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ECEN 380
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PS07: 3.48(a), 3.49(b), 3.50(d), 3.51(d), 3.53(b), 3.54(c), MATLAB 3.102

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3.48(a) Use the definition equation for DTFS coefficients to evaluate the DTFS representation of the following signal:

$$x[n] = \cos\left(\frac{6\pi}{17}n + \frac{\pi}{3}\right)$$

$$X[k] \overset{?}{=} \frac{1}{N}\sum_{n=\langle N\rangle} x[n]e^{jk\Omega_0 n} \quad ; \quad \frac{6\pi}{17} = 3\left(\frac{2\pi}{17}\right) \rightarrow m=3,\ \Omega_0 = \frac{2\pi}{17},\ N=17$$

$$X[k] = \sum_{-8}^{8} x[k]e^{jk\Omega_0 n} = x[-8]e^{j(-8)\Omega_0 n} + \cdots + x[8]e^{j(8)\Omega_0 n}$$

$$\cos\phi = \tfrac{1}{2}e^{j\phi} + \tfrac{1}{2}e^{-j\phi}$$

$$\cos\left(3\Omega_0 n + \tfrac{\pi}{3}\right) = \tfrac{1}{2}e^{j(3\Omega_0 n + \frac{\pi}{3})} + \tfrac{1}{2}e^{-j(3\Omega_0 + \frac{\pi}{3})}$$

$$= \tfrac{1}{2}e^{j(3\Omega_0 n)} \cdot e^{j\frac{\pi}{3}} + \tfrac{1}{2}e^{-j(3\Omega_0)} \cdot e^{-j\frac{\pi}{3}}$$

$$= \left[\tfrac{1}{2}e^{j\frac{\pi}{3}}\right]e^{j(3\Omega_0 n)} + \left[\tfrac{1}{2}e^{-j(\pi/3)}\right]e^{-j(3\Omega_0)}, \text{ where } \Omega_0 = \frac{2\pi}{17}$$

$$X[-3] = \tfrac{1}{2}e^{j\frac{\pi}{3}} \ ; \ X[3] = \tfrac{1}{2}e^{j\pi/3}$$

$$X[k] = \begin{cases} \frac{1}{2}e^{j\frac{\pi}{3}}, & k=-3 \\ \frac{1}{2}e^{-j\frac{\pi}{3}}, & k=3 \\ 0, & \text{otherwise} \end{cases} \Rightarrow \boxed{X[k] = \begin{cases} \frac{1}{2}e^{j\pi/3}, & 3+17\gamma \\ \frac{1}{2}e^{-j\pi/3}, & -3+17\gamma \quad \gamma = 0,1,2\ldots \\ 0, & \text{otherwise} \end{cases}}$$

3.49(b) Use the definition of the DTFS to determine the time-domain signals represented by the following DTFS coefficients: $X[k] = \cos\left(\frac{10\pi}{19}k\right) + j2\sin\left(\frac{4\pi}{19}k\right)$

$\frac{2}{6}$

DTFS: $x[n] = \sum_{k=0}^{N-1} X[k] e^{jk\Omega_0 n}$; $X[k] = \frac{1}{N}\sum_{n=0}^{N-1} x[n] e^{-jk\Omega_0 n}$

$x[n]$ and $X[k]$ have period N, $\Omega_0 = \frac{2\pi}{N}$

$\cos\left(\frac{10\pi}{19}k\right) \rightarrow m=5, N=19, \Omega_0 = \frac{2\pi}{19}$

$\cos(5\Omega_0 k) = \frac{1}{2}e^{j5\Omega_0 k} + \frac{1}{2}e^{-j5\Omega_0 k}$

$= \frac{1}{5}\left[\frac{5}{2}e^{j5\Omega_0 k} + \frac{5}{2}e^{-j5\Omega_0 k}\right]$

$x[5] = \frac{5}{2}$; $x[-5] = \frac{5}{2}$

$j2\sin\left(\frac{4\pi}{19}k\right) \rightarrow m=2, N=19, \Omega_0 = \frac{2\pi}{19}$

$j2\sin(2\Omega_0 k) = j2\left[\frac{1}{j2}e^{j2\Omega_0 k} - \frac{1}{j2}e^{-j2\Omega_0 k}\right]$

