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ECEN 380

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1.81, 1.84(a), 1.84(b)
1.89, MATLAB 1.97

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1.81 Linear, time-invariant system below. $x(t)$ is periodic with period T. Show that $y(t)$ is also periodic w/ T.

$$x(t) \longrightarrow \boxed{H} \longrightarrow y(t)$$

$$x(t) = x(t+T) \longrightarrow y(t) = y(t+T) ?$$

$$y(t+T) \overset{?}{=} H\{x(t+T)\}$$

$$\boxed{y(t+T) = H\{x(t)\} = y(t)} \checkmark$$

1.84 Block diagram of linear time-varying system below.

$$y(t) = A_0 \cos(\omega_0 t + \phi)\, x(t)$$

$$x(t) \longrightarrow \otimes \longrightarrow y(t)$$

$$\uparrow$$

$$A_0 \cos(\omega_0 t + \phi)$$

a. Demonstrate that the system is linear.

$$x(t) = a x_1(t) + b x_2(t) \; ; \; y(t) = A_0 \cos(\omega_0 t + \phi)[a x_1(t) + b x_2(t)]$$

$$a x_1(t) \rightarrow y_1(t) = A_0 \cos(\omega_0 + \phi)[a x_1(t)]$$

$$a x_2(t) \rightarrow y_2(t) = A_0 \cos(\omega_0 + \phi)[b x_2(t)]$$

$$y_1(t) + y_2(t) = y(t) ?$$

$$A_0 \cos(\omega_0 + \phi)[a x_1(t)] + A_0 \cos(\omega_0 + \phi)[b x_2(t)]$$

$$\boxed{A_0 \cos(\omega_0 + \phi)[a x_1(t) + b x_2(t)] = y(t)} \checkmark$$

(b.) Demonstrate that the system is time-variant.

use $x(t) = \delta(t)$

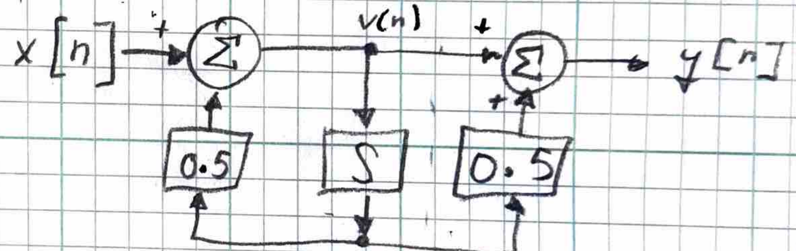
$$y(t) = A_0 \cos(\omega_0 t + \phi)\delta(t)$$

$$y_2(t) = A_0 \cos(\omega_0 t + \phi)\,\delta(t - t_0)$$

$$y_1(t - t_0) = A_0 \cos(\omega_0 (t - t_0) + \phi)\,\delta(t - t_0)$$

$$\boxed{y_1(t - t_0) \neq y_2(t) \Rightarrow \text{Time variant}}$$

(1.89) Block diagram below, first-order recursive discrete-time filter. Derive expression for $y[n]$ in terms of $x[n]$



Left: $a[n] = x[n] + 0.5a[n-1]$

Right: $a[n] + 0.5a[n-1] = y[n]$

$$y[n] = x[n] + 0.5\,a[n-1] + 0.5\,a[n-1]$$

$$y[n] = x[n] + a[n-1]$$

$$a[n-1] = x[n-1] + 0.5\,a[n-2]$$

$$\cancel{a[n-2] = x[n-2] + 0.5\,a[n-3]} \ \ldots$$

$$y[n-1] = x[n-1] + a[n-2]$$

$$a[n-2] = y[n-1] - x[n-1]$$

$$y[n] = x[n] + \left[x[n-1] + 0.5\left[y[n-1] - x[n-1]\right]\right]$$

$$y[n] = x[n] + x[n-1] + 0.5\,y[n-1] - 0.5x[n-1]$$

$$\boxed{y[n] = x[n] + 0.5x[n-1] + 0.5\,y[n-1]}$$