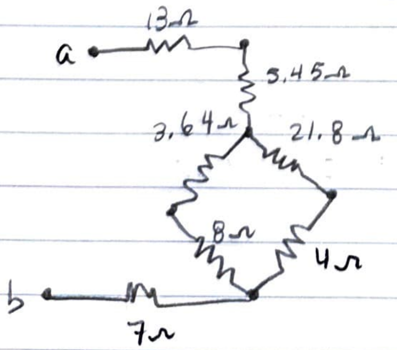


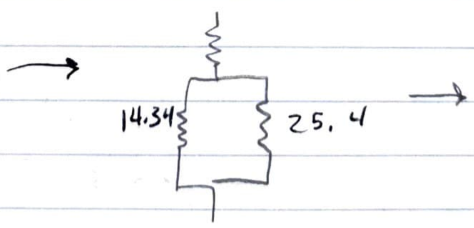
$$R_{Y1} = \frac{(R_2)(R_3)}{R_4 + R_3 + R_2} = \frac{(10)(60)}{40+60+10} = \frac{600}{110} = 5.45\,\Omega$$

$$R_{Y3} = \frac{(R_3)(R_4)}{R_4 + R_3 + R_2} = \frac{(60)(40)}{110} = \frac{2400}{110} = 21.8\,\Omega$$

$$R_{Y2} = \frac{(R_2)(R_4)}{R_4 + R_3 + R_2} = \frac{(10)(40)}{110} = 3.64\,\Omega$$

Part B Find equivalent resistance R_{ab}





$$\frac{(14.34)(25.4)}{14.34+25.4} = 9.22\,\Omega$$

$$R_{eq} = 13 + 5.45 + 9.22 + 7$$

$$= 34.67\,\Omega$$

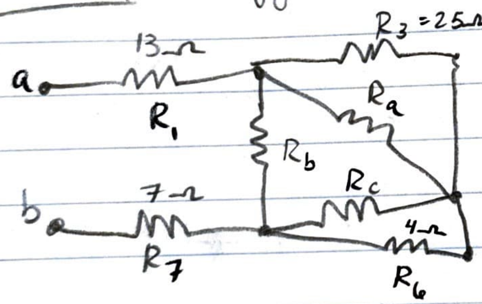
$$\approx \boxed{33.5\,\Omega}$$

Part C | Simplify by Y-to-Δ w/ R_2, R_4, R_5

$$R_a = \frac{R_2R_4 + R_4R_5 + R_5R_2}{R_5} = \frac{(10)(40)+(40)(8)+(8)(10)}{8} = \boxed{100\,\Omega}$$

$$R_b = \frac{800}{R_4} = \frac{800}{40} = \boxed{20\,\Omega} \qquad R_c = \frac{800}{R_2} = \frac{800}{10} = \boxed{80\,\Omega}$$

Part D | Suppose $R_3 = 25\,\Omega$. Find R_{ab}.



$$R_3 \,//\, R_a = \frac{(25)(100)}{125} = 20\,\Omega$$

$$+\; R_6 \,//\, R_c = \frac{(4)(80)}{84} = 3.81\,\Omega$$

$23.81\,\Omega$

$$\underbrace{23.81 \,//\, R_b}_{\downarrow} = \frac{(23.81)(20)}{43.81} = 10.87\,\Omega$$

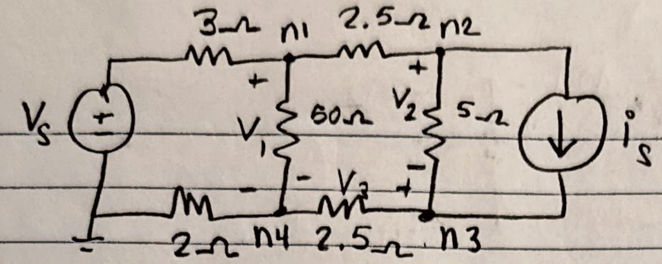
$$R_1 + \downarrow + R_7 = 30.87\,\Omega = \boxed{30.9\,\Omega}$$

Part E | Two Δ-to-Y or Y-to-Δ that can be used:

$$\boxed{\begin{array}{c} R_4 - R_5 - R_6 \\ R_3 - R_4 - R_6 \end{array}}$$

HP Prime

4.12



$V_s = 830$; $i_s = 12.0\,A$

Part A Use the node voltage method to find V_1

Node 1: $\frac{n1 - V_s}{3} + \frac{n1 - n4}{50} + \frac{n1 - n2}{2.5} = 0 \longrightarrow \frac{n1 - 830}{3} + \frac{n1 - n4}{50} + \frac{n1 - n2}{2.5} = 0$

$\left(\frac{1}{3} + \frac{1}{50} + \frac{1}{2.5}\right) - \frac{1}{2.5} + 0 - \frac{1}{50} = \frac{830}{3}$

Node 2: $\frac{n2 - n1}{2.5} + \frac{n2 - n3}{5} + i_s = 0 \longrightarrow \frac{n2 - n1}{2.5} + \frac{n2 - n3}{5} + 12.0 = 0$

$-\frac{1}{2.5} + \left(\frac{1}{2.5} + \frac{1}{5}\right) - \frac{1}{5} + 0 = -12$

Node 3: $\frac{n3 - n2}{5} + \frac{n3 - n4}{2.5} - i_s = 0 \longrightarrow \frac{n3 - n2}{5} + \frac{n3 - n4}{2.5} - 12 = 0$

$0 + \left(-\frac{1}{5}\right) + \left(\frac{1}{5} + \frac{1}{2.5}\right) - \frac{1}{2.5} = 12$

Node 4: $\frac{n4}{2} + \frac{n4 - n1}{50} + \frac{n4 - n3}{2.5} = 0 \longrightarrow \frac{n4}{2} + \frac{n4 - n1}{50} + \frac{n4 - n3}{2.5} = 0$

$-\frac{1}{50} + 0 - \frac{1}{2.5} + \left(\frac{1}{2} + \frac{1}{50} + \frac{1}{2.5}\right) = 0$

$n1 = 632 \quad n3 = 272$

$n2 = 492 \quad n4 = 132$

$V_1 = n_1 - n_4 = \boxed{500V}$

Part B Use the node voltage method to find V_2

$V_2 = (n2 - n3) + (n3 - n4)$

$= (492 - 272) + (272 - 132) = \boxed{360V}$

Part C Use the node voltage method to find V_3

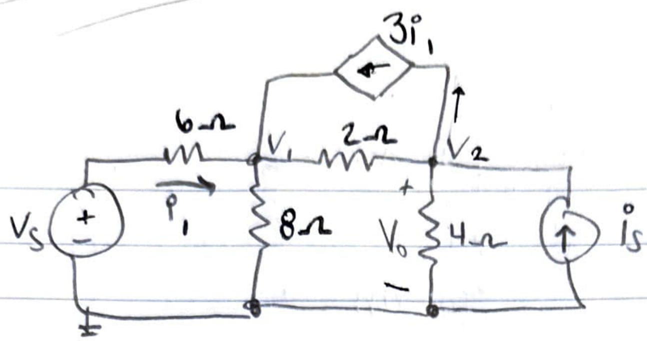
$V3 = n3 - n4 = 272 - 132 = \boxed{140V}$

Part D How much power does the 830V voltage source deliver to the circuit?

$i = \frac{V4}{2} = \frac{132}{2} = 66A$

$P = 830 \cdot 66 = 54780 = \boxed{54.8\,KW}$

4.21



$V_s = 78V$, $i_s = 6A$

Part A Use node-voltage method to find V_o

Node 1 : $\frac{V_1 - V_s}{6} - 3i_1 + \frac{V_1 - V_2}{2} + \frac{V_1}{8} = 0$

Node 2 : $\frac{V_2 - V_1}{2} + \frac{V_2}{4} - i_s + 3i_1 = 0$

$i_1 = \frac{V_s - V_1}{6}$

Equation solver → $V_1 = 48$ $i_1 = 5$
$V_2 = 20$
$\boxed{V_o = 20V}$

Part B Find power absorbed by dependent source

$V_1 - V_2 = V_D$
$48 - 20 = 28V$

$P = 28V \cdot 3(5)A = 420W$

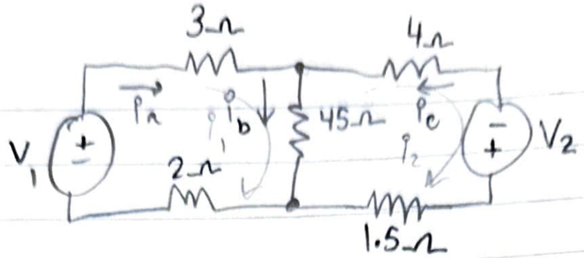
Part C Find the total power developed by the independent sources

$P_{cs} = V_o \cdot i_s = 20 \cdot 6 = 120W$

$P_{vs} = V_s \cdot i_1 = 78V \cdot 5A = 390W$

$120 + 390 = \boxed{510W}$

4.37



$V_1 = 55V$
$V_2 = 88V$

Part A Find the value of i_a

Mesh 1 : $-55 + 3i_1 + 45(i_1 - i_2) + 2i_1 = 0$

Mesh 2 : $-88 + 1.5i_2 + 45(i_2 - i_1) + 4i_2 = 0$

$-55 + 3i_1 + 45i_1 - 45i_2 + 2i_1 = 0$
$50i_1 - 45i_2 - 55 = 0$

$-88 + 1.5i_2 + 45i_2 - 45i_1 + 4i_2 = 0$
$-45i_1 + 50.5i_2 - 88 = 0$

* Equation Solver
$\rightarrow \boxed{i_1 = i_a = 13.5A}$

Part B Find the value of i_b

$i_a = 13.5A$ $\quad i_a < i_c$
$i_c = 13.75A$
$|i_a| + |i_b| = i_c$
$|i_b| = 0.275$
$\boxed{i_b = -0.275A}$

Part C Find the value of i_c

$$(50i_1 - 45i_2 - 55 = 0)\cdot 45$$
$$(-45i_1 + 50.5i_2 - 88 = 0)\cdot 50$$
$$2250i_1 - 2025i_2 - 2475 = 0$$
$$-2250i_1 + 2525i_2 - 4400 = 0$$
$$500i_2 - 6875 = 0$$
$$i_2 = 13.75$$
$$i_c = -i_2 = \boxed{-13.8A}$$

Part D Find i_a if V_2 is reversed

$$(50i_1 - 45i_2 - 55 = 0)\cdot 45$$
$$(-45i_1 + 50.2i_2 + 88 = 0)\cdot 50$$
$$2250i_1 - 2025i_2 - 2475 = 0$$
$$-2250i_1 + 2525i_2 + 4400 = 0$$
$$500i_2 + 1925 = 0$$
$$i_2 = -3.85$$
$$50i_1 - 45(-3.85) - 55 = 0$$
$$50i_1 + 118.25 = 0$$
$$i_1 = \boxed{-2.365 = i_a}$$

Part E Find i_c if V_2 is reversed

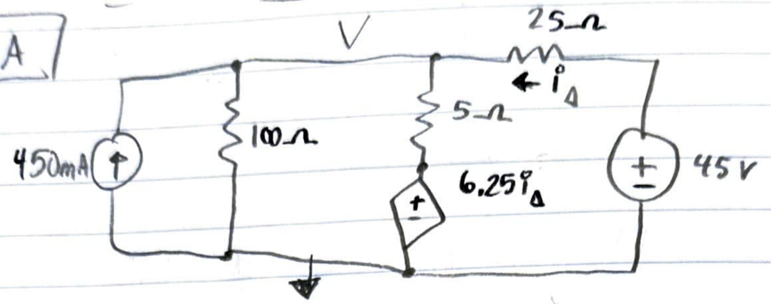
$$i_b = i_c - i_a$$
$$= 3.85 - 2.365 = \boxed{1.49A}$$

Part F Find i_c if V_2 is reversed

$$-i_2 = i_c = \boxed{3.85A}$$

Video Solution Problem

Part A



For the circuit analyzed in the video, what is the power dissipated by the 25Ω resistor?

$$-0.45 + \frac{V}{100} + \frac{V - 6.25 i_\Delta}{5} + \frac{V-45}{25} = 0$$

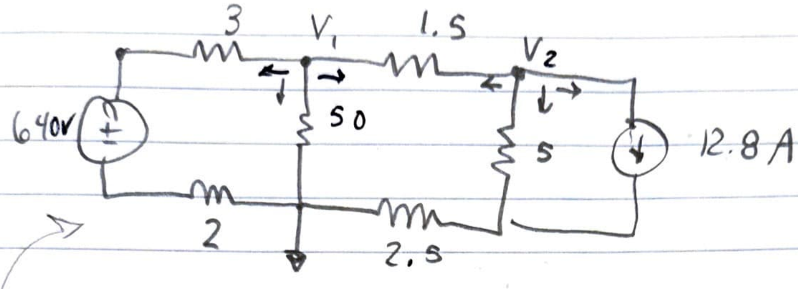
$$i_\Delta = \frac{45-V}{25}$$

$$-0.45 + \frac{V}{100} + \frac{V - 6.25\left(\frac{45-V}{25}\right)}{5} + \frac{V-45}{25} = 0$$

$$V = 15$$

$$P_R = V_R \cdot i_R = (45-15) \cdot \left(\frac{45-15}{25}\right) = 30 \cdot 1.2 = \boxed{36.0W}$$

4.12 → class

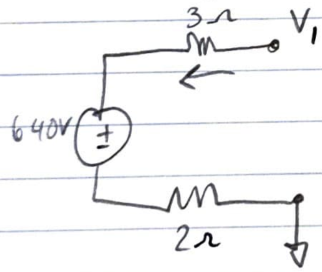


5 is the total resistance in the mesh

$$\frac{V_1 - 640}{5} + \frac{V_1}{50} + \frac{V_1 - V_2}{2.5} = 0$$

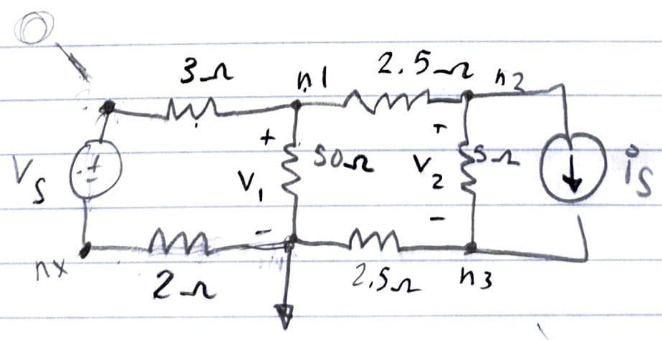
$$\frac{V_2 - V_1}{2.5} + \frac{V_2 - V_3}{5} + 12.8 = 0$$

Voltage across 3Ω?



??

$V_s = 830$
$i_s = 12.0A$



n1 = 632

n1: $\frac{n1 - 830}{5} + \frac{n1}{50} + \frac{n1 - n2}{2.5} = 0$

n2: $\frac{n2 - n1}{2.5} + \frac{n2 - n3}{5} + 12 = 0$

n3: $\frac{n3}{2.5} + \frac{n3 - n2}{5} - 12 = 0$

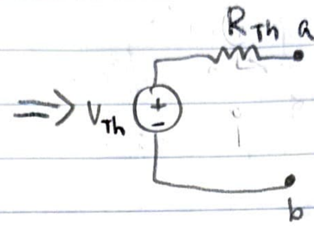
$V_{3\Omega} - V_s + V_{2\Omega} = 0$

$V_s = V_{3\Omega} + V_{2\Omega}$

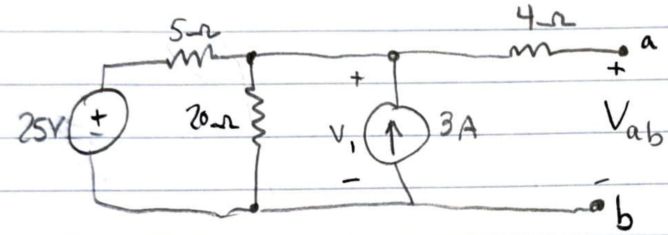
$632 = 2(V_{3\Omega} + V_{2\Omega})$

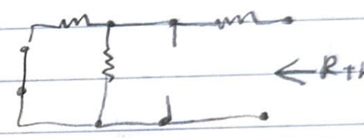
4.10 Thevenin and Norton Equivalents

A resistive network containing independent & dependent sources •a •b



ex;

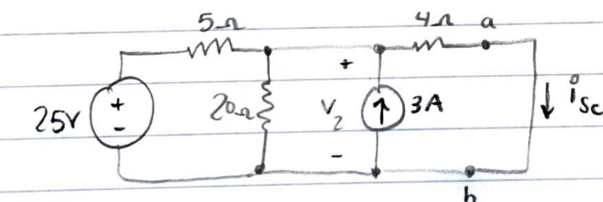




$$\frac{5 \cdot 20}{25} + 4 = 8\Omega$$

Step1 KCL: $\frac{V_1 - 25}{5} + \frac{V_1}{20} - 3 = 0 \longrightarrow V_1 = 32 = V_{ab} = V_{Th} = 32$

Step2 Place short circuit across ab



Find V_2

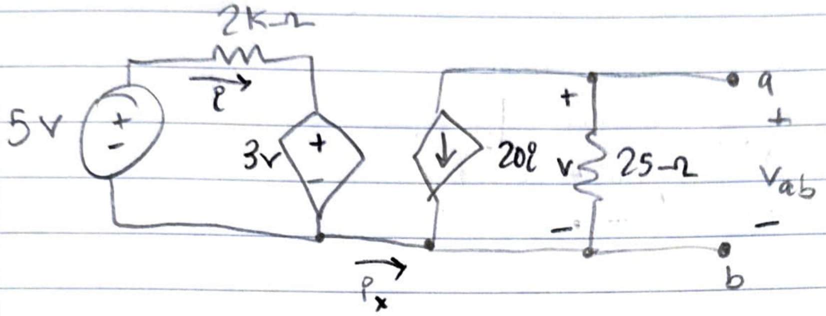
$$\frac{V_2 - 25}{5} + \frac{V_2}{20} - 3 + \frac{V_2}{4} = 0$$

$$V_2 = 16V$$

So $i_{sc} = \frac{16}{4} = 4A$

Step 3 Find $R_{Th} = \frac{V_{Th}}{i_{sc}} = \frac{32}{4} = 8\Omega$

Thevenin Equivalent w/ Dependent Source

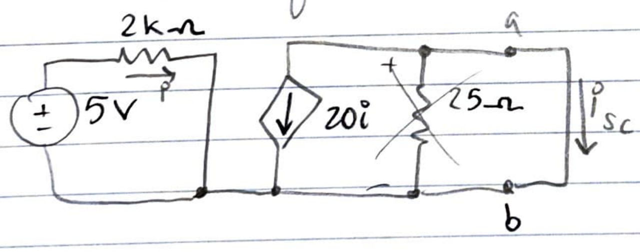


i_x must be zero bc there is no return path

$V_{Th} = V_{ab} = (-20i)(25) = -500i$ ($-20i$ bc current flows from + to -)

$i = \frac{5-3}{2000} = \frac{5-3V_{Th}}{2000}$

* Combine two equations → $V_{Th} = -5V$

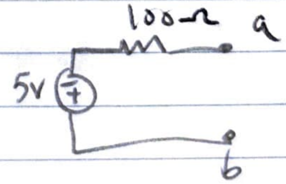


(take away dependent voltage source bc v=0)

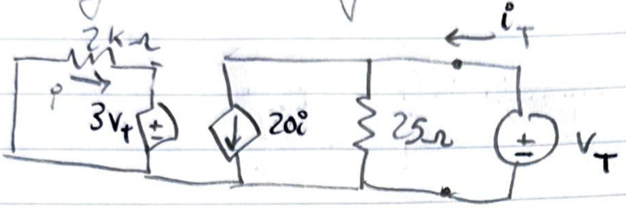
$i_{sc} = -20i$

$i = \frac{5}{2000} = 2.5mA$ $i_{sc} = -50mA$

$R_{Th} = \frac{V_{Th}}{i_{sc}} = \frac{-5}{-0.05} = 100\Omega$ →



Finding R_{TH} using a test source

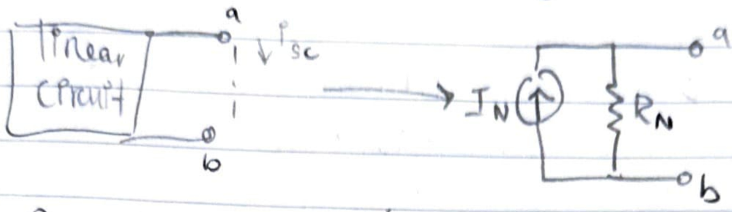


$$i_T = \frac{V_T}{25} + 20i \qquad i = \frac{-3V_T}{2000}$$

$$i_T = \frac{V_T}{25} + 20\left(\frac{-3V_T}{2000}\right)$$

$$\frac{V_T}{i_T} = 100\Omega = R_{TH}$$

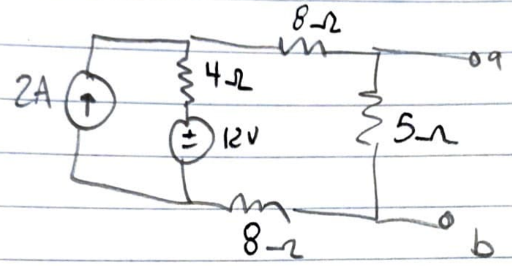
Norton's Theorem

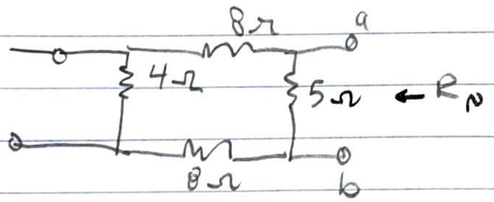


$R_N = R_{TH}$ $\qquad I_N = \frac{V_{TH}}{R_{TH}}$ $\qquad$ If you short ab, $i_{sc} = I_N$

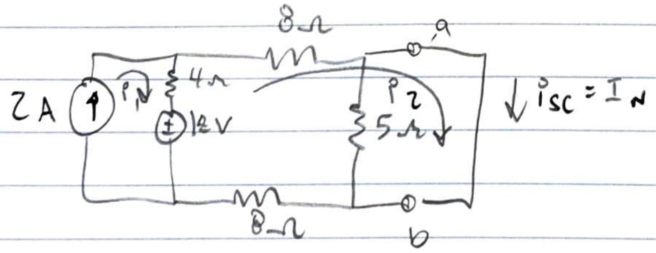
$V_{oc} = V_{TH}$ $\qquad R_{TH} = \frac{V_{oc}}{i_{sc}} = R_N$

ex;





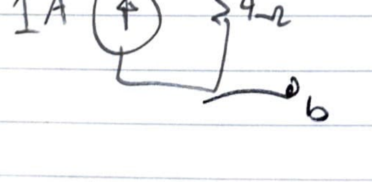
$R_N = 4\Omega$



$\downarrow i_{sc} = I_N$

$i_1 = 2A$ $\qquad 20i_2 - 4i_1 - 12 = 0$

$i_2 = 1A = i_{sc} = I_N$



Summary:

Thevenin:

w/ no dep source → turn off indep. Sources
find R_{TH}
find V_{TH} (Nodal Analysis)

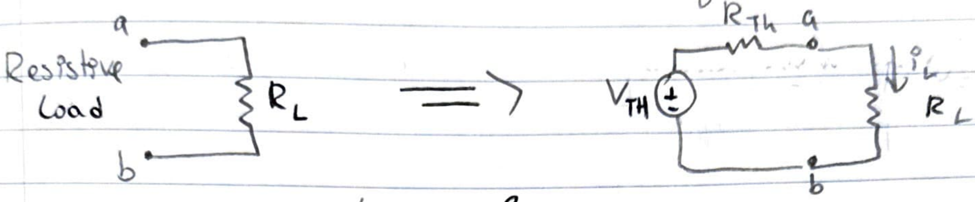
w/ dep source → turn off indep. Sources
add simple voltage to ab
find R_{TH}
find V_{TH}

Norton:

w/ no dep source → turn off indep. Sources
find R_N
Short circuit
find I_N (current Analysis)
I_{sc}

w/ dep → "
add simple voltage sourse
find R_N
Short circuit
find $I_N = I_{sc}$

4.12 Maximum Power Transfer

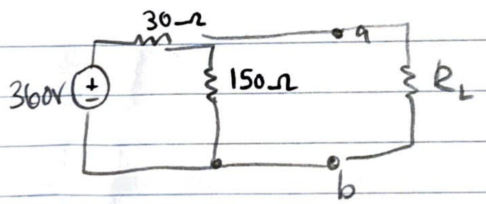


$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 R_L$$

Maximum Power → $R_L = R_{Th}$

$$P_{max} = \frac{V_{Th}^2}{4R_L}$$

ex:



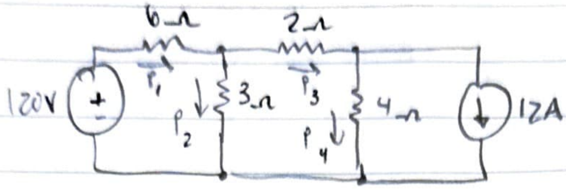
$$R_{Th} = \frac{(30)(150)}{180} = 25\,\Omega$$

$$V_{Th} = V_{150\Omega} = 360\left(\frac{150}{180}\right) = 300\,V$$

$$P_{max} = \frac{300^2}{100} = 900\ W$$

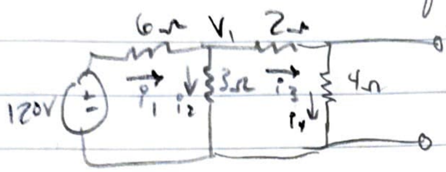
4.13 Superposition

ex;



Find branch currents

* Take out 12A source first

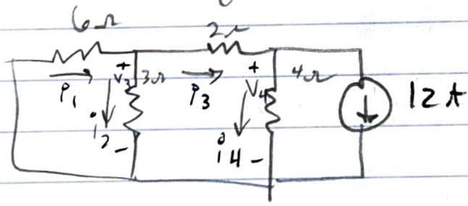


$$\frac{V_1-120}{6}+\frac{V_1}{3}+\frac{V_1}{2+4}=0$$

$$V_1=30V$$

$$i_1=\frac{120-30}{6}=15A$$
$$i_2=\frac{30}{3}=10A$$
$$i_3=i_4=30/6=5A$$

* Take out voltage source



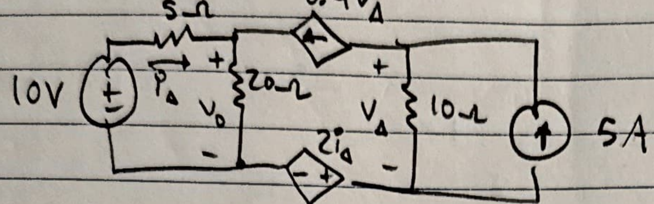
$$\left.\begin{array}{l}\dfrac{V_3}{3}+\dfrac{V_3}{6}+\dfrac{V_3-V_4}{2}=0\\[2ex]\dfrac{V_4-V_3}{2}+V_4/4+12=0\end{array}\right\}\rightarrow \begin{array}{l}V_3=-12V\\V_4=-24V\end{array}$$

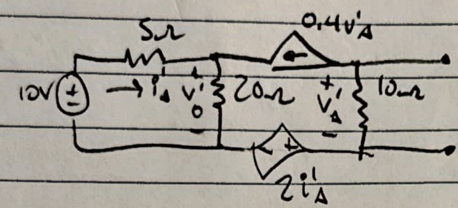
$$i_1=-V_3/6=12/6=2A$$
$$i_2=V_3/3=-4A$$
$$i_3=\frac{-12+24}{2}=6A$$
$$i_4=V_4/4=-6A$$

* add together $i_1=17A$ $i_2=6A$
$i_3=11A$ $i_4=-1A$

Superposition w/ dependent source

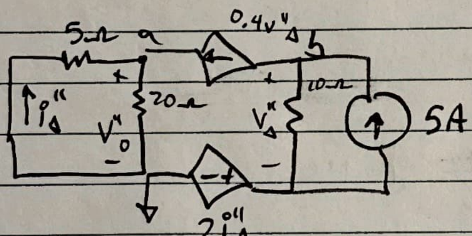
Find V_o





$0.4V'_\Delta = (0.4V'_\Delta)(10) \longrightarrow V'_\Delta$ must be zero

so, $V'_o = \frac{20}{25}10 = 8V$



node a: $\frac{V''_o}{20} + \frac{V''_o}{5} - 0.4V''_\Delta = 0 \longrightarrow 5V''_o - 8V''_\Delta = 0$

node b: $0.4V''_\Delta + \frac{V_b - 2i''_\Delta}{10} - 5 = 0 \longrightarrow 4V''_\Delta + V_b - 2i''_\Delta = 50$

also: $V_b = V''_\Delta + 2i''_\Delta$

$5V''_\Delta = 50 \longrightarrow V''_\Delta = 10$

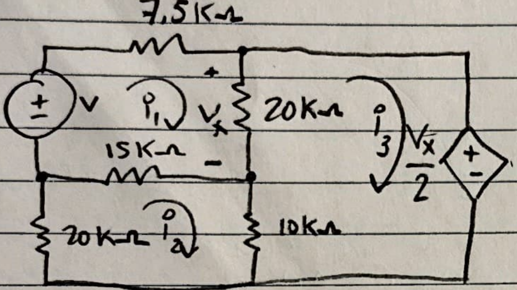
$5V''_o = 80 \longrightarrow V''_o = 16V$

$V_o = V'_o + V''_o = 8 + 16 = \underline{24V}$

Seth Ricks | ECEN 250 1/23/2024 | HW #3 1/10

4.40 Use the mesh-current method to find the power delivered by the 280 V source in the circuit.

Suppose V = 280 V



Mesh 1: $-V + 7.5i_1 + 20(i_1 - i_3) + 15(i_1 - i_2) = 0$

$-V + 7.5i_1 + 20i_1 - 20i_3 + 15i_1 - 15i_2 = 0$

$\underbrace{(7.5 + 20 + 15)}_{42.5} i_1 - 15i_2 - 20i_3 = V = 280$

Mesh 2: $15(i_2 - i_1) + 10(i_2 - i_3) + 20i_2 = 0$

$15i_2 - 15i_1 + 10i_2 - 10i_3 + 20i_2 = 0$

$-15i_1 + \underbrace{(15 + 10 + 20)}_{45} i_2 - 10i_3 = 0$

Mesh 3: $\frac{V_x}{2} + 10(i_3 - i_2) + 20(i_3 - i_1) = 0$

$V_x = 20(i_1 - i_3)$ $\quad 10(i_1 - i_3) + 10(i_3 - i_2) + 20(i_3 - i_1) = 0$

$i_1 = 0.014$

$i_2 = 0.007$

$i_3 = 0.0105$

$P_V = 0.014 \cdot 280 = \boxed{3.92\ W}$

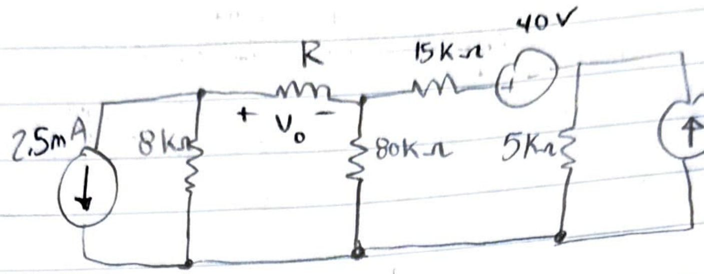
$10i_1 - 10i_3 + 10i_3 - 10i_2 + 20i_3 - 20i_1 = 0$

$-10i_1 - 10i_2 + 20i_3 = 0$

2/10

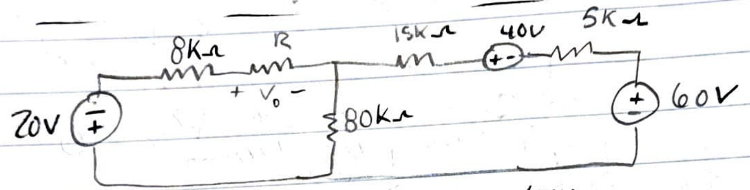
$\boxed{4.62}$

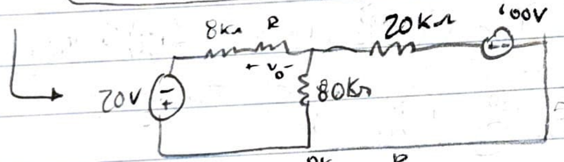
$\boxed{\text{Part A}}$

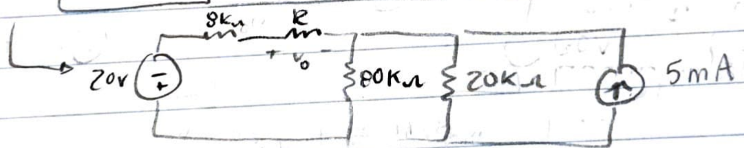


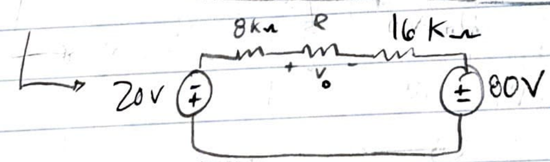
$R = 12\ K\Omega$ $i_s = \frac{V_s}{R}$ $V_s = i_s R$

Make a series of transformations to find the voltage V_o.

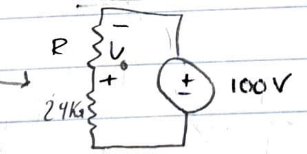








$V_s = 5mA \cdot 16K\Omega =$



$V_o = 100\left(\frac{12}{12K\Omega + 24K\Omega}\right)$

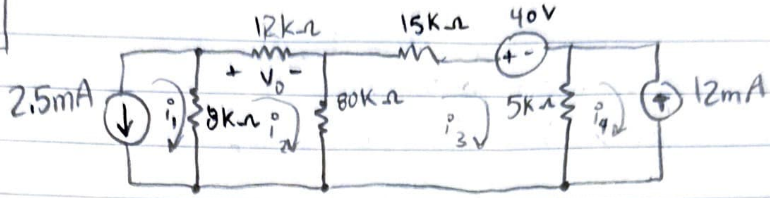
$\boxed{V_o = -33.3\ V}$

5:30

243

3/10

Part B



Find the voltage V_0 in the circuit using the mesh-current method.

