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ECEN 380
2/23/2026

PS07: 3.48(a),
3.49(b), 3.50(d), 3.51(d),
3.53(b), 3.54(c), MATLAB
3.102

V₆

3.48(a)

Use the definition equation for 1D TFS coefficients to evaluate the DTF S representation of the following signal:

$$x[n] = \cos\left(\frac{6\pi}{17}n + \frac{\pi}{3}\right)$$

$$X[k] = \frac{1}{N} \sum_{n=-N}^{N} x[n] e^{-j k n \omega_0} ; \quad \frac{6\pi}{17} = 3\left(\frac{2\pi}{17}\right) \rightarrow m=3, \omega_0 = \frac{2\pi}{17}, N=17$$

$$X[k] = \sum_{n=-8}^8 x[n] e^{-j k n \omega_0} = x[-8] e^{j(-8)\omega_0 n} + \dots + x[8] e^{j(8)\omega_0 n}$$

$$\cos \theta = \frac{1}{2} e^{j\theta} + \frac{1}{2} e^{-j\theta}$$

$$\cos\left(3\omega_0 n + \frac{\pi}{3}\right) = \frac{1}{2} e^{j(3\omega_0 n + \frac{\pi}{3})} + \frac{1}{2} e^{-j(3\omega_0 n + \frac{\pi}{3})}$$

$$= \frac{1}{2} e^{j(3\omega_0 n)} e^{j\frac{\pi}{3}} + \frac{1}{2} e^{-j(3\omega_0 n)} e^{-j\frac{\pi}{3}}$$

$$= \left[\frac{1}{2} e^{j\frac{\pi}{3}} \right] e^{j(3\omega_0 n)} + \left[\frac{1}{2} e^{-j\frac{\pi}{3}} \right] e^{-j(3\omega_0 n)}$$

, where $\omega_0 = \frac{2\pi}{17}$

$$x[-3] = \frac{1}{2} e^{j\frac{\pi}{3}} ; \quad x[3] = \frac{1}{2} e^{-j\frac{\pi}{3}}$$

$$x[k] = \begin{cases} \frac{1}{2} e^{j\frac{\pi}{3}}, & k=-3 \\ \frac{1}{2} e^{-j\frac{\pi}{3}}, & k=3 \\ 0, & \text{otherwise} \end{cases}$$

$$\Rightarrow x[k] = \begin{cases} \frac{1}{2} e^{j\frac{\pi}{3}}, & 3+17\gamma \\ \frac{1}{2} e^{-j\frac{\pi}{3}}, & -3+17\gamma \\ 0, & \text{otherwise} \end{cases} \quad \gamma = 0.52$$

3.49(b) Use the definition of the DTFS to determine the time-domain signals represented by the following

$$\text{DTFS coefficients: } X[k] = \cos\left(\frac{10\pi}{19}k\right) + j2\sin\left(\frac{4\pi}{19}k\right)$$

$$\text{DTFS: } x[n] = \sum_{k=0}^{N-1} X[k] e^{j k n_0} ; \quad X[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-j k n_0}$$

$x[n]$ and $X[k]$ have period N , $n_0 = \frac{2\pi}{N}$

$$\cos\left(\frac{10\pi}{19}k\right) \rightarrow m=5, N=19, -n_0 = \frac{2\pi}{19}$$

$$\cos(5n_0 k) = \frac{1}{2} e^{j 5n_0 k} + \frac{1}{2} e^{-j 5n_0 k}$$

$$x[n] = \frac{1}{2} \left[\frac{5}{2} e^{j 5n_0 k} + \frac{5}{2} e^{-j 5n_0 k} \right]$$

$$x[5] = \frac{5}{2} ; \quad x[-5] = \frac{5}{2}$$

$$j2\sin\left(\frac{4\pi}{19}k\right) \rightarrow m=2, N=19, -n_0 = \frac{2\pi}{19}$$

$$j2\sin(2n_0 k) = j2 \left[\frac{1}{2} e^{j 2n_0 k} - \frac{1}{2} e^{-j 2n_0 k} \right]$$

$$= e^{j 2n_0 k} - e^{-j 2n_0 k}$$

$$= \frac{1}{19} \left[19 e^{j 2n_0 k} - 19 e^{-j 2n_0 k} \right]$$

$$x[2] = -19 ; \quad x[-2] = 19$$

$$x[n] = \begin{cases} -19, & 2 \\ 19, & -2 \\ 0, & \text{otherwise} \end{cases}$$

$$x[n] = x_1[n] + x_2[n] = \begin{cases} \frac{5}{2}, & 5 \\ -19, & 2 \\ 19, & -2 \\ \frac{5}{2}, & -5 \\ 0, & \text{otherwise} \end{cases}$$

