

- 1.64b. Determine whether
- 1.) memoryless
  - 2.) stable
  - 3.) causal
  - 4.) linear
  - 5.) time variant

b.)  $y[n] = 2x_1[n]u[n]$  + unit step

Doesn't depend on past or future  $\rightarrow$  [Memoryless]

Depends only on present  $\rightarrow$  [Causal]

Output is finite  $\rightarrow$  [Stable]

$$x[n] = ax_1[n] + bx_2[n]$$

$$y[n] = 2(ax_1[n] + bx_2[n])u[n]$$

$$x[n] = ax_1[n] \rightarrow y[n] = 2ax_1[n]u[n]$$

$$x[n] = bx_2[n] \rightarrow " = 2bx_2[n]u[n]$$

$$2ax_1[n]u[n] + 2bx_2[n]u[n]$$

$$2u[n](ax_1[n] + 2bx_2[n])$$

$\rightarrow$  [Linear]

$$y_2[n] = 2x_1[n-n_0]u[n]$$

$$y_1[n-n_0] = 2x_1[n-n_0]u[n-n_0] \quad y_1[n-n_0] \neq y_2[n] \rightarrow$$

[Time Variant]

1.67h

Determine:

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- 1.) Memoryless
- 2.) Stable
- 3.) Causal
- 4.) Linear
- 5.) Time Variant

$$y(t) = \frac{d}{dt} x(t)$$

Relies on past & future  $\rightarrow$  NOT Memoryless

$$|x(t)| \leq M < \infty \rightarrow x(t) = \sin(t^2) \rightarrow y(t) = 2t^2 \cos(t^2) \rightarrow \boxed{\text{UNstable}}$$

Relies on future  $\rightarrow$  NOT Causal

$$x(t) = a x_1(t) + b x_2(t)$$

$$H\{x(t)\} \rightarrow y(t) = \frac{d}{dt} (a x_1(t) + b x_2(t)) \\ = \frac{d}{dt} a x_1(t) + \frac{d}{dt} b x_2(t)$$

$$H\{a x_1(t)\} \rightarrow y(t) = \frac{d}{dt} a x_1(t)$$

$$H\{b x_2(t)\} \rightarrow y(t) = \frac{d}{dt} b x_2(t)$$

$$+ H\{x(t)\} = H\{a x_1(t)\} + H\{b x_2(t)\}$$

$\hookrightarrow$  Linear

$$y_1(t) = \frac{d}{dt} x(t)$$

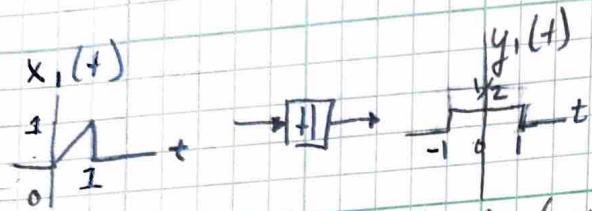
$$y_1(t-t_0) = \frac{d}{dt} x(t-t_0) \rightsquigarrow$$

$$H\{x(t-t_0)\} \rightarrow y(t) = \frac{d}{dt} x(t-t_0) \rightarrow \boxed{\text{Time Invariant}}$$

1.75b

A system  $H$  has input-output pairs given. Determine:

- 1.) Memoryless
- 2.) Causal
- 3.) Linear
- 4.) Time variant



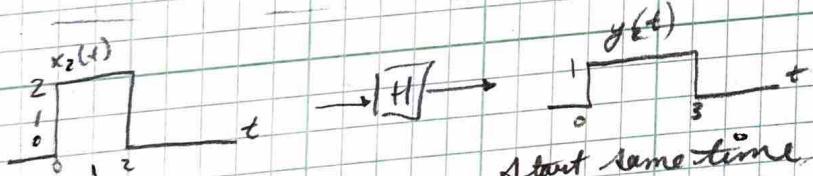
Output starts before input → NOT memoryless  
NOT causal