$i_1 = -2.5mA$

$i_4 = -12mA$

$8(i_2 - i_1) + 12(i_2) + 80(i_2 - i_3) = 0$

$8i_2 - 8i_1 + 12i_2 + 80i_2 - 80i_3 = 0$

$-8i_1 + 100i_2 - 80i_3 = 0$

$20 + 100i_2 - 80i_3 = 0$

$100i_2 - 80i_3 = -20$

$15i_3 + 40 + 5(i_3 - i_4) + 80(i_3 - i_2) = 0$

$15i_3 + 40 + 5i_3 - 5i_4 + 80i_3 - 80i_2 = 0$

$-80i_2 + 100i_3 - 5i_4 = -40$

$-80i_2 + 100i_3 + 60 = -40$

$-80i_2 + 100i_3 = -100$

$i_2 = 0.002777$

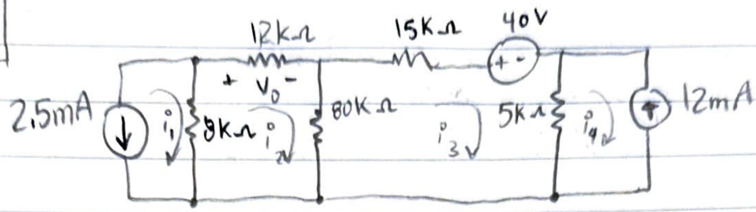
$V_0 = 12 \times 10^3 \cdot i_2 = \boxed{33.3V}$

5:30

243

3/10

Part B



Find the voltage V_o in the circuit using the mesh-current method.

$i_1 = -2.5mA$

$i_4 = -12mA$

$8(i_2 - i_1) + 12(i_2) + 80(i_2 - i_3) = 0$

$8i_2 - 8i_1 + 12i_2 + 80i_2 - 80i_3 = 0$

$-8i_1 + 100i_2 - 80i_3 = 0$

$20 + 100i_2 - 80i_3 = 0$

$100i_2 - 80i_3 = -20$

$15i_3 + 40 + 5(i_3 - i_4) + 80(i_3 - i_2) = 0$

$15i_3 + 40 + 5i_3 - 5i_4 + 80i_3 - 80i_2 = 0$

$-80i_2 + 100i_3 - 5i_4 = -40$

$-80i_2 + 100i_3 + 60 = -40$

$-80i_2 + 100i_3 = -100$

$i_2 = 0.002777$

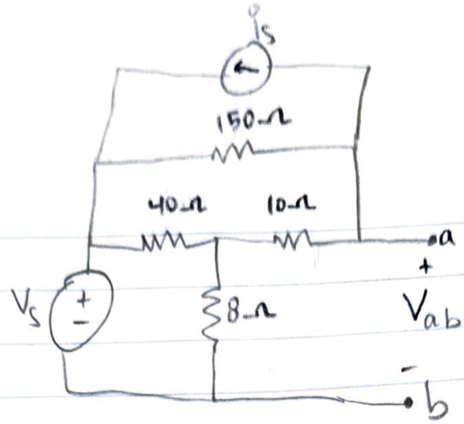
$V_o = 12 \times 10^3 \cdot i_2 = \boxed{33.3V}$

4.68

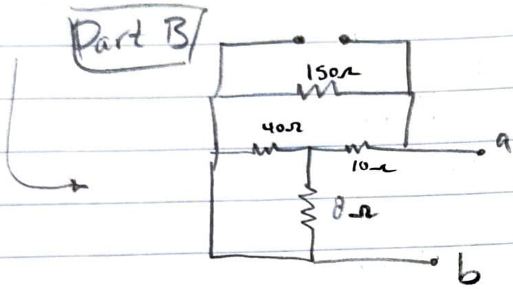
$V_s = 380V$

$i_s = 2A$

4/6



Part B

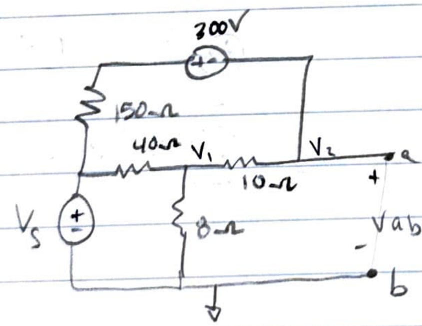


$$\frac{40 \cdot 8}{48} = \frac{20}{3} \longrightarrow \frac{20}{3} + 10 = \frac{50}{3}$$

$$\frac{\frac{50}{3} \cdot 150}{\frac{50}{3} + 150} = 15\Omega$$

$$\boxed{R_{Th} = 15\Omega}$$

Part A



$V_s = i_s R$

$V_s = 2 \cdot 150 = 300V$

$$\frac{V_1 - 380}{40} + \frac{V_1}{8} + \frac{V_1 - V_2}{10} = 0$$

$$\frac{V_2 - 380 + 300}{150} = 0$$

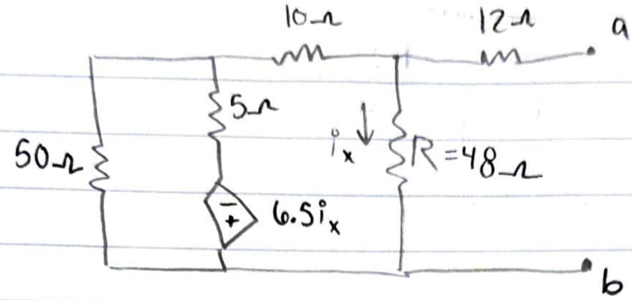
$$\frac{V_2 - V_1}{10} + \frac{V_2 - 80}{150} = 0$$

$V_1 = 64V$

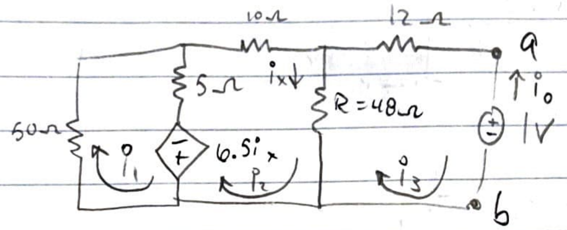
$\boxed{V_2 = 65.0V}$

5/10

4.81



Part B — Find Thevenin Resistance



Mesh 1: $-6.5i_x + 50i_1 + 5(i_1 - i_2) = 0$ $\qquad i_x = i_2 - i_3$

$-6.5(i_2 - i_3) + 50i_1 + 5(i_1 - i_2) = 0$

$-6.5i_2 + 6.5i_3 + 50i_1 + 5i_1 - 5i_2 = 0 \rightarrow 55i_1 - 11.5i_2 + 6.5i_3 = 0$

Mesh 2: $6.5i_x + 5(i_2 - i_1) + 10i_2 + 48(i_2 - i_3) = 0$

$6.5(i_2 - i_3) + 5(i_2 - i_1) + 10i_2 + 48(i_2 - i_1) = 0$

$6.5i_2 - 6.5i_3 + 5i_2 - 5i_1 + 10i_2 + 48i_2 - 48i_1 = 0 \rightarrow -53i_1 + 69.5i_2 - 6.5i_3 = 0$

Mesh 3: $48(i_3 - i_2) + 12i_3 + 1 = 0$

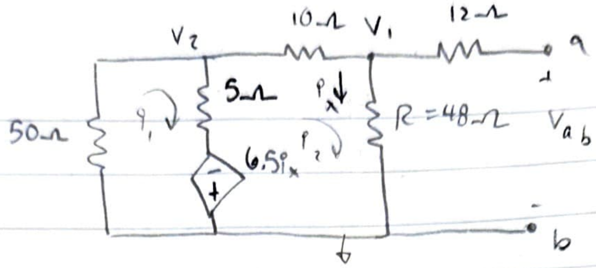
$48i_3 - 48i_2 + 12i_3 + 1 = 0 \rightarrow -48i_2 + 60i_3 = -1$

$i_3 = -0.045\text{ A}$ $\qquad R_{TH} = 1/i_o = \boxed{22.2\,\Omega}$

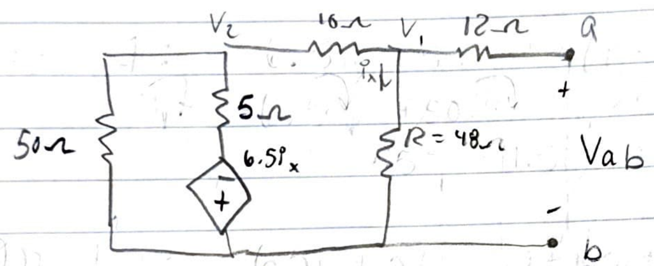
$i_o = 0.045\text{ A}$

6/10

4.81



Part A Find Thevenin Voltage



$$\frac{V_2}{50} + \frac{V_2 + 6.5 i_x}{5} + \frac{V_2 - V_1}{10} = 0$$

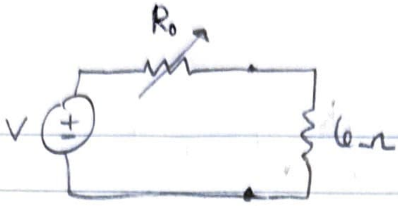
$$\frac{V_1 - V_2}{10} + \frac{V_1}{48} = 0$$

$V_1 = 0$; $V_2 = 0$

$V_{ab} = V_1 = \boxed{0 \text{ Volts}}$

7/10

4.84



$V = 29V$

Part A: Find the value of the variable resistor R_o that will result in maximum power dissipation in the 6Ω resistor

Max power $\rightarrow$ $\boxed{R_o = 0\,\Omega}$

Part B: What is the maximum power that can be delivered to the 6Ω resistor?

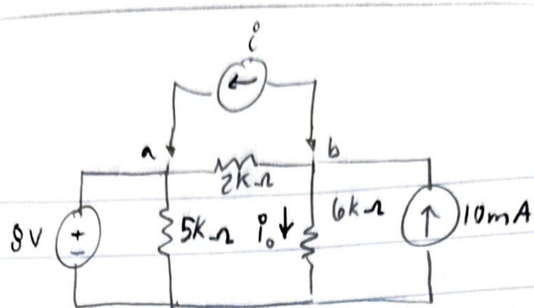
$P_{max} = \frac{V_{Th}^2}{4R_L}$ $\quad$ $V_{Th} = 29\left(\frac{6}{12}\right) = 14.5$

$i = V/R = 29/6$ $\quad$ $P = Vi = \frac{29}{6} \cdot 29 = \boxed{140\ W}$

★ I don't understand why we learned this if it's not applicable

8/10

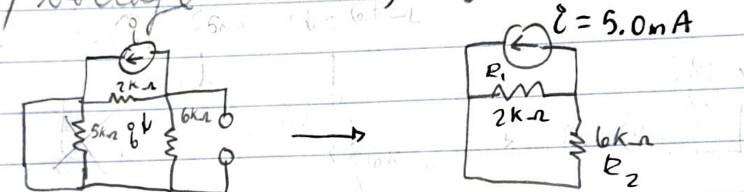
4.96



Before the current source i is attached, the current i_o is calculated to be 3.5 mA. Take i = 5.0mA.

Part A Use superposition to find the value of i_o after the current source is attached.

w/ voltage & 10mA CS, $i_o = 3.5mA$



$i_{R2} = i_{in}\left(\frac{R_1}{R_1+R_2}\right) = 5.0mA\left(\frac{2000}{2000+6000}\right) = 0.00125$
$= 1.25\,mA$

$i_o = -1.25\,mA$

$i_T = 3.5mA - 1.25mA$
$= \boxed{2.25mA}$

Part B Find i_o when all sources are on

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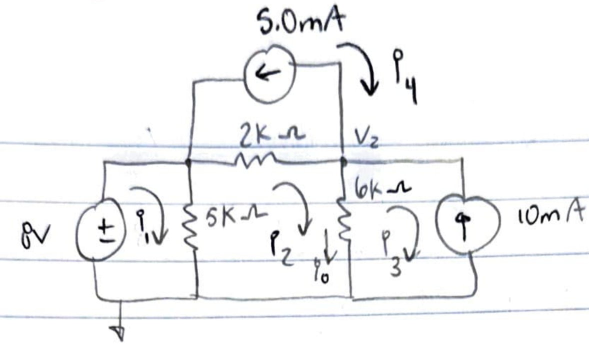

$$\frac{V_2}{6000} + \frac{V_2 - 8}{2000} - 0.01 + 0.005 = 0$$

$$\left(\frac{1}{6000} + \frac{1}{2000}\right) V_2 - \frac{8}{2000} - 0.01 + 0.005 = 0$$

$$\left(\frac{1}{6000} + \frac{1}{2000}\right) V_2 = 8/2000 + 0.01 - 0.005$$

$$= 0.009$$

$$V_2 = 13.5\,V$$

$$i_o = 13.5\,V / 6000\,\Omega = \boxed{2.25\,mA}$$

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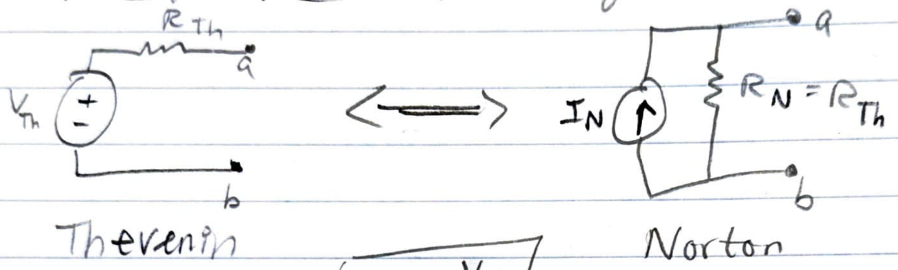
Video Solution Problem: Finding a Norton Equivalent

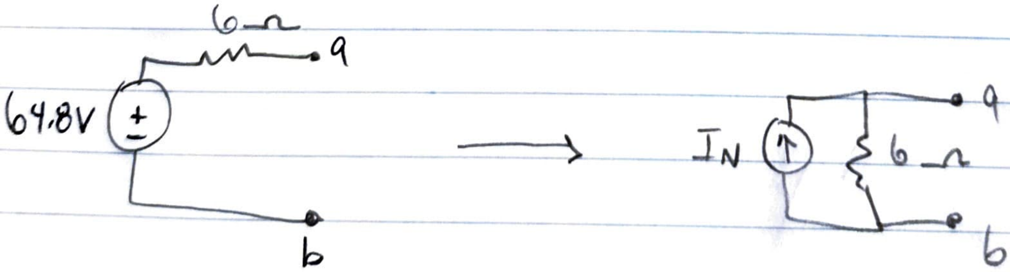
Once the Thevenin equivalent circuit is found, which formula is used to find the Norton equivalent source?



$$\boxed{I_N = \frac{V_{Th}}{R_{Th}}}$$

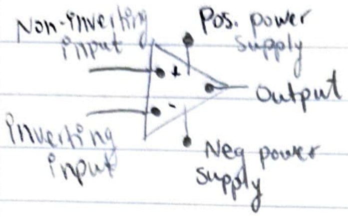
Video Solution Problem: Finding a Thevenin Equivalent

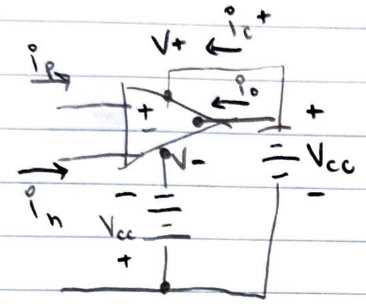
What is current source value for the Norton equivalent of the circuit in this Assessment Problem?



$$I_N = \frac{V_{Th}}{R_{Th}} = \frac{64.8V}{6\Omega} = \boxed{10.8A}$$

Chapter 5: Operational Amps





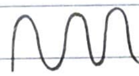
OP amp rules:

$V_p = V_n$

$I_p = i_n = 0$

Analyzing an Ideal op amp

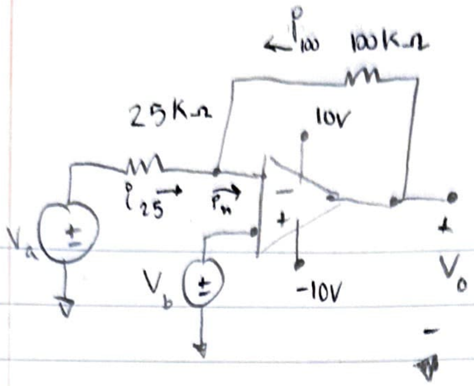
1. Check for a negative feedback path
2. Write KCL equation at inverting input terminal
3. Solve KCL equation and use solution to find op amp's output voltage
4. Compare the op amp's output voltage to the power supply voltage

if input is  but V_{cc} isn't high enough → 

"saturated"

★ Positive terminal is the leader, negative is the follower ★

ex;



$V_a = 1V$; $V_b = 0V$

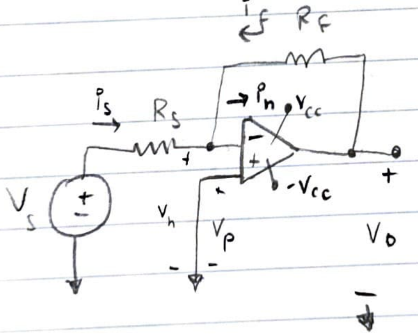
$$i_{25} + i_{100} - i_n = 0$$

$$i_{25} = \frac{V_a - V_n}{25{,}000} = \frac{1-0}{25{,}000} = \frac{1}{25{,}000}$$

$$i_{100} = \frac{V_o}{100{,}000}$$

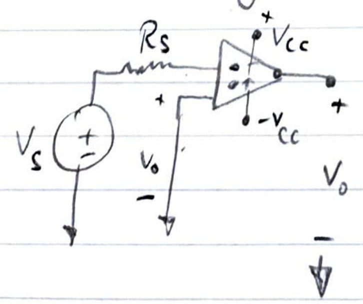
$$\frac{1}{25{,}000} + \frac{V_o}{100{,}000} = 0 \qquad V_o = -4V$$

$\boxed{5.3}$ The Inverting-Amp Circuit



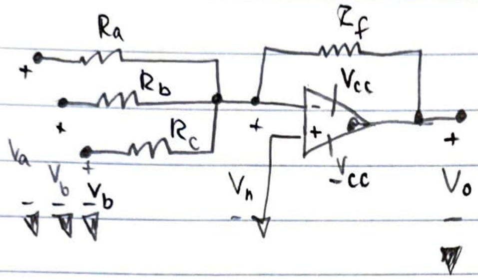
$$V_o = \frac{-R_f}{R_s} V_s \qquad A_v = \frac{-R_F}{R_S}$$

Open-loop gain → = ∞



$$V_o = -AV_n$$

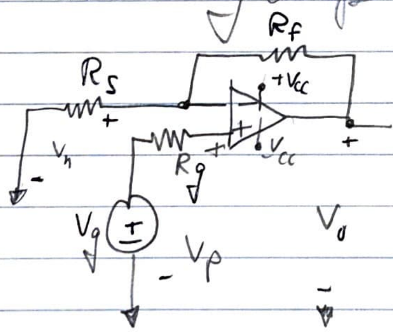
5.4 The summing-amplifier circuit



$$V_o = -\left(\frac{R_f}{R_a}V_a + \frac{R_f}{R_b}V_b + \frac{R_f}{R_c}V_s\right)$$

The output ~~is~~ an inverted, scaled sum of the inputs

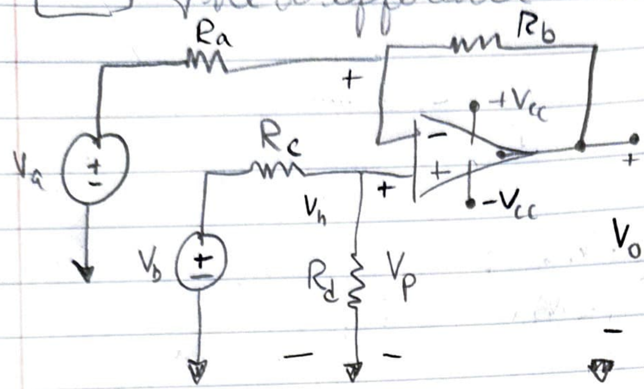
5.5 Non-inverting Amplifier circuit



$$V_o = \frac{R_s + R_f}{R_s} V_g$$

* If $R_s \to 0$, $A = 1$

5.6 The Difference Amplifier Circuit



★ Takes the difference times the gain $= V_o$ ★

$$V_o = \frac{R_d(R_a+R_b)}{R_a(R_c+R_d)} V_b - \frac{R_b}{R_a} V_a$$

ideal

★ If $\frac{R_a}{R_b} = \frac{R_c}{R_d}$ ★ $\longrightarrow V_o = \frac{R_b}{R_a}(V_b - V_a)$

Let's look at this circuit with respect to two other voltages

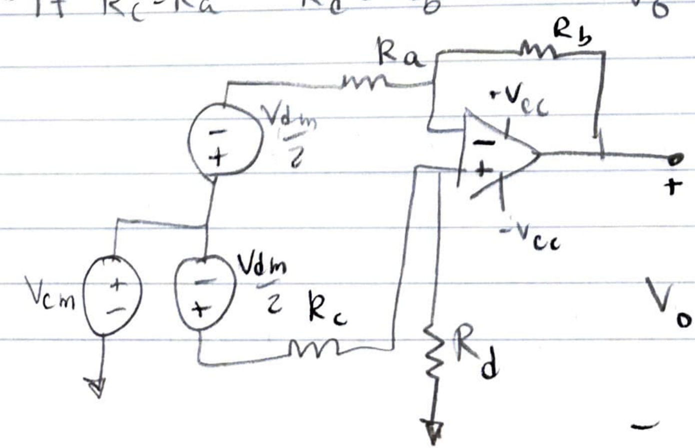
$V_{dm} = V_b - V_a \rightarrow$ differential mode input

$V_{cm} = (V_a + V_b)/2 \rightarrow$ common mode input

$$V_o = \left[\frac{R_aR_d - R_bR_c}{R_a(R_c+R_d)}\right] V_{cm} + \left[\frac{R_d(R_a+R_b)+R_b(R_c+R_d)}{2R_a(R_c+R_d)}\right] V_{dm}$$

$$= A_{cm}V_{cm} + A_{dm}V_{dm}$$

★ if $R_c = R_a$ & $R_d = R_b \longrightarrow V_o = 0V_{cm} + \left(\frac{R_b}{R_a}\right)V_{dm}$



[redacted]

Common mode rejection ratio (CMRR)

$$CMRR = \left|\frac{A_{dm}}{A_{cm}}\right|$$, larger CMRR → larger to ideal

ex; Design difference amplifier that has a gain of 8, and ±8V power supplies, using ideal op amp

$$V_o = \frac{R_b}{R_a}(V_b - V_a) = 8(V_b - V_a) \quad \text{so} \quad \frac{R_b}{R_a} = 8$$

$\boxed{\text{example Solution: } R_c = R_a = 1.5k\Omega \ ; \ R_d = R_b = 12k\Omega}$

If $V_a = 1V$, what range for V_b will allow to remain in its linear operating region?

$$V_o = \frac{R_b}{R_a}(V_b - V_a) \rightarrow \frac{V_o}{8} + 1 = V_b \rightarrow$$

max ++

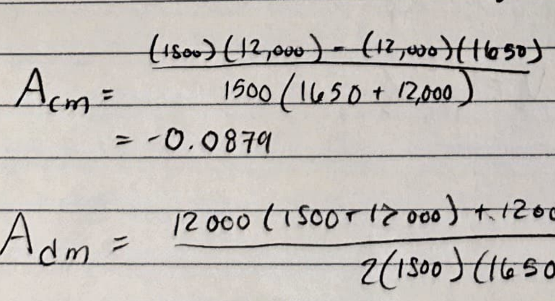
$$\frac{8}{8} + 1 = 2V$$

$$-\frac{8}{8} + 1 = 0V$$

min --

$\boxed{0V \leq V_b \leq 2V}$

ex,



$$A_{cm} = \frac{(1500)(12{,}000) - (12{,}000)(1650)}{1500(1650 + 12{,}000)}$$

$$= -0.0879$$

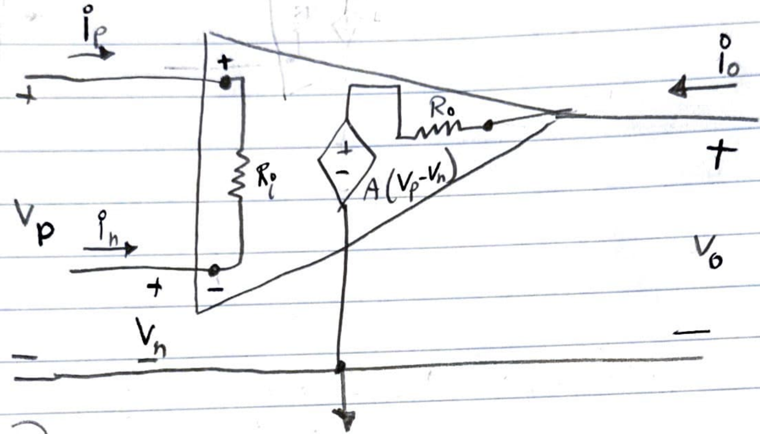
$$A_{dm} = \frac{12000(1500 + 12000) + 12000(1650 + 12000)}{2(1500)(1650 + 12000)}$$

$$= 7.956$$

$$CMRR = \left|\frac{7.956}{-0.0879}\right| = 90.5$$

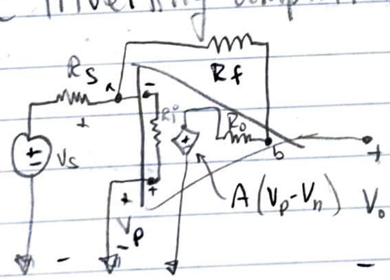
yayyy

5.7 Realistic Model for the Op Amp



$$\left.\begin{array}{l} V_n = V_p \\ i_n = i_p = 0 \end{array}\right\} \text{invalid assumptions}$$

Realistic inverting amplifier:



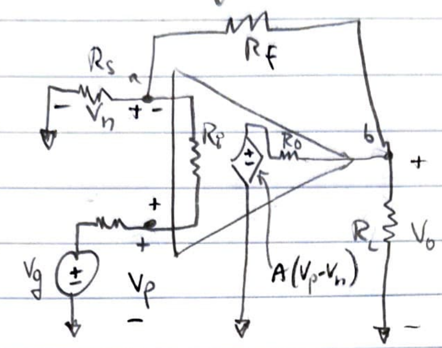
$$V_o = \frac{-A + (R_o/R_f)}{\frac{R_s}{R_f}\left(1 + A + \frac{R_o}{R_i}\right) + \left(\frac{R_s}{R_i} + 1\right) + \frac{R_o}{R_f}} V_s$$

$$\text{as} \left.\begin{array}{l} R_o \to 0 \\ R_i \to \infty \\ A \to \infty \end{array}\right\} \longrightarrow V_o = \frac{-R_f}{R_s} V_s$$

*If R_L Load at V_o, then

$$V_o = \frac{-A + (R_o/R_f)}{\frac{R_s}{R_f}\left(1 + A + \frac{R_o}{R_i} + \frac{R_o}{R_L}\right) + \left(1 + \frac{R_o}{R_L}\right)\left(1 + \frac{R_s}{R_i}\right) + \frac{R_o}{R_f}} V_s$$

Realistic non-inverting amplifier



$$V_o = \frac{\left[(R_f + R_s) + (R_s R_o / A R_i)\right] V_g}{R_s + \frac{R_o}{A}(1 + K_r) + \frac{R_f R_s + (R_f + R_s)(R_i + R_g)}{A R_i}}$$

where $K_r = \frac{R_s + R_g}{R_i} + \frac{R_f + R_s}{R_L} + \frac{R_f R_s + R_f R_g + R_g R_s}{R_i R_L}$

★ reduces to $V_o = \frac{R_s + R_f}{R_s} V_g$ when → $\begin{cases} R_o \to 0 \\ A \to \infty \\ R_i \to \infty \end{cases}$

★ when no R_L →

$$V_o = \frac{\left[(R_f + R_s) + R_s R_o / A R_i\right] V_g}{R_s + \frac{R_o}{A}\left(1 + \frac{R_s + R_g}{R_i}\right) + \frac{1}{A R_i}\left[R_f R_s + (R_f + R_s)(R_i + R_g)\right]}$$

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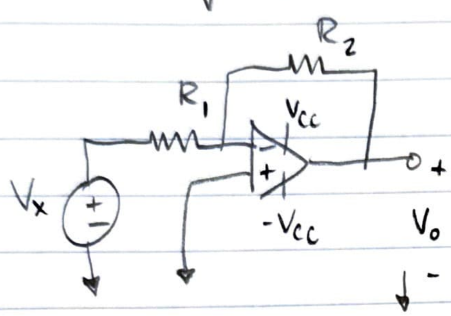
Inverting OpAmpt Circuits



Part A Determine V_o when $R_1 = 7.4\,k\Omega$; $R_2 = 6.8\,k\Omega$; $V_x = 320\,\mu V$; $V_{cc} = 10V$

$$i_{R1} = \frac{V_x}{R_1} \qquad i_{R2} = \frac{V_o}{R_2}$$

$$\frac{V_x}{R_1} + \frac{V_o}{R_2} = 0$$

$$V_o = -\frac{V_x}{R_1} \cdot R_2 = \frac{-320 \times 10^{-6}}{7.4 \times 10^{3}} \cdot 6.8 \times 10^{3} = \boxed{-294\,\mu V}$$

Part B Determine R_2 such that $V_o = -m \times V_x$. Assume $m = 69$, $R_1 = 4.6\,k\Omega$, $V_{cc} = 15V$ and that the opamp is in its linear region of operation.

$$V_o = -\frac{R_2}{R_1} V_x = -mV_x = -69V_x$$

$$-69 = -\frac{R_2}{R_1} \qquad R_2 = 69R_1 = 317{,}400\,V = \boxed{317.4\,k\Omega}$$

Part C What positive value of V_x will lead to saturation of the Opamp?

$$V_o = -\frac{R_2}{R_1} V_x = 69V_x$$

$$15V = 69V_x$$

$$\boxed{V_x = 0.217\,V}$$

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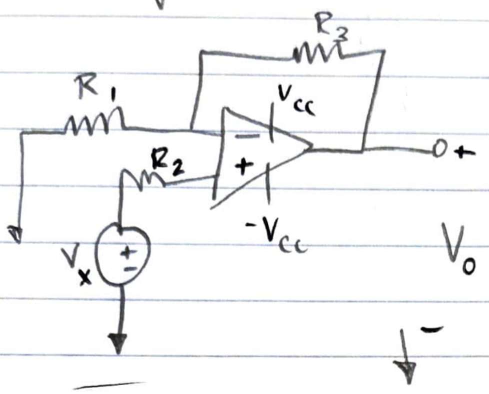
Non-inverting Op-Amp Circuit



Part A Determine V_o when $R_1 = 4.6\,k\Omega$, $R_2 = 1.8\,k\Omega$, $R_3 = 60\,k\Omega$, $V_x = 90\,mV$, $V_{cc} = 10V$

$$V_o = \frac{R_1 + R_3}{R_1} V_x = \frac{4.6\times10^3 + 60\times10^3}{4.6\times10^3} \cdot 90\times10^{-3} = \boxed{1260\ mV}$$

Part B Determine R_1 such that $V_o = m \times V_x$.
$m = 18$, $R_2 = 4\,k\Omega$, $R_3 = 25\,k\Omega$

$$V_o = 18V_x = \frac{R_1 + 25{,}000}{R_1} V_x$$

$$18 = \frac{R_1 + R_3}{R_1}$$

$$18R_1 - R_1 = R_3$$

$$17R_1 = R_3$$

$$R_1 = \frac{25{,}000}{17} = 1470\,\Omega = \boxed{1.47\,k\Omega}$$

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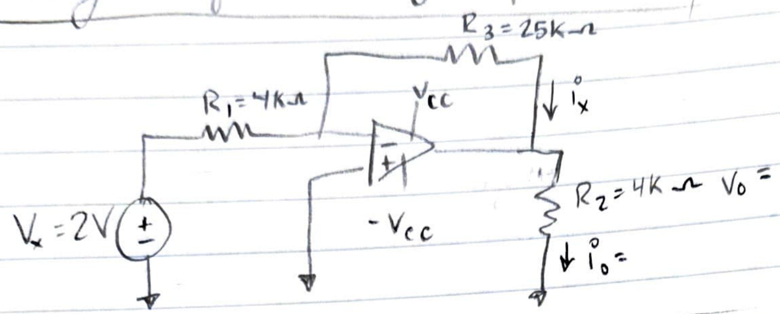
Analysis of Ideal Op Amp Circuits



Part A $V_0 = ?$; $i_x = ?$; $i_0 = ?$

$$\frac{0 - V_0}{R_3} + \frac{0 - V_x}{R_1} = 0$$

$$\frac{-V_0}{R_3} + \frac{-V_x}{R_1} = 0$$

$$\frac{-V_0}{R_3} = \frac{V_x}{R_1}$$

$$V_0 = -\frac{V_x}{R_1} \cdot R_3$$

$$= -\frac{2}{4{,}000} \cdot 25{,}000$$

$$V_0 = \boxed{-12.5V}$$

$$i_x = \frac{0 - V_0}{R_3} = 0.0005\ A \quad i_x = \boxed{0.5mA}$$

$$i_0 = \frac{V_0 - 0}{R_2} = 0.003125A \quad i_0 = \boxed{-3.125mA}$$

* How can there be current traveling up from ground if there is no current source?
* How can i_x be + if V_0 is − ?

Part B Determine the range of V_x so that in linear region.

