

Structure of the FITS File:

- **Frequency channels** Y : GHz Y axis represent different radio frequencies.
- **Time bins** X ms : X axis represents the time steps.
- **Intensity**: Pixel brightness represent the strength of the signal (flux) at each time-frequency point.

Fast Radio Bursts (FRBs) spread over multiple frequencies primarily due to **dispersion** caused by their propagation through the interstellar and intergalactic medium. This dispersion occurs because the radio waves that make up an FRB travel through a medium filled with free electrons, and the propagation speed of these radio waves depends on their frequency.

Dispersion Measure (DM)

The **dispersion measure (DM)** is a key quantity used in radio astronomy to describe the amount of plasma (free electrons) that an electromagnetic signal, such as a fast radio burst (FRB), has traveled through. The DM determines the time delay between different frequencies due to the dispersive nature of plasma.

$$DM = \frac{t_2 - t_1}{k_{DM} (v_2^{-2} - v_1^{-2})}$$

- DM - dispersion measure.
- v_1 and v_2 - earlier and later [frequencies](#) received.
- t_1 and t_2 - earlier and later reception times (in ms).
- k_{DM} - **dispersion constant**: $4.149 \text{ GHz}^2 \text{ pc}^{-1} \text{ cm}^3 \text{ ms}$ (given the time-difference is in ms)

Dispersion Measure Equation:

$$\Delta t = k \cdot DM \cdot \left(\frac{1}{f_1^2} - \frac{1}{f_2^2} \right)$$

Where:

- Δt is the time delay between two frequencies, f_1 and f_2 .
- f_1 and f_2 are the two radio frequencies in MHz or GHz (often measured in MHz in practice).
- DM is the **dispersion measure** in units of pc/cm^3 , which indicates the total column density of free electrons along the line of sight.

- k is a constant, approximately $4.148808 \times 10^3 \text{ ms MHz}^2 \text{ pc}^{-1} \text{ cm}^3$, for converting units.

The **DM** itself is calculated as:

$$\text{DM} = \int_0^d n_e dl$$

Where:

- n_e is the electron density (number of free electrons per unit volume) in cm^{-3} .
- dl is a small distance element along the line of sight in parsecs (pc).
- d is the total distance to the source (in parsecs).

Terms:

- **DM** quantifies the number of electrons that the signal passes through between its source and the observer.
- The equation for Δt shows that the time delay between frequencies is inversely proportional to the square of the frequencies. Lower frequencies take longer to arrive than higher frequencies.
- By measuring Δt at different frequencies, astronomers can calculate the DM and infer information about the interstellar or intergalactic medium the signal has passed through.
- **Higher DM** means more electrons along the path, implying either a longer distance to the source or denser regions of plasma.

Brute Force Algorithm

Dedispersed Time series

The main objective of these dedispersion algorithms is to be able to **generate the dedispersed time series**

A 2D array, where the y-axis contains all the DMs searched through by the algorithm and the x-axis contains the arrival times for each dispersion sweep.

For a specific **entry** in the dedispersed time series:

$S_{d,t}$ is the sum (or average) of all flux densities in the dynamic spectrum along the dispersion path corresponding to $\text{DM} = d$ and arrival time, t

Input Array: $I(v, t)$

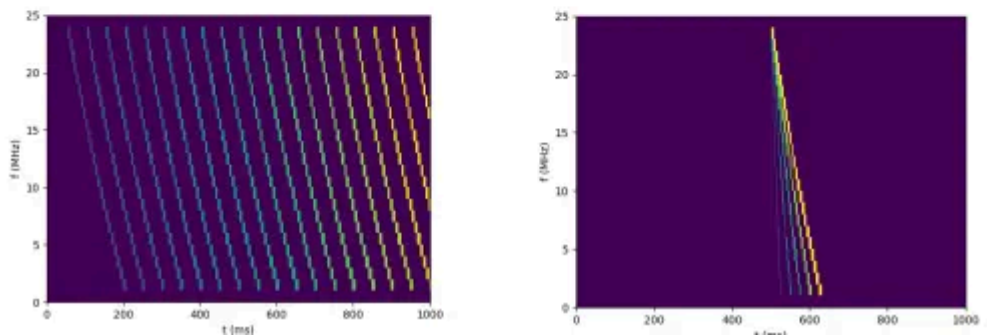
Frequency Channels	v_n											
	...											
	v_4											
	v_4											
	v_2											
	v_1											
	v_0											
		t_0	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	...	t_n
		Time Bins										

Figure 4: Input array convention, a pixel or single flux value in the input array I is denoted as $I(v, t)$ and frequency channels are denoted by v_n and time bins denoted by t_n .

This function will iteratively apply the discretised dispersion equation (Equation 4)

$$\Delta t(d, v) = \text{round} \left(d \frac{k_{DM}}{\Delta \tau} \left(\frac{1}{(v_0 - v \Delta v)^2} - \frac{1}{v_0^2} \right) \right),$$

using this equation, the number of discrete time bins required to iterate through while “moving down” in each frequency channel can be calculated - following the quadratic sweep. This calculation happens for each DM, each arrival time, and each frequency channel step, following very closely to the pseudocode of Section 2.4.1. A visualisation of these paths is shown in Figure 12.



Implementation

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The brute force algorithm essentially iterates through all trial DMs within a selected range; for each DM, it will iterate through each time bin; the algorithm will iteratively apply Equation 2 to determine the step sizes in time between each frequency channel, forming a dispersion path; and then, following the path, the algorithm will add the flux value of the input dynamic spectrum for each pixel .

For each DM -> each time bin -> find delta T using frequency channels

the key idea was to first generate a hash table with keys corresponding to each DM and start time. This hash table will then contain each dispersed path for each DM and each start time. Then, using the table of paths, the algorithm can dedisperse the input array by returning the mean flux of each path for each DM and each start time.