$= e^{j2\Omega_0 k} - e^{-j2\Omega_0 k}$

$= \frac{1}{19}\left[19e^{j2\Omega_0 k} - 19e^{-j2\Omega_0 k}\right]$

$x[2] = -19$; $x[-2] = 19$

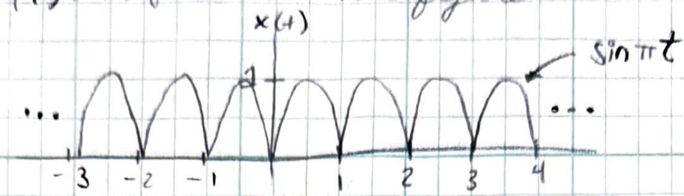
$$x[n] = \begin{cases} -19, & 2 \\ 19, & -2 \\ 0, & \text{otherwise} \end{cases}$$

$$x[n] = x_1[n] + x_2[n] = \begin{cases} \frac{5}{2}, & 5 \\ -19, & 2 \\ 19, & -2 \\ \frac{5}{2}, & -5 \\ 0 & \text{otherwise} \end{cases}$$

$$x[n] = \begin{cases} \frac{5}{2}, & k = 5 + 19\gamma \\ -19, & k = 2 + 19\gamma \\ 19, & k = -2 + 19\gamma, \quad \gamma = 0, \pm 1, \pm 2 \ldots \\ \frac{5}{2}, & k = -5 + 19\gamma \\ 0, & \text{otherwise} \end{cases}$$

3.50 (d) Use the defining equation for FS coeffs to evaluate the FS representation of following signal:

$x(t)$ as depicted in the figure:



$$x(t)=\sum_{k=-\infty}^{\infty}X[k]e^{jk\omega_0 t};\quad X[k]=\frac{1}{T}\int_0^T x(t)e^{-jk\omega_0 t}\,dt,\quad x(t) \text{ has period } T,\ \omega_0=\frac{2\pi}{T}$$

$$T=1,\ \omega_0=2\pi$$

$$X[k]=\frac{1}{1}\int_0^1 |\sin(\pi t)|\,e^{-j2\pi kt}\,dt$$

$$=\int_0^1 \sin(\pi t)e^{-j2\pi kt}\,dt=\int_0^1\left(\frac{1}{2j}\left(e^{j\pi t}-e^{-j\pi t}\right)\right)e^{-j2\pi kt}\,dt$$

$$=\int_0^1\left(\frac{1}{2j}\left(e^{j\pi t}\cdot e^{-j2\pi kt}-e^{-j\pi t}\cdot e^{-j2\pi kt}\right)\right)dt$$

$$=\frac{1}{2j}\int_0^1 e^{j\pi t-j2\pi kt}-e^{-j\pi t-j2\pi kt}\,dt$$

$$=\frac{1}{2j}\int_0^1\left[e^{-j2\pi(k-\frac{1}{2})t}-e^{-j2\pi(k+\frac{1}{2})t}\right]dt$$

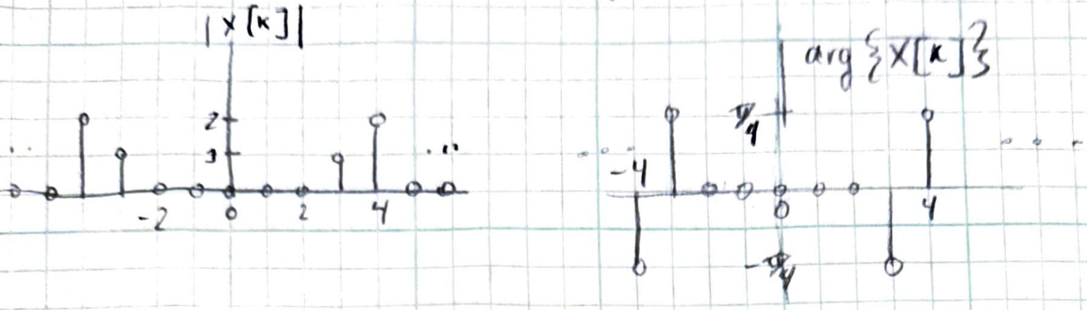
$$\int_0^1 e^{-j2\pi(a)t}\,dt=\left.\frac{e^{-j2\pi(a)t}}{-j2\pi(a)}\right|_0^1=\frac{e^{-j2\pi(a)}-1}{-j2\pi(a)}$$

$$X[k]=\frac{1}{2j}\left[\frac{e^{-j2\pi(k-\frac{1}{2})}-1}{-j2\pi(k-\frac{1}{2})}-\left[\frac{e^{-j2\pi(k+\frac{1}{2})}-1}{-j2\pi(k+\frac{1}{2})}\right]\right]$$

$$X[k]=\frac{e^{-j2\pi(k-\frac{1}{2})}-1}{4\pi(k-\frac{1}{2})}-\frac{e^{-j2\pi(k+\frac{1}{2})}-1}{4\pi(k+\frac{1}{2})}$$

3.51(d) Use definition of FS to determine time-domain signal represented by the following FS coeff's:

$X[k]$ in the figure, $\omega_0 = \pi$



$$X[k] = \begin{cases} 2e^{j(-\pi/4)}, & k=-4 \\ 1e^{j(\pi/4)}, & k=-3 \\ 1e^{j(-\pi/4)}, & k=3 \\ 2e^{j(\pi/4)}, & k=4 \\ 0, & \text{otherwise} \end{cases}$$

