Bootstrapping Looping Strategies

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Abstract

Although very profitable for users, looping strategies are resource-intensive and require skillful management at chain level. This paper presents a formalization of the bootstrapping optimization of a looping ecosystem, which allows to optimize the chain sponsor's resource consumption.

We only formalize the equilibrium state of the ecosystem, with strong but intuitive assumptions that lead to a closed form solution.

The optimization problem is formulated as a constrained optimization problem and solved using the Lagrange multiplier method.

1 Looping: a 4 Dapp Ecosystem

Looping is made up of components:

- a yield-bearing asset with yield r, correlated to a base asset (eg mRe7 correlated to USDC).
- a Lending protocol (eg Gearbox passive pool):
  - supply by users: N_l, and sponsor: N_l^*
  - borrowed amount: $N_b <= (N_l + N_l^*)$
  - the protocol's risk manager sets the borrow cap proportional to dex liquidity: $N_b <= g(N_d + N_d^*)$
  - borrow rate curve: $r_b = IR(\frac{N_b}{N_l+N_l^*})$
  - lending rate curve: $r_b(1-\epsilon)$
  - reward[1] paid by sponsor: R_l
- a DEX protocol (eg Curve)
  - liquidity[2] supplied by users: N_d, and sponsor: N_d^*
  - fee APY: $r_d \approx 0$

[1]Rewards are defined as a fixed annual budget and the sponsor is blacklisted, ie $APY = \frac{R}{N}$

[2]DEX liquidity is defined as the sum of the two pools

- reward paid by sponsor: R_d

- a looping vault (eg Gearbox credit account)
  - deposit by users only: N_v (and $N_v^* = 0$)
  - We assume r is high enough so that loopers seek maximum leverage l, so that $N_b = N_v(l-1)$
  - reward paid by sponsor: R_v

1.1 Borrowed amount: Lending + DEX constraint

The interest rate curve is parametrized by:

$$IR(U) = \begin{cases} r_0 + s_0 U & \text{if } U <= U_1 \\ r_1 + s_1(U - U_1) & \text{if } U_1 < U <= U_2 \\ r_2 + s_2(U - U_2) & \text{if } U_2 < U <= 1 \end{cases}$$

In a properly configured lending market, borrow rate naturally hovers around the kink[3] , which we set at a typical value of 90%. [4]. Assuming equilibrium is achieved at the kink greatly simplifies the problem: we can parametrize the IR curve only by $r_b(90\%)$. [5].

$$\begin{cases} U = 0.9 \\ N_b = U(N_l + N_l^*) \\ r_b = IR(U) \end{cases}$$

Also DEX liquidity is expensive for the sponsor, as it does not earn rewards but only small pool fees. Hence it's optimal for sponsor to let DEX liquidity be just enough to cover the leverage loopers seek:

$$\begin{cases} N_v(l-1) = \alpha(N_d + N_d^*), \text{where } \alpha = 1^6 \\ N_v(l-1) = U(N_l + N_l^*), \text{where } U = 0.9 \end{cases} \tag{1}$$

Sponsor liquidity (N_l^*, N_d^*) are thus determined by 1.

$$N_l^* = \frac{\beta_v R_v(l-1)}{U} - \beta_l R_l$$
$$N_d^* = \frac{\beta_v R_v(l-1)}{\alpha} - \beta_d R_d$$

[3]Gearbox has two kinks, we focus on the second kink

[4]See Morpho blog post

[5]keeping a piecewise linear IR curve leads to a partitioned quadratic problem, which is still solvable but more complex

1.2 Equilibrium APYs: empirical observations

Expressing each personna's APY:

$$\begin{cases} APY_l = r_b(1-\epsilon) + \dfrac{R_l}{N_l} \\ APY_d \approx \dfrac{r}{2} + \dfrac{R_d}{N_d} \\ APY_v = lr - (l-1)r_b + \dfrac{R_v}{N_v} \end{cases}$$

Market APYs can be estimated by AB testing, ie shocking reward budget every epoch and observing where TVL stabilitizes [7].
For stable lending and DEX, we observed $APY_l = 15\%$ and $APY_d = 20\%$.
For looping we assess the mimimum APY with high marketing impact to be $APY_v = 75\%$.
Given assumptions about user TVL's, we can now solve for user TVL:

$$\begin{cases} N_l = \beta_l R_l & \beta_l = \dfrac{1}{APY_l - r_b(1-\epsilon)} \\ N_d = \beta_d R_d & \beta_d = \dfrac{1}{APY_d - \frac{r}{2}} \\ N_v = \beta_v R_v & \beta_v = \dfrac{1}{APY_v - lr + (l-1)r_b} \end{cases} \tag{2}$$