$R_1 = 6k\Omega$; $R_2 = 9k\Omega$; $R_3 = 40k\Omega$; $V_{cc} = 15V$

$$V_0 = -\frac{R_3}{R_1} V_x$$

Min : $V_0 = -15V$ $\quad V_x = V_0 \cdot \frac{-R_1}{R_3} = (-15) \cdot \frac{-6{,}000}{40{,}000} = \boxed{2.25V}$

Max : $V_0 = 15V$ $\quad V_x = V_0 \cdot \frac{-R_1}{R_3} = 15 \cdot \frac{-6{,}000}{40{,}000} = \boxed{-2.25V}$

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Part C Determine V_o when $V_x = 3V$, $I_x = 1.0mA$, $R_1 = 5k\Omega$, $R_2 = 45k\Omega$
$R_3 = 3k\Omega$

$$V_o = -\frac{R_3}{R_1} V_x = \frac{-3000}{5000} \cdot 3 = \boxed{-1.8V}$$

$$i_x = 1.0 \times 10^{-3} = \frac{-V_o}{R_3} \qquad V_o = -i_x \cdot R_3 = -(1.0 \times 10^{-3})(3{,}000) = -3.0V$$

?? hint: $I_x + \frac{V_o}{R_2} - \frac{V_x}{R_2} = 0$

$$V_o = \left(\frac{V_x}{R_2} - I_x\right) R_2$$
$$= V_x - I_x R_2 = 3 - (1.0 \times 10^{-3})(45{,}000)$$
$$= \boxed{-42V}$$

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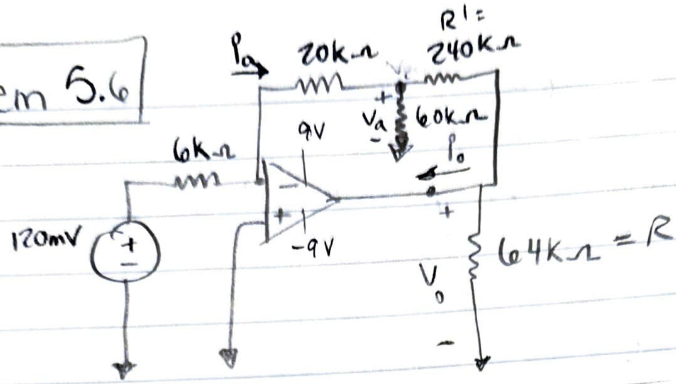
Problem 5.6



Part B Calculate V_a

$$\frac{0-120mV}{6k\Omega} + \frac{0-V_a}{20k\Omega} = 0$$

$$V_a = -\frac{120mV}{6\cancel{k\Omega}} \cdot 20\cancel{k\Omega} = \boxed{-0.40V}$$

Part C Calculate V_o

$$\frac{V_a}{6k\Omega} + \frac{V_a}{20k\Omega} + \frac{V_a - V_o}{240k\Omega} = 0$$

$$240k\Omega\left(\frac{V_a}{6k\Omega} + \frac{V_a}{20k\Omega} + \frac{V_a}{240k\Omega}\right) = V_o = \boxed{-6.80V}$$

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5.6

Part A Calculate i_a

$$\frac{120mV}{6K\Omega} = i_a = \boxed{20.0\mu A}$$

Part D Calculate i_o

$$\frac{V_o}{R} + \frac{V_o - V_a}{240k\Omega} + i_o = 0$$

$$i_o = -\frac{V_o}{R} - \frac{V_o - V_a}{240k\Omega} = \boxed{133\mu A}$$

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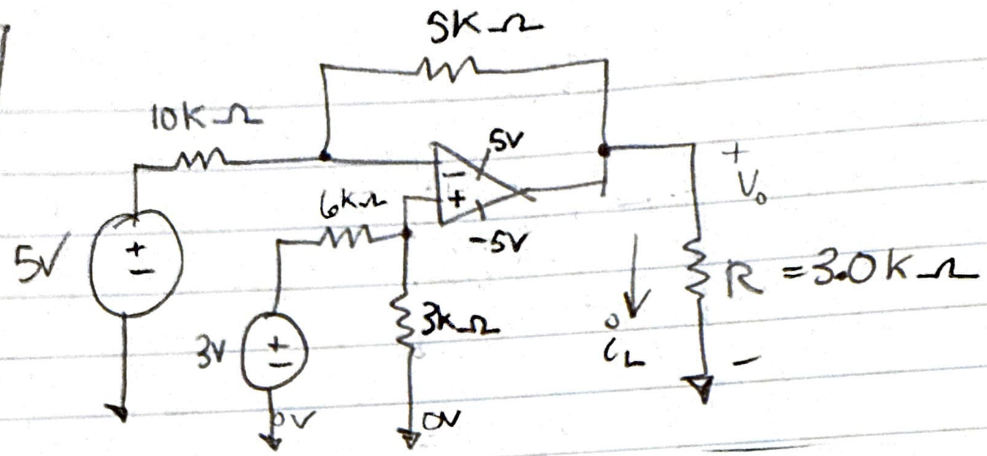
5.7



Part A

Find i_L

Voltage divider $\quad V_p = \left(\frac{3k\Omega}{6k\Omega + 3k\Omega}\right) 3V = 1V$

$V_p = V_n = 1V$

$$\frac{1-5}{10k\Omega} + \frac{1-V_o}{5k\Omega} = 0$$

$$\frac{-4}{10k\Omega} + \frac{1}{5k\Omega} = \frac{V_o}{5k\Omega}$$

$$V_o = 5k\Omega\left(\frac{-4}{10k\Omega} + \frac{1}{5k\Omega}\right)$$

$$= -1V$$

$$i_L = -1V/3{,}000 = \boxed{-0.333mA}$$

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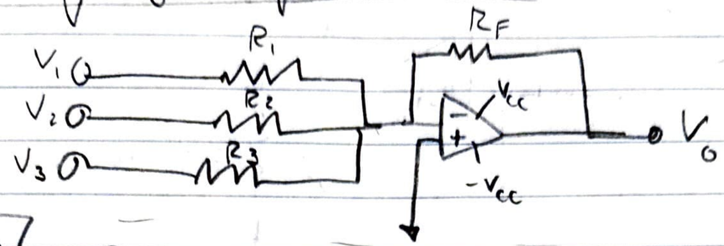
Summing Op Amp Circuit



Part A Determine V_o when $R_1 = 1.2k\Omega$, $R_2 = 9.8k\Omega$, $R_3 = 7.2k\Omega$, $R_F = 100k\Omega$, $V_1 = 20mV$, $V_2 = 70mV$, $V_3 = 150mV$, $V_{cc} = 5V$

$$V_o = -\left(\frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3\right)$$

$$= -\left(\left(\frac{100,000}{1,200} \cdot 20\times10^{-3}\right) + \left(\frac{100,000}{9800} \cdot 70\times10^{-3}\right) + \left(\frac{100,000}{7200} \cdot 150\times10^{-3}\right)\right)$$

$$= \boxed{-4.464V}$$

Part B Determine R_1; R_2; R_3 so

$$V_o = -(m_1 \cdot V_1 + m_2 \cdot V_2 + m_3 \cdot V_3)$$

$m_1 = 5$; $m_2 = 8$; $m_3 = 3$; $R_F = 120k\Omega$

$m_1 = 5 = \frac{R_f}{R_1} \rightarrow R_1 = \frac{R_f}{5} = \frac{120,000}{5} = 24,000\Omega = \boxed{24k\Omega}$

$m_2 = 8 = \frac{R_F}{R_2} \rightarrow R_2 = \frac{R_f}{8} = 15k\Omega$

$m_3 = 3 = \frac{R_F}{R_3} \rightarrow R_3 = \frac{R_F}{3} = \boxed{40k\Omega}$

Part C Determine range of V_1 so op amp => linear region

$R_1 = 3.4k\Omega$, $R_2 = 7.2k\Omega$, $R_3 = 7.8k\Omega$, $R_F = 200k\Omega$, $V_2 = 60mV$, $V_3 = 210mV$, $V_{cc} = 10V$

Min: $-10 = -\left(\left(\frac{200,000}{3,400}\right)V_1 + \left(\frac{200,000}{7,200}\right)60\times10^{-3} + \left(\frac{200,000}{7,800}\right)210\times10^{-3}\right)$

$V_1 = 0.6501V = \boxed{601 mV}$

Max: $10 =$ "

$V_1 = -0.290 = \boxed{-290mV}$

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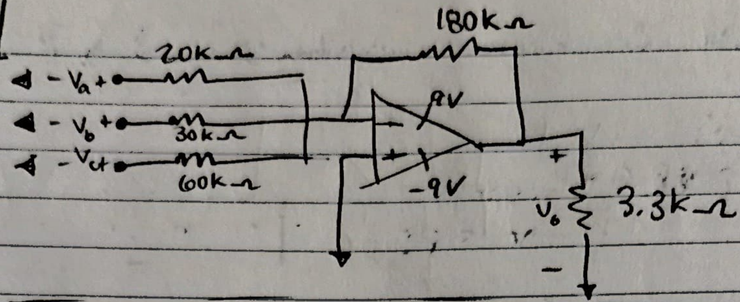
5.12



Part A The circuit is an example of an inverting summing amplifier.

Part B Find V_o if $V_a = 0.5V$, $V_b = 1.5V$, $V_c = -3.3V$

$$V_o = -\left(\frac{R_F}{R_a}V_a + \frac{R_F}{R_b}V_b + \frac{R_F}{R_c}V_c\right)$$

$$= -\left(\frac{180{,}000}{20{,}000}\cdot 0.5 + \frac{180{,}000}{30{,}000}\cdot 1.5V + \frac{180{,}000}{60{,}000}\cdot -3.3V\right)$$

$$\boxed{V_o = -3.6V}$$

Part C If $V_a = 0.5V$, $V_b = 1.5V$, min V_c?

$$9 = -\left(\frac{180{,}000}{20{,}000}\cdot 0.5 + \frac{180{,}000}{30{,}000}\cdot 1.5 + \frac{180{,}000}{60{,}000}V_c\right)$$

$$\boxed{V_c = -7.50V}$$

Part D

$$-9 = -\left(\frac{180{,}000}{20{,}000}\cdot 0.5 + \frac{180{,}000}{30{,}000}\cdot 1.5 + \frac{18{,}000}{60{,}000}V_c\right)$$

$$\boxed{V_c = -1.50V}$$

10/13

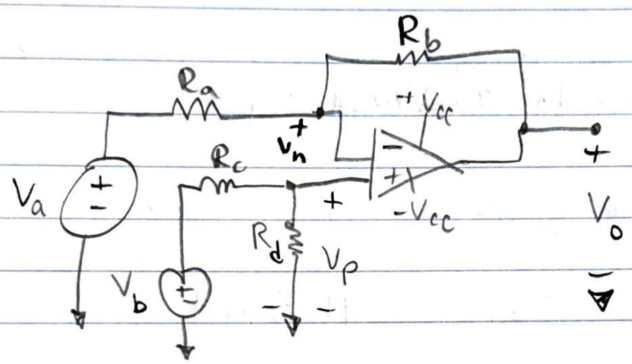
Video: Analyzing Non-Inverting Amplifier

$-3.97V \le V_g \le 3.97V$

$V_o = 2.52 V_g = 10V$

$100^2 \cdot 30{,}000 = P_{30k\Omega} = \boxed{3.33mW}$

5.27



$R_a = 20k\Omega$; $R_b = 80k\Omega$; $R_c = 47k\Omega$; $R_d = 33k\Omega$

$V_a = 0.45V$; $V_b = 0.9V$; $V_{cc} = \pm 9V$

Part A

Find V_o

$$V_o = \frac{R_d(R_a+R_b)}{R_a(R_c+R_d)} V_b - \frac{R_b}{R_a} V_a$$

$$= \frac{33{,}000(20{,}000+80{,}000)}{(20{,}000)(47{,}000+33{,}000)} \cdot 0.9 - \frac{80{,}000}{20{,}000} \cdot 0.45 = \boxed{0.056V}$$

11/13

5.27 Part B / Calculate resistance in V_a, using current passing through it

$$V_p = \left(\frac{R_d}{R_c+R_d}\right) V_b = \left(\frac{33k\Omega}{47k\Omega+33k\Omega}\right) \cdot 0.9V$$
$$= 0.37125\,V$$

$$i_{Ra} = \frac{0.45 - V_n}{R_a} = 3.94\,\mu A$$

$$R_{Va} = V_a / i_{Ra} = 0.45 / 3.94\mu A = \boxed{114\,K\Omega}$$

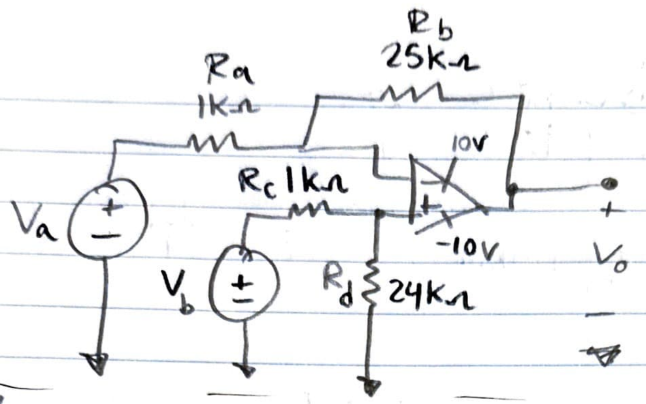
Part C Calculate resistance in V_b

$$i_{Rc} = \frac{0.9V - 0.37125V}{47{,}000} = 11.3\,\mu A$$

$$R_{V_b} = V_b / i_{Rc} = 80{,}000\,\Omega = \boxed{80\,K\Omega}$$

12/13

5.34



Part A Compute differential mode gain

$$A_{dm} = \left[\frac{R_d(R_a+R_b)+R_b(R_c+R_d)}{2R_a(R_c+R_d)}\right]$$

$$= \boxed{24.98}$$

Part B Compute Common mode gain

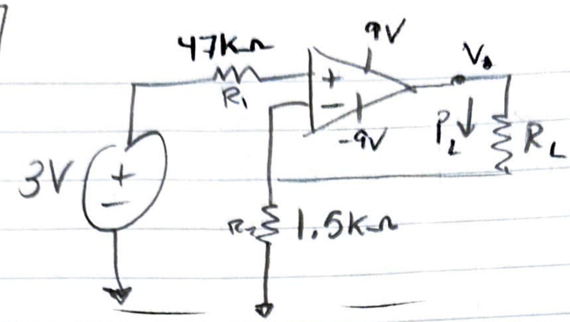
$$A_{cm} = \left[\frac{R_aR_d - R_bR_c}{R_a(R_c+R_d)}\right]$$

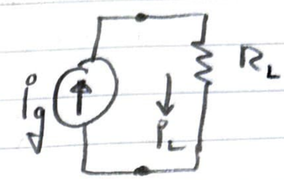
$$= \boxed{-0.04}$$

Part C Compute the CMRR

$$CMRR = \left|\frac{A_{dm}}{A_{cm}}\right| = \left|\frac{24.98}{-0.04}\right| = \boxed{624.5}$$

5.39





Part A/ Find value of i_L for $R_L = 2.5k\Omega$

$V_p = V_n$ $\qquad \frac{V_p - 3V}{47} = 0A \rightarrow V_p = 3V$

$$\frac{V_n}{R_2} + \frac{V_n - V_o}{R_L} = 0$$

$$V_o = \left(\frac{V_n}{R_2} + \frac{V_n}{R_L}\right) R_L = \left(\frac{3}{1.5k\Omega} + \frac{3}{2.5k\Omega}\right) 2.5k\Omega$$

$$= 8V$$

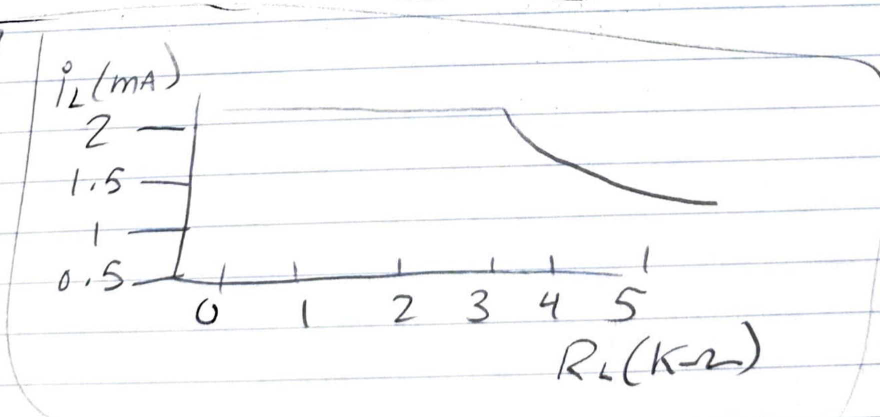
$$i_L = \frac{V_o - V_n}{R_L} = \boxed{2.00mA}$$

Part B/ Find the maximum value for R_L for which i_L will have the value in Part A

$i_L = 2.00mA$ $\quad V_o = ?$ $\quad R_L = ?$

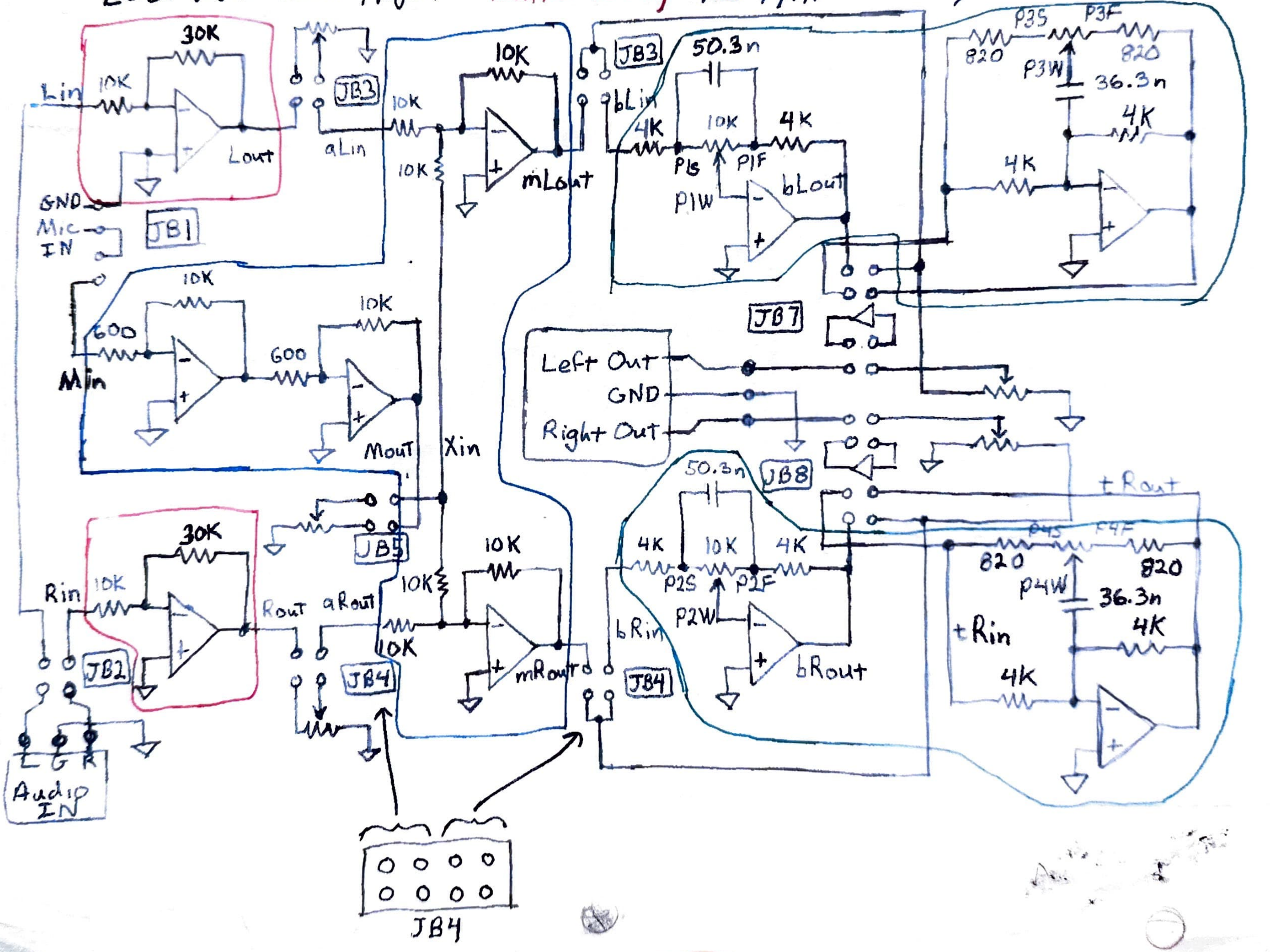
Max $V_o = 9V$ $\qquad (9V - 3V)/2.00mA = \boxed{3.00k\Omega}$

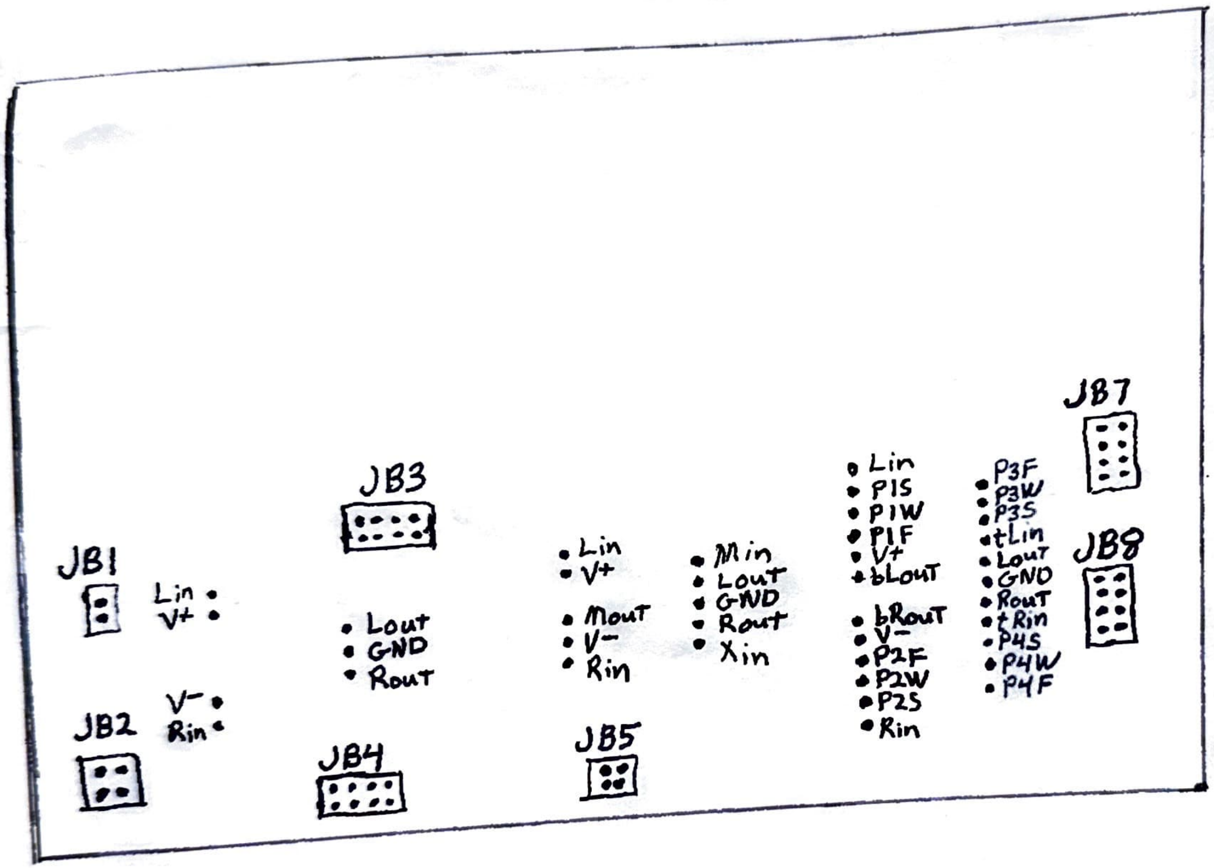
Part C/



Choose the correct image

ECEN250 Audio Project - Buffer Board, Preamp/Mixer Board, Treble and Bass Board



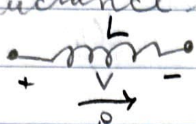


Mother Board

Chapter 6

6.1 The Inductor

- Opposes change in current
- inductance measured in Henrys (H)



$$V = L\,\frac{di}{dt}$$

Voltage is constant → voltage across = 0

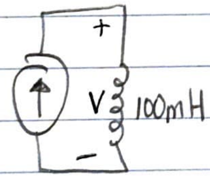
* Current can NOT change instantaneously

$$i(t) = \frac{1}{L}\int_0^t v\,d\tau + i(0)$$: Current

Power : $p = Li\frac{di}{dt}$

Energy : $w = \frac{1}{2}Li^2$

ex;



$i = 0 \quad , \; t < 0$

$i = 10te^{-5t}\,A \quad , \; t > 0$

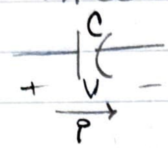
At what time is current max?

$$di/dt = 10(-5te^{-5t} + e^{-5t}) = 10e^{-5t}(1-5t)$$

when $di/dt = 0 \rightarrow t = 0.2s$

$\boxed{6.2}$ The capacitor — stores electrical charge

→ measured in Farads (F)



$$F = C/V$$

Current: $i = C\,dv/dt$

- capacitor behaves like an open circuit in constant voltage

↳ voltage can't change instantaneously

Voltage: $v(t) = \frac{1}{C}\int_0^t i\,d\tau + v(t_0)$

Power: $p = vi = Cv\frac{dv}{dt}$

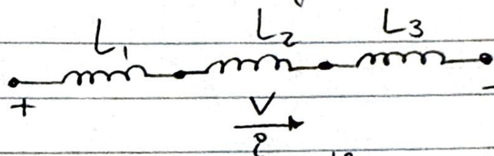
Energy: $w = \frac{1}{2}Cv^2$

When power is positive → energy being stored

negative → energy being dissipated

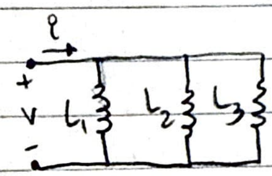
6.3 Series-Parallel Combination of Inductance and Capacitance

Series



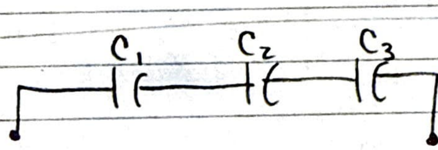
$$v = v_1 + v_2 + v_3 = (L_1 + L_2 + L_3)\frac{di}{dt} \longrightarrow L_{eq} = \sum_{i=1}^{n} L_i$$

Parallel



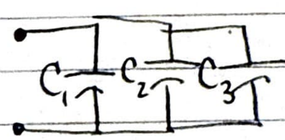
$$\frac{1}{L_{eq}} = \sum_{i=1}^{n} \frac{1}{L_i} \qquad i(t_0) = \sum_{j=1}^{n} i_j(t_0)$$

Series



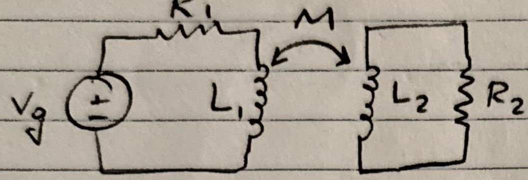
$$\frac{1}{C_{eq}} = \sum_{i=1}^{n} \frac{1}{C_i} \qquad v(t_0) = \sum_{j=1}^{n} v_j(t_0)$$

Parallel



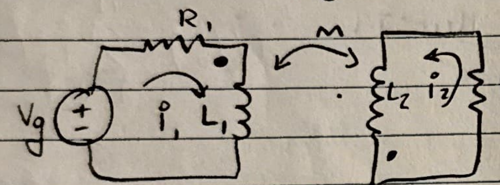
$$C_{eq} = \sum_{i=1}^{n} C_i$$

6.4 Mutual Inductance



Voltage across $L_1 \rightarrow L_1\,(di_1/dt)$

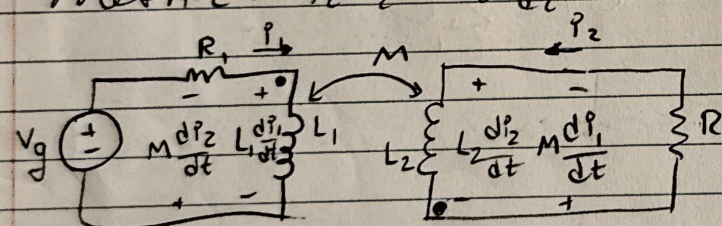
" " $L_2 \rightarrow M\,(di_2/dt)$



$\rightarrow$ L_2 causes "extra" voltage across L_1

Mesh 1: $-V_g + i_1 R_1 + L_1 \frac{di_1}{dt} - M\frac{di_2}{dt} = 0$

Mesh 2: $i_2 R_2 + L_2 \frac{di_2}{dt} - M\frac{di_1}{dt} = 0$



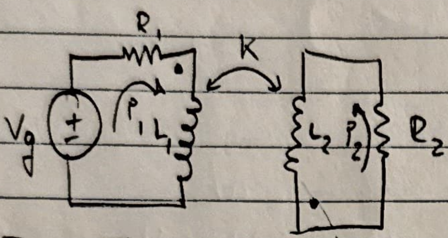
Dot convention: When the reference direction for a current enters the dotted terminal of a coil, the reference polarity of the voltage that it induces in the other coil is positive at its dotted terminal

$M = K\sqrt{L_1 L_2}$ $\qquad K \rightarrow$ "coefficient of coupling"

$w(t) = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 \pm M i_1 i_2$

Seth Ricks | ECEN 250 | HW #6 1/14

[A Closer look at Mutual Inductance]



Part A Determine mutual inductance M

$L_1 = 3.0H$; $L_2 = 2.0H$; $R_1 = 9.0\Omega$; $R_2 = 8.0\Omega$; $k = 0.9$

$M = k\sqrt{L_1 L_2} = \boxed{2.20}$

Part B Determine the energy stored in the coupled inductors at $t = 0.2s$

$K = 0.9$; $L_1 = 3.0H$; $L_2 = 2.0H$; $R_1 = 9.0\Omega$; $R_2 = 8.0\Omega$; $M = 0.9\sqrt{6}\ H$

$i_1 = 6.04A$; $i_2 = 4.25A$

$W(t) = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 - M i_1 i_2$

(since no t in i_1 & i_2 → doesn't matter)

$= \boxed{16.2\ J}$

Part C Determine energy stored at $t = \infty$

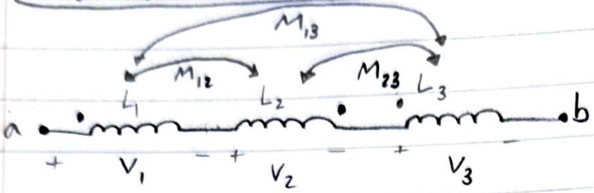
$i_1 = 4.74A$; $i_2 = 2.11A$; $K = 0.9$; $L_1 = 4.0H$; $L_2 = 3.0H$; $R_1 = 9.0\Omega$; $R_2 = 8.0\Omega$

$w(t) = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 - M i_1 i_2$

$= \boxed{20.4\ J}$

2/14

Mutual Inductance



Part A Calculate equivalent inductance between ab

Assume $M_{12} = M_{21} = 10.0\,mH$; $M_{13} = M_{31} = 10.0\,mH$; $M_{23} = M_{32} = 10.0\,mH$;
$L_1 = 3.00\,mH$; $L_2 = 1.00\,mH$; $L_3 = 6.00\,mH$

$$V_1 = L_1\frac{di}{dt} - M_{12}\frac{di}{dt} + M_{13}\frac{di}{dt}$$
$$V_2 = L_2\frac{di}{dt} - M_{12}\frac{di}{dt} - M_{23}\frac{di}{dt}$$
$$V_3 = L_3\frac{di}{dt} + M_{13}\frac{di}{dt} - M_{23}\frac{di}{dt}$$

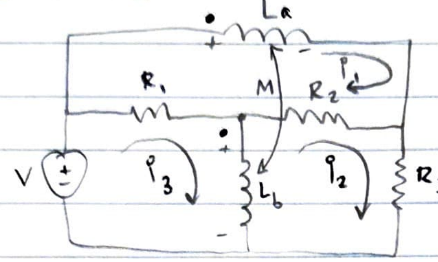
★ For the inductor you're on, if current enters dot, dot is +
↳ then all the other dots follow sign
↳ if current enters − terminal, subtract voltage & vice versa

$$V_T = (L_1 + L_2 + L_3)\frac{di}{dt} + (-2M_{12} + 2M_{13} - 2M_{23})\frac{di}{dt}$$

$V = L\frac{di}{dt}$

$L = V/(di/dt) = L_1 + L_2 + L_3 - 2M_{12} + 2M_{13} - 2M_{23} = \boxed{-10.0\,mH}$

Part B | Determine KVL equation for mesh 1

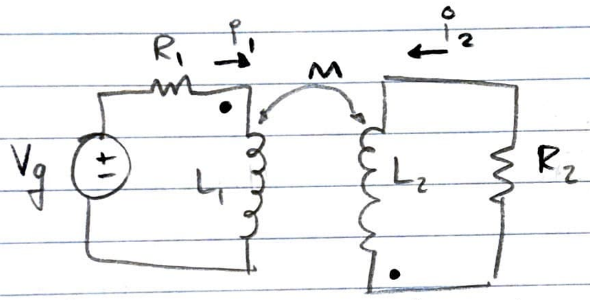


Current through inductor that induces voltage → L_b

Mesh 1 : $\boxed{L_a \frac{di_1}{dt} + R_2(i_1 - i_2) + R_1(i_1 - i_3) - M\frac{d(i_2 - i_3)}{dt} = 0}$

minus bc respect to i_2

Part C



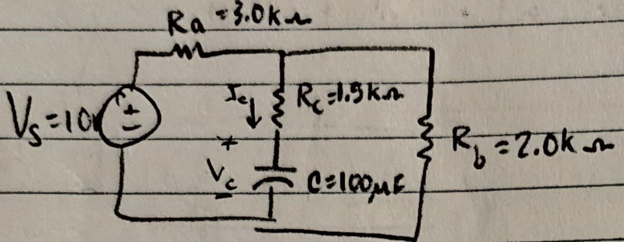
Determine mesh equation for mesh 1.

$$\boxed{V_g = R_1 i_1 + L_1\frac{di_1}{dt} - M\frac{di_2}{dt}}$$

4/14

The Capacitor

Part A Connected for a long time. Current + voltage across capacitor? $R_a = 3.0k\Omega$; $R_b = 2.0k\Omega$; $R_c = 1.5k\Omega$; $C = 100\mu F$; $V_s = 10V$

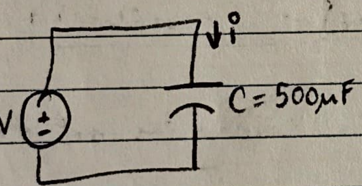


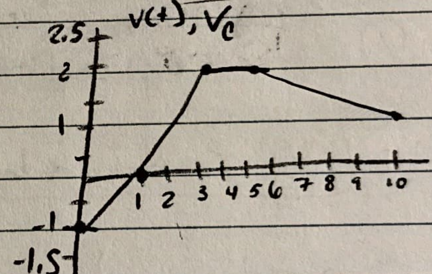
Capacitor under steady state → open circuit

$\boxed{I_c = 0A}$ $\quad V_{R_b} = V_c \rightarrow I_{R_c} = 0$

$$V_{R_b} = 10V\left(\frac{2.0k\Omega}{5.0k\Omega}\right) = \boxed{4.0V}$$

Part B

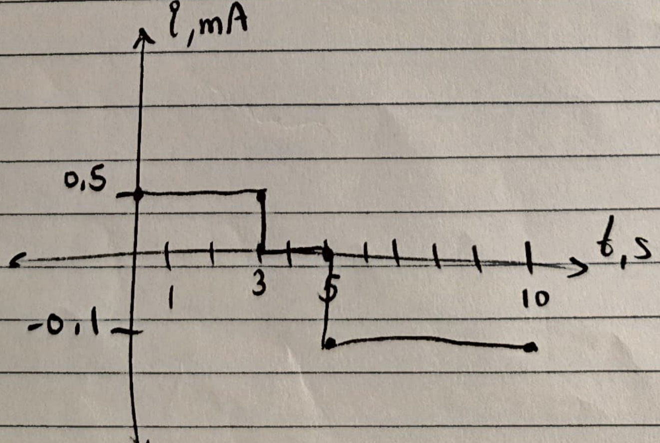




Create a sketch of current across the capacitor

$$i = C\frac{dv}{dt}$$

$\frac{d(v_0)}{dt} = 1$ $\quad \frac{d(v_1)}{dt} = 1$ $\quad \frac{d(v_3)}{dt} = 0$ $\quad \frac{d(v_5)}{dt} = 0$ $\quad \frac{d(v_{10})}{dt} = -1/5$



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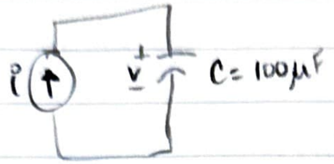
Part C



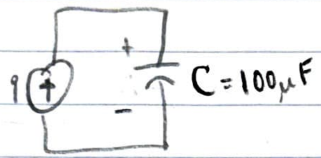
$V_o = 1V$. If $i(t) = 10\cos(100t)$A, what is $v(t)$ across capacitor?

