Wireless Networks
Assignment-1

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BTech ECE

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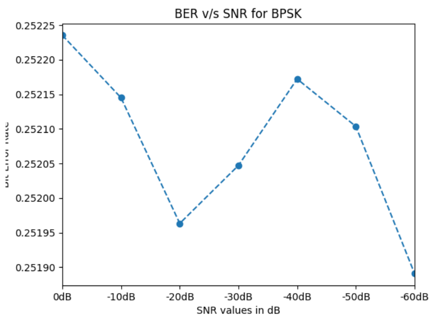
Q1

The BER (Bit Error Rate) versus SNR (Signal to Noise Ratio) plots for all the 6 modulation techniques are shown in Figure 1. The code for this simulation can be found in the `Q1` directory. The `README` file in that directory describes how to run the code. Note that you may need to install the dependencies before running the code. The dependencies and the references are also mentioned in the `README` file itself. Please note that the signal power considered for QAM for the SNR is the maximum signal power, i.e. the power of the corner points in the constellation diagram.

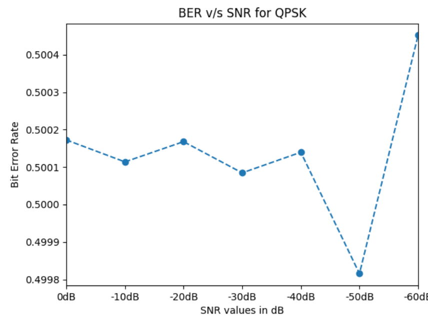
One would expect to see higher BER values as the SNR is decreased. However, in my simulation, I did not observe this behaviour. This can be attributed to the fact that I averaged the error in bits over 120 megabits. If one runs the code for longer than this (for instance maybe 12 gigabits), one might observe the expected behaviour. The noise due to insufficient number of bit samples may be contributing to the observed behaviour. I was not able to run the experiment for larger bit strings due to the limitations of my system. The code supports parallel simulations and even while running the simulation on 16 threads, the simulation time was around 5-6 hours.

(i) The BER of 4-QAM is almost the same as the BER of QPSK. Therefore both of these schemes will perform equally well.

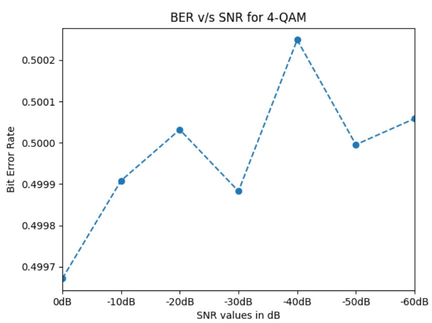
(ii) The BER of 16-PSK is almost same as the BER of 16-QAM. However, if one were to theoretically derive the formula for BER of both of the modulation schemes, one would find that the BER of 16-PSK is larger than the BER of 16-QAM, which would make 16-QAM a better choice as compared to 16-PSK. I attribute this discrepancy between the theoretical values and the simulation results to the low number of simulated bits.