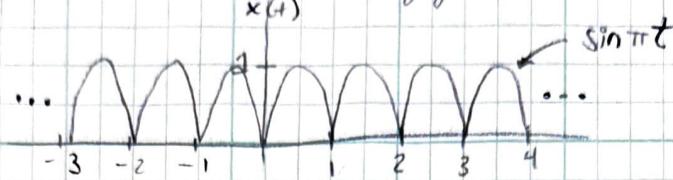
$$x[n] = \begin{cases} \frac{5}{2}, & k = 5 + 19\gamma \\ -19, & k = 2 + 19\gamma \\ 19, & k = -2 + 19\gamma \\ \frac{5}{2}, & k = -5 + 19\gamma \\ 0, & \text{otherwise} \end{cases} \quad \gamma = 0, \pm 1, \pm 2, \dots$$

3.50(d)

Use the defining equation for FS coeffs to evaluate the FS representation of following signal:

3/6

$x(t)$ as depicted in the figure:



$$x(t) = \sum_{k=-\infty}^{\infty} X[k] e^{jk\omega_0 t}; X[k] = \frac{1}{T} \int_0^T x(t) e^{-jk\omega_0 t} dt, x(t) \text{ has period } T, \omega_0 = \frac{2\pi}{T}$$

$$T = 1, \omega_0 = 2\pi$$

$$X[k] = \frac{1}{1} \int_0^1 |\sin(\pi t)| e^{-jk2\pi k t} dt$$

$$= \int_0^1 \sin(\pi t) e^{-jk2\pi k t} dt = \int_0^1 \left(\frac{1}{2j} (e^{j\pi t} - e^{-j\pi t}) \right) e^{-jk2\pi k t} dt$$

$$= \int_0^1 \left(\frac{1}{2j} (e^{j\pi t} \cdot e^{-jk2\pi k t} - e^{-j\pi t} \cdot e^{-jk2\pi k t}) \right) dt$$

$$= \frac{1}{2j} \int_0^1 [e^{j\pi t - j2\pi k t} - e^{-j\pi t - j2\pi k t}] dt$$

$$= \frac{1}{2j} \int_0^1 [e^{-j2\pi(k+\frac{1}{2})t} - e^{-j2\pi(k-\frac{1}{2})t}] dt$$

$$\int_0^1 e^{-j2\pi(a)t} dt = \frac{e^{-j2\pi(a)t}}{-j2\pi(a)} \Big|_0^1 = \frac{e^{-j2\pi(a)} - 1}{-j2\pi(a)}$$

$$X[k] = \frac{1}{2j} \left[\frac{e^{-j2\pi(k+\frac{1}{2})} - 1}{-j2\pi(k+\frac{1}{2})} \right] \left[\frac{e^{-j2\pi(k-\frac{1}{2})} - 1}{-j2\pi(k-\frac{1}{2})} \right]$$

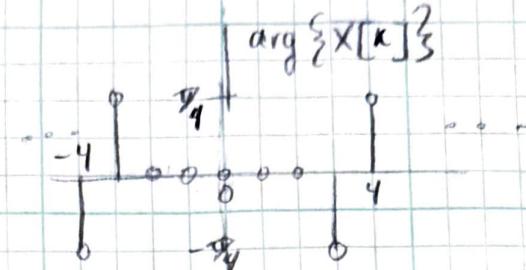
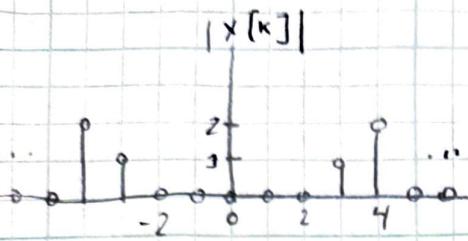
$$X[k] = \frac{e^{-j2\pi(k+\frac{1}{2})} - 1}{4\pi(k+\frac{1}{2})} \quad e^{-j2\pi(k-\frac{1}{2})} - 1$$

3.51(d)

Use definition of FS to determine
time-domain signal represented by
the following FS coeffs:

4/6

$x[k]$ in the figure, $\omega_0 = \pi$



$$x[k] = \begin{cases} 2 e^{j\frac{\pi}{4}}, & k = -4 \\ 1 e^{j\frac{\pi}{4}}, & k = -3 \\ 1 e^{j(-\frac{\pi}{4})}, & k = 3 \\ 2 e^{j(\frac{\pi}{4})}, & k = 4 \\ 0, & \text{otherwise} \end{cases}$$

Freq, nonperiodic, discrete

$$x(t) = \sum_{k=-\infty}^{\infty} x[k] e^{jk\omega_0 t} = x[-4] e^{j(-4)\pi t} + x[-3] e^{j(-3)\pi t} + x[3] e^{j(3)\pi t} + x[4] e^{j(4)\pi t}$$

$$x(t) = 2 e^{j(-\frac{\pi}{4})} \cdot e^{j(-4)\pi t} + e^{j\frac{\pi}{4}} \cdot e^{j(-3)\pi t} + e^{j(-\frac{\pi}{4})} e^{j(3)\pi t} + 2 e^{j(\frac{\pi}{4})} e^{j(4)\pi t}$$

$$x(t) = 2 e^{(j\frac{\pi}{4} - j4\pi t)} + e^{(j\frac{\pi}{4} - j3\pi t)} + e^{(j\frac{\pi}{4} + j3\pi t)} + 2 e^{(j\frac{\pi}{4} + j4\pi t)}$$

3.53(b)

use DTFT to determine time-domain
signals corresponding to:

$$X(e^{j\omega}) = \sin(-\omega) + \cos\left(\frac{\pi}{2}\right)$$

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{jn\omega} d\omega$$

$$X(e^{jn\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-jn\omega} ; \quad X(e^{jn\omega}) \text{ has period } (2\pi)$$

$$\sin\theta = \frac{1}{j2} e^{j\theta} - \frac{1}{j2} e^{-j\theta} ; \quad \cos(\theta) = \frac{1}{2} e^{j\theta} + \frac{1}{2} e^{-j\theta}$$

$$X(e^{jn\omega}) = \frac{1}{j2} e^{jn\omega} - \frac{1}{j2} e^{-jn\omega} + \frac{1}{2} e^{j\frac{\pi}{2}} + \frac{1}{2} e^{-j\frac{\pi}{2}}$$

$$= \frac{1}{j2} e^{-jn\omega(-1)} - \frac{1}{j2} e^{-jn\omega(1)} + \frac{1}{2} e^{-jn\omega(\frac{1}{2})} \quad \frac{1}{2} e^{-jn\omega(\frac{1}{2})}$$

\downarrow
n not integer

$$x[-1] = \frac{1}{j2}$$

$$x[1] = -\frac{1}{j2} \quad \omega_0 = \frac{2\pi}{N} = 2\pi \Rightarrow N = 1$$

$$x[n] = \begin{cases} \frac{1}{j2}, & n \neq 1 \\ -\frac{1}{j2}, & n = 1 \end{cases} \quad \omega = 0, \pm 1, \pm 2, \dots$$

5/6

3.54(c)

Use defining equation for FT to eval
freq-dom. resp of:

6/6

$$x(t) = 2e^{-t} u(t)$$

$$X(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt = \int_0^{\infty} (t e^{-t} \cdot e^{-j\omega t}) dt$$

$$= \int_0^{\infty} t \cdot e^{(a-j\omega)t} dt = \int_0^{\infty} t \cdot e^{(-1-j\omega)t} dt$$

$$\int_0^{\infty} t \cdot e^{at} dt, a = -1 - j\omega$$

* integration by parts: $\int u dv = uv - \int v du$

$$u = t; dv = e^{at} dt$$

$$\int t \cdot e^{at} dt = \frac{te^{at}}{a} - \int \frac{e^{at}}{a} dt$$

$$= \frac{te^{at}}{a} - \frac{e^{at}}{a^2}$$

$$\int_0^{\infty} t \cdot e^{at} dt = \left[\frac{te^{at}}{a} - \frac{e^{at}}{a^2} \right] \Big|_0^{\infty}$$

$$= \left[0 - \frac{e^{at}}{a} \right]_0^{\infty} = \left[0 - \frac{e^{(-1-j\omega)t}}{a^2} \right]_0^{\infty} = \left[0 - \frac{e^{-t} e^{-j\omega t}}{a^2} \right] \Big|_0^{\infty}$$