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i\,d\tau + v(t_0)$$

$$= \frac{1}{100\times10^{-6}}\int_0^t 10\cos 100t + 1 = \boxed{\frac{10}{100\times10^{-6}} \cdot \frac{\sin 100t}{100} + 1 = v(t)}$$

$$= \boxed{100\sin(100t) + 1}$$

Part D



What is the instantaneous power?

$$\boxed{P = vi = (100\sin(100t)+1)(10\cos(100t))}$$

Part E Energy stored in capacitor at t=0?

$$W = \frac{1}{2}CV^2$$

$$= \frac{1}{2}(100\mu F)(100\sin(100t)+1)$$

$$W(0) = \frac{1}{2}(100\times10^{-6})(1) = \boxed{50\,\mu J}$$

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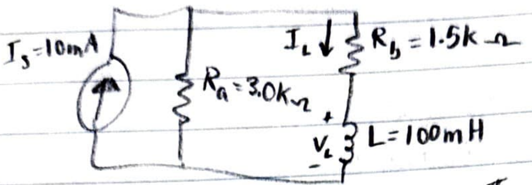
The Inductor

Part A



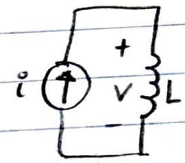
$R_a = 3.0k\Omega$; $R_b = 1.5k\Omega$; $L = 100mH$; $I_s = 10mA$

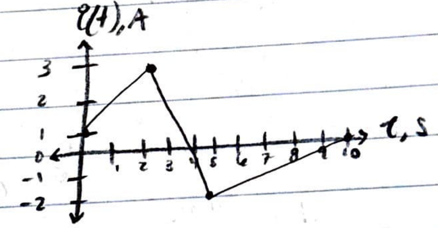
$I_L = ?$ $V_L = ?$

$\boxed{V_L = L\frac{di}{dt} = 0} \rightarrow \frac{di}{dt} = 0$

$I_L = I_s\left(\frac{R_a}{R_a + R_b}\right) = 10mA\left(\frac{3.0k\Omega}{3.0k\Omega + 1.5k\Omega}\right) = \boxed{6.67mA}$

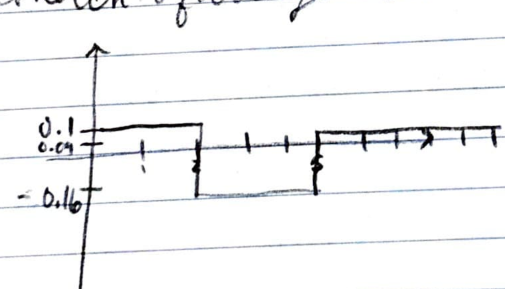
Part B



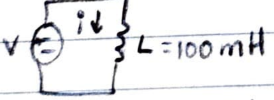


Create a sketch of voltage across inductor

$V = L\frac{di}{dt}$



Part C



$I_0 = 10mA$, $v(t) = 1e^{-10t} + 2e^{-5t}$ V, what is $i(t)$ through inductor?

$i(t) = \frac{1}{L}\int_0^t v(t)dt + i(0)$

$= \frac{1}{100\times10^{-3}} \cdot \int_0^t \left[1e^{-10t} + 2e^{-5t}\right]dt + 10\times10^{-3}$ A

$$= \frac{1}{100\times10^{-3}} \cdot \left[\frac{-e^{-10t}}{10} + \frac{-2e^{-5t}}{5}\right]_0^t + 10\times10^{-3}\ A$$

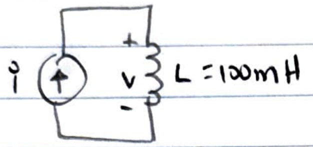
$$= \frac{1}{100\times10^{-3}} \cdot \left[\left(-\frac{e^{-10t}}{10} - \frac{2e^{-5t}}{5}\right) - \left(-\tfrac{1}{10} - \tfrac{2}{5}\right)\right] + (10\times10^{-3})$$

$$= \frac{1}{100\times10^{-3}} \cdot \left[\tfrac{1}{10}\left(-e^{-10t} - 4e^{-5t} + 5\right)\right] + (10\times10^{-3})$$

$$= -e^{-10t} - 4e^{-5t} + 5.01$$

$$\boxed{i(t) = 1000(-e^{-10t} - 4e^{-5t} + 5.01)\ mA}$$

Part D



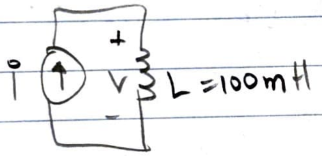
$i = 2t^2\ mA = 0.002t^2\ A$

Power absorbed?

$P = Vi \qquad V = L\frac{di}{dt} = (100\times10^{-3}) \cdot (0.004t)\ V$

$$\boxed{P = 0.8t^3\ \mu W}$$

Part E



$i = 2(2-e^{-t/100})\ mA$

$= 2(2-e^{-t/100})/1000\ A$

$$W = \tfrac{1}{2}Li^2 = \tfrac{1}{2}(100\times10^{-3})\left(\frac{4-2e^{-t/100}}{1000}\right)^2$$

$$= \tfrac{1}{2}(100\times10^{-3})\left(\frac{2}{1000}\right)^2 = \boxed{200\ nJ}$$

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Problem 6.3

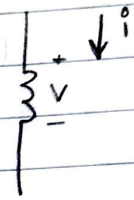
The current in a 15 mH inductor is

$i = 1A,\ t \le 0$

$i = A_1 e^{-2000t} + A_2 e^{-8000t} A,\ t \ge 0$

and $V(0) = 60V$

Part A Choose expression for voltage for $t > 0$



$V = L\frac{di}{dt}$

$i(0) = 1A = A_1 + A_2$

$v(t) = L\left[-2000A_1 e^{-2000t} - 8000A_2 e^{-8000t}\right]$

$V(t) = -30A_1 e^{-2000t} - 120A_2 e^{-8000t}$

$V(0) = 60 = -30A_1 - 120A_2$

$A_1 = 2\ ;\ A_2 = -1$

$\boxed{v(t) = -60e^{-2000t} + 120e^{-8000t}}$

Part B Find the time greater than zero, when the power at the terminals of the inductor is zero

$P = vi = \left(-60e^{-2000t} + 120e^{-8000t}\right)\left(2e^{-2000t} - e^{-8000t}\right)$

$P = 0 \rightarrow \boxed{t = 116\mu s}$

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Problem 6.5

Current in a 2H inductor is

$i = 25\text{A} \quad , \quad t \le 0$

$i = (B_1 \cos 5t + B_2 \sin 5t)e^{-t} \quad , \quad t \ge 0$

Voltage across is 100V at $t=0$

Part A Calculate power at the terminals of the inductor at $t = 490\text{ms}$.

$25 = B_1$

$i = (25\cos 5t + B_2 \sin 5t)e^{-t}$

$i = 25\cos 5t e^{-t} + B_2 \sin 5t e^{-t}$

$v = L\frac{di}{dt}$

$\frac{di}{dt} = -25\cos 5t e^{-t} - 125 e^{-t}\sin 5t + B_2\left[5\cos 5t e^{-t} - e^{-t}\sin 5t\right]$

$V = 2\left[-25\cos 5t e^{-t} - 125 e^{-t}\sin 5t + 5B_2 \cos 5t e^{-t} - B_2 e^{-t}\sin 5t\right]$

$V = -50\cos 5t e^{-t} - 250 e^{-t}\sin 5t + 10B_2 \cos 5t e^{-t} - 2B_2 e^{-t}\sin 5t$

$V(0) = 100 = -50 + 10B_2$

$150 = 10B_2$

$B_2 = 15$

HAPPY VALENTINES DAY!

$V = -50\cos 5t e^{-t} - 250 e^{-t}\sin 5t + 150\cos 5t e^{-t} - 30 e^{-t}\sin 5t$

$i = (25\cos 5t + 15\sin 5t)e^{-t}$

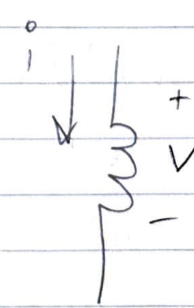
$p = vi$

$p(490\text{ms}) = \boxed{929\text{ W}}$

Part B

Ruth Ricky + Isabel Mooney

The inductor is $\boxed{\text{absorbing}}$ power.



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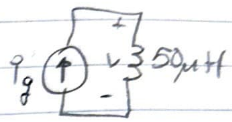
Video Solution Problem: Voltage, Current, Power, and Energy in an Inductor

Find the time when the power is maximum.



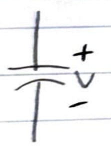
$i_g = 18te^{-10t}$ $\quad v = 0.9e^{-10t}(1-10t)$

$P = vi$

$\boxed{P_{max} = 0.0295}$

Problem 6.14 The voltage of 0.6μF capacitor is zero for $t<0$. For $t \geq 0$, the voltage is $40e^{-15000t}\sin 30000t$ V

Part A Find the initial current in the capacitor.



$v = \begin{cases} 0, & t<0 \\ 40e^{-15000t}\sin 30000t, & t \geq 0 \end{cases}$

$i = C\,dv/dt \qquad i(0) = C\frac{dv}{dt}(0) = \boxed{0.72\text{ A}}$

Part B Find the power delivered to the capacitor at $t = \pi/80$ ms

$P(\pi/80000) = v(t)\,i(t) = \boxed{649\text{ mW}}$

Part C Find energy stored at $t = \frac{\pi}{80}$ ms

$w = \frac{1}{2}Cv^2$

$= \frac{1}{2}(0.6\times10^{-6})(40e^{-15{,}000t}\sin 30{,}000t)^2$

$w\left(\frac{\pi}{80{,}000}\right) = \boxed{126\,\mu J}$

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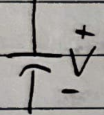
Problem 6.17

The voltage across the terminals of 400nF capacitor is

$$v = \begin{cases} 25V, & t \le 0 \\ A_1 t e^{-1500t} + A_2 e^{-1500t}, & t \ge 0 \end{cases}$$

The initial current is 90mA

Part A What is the initial energy stored in the capacitor.



$$W = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(400 \times 10^{-9})(25)^2 = \boxed{125\mu J}$$

Part C Evaluate the coefficient A_2

$$25 = \cancel{A_1(0)e^{-1500(0)}} + A_2 \cancel{e^{-1500(0)}} \qquad \boxed{A_2 = 25.0V}$$

Part B Evaluate the coefficient A_1

$$i = C\frac{dv}{dt}$$

$$\frac{dv}{dt} = A_1\left(\underbrace{-1500xe^{-1500x} + e^{-1500x}}\right) + 25\left(-1500e^{-1500x}\right)$$

$$i = (400 \times 10^{-9})A_1(\ \ast\ ) + (400 \times 10^{-9})25(-1500e^{-1500x})$$

$$i(0) = 90 \times 10^{-3} = (400 \times 10^{-9})A_1 + (400 \times 10^{-9})(25)(-1500)$$

$$225{,}000 = A_1 + 25(-1500)$$

$$\boxed{A_1 = 262{,}500\ V/s}$$

Part D Equation for capacitor current

$$i = (400 \times 10^{-9})(262{,}500)(-1500xe^{-1500x} + e^{-1500x}) + (400 \times 10^{-9})(25)(-1500e^{-1500x})$$

$$= -157.5xe^{-1500x} + 0.105e^{-1500x} + -0.015e^{-1500x}$$

$$\boxed{i = -157.5xe^{-1500x} + 0.09e^{-1500x}\ A}$$

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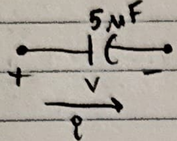
Video Solution: Voltage, Current, Power in Capacitor



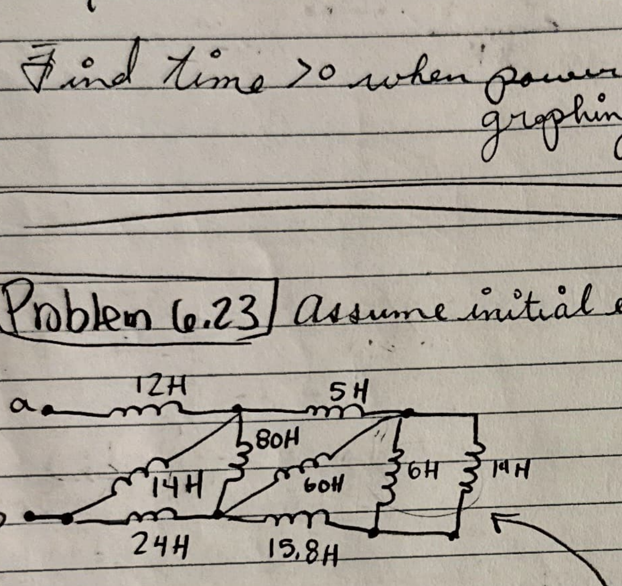
$$v = 500te^{-2500t}\ V$$
$$i = 2.5e^{-2500t}(1-2500t)\ mA$$

Find time > 0 when power is zero

graphing → $\boxed{t = 0.400\,mS}$

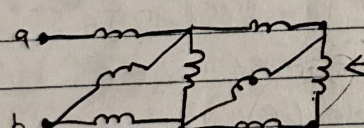
Problem 6.23

Assume initial energy in inductors is zero

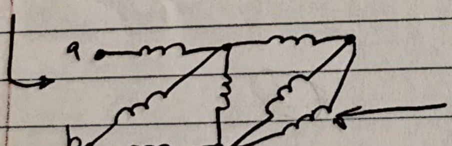


Part A

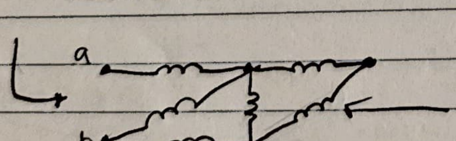
Find equivalent inductance



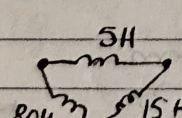
$$\frac{6 \cdot 14}{6+14} = 4.2H$$



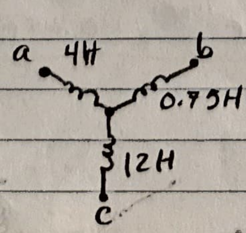
$$4.2 + 15.8 = 20H$$



$$\frac{20 \cdot 60}{20+60} = 15H$$

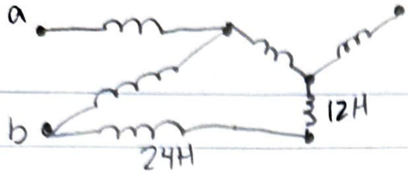


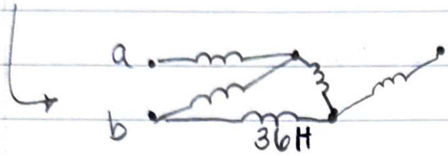
→

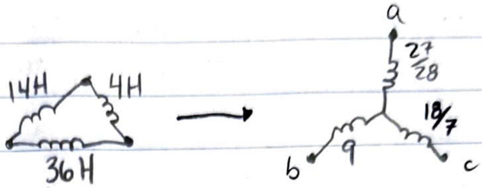


$$R_a = \frac{5 \cdot 80}{5+80+15} = 4H$$
$$R_b = \frac{5 \cdot 15}{100} = 0.75H$$
$$R_c = \frac{80 \cdot 15}{100} = 12H$$

13/14



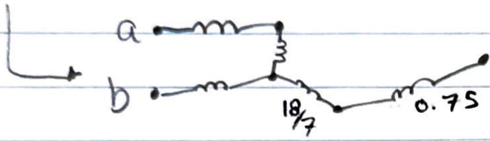


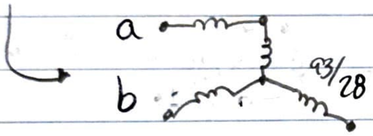


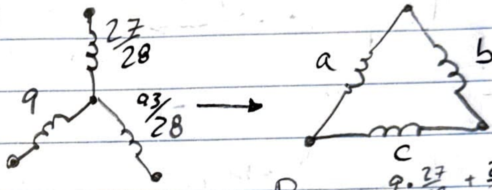
$R_a = \frac{14 \cdot 4}{14+4+36} = \frac{27}{28}$

$R_b = 9$

$R_c = \frac{18}{7}$





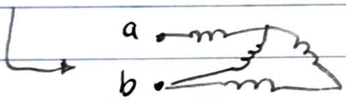


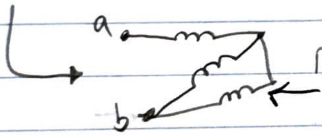
$R_a = \frac{9 \cdot \frac{27}{28} + \frac{27}{28} \cdot \frac{93}{28} + 9 \cdot \frac{93}{28}}{9}$

$= \frac{10917}{868}$

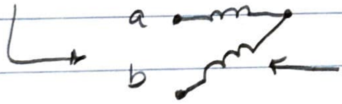
$R_b = \frac{3639}{784}$

$R_c = \frac{1213}{28}$

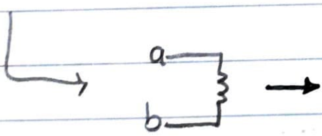




$R_{b+c} = \frac{37603}{784}$

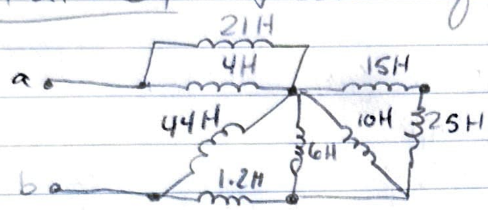


$$\frac{\frac{37603}{784} \cdot \frac{10917}{868}}{\frac{37603}{784} + \frac{10917}{868}} = \frac{279}{28}$$

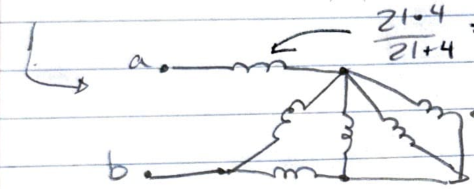


$\rightarrow \frac{615}{28} = \boxed{22.0\ H}$

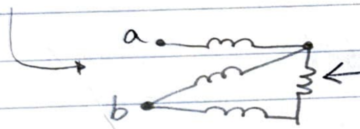
Part B Find equivalent inductance



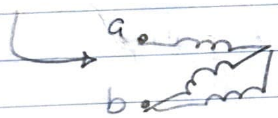
$$\frac{21 \cdot 4}{21+4} = 3.36\,H$$



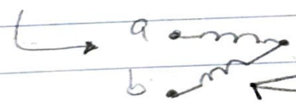
$$15+25 = 40\,H$$



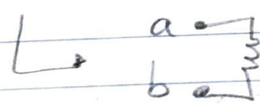
$$\frac{1}{L} = \frac{1}{40} + \frac{1}{10} + \frac{1}{6} \rightarrow L = \frac{24}{7}\,H$$



$$\frac{24}{7} + 1.2 = \frac{162}{35}\,H$$



$$\frac{1}{L} = \frac{1}{44} + \frac{1}{\frac{162}{35}} \rightarrow L = \frac{3564}{851}\,H$$



$$\frac{3564}{851} + 3.36 = \boxed{7.55\,H}$$

Chapter 9 introduction

9.1 The Sinusoidal Source

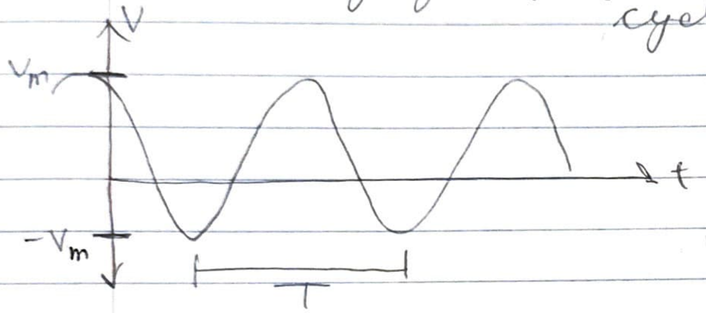
Sinusoidal voltage : $v = V_m \cos(\omega t + \phi)$

amplitude : $\pm V_m$

Period : T (seconds)

frequency (cycles per second) : $f = \frac{1}{T}$

cycle per second → hertz (Hz)



angular frequency : $\omega = 2\pi f = \frac{2\pi}{T}$ (rads/sec)

phase angle : ϕ

↳ determines $t=0$

↳ positive → shift right

negative → shift left

$$(\#\ \text{degrees}) = \frac{180^\circ}{\pi}(\#\ \text{rads})$$

RMS value : $V_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} V_m^2 \cos^2(\omega t+\phi)\,dt}$

{ Simplified } → $V_{rms} = \frac{V_m}{\sqrt{2}}$ (used to calculate power)

→ "root, mean, square"

rms of i → $I_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} i^2\,dt}$

$$\sin(\omega t+\phi) = \cos(\omega t+\phi-90^\circ)$$

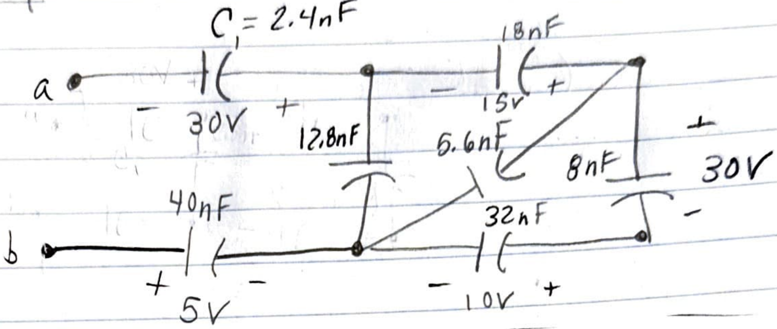
Seth Lewis Ricks

loves

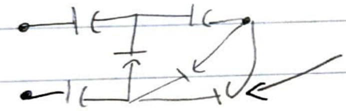
Isabel Elisabeth Toomey Ricks

Seth Ricks | ECEN 250 | HW#6 | 1/10

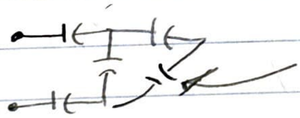
Problem 6.30



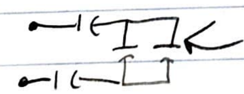
Part A Find equivalent resistance



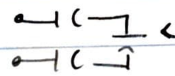
$\frac{32 \cdot 8}{40} = 6.4nF$



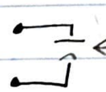
$6.4 + 5.6 = 12nF$



$\frac{12 \cdot 18}{12+18} = 7.2nF$



$7.2 + 12.8 = 20nF$



$\frac{1}{C} = \frac{1}{20} + \frac{1}{40} + \frac{1}{2.4} = \boxed{2.03nF = C_{eq}}$

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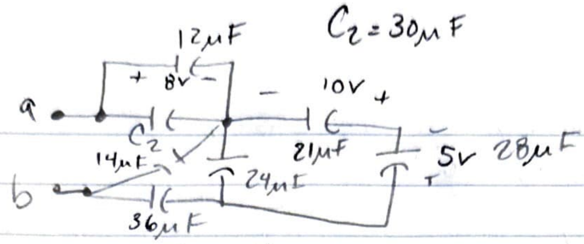
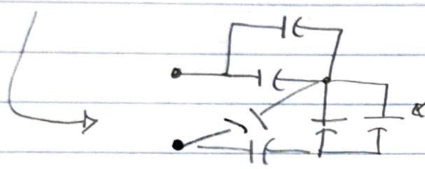
Part B

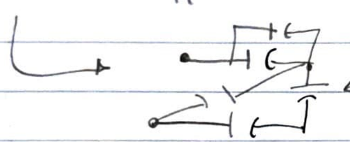
$C_2 = 30\mu F$



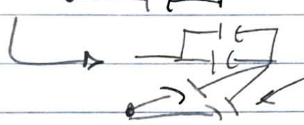
Find equivalent capacitance



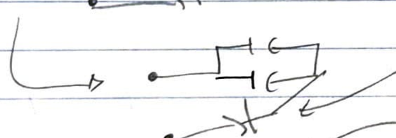
$$\frac{21 \cdot 28}{21+28} = 12\mu F$$



$$12 + 24 = 36\mu F$$

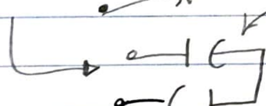


$$\frac{36 \cdot 36}{36+36} = 18\mu F$$



$$18 + 14 = 32\mu F$$

$$12 + 30 = 42\mu F$$



$$= \boxed{18.28\mu F = C_{eq}}$$

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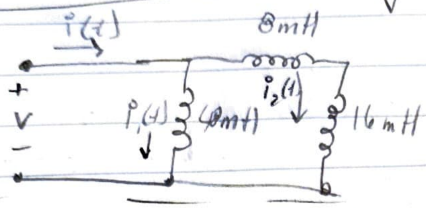
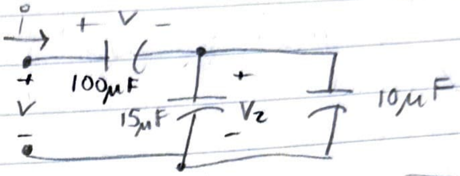
Video: Combining Capacitors





Part A
Find initial energy stored in L_{eq}

$$L_{eq} = \frac{(8+16)\cdot 48}{8+16+48} = 16\,mH$$

$W = \frac{1}{2}Li^2 \quad i = -0.4A \longrightarrow W = \frac{1}{2}(16\times10^{-3})(-0.4)^2 = \boxed{1.28\,mJ}$

Part B
Find initial energy stored in C_{eq}

$$C_{eq} = \frac{(15+10)\cdot 100}{15+10+100} = 20\mu F$$

$W = \frac{1}{2}CV^2 \quad V = 5V \longrightarrow W = \frac{1}{2}(20\times10^{-6})(5^2)$

$= \boxed{250\,\mu J}$

Series-Parallel Combination of Inductance & Capacitance

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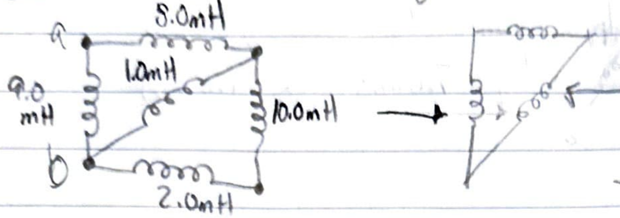
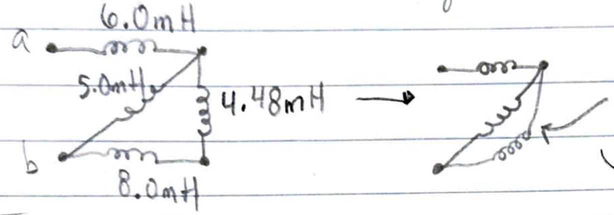
Part A Determine L_{eq}



$\frac{12\cdot 1}{13} = 0.923\,mH$

$L_{eq} = \frac{(0.923+8)\cdot 9}{0.923+8+9} = \boxed{4.48\,mH}$

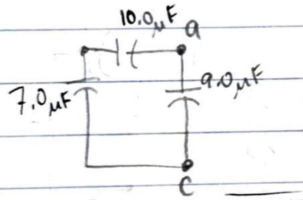
Part B Determine L_{eq}



$4.48+8.0 = 12.48\,mH$

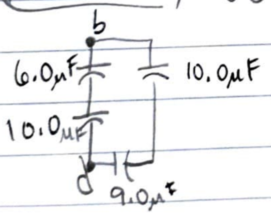
$L_{eq} = \frac{12.48\cdot 5}{12.48+5} + 6 = \boxed{9.57\,mH}$

Part C Determine C_{ab}



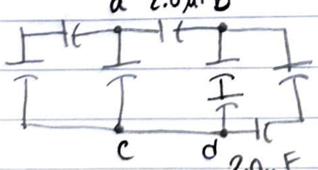
$C_{eq} = \frac{70}{17} + 9 = \boxed{13.1\,\mu F}$

Part D Determine C_{bd}

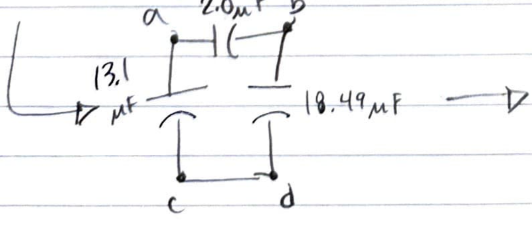


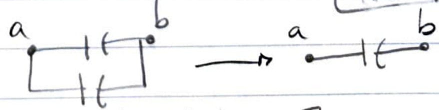
$C_{eq} = \frac{60}{16} + \frac{90}{19} = \boxed{8.49\,\mu F}$

Part E Determine C_{ab}



$\boxed{7.15\,\mu F}$

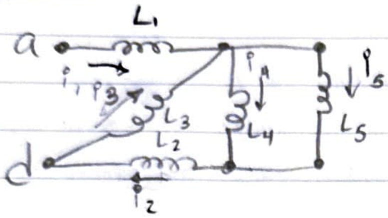




$\frac{13.1\cdot 8.49}{13.1+8.49} = 5.15\,\mu F$

5/10

Part F

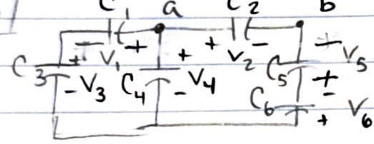


$i_3 = -2.0mA$, $i_4 = 6.0mA$, $i_5 = 8.0mA$, $i_2 = ?$
$L_1 = 6.0mH$, $L_2 = 10.0mH$, $L_3 = 1.0mH$,
$L_4 = 7.0mH$, $L_5 = 1.0mH$

$i_2 = i_4 + i_5 = 14.0mA$

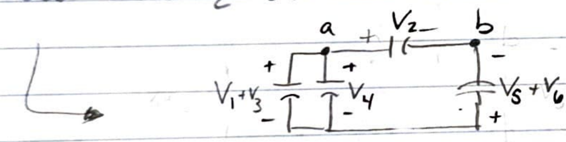
$i_1 + i_3 = i_4 + i_5 \longrightarrow i_1 = i_4 + i_5 - i_3 = 6.0 + 8.0 - 2.0 = \boxed{12.0mA}$

Part G



$V_1 = -2.0V$, $V_3 = 1.0V$, $V_5 = -6.0V$
$V_6 = 7.0V$, $C_1 = 8.0\mu F$, $C_2 = 3.0\mu F$,
$C_3 = 10.0\mu F$, $C_4 = 6.0\mu F$, $C_5 = 5.0\mu F$,
$C_6 = 6.0\mu F$ $\quad V_4 = ?$

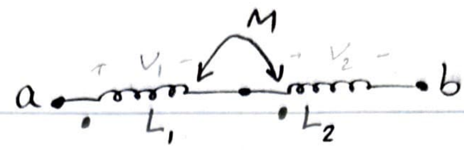
Determine V_2 across ab



$V_2 = V_1 + V_3 + V_5 + V_6 = \boxed{0V}$

6/10

Problem 6.38



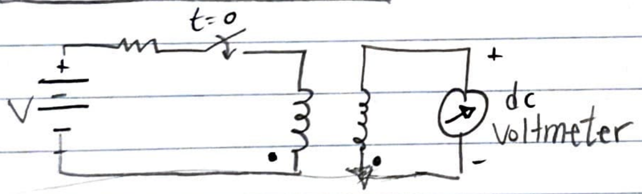
Part A The two coupled coils can be replaced by a single coil having an inductance of L_{ab}. Express L_{ab} in terms of L_1, L_2, and M (Express V_{ab} as a function of i_{ab})

$$V_1 = L_1\frac{di}{dt} + M\frac{di}{dt} \quad V_2 = L_2\frac{di}{dt} + M\frac{di}{dt}$$
$$V_{ab} = (L_1 + L_2 + 2M)\frac{di}{dt}$$
$$\boxed{L_{ab} = L_1 + L_2 + 2M}$$

Part B If L_2 is Reversed, find L_{ab}

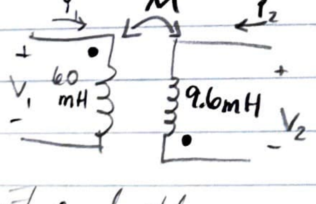
$$\boxed{L_{ab} = L_1 + L_2 - 2M}$$

Problem 6.42, Part A



When t=0, the voltmeter is upscale. So place the dot on the same terminal as the other, because of their positive relationship.

Video: Circuit w/ Mutually Coupled Coils



Find the energy when mutual inductance is max. (K=1)

$$w(t) = \tfrac{1}{2}L_1 i_1^2 + \tfrac{1}{2}L_2 i_2^2 - M i_1 i_2 \qquad i_1 = 10A \; ; \; i_2 = 15A$$

$$M = K\sqrt{L_1 L_2}$$
$$= (1)\sqrt{60 \cdot 9.6 \cdot 10^{-6}} = 0.024\,H$$

$$= \tfrac{1}{2}(60\times10^{-3})(10)^2 + \tfrac{1}{2}(9.6\times10^{-3})(15)^2 - (0.024)(10)(15) = \boxed{0.48\,J}$$

1.2 = ✓ − x (10)(15)

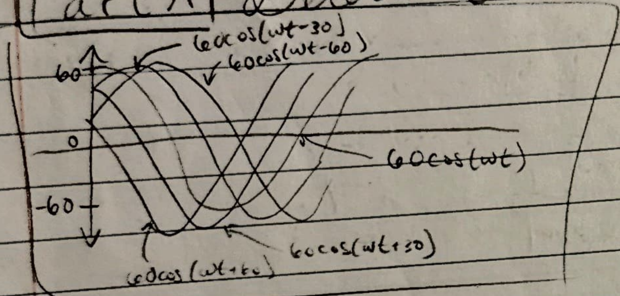
7/10

Video : Analyzing Mutually Coupled Coils

Purpose of Dot Convention - establish reference polarity

Problem 9.1

Part A Determine sketches for $i = 60\cos(\omega t + \phi)$ for $\phi = 60°, 30°, 0°, -30°, -60°$



Part B State whether the current function is shifting to the right or left as ϕ becomes more negative.

