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ECEN 380
2/4/2026

PS05: 2.50(e), 2.57(a)
2.59(b), 2.60 for 2.59(b),
2.65(b), MATLAB 2.86

1/5

2.50(e) Evaluate the step response for the LTI system with the given impulse response:

e.) $h(t) = e^{-|t|}$

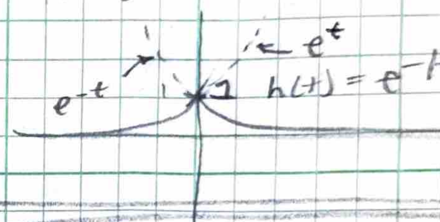
$\delta(t) \rightarrow \boxed{H} \rightarrow h(t)$

$u(t) \rightarrow \boxed{H} \rightarrow s(t)$

$$s(t) = \int_{-\infty}^{t} h(\tau)d\tau$$

$$s(t) = \int_{-\infty}^{t} e^{-|\tau|}d\tau = \begin{cases} \int_{-\infty}^{t} e^{\tau}d\tau, \ \tau < 0 = e^{\tau}\Big|_{-\infty}^{t} = e^{t} - e^{-\infty} = e^{t} - \frac{1}{e^{\infty}} \\ \int_{-\infty}^{0} e^{\tau}d\tau + \int_{0}^{t} e^{-\tau}d\tau, \ \tau \geq 0 = e^{\tau}\Big|_{-\infty}^{0} + -e^{-\tau}\Big|_{0}^{t} = \overset{1}{e^{0}} - \overset{0}{e^{-\infty}} + -e^{-t} + \overset{1}{e^{-0}} \end{cases}$$

$$\boxed{s(t) = \begin{cases} e^{t}, & t < 0 \\ -e^{-t} + 2, & t \geq 0 \end{cases}}$$



2.57(a) Determine the output of the system described by the following differential equation w/ input & initial conditions.

$$\frac{d}{dt}y(t) + 10y(t) = 2x(t); \; y(0^-) = 1$$

$$x(t) = u(t)$$

homogeneous eq: $r + 10 = 0$

$r = -10$

$y^{(h)}(t) = C_1 e^{-10t}$

particular eq: $y^{(p)}(t) = K \;, \; t \geq 0$

$\frac{d}{dt}(K) + 10(K) = 2 \;, \; t \geq 0$

$0 + 10K = 2$

$10K = 2$

$K = 2/10$

$y^{(p)}(t) = 1/5 \;, \; t \geq 0$

$y(t) = y^{(h)}(t) + y^{(p)}(t) = C_1 e^{-10t} + 1/5$

$y(0^-) = 1 \longrightarrow y(0^-) = C_1(e^{-10(0^-)}) + \frac{1}{5} = 1 \longrightarrow C_1 + \frac{1}{5} = 1 \longrightarrow C_1 = 4/5$

$\boxed{y(t) = 4/5 e^{-10t} + 1/5}$

2.59 (b) Determine output of systems described by the following difference equation w/ input & initial conditions.

$$y[n] - \tfrac{1}{9}y[n-2] = x[n-1] \; ; \; y[-1] = 1 \, , \; y[-2] = 0$$
$$x[n] = u[n]$$

