Seth Ricks

ECEN 380
2/4/2026

PS05: 2.50(e), 2.57(a)
2.59(b), 2.60 for 2.59(b),
2.65(b), MATLAB 2.86

1/5

(2.50(e)) Evaluate the step response for the LTI system with the given impulse response:

e.) $h(t) = e^{-|t|}$

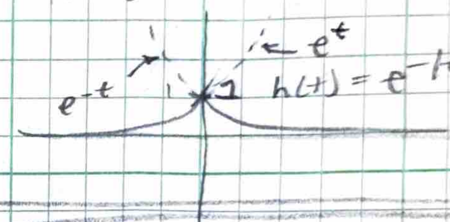
$\delta(t) \longrightarrow \boxed{H} \longrightarrow h(t)$

$u(t) \longrightarrow \boxed{H} \longrightarrow s(t)$

$$s(t) = \int_{-\infty}^{t} h(\tau)\,d\tau$$

$$s(t) = \int_{-\infty}^{t} e^{-|\tau|}\,d\tau = \begin{cases} \int_{-\infty}^{t} e^{\tau}\,d\tau, & \tau < 0 = e^{\tau}\Big|_{-\infty}^{t} = e^{t} - e^{-\infty} = e^{t} - \frac{1}{e^{\infty}} \\ \int_{-\infty}^{0} e^{\tau}\,d\tau + \int_{0}^{t} e^{-\tau}\,d\tau, & \tau \geq 0 = e^{\tau}\Big|_{-\infty}^{0} + -e^{-\tau}\Big|_{0}^{t} = \cancel{e^{0}}^{1} - \cancel{e^{-\infty}}^{0} + -e^{-t} + \cancel{e^{-0}}^{1} \end{cases}$$

$$\boxed{s(t) = \begin{cases} e^{t}, & t < 0 \\ -e^{-t} + 2, & t \geq 0 \end{cases}}$$



2.57(a) Determine the output of the system described by the following differential equation w/ input & initial conditions.

$$\frac{d}{dt}y(t) + 10y(t) = 2x(t); \; y(0^-) = 1$$

$$x(t) = u(t)$$

homogeneous eq: $r + 10 = 0$

$r = -10$

$y^{(h)}(t) = C_1 e^{-10t}$

particular eq: $y^{(p)}(t) = K \quad , t \geq 0$

$\frac{d}{dt}(K) + 10(K) = 2 \quad , t \geq 0$

$0 + 10K = 2$

$10K = 2$

$K = 2/10$

$y^{(p)}(t) = 1/5 \quad , t \geq 0$

$$y(t) = y^{(h)}(t) + y^{(p)}(t) = C_1 e^{-10t} + 1/5$$

$$y(0^-) = 1 \longrightarrow y(0^-) = C_1\left(e^{-10(0^-)}\right) + \tfrac{1}{5} = 1 \longrightarrow C_1 + \tfrac{1}{5} = 1 \longrightarrow C_1 = 4/5$$

$$\boxed{y(t) = \tfrac{4}{5}e^{-10t} + \tfrac{1}{5}}$$

2.59 (b) Determine output of systems described by the following difference equation w/ input & initial conditions.

$$y[n] - \tfrac{1}{9}y[n-2] = x[n-1] \; ; \; y[-1] = 1 \; , \; y[-2] = 0$$

$$x[n] = u[n]$$

homogeneous eq: $r^2 + 0r - \tfrac{1}{9} = 0$

$r^2 = \tfrac{1}{9}$

$r = \pm \tfrac{1}{3}$

$y^{(h)}[n] = C_1(-\tfrac{1}{3})^n + C_2(\tfrac{1}{3})^n$

particular eq: $y^{(p)}[n] = K, \; n \geq 0$

$y^p[n] - \tfrac{1}{9}y^p[n-2] = x[n-1], \; n \geq 0$

*? $y^p[n+2] - \tfrac{1}{9}y^p[n] = x[n+1]$

$K - \tfrac{1}{9}K = 1$

$\tfrac{8}{9}K = 1$

$K = \tfrac{9}{8}$

$$y[n] = C_1(-\tfrac{1}{3})^n + C_2(\tfrac{1}{3})^n + \tfrac{9}{8}$$

Recursive eq: $y[n] = \tfrac{1}{9}y[n-2] + x[n-1]$

$y[0] = \tfrac{1}{9}y[-2] + \cancel{x[-1]}$

$y[0] = 0$

$y[1] = \tfrac{1}{9}y[-1] + x[0]$

$= \tfrac{1}{9}(1) + 1 = \tfrac{10}{9}$

$y[1] = \tfrac{10}{9}$

$y[0] = 0 = C_1(-\tfrac{1}{3})^0 + C_2(\tfrac{1}{3})^0 + \tfrac{9}{8}$

$C_1 + C_2 = -\tfrac{9}{8}$

$y[1] = \tfrac{10}{9} = C_1(-\tfrac{1}{3})^1 + C_2(\tfrac{1}{3})^1 + \tfrac{9}{8}$

$-\tfrac{1}{3}C_1 + \tfrac{1}{3}C_2 = -\tfrac{1}{72}$

$$\begin{bmatrix} 1 & 1 & -\tfrac{9}{8} \\ -\tfrac{1}{3} & \tfrac{1}{3} & -\tfrac{1}{72} \end{bmatrix}$$

$\Downarrow$ RREF

$C_1 = -\tfrac{13}{24} \; ; \; C_2 = -\tfrac{7}{12}$

$$\boxed{y[n] = \frac{-13}{24}(-\tfrac{1}{3})^n + \frac{-7}{12}(\tfrac{1}{3})^n + \tfrac{9}{8}}$$