→ Right as ϕ becomes more negative

Part C What is the direction of the shift if ϕ goes from $0°$ to $30°$?

→ left

8/10

Problem 9.5 Consider the sinusoidal voltage
$v(t) = 290\cos(120\pi t - 60°)$ V

Part A What is the maximum amplitude of the voltage?
↳ $\boxed{290V}$

Part B What is the frequency in hertz?
↳ $\omega = 2\pi f \rightarrow f = \frac{\omega}{2\pi} = \frac{120\pi}{2\pi} = \boxed{60.0\ Hz}$

Part C What is the frequency in radians per second?
↳ $\omega = \frac{2\pi}{T} = \frac{2\pi}{1/f} = \boxed{377\ rad/s}$

Part D What is the phase angle in radians?
$-60° \cdot \frac{\pi}{180} = \boxed{-1.05\ rad}$

Part E Phase angle in degrees? $\rightarrow \boxed{-60.0°}$

Part F Period in milliseconds? $T = \frac{1}{f} = \boxed{16.7\ mS}$

Part G First time after t=0 that v=290?
↳ $v(t) = 290\cos(120\pi t - 60 \cdot \frac{\pi}{180})$
$v(t) = 290 \rightarrow \boxed{2.78\,ms}$

Part H The sinusoidal function is shifted 125/18 ms to the right.
What is the expression for v(t)?

$$v(t) = 290\cos\left(120\pi\left(t - \tfrac{125}{18}\right) - 60 \cdot \tfrac{\pi}{180}\right)$$
$$= 290\cos\left(120\pi t - \tfrac{5}{6}\pi - \tfrac{1}{3}\pi\right)$$
$$= 290\cos\left(120\pi t - \tfrac{7}{6}\pi\right) = 290\cos\left(120\pi t - \pi - \tfrac{1}{6}\pi\right)$$
$$\boxed{v(t) = -290\cos\left(120\pi t - \tfrac{\pi}{6}\right)}$$

Part I Min ms shifted to the left for $v(t) = 290\sin(120\pi t)$

$$v(t) = 290\cos\left(120\pi(t + x) - \tfrac{\pi}{3}\right)$$
$$= 290\cos\left(120\pi t + 120\pi x - \tfrac{\pi}{3}\right)$$
$$120\pi x - \tfrac{\pi}{3} = -\tfrac{\pi}{2}$$
$$120\pi x = -\tfrac{\pi}{6}$$
$$x = -\tfrac{1}{720} = -1.39\,ms \rightarrow +16.7\,ms = \boxed{15.3\,ms}$$

9/10

Problem 9.6 $V = 100\cos(240\pi t + 45°)V$

$V = 100\cos(240\pi t + \pi/4)V$

Part A Find f in hertz

↳ $\omega = 2\pi f$ $240\pi = 2\pi f \rightarrow \boxed{f = 120hz}$

Part B Find T

↳ $T = \frac{1}{f} = \frac{1}{120} = \boxed{8.33ms}$

Part C Find V_m

↳ $\boxed{V_m = 100V}$

Part D Find V(0)

↳ $\boxed{V(0) = 70.7V}$

Part E Find ϕ in degrees

↳ $\boxed{\phi = 45°}$

Part F Find ϕ in radians

↳ $\boxed{\phi = \pi/4 \, rad}$

Part G Find smallest positive t which $V=0$

$0 = 100\cos(240\pi t + \pi/4)$

$\pi/2 = 240\pi t + \pi/4$

$\pi/4 = 240\pi t$

$\frac{1}{960} = \boxed{t = 1.04ms}$

Part H Find smallest positive t which $dv/dt = 0$

$\frac{dv}{dt} = -100\sin(240\pi t + \pi/4) \cdot 240\pi$

$= -24000\pi \sin(240\pi t + \pi/4) = 0$

$\sin(240\pi t + \pi/4) = 0$

$240\pi t + \pi/4 = 0$

$240\pi t = -\pi/4$

$t = -1.04ms + T/2$

↳ $\boxed{t = 3.13ms}$

10/10

The Sinusoidal Source

$v(t) = 50\cos(2000t - \pi/4)$ mV

Part A Find amplitude $\rightarrow$ $\boxed{50\,mV}$

Part C Find frequency in hertz

$\hookrightarrow$ $2000 = 2\pi f \rightarrow f = \frac{1000}{\pi}$ Hz

Part B Find Frequency in rad/sec

$\hookrightarrow$ $\boxed{2000\ rad/sec}$

Part D Find period $\rightarrow$ $T = \frac{1}{f} = \frac{\pi}{1000}$ s $\rightarrow$ $\boxed{T = 3.14\,ms}$

Part E Find phase angle in degrees $\rightarrow$ $\boxed{-45^\circ}$

Part F Find the value of the voltage source at $t = 392.7\,\mu s$

$v(392.7\mu s) = \boxed{50.0\,mV}$

Part G Construct a cosine function to match the given plot.

$\hookrightarrow$ $\boxed{i(t) = 25\cos(1000t + 30^\circ)\,mA,\ t \geq 0\,ms}$

$i(0) = 21.651$

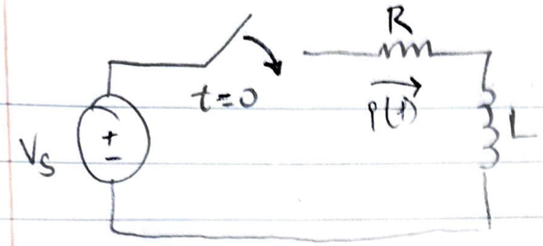
$21.651 = 25(\cos x)$

$x = 30^\circ$

$T = (7.330 - 1.047) \times 10^{-3}$?

$\omega = \frac{2\pi}{T} = 1000$

9.2 The Sinusoidal Response



$v_s = V_m \cos(\omega t + \phi)$

$L\frac{di}{dt} + Ri = V_m \cos(\omega t + \phi)$

$\hookrightarrow \; i = \underbrace{\frac{-V_m}{\sqrt{R^2+\omega^2L^2}} \cos(\phi - \theta) e^{-(R/L)t}}_{\text{steady-state component}} + \underbrace{\frac{V_m}{\sqrt{R^2+\omega^2L^2}} \cos(\omega t + \phi - \theta)}_{\text{transient component}}$

where $\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$

Steady state: cosine function, like circuit source
: frequency of solution = frequency of source
: max current amplitude = $V_m / \sqrt{R^2+\omega^2L^2}$
: phase angle of current = $\phi - \theta$, voltage = ϕ

9.3 The Phasor

Euler's identity $e^{\pm j\theta} = \cos\theta \pm j\sin\theta$

"frequency" domain

Phasor transform: $V = V_m e^{j\phi} = \mathcal{P}\{V_m \cos(\omega t + \phi)\}$

the phasor transform of

Rectangular form: $V = V_m \cos\phi + jV_m \sin\phi$

Notes from 150

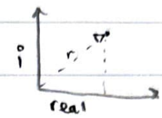
Representations of Complex Numbers

$Z = x + jy$, where $j = \sqrt{-1}$

polar: $Z = r\angle\theta$, where $r = \sqrt{x^2+y^2}$

$\theta = \tan^{-1} y/x$

$x = r\cos\theta$, $y = r\sin\theta$



$a = r\cos\theta$

$b = r\sin\theta$

$Z = a + jb$

$Z = x + jy = r\angle\theta = r\cos\theta + jr\sin\theta$

$Z = x + jy \quad \theta = \tan^{-1} y/x \rightarrow$ 1st Quad

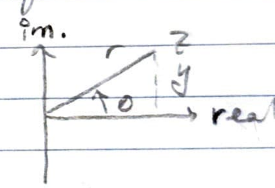
$Z = -x + jy \quad \theta = \tan^{-1} y/x \rightarrow$ 2nd Quad

$Z = -x - jy \quad \theta = 180° + \tan^{-1} y/x \rightarrow$ 3rd

$Z = x - jy \quad \theta = 360° - \tan^{-1} y/x \rightarrow$ 4th

exponential: $Z = re^{j\theta}$, $r = \sqrt{x^2+y^2}$, $\theta = \tan^{-1} y/x$

Phasors: complex number that represents amplitude and phase of a sinusoid



$Z = x + jy = r\angle\theta = r(\cos\theta + j\sin\theta)$

Operations:

addition: $Z_1 + Z_2 = (x_1 + x_2) + j(y_1 + y_2)$

subtraction: $Z_1 - Z_2 = (x_1 - x_2) + j(y_1 - y_2)$

Mult: $Z_1 Z_2 = r_1 r_2 \angle \theta_1 + \theta_2$

Div: $Z_1/Z_2 = r_1/r_2 \angle \theta_1 + \theta_2$

Reciprocal: $1/Z = 1/r \angle -\theta$

Square root: $\sqrt{Z} = \sqrt{r} \angle \theta/2$

Complex conjugate: $Z^* = x - jy = r\angle -\theta = re^{-j\theta}$

$\hookrightarrow 1/j = -j$

$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$

$\cos\theta = \text{Re}(e^{j\theta}) \rightarrow$ "real" part

$\sin\theta = \text{Im}(e^{j\theta}) \rightarrow$ "imaginary" part

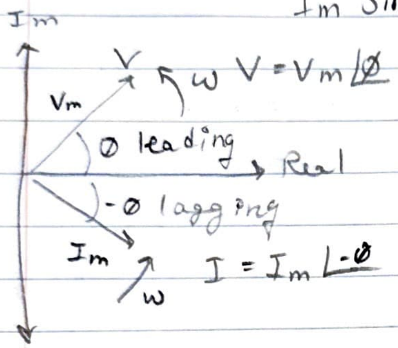
$v(t) = V_m \cos(\omega t + \theta)$

$\hookrightarrow v(t) = \text{Re}(Ve^{j\omega t})$, where $V = V_m e^{j\theta} = V_m \angle\theta$

time-domain | phasor-domain

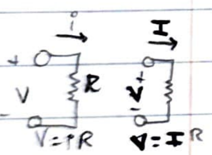
$v(t) = V_m \cos(\omega t + \phi) \longleftrightarrow V = V_m \angle \phi$

time-domain	phasor-domain
$V_m \cos(\omega t + \phi)$	$V_m \angle \phi$
$V_m \sin(\omega t + \phi)$	$V_m \angle \phi - 90°$
$I_m \cos(\omega t + \phi)$	$I_m \angle \phi$
$I_m \sin(\omega t + \phi)$	$I_m \angle \phi - 90°$



☆ Can only convert cos to a+jb

9.4 Phasor relationships for circuit elements



Element	Time Domain	Frequency Domain
R	$v = Ri$	$V = RI$
L	$v = L\frac{di}{dt}$	$V = j\omega L I$
C	$i = C\frac{dv}{dt}$	$V = \frac{I}{j\omega C}$

9.5 Impedance & Admittance

$V = RI$ $\quad V = j\omega L I$ $\quad V = \frac{I}{j\omega C}$

→ $\frac{V}{I} = R$ $\quad \frac{V}{I} = j\omega L$ $\quad \frac{V}{I} = \frac{1}{j\omega C}$

→ $Z = \frac{V}{I}$ or $V = ZI$

Z is a frequency-dependent quantity measured in ohms

→ impedance

$R = Re(Z)$ → resistance

$X = Im(Z)$ → reactance

$Z = R \pm jX = |Z|\angle\theta$, where $|Z| = \sqrt{R^2+X^2}$, $\theta = \tan^{-1}\left(\frac{\pm X}{R}\right)$

$R = |Z|\cos\theta$, $X = |Z|\sin\theta$

Reciprocal of impedance → admittance, (siemens (S))
→ or mhos

$Y = G + jB$ — B → susceptance
G → conductance

$G + jB = \frac{1}{R + jX}$ or $G = \frac{R}{R^2+X^2}$ and $B = \frac{-X}{R^2+X^2}$

Element	Impedance	Admittance
R	$Z = R$	$Y = \frac{1}{R}$
L	$Z = j\omega L$	$Y = \frac{1}{j\omega L}$
C	$Z = \frac{1}{j\omega C}$	$Y = j\omega C$

always $X\angle 90^\circ$ →
Short at DC
$\omega = 0$
open at high freq.
$\omega = \infty$

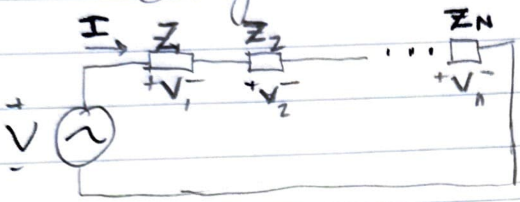
always $X\angle -90^\circ$ →
open at DC
$\omega = 0$
short at high frequencies
$\omega = \infty$

9.6 Kirchoff's Laws in the Frequency Domain

KVL: $v_1+v_2+\dots+v_n=0 \longrightarrow \mathbf{V}_1+\mathbf{V}_2+\dots+\mathbf{V}_n=0$

KCL: $i_1+i_2+\dots+i_n=0 \longrightarrow \mathbf{I}_1+\mathbf{I}_2+\dots+\mathbf{I}_n=0$

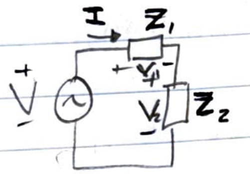
9.7 Impedance Combinations



KVL: $\mathbf{V}=\mathbf{V}_1+\mathbf{V}_2+\dots+\mathbf{V}_N=\mathbf{I}(\mathbf{Z}_1+\mathbf{Z}_2+\dots+\mathbf{Z}_N)$

$\mathbf{Z}_{eq}=\frac{\mathbf{V}}{\mathbf{I}}=\mathbf{Z}_1+\mathbf{Z}_2+\dots+\mathbf{Z}_N$

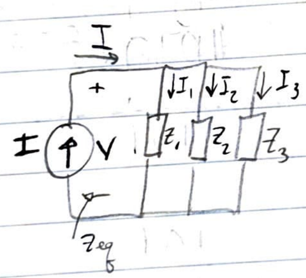
If $N=2$



(Voltage Division)

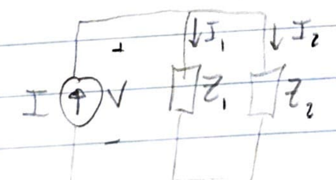
$\mathbf{V}_1=\frac{\mathbf{Z}_1}{\mathbf{Z}_1+\mathbf{Z}_2}\mathbf{V}$, $\mathbf{V}_2=\frac{\mathbf{Z}_2}{\mathbf{Z}_1+\mathbf{Z}_2}\mathbf{V}$

If in $\parallel$:



$\mathbf{Y}_{eq}=\mathbf{Y}_1+\mathbf{Y}_2+\dots+\mathbf{Y}_N$

If $N=2$



(current Division)

$\mathbf{I}_1=\frac{\mathbf{Z}_2}{\mathbf{Z}_1+\mathbf{Z}_2}\mathbf{I}$, $\mathbf{I}_2=\frac{\mathbf{Z}_1}{\mathbf{Z}_1+\mathbf{Z}_2}\mathbf{I}$

$Y_{eq}=Y\angle\theta^\circ$

$\frac{1}{Y_{eq}}=\frac{1\angle 0^\circ}{Y\angle\theta^\circ}=\frac{1}{Y}\angle 0-\theta^\circ$

★ Delta-to-WYE is the same as before ★

Seth Ricks | ECEN 250 | HW#7 1/8

Problem 9.7

$V = V_m \sin\frac{2\pi}{T}t, \quad 0 \le t \le T/2$

Part A Find the rms value

$$\boxed{V_{rms} = \frac{V_m}{\sqrt{2}}}$$

Video: Using the Phasor transform

Part A What does NOT appear in the phasor representation?

↳ $\boxed{\text{frequency}}$

Video: Frequency Domain of Inductor

Part A The phasor representation of an inductance corresponds to

↳ $\boxed{\text{an imaginary positive impedance}}$

Problem 9.11

Part A

$$y = 100\cos(300t + 45°) + 500\cos(300t - 60°)$$

$$100\angle 45° \quad + \quad 500\angle -60°$$

$x_1 = r\cos\theta = 100\cos 45° = \frac{100}{\sqrt{2}}$ $\qquad$ $x_2 = 500\cos(-60°) = 250$

$y_1 = r\sin\theta = 100\sin 45° = \frac{100}{\sqrt{2}}$ $\qquad$ $y_2 = 500\sin(-60°) = -250\sqrt{3}$

$Z_1 = \frac{100}{\sqrt{2}} + j\frac{100}{\sqrt{2}}$ $\qquad$ $Z_2 = 250 + j(-250\sqrt{3})$

$$Z_1 + Z_2 = 320.71 + j(-362.30)$$

$$r = \sqrt{x^2 + y^2} = 483.86$$

$$\theta = \tan^{-1}\left(\frac{-362.30}{320.71}\right) = -48.48°$$

$$\boxed{y = 483.86\cos(300t - 48.48)}$$

2/8

Part B $y = 250\cos(377t + 30^\circ) - 150\sin(377t + 140^\circ)$

$-150\cos(377t + 50^\circ)$

$x_1 = 250\cos 30^\circ = 125\sqrt{3}$

$y_1 = 250\sin 30^\circ = 125$

$z_1 = 125\sqrt{3} + j125$

$x_1 = -150\cos 50^\circ = -96.42$

$y_1 = -150\sin 50^\circ = -114.91$

$z = -96.42 - 114.91j$

$z = 120.09 + 10.09j$

$r = 120.51 \quad \theta = \tan^{-1}(y/x) = 4.80^\circ$

$\boxed{y = 120.51\cos(377t + 4.80^\circ)}$

Part C $y = 60\cos(100t + 60^\circ) - 120\sin(100t - 125^\circ) + 100\cos(100t + 90^\circ)$

$\hookrightarrow -120\cos(100t - 215)$

$x_1 = 60\cos 60^\circ = 30$

$y_1 = 60\sin 60^\circ = 30\sqrt{3}$

$z_1 = 30 + 30\sqrt{3}j$

$x_2 = -120\cos 215 = 98.30$

$y_2 = -120\sin 215^\circ = -68.83$

$z_2 = 98.30 - 68.83j$

$x_3 = 100\cos 90^\circ = 0$

$y_3 = 100\sin 90^\circ = 100$

$z_2 = 0 + 100j$

$z = 128.3 + 83.13j$

$r = 152.88 \quad \theta = 32.94$

$\boxed{y = 152.88\cos(100t + 32.94^\circ)}$

Part D $y = 100\cos(\omega t + 40^\circ) + 100\cos(\omega t + 160^\circ) + 100\cos(\omega t - 80^\circ)$

$x_1 = 100\cos 40^\circ = 76.60$

$y_1 = 100\sin 40^\circ = 64.28$

$x_2 = -93.97$

$y_2 = 34.20$

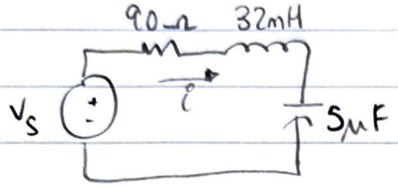
$x_3 = 17.36$

$y_3 = -98.48$

$z = -0.01 + 0j$

$r \approx 0, \quad \theta = 0 \quad \boxed{y = 0}$

Video : Circuit in Frequency Domain



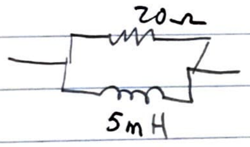
$V_s = 125\angle{-60}\,V$

a.) Value of capacitance st current i w/ phase $-105°$

$i = I_m\angle{-105°}\,A \qquad i = \frac{125\angle{-60°}}{|Z|\angle\theta_z} = \frac{125}{|Z|}\angle{-60-\theta_z}$

$-105° = -60-\theta_z \rightarrow \theta_z = \boxed{45°}$

Video : Impedance in Series and Parallel



Combined resistance & reactance

$\hookrightarrow Z = (Y_1 + Y_2)^{-1} = \left(\frac{1}{20} + \frac{1}{j\omega L}\right)^{-1} = \frac{j\omega 20L}{20+j\omega L} \times \frac{20-j\omega L}{20-j\omega L}$

x by Fancy 1

Video : Δ-to-Y in Frequency Domain

(The transformation involves only resistances)

Problem 9.13 / The voltage across the terminals of a $5\mu F$ capacitor is $22\cos(4000t + 25°)V$

Part A / Calculate the capacitive reactance.

$$Z = \frac{1}{j\omega C} = \frac{1}{j(4000)(5\times10^{-6})} = -50j \rightarrow \boxed{-50.0\,\Omega}$$

Part B / Calculate impedance of capacitor

$$\boxed{0 - 50.0j\,\Omega}$$

Part C / Calculate phasor current I

$$V = \frac{I}{j\omega C} \quad I = Vj\omega C = 22\angle 25° \cdot j(4000)(5\times10^{-6})$$

negative

$$= 22\angle 25° \cdot 0.02\angle -90$$

$$\boxed{= 0.44\angle -65°}$$

Part D / Find the steady state expression for the capacitor

$$i = 0.44\angle 115° \quad (\text{add } 180°)\ (\text{switches sign})$$

$$\boxed{i = 0.44\cos(4000t + 115°)A}$$

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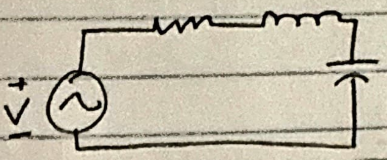
Problem 9.20 / a 400-Ω resistor, a 87.5 mH inductor, and a 3125 nF capacitor are connected in series. $V = 500\cos(8000t + 60^\circ)$ V



Part A / Determine impedance of elements in frequency-domain

$Z_L = j\omega L = j\,8000L = 700j\ \Omega$; $Z_C = \frac{1}{j\omega C} = 400\frac{1}{j} = -400j\ \Omega$; $Z_R = 400\ \Omega$

$\boxed{Z = 400 + 300j} \rightarrow 500\angle 36.87^\circ$

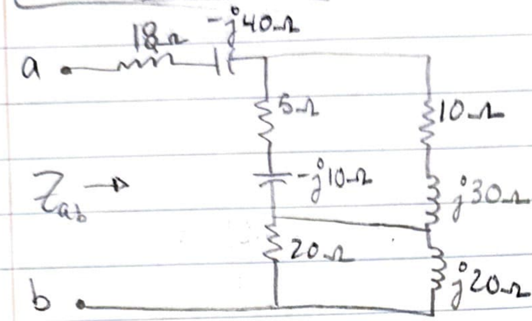
Part B / Find the Phasor current

$I_g = V/Z = 500\angle 60^\circ / 500\angle 36.87^\circ = \boxed{1\angle 23.13^\circ\ A}$

Part C $\boxed{i(t) = 1\cos(8000t + 23.1^\circ)\,A}$

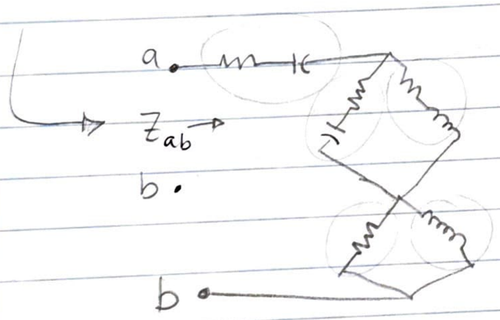
6/8

Problem 9.29



Part A

Express Z_{ab} in polar form



$$\frac{(20)(j20)}{20+j20} + \frac{(5-j10)(10+j30)}{5-j10+10+j30} + 18 - j40 = Z_{ab}$$

$$\frac{400j}{20+j20} + \frac{50+j150-j100+300}{15+j20} + 18 - j40 = Z_{ab}$$

$$\frac{20j}{1+j} + \frac{350+j50}{15+j20} + 18 - j40 = Z_{ab}$$

$$\frac{1-j}{1-j} \cdot \frac{20j}{1+j} + \frac{70+j10}{3+j4} + 18 - j40 = Z_{ab}$$

$$\frac{20+20j}{2} + \frac{70+j10}{3+j4} + 18 - j40 = Z_{ab}$$

$$10+10j + \frac{250-j250}{25} + 18 - j40 = Z_{ab}$$

$$10+10j + 10-10j + 18 - j40 = Z_{ab}$$

$$38 - j40 = Z_{ab}$$

$$r = \sqrt{38^2 + (-40)^2} = 55.2$$

$$\theta = \tan^{-1}\left(\frac{-40}{38}\right) = -46.5°$$

$$\boxed{Z_{ab} = 55.2 \angle -46.5°}$$

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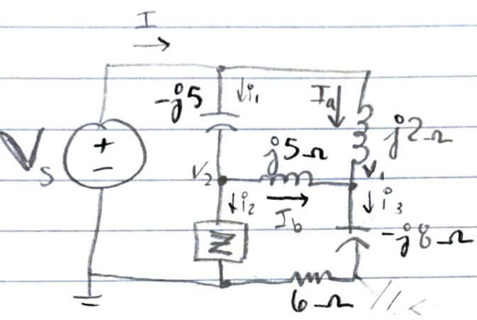
$\boxed{\text{Part B}}$ Express Z_{ab} in rectangular form

$\boxed{Z_{ab} = 38 - j40}$

$\boxed{\text{Problem 9.37}}$



$V_s = 80\angle 0^\circ$ V

$I_a = 5\angle -90^\circ$ A

$x = 80\cos 0 = 80$
$y = 80\sin 0 = 0$
$V = 80 + 0j$

$x = 5\cos(-90) = 0$
$y = 5\sin(-90) = -5j$

$\boxed{\text{Part A}}$ Find I_b

$$V_s - V_1 = (-j5)(j2) = 10V$$
$$V_1 = 70V$$

$$\frac{6+j8}{6+j8} \cdot \frac{V_1 - 0}{6-j8} = i_3$$

$$\frac{420 + 560j}{100} = i_3 = 4.2 + 5.6j$$

$$i_3 = I_b + I_a$$
$$I_b = i_3 - I_a$$
$$= 4.2 + 5.6j + (+5j)$$
$$I_b = \boxed{4.2 + 10.6j \text{ A}}$$

$\boxed{\text{Part B}}$

$$V_2 - V_1 = (4.2 + 10.6j) \cdot (j5)$$
$$= 21j - 53 = -53 + 21j$$
$$V_2 = -53 + 21j + 70$$
$$V_2 = 17 + 21j$$

$$\frac{V_s - V_2}{-j5} = i_1$$

$$\frac{j}{j} \cdot \frac{80 - 17 - 21j}{-j5} = i_1 \quad \frac{80j - 17j + 21}{5} = \frac{21 + 63j}{5} = 4.2 + 12.6j$$

8/8

$\mathring{i}_1 = \mathring{i}_2 + I_b$

$\mathring{i}_1 = 4.2 + \mathring{j}12.6$

$I_b = 4.2 + 10.6\mathring{j}$

$\mathring{i}_2 = \mathring{i}_1 - I_b = 2\mathring{j}$

$$Z = \frac{V_2}{\mathring{i}_2} = \frac{17+21\mathring{j}}{2\mathring{j}} \cdot \frac{-\mathring{j}}{-\mathring{j}} = \frac{21-17\mathring{j}}{2} = \boxed{10.5-8.5\mathring{j}\ \Omega}$$

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Polar		Rectangular	
$20\angle 30°$	$\longrightarrow$	$x = 20\cos 30° = 10\sqrt{3}$ $y = 20\sin 30° = 10$	$10\sqrt{3} + 10\mathring{j}$
$16\angle -130°$	$\longrightarrow$	$-10.28 - 12.26\mathring{j}$	
$12 + \mathring{j}30$	$\longrightarrow$	$32.31\angle 68.20°$	
$16 + \mathring{j}20$ $-12 - \mathring{j}2$	$\longrightarrow$	$2.11\angle -138.12°$	
$13\angle 12°$ $-6\angle 16°$	$\longrightarrow$	$-2.16 + 0.15\mathring{i}$	

Voltage Sources	Phasor form
$120\sin(60(2\pi)t + 30°)$	
$120\cos(60(2\pi)t - 60°)$	$120\angle -60°$
$120\cos(120\pi t + 60°)$	$120\angle 60°$

$22\cos(100\pi t + 20°) + 16\sin(100\pi t - 60°)$

$= 22\cos(100\pi t + 20°) + 16\cos(100\pi t - 150°)$

$= 22\angle 20° + 16\angle -150° = 5.07\angle -13.22° = 6.83\angle -3.99°$

$= \boxed{5.07\cos(100\pi t - 13.22°)} \quad = 6.83\cos(100\pi t - 3.99°)$

$15\cos(100t + 16°) + 20\cos(200t + 22°)$

$15\angle 16° + 20\angle 22° = 34.95\angle 19.43°$

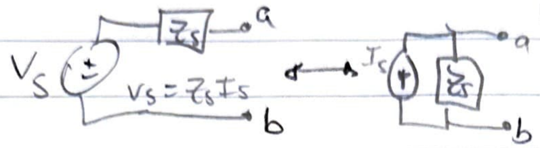
$= \boxed{34.95\cos(100\pi t + 19.43°)}$

$= 0.200\cos(100t - 180°) - 0.100\cos(100t + 12°)$

$= 0.2\angle -180° + -0.100\angle 12° = 0.30\angle -176°$

$= 0.30\cos(100t - 176°)$

9.7 Source Transformations & TH equivalents



Same as before (so is THeq) lol

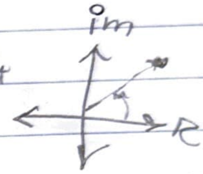
9.8 Node voltage w/ frequencies

↳ same as before lol ← Just remember to convert!

9.9 Mesh Current Method

↳ same as before lol

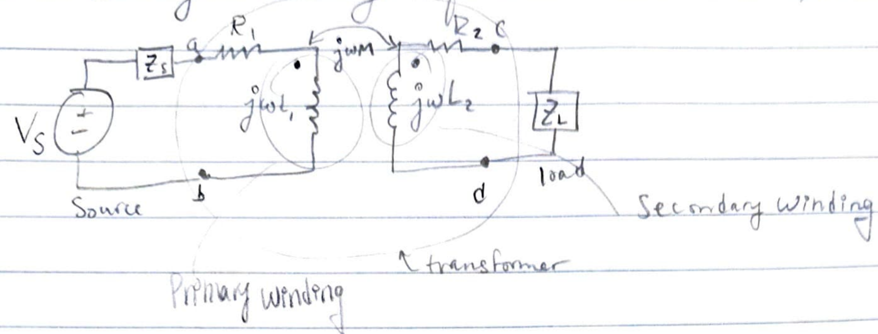
9.12 Phasor Diagrams → you know this



$y = 10\angle 45°$ or $y = 7.07 + 7.07j$

9.10 The Transformer (Linear)

- Used to eliminate DC signals
- Change voltage for transmission, distribution, & consumption



$$V_S = (Z_s + R_1 + j\omega L_1)I_1 - j\omega M I_2$$
$$0 = -j\omega M I_1 + (R_2 + j\omega L_2 + Z_L)I_2$$
$$Z_{11} = Z_S + R_1 + j\omega L_1$$
$$Z_{22} = R_2 + j\omega L_2 + Z_L$$
$$I_1 = \frac{Z_{22}}{Z_{11}Z_{22} + \omega^2 M^2} V_S$$
$$I_2 = \frac{j\omega M}{Z_{22}} I_1$$

$$\boxed{Z_{internal} = Z_{11} + \frac{\omega^2 M^2}{Z_{22}}}$$

...

$$Z_{ab} = Z_{11} + \frac{\omega^2 M^2}{Z_{22}} - Z_S = \boxed{R_1 + j\omega L_1 + \frac{\omega^2 M^2}{(R_2 + j\omega L_2 + Z_L)}}$$

$$Z_r \text{ (reflected imp.)} = \frac{\omega^2 M^2}{|Z_{22}|^2}\left[(R_2 + R_L) - j(\omega L_2 + X_L)\right]$$

"Reflected impedance"

Scaling factor for reflected impedance:

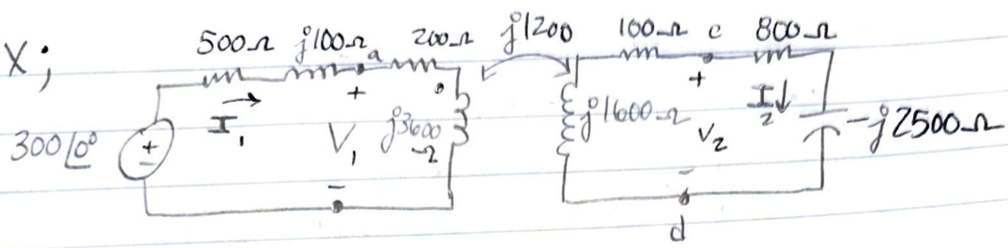
$$Z^*_{22} = \frac{\omega^2 M^2}{|Z_{22}|^2} = \frac{\omega^2 M^2}{|Z_{22}|^2}$$

$$Z_{22} = R_2 + R_L + j(\omega L_2 + X_L)$$

(Z_{22} is a factor of Z_r to some degree)

ex;

a.)



b.) Self-impedance of primary circuit:

$Z_{11} = 500 + j100 + 200 + j3600 = \boxed{700 + j3700\,\Omega}$

c.) Self impedance of secondary circuit:

$Z_{22} = 100 + j1600 + 800 - j2500 = \boxed{900 - j900\,\Omega}$

d.) Impedance reflected into primary winding:

$Z_r = \left(\dfrac{1200}{|900 - j900|}\right)^2 \cdot (900 + j900) = 800 + j800\,\Omega$

e.) Scaling factor : $Z_{22}^* = 8/9$

f.) Impedance looking into terminals

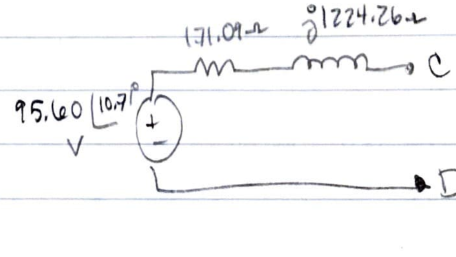
$Z_{ab} = 200 + j3600 + 800 + j800 = \boxed{1000 + j4400\,\Omega}$

g.) Calculate the Thevenin equivalent w/respect to the terminals of the load impedance

$I_1 = \dfrac{300\angle 0^\circ}{700 + j3700} = 79.67\angle -79.29^\circ\ mA$

? $V_{Th} = j1200\,(79.67\angle -79.29^\circ) \times 10^{-3}$
$= 95.60\angle 10.71^\circ\ V$

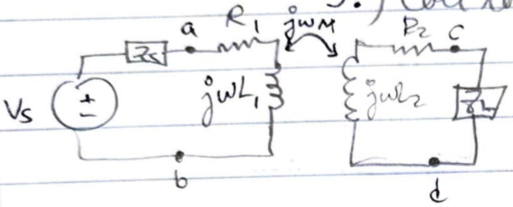
$Z_{Th} = 100 + j1600 + \left(\dfrac{1200}{|700 + j3700|}\right)^2 (700 - j3700)$
$= 171.09 + j1224.26\,\Omega$



9.11 The Ideal Transformer

ideal transformer - two coupled coils w/ N_1 & N_2 turns

1.) Coefficient of coupling $\rightarrow k=1$
2.) Self-inductance of each coil is $\infty \rightarrow L_1 = L_2 = \infty$
3.) Coil losses due to parasitic resistance are negligible



$$Z_{ab} = R_1 + j\omega L_1 + \frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_1} \qquad Z_{22} = R_2 + R_L + j(\omega L_2 + X_L) = R_{22} + jX_{22}$$

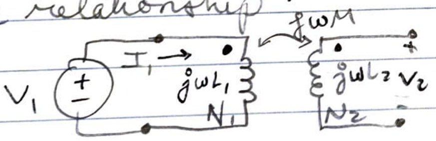
As $L_1 \to \infty$, $L_2 \to \infty$, $k \to 1$

$$X_{ab} = \left(\frac{N_1}{N_2}\right)^2 X_L$$

$$\frac{\omega^2 M^2 R_{22}}{R_{22}^2 + X_{22}^2} = \frac{L_1}{L_2} R_{22} = \left(\frac{N_1}{N_2}\right)^2 R_{22}$$

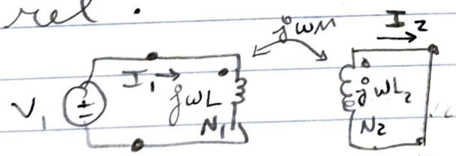
$$Z_{ab} = R_1 + \left(\frac{N_1}{N_2}\right)^2 R_2 + \left(\frac{N_1}{N_2}\right)^2 (R_L + jX_L)$$

Voltage relationship:

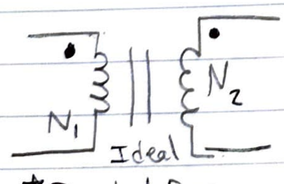


$$\frac{V_1}{N_1} = \frac{V_2}{N_2}$$

Current rel:



$$I_1 N_1 = I_2 N_2$$



*Symbol for Ideal transformer

Dot convention for Ideal Transformers

- If the coil voltages V_1 & V_2 are both + or both −, $\frac{V_1}{N_1} = \frac{V_2}{N_2}$
- If I_1 & I_2 are both directed into or out of dots, $I_1 N_1 = -I_2 N_2$

ex;

$\frac{V_1}{N_1} = \frac{V_2}{N_2}$; $N_1 I_1 = -N_2 I_2$

$\frac{V_1}{N_1} = -\frac{V_2}{N_2}$; $N_1 I_1 = N_2 I_2$

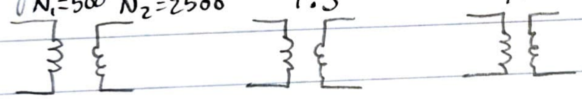
$\frac{V_1}{N_1} = \frac{V_2}{N_2}$; $N_1 I_1 = N_2 I_2$

$\frac{V_1}{N_1} = -\frac{V_2}{N_2}$; $N_1 I_1 = -N_2 I_2$

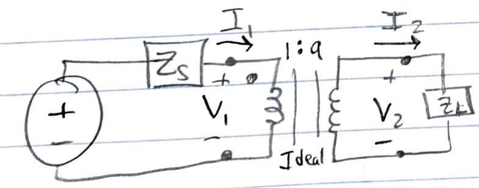
turn ratio : $a = \frac{N_2}{N_1}$

3 ways to show turn ratio

$N_1 = 500$ $N_2 = 2500$ 1:5   1/5 : 1

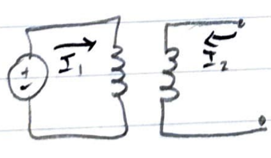


Using an Ideal Transformer for Impedance Matching

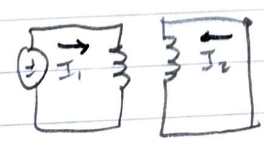


$V_1 = \frac{V_2}{a}$; $I_1 = aI_2$; $Z_{in} = \frac{1}{a^2} Z_L$

☆



← I_1 not affected bc $I_2 = 0$



← I_1 affected bc $I_2 \neq 0$ w/ the short

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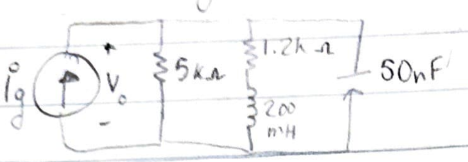
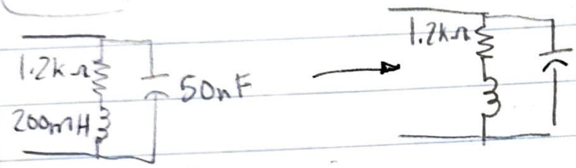
Problem 9.35 The frequency of the source voltage in the circuit is adjusted so V_o is in phase with i_g.