homogeneous eq: $r^2 + 0r - \frac{1}{9} = 0$

$r^2 = \frac{1}{9}$

$r = \pm\frac{1}{3}$

$y^{(h)}[n] = C_1(-\frac{1}{3})^n + C_2(\frac{1}{3})^n$

particular eq: $y^{(p)}[n] = K, \; n \geq 0$

$y^p[n] - \frac{1}{9}y^p[n-2] = x[n-1], \; n \geq 0$

$y^p[n+2] - \frac{1}{9}y^p[n] = x[n+1]$

$K - \frac{1}{9}K = 1$

$\frac{8}{9}K = 1$

$K = \frac{9}{8}$

$$y[n] = C_1(-\tfrac{1}{3})^n + C_2(\tfrac{1}{3})^n + \tfrac{9}{8}$$

Recursive eq: $y[n] = \frac{1}{9}y[n-2] + x[n-1]$

$y[0] = \frac{1}{9}y[-2] + \cancel{x[-1]}$

$y[0] = 0$

$y[1] = \frac{1}{9}y[-1] + x[0]$

$= \frac{1}{9}(1) + 1 = \frac{10}{9}$

$y[1] = \frac{10}{9}$

$y[0] = 0 = C_1(-\frac{1}{3})^0 + C_2(\frac{1}{3})^0 + \frac{9}{8}$

$C_1 + C_2 = -\frac{9}{8}$

$y[1] = \frac{10}{9} = C_1(-\frac{1}{3})^1 + C_2(\frac{1}{3})^1 + \frac{9}{8}$

$-\frac{1}{3}C_1 + \frac{1}{3}C_2 = -\frac{1}{72}$

$$\begin{bmatrix} 1 & 1 & -\frac{9}{8} \\ -\frac{1}{3} & \frac{1}{3} & -\frac{1}{72} \end{bmatrix}$$

$\Downarrow$ RREF

$C_1 = -\frac{13}{24} \; ; \; C_2 = -\frac{7}{12}$

$$\boxed{y[n] = \frac{-13}{24}(-\tfrac{1}{3})^n + \frac{-7}{12}(\tfrac{1}{3})^n + \tfrac{9}{8}}$$

2.60 for 2.59(b) Identify the natural and forced responses for: 4/5

$$y[n] - \tfrac{1}{9} y[n-2] = x[n-1]; \quad y[-1]=1, \; y[-2]=0$$

$$x[n] = u[n]$$

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homo eq: (from 2.59(b)) $\longrightarrow y^{(h)}[n] = C_1(-\tfrac{1}{3})^n + C_2(\tfrac{1}{3})^n$

$$y[-1] = 1 = C_1(-\tfrac{1}{3})^{-1} + C_2(\tfrac{1}{3})^{-1}$$

$$= C_1(-3) + C_2(3) = 1$$

$$-3C_1 + 3C_2 = 1$$

$$y[-2] = 0 = C_1(-\tfrac{1}{3})^{-2} + C_2(\tfrac{1}{3})^{-2}$$

$$9C_1 + 9C_2 = 0$$

$$\begin{bmatrix} -3 & 3 & 1 \\ 9 & 9 & 0 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & -\tfrac{1}{6} \\ 0 & 1 & \tfrac{1}{6} \end{bmatrix}$$

$$\boxed{y^{(n)}[n] = -\tfrac{1}{6}(-\tfrac{1}{3})^n + \tfrac{1}{6}(\tfrac{1}{3})^n}$$

particular eq: (from 2.59(b)) $y^{(p)}[n] = \tfrac{9}{8}$

Recursive: $y[n] = \tfrac{1}{9} y[n-2] + x[n-1]$

set initials to 0: $y[-1] = 0; \; y[-2] = 0$

$$y[0] = \tfrac{1}{9}(0) + 0 = 0$$

$$y[1] = \tfrac{1}{9} y[-1] + x[0] = 1$$

$$y^{(f)}[n] = C_1(-\tfrac{1}{3})^n + C_2(\tfrac{1}{3})^n + \tfrac{9}{8}$$

$$y^{(f)}[0] = 0 = C_1 + C_2 + \tfrac{9}{8} \rightarrow C_1 + C_2 = -\tfrac{9}{8}$$

$$y^{(f)}[1] = 1 = -\tfrac{1}{3}C_1 + \tfrac{1}{3}C_2 + \tfrac{9}{8} \rightarrow -\tfrac{1}{3}C_1 + \tfrac{1}{3}C_2 = -\tfrac{1}{8}$$

$$\begin{bmatrix} 1 & 1 & -\tfrac{9}{8} \\ -\tfrac{1}{3} & \tfrac{1}{3} & -\tfrac{1}{8} \end{bmatrix} \Downarrow \quad C_1 = -\tfrac{3}{8}; \; C_2 = -\tfrac{3}{4}$$

$$\boxed{y^{(f)}[n] = -\tfrac{3}{8}(-\tfrac{1}{3})^n + -\tfrac{3}{4}(\tfrac{1}{3})^n + \tfrac{9}{8}}$$