$$= \left[0 - 0 \right] - \left[0 - \frac{1}{a^2} \right] = \frac{1}{a^2} = \frac{1}{(-1-j\omega)^2} = \boxed{X(j\omega) = \frac{1}{(-1-j\omega)^2}}$$

$$t e^{t(-1-j\omega)}$$

→ approaches zero

$$\lim_{t \rightarrow \infty} e^{(a+j\beta)t} \quad \begin{array}{l} \text{if } x < 0 \rightarrow 0 \\ \text{if } x > 0 \rightarrow \infty \end{array}$$

$$\frac{te^{at}}{a} = \frac{te^{-t} e^{-j\omega t}}{-1-j\omega}$$

$$\lim_{t \rightarrow \infty} te^{-t} = \lim_{t \rightarrow \infty} \frac{1}{e^t} = 0$$

Problem 3.102 in PS07

Use MATLAB to repeat Example 3.7 for N=50 and (a) M = 12, (b) M=5, and (c) M=20

Example 3.7: Building A Square Wave From DTFS Coefficients

The contribution of each term to the square wave may be illustrated by defining the partial-sum approximation to $x[n]$ in Eq. (3.17) as

$xhat_J[n] = \sum(k=0 \text{ to } J) [B[k] \cos(k \Omega_0 n)]$, where $J \leq N/2$. This approximation contains the first $2J+1$ terms centered on $k=0$ in Eq. (3.10). Assume a square wave has period $N=50$, $M=12$. Evaluate on period of the J th term in Eq. (3.18) and $2J+1$ term approximation $xhat_J[n]$ for $J = 1, 3, 5, 23, 25$.

From page 208-210:

Square wave: $x[n] = \begin{cases} 1, & -M \leq n \leq M \\ 0, & M < n < N-M \end{cases}$

$$X[k] = 1/N * (\sin(k * \pi * (2M+1)/N)) / (\sin(k * \pi / N)) \text{ for } k \neq 0, \pm N, \pm 2N$$

$$= (2M+1)/N \text{ for } k = 0, \pm N, \pm 2N$$

```

% For J = 1, 3, 5, 23, 25
% (a) M = 12
% (b) M = 5
% (c) M = 20

N = 50; % Constant
n = -25:24; % n is the dependent variable, to be graphed
M_array = [12, 5, 20]; % width of square wave
J_array = [1, 3, 5, 23, 25]; % partial sum limit

for mIndex = 1:length(M_array)
    M = M_array(mIndex);
    figure
    for jIndex = 1:length(J_array)
        J = J_array(jIndex);

        subplot(5, 2, 1+2*(jIndex-1));
        stem(n(1:50), DTFS_Coeff(N, M, J)*cos(J*2*pi/N*n(1:50)), 'filled', 'r')
        title('DTFS Term')
        xlabel('n');
        ylabel(sprintf('B[%d]cos(\omega_0n)', J))
        grid on;

        subplot(5, 2, 2+2*(jIndex-1));
        stem(n, sum_of_DTFS_Coeffs(N, M, J, n), 'filled', 'b')
        title('Partial Sum')
        xlabel('n');
        ylabel(sprintf('x_{%d}[n]', J))
        grid on;
        sgttitle(sprintf('\underline{Square Wave From DTFS Coefficients, M=%d: Seth Ricks}', M), 'Interpreter', 'latex'); %Subgraph title
    end
end