Freq, nonperiodic, discrete

$$x(t) = \sum_{k=-\infty}^{\infty} X[k]e^{jk\omega_0 t} = X[-4]e^{j(-4)\pi t} + X[-3]e^{j(-3)\pi t} + X[3]e^{j(3)\pi t} + X[4]e^{j(4)\pi t}$$

$$x(t) = 2e^{j(-\pi/4)} \cdot e^{j(-4)\pi t} + e^{j\pi/4} \cdot e^{j(-3)\pi t} + e^{j(-\pi/4)}e^{j(3)\pi t} + 2e^{j(\pi/4)}e^{j(4)\pi t}$$

$$\boxed{x(t) = 2e^{(-j\pi/4 - j4\pi t)} + e^{(j\pi/4 - j3\pi t)} + e^{(-j\pi/4 + j3\pi t)} + 2e^{(j\pi/4 + j4\pi t)}}$$

3.53(b) Use DTFT to determine time-domain signals corresponding to:

$$X(e^{j\Omega}) = \sin(\Omega) + \cos\left(\frac{\Omega}{2}\right)$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\Omega})e^{j\Omega n}\,d\Omega$$

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$$; $X(e^{j\Omega})$ has period 2π

$\sin\theta = \frac{1}{j2}e^{j\theta} - \frac{1}{j2}e^{-j\theta}$; $\cos(\theta) = \frac{1}{2}e^{j\theta} + \frac{1}{2}e^{-j\theta}$

$$X(e^{j\Omega}) = \frac{1}{j2}e^{j\Omega} - \frac{1}{j2}e^{-j\Omega} + \frac{1}{2}e^{j\frac{\Omega}{2}} + \frac{1}{2}e^{-j\frac{\Omega}{2}}$$

$$= \frac{1}{j2}e^{-j\Omega(-1)} - \frac{1}{j2}e^{-j\Omega(1)} + \frac{1}{2}e^{-j\Omega(-\frac{1}{2})} + \frac{1}{2}e^{-j\Omega(\frac{1}{2})}$$

n not int

$x[-1] = \frac{1}{j2}$

$x[1] = -\frac{1}{j2}$ $\qquad \Omega_0 = \frac{2\pi}{N} = 2\pi \rightarrow N = 1$

$$\boxed{x[n] = \begin{cases} \frac{1}{j2}, & -1+\gamma \\ -\frac{1}{j2}, & 1+\gamma \end{cases} \qquad \gamma = 0, \pm 1, \pm 2 \ldots}$$

3.54(c) Use defining equation for FT to eval freq-dom. repr of:

$$x(t) = te^{-t}u(t)$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\,d\omega$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt = \int_0^{\infty}(te^{-t}\cdot e^{-j\omega t})\,dt$$

$$= \int_0^{\infty} t\cdot e^{-t-j\omega t}\,dt = \int_0^{\infty} t\cdot e^{(-1-j\omega)t}\,dt$$

$$\int_0^{\infty} t\cdot e^{at}\,dt,\quad a = -1-j\omega$$

* integration by parts: $\int u\,dv = uv - \int v\,du$

$u = t;\ dv = e^{at}dt$

$$\int t\cdot e^{at}dt = \frac{te^{at}}{a} - \int\frac{e^{at}}{a}dt$$

$$= \frac{te^{at}}{a} - \frac{e^{at}}{a^2}$$

$te^{t(-1-j\omega)}$ ↳ approaches zero

$\lim_{t\to\infty} e^{(\alpha+j\beta)t}$ if $\alpha<0 \to 0$; if $\alpha>0 \to \infty$

$$\frac{te^{at}}{a} = \frac{te^{-t}e^{-j\omega t}}{-1-j\omega}$$

$$\int_0^{\infty} t\cdot e^{at}dt = \left[\frac{te^{at}}{a} - \frac{e^{at}}{a^2}\right]\Bigg|_0^{\infty}$$

$$\lim_{t\to\infty} te^{-t} = \lim_{t\to\infty}\frac{1}{e^t} = 0$$

$$= \left[0 - \frac{e^{at}}{a^2}\right]_0^{\infty} = \left[0 - \frac{e^{(-1-j\omega)t}}{a^2}\right]_0^{\infty} = \left[0 - \frac{e^{-t}e^{-j\omega t}}{a^2}\right]_0^{\infty}$$

$$= [0-0] - \left[0 - \frac{1}{a^2}\right] = \frac{1}{a^2} = \frac{1}{(-1-j\omega)^2}$$

$$\boxed{X(j\omega) = \frac{1}{(-1-j\omega)^2}}$$