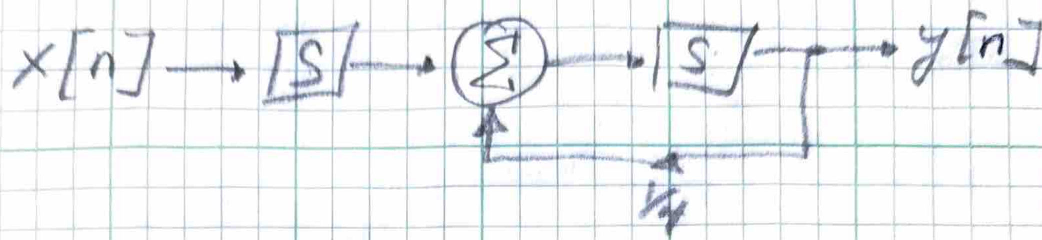
Check: $y^{(t)}[n] = y^{(f)}[n] + y^{(n)}[n] \overset{?}{}$

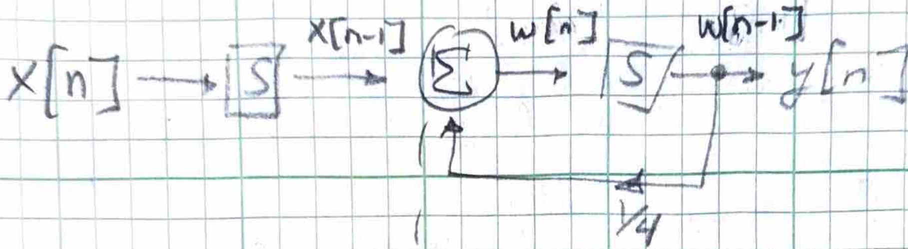
$$-\tfrac{13}{24}(-\tfrac{1}{3})^n + -\tfrac{7}{12}(\tfrac{1}{3})^n + \tfrac{9}{8} \overset{?}{=} -\tfrac{3}{8}(-\tfrac{1}{3})^n + -\tfrac{3}{4}(\tfrac{1}{3})^n + \tfrac{9}{8}$$

$$+ -\tfrac{1}{6}(-\tfrac{1}{3})^n + \tfrac{1}{6}(\tfrac{1}{3})^n$$

$$= -\tfrac{13}{24}(-\tfrac{1}{3})^n + -\tfrac{7}{12}(\tfrac{1}{3})^n + \tfrac{9}{8} \checkmark$$

2.65(b) Find the difference equation description for the system below:





$w[n] = x[n-1] + \frac{1}{4} y[n]$

$y[n] = w[n-1] = x[n-2] + \frac{1}{4} y[n-1]$

$\boxed{y[n] = x[n-2] + \frac{1}{4} y[n-1]}$

C: ▸ Users ▸ sethr ▸ OneDrive ▸ Desktop ▸ ECEN380 ▸ Homework ▸ Homework 3

Live Editor - C:\Users\sethr\OneDrive\Desktop\ECEN380\Homework\Homework 5\problem_2_point_86.mlx

homework1.m | MATLAB_review.m | homework3_question197.mlx | Homework2.mlx | Problem_2.83.mlx | problem_2_point_86.mlx

```
% xlabel('n');
% ylabel('s[n]');
% grid on;
```

Problem 2.86:

```
subplot(2, 1, 1);
n = 0:19;
u = ones(1,20);
h1 = zeros(size(n));
h1(n >= 0 & n <= 3) = 1/4;
s = conv(u,h1);
stem(0:19, s(1:20),'filled','Color','r') % step response 1
% stem(0:19, h1,'filled','Color','r') % Impulse response 1
xlabel('n');
ylabel('s1[n]');
grid on;

subplot(2, 1, 2);
h2 = zeros(size(n));
h2(n == 0 | n == 2) = 1/4;
h2(n == 1 | n == 3) = -1/4;
s = conv(u,h2);
stem(0:19, s(1:20),'filled','Color','b') % step response 2
% stem(0:19, h2,'filled','Color','b') % Impulse response 2
xlabel('n');
ylabel('s2[n]');
grid on;
```