```

N = period, M = width of square wave, k = coefficient number

Function for calculating sum of DTFS terms

```
function x_n = sum_of_DTFS_Coeffs(N, M, J, n)
    x_n = zeros(1, length(n)); % initialize output

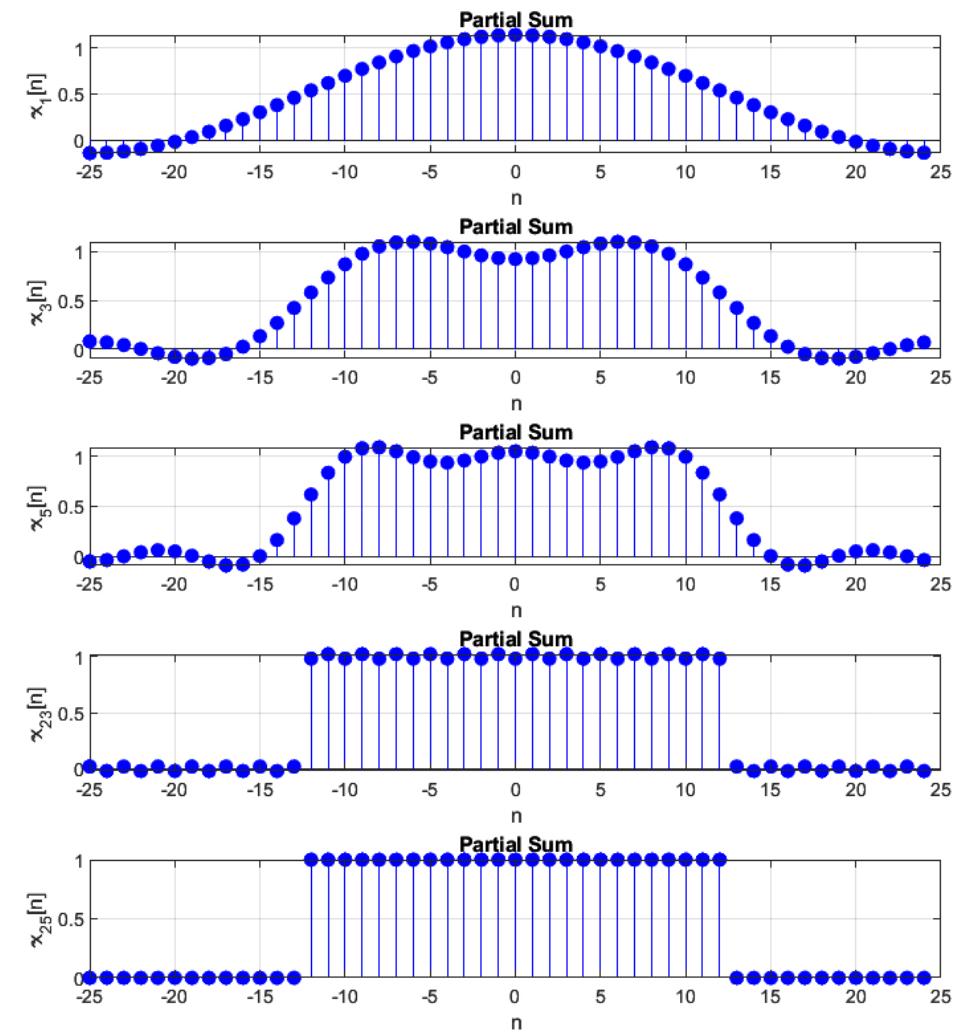
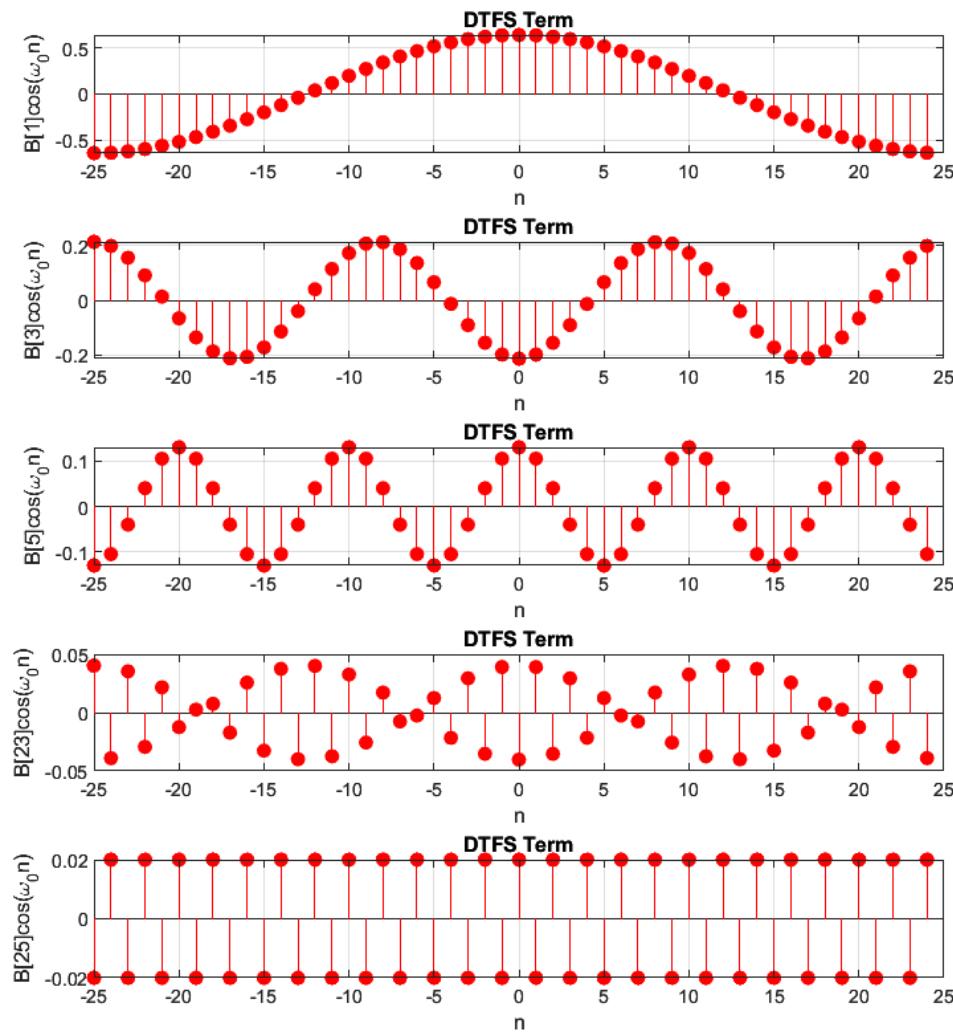
    for k = 0:J
        B_k = DTFS_Coeff(N, M, k);
        x_n = x_n + B_k * cos(k*2*pi/N*n); % add each term
    end
end
```