2 Optimization problem

2.1 Contraints

The sponsor is bound by a liquidity budget and a reward target. The latter includes reward paid, cost of capital (0 for now), offset by APY accrued:

$$\begin{cases} N_l^* + N_d^* <= N \\ (R_l + R_d + R_v) - N_l^* r_b(1-\epsilon) - N_d^* \dfrac{r}{2} = R \end{cases}$$

[7]In further studies we may estimate and use the half-life of this equilibrium and the present study dynamic

Also, all rewards and notional are positive. So:

$$
\begin{cases}
-\dfrac{R_l}{APY_l - r_b(1-\epsilon)} - \dfrac{R_d}{APY_d - \frac{r}{2}} + \dfrac{R_v(l-1)}{APY_v - lr + (l-1)r_b}\left(\dfrac{1}{U} + \dfrac{1}{\alpha}\right) \leq N \\
R_l\left[1 + \dfrac{r_b(1-\epsilon)}{APY_l - r_b(1-\epsilon)}\right] + R_d\left[1 + \dfrac{\frac{r}{2}}{APY_d - \frac{r}{2}}\right] + \dots \\
R_v\left[1 - \dfrac{(l-1)r_b(1-\epsilon)}{(APY_v - lr + (l-1)r_b)U} - \dfrac{(l-1)r}{2\alpha(APY_v - lr + (l-1)r_b)}\right] = R \\
R_l \geq 0 \\
R_d \geq 0 \\
R_v \geq R_l\dfrac{U(APY_v - lr + (l-1)r_b)}{(l-1)(APY_l - r_b(1-\epsilon))} \\
R_v \geq R_d\dfrac{\alpha(APY_v - lr + (l-1)r_b)}{(l-1)(APY_d - \frac{r}{2})}
\end{cases}
\tag{3}
$$

Note that positive user TVL directly constrains input parameters:

$$
\begin{cases}
\text{User vault:} & APY_v - lr + (l-1)r_b \geq 0 \\
\text{User lending:} & APY_l - r_b(1-\epsilon) \geq 0 \\
\text{User DEX:} & APY_d - \dfrac{r}{2} \geq 0
\end{cases}
\tag{4}
$$

Note we can use the budget constraint to express R_v in terms of R_l and R_d:

$$
R_v = \frac{R - R_l\left[1 + \frac{r_b(1-\epsilon)}{APY_l - r_b(1-\epsilon)}\right] - R_d\left[1 + \frac{\frac{r}{2}}{APY_d - \frac{r}{2}}\right]}{1 - \frac{(l-1)r_b(1-\epsilon)}{(APY_v - lr + (l-1)r_b)U} - \frac{(l-1)r}{2\alpha(APY_v - lr + (l-1)r_b)}}
\tag{5}
$$

2.2 Objective function

Substituting R_v into the TVL objective:

$$
\max \quad TVL = \alpha_l R_l + \alpha_d R_d
\tag{6}
$$

where:

$$
\begin{cases}
\alpha_l = \beta_l - \dfrac{\beta_v\left[1 + \frac{r_b(1-\epsilon)}{APY_l - r_b(1-\epsilon)}\right]}{1 - \frac{(l-1)r_b(1-\epsilon)}{(APY_v - lr + (l-1)r_b)U} - \frac{(l-1)r}{2\alpha(APY_v - lr + (l-1)r_b)}} \\
\alpha_d = \beta_d - \dfrac{\beta_v\left[1 + \frac{\frac{r}{2}}{APY_d - \frac{r}{2}}\right]}{1 - \frac{(l-1)r_b(1-\epsilon)}{(APY_v - lr + (l-1)r_b)U} - \frac{(l-1)r}{2\alpha(APY_v - lr + (l-1)r_b)}}
\end{cases}
\tag{7}
$$

With inequality contraints:

$$
\gamma_l R_l + \gamma_d R_d \leq N + \gamma_R R
\tag{8}
$$

where:

$$
\begin{cases}
\gamma_l = -\dfrac{1}{APY_l - r_b(1-\epsilon)} - \dfrac{(l-1)\left[1 + \frac{r_b(1-\epsilon)}{APY_l - r_b(1-\epsilon)}\right]}{(APY_v - lr + (l-1)r_b)\left[1 - \frac{(l-1)r_b(1-\epsilon)}{(APY_v - lr + (l-1)r_b)U} - \frac{(l-1)r}{2\alpha(APY_v - lr + (l-1)r_b)}\right]}\left(\dfrac{1}{U} + \dfrac{1}{\alpha}\right) \\
\gamma_d = -\dfrac{1}{APY_d - \frac{r}{2}} - \dfrac{(l-1)\left[1 + \frac{\frac{r}{2}}{APY_d - \frac{r}{2}}\right]}{(APY_v - lr + (l-1)r_b)\left[1 - \frac{(l-1)r_b(1-\epsilon)}{(APY_v - lr + (l-1)r_b)U} - \frac{(l-1)r}{2\alpha(APY_v - lr + (l-1)r_b)}\right]}\left(\dfrac{1}{U} + \dfrac{1}{\alpha}\right) \\
\gamma_R = \dfrac{(l-1)}{(APY_v - lr + (l-1)r_b)\left[1 - \frac{(l-1)r_b(1-\epsilon)}{(APY_v - lr + (l-1)r_b)U} - \frac{(l-1)r}{2\alpha(APY_v - lr + (l-1)r_b)}\right]}\left(\dfrac{1}{U} + \dfrac{1}{\alpha}\right)
\end{cases}
\tag{9}
$$

