Ayudantía Álgebra N.2

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- 1. Determine si las siguientes proposiciones corresponden a una tautología, contradicción o contingencia:
 - (a) $[(p \Rightarrow q) \land (p \land \neg q \land r)] \Rightarrow (\neg p \lor q)$
 - (b) $[\{(p \lor q) \land \neg p\} \Rightarrow q] \Leftrightarrow q$
- 2. Simplificar, aplicando propiedades:

$$[A \cap B^c \cap (A - B^c)]^c \cup A^c$$

3. Dados los conjuntos A, B y C, simplificar al máximo la siguiente expresión:

$$[A \cap (A^c \cup B)] \cup [B \cap (B \cup C)] \cup B$$

4. Considere el conjunto referencial $E = \mathbb{N}$ y las funciones proposicionales:

$$p(x): x \text{ es par, } q(x): x \ge 5$$

Decida si las proposiciones son ambas verdaderas:

$$\forall x \in E : p(x) \Rightarrow \neg q(x) ; \exists x \in E : p(x)$$

5. Sean $A = \{-1, 2, 3, 4, 1\}$ y $B = \{1, \pi\}$. Determine el valor de verdad de la afirmación. Justifique.

$$\exists x \in A, \forall y \in B : x + y < y$$

- 6. Determine el valor de verdad de las siguientes proposiciones:
 - (a) $(\exists x \in \{2, 3, 4\})(\forall y \in \{-1, 0, 1\})(y \ge 0 \Rightarrow x + y \ge 3)$
 - (b) $(\forall x \in \{1, -2, \frac{1}{2}\})(\exists y \in \{-1, 1, 2, 3\})(xy > \frac{3}{2})$

$\mathbf{Tips:} \ \ \mathbf{Sean} \ A, B \neq C \ \mathbf{conjuntos:}$

Identidad $A \cap \mathcal{U} = A$ $A \cap \mathcal{Q} = \emptyset$ $A \cup \mathcal{U} = \mathcal{U}$ $A \cup \mathcal{Q} = A$ Idempotencia $A \cap A = A$ $A \cup A = A$ Involución $(A^c)^c = A$ $Complemento A \cap A^c = \emptyset A \cup A^c = \mathcal{U} Conmutatividad A \cap B = B \cap A A \cup B = B \cup A A \cup B = B \cup A Asociatividad A \cap (B \cap C) = (A \cap B) \cap C A \cup (B \cup C) = (A \cup B) \cup C A \cup (B \cap C) = (A \cup B) \cup (A \cap C) A \cup (B \cap C) = (A \cup B) \cap (A \cup C) A \cup (B \cap C) = (A \cup B) \cap (A \cup C) A \cup (B \cap C) = (A \cup B) \cap (A \cup C) A \cup (B \cap C) = (A \cup B) \cap (A \cup C) A \cap (A \cup B)^c = A^c \cap B^c A \cap (A \cup B) = A A \cup (A \cap B) = A$		
Identidad $A \cup \mathcal{U} = \mathcal{U}$ $A \cup \mathcal{O} = A$ $A \cap A = A$ $A \cup A = A$ Involución $(A^c)^c = A$ $Complemento \qquad A \cap A^c = \mathcal{O}$ $A \cup A^c = \mathcal{U}$ $A \cap B = B \cap A$ $A \cup B = B \cup A$ $A \cap (B \cap C) = (A \cap B) \cap C$ $A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cup (A \cap C)$ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cup (B \cap C) = A^c \cup B^c$ $(A \cup B)^c = A^c \cap B^c$ $A \cap (A \cup B) = A$ $A \cup (A \cap B) = A$	Identidad	$A \cap \mathcal{U} = A$
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Resta $A - B = A \cap B^c$		$A \cup (A \cap B) = A$
11 2 11 12	Resta	$A - B = A \cap B^c$

1. Recuerda que el símbolo \subseteq , significa que el conjunto de la izquierda es un subconjunto del de la derecha.