Linear:  $\begin{aligned} x_1(t) = \\ y_1(t) = ? \end{aligned}$

→ don't know, only one signal

Time variant:

$$y(t-t_0) = H\{x(t-t_0)\} \rightarrow \text{don't know w/out equations}$$

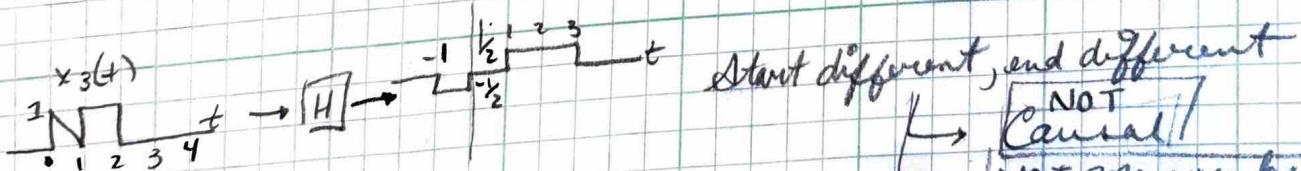


Start same time  
End diff. times

→ NOT causal  
Not memoryless

Linear → Don't know, only one signal

Time variant → Don't know

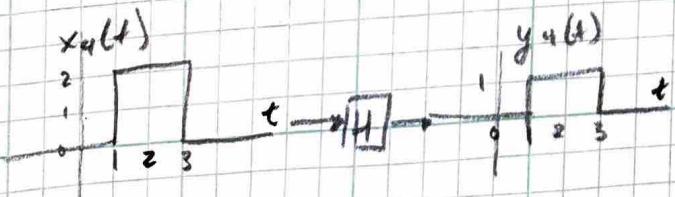


Start different, end different

→ NOT causal  
NOT memoryless

Linear/Time variant

→ Don't know



Start/end same

→ Causal

Linear/Time → Memoryless

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(1.77 b)

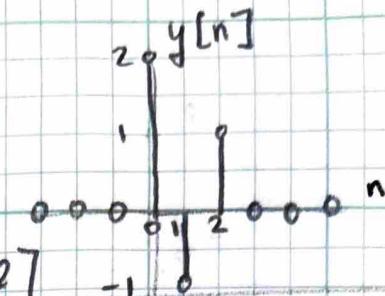
A discrete-time system is both linear and in-variant.

$$\text{Input: } x[n] = \delta[n]$$

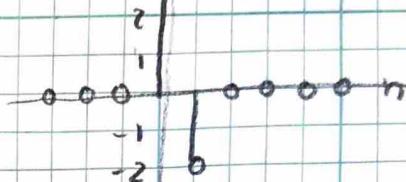
Find output due to  
input:

$$x[n] = 2\delta[n] - \delta[n-2]$$

$$\begin{aligned} x[0] &= 2(2) - 0 = 4 \\ x[1] &= 2(-1) - 0 = -2 \\ x[2] &= 2(1) - 2 = 0 \end{aligned}$$



$$y[n] = H\{2\delta[n] - \delta[n-2]\}$$



(1.78b)

$$x(t) = x_e(t) + x_o(t)$$

↳ even and odd components of  $x(t)$ ,  $-00 < t < 00$

$$x[n] = x_e[n] + x_o[n]$$

Show that:

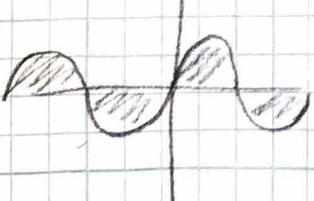
$$\sum_{n=-\infty}^{\infty} x^2[n] = \sum_{n=-\infty}^{\infty} x_e^2[n] + \sum_{n=-\infty}^{\infty} x_o^2[n]$$

$$(x[n])^2 = (x_e[n] + x_o[n])^2$$

$$x^2[n] = x_e^2[n] + 2x_e[n]x_o[n] + x_o^2[n]$$

$$y[n] = 2x_e[n]x_o[n]$$

$$\sum_{n=-\infty}^{\infty} y[n] = 0$$



$$y[-n] = 2x_e[-n]x_o[n]$$

$$= 2x_e[n](-x_o[n])$$

$$= -2x_e[n]x_o[n] = -y[n] \rightarrow \text{odd}$$

\* sum of odd of y-axis symmetric functions

$$x^2[n] = x_e^2[n] + x_o^2[n]$$

$$\sum_{n=-\infty}^{\infty} x^2[n] = \sum_{n=-\infty}^{\infty} x_e^2[n] + \sum_{n=-\infty}^{\infty} x_o^2[n]$$

