Seth Ricks ECEN 380 1/23/2026

1.81, 1.84(a), 1.84(b)
1.89, MATLAB 1.97

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(1.81) Linear, time-invariant system below. $x(t)$ is periodic with period T. Show that $y(t)$ is also periodic w/ T.

$$x(t) \longrightarrow \boxed{H} \longrightarrow y(t)$$

$$x(t) = x(t+T) \longrightarrow y(t) = y(t+T) \,?$$

$$y(t+T) \overset{?}{=} H\{x(t+T)\}$$

$$\boxed{y(t+T) = H\{x(t)\} = y(t)} \checkmark$$

(1.84) Block diagram of linear time-varying system below.

$$y(t) = A_0 \cos(\omega_0 t + \phi)\, x(t)$$

$$x(t) \longrightarrow \bigotimes \longrightarrow y(t)$$

$$\uparrow$$

$$A_0 \cos(\omega_0 t + \phi)$$

(a.) Demonstrate that the system is linear.

$$x(t) = a x_1(t) + b x_2(t) \;;\; y(t) = A_0 \cos(\omega_0 t + \phi)[a x_1(t) + b x_2(t)]$$

$$a x_1(t) \rightarrow y_1(t) = A_0 \cos(\omega_0 + \phi)[a x_1(t)]$$

$$a x_2(t) \rightarrow y_2(t) = A_0 \cos(\omega_0 + \phi)[b x_2(t)]$$

$$y_1(t) + y_2(t) = y(t) \,?$$

$$A_0 \cos(\omega_0 + \phi)[a x_1(t)] + A_0 \cos(\omega_0 + \phi)[b x_2(t)]$$

$$\boxed{A_0 \cos(\omega_0 + \phi)[a x_1(t) + b x_2(t)] = y(t)} \checkmark$$

(b.) Demonstrate that the system is time-variant.

use $x(t) = \delta(t)$

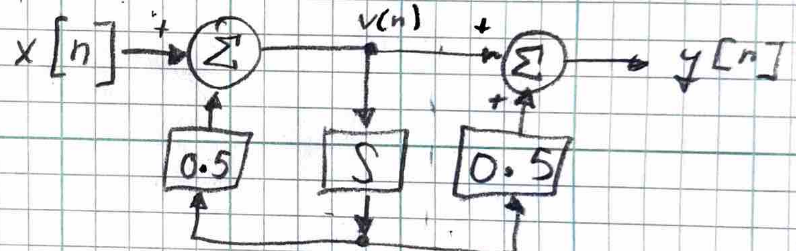
$$y(t) = A_0 \cos(\omega_0 t + \phi)\delta(t)$$

$$y_2(t) = A_0 \cos(\omega_0 t + \phi)\,\delta(t - t_0)$$

$$y_1(t - t_0) = A_0 \cos(\omega_0 (t - t_0) + \phi)\,\delta(t - t_0)$$

$$\boxed{y_1(t - t_0) \neq y_2(t) \Rightarrow \text{Time variant}}$$

(1.89) Block diagram below, first-order recursive discrete-time filter. Derive expression for $y[n]$ in terms of $x[n]$



Left: $a[n] = x[n] + 0.5a[n-1]$

Right: $a[n] + 0.5a[n-1] = y[n]$

$$y[n] = x[n] + 0.5\,a[n-1] + 0.5\,a[n-1]$$

$$y[n] = x[n] + a[n-1]$$

$$a[n-1] = x[n-1] + 0.5\,a[n-2]$$

$$\cancel{a[n-2] = x[n-2] + 0.5\,a[n-3]} \ \ldots$$

$$y[n-1] = x[n-1] + a[n-2]$$

$$a[n-2] = y[n-1] - x[n-1]$$

$$y[n] = x[n] + \left[x[n-1] + 0.5\left[y[n-1] - x[n-1]\right]\right]$$

$$y[n] = x[n] + x[n-1] + 0.5\,y[n-1] - 0.5\,x[n-1]$$

$$\boxed{y[n] = x[n] + 0.5\,x[n-1] + 0.5\,y[n-1]}$$

Define $v[n] = x[n] + 0.5x[n-1]$

$$y[n] = v[n] + 0.5y[n-1]$$

$$y[n] = \sum_{k=0}^{\infty} (0.5)^k v[n-k] \quad (\text{eq } 1.115/1.116)$$

$$y[n] = \sum_{k=0}^{\infty} (0.5)^k \left(x[n-k] + 0.5x[n-k-1]\right)$$

$$y[n] = \sum_{k=0}^{\infty} (0.5)^k x[n-k] + \underbrace{\sum_{k=0}^{\infty} \left[(0.5)^k \, 0.5x[n-k-1]\right]}$$

$$\sum_{k=0}^{\infty} \left[0.5^{(k+1)} x[n-(k+1)]\right]$$

$m = k+1$

$$\sum_{m=1}^{\infty} \left[0.5^m x[n-m]\right]$$

$$\underbrace{\sum_{k=1}^{\infty} \left[0.5^k x[n-k]\right]}$$

$$y[n] = \sum_{k=0}^{\infty} (0.5)^k x[n-k] +$$

$$y[n] = (0.5)^{k=0} x[n-(k=0)] + 2\sum_{k=1}^{\infty} 0.5^k x[n-k]$$

$$\boxed{y[n] = x[n] + 2\sum_{k=1}^{\infty} 0.5^k x[n-k]}$$

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homework1.m | MATLAB_review.m | homework3_question197.mlx | Homework2.m

Problem 1.97

A rectangular pulse x(t) is defined by

x(t) =

10, 0 <= t <= 5

0, otherwise

Generate x(t), using

(a) A pair of time-shifted step functions

(b) An M-file (numerically)

Part (a)

```
syms t;
subplot(2,1,1)
unit_step = 10 * (heaviside(t) - heaviside(t - 5));
fplot(unit_step, [-5 15]);
ylim([-5 15])
title('Problem 1.97a: Seth Ricks');
xlabel('t');
ylabel('x(t)');
grid on;
```

Part (b)

```
t = -5:0.01:15;     % Time
x = 10 * (t >= 0 & t <= 5); % True for x>=0 and t <=5, false otherwise
subplot(2,1,2)
plot(t, x);
ylim([-5 15])
xlim([-5 15])
title('Problem 1.97b: Seth Ricks');
xlabel('t');
ylabel('x(t)');
grid on;
```

Figure

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