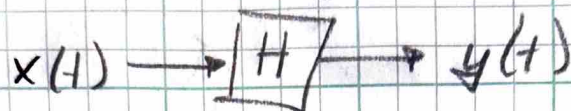


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1/2

- 1.81 Linear, time-invariant system below. $x(t)$ is periodic with period T . Show that $y(t)$ is also periodic w/ T .



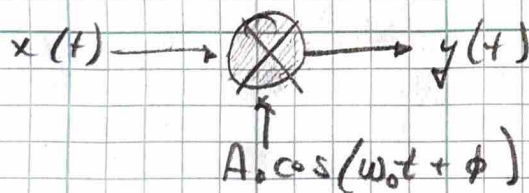
$$x(t) = x(t+T) \rightarrow y(t) = y(t+T)?$$

$$y(t+T) \stackrel{?}{=} H\{x(t+T)\}$$

$$y(t+T) = H\{x(t)\} = y(t) \quad \checkmark$$

- 1.84 Block diagram of linear time-varying system below.

$$y(t) = A_0 \cos(\omega_0 t + \phi) x(t)$$



- a. Demonstrate that the system is linear.

$$x(t) = a x_1(t) + b x_2(t) ; y(t) = A_0 \cos(\omega_0 t + \phi) [a x_1(t) + b x_2(t)]$$

$$a x_1(t) \rightarrow y_1(t) = A_0 \cos(\omega_0 t + \phi) [a x_1(t)]$$

$$b x_2(t) \rightarrow y_2(t) = A_0 \cos(\omega_0 t + \phi) [b x_2(t)]$$

$$y_1(t) + y_2(t) = y(t)?$$

$$A_0 \cos(\omega_0 t + \phi) [a x_1(t)] + A_0 \cos(\omega_0 t + \phi) [b x_2(t)]$$

$$A_0 \cos(\omega_0 t + \phi) [a x_1(t) + b x_2(t)] = y(t) \quad \checkmark$$

b. Demonstrate that the system is time-variant. 2/2
 Use $x(t) = \delta(t)$

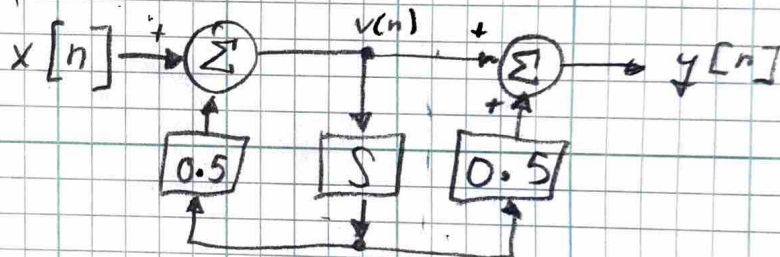
$$y_1(t) = A_0 \cos(\omega_0 t + \phi) \delta(t)$$

$$y_2(t) = A_0 \cos(\omega_0 t + \phi) \delta(t - t_0)$$

$$y_1(t - t_0) = A_0 \cos(\omega_0(t - t_0) + \phi) \delta(t - t_0)$$

$$y_1(t - t_0) \neq y_2(t) \Rightarrow \text{Time Variant}$$

1.89 Block diagram below, first-order recursive discrete-time filter. Derive expression for $y[n]$ in terms of $x[n]$



Left: $a[n] = x[n] + 0.5a[n-1]$

Right: $a[n] + 0.5a[n-1] = y[n]$

$$y[n] = x[n] + 0.5a[n-1] + 0.5a[n-1]$$

$$y[n] = x[n] + a[n-1]$$

$$a[n-1] = x[n-1] + 0.5a[n-2]$$

$$a[n-2] = x[n-2] + 0.5a[n-3] \dots$$

$$y[n-1] = x[n-1] + a[n-2]$$

$$a[n-2] = y[n-1] - x[n-1]$$

$$y[n] = x[n] + [x[n-1] + 0.5[y[n-1] - x[n-1]]]$$

$$y[n] = x[n] + x[n-1] + 0.5y[n-1] - 0.5x[n-1]$$

$$y[n] = x[n] + 0.5x[n-1] + 0.5y[n-1]$$

Problem 1.97

A rectangular pulse $x(t)$ is defined by

$$x(t) =$$

$$10, 0 \leq t \leq 5$$

$$0, \text{ otherwise}$$

Generate $x(t)$, using

(a) A pair of time-shifted step functions

(b) An M-file (numerically)

Part (a)

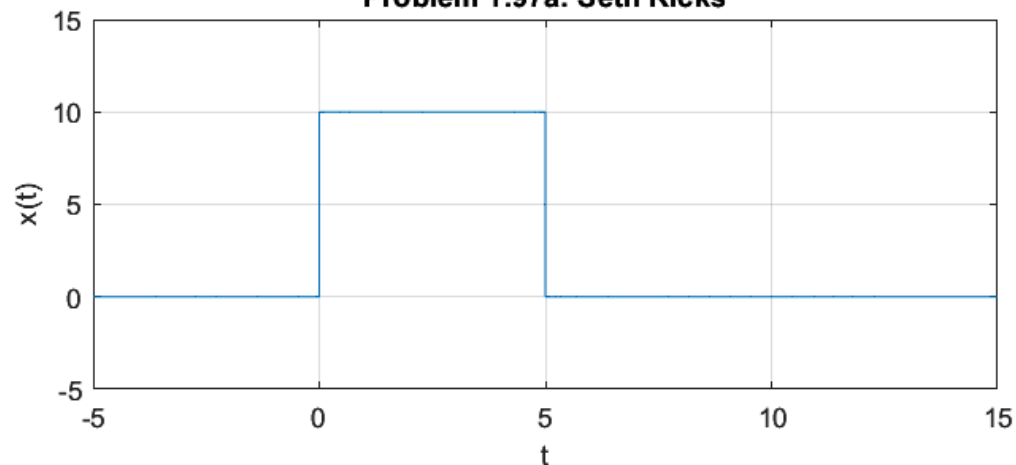
```
1 syms t;
2 subplot(2,1,1)
3 unit_step = 10 * (heaviside(t) - heaviside(t - 5));
4 fplot(unit_step, [-5 15]);
5 ylim([-5 15])
6 title('Problem 1.97a: Seth Ricks');
7 xlabel('t');
8 ylabel('x(t)');
9 grid on;
```

Part (b)

```
10 t = -5:0.01:15; % Time
11 x = 10 * (t >= 0 & t <= 5); % True for x>=0 and t <=5, false otherwise
12 subplot(2,1,2)
13 plot(t, x);
14 ylim([-5 15])
15 xlim([-5 15])
16 title('Problem 1.97b: Seth Ricks');
17 xlabel('t');
18 ylabel('x(t)');
19 grid on;
```



Problem 1.97a: Seth Ricks



Problem 1.97b: Seth Ricks