Function for calculating DTFS coefficients

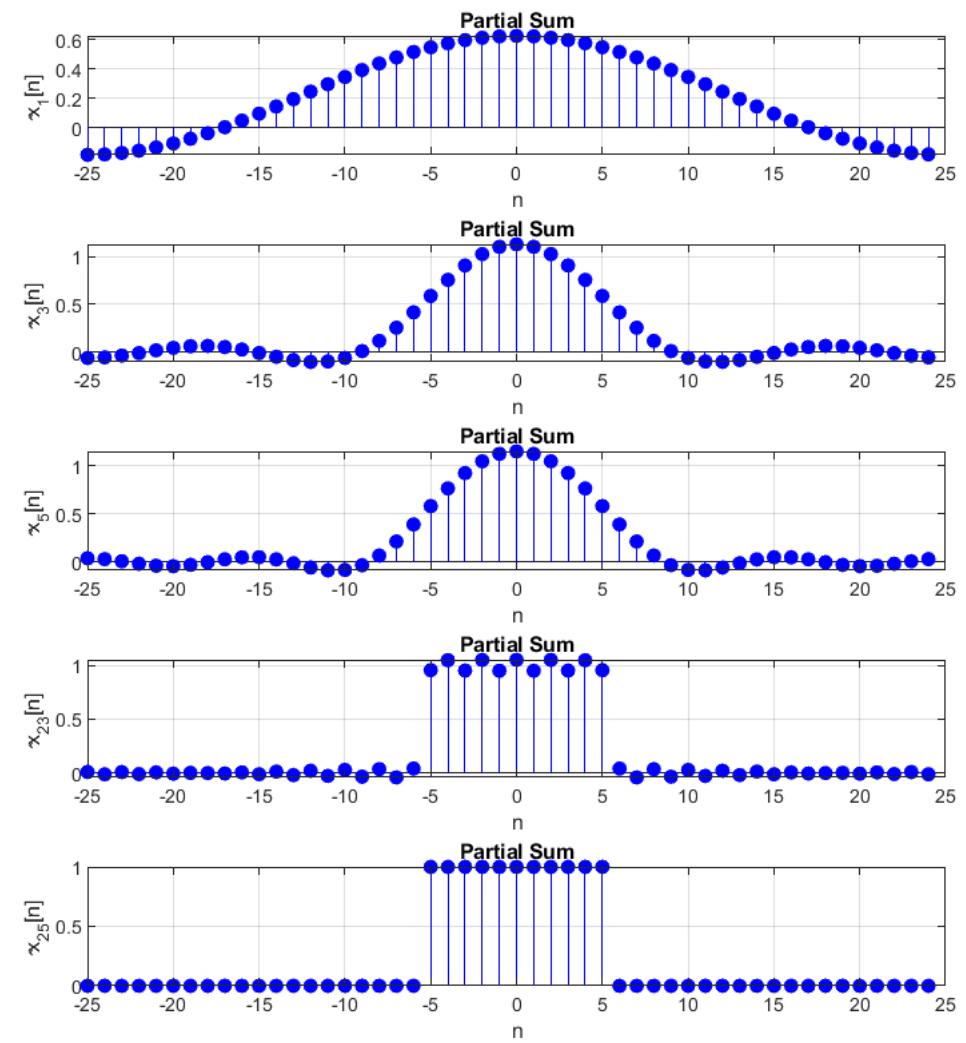
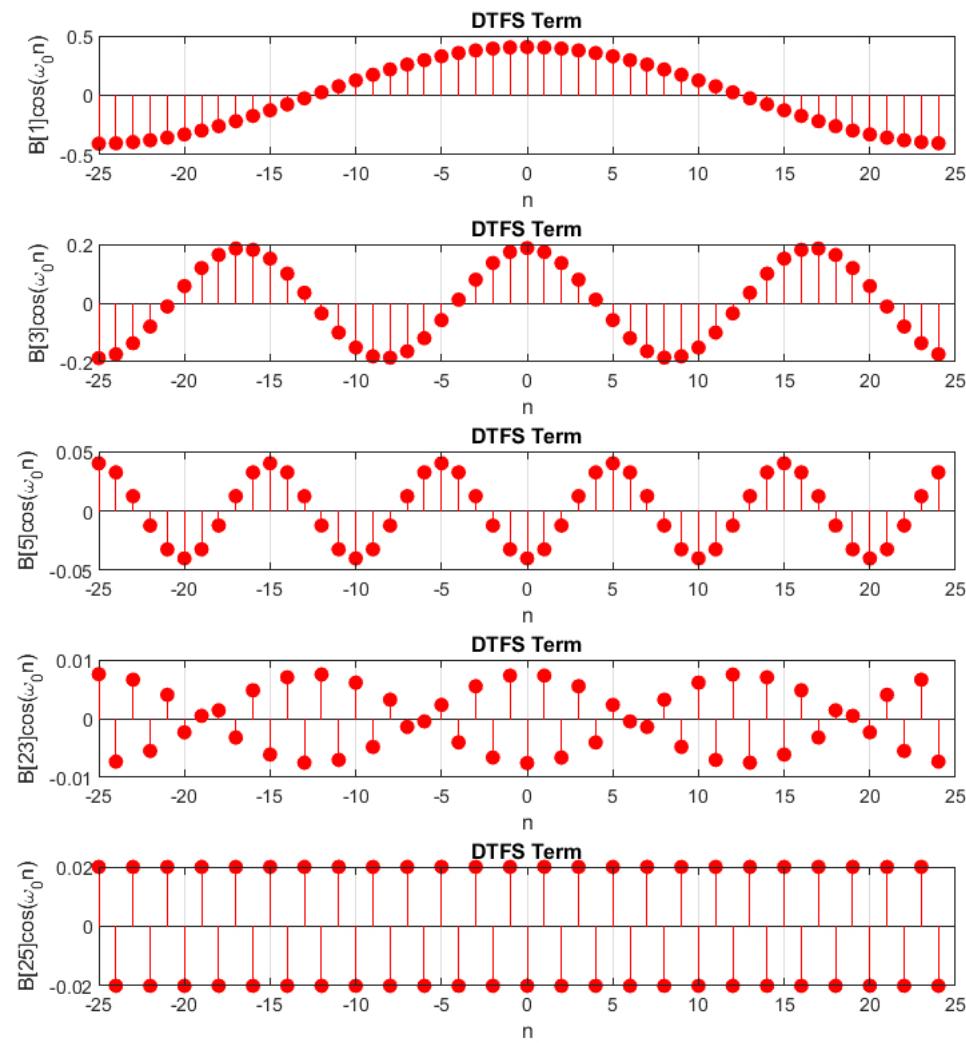
```
function B_k = DTFS_Coeff(N, M, J)
    if (J == 0)
        x_k = (2*M+1)/N; % handle indeterminate 0/0 term
    else
        x_k = 1/N * sin((J*pi)*(2*M+1)/N) / sin((J*pi)/N);
    end

    if (J == 0 || J == N/2)
        B_k = x_k;
    else
        B_k = 2 * x_k;
    end
end
```

Square Wave From DTFS Coefficients, M=12: Seth Ricks



Square Wave From DTFS Coefficients, M=5: Seth Ricks



Square Wave From DTFS Coefficients, M=20: Seth Ricks