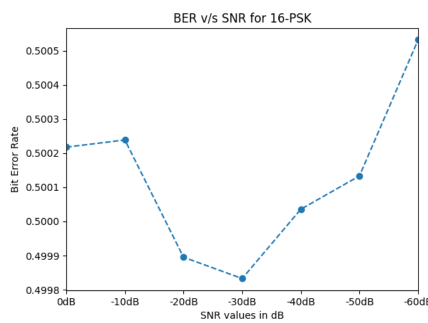
BPSK



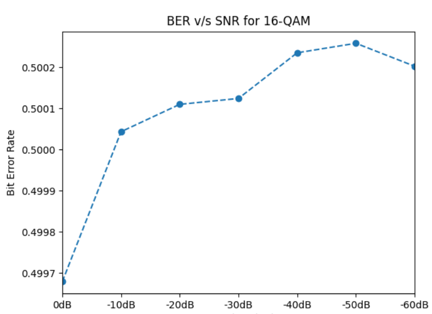
QPSK



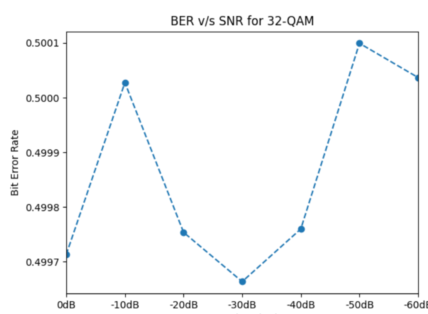
4-QAM



16-PSK



16-QAM



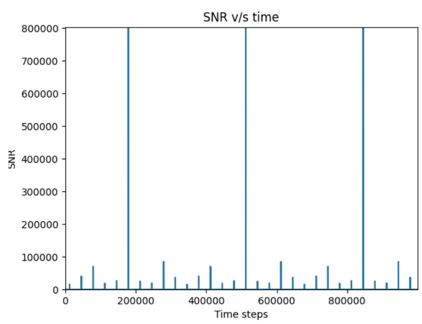
32-QAM

Figure 1: Bit error rate v/s SNR values for various modulation schemes

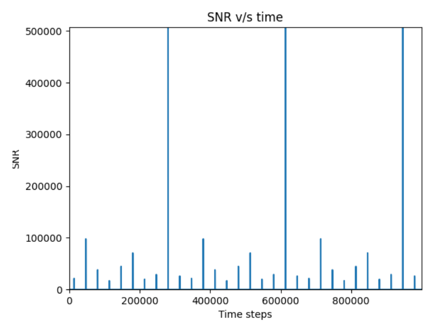
Q2

The SNR (Signal to Noise Ratio) versus time plots for both the cases are shown in Figure 2. The code for this simulation can be found in the `Q2` directory. The `README` file in that directory describes how to run the code. Note that you may need to install the dependencies before running the code. The dependencies and the references are also mentioned in the `README` file itself.

Note that for the simulation, I have considered an attenuation factor of 1.1 and a phase deviation factor of 0.5 radians.



10 independent sinusoids



100 independent sinusoids

Figure 2: SNR v/s time in presence of Rayleigh fading

Q3

The code for this experiment can be found in the `Q3` directory. The `README` file in that directory describes how to build and run the code. Note that you need to run this code on a UNIX based system like Linux, FreeBSD, etc. The references are also mentioned in the `README` file itself. The output files `wired.csv` and `wireless.csv` are present in the `Q3` directory itself. Each of these files have 100 lines depicting the latency of the 100 packets sent over the wired and the wireless channels. More details of the experiment methodology can be found in the `README` file. Please note that the latency is in microseconds.

Q4: Theoretical Approach

The code for this simulation can be found in the `Q4` directory. The `README` file in that directory describes how to run the code. Note that you may need to install the dependencies before running the code. The dependencies and the references are also mentioned in the `README` file itself.

Let $P[1]$ denote the probability of detecting 1, and $P[0]$ denote the probability of detecting 0. At the threshold, the probability of detecting 1, $P[1]$, and the probability of detecting 0, $P[0]$, should be equal. Let T denote the optimal threshold, r denote the received signal and t denote the transmitted signal. Moreover let $\mathcal{N}(\mu, \sigma)$ denote a normal distribution with mean μ and standard deviation σ. Since the variance of AWGN is 4, its standard deviation will be 2.

The formula of $P[0]$ and $P[1]$ is as follows:

$$\begin{aligned}
P[1] &= P[t=1]P[r > T \mid r \sim \mathcal{N}(5,2)] \\
&= \frac{2}{3}P[\frac{r-5}{2} > \frac{T-5}{2} \mid r \sim \mathcal{N}(5,2)] \\
&= \frac{2}{3}P[z > \frac{T-5}{2} \mid z \sim \mathcal{N}(0,1)] \\
&= \frac{2}{3}Q(\frac{T-5}{2}) \\
P[0] &= P[t=0]P[r < T \mid r \sim \mathcal{N}(-5,2)] \\
&= \frac{1}{3}P[\frac{r+5}{2} < \frac{T+5}{2} \mid r \sim \mathcal{N}(-5,2)] \\
&= \frac{1}{3}P[z < \frac{T+5}{2} \mid z \sim \mathcal{N}(0,1)] \\
&= \frac{1}{3}P[z > -\frac{T+5}{2} \mid z \sim \mathcal{N}(0,1)]) \\
&= \frac{1}{3}Q(-\frac{T+5}{2})
\end{aligned}$$