Part A What is the value of ω in rad/sec



$Z_L = j\omega L$

$C = \frac{1}{j\omega C}$

$$Y_{eq} = \frac{1}{R+j\omega L} + j\omega C$$

$$= \frac{1 + (j\omega C)(R+j\omega L)}{R+j\omega L} \cdot \frac{(R-j\omega L)}{(R-j\omega L)} = \frac{R-j\omega L + (j\omega C)(R+j\omega L)(R-j\omega L)}{R^2+\omega^2L^2}$$

$$= \frac{R-j\omega L + (j\omega C)(R^2+\omega^2L^2)}{R^2+\omega^2L^2}$$

$$= \frac{R}{R^2+\omega^2L^2} + \frac{-j\omega L + (j\omega C)(R^2+\omega^2L^2)}{R^2+\omega^2L^2}$$

$$0 = -j\omega L + (j\omega C)(R^2+\omega^2L^2)$$

$$= -j\omega L + j\omega CR^2 + j\omega^3L^2C$$

$$= \omega L + \omega CR^2 + \omega^3L^2C$$

$$0 = -L + CR^2 + \omega^2L^2C$$

$$\omega^2L^2C = -L - CR^2$$

$$\omega = \sqrt{\frac{L-CR^2}{L^2C}}$$

$$= \sqrt{\frac{(200\times10^{-3}) - (50\times10^{-9})(1.2\times10^3)}{(200\times10^{-3})^2(50\times10^{-9})}}$$

$$= \boxed{8{,}000 \text{ rad/s}}$$

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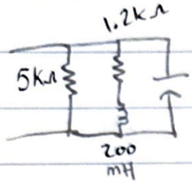
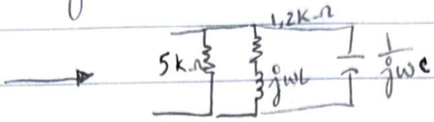
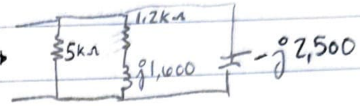
Part B

If $i_g = 7\cos\omega t$ mA & $V_o(t) = V_o\cos(\omega t + \phi)$, find V_o, ω, ϕ







$$Z_{eq} = 5k\Omega \,//\, 1.2k\Omega + j1{,}600 \,//\, -j2{,}500$$

$$Z_{eq} = 2{,}000$$

$$V_o = 14\cos(8{,}000t + 0)$$

$$\boxed{14, 8000, 0}$$

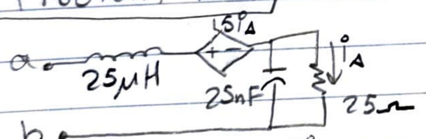
Video : Node voltage w/ frequency

Finding V as a function of t?

↳ **Write node-voltage equation**

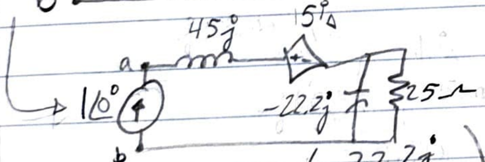
Problem 9.51

Find Z_{ab} when frequency = 1.8 M rad/s



$$Z_C = \frac{1}{j\omega C} = \frac{1}{j(1.8)(10^6)(25)(10^{-9})} = -22.2j$$

$$Z_L = j\omega L = j(1.8)(10^6)(25)(10^{-6}) = 45j$$



$$i_\Delta = 1\left(\frac{-22.2j}{25-22.2j}\right) = 0.441 - 0.496j$$

$$15i_\Delta = 6.61 - 7.45j = V_{DS}$$

$$15i_\Delta / 1\angle 0° = Z_{DS} = 6.61 - 7.45j$$

$$Z_{eq} = \frac{(25)(-22.2j)}{25-22.2j} + 6.61 - 7.45j + 45j = 17.6 + 25.1j = \boxed{30.7\angle 55.0°}$$

$\boxed{\text{Video: Mesh method w/ frequency}}$

Alternative for mesh-current method

↳ $\boxed{\text{combine the two middle branches}}$

$\boxed{\text{Video: Source trans.}}$

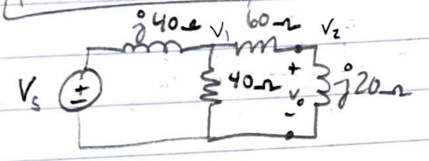
$\boxed{\text{Part A}}$ Find voltage across 30-Ω resistor

$I = 4.47\angle 26.565°A$

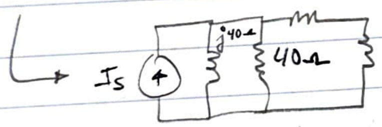
$V_o(t) = I \cdot -30 = \boxed{-134.1\cos(500t + 26.565)\ V}$

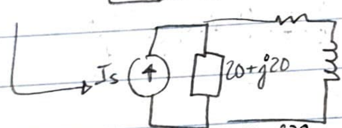
$\boxed{\text{Problem 9.54}}$ $V_s = 80\angle 0°\ V$

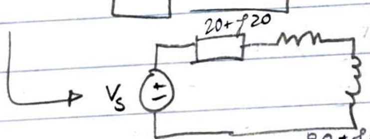
Use node-voltage to find V_o



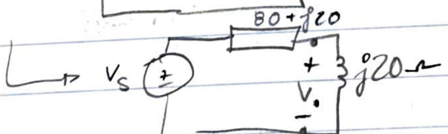
$I_s = \frac{V_s}{Z} = \frac{80\angle 0°}{j40} = 2\angle -90°$







$V_s = I_s Z = 2\angle -90° \cdot (20 + j20)$
$= 56.57\angle -45°$

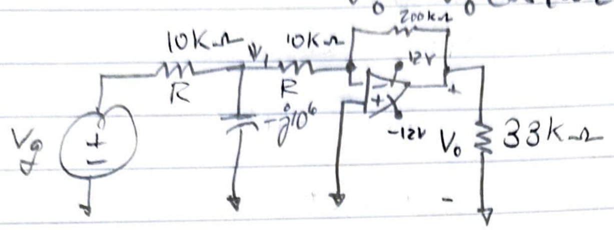


$V_o = V_s\left(\frac{j20}{80 + j20 + j20}\right) = \boxed{12.65\angle 18.4}$

4/7

$\boxed{\text{Problem 9.69}}$ $V_g = 2.8\cos 100t\ V$

$V_o(t) = V_o\cos(\omega t + \phi)$, where $-180° < \phi < 180°$



$1\mu F = \frac{1}{j\omega C} = -j10^4$

$V_g = 2.8\angle 0°$
$= 2.8 + 0j$

$\boxed{\text{Part A}}$ Find V_o

$$\frac{V_1 - V_g}{R} + \frac{V_1}{C} + \frac{V_1}{R} = 0$$

$$\frac{V_1}{R} - \frac{V_g}{R} + \frac{V_1}{C} + \frac{V_1}{R} = 0$$

$$\frac{V_1}{R} + \frac{V_1}{C} + \frac{V_1}{R} = \frac{V_g}{R}$$

$$V_1\left(\frac{1}{R} + \frac{1}{C} + \frac{1}{R}\right) = \frac{V_g}{R} \rightarrow V_1 = \frac{V_g}{R} \Big/ \left(\frac{1}{R} + \frac{1}{C} + \frac{1}{R}\right) = 1.12 - 0.56j$$

$$\frac{0 - V_1}{10k\Omega} + \frac{0 - V_o}{200k\Omega} = 0$$

$$\frac{-V_o}{200k\Omega} = \frac{V_1}{10k\Omega}$$

$$V_o = \frac{-V_1}{10\times10^3} \cdot 200\times10^3 = \boxed{25.0\angle 153°}$$

$\boxed{\text{Part B}}$ Find $\phi \rightarrow \boxed{\phi = 153°}$

$\boxed{\text{Part C}}$ Find $\omega \rightarrow \boxed{\omega = 100\ rad/s}$

5/7

Video: Mesh method w/ frequency

Find the voltage across 60mH inductor, + on left, with $I = -0.8 - j1.6$ A

$$Z = j\omega L = j300\,\Omega$$

$$V = IZ = (-0.8 - j1.6)(j300) = -480 - 240j = 536.7\angle{-26.7^\circ}$$

$$\boxed{v(t) = 536.7\cos(5000t - 26.7^\circ)}$$

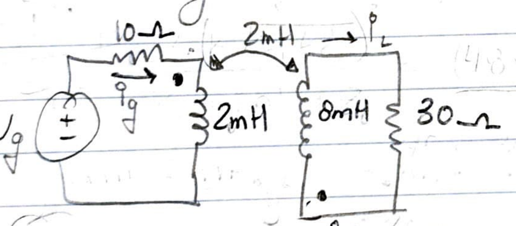
Video: Analyzing circuit w/ ideal transformer

Which pair of equations is from the video?

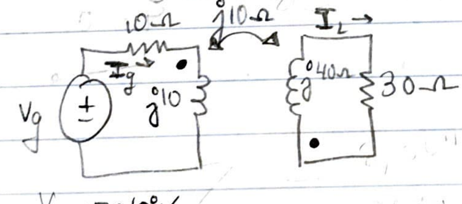
$$\boxed{\frac{V_1}{N_1} = -\frac{V_2}{N_2} \quad N_1 I_1 = -N_2 I_2}$$

Video: Linear Transformer

Voltage across 30Ω resistor, positive at top



$V_g = 70\cos 5000t$ V



$V_g = 70\angle 0^\circ$ V

$$-70\angle 0^\circ + 10I_g + j10I_g + j10I_L = 0$$

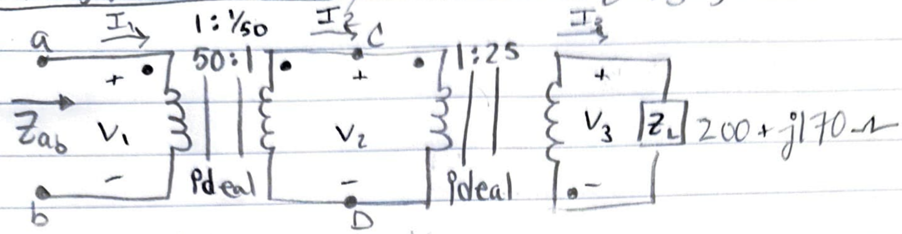
$$30I_L + j40I_L + j10I_g = 0$$

$$I_g = 4 - j3 = 5\angle{-36.87^\circ}$$

$$I_L = -1\text{A} = 1\angle 180^\circ\text{ A}$$

$$\boxed{V_{30\Omega} = 30\cos(5000t + 180^\circ)}$$

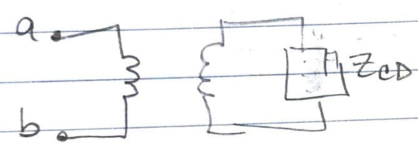
Problem 9.80 Find Z_{ab}; $Z_L = 200 + j170\,\Omega$



$V_1 = 50V_2$ $\qquad V_2 = \frac{-V_3}{25}$

$I_1 = \frac{1}{50} I_2$ $\qquad I_2 = -25I_3$

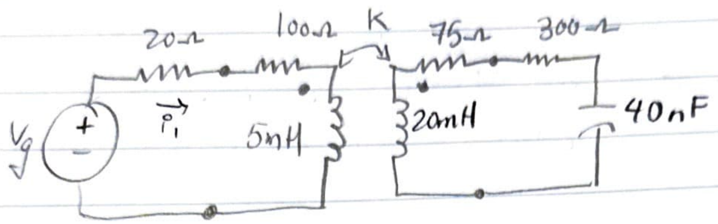
$$Z_{CD} = \left(\frac{N_1}{N_2}\right)^2 (200 + j170\,\Omega) = \left(\frac{1}{25}\right)^2 (200 + j170\,\Omega) = 0.32 + 0.272j$$



$$Z_{ab} = \left(\frac{N_1}{N_2}\right)^2 (0.32 + 0.272j) = (50)^2 \cdot (0.32 + 0.272j) = \boxed{800 + 680j\,\Omega}$$

7/7

Problem 9.74



V_g frequency : 48 k rad/s

$M = K\sqrt{L_1 L_2}$

Part A / What is the value of k?

$Z_{L_1} = j(5mH)(48k) = j240\Omega$

$Z_{L_2} = j(20mH)(48k) = j960\Omega$

$Z_M = jk\sqrt{(5mH)(20mH)} \cdot 48k = jk\,480\Omega$

$Z_C = -j\frac{1}{48k\,(40nF)} = -j520.83\Omega$

Mesh 1: $-V_g + 120i_1 + j240i_1 - jk480i_2 = 0$

Mesh 2: $375i_2 - j520.83i_2 + j960i_2 - jk480i_1 = 0$

$i_2(375 - j520.83 + j960) = jk480i_1$

$i_2 = \frac{jk480i_1}{375 + j439.17}$

$\frac{V_g}{i_1} = Z_{in} = 120 + j240 + \frac{k^2\,480^2}{375 + j439.17} = 120 + j240 + k^2(259.07 - j303.41)$

? → $|Z_{in}| = \sqrt{(120 + 259.07k^2)^2 + (240 - 303.41k^2)^2}$

$\frac{d|Z_{in}|}{dk} = 0 \rightarrow \boxed{k = 0.512}$

Part B / What is the peak amplitude of i_1 when $V_g = 429\cos(4.8\times10^4 t)$ V?

$i_1 = \frac{V_g}{Z_{in}} = \frac{429\cos(48{,}000t)}{} = \frac{429}{120 + j240 + (0.512)^2(259.07 - 303.41)}$

$= 1.74\angle -40.5°$

$\boxed{i_{1peak} = 1.74\text{ A}}$

12.1 Definition of Laplace Transform

$$F(s) = \mathcal{L}[f(t)]$$

↳ transforms from time to frequency domain

- We solve problem in frequency and then convert it back

Functional transform → of a specific function

Operational " → specific math stuff

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt$$

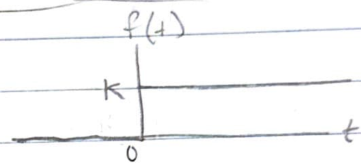
t → time domain

s → frequency domain

12.2 The Step Function

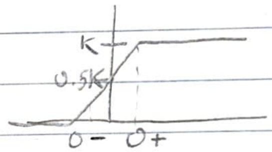
$Ku(t) = 0, \quad t<0$

$Ku(t) = K, \quad t>0$



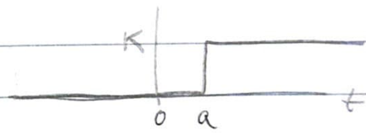
* Not defined at $t=0$. If we need to, then we can say

$Ku(0) = 0.5K$



$Ku(a-t) = K, \quad t<a$

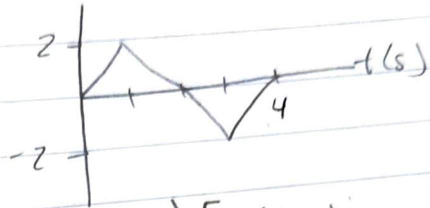
$Ku(a-t) = 0, \quad t>a$



$$K\Big[\underbrace{u(t-1)}_{\text{"turning on"}} - \underbrace{u(t-3)}_{\text{"turning off"}}\Big] = K, \quad 1<t<3$$

ex; Using Step function for Finite Duration

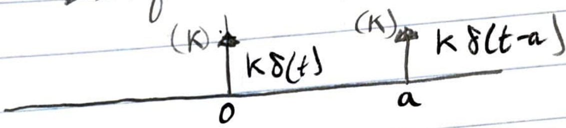
$$f(t) = \begin{cases} 0 & t \leq 0 \\ 2t & 0 \leq t \leq 1 \\ -2t+4 & 1 \leq t \leq 3 \\ 2t-8 & 3 \leq t \leq 4 \\ 0 & t \geq 4 \end{cases}$$



$$f(t) = 2t\left[u(t) - u(t-1)\right] + (-2t+4)\left[u(t-1) - u(t-3)\right] + (2t-8)\left[u(t-3) - u(t-4)\right]$$

12.3 The Impulse Function

→ infinite amplitude, zero duration



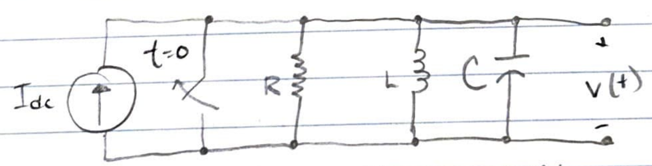
12.4 Functional Transform

Type	$f(t)\ (t > 0^-)$	$F(s)$
impulse	$\delta(t)$	1
step	$u(t)$	$1/s$
ramp	t	$1/s^2$
exponential	e^{-at}	$\frac{1}{s+a}$
sine	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
cosine	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
damped ramp	te^{-at}	$\frac{1}{(s+a)^2}$
damped sine	$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
damped cosine	$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
	$\cosh at$	$= \frac{s}{s^2-a^2}$
	$\sinh at$	$= \frac{a}{s^2-a^2}$

12.5 Operational Transforms

Operation	$f(t)$	$F(s)$
Mult. by a constant	$Kf(t)$	$KF(s)$
Add/Sub	$f_1(t) \pm f_2(t) \ldots$	$F_1(s) \pm F_2(s) \ldots$
First der.	$f'(t)$	$sF(s) - f(0)$
Sec der.	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
nth der.	$f^n(t)$	$s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - s^{n-3}f''(0) - \ldots - f^{n-1}(0)$
integral	$\int_0^t f(x)\,dx$	$F(s)/s$
translation in time	$f(t-a)u(t-a), a>0$	$e^{-as}F(s)$
translation in frequency	$e^{-at}f(t)$	$F(s+a)$
Scale changing	$f(at), a>0$	$\frac{1}{a}F(s/a)$
First der. (s)	$tf(t)$	$-F'(s)$
nth der. (s)	$t^nf(t)$	$(-1)^nF^n(s)$
s integral	$f(t)/t$	$\int_s^\infty F(u)\,du$

12.6 Applying the Laplace Transform



step function, current turns on at t=0

Sum of currents: $\frac{v(t)}{R} + \frac{1}{L}\int_0^t v(x)\,dx + C\frac{dv(t)}{dt} = I_{dc}u(t)$

Laplace Transform: $\frac{V(s)}{R} + \frac{1}{L}\frac{V(s)}{s} + C[sV(s) - v(0)] = I_{dc}\left(\frac{1}{s}\right)$

$$V(s) = \frac{I_{dc}/C}{s^2 + (1/RC)s + (1/LC)}$$

$v(t) = \mathcal{L}^{-1}[V(s)]$ ← Next Section

time domain

$\mathcal{L}[v] = V$ or $v = \mathcal{L}^{-1}[V]$

$\mathcal{L}[i] = I$ or $i = \mathcal{L}^{-1}[I]$

$\mathcal{L}[f] = F$ or $f = \mathcal{L}^{-1}[F]$

frequency domain

Seth Ricks | ECEN 250 | HW #9

1

The Step Function

Part A plot function $f(t) = 2tu(t) - 2(t-1)u(t-1) - 2(t-3)u(t-3) + 2(t-4)u(t-4)$ over $0 \le t \le 5$

when $t=0$, all terms are 0

$t=1$, only $2tu(t)$ is nonzero (all others not turned on yet, $2tu(t)$ still on)

↳ $f(1)=2$

$t=3$; $2tu(t)$ & $-2(t-1)u(t-1)$ are on

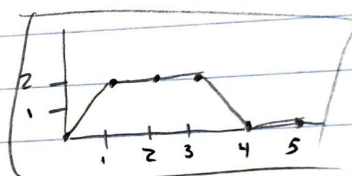
↳ $f(3)=2$

$t=4$, only $2(t-4)u(t-4)$ is off (all other are on)

↳ $f(4)=0$

$t=5$; all terms are on

↳ $f(5)=0$



Part B plot $f(t) = 4u(t) - 7tu(t) + 12(t-1)u(t-1) - 8(t-2)u(t-2) + 4(t-3)u(t-3) - (t-4)u(t-4)$ over $0 \le t \le 5$

$t=0$; $f(0)=4$

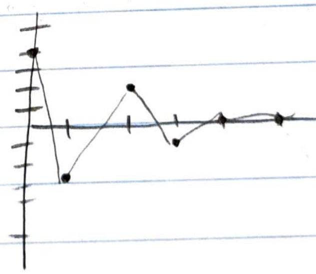
$t=1$; $f(1)=-3$

$t=2$; $f(2)=2$

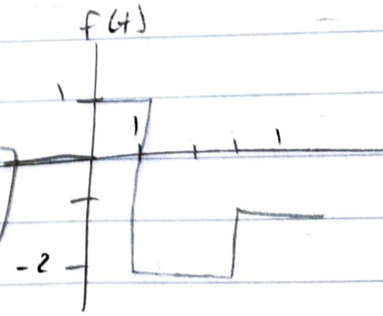
$t=3$; $f(3)=-1$

$t=4$; $f(4)=0$

$t=5$; $f(5)=0$

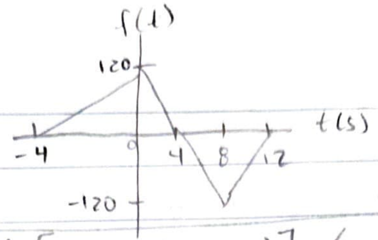


Part C Make function for graph



$$\boxed{f(t) = 1u(t) - 1u(t-1) + -2u(t-1) + 2u(t-3) - 1u(t-3)}$$

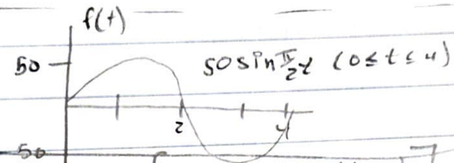
Problem 12.1



Part A

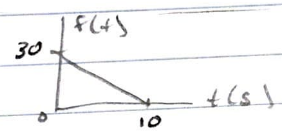
$$\boxed{f(t) = (120+30t)[u(t+4)-u(t)] + (120-30t)[u(t)-u(t-8)] + (30t-360)[u(t-8)-u(t-12)]}$$

Part B



$$\boxed{f(t) = (50\sin\tfrac{\pi}{2}t)[u(t)-u(t-4)]}$$

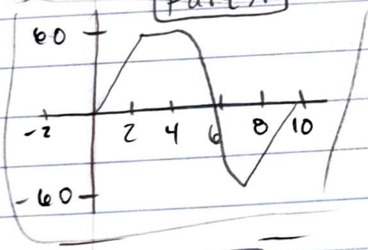
Part C



$$\boxed{f(t) = (30-3t)[u(t)-u(t-10)]}$$

Problem 12.4

Part A



$$
\begin{aligned}
f(t) &= 0 && t \le 0; \\
&= 30t && 0 \le t \le 2s \\
&= 60 && 2s \le t \le 4s \\
&= 60\cos(\tfrac{\pi}{4}t-\pi) && 4s \le t \le 8s \\
&= 30t-300 && 8s \le t \le 10s \\
&= 0 && 10s \le t \le \infty
\end{aligned}
$$

Part B

$$\boxed{f(t) = 30t[u(t)-u(t-2)] + 60[u(t-2)-u(t-4)] + 60\cos(\tfrac{\pi}{4}t-\pi)[u(t-4)-u(t-8)] + (30t-300)[u(t-8)-u(t-10)]}$$

3

The Impulse Function

Part A

$$f(t) = \begin{cases} 0, & t < -\epsilon \\ \frac{1}{2\epsilon} t, & -\epsilon < t < \epsilon \\ 1, & t > \epsilon \end{cases}$$

The total area under the first derivative is 1
In the limit $\epsilon \to 0$, the first derivative is zero everywhere except $t = 0$

Part B

$$\int_{-\infty}^{\infty} (8t + 7)\,\delta(t - 4.0)\,dt$$
$$= 8(4.0) + 7 = \boxed{39}$$

Problem 12.6 Explain why the following function generates an impulse function as $\epsilon \to 0$

$$f(t) = \frac{\epsilon/\pi}{\epsilon^2 + t^2}, \quad -\infty \le t \le \infty$$

* As $\epsilon \to 0$ the amplitude $\to \infty$; and the area is independent of ϵ
i.e. $A = \int_{-\infty}^{\infty} \frac{\epsilon}{\pi} \frac{1}{\epsilon^2 + t^2} dt = 1$

$\boxed{\text{Problem 12.13}}$ Find Laplace Transform

$\boxed{\text{Part A}}$ $f(t) = t$

$\mathcal{L}[f(t)] = \boxed{\frac{1}{s^2}}$

$\boxed{\text{Part B}}$ $f(t) = te^{-at}$

$\mathcal{L}[f(t)] = \boxed{\frac{1}{(s+a)^2}}$

$\boxed{\text{Part C}}$ $f(t) = \sin \omega t$

$\mathcal{L}[f(t)] = \boxed{\frac{\omega}{s^2+\omega^2}}$

$\boxed{\text{Part D}}$ $f(t) = \cosh \beta t$

$\mathcal{L}[f(t)] = \boxed{\frac{s}{s^2-\beta t}}$

$\boxed{\text{Part E}}$ $f(t) = \sinh \beta t$

$\mathcal{L}[f(t)] = \boxed{\frac{\beta}{s^2-\beta^2}}$

$\boxed{\text{Video: Partial Fraction}}$

If $F(s)$ is a rational function where the num. and denom. are of the same order, the $\mathcal{L}^{-1}[F(s)]$ will always contain

↳ $\boxed{\text{an impulse function}}$

$\boxed{\text{Video: Partial Fraction}}$

When the denominators of two terms in a partial fraction are complex conjugates, the associated

↳ $\boxed{\text{also complex conjugates}}$

$\boxed{\text{Video: Partial Fraction}}$

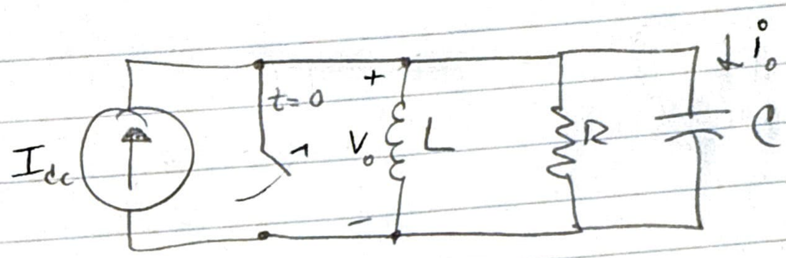
Evaluating $F(s)(s+1)$ at $s=-1$ → why NOT zero?

↳ $\boxed{\text{because } F(s) \text{ has } (s+1) \text{ in denominator}}$

$\boxed{\text{Video}}$ when $(s+a)^2$ in denominator, what denominators in expansion?

↳ $\boxed{(s+a) \text{ and } (s+a)^2}$

Problem 12.28



Part A Derive the integrodifferential equation that governs the behavior of the voltage V_o

$$\boxed{\frac{1}{L}\int_0^t V_o(x)\,dx + \frac{V_o(t)}{R} + C\frac{dV_o(t)}{dt} = I_{dc}\,u(t)}$$

Part B Find $V_o(s)$

$$\mathcal{L}[\,] \longrightarrow \frac{V(s)}{R} + \frac{1}{L}\frac{V(s)}{s} + C[sV(s) - V(0)] = I_{dc}\left(\frac{1}{s}\right)$$

$$\boxed{V(s) = \frac{I_{dc}/C}{s^2 + (1/RC)s + (1/LC)}}$$

Part C

$$I_o(s) = C\frac{dV_o(t)}{dt}$$

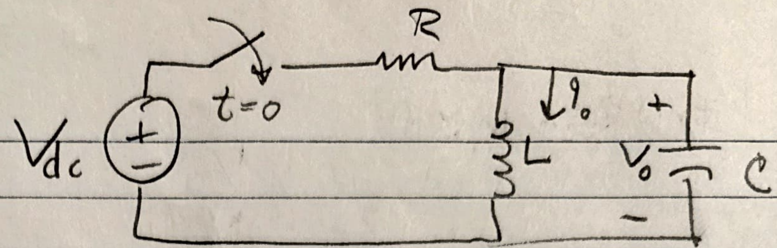
$$= C[sV(s) - V_o]$$

$$= \boxed{\frac{I_{dc}\,s}{s^2 + (1/RC)s + (1/LC)}}$$

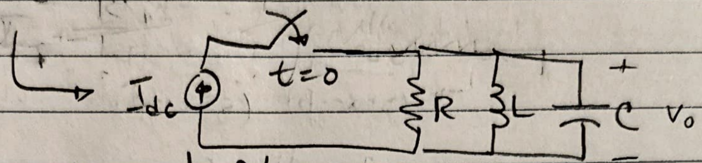
6

Problem 12.29

Part A



Find V_o for $t \geq 0$



$$I_{dc} = \frac{V_o}{R} + \frac{1}{L}\int_0^t v_o(x)\,dx + C\frac{dv_o}{dt}$$

$$\boxed{V_{dc} = V_o + \frac{R}{L}\int_0^t V_o\,d\tau + RC\frac{dV_o}{dt}} \leftarrow (V_{dc}u(t))$$

Part B

Find $V_o(s)$

$$\mathcal{L}[\,] \rightarrow \frac{V_{dc}}{s} = V(s) + R/L\,\frac{V(s)}{s} + RC\,[sV(s) - \cancel{V(0)}]$$

$$\frac{V_{dc}}{s} = V(s) + V(s)\left(\frac{R}{Ls}\right) + V(s)(RCs) - RCV(0)$$

$$\frac{V_{dc}}{s} = V(s)\left[1 + R/Ls + RCs\right]$$

$$\boxed{V(s) = \frac{V_{dc}}{s + \frac{R}{L} + RCs^2}}$$

Part C

Find $I_o(s)$

$$i_o = \frac{1}{L}\int_0^t v_o(x)\,dx$$

$$\mathcal{L}[i_o] = I_o = \frac{1}{L}\frac{V(s)}{s} = \frac{V_{dc}}{s+\frac{R}{L}+RCs^2}\cdot\frac{1}{Ls} = \boxed{\frac{V_{dc}}{Ls^2+Rs+RCLs^3}}$$

Video: Initial & Final Value Theorem

If $f(t)$ is the $\mathcal{L}^{-1}[F(s)]$, then by the final value theorem, the long-term, steady-state value of $f(t)$ equals

→ $\boxed{\text{the value of } sF(s) \text{ as } s \text{ approaches } 0}$

12.7 inverse laplace

$u(t)$ is bc it is a real-world problem that starts at $t=0$!

