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ECEN 380
1/18/2025

PS02: 1.64(b), 1.64(h), 1.75(b), 1.77(b), 1.78(b), 1.95

1.64 b. Determine whether 1.) memoryless
2.) stable
3.) causal
4.) linear
5.) time variant

b.) $y[n] = 2x[n]u[n]$ ← unit step

Doesn't depend on past or future → $\boxed{\text{Memoryless}}$ ✓

Depends only on present → $\boxed{\text{Causal}}$ ✓

Output is finite → $\boxed{\text{Stable}}$ ✓

$x[n] = ax_1[n] + bx_2[n]$

$y[n] = 2(ax_1[n] + bx_2[n])\,u[n]$

$x[n] = ax_1[n] \rightarrow y[n] = 2ax_1[n]u[n]$

$x[n] = bx_1[n] \rightarrow$ " $= 2bx_2[n]u[n]$

$2ax_1[n]u[n] + 2bx_2[n]u[n]$

$2u[n](ax_1[n] + 2bx_2[n])$ → $\boxed{\text{Linear}}$ ✓

$y_2[n] = 2x_1[n-n_0]u[n]$

$y_1[n-n_0] = 2x_1[n-n_0]u[n-n_0]$ $\quad y_1[n-n_0] \neq y_2[n]$ → $\boxed{\text{Time Variant}}$ ✓

1.64h Determine:

1.) Memoryless
2.) Stable
3.) Causal
4.) Linear
5.) Time Varient

$$y(t) = \frac{d}{dt}x(t)$$

Relies on past & future $\longrightarrow$ $\boxed{\text{NOT Memoryless}}$

$|x(t)| \leq M < \infty \rightarrow x(t) = \sin(t^3) \rightarrow y(t) = 3t^2\cos(t^3) \rightarrow \boxed{\text{UN-stable}}$

Relies on future $\longrightarrow$ $\boxed{\text{NOT Causal}}$

$$x(t) = a\,x_1(t) + b\,x_2(t)$$

$$H\{x(t)\} \rightarrow y(t) = \frac{d}{dt}(ax_1(t) + bx_2(t))$$
$$= \frac{d}{dt}ax_1(t) + \frac{d}{dt}bx_2(t)$$

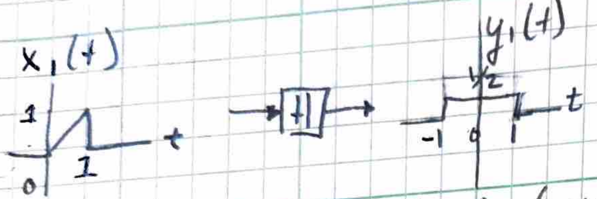
$$H\{ax_1(t)\} \rightarrow y(t) = \frac{d}{dt}ax_1(t)$$
$$H\{bx_2(t)\} \rightarrow y(t) = \frac{d}{dt}bx_2(t)$$

$$H\{x(t)\} = H\{ax_1(t)\} + H\{bx_2(t)\}$$
$\hookrightarrow \boxed{\text{Linear}}$

$$y_1(t) = \frac{d}{dt}x(t)$$
$$y_1(t-t_0) = \frac{d}{dt}x(t-t_0)$$
$$H\{x(t-t_0)\} \rightarrow y(t) = \frac{d}{dt}x(t-t_0) \longrightarrow \boxed{\text{Time Invariant}}$$

1.75b A system H has input-output pairs given. Determine:

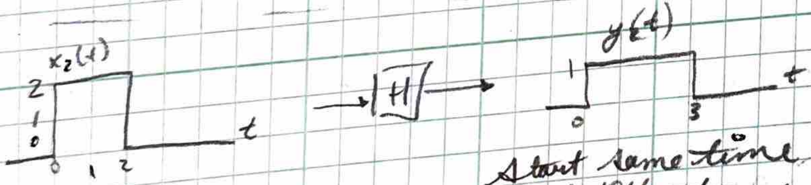
1.) Memoryless
2.) Causal
3.) Linear
4.) Time variant