Now we can equate $P[0]$ and $P[1]$.

$$\begin{aligned}
& P[0] = P[1] \\
\implies & \frac{2}{3}Q(\frac{T-5}{2}) = \frac{1}{3}Q(-\frac{T+5}{2}) \\
\implies & 2Q(\frac{T-5}{2}) - Q(-\frac{T+5}{2}) = 0
\end{aligned}$$

This equation cannot be solved analytically. Therefore, I have written a python script to find the value of T numerically. The path to this script is `Q4/thresh_calc.py`. The script gives an estimate of $T = 4.99755859375$. This value of T is then fed to the `Q4/main.py` which gives the simulated BER of 0.333526.

The analytic derivation of BER is as follows:

$$\begin{aligned}
BER &= P[t=1]P[r < T \mid r \sim \mathcal{N}(5,2)] + P[t=0]P[r > T \mid r \sim \mathcal{N}(-5,2)] \\
&= \frac{2}{3}P[\frac{r-5}{2} < \frac{T-5}{2} \mid r \sim \mathcal{N}(5,2)] + \frac{1}{3}P[\frac{r+5}{2} > \frac{T+5}{2} \mid r \sim \mathcal{N}(-5,2)] \\
&= \frac{2}{3}P[z < \frac{T-5}{2} \mid z \sim \mathcal{N}(0,1)] + \frac{1}{3}P[z > \frac{T+5}{2} \mid z \sim \mathcal{N}(0,1)] \\
&= \frac{2}{3}P[z > -\frac{T-5}{2} \mid z \sim \mathcal{N}(0,1)] + \frac{1}{3}P[z > \frac{T+5}{2} \mid z \sim \mathcal{N}(0,1)] \\
&= \frac{2}{3}Q(-\frac{T-5}{2}) + \frac{1}{3}Q(\frac{T+5}{2}) \\
&= \frac{2}{3}Q(0.001220703125) + \frac{1}{3}Q(4.998779296875) \\
&= 0.33266666666666667
\end{aligned}$$

As expected the simulated value and the theoretical values are close to each other.

Q4: Numerical Approach

Note that the value of threshold found in the previous approach does not seem to make sense at all. I don't know where the error is in my calculations. Therefore, I wrote a script to find the optimal threshold numerically which can be found in `Q4_numerical` directory. Please refer to the `README` file in the same directory to understand how to execute the script. As per the numerical method, I got a bit error rate of 0.016103 at the threshold value of $\frac{-5}{3}$. This seems to make intuitive sense as the symbol "1" at +5V is twice as more likely than the symbol "0" at -5V.

Q5

The failure of a single transmission can occur due to two reasons: symbol corruption and collision. We can represent these two events as separate event sets. Let SC represent the event set of symbol corruption events and C represent the set of collision events. We know that $P[C] = 0.2$ as the channel is busy 20% of the time uniformly. Therefore the event set of failed transmission is given by $F = SC \cup C$. Note that the sets SC and C are independent of one another.

Although we know the value of $P[C]$, the value of $P[SC]$ is not known. Therefore, I simulated a 64-QAM transmission over an AWGN channel with SNR = -10dB and I found out that $P[SC] = 0.931417$. The code for this simulation is available in `Q5`.

Therefore,

$$\begin{aligned}
P[F] &= P[SC \cup C] \\
&= P[SC] + P[C] - P[SC \cap C] \\
&= P[SC] + P[C] - P[SC]P[C] \\
&= 0.931417 + 0.2 - (0.931417 \times 0.2) \\
&= 0.9451336000000001
\end{aligned}$$