Root type		$F(s)$	$f(t)$
Distinct real	polar	$K/_{s+a}$	$Ke^{-at}u(t)$
repeated real		$K/_{(s+a)^2}$	$Kte^{-at}u(t)$
Distinct complex		$\frac{K}{s+\alpha-j\beta}+\frac{K^*}{s+\alpha+j\beta}$	$2\lvert K\rvert e^{-\alpha t}\cos(\beta t+\theta)u(t)$
Repeated complex		$\frac{K}{(s+\alpha-j\beta)^2}+\frac{K^*}{(s+\alpha+j\beta)^2}$	$2t\lvert K\rvert e^{-at}\cos(\beta t+\theta)u(t)$
	General:	$\frac{K}{(s+a)^r}$	$\frac{Kt^{r-1}e^{-at}}{(r-1)!}u(t)$

Partial Fractions:

1.) $\frac{px+q}{(x-a)(x-b)}$, $a\neq b$ $\quad \frac{A}{x-a}+\frac{B}{x-b}$

2.) $\frac{px+q}{(x-a)^2}$ $\quad \frac{A}{x-a}+\frac{B}{(x-a)^2}$

3.) $\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$ $\quad \frac{A}{x-a}+\frac{B}{x-b}+\frac{C}{x-c}$

4.) $\frac{px^2+qx+r}{(x-a)^2(x-b)}$ $\quad \frac{A}{x-a}+\frac{B}{(x-a)^2}+\frac{C}{x-b}$

5.) $\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$ $\quad \frac{A}{x-a}+\frac{Bx+C}{x^2+bx+c}$

Non-factorable

6.) $$\frac{A(s+z_1)}{(s+\alpha-j\beta)(s+\alpha+j\beta)}=\frac{K}{s+\alpha-j\beta}+\frac{K^*}{s+\alpha+j\beta}$$

7.) $$\frac{A(s+z_1)}{(\alpha+j\beta)^r}=\frac{K}{(s+\alpha-j\beta)^r}+\frac{K^*}{(s+\alpha+j\beta)^r}$$

Ilsabel Ricks

ex; $F(s) = \dfrac{96(s+5)(s+12)}{s(s+8)(s+6)} = \dfrac{120}{x} + \dfrac{48}{x+6} - \dfrac{72}{x+8}$

$f(t) = \mathcal{L}^{-1}[F(s)] = (120 - 78e^{-8t} + 48e^{-6t})u(t)$

ex; $F(s) = \dfrac{100(s+3)}{(s+6)(s^2+6s+25)} = \dfrac{-12}{x+6} + \dfrac{6+8i}{x+3+4i} + \dfrac{6-8i}{x+3-4i} = \dfrac{-12}{x+6} + \dfrac{10\angle -53.13^\circ}{x+3+4i} + \dfrac{10\angle 53.13^\circ}{x+3-4i}$

$f(t) = \mathcal{L}^{-1}[F(s)] = [-12e^{-6t} + 20e^{-3t}\cos(4t - 53.13^\circ)]u(t)$

ex; $F(s) = \dfrac{100(s+25)}{s(s+5)^3} = \dfrac{20}{x} - \dfrac{400}{(x+5)^3} - \dfrac{100}{(x+5)^2} - \dfrac{20}{x+5}$

$f(t) = \mathcal{L}^{-1}[F(s)] = [20 - 200t^2e^{-5t} - 100te^{-5t} - 20e^{-5t}]u(t)$

ex; $F(s) = \dfrac{768}{(s^2+6s+25)^2} = \left[\dfrac{-12}{(s+3-j4)^2} + \dfrac{-12}{(s+3+j4)^2}\right] + \left[\dfrac{3\angle -90^\circ}{s+3-j4} + \dfrac{3\angle 90^\circ}{s+3+j4}\right]$

$f(t) = \mathcal{L}^{-1}[F(s)] = [-24te^{-3t}\cos 4t + 6e^{-3t}\cos(4t - 90^\circ)]u(t)$

12.8 Poles and Zeros of F(s)

$F(s) = \dfrac{k(s+z_1)(s+z_2)\dots(s+z_n)}{(s+p_1)(s+p_2)\dots(s+p_m)}$

$-p_1, -p_2, -p_3, \dots -p_m \longrightarrow$ poles of F(s)

$-z_1, -z_2, -z_3, \dots -z_n \longrightarrow$ zeros of F(s)

12.9 Initial-and Final-Value Theorems

Initial Value Theorem: $\lim_{t \to 0^+} f(t) = \lim_{s \to \infty} sF(s)$

Final-Value Theorem: $\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$

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Problem 12.42

Part A

Find $f(t)$ for $F(s) = \frac{6s^2+26s+26}{(s+1)(s+2)(s+3)} = \frac{3}{x+1} + \frac{2}{x+2} + \frac{1}{x+3}$

$$\mathcal{L}^{-1}[F(s)] = \boxed{\left[3e^{-t} + 2e^{-2t} + e^{-3t}\right]u(t) = f(t)}$$

Part B

Find $f(t)$ for $F(s) = \frac{13s^3+134s^2+392s+288}{s(s+2)(s^2+10s+24)} = \frac{6}{x} + \frac{4}{x+2} + \frac{2}{x+4} + \frac{1}{x+6}$

$$\mathcal{L}^{-1}[F(s)] = \boxed{\left[6 + 4e^{-2t} + 2e^{-4t} + e^{-6t}\right]u(t) = f(t)}$$

Part C

Find $f(t)$ for $F(s) = \frac{10(s^2+119)}{(s+5)(s^2+10s+169)} =$

$$= \frac{10}{x+5} + \frac{\frac{-25j}{6}}{x+5+12j} + \frac{\frac{25j}{6}}{x+5-12j} = \frac{10}{x+5} + \frac{4.17\angle -90^\circ}{x+5+12j} + \frac{4.17\angle 90^\circ}{x+5-12j}$$

$$\mathcal{L}^{-1}[F(s)] = \boxed{f(t) = \left[10e^{-5t} - 8.33e^{-5t}\cos(12t + 90)\right]u(t)}$$

Part D

Find $f(t)$ for $F(s) = \frac{56s^2+1125s+5000}{s(s^2+14s+625)} = \frac{8}{x} + \frac{\frac{25\angle -16.3^\circ}{24-7j}}{x+7+24j} + \frac{\frac{25\angle 16.3^\circ}{24+7j}}{x+7-24j}$

$$\mathcal{L}^{-1}[F(s)] = \boxed{f(t) = \left[8 + 50e^{-7t}\cos(24t - 16.3^\circ)\right]u(t)}$$

2

RADIANS!!

Problem 12.45

Part A Find $f(t)$ for $F(s) = \dfrac{100(s+1)}{s^2(s^2+2s+5)}$

$$= \frac{20}{s^2} + \frac{12}{s} + \frac{-6-8j}{x+1+2j} + \frac{-6+8j}{x+1-2j} = \frac{20}{s^2} + \frac{12}{s} + \frac{10\angle -2.21}{x+1+2j} + \frac{10\angle 2.21}{x+1-2j}$$

$$\mathcal{L}^{-1}[F(s)] = f(t) = \left[20t + 12 + 20e^{-t}\cos(2t+2.21)\right]\cdot u(t)$$

Part B Find $f(t)$ for $F(s) = \dfrac{40(s+2)}{s(s+1)^3}$

$$= \frac{80}{x} - \frac{40}{(x+1)^3} - \frac{80}{(x+1)^2} - \frac{80}{x+1}$$

$$\mathcal{L}^{-1}[F(s)] = \boxed{f(t) = \left[80 - 20t^2e^{-t} - 80te^{-t} - 80e^{-t}\right]u(t)}$$

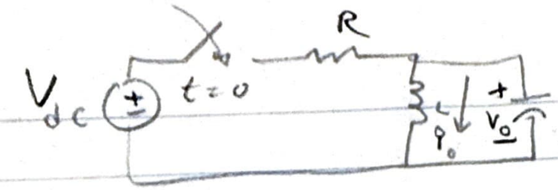
Part C Find $f(t)$ for $F(s) = \dfrac{5s^2+29s+32}{(s+2)(s+4)}$

$$= 5 - \frac{3}{x+2} + \frac{2}{x+4} \qquad \mathcal{L}^{-1}[F(s)] = \boxed{f(t) = \left[5\delta t - 3e^{-2t} + 2e^{-4t}\right]u(t)}$$

Part D $F(s) = \dfrac{2s^3+8s^2+2s-4}{s^2+5s+4} = 2x - 2 + \dfrac{4}{x+4}$

$$\mathcal{L}^{-1}[F(s)] = f(t) = \left[2\delta'(t) - 2\delta(t) + 4e^{-4t}\right]\cdot u(t)$$

Problem 12.5 /



$$V_o(s) = \frac{V_{dc}/RC}{s^2 + (1/RC)s + (1/LC)} \quad ; \quad I_o(s) = \frac{V_{dc}/RLC}{s[s^2+(1/RC)s+(1/LC)]}$$

Part A | Find final value of the voltage

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s) = \boxed{0}$$

Part B / Find initial value of the voltage

$$\lim_{t\to 0^+} f(t) = \lim_{s\to\infty} sF(s) = \boxed{0}$$

Part D | Find initial value of the current

$$\lim_{t\to 0^+} f(t) = \lim_{s\to\infty} sF(s) = \boxed{0}$$

Part C / Find final value of the current

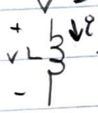
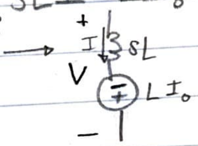
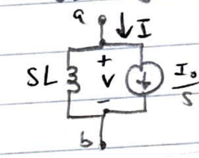
$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s) = \boxed{V_{dc}/R}$$

13.1 Circuit Elements in the S Domain

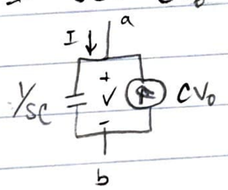
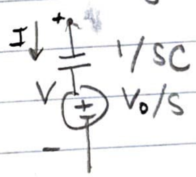
S-domain voltage → Volt-seconds
" Current → A-seconds
" ohms → ohms

Resistor → the same

Inductor → $V = SLI - LI_0$ $\qquad I = \frac{V}{SL} + \frac{I_0}{S}$

Capacitor → $I = SCV - CV_0$ $\qquad V = \left(\frac{1}{SC}\right) I + \frac{V_0}{S}$

13.2 Circuit Analysis in the s Domain

- If no energy is stored in the capacitor or inductor, then $V = ZI$
  - ↳ so $Z_L = sL$ ohms,
    $Z_C = 1/sC$ ohms

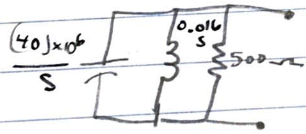
Laplace Transform Method

1.) Determine initial conditions for Capacitors & Inductors
2.) Laplace-transform independent v & i functions
3.) transform time domain to s domain
4.) analyze s-domain circuit
5.) use initial & final value theorems to check voltages & currents
6.) Inverse-Laplace

ex; 500 Ω resistor, 16 mH inductor, 25 nF capacitor in //

a.) Express total admittance

★ No initial conditions



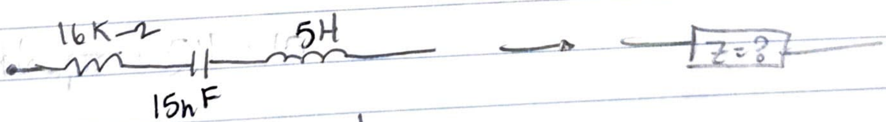
$Z_R = 500\,\Omega$

$Z_L = 0.016s\,\Omega$

$Z_C = \frac{40\times10^6}{s}\,\Omega$

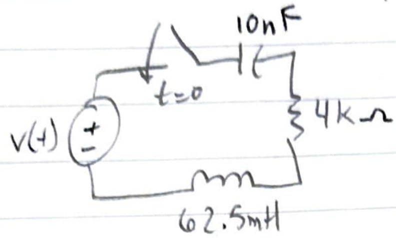
$$Y_{eq} = \frac{1}{500} + \frac{1}{0.016s} + \frac{s}{40\times10^6} = \frac{s^2 + 80{,}000s + 25\times10^8}{40\times10^6 s}\ S$$

ex;



$$Z = 16K\,\Omega + \frac{1}{(15\times10^{-9})s} + 5s$$

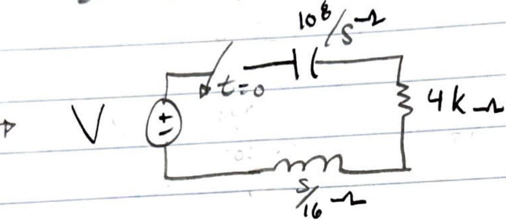
13.3.4 Circuit w/ a sinusoidal source — find i(t)



$v(t) = 15\cos 40{,}000t \text{ V}$

* Initial energy is zero

$$V = \mathcal{L}[v(t)] = \frac{15s}{s^2+40{,}000^2} \text{ V-s}$$

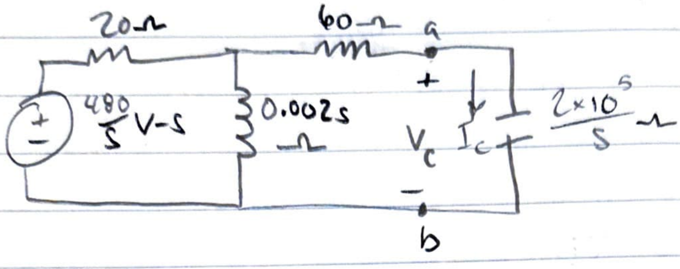


$$I = \frac{V}{Z} = \frac{240s^2}{(s^2+40{,}000^2)(s^2+64{,}000s+16\times10^8)} \text{ A-s}$$

$$= \frac{1.875\times10^{-3}}{s-j40{,}000} + \frac{1.875\times10^{-3}}{s+j40{,}000} + \frac{3.125\times10^{-3}\angle -126.87^\circ}{s+32{,}000-j24{,}000} + \frac{3.125\times10^{-3}\angle 126.87^\circ}{s+32{,}000+j24{,}000}$$

$$\mathcal{L}^{-1}[I] = \left[3.7\cos 40{,}000t + 6.25e^{-32{,}000t}\cos(24{,}000t - 126.87^\circ)\right]u(t) \text{ mA}$$

13.3.6 Creating a Thevenin Equivalent in S Domain



− If ab is open, 60Ω resistor is turned off

(Voltage Divider) $V_{Th} = \frac{(480/s)(0.002s)}{20+0.002s} = \frac{480}{S+10^4}$

$$Z_{Th} = 60 + \frac{(0.002s)(20)}{20+0.002S} = \frac{80(S+7500)}{S+10^4}$$

$$I_c = \frac{480/S+10^4}{\frac{80(S+7500)}{S+10^4} + \frac{2\times10^5}{S}} = \frac{6s}{(S+5000)^2} = \frac{-30{,}000}{(S+5000)^2} + \frac{6}{S+5000}$$

$$i_c = (-30{,}000te^{-5000t} + 6e^{-5000t})u(t)A$$

$$V_c = \frac{1}{sC} I_c = \frac{2\times10^5}{S} \cdot \frac{6s}{(S+5000)^2} = \frac{12\times10^5}{(S+5000)^2}$$

$$V_c = 12\times10^5 te^{-5000t}u(t)V$$

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Circuit Elements in the s-Domain

Part A)

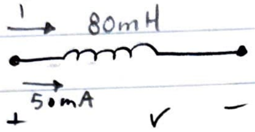
Find $I(s)$

$V(s) = \frac{50}{s+5000}$

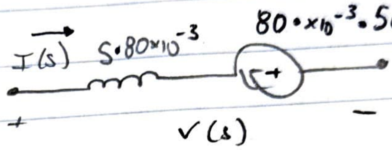
$Z = 2.5\,k\Omega$

$\boxed{I(s) = \frac{V(s)}{Z} = \frac{0.02}{s+5000}}$

Part B



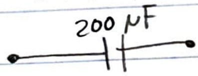
$I_0 = 50mA$

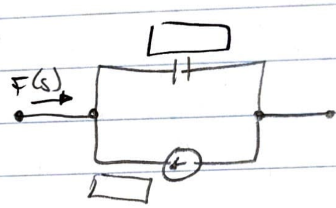


$\boxed{0.08s,\ 0.004\,V}$

Part C

$V_0 = 25V$





$\frac{1}{sC} = \frac{1}{s(200\times10^{-6})} = \boxed{\frac{1}{s}\cdot 5{,}000}$

$CV_0 = (200\times10^{-6})\cdot 25 = \boxed{0.005\,A}$

Part D

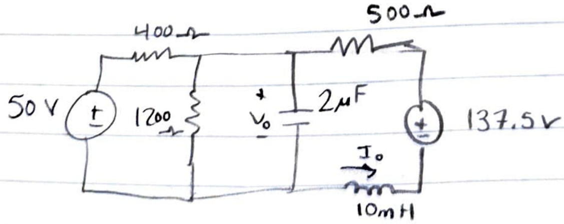
$v(t) = 5\cos 2000t\ V$ $\quad C = 2.5\mu F \quad L = 50mH$

$\mathcal{L}[v(t)] = \boxed{\frac{5s}{s^2+4\times10^6}}$

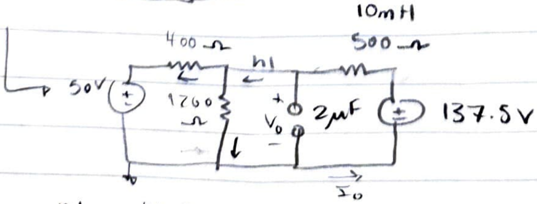
$\mathcal{L}[C] = \frac{1}{sC} = \boxed{\frac{400000}{s}\ \Omega}$

$\mathcal{L}[L] = sL = \boxed{s50\times10^{-3}}$

Part E



$\frac{75}{1200} + \frac{75-50}{400} = \boxed{I_0 = 125mA}$



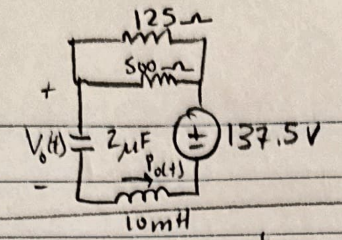
$\frac{n1}{1200} + \frac{n1-50}{400} + \frac{n1-137.5}{500} = 0$

$\frac{n1}{1200} + \frac{n1}{400} - \frac{50}{400} + \frac{n1}{500} - \frac{137.5}{500} = 0$

$\frac{2n}{375} = 0.4$

$\boxed{n1 = 75V}$

Part E



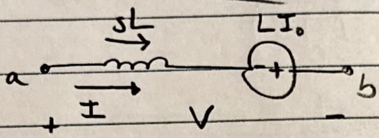
$Z_R = 100$ $\quad Z_C = \frac{5 \times 10^6}{s}$ $\quad Z_L = sL = 10 \times 10^{-3} s$

$V_c(s) = \frac{V_o}{s} = \frac{75V}{s}$

$V_L(s) = LI_o = 1.25 mV$ $\quad V(t) = 137.5 u(t)$ (bc of switch)

$V(s) = \frac{137.5}{s}$

Problem 13.2



Part A

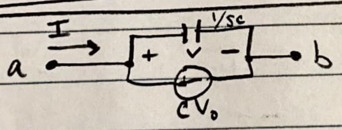
Find Norton equivalent of the circuit

$$I_n = \frac{-LI_o}{sL} = \boxed{\frac{-I_o}{s}}$$

Part B Calculate impedance equivalent

$$Z_n = \boxed{sL}$$

Problem 13.3



Part A Find equivalent voltage

$$V_{Th} = CV_o \cdot \frac{1}{sC} = \boxed{\frac{V_o}{s}}$$

Part B Calculate equivalent impedance

$$Z_{Th} = \boxed{\frac{1}{sC}}$$

★ Why do the voltage sources not have resistance?

Problem 13.5 16k-Ω, 5 H, 15nF in series

Part A Find s-domain impedance total

$Z_R = 16k\Omega$

$Z_L = 5s$

$Z_C = \frac{6.7\times10^7}{s}$

$\boxed{Z_T = \frac{5}{s}(s^2+3200s+1.3\times10^7)}$

Part B Numerical pole

$\boxed{p = 0 \text{ rad/s}}$

Part C Numerical zeros

$\boxed{-z_1, -z_2 = -1600+3280j, -1600-3280j \text{ rad/s}}$

$$F(s) = \frac{100(s+1)}{s^2(s^2+2s+5)} = \frac{K_1}{s^2} + \frac{K_2}{s} + \frac{K_3}{s+1+2j} + \frac{K_4}{s+1-2j}$$

$$100(s+1) = K_1\left[(s^2+2s+5)\right] + K_2\left[(s)(s^2+2s+5)\right]$$

$$+ K_3\left[(s^2)(s+1-2j)\right] + K_4\left[(s^2)(s+1+2j)\right]$$

$$100s+100 = K_1\left[s^2+2s+5\right] + K_2\left[s^3+2s^2+5s\right] + K_3\left[s^3+s^2-2s^2j\right] + K_4\left[s^3+s^2+2s^2j\right]$$

$$K_3 s^3 + s^2\left[K_3 - 2K_3 j\right]$$

$$K_3(1-2j)$$

$K_2 + K_3 + K_4 = 0$

$K_1 + 2K_2 + (1-2j)K_3 + (1+2j)K_4 = 0$

$2K_1 + 5K_2 = 100$

$K_1 5 = 100$

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 2 & 1-2i & 1+2i & 0 \\ 2 & 5 & 0 & 0 & 100 \\ 5 & 0 & 0 & 0 & 100 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 20 \\ 0 & 1 & 0 & 0 & 12 \\ 0 & 0 & 1 & 0 & -6-8j \\ 0 & 0 & 0 & 1 & -6+8j \end{bmatrix}$$

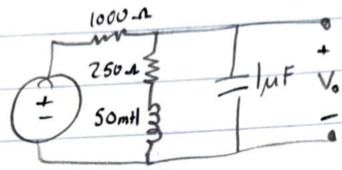
13.4 The Transfer Function

→ The s-domain ratio of the Laplace transform of the output to the Laplace transform of the input

$$H(s) = \frac{Y(s)}{X(s)}$$ ← output signal / ← input signal

→ Multiple equations can represent circuits, depending on what output and input signals are chosen.

ex; Input V_g
Output V_o



Nodal → $V_o = \frac{1000(s+5000)V_g}{s^2+6000s+25\times10^6}$

$H(s) = \frac{V_o}{V_g} = \frac{1000(s+5000)}{s^2+6000s+25\times10^6}$

13.5 The Transfer Function in Partial Fraction Expansion

$Y(s) = H(s)X(s)$

– Expanding $H(s)$ and $X(s)$ both into partial fractions leads to their holes and poles.

↳ $H(s)$ → transient response

↳ $X(s)$ → steady-state response

- If circuit is delayed by a seconds, then

↳ $Y(s) = H(s)X(s)e^{-at}$

- If circuit is unit impulse function

↳ $Y(s) = H(s)$

ex; where $V_g = 50tu(t)$ V $\mathcal{L}[V_g] = 50/s^2$

$H(s) = \frac{1000(s+5000)}{s^2+6000s+25\times10^6}$ → $V_o = \frac{1000(s+5000)}{(s^2+6000s+25\times10^6)}\cdot\frac{50}{s^2}$

Partial fraction, $\mathcal{L}^{-1}$ → $V_o = [\underbrace{10\sqrt{5}\times10^{-4}e^{-3000t}\cos(4000t+79.70^\circ)}_{\text{transient}} + \underbrace{10t - 4\times10^{-4}}_{\text{Steady-State}}]u(t)$ V

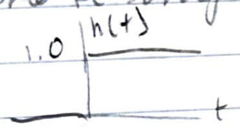
13.6 The Transfer Function and the Convolution Integral

→ Relates the output $y(t)$ to the input $x(t)$ of the circuit and the circuit's impulse $h(t)$

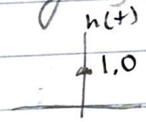
$$y(t) = \int_{-\infty}^{\infty} h(\lambda)\, x(t-\lambda)\, d\lambda = \int_{-\infty}^{\infty} h(t-\lambda)\, x(\lambda)\, d\lambda$$

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$x(t)$ can be represented w/ impulses, and the sum of those impulses $= y(t)$

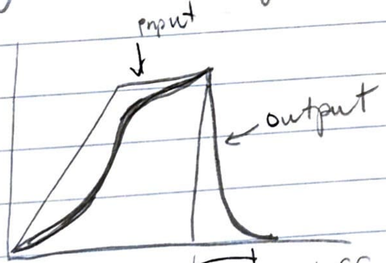


Perfect memory



No memory

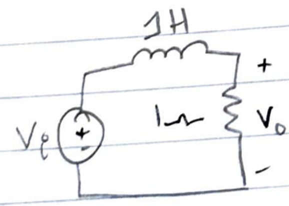
→ The more memory, the more distortion between input and output

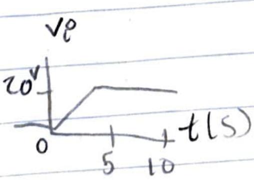


→ difference = distortion

t domain of s domain transfer function
→ "impulse response"

ex;





$$V_o = \frac{1}{s+1} V$$

when V_i is the unit impulse $\delta(t)$, $V_i = 1$

$H(s) = \frac{V_o}{V_i} \longrightarrow V_o = h(t) = e^{-t}u(t)$

$h(\lambda) = e^{-\lambda}u(\lambda)$

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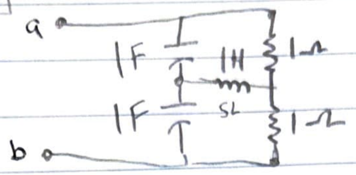
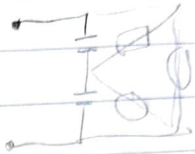
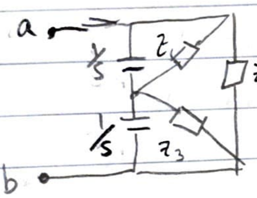
Problem 13.8



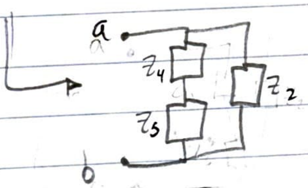


Part A Express equivalent s-domain impedance



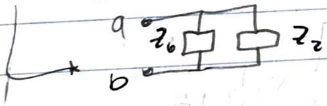
$$Z_1 = \frac{1+s+s}{1} = 2s+1 = Z_3$$

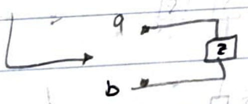
$$Z_2 = \frac{2s+1}{s}$$



$$Z_4 = \frac{(2s+1)(\frac{1}{s})}{2s+1+\frac{1}{s}} = \frac{2+\frac{1}{s}}{2s+1+\frac{1}{s}} = \frac{\frac{2s+1}{s}}{\frac{2s^2+s+1}{s}} = \frac{2s+1}{2s^2+s+1} = Z_5$$

$$Z_6 = 2Z_4 = \frac{4s+2}{2s^2+s+1}$$





$$Z = Z_2 || Z_6 = \frac{\left(\frac{4s+2}{2s^2+s+1}\right)\left(\frac{2s+1}{s}\right)}{\frac{4s+2}{2s^2+s+1} + \frac{2s+1}{s}} = \boxed{\frac{2}{s+1}}$$

Part B Find the poles

$\boxed{p = -1}$

Part C Find the zeros

$\boxed{\text{no zeros}}$

13.7 The transfer function and the steady-state sinusoidal response

assuming the input $x(t) = A\cos(\omega t + \phi)$

and $Y(s) = H(s)X(s)$

$$y_{ss}(t) = A\,|H(j\omega)|\cos[\omega t + \phi + \theta(\omega)]$$

ex; given $V_g = 120\cos(5000t + 30°)\,V$

and $H(s) = \dfrac{1000(s+5000)}{s^2 + 6000s + 25\times10^6}$

$$H(j\omega) = H(j5000) = \frac{1 - j1}{6} = \frac{\sqrt{2}}{6}\angle -45°$$

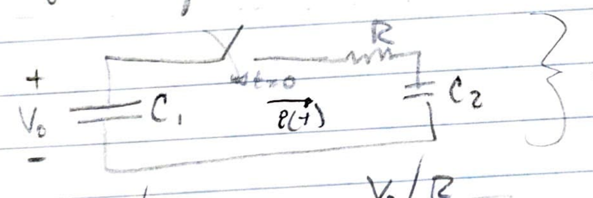
$$y_{ss}(t) = \frac{(120)\sqrt{2}}{6}\cos(5000t + 30° - 45°)$$

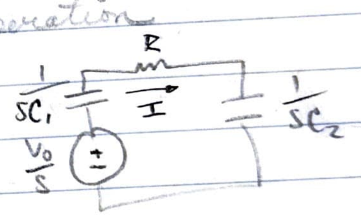
$$= 20\sqrt{2}\cos(5000t - 15°)\,V$$

13.8 The Impulse Function in Circuit Analysis

We use 1.) capacitor circuit 2.) inductor series circuit to create an impulse function with a switching operation

ex;





$$C_e = \frac{C_1C_2}{C_1+C_2}$$

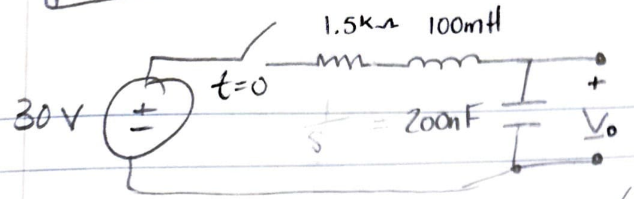
$$I = \frac{V_0/s}{R + 1/sC_1 + \frac{1}{sC_2}} = \frac{V_0/R}{s + (1/RC_e)}$$

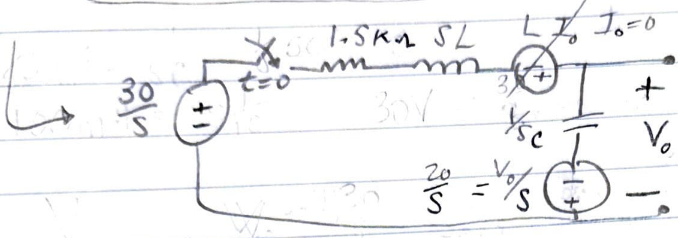
$$\mathcal{L}^{-1}[I] = \left(\frac{V_0}{R}e^{-t/RC_e}\right)u(t)$$

$$q(\text{charge}) = \int_{0^-}^{\infty}\frac{V_0}{R}e^{-t/RC_e}\,dt = V_0C_e$$

as $R \to 0$; $i \to V_0 C_e \delta(t)$

Problem 13.14 Part A





$$\frac{\frac{50}{s}}{\left(1.5\times10^3 + sL + \frac{1}{sC}\right)} = I$$

$$I\Big/\tfrac{1}{sC}\ \text{(sic)} = \frac{50}{s} - \frac{100}{s+5000} + \frac{50}{s+10000}$$

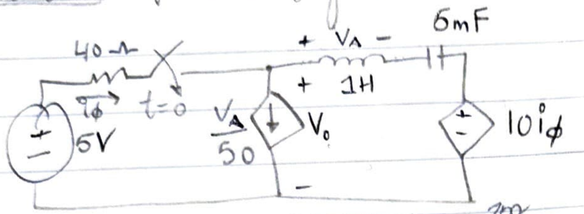
$$V_o = \underbrace{\quad}_{} + \frac{20}{s} \qquad \boxed{V_o = \frac{30}{s} - \frac{100}{s+5000} + \frac{50}{s+10000}}$$

Part B

$$v_o(t) = \mathcal{L}^{-1}[V_o] = \boxed{\left[30 - 100e^{-5000t} + 50e^{-10000t}\right]u(t)\ V}$$

Problem 13.26 | There is no initial energy @ $t=0$

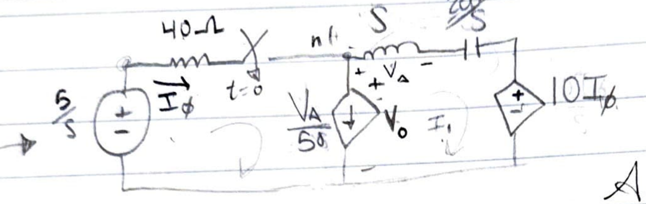
Part A | Find V_o for $t \geq 0$



$SL = S$

$\frac{1}{sC} = 200/s$

S domain



Supermesh: $I_\phi - I_1 = \frac{V_A}{50}$

$I_1 = I_\phi - \frac{V_A}{50}$

$I_1 = I_\phi - \frac{I_1 S}{50}$

$I_1 + \frac{I_1 S}{50} = I_\phi$

$I_1\left[1 + \frac{S}{50}\right] = I_\phi$

$I_1 = I_\phi / \frac{S+50}{50}$

$I_1 = \frac{50 I_\phi}{S+50}$

$$\frac{\frac{5}{S} - V_o}{40} = I_\phi$$

$$10 I_\phi = \frac{\frac{5}{S} - V_o}{4} = \frac{5}{4S} - \frac{V_o}{4}$$

$$\frac{V_o - 10 I_\phi}{S + \frac{200}{S}} = I_1$$

$$I_1 = \frac{50 I_\phi}{S+50} = \frac{50\left[\frac{\frac{5}{S} - V_o}{40}\right]}{S+50}$$

$$\frac{V_o - 10 I_\phi}{S + \frac{200}{S}} = \frac{50\left[\frac{\frac{5}{S} - V_o}{40}\right]}{S+50}$$

$$\frac{V_o - 10\left[\frac{\frac{5}{S} - V_o}{40}\right]}{S + \frac{200}{S}} = \frac{50\left[\frac{\frac{5}{S} - V_o}{40}\right]}{S+50}$$

$$(S+50) \cdot \left[V_o - 10\left[\frac{\frac{5}{S} - V_o}{40}\right]\right] = \left(S + \frac{200}{S}\right) \cdot \left[50\left[\frac{\frac{5}{S} - V_o}{40}\right]\right]$$

$$(S+50) \cdot \left[V_o - \frac{5}{4S} + \frac{V_o}{4}\right] = \left(S + \frac{200}{S}\right) \cdot \left[\frac{25}{4S} - \frac{5V_o}{4}\right]$$

$$\frac{S+50}{S + \frac{200}{S}} = \frac{5 - V_o S}{-1 + V_o S}$$

$$\frac{-V_o S^3 - 200 V_o S + 5S^2 + 1000}{S} = V_o S^2 + 50 V_o S - S - 50$$

$$-V_o S^3 - 200 V_o S + 5S^2 + 1000 = V_o S^3 + 50 V_o S^2 - S^2 - 50S$$

$$-2V_o s^3 - 50 V_o s^2 - 200 V_o S = -6s^2 - 50s - 1000$$

$$V_o = \frac{-6s^2 - 50s - 1000}{-2s^3 - 50s^2 - 200s} = \frac{5}{x} - \frac{6}{x+5} + \frac{4}{x+20}$$

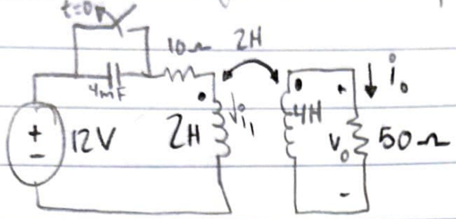
$$\boxed{\mathcal{L}^{-1}[V_o] = \left[5 - 6e^{-5t} + 4e^{-20t}\right] u(t)\,V}$$

Part B | Does it make sense?