2.60 for 2.59(b) Identify the natural and forced responses for: 4/5

$$y[n] - \frac{1}{9}y[n-2] = x[n-1]; \quad y[-1]=1, \quad y[-2]=0$$

$$x[n] = u[n]$$

---

homo eq: (from 2.59(b)) $\longrightarrow y^{(h)}[n] = C_1(-\frac{1}{3})^n + C_2(\frac{1}{3})^n$

$$y[-1] = 1 = C_1(-\tfrac{1}{3})^{-1} + C_2(\tfrac{1}{3})^{-1}$$

$$= C_1(-3) + C_2(3) = 1$$

$$-3C_1 + 3C_2 = 1$$

$$y[-2] = 0 = C_1(-\tfrac{1}{3})^{-2} + C_2(\tfrac{1}{3})^{-2}$$

$$9C_1 + 9C_2 = 0$$

$$\boxed{y^{(n)}[n] = -\tfrac{1}{6}(-\tfrac{1}{3})^n + \tfrac{1}{6}(\tfrac{1}{3})^n}$$

$$\begin{bmatrix} -3 & 3 & 1 \\ 9 & 9 & 0 \end{bmatrix} \Downarrow \text{RREF} \begin{bmatrix} 1 & 0 & -\frac{1}{6} \\ 0 & 1 & \frac{1}{6} \end{bmatrix}$$

particular eq: (from 2.59(b)) $y^{(p)}[n] = \frac{9}{8}$

Recursive: $y[n] = \frac{1}{9}y[n-2] + x[n-1]$

set initials to 0: $y[-1] = 0$; $y[-2] = 0$

$$y[0] = \tfrac{1}{9}(0) + 0 = 0$$

$$y[1] = \tfrac{1}{9}y[-1] + x[0] = 1$$

$$y^{(f)}[n] = C_1(-\tfrac{1}{3})^n + C_2(\tfrac{1}{3})^n + \tfrac{9}{8}$$

$$y^{(f)}[0] = 0 = C_1 + C_2 + \tfrac{9}{8} \rightarrow C_1 + C_2 = -\tfrac{9}{8}$$

$$y^{(f)}[1] = 1 = -\tfrac{1}{3}C_1 + \tfrac{1}{3}C_2 + \tfrac{9}{8} \rightarrow -\tfrac{1}{3}C_1 + \tfrac{1}{3}C_2 = -\tfrac{1}{8}$$

$$\boxed{y^{(f)}[n] = -\tfrac{3}{8}(-\tfrac{1}{3})^n + -\tfrac{3}{4}(\tfrac{1}{3})^n + \tfrac{9}{8}}$$

$$\begin{bmatrix} 1 & 1 & -\frac{9}{8} \\ -\frac{1}{3} & \frac{1}{3} & -\frac{1}{8} \end{bmatrix} \Downarrow \quad C_1 = -\tfrac{3}{8}; \; C_2 = -\tfrac{3}{4}$$

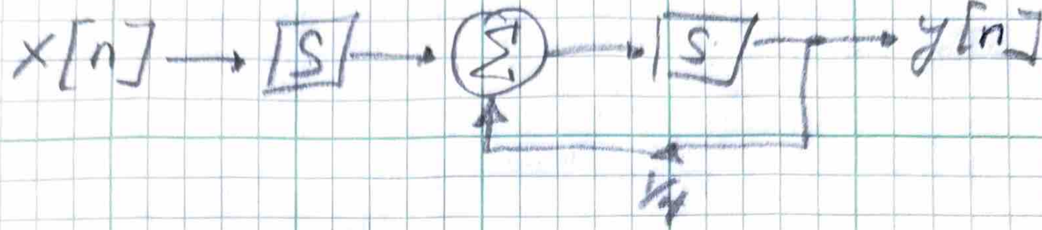
Check: $y^{(t)}[n] = y^{(f)}[n] + y^{(n)}[n]$?

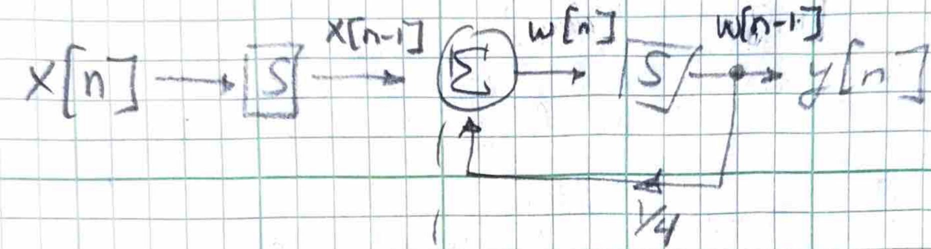
$$-\tfrac{13}{24}(-\tfrac{1}{3})^n + -\tfrac{7}{12}(\tfrac{1}{3})^n + \tfrac{9}{8} \stackrel{?}{=} -\tfrac{3}{8}(-\tfrac{1}{3})^n + -\tfrac{3}{4}(\tfrac{1}{3})^n + \tfrac{9}{8}$$

$$+ -\tfrac{1}{6}(-\tfrac{1}{3})^n + \tfrac{1}{6}(\tfrac{1}{3})^n$$

$$= -\tfrac{13}{24}(-\tfrac{1}{3})^n + -\tfrac{7}{12}(\tfrac{1}{3})^n + \tfrac{9}{8} \checkmark$$

2.65(b) Find the difference equation description for the system below:





$w[n] = x[n-1] + \frac{1}{4} y[n]$

$y[n] = w[n-1] = x[n-2] + \frac{1}{4} y[n-1]$

$$\boxed{y[n] = x[n-2] + \tfrac{1}{4} y[n-1]}$$