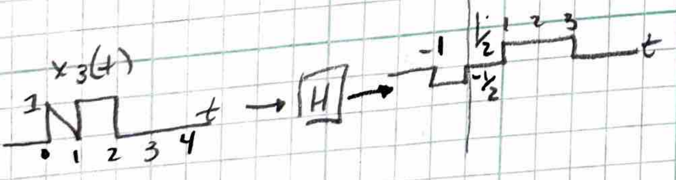
Output starts before input → NOT Memoryless
→ NOT Causal

Linear: $x_1(t) = ax_1(t) + bx_2(t)$ → don't know, only one signal
$y_1(\ ) = ?$

Time Variant:
$y(t-t_0) \overset{?}{=} H\{x(t-t_0)\}$ → don't know w/out equations



Start same time → NOT Causal
End diff. times → NOT memoryless
Linear → Don't know, only one signal
Time Variant → Don't know



Start different, end different
→ NOT Causal
→ NOT memoryless

Linear/Time Variant
→ DON'T KNOW

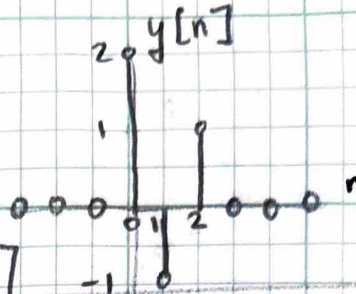
Start/end same
→ Causal
→ Memoryless

Linear/Time → DON'T KNOW

1.77 b

a discrete-time system is both linear and in-variant.

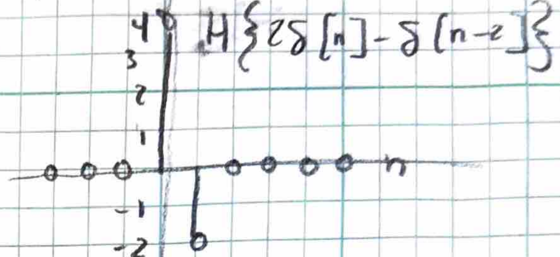
Input: $x[n] = \delta[n]$



Find output due to input:

$x[n] = 2\delta[n] - \delta[n-2]$

$x[0] = 2(2) - 0 = 4$
$x[1] = 2(-1) - 0 = -2$
$x[2] = 2(1) - 2 = 0$



1.78b

$x(t) = x_e(t) + x_o(t)$

↳ even and odd components of $x(t)$, for all t $-\infty < t < \infty$

$x[n] = x_e[n] + x_o[n]$

show that:

$$\sum_{n=-\infty}^{\infty} x^2[n] = \sum_{n=-\infty}^{\infty} x_e^2[n] + \sum_{n=-\infty}^{\infty} x_o^2[n]$$

$$(x[n])^2 = (x_e[n] + x_o[n])^2$$

$$x^2[n] = x_e^2[n] + 2x_e[n]\,x_o[n] + x_o^2[n]$$

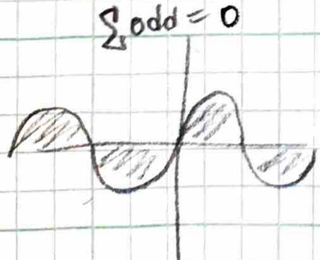
$$y[n] = 2x_e[n]\,x_o[n]$$

$$y[-n] = 2x_e[-n]\,x_o[n]$$

$$= 2x_e[n](-x_o[n])$$

$$= -2x_e[n]x_o[n] = -y[n] \rightarrow \text{odd}$$

$\sum \text{odd} = 0$



* sum of odd of y-axis symmetric function ↳ 0

$$\boxed{\begin{aligned} x^2[n] &= x_e^2[n] + x_o^2[n] \\ \sum_{n=-\infty}^{\infty} x^2[n] &= \sum_{n=-\infty}^{\infty} x_e^2[n] + \sum_{n=-\infty}^{\infty} x_o^2[n] \end{aligned}}$$