Yes, it does!!

Problem 13.38

Part A) Use Laplace transform method to find i_o



* Switch has been closed for a long time before opening at $t=0$.

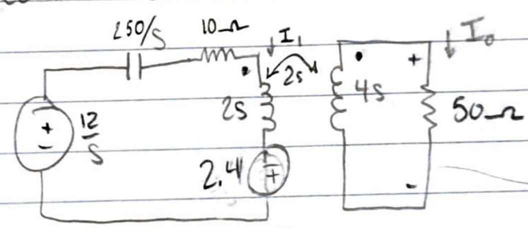
Because it has been closed for a long time, both inductors act like a short.

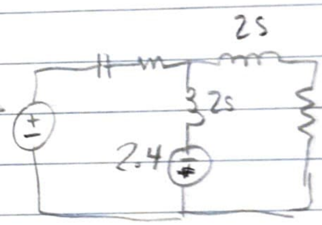
↳ $i_1(0^-) = \cancel{12/10 = 1.2A}$ $\qquad LI_o = 2.4$ $\qquad \frac{1}{sC} = \frac{250}{s}$

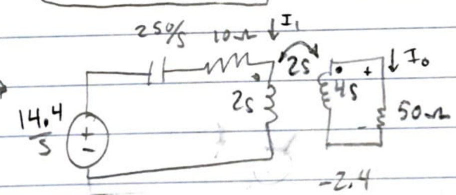
$i_o(0^-) = 0\,A$ $\qquad sL_1 = 2s$ $\qquad sM = 2s$

$V_c(0^-) = 0V$ (open, no charge) $\qquad sL_2 = 4s$

S-domain, $t \geq 0$







Mesh 1: $-\frac{12}{s} + \frac{250}{s}I_1 + 10I_1 + 2sI_1 - 2sI_o = 0$ $\qquad \left(\frac{250}{s} + 10 + 2s\right)I_1 - 2sI_o = \frac{12}{s} + 2.4$

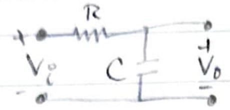
Mesh 2: $50I_o + 4sI_o - 2sI_1 = -2.4$ $\qquad -2sI_1 + (50+4s)I_o = -2.4$

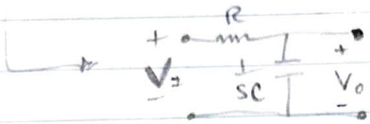
$$I_o = \frac{-\frac{3}{10}}{x+25} + \frac{\frac{3-6i}{20}}{x+5+10i} + \frac{\frac{3+6i}{20}}{x+5-10i}$$

$$\boxed{\mathcal{L}^{-1}[I_o] = i_o = \left[-300e^{-25t} + 300\sqrt{5}\,e^{-5t}\cos(10t + 63.43^\circ)\right]u(t)\ \text{mA}}$$

Problem 13.49

Part A) Find $H(s) = V_o/V_i$

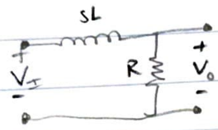




$$\frac{V_I}{R+\frac{1}{sC}} = \frac{V_o}{\frac{1}{sC}} \qquad \frac{V_o}{V_I} = \frac{\frac{1}{sC}}{R+\frac{1}{sC}} = \frac{1}{sCR+1}$$

$$\boxed{H(s) = \frac{1}{sCR+1}}$$

Part B



$$\frac{V_I}{SL+R} = \frac{V_o}{R} \qquad \frac{V_o}{V_I} = \boxed{\frac{R}{SL+R} = H(s)}$$

Part C If $R = 47\,\Omega$ and $H(s) = V_o/V_i = \frac{5000}{S+5000}$, find C

$$\frac{1}{sCR+1} = \frac{5000}{S+5000} = \frac{1}{sC(47)+1} = \frac{5000}{C\cdot 235000s+5000}$$

$C \cdot 235000 = 1$ $\quad \boxed{C = 4.26\,\mu F}$

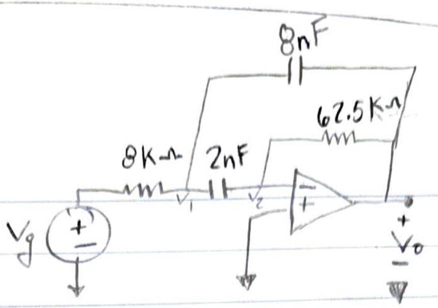
Part D If $R = 47\,\Omega$ and $H(s) = \frac{5000}{S+5000}$, find L

$$\frac{R}{SL+R} = \frac{5000}{S+5000} = \frac{47}{SL+47} \cdot \frac{\frac{5000}{47}}{\frac{5000}{47}}$$

$\cancel{S}L\,\frac{5000}{47} = \cancel{S}1$ $\quad \boxed{L = 9.4\,mH}$

6

Problem 13.56



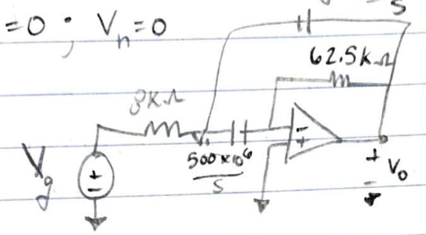
Part A Derive $H(s) = V_o / V_g$

$V_p = 0 \; ; \; V_n = 0$

$\frac{1}{sC_1} = \frac{500\times10^6}{s}$ $\quad \frac{1}{sC_2} = \frac{125\times10^6}{s}$

$R_1 = 8k\Omega$

$R_2 = 62.5k\Omega$



$$\frac{V_1 - V_g}{R_1} + \frac{V_1 - V_o}{\frac{1}{sC_1}} + \frac{V_1}{\frac{1}{sC_2}} = 0 \qquad \frac{-V_o}{R_2} + \frac{-V_1}{\frac{1}{sC_1}} = 0$$

$$\frac{V_1}{R_1} - \frac{V_g}{R_1} + V_1 sC_1 + V_1 sC_2 = 0 \qquad -V_o sC_1$$

$$\frac{-V_o}{R_2} - V_1 sC_1 = 0$$

$$V_1 sC_1 = -\frac{V_o}{R_2} \qquad V_1 = \frac{-V_o}{R_2 sC_1}$$

$$-V_1 = \frac{500\times10^6}{s} \cdot \frac{V_o}{62.5\times10^3}$$

$$-V_1 = \frac{8000\,V_o}{s}$$

$$\frac{-V_1}{\frac{500\times10^6}{s}} + \frac{V_o - V_1}{\frac{125\times10^6}{s}} + \frac{V_g - V_1}{8000} = 0$$

$$\frac{\frac{8000V_o}{s}}{\frac{500\times10^6}{s}} + \frac{V_o + \frac{8000V_o}{s}}{\frac{125\times10^6}{s}} + \frac{V_g + \frac{8000V_o}{s}}{8000} = 0$$

$$\frac{V_o}{62500} + \frac{V_o s}{125\times10^6} + \frac{V_o}{15625} + \frac{V_g}{8000} + \frac{V_o}{s} = 0$$

$$V_o\left[\frac{1}{62500} + \frac{s}{125\times10^6} + \frac{1}{15625} + \frac{1}{s}\right] = \frac{-V_g}{8000}$$

$$\boxed{\frac{V_o}{V_g} = -\frac{15625s}{s^2 + 10000s + 125{,}000{,}000}}$$

Part B $-P_1, -P_2 = -5000 + 1.00\times10^4 j, \; -5000 - 1.00\times10^4 j$

Part C Unit step response in milliseconds?

$$V_o(t) = -1.5625\, e^{-5t} \cos(10t - 90^\circ)$$

Chapter 14: Intro to Frequency-Selective Circuits

- Now ω changes

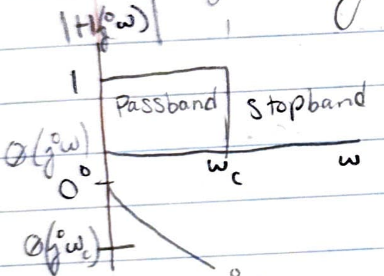
$$H(s) = V_o(s) / V_i(s)$$

Passband - signals that pass from input to output

Stopband - signals that are blocked

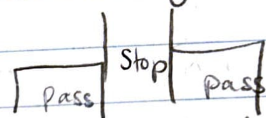
Frequency response plot $\rightarrow |H(j\omega)|$ vs. ω $\rightarrow$ magnitude plot

$\hookrightarrow \theta(j\omega)$ vs. ω $\rightarrow$ phase angle plot



★ High-pass

★ low pass



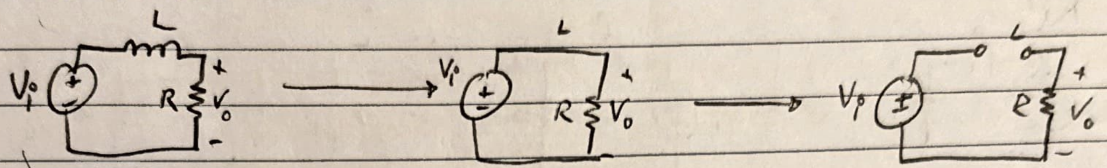
★ band reject

★ band pass

14.2 Low-pass Filters

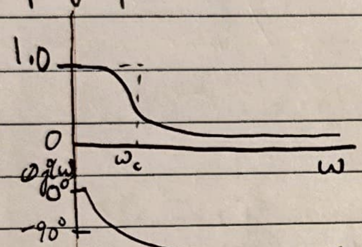
RL circuit & series RC circuit behave as low-pass

RL circuit



$\omega = 0$ $\omega = \infty$

$|H(j\omega)|$



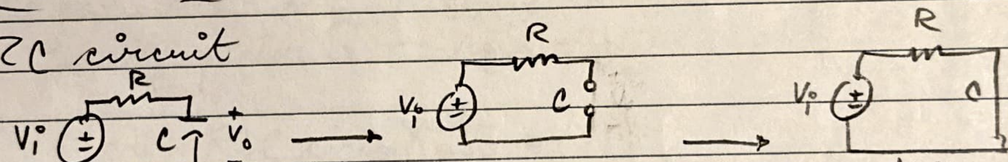
Cutoff Frequency Definition:

$$|H(j\omega_c)| = \frac{1}{\sqrt{2}} H_{max}$$

Cutoff Frequency for RL Filters:

$$\omega_c = \frac{R}{L}$$

RC circuit

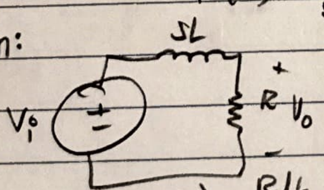


$\omega = 0$ $\omega = \infty$

Transfer Function for Low-pass Filters

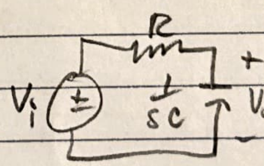
$$H(s) = \frac{\omega_c}{s+\omega_c}$$

S domain:



$$H(s) = \frac{R/L}{s+R/L}$$

$$\omega_c = R/L$$



$$H(s) = \frac{1/RC}{s+1/RC}$$

$$\omega_c = 1/RC$$

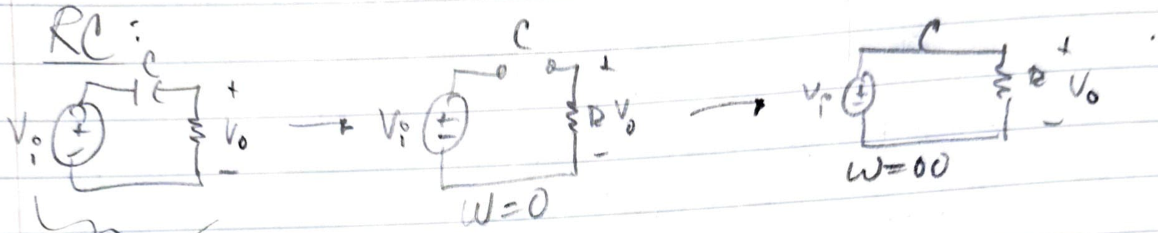
$$\tau = 1/\omega_c$$

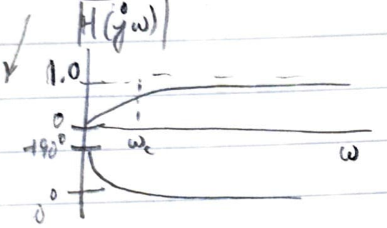
$$\frac{\frac{1}{sC}}{R+\frac{1}{sC}} = \frac{\frac{1}{\cancel{sC}}}{\frac{RsC+1}{\cancel{sC}}} = \frac{1}{RsC+1} = \frac{1/RC}{s+1/RC}$$

14.3 High-Pass Filter

RL circuit & RC circuit again, but $V_o(s)$ is defined in a different place

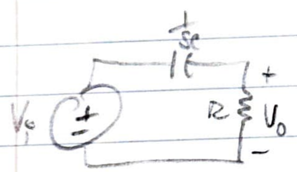
RC:





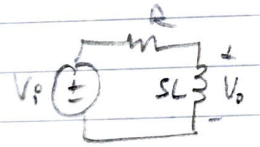
Transfer Function for High-Pass Filter

$$H(s) = \frac{s}{s+\omega_c}$$



$$H(s) = \frac{s}{s+1/RC}$$

$$\omega_c = 1/RC$$

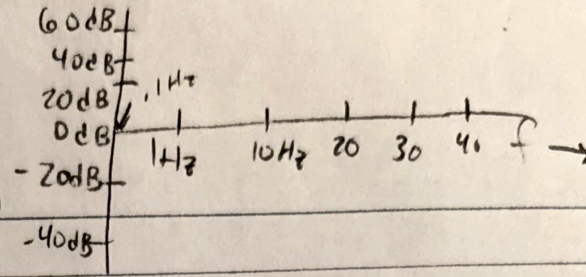


$$H(s) = \frac{s}{s+R/L}$$

$$\omega_c = R/L$$

Class notes

Bode Plots



$\frac{V_o^2}{V_i^2}$

$dB \rightarrow 10\log_{10}\frac{P_o}{P_i} \rightarrow 20\log_{10}\frac{V_o}{V_i}$

-20dB → reduced by factor of 10

Zero → increases slope

pole → decreases the slop

$H(j\omega) = \frac{R/L}{j\omega + R/L}$

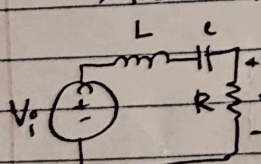
Magnitude: $|H(j\omega)| = \frac{R/L}{\sqrt{\omega^2 + (R/L)^2}}$ phase: $\theta(j\omega) = -\tan^{-1}\left(\frac{\omega L}{R}\right)$

$|H(j\omega_c)| = \frac{1}{\sqrt{2}}|1| = \frac{R/L}{\sqrt{\omega_c^2 + (R/L)^2}}$ } when hit ω_c, plot goes down and starts approaching zero

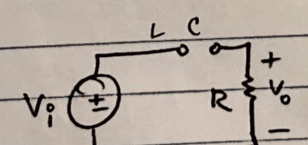
14.4 Bandpass Filters

- two cutoff frequencies, ω_{c1} & ω_{c2}
- Center frequency - ω_o → frequency where $H(s)$ is purely real
- ↳ (resonant frequency) → where Z_c & Z_L cancel out
- $\omega_o = \sqrt{\omega_{c1}\,\omega_{c2}}$
- $H_{max} = |H(j\omega_o)|$
- Bandwidth - β → width of the passband
- quality factor Q → ratio of center frequency to bandwidth

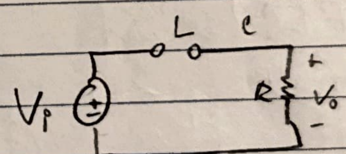
Series RLC circuit



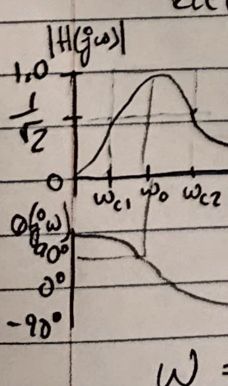
RLC circuit



$\omega = 0$



$\omega = \infty$



$$H(s) = \frac{(R/L)s}{s^2 + (R/L)s + (1/LC)}$$

$$\theta(j\omega) = 90° - \tan^{-1}\left[\frac{\omega(R/L)}{(1/LC) - \omega^2}\right]$$

$$|H(j\omega)| = \frac{\omega(R/L)}{\sqrt{\left[(1/LC) - \omega^2\right]^2 + \left[\omega(R/L)\right]^2}}$$

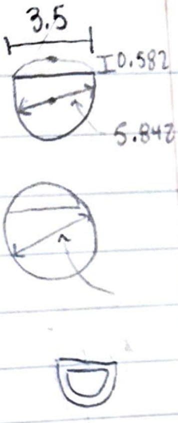
$$\omega_o = \sqrt{\frac{1}{LC}} \qquad \omega_{c1} = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}$$

$$\omega_{c2} = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}$$

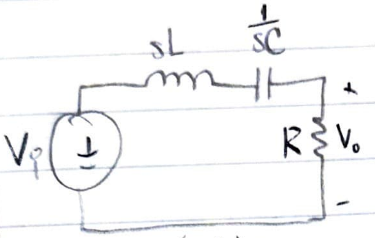
$$\omega_o = \sqrt{\omega_{c1} + \omega_{c2}} \qquad \beta = f_{c2} - f_{c1} \qquad \beta = \frac{R}{L}$$

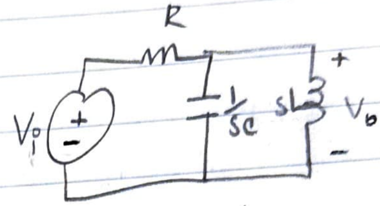
$= \sqrt{\ }$

??



$$Q = \frac{\omega_0}{\beta} \qquad Q = \sqrt{\frac{L}{R^2 C}} \qquad H(s) = \frac{\beta s}{s^2 + \beta s + \omega_0^2}$$





$$H(s) = \frac{(R/L)s}{s^2 + (R/L)s + 1/LC}$$

$$\omega_0 = \sqrt{1/LC}$$

$$\beta = R/L$$

$$H(s) = \frac{s/RC}{s^2 + s/RC + 1/LC}$$

$$\omega_0 = \sqrt{1/LC}$$

$$\beta = 1/RC$$

$$\triangleright\ H(s) = \frac{K\beta s}{s^2 + \beta s + \omega_0^2}$$

Where K & β depend on whether the series resistance of the voltage source is zero or nonzero

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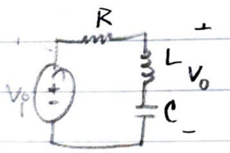
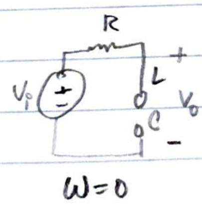
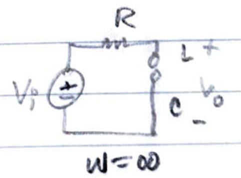
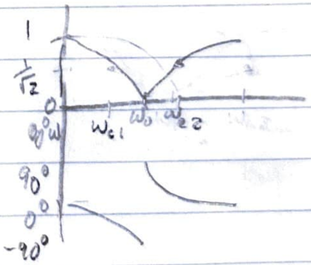
14.5 Bandreject Filters

RLC circuit

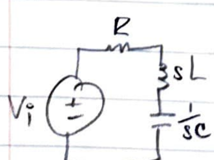


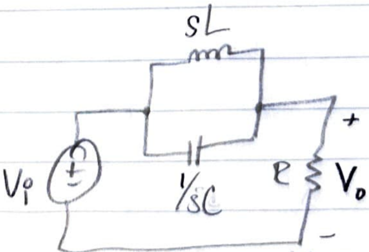






$$H(s) = \frac{s^2 + \omega_0^2}{s^2 + \beta s + \omega_0^2}$$





$$H(s) = \frac{s^2 + 1/LC}{s^2 + (R/L)s + 1/LC}$$

$$\omega_0 = \sqrt{1/LC}$$

$$\beta = R/L$$

$$H(s) = \frac{s^2 + 1/LC}{s^2 + s/RC + 1/LC}$$

$$\omega_0 = \sqrt{1/LC} \qquad \beta = 1/RC$$

Seth Ricks	ECEN 250	HW #12

Problem 14.6 Use 25 mH inductor to design a low-pass RL passive filter with a cutoff frequency of 2.5 kHz

Part A Resistor?

$\omega_c = R/L \rightarrow \quad R = L \cdot \omega_c = L \cdot 2\pi f = \boxed{393\Omega}$

Part B Load w/ 750Ω, what is cuttoff frequency?

$750 // 393 = 257.87\Omega$

$f_c = R/L \cdot \frac{1}{2\pi} = \boxed{1,642\,Hz}$

Part C

Use $\boxed{390\Omega}$ resistor from the given table

Part D If you use, then what is f_c?

$f_c = R/L \cdot \frac{1}{2\pi} = \boxed{2,483\,Hz}$

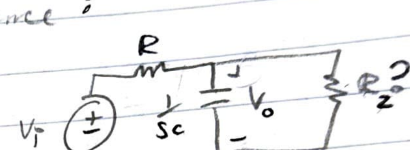
(2)

Problem 14.10 Use 35nF capacitor to design a low-pass passive filter w/ a cutoff frequency of 160 krad/s

Part A f_c? $f_c = \frac{\omega_0}{2\pi} = \boxed{25.5\text{kHz}}$

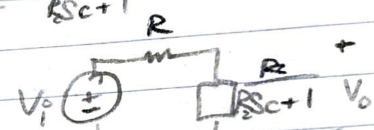
Part B Filter resistor? $R = \frac{1}{\omega_c C} = \boxed{179\,\Omega}$

Part C Assume cutoff frequency can't increase more than 8%. Smallest load resistance?



$$1.08 \cdot f_c = 27{,}540\,\text{Hz}$$

$$H(S) = \frac{V_o}{V_i} \qquad \frac{1}{SC} \,//\, R_2 = \frac{\frac{R_2}{SC}}{\frac{1}{SC} + R_2} = \frac{\frac{R_2}{SC}}{\frac{R_2SC+1}{SC}} = \frac{R_2}{R_2SC+1}$$



$$V_o = V_i\left[\frac{\frac{R_2}{R_2SC+1}}{R + \frac{R_2}{R_2SC+1}}\right]$$

$$H(s) = \left[\frac{\frac{R_2}{R_2SC+1}}{R + \frac{R_2}{R_2SC+1}}\right]$$

$$= \frac{\frac{R_2}{\cancel{R_2SC+1}}}{\frac{R(R_2SC+1)+R_2}{\cancel{R_2SC+1}}} = \frac{R_2}{R(R_2SC+1)+R_2} = \frac{R_2}{RR_2SC+R+R_2}$$

$$= \frac{\frac{R_2}{RR_2C}}{S + \frac{R+R_2}{RR_2C}}$$

$$2\pi f_c = \omega_c = \frac{R+R_2}{RR_2C} = \frac{1}{R_2C} + \frac{1}{RC}$$

$$-\frac{1}{R_2C} = \frac{1}{RC} - 2\pi f_c$$

$$R_2C = \frac{-1}{\frac{1}{RC} - 2\pi f_c}$$

$$R_2 = \frac{-1}{\frac{1}{RC} - 2\pi f_c} \Big/ C = \boxed{2128.7\,\Omega}$$

Part D $H(j\omega) = ?$ when $\omega = 0$? (For part C)

$$H(j\omega) - H(j0) = \frac{R_2}{R+R_2} = \boxed{0.922}$$

3

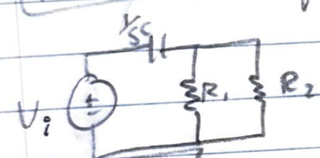
Problem 14.17 Design RC high-pass filter w/ cutoff frequency of 240 Hz using 100nF capacitor.

Part A What is the ω_{co}? $\omega_c = 2\pi f_c = \boxed{510 \text{ rad/s}}$

Part B Resistor value? $\omega_c = \frac{1}{RC} \rightarrow R = \frac{1}{\omega_c C} = \boxed{6{,}631\,\Omega}$

Part C Transfer function, $H(s)$? $\boxed{H(s) = \frac{s}{s + 1/RC}}$

Part D If loaded w/ same resistor as part B, $H(s) = ?$



$R_{eq} = 3{,}316\,\Omega$ $H(s) = \frac{s}{s + \frac{1}{\frac{R}{2}C}}$

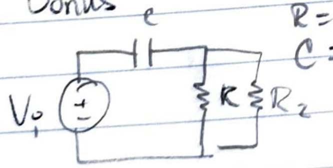
Part E ω_c from part D?

$\omega_c = \frac{1}{\frac{R}{2}\cdot C} = \frac{2}{RC} = \boxed{3{,}016 \text{ rad/s}}$

Part F What is the gain in the passband from part D?

$H(\infty) = \frac{\infty}{\infty + \frac{1}{\frac{R}{2}\cdot C}} \longrightarrow \frac{1}{1+0} = \boxed{1}$

Bonus



$R = 50\,\Omega$
$C = 47\,\mu F$

$\frac{R \cdot R_2}{R + R_2} = 45.45$

$50R_2 = 45.45(50 + R_2)$
$= 2272.74 + 45.45R_2$

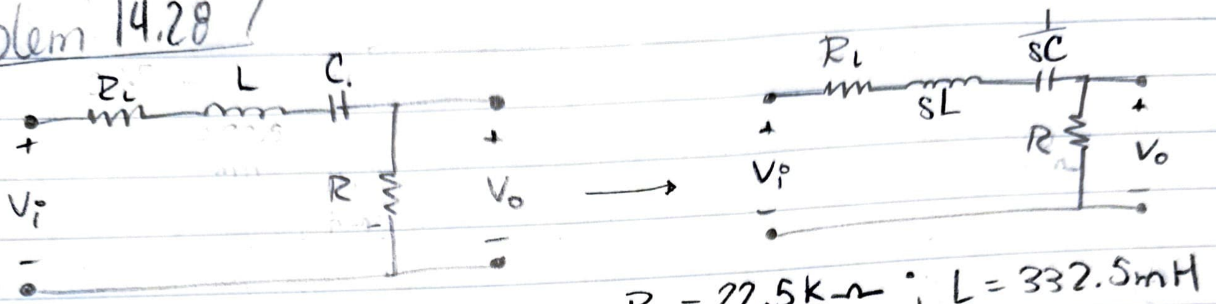
$R_2 = 500$

$\omega_c = 425.53$

$\omega_{c+} = 468.083 = 1/RC$

$R = \frac{1}{\omega_{c+} \cdot C} = 45.45$

Problem 14.28



$R_L = 22.5\,k\Omega$; $L = 332.5\,mH$
$C = 1\,pF$; $R = 50\,k\Omega$

Part A Calculate f_o

$$H(s) = \frac{V_o}{V_i} = \frac{R}{R_L + sL + \frac{1}{sC} + R} = \frac{Rs}{s^2L + (R+R_L)s + \frac{1}{C}} = \frac{\frac{R}{L}s}{s^2 + \frac{(R+R_L)}{L}s + \frac{1}{LC}}$$

$$\omega_o = \sqrt{1/LC} \qquad f_o = \frac{\sqrt{1/LC}}{2\pi} = \boxed{276\,kHz}$$

Part C Calculate f_{c1}

$$f_{c1} = \frac{\omega_{c1}}{2\pi} = \frac{-\frac{R+R_L}{2L} + \sqrt{\left(\frac{R+R_L}{2L}\right)^2 + \frac{1}{LC}}}{2\pi} = \boxed{259\,kHz}$$

Part D Calculate f_{c2}

$$f_{c2} = \frac{\omega_{c2}}{2\pi} = \frac{R+R_L}{2L} + \sqrt{\left(\frac{R+R_L}{2L}\right)^2 + \frac{1}{LC}} = \boxed{294\,kHz}$$

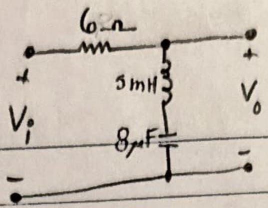
Part E Calculate β (Hz)

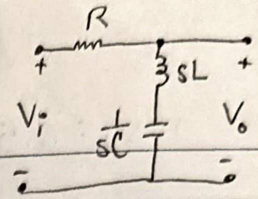
$$\beta = f_{c2} - f_{c1} = 35\,kHz$$

Part B $Q = ?$

$$Q = \frac{\omega_o}{\beta} = \frac{276}{35} = 7.89$$

Problem 14.40





Part A Find the center frequency of the filter.

$H(s) = \frac{V_o}{V_i} = \frac{sL + \frac{1}{sC}}{sL + \frac{1}{sC} + R} = \frac{s^2L + \frac{1}{C}}{s^2L + sR + \frac{1}{C}} = \frac{s^2 + \frac{1}{CL}}{s^2 + \frac{sR}{L} + \frac{1}{CL}}$ $\quad \omega_o = \sqrt{1/LC} = \boxed{5{,}000 \text{ rad/s}}$

Part B Calculate center frequency in hertz. $f_o = \frac{\omega_o}{2\pi} = \boxed{796 \text{ Hz}}$

Part D Bandwidth β? $\beta = R/L = 1200$ rad/s $\rightarrow \boxed{191 \text{ Hz}}$

Part C Quality factor Q? $Q = \frac{\omega_o}{\beta} = \boxed{4.17}$

Part E Find lower cutoff frequency $\omega_c 1 = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)} = \boxed{4{,}436 \text{ rad/s}}$

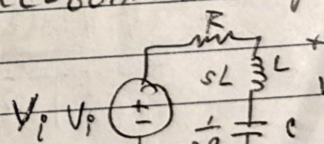
$= 5{,}636$ rad/s

Part G Find higher cutoff frequency $\omega_c 2 = +$

Part H Find $f_c 2$ $\quad f_c 2 = \omega_c 2/2\pi = \boxed{897 \text{ Hz}}$

Part F Find $f_c 1$: $f_c 1 = \omega_c 1/2\pi = \boxed{706 \text{ Hz}}$

Problem 14.44 Consider the RLC band reject filter. The quality factor of the filter is 2/3, the center frequency is 4 krad/s, and C = 80nF. The input to the filter is $175\cos\omega t$ V



Part A Find voltage drop across the resistor when $\omega = \omega_o$. Assume $V_R(t) = A_R\cos(\omega t + \phi)$

$\boxed{A_R = 175 \text{ V}; \ \omega = 4 \text{ krad/s}; \ \phi = 0°}$

Part B Same thing, but $\omega = \omega_c 1$.

$\beta = \frac{\omega_o}{Q} = 6$ krad/s $\quad \omega_o = 4$ krads

$Q = \sqrt{\frac{L}{R^2C}} = \sqrt{\frac{R/\beta}{R^2C}} \quad Q = 2/3$

$Q^2 = \frac{1}{R\beta C} \rightarrow Q^2\beta C = \frac{1}{R} \rightarrow R = \frac{1}{Q^2\beta C} = \frac{1}{\left(\frac{2}{3}\right)^2 \cdot (6 \times 10^3) \cdot (80 \times 10^{-9})} = 4{,}688\ \Omega \quad L = R/\beta = \frac{4{,}688}{6{,}000} = 0.781$ H

$\omega_c 1 = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)} = 8000$

$H(s) = \frac{s^2 + 1/LC}{s^2 + (R/L)s + 1/LC} = \frac{s^2 + \omega_o^2}{s^2 + \beta s + \omega_o^2} = \frac{(j\omega_{c1})^2 + \omega_o^2}{(j\omega_{c1})^2 + \beta(j\omega_{c1}) + \omega_o^2} = 0.5 + 0.5j = 0.707\angle 45°$

$\frac{V_o}{V_i} = 0.707\cos(2000t + 45°)$

$\boxed{V_o = 124\cos(2000t + 45°)}$

$\boxed{\text{Part C}}$ Same thing, $\omega = \omega_{c2}$

$$\omega_{c2} = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} = 8000 \text{ rad/s}$$

$$H(s) = 0.5 + 0.5j \qquad V_R(t) = 174\cos(8000t - 45°)$$

$\boxed{\text{Part D}}$ Same thing, $\omega = 0.250\omega_0 \longrightarrow \omega = 1{,}000 \text{ rad/s}$

$$H(s) = \frac{s^2+\omega_0^2}{s^2+\beta s+\omega_0^2} = \frac{(j1000)^2 + (4000)^2}{(j1000)^2 + (6000)(j1000) + (4000)^2} = 0.862 - 0.35j = 0.928\angle -21.80 = \frac{V_o}{V_i}$$

$$V_o = 162.5\angle -21.8°$$

$$V_R = V_i - V_o = 65.0\angle 68.2°$$

$$V_R(t) = 65.0\cos(1000t + 68.2°)$$

$\boxed{\text{Part E}}$ Same thing, $\omega = 8\omega_0 = 32{,}000$

$$H(s) = 0.982\angle 10.784°$$

$$V_o = V_i \cdot H(s) = 172\angle 10.78°$$

$$V_R = V_i - V_o = 32.7\angle -79.2$$

$$\boxed{V_R(t) = 32.7\cos(32{,}000t - 79.2°)}$$

LAB:

$\omega \to 1$

$$\frac{\sqrt{1+\left(\frac{\omega}{\omega_n}\right)^2}}{\sqrt{1+\left(\frac{\omega}{\omega_o}\right)^2}} = \sqrt{\frac{1+\left(\frac{1}{\omega_n}\right)^2}{1+\left(\frac{1}{\omega_d}\right)^2}} = 1$$

$$\sqrt{\frac{1+\left(\frac{\omega}{\omega}\right)^2}{1+\left(\frac{\omega}{\omega_d}\right)^2}} = \sqrt{\frac{1+\frac{\omega^2}{\omega_n^2}}{1+\frac{\omega^2}{\omega_d^2}}} = \sqrt{\frac{\frac{\omega_n^2+\omega^2}{\omega_n^2}}{\frac{\omega_d^2+\omega^2}{\omega_d^2}}}$$