And non-negativity constraints after substituting R_v:

$$R_l \geq 0 \tag{10}$$

$$R_d \geq 0 \tag{11}$$

$$\delta_l R_l + \delta_d R_d \leq \delta_R R \tag{12}$$

$$\epsilon_l R_l + \epsilon_d R_d \leq \epsilon_R R \tag{13}$$

where:

$$
\begin{cases}
\delta_l = \dfrac{1 + \frac{r_b(1-\epsilon)}{APY_l - r_b(1-\epsilon)}}{1 - \frac{(l-1)r_b(1-\epsilon)}{(APY_v - lr + (l-1)r_b)U} - \frac{(l-1)r}{2\alpha(APY_v - lr + (l-1)r_b)}} + \dfrac{U(APY_v - lr + (l-1)r_b)}{(l-1)(APY_l - r_b(1-\epsilon))} \\
\delta_d = \dfrac{1 + \frac{\frac{r}{2}}{APY_d - \frac{r}{2}}}{1 - \frac{(l-1)r_b(1-\epsilon)}{(APY_v - lr + (l-1)r_b)U} - \frac{(l-1)r}{2\alpha(APY_v - lr + (l-1)r_b)}} \\
\delta_R = \dfrac{1}{1 - \frac{(l-1)r_b(1-\epsilon)}{(APY_v - lr + (l-1)r_b)U} - \frac{(l-1)r}{2\alpha(APY_v - lr + (l-1)r_b)}} \\
\epsilon_l = \dfrac{1 + \frac{r_b(1-\epsilon)}{APY_l - r_b(1-\epsilon)}}{1 - \frac{(l-1)r_b(1-\epsilon)}{(APY_v - lr + (l-1)r_b)U} - \frac{(l-1)r}{2\alpha(APY_v - lr + (l-1)r_b)}} \\
\epsilon_d = \dfrac{1 + \frac{\frac{r}{2}}{APY_d - \frac{r}{2}}}{1 - \frac{(l-1)r_b(1-\epsilon)}{(APY_v - lr + (l-1)r_b)U} - \frac{(l-1)r}{2\alpha(APY_v - lr + (l-1)r_b)}} + \dfrac{\alpha(APY_v - lr + (l-1)r_b)}{(l-1)(APY_d - \frac{r}{2})} \\
\epsilon_R = \dfrac{1}{1 - \frac{(l-1)r_b(1-\epsilon)}{(APY_v - lr + (l-1)r_b)U} - \frac{(l-1)r}{2\alpha(APY_v - lr + (l-1)r_b)}}
\end{cases}
\tag{14}
$$

2.3 Parameters values

$$\begin{cases} \epsilon = 0.2 \\ r = 0.2 \\ l = 10 \\ APY_l = 0.15 \\ APY_d = 0.20 \\ APY_v = 0.75 \\ r_b = 0.15 \\ R = 8000000 \\ r^* = 0.10 \\ N = 30000000 \\ U = 0.9 \\ \alpha = 1 \end{cases} \tag{15}$$

3 Closed-form solution for R_l, R_d

We consider the linear program

$$\begin{aligned} \text{maximize} \quad & \text{TVL} \ = \ \alpha_l R_l + \alpha_d R_d, \\ \text{subject to} \quad & \gamma_l R_l + \gamma_d R_d \le G := N + \gamma_R R \qquad \text{(C0)} \\ & \delta_l R_l + \delta_d R_d \le \delta_R R \qquad \text{(C1)} \\ & \epsilon_l R_l + \epsilon_d R_d \le \epsilon_R R \qquad \text{(C2)} \\ & R_l \ge 0, \quad R_d \ge 0, \end{aligned}$$

where all coefficients $\alpha., \gamma., \delta., \epsilon.$ and right-hand sides are given constants (derived from the model). The feasible region is a polygon in the first quadrant bounded by at most the three lines $C0, C1, C2$ and the coordinate axes. Since the objective is linear, the optimum is attained at a vertex of the feasible polygon. The possible vertex types are:

1. Axis vertices: $(0, 0)$, $\left(\frac{G}{\gamma_l}, 0\right)$ (if $\gamma_l > 0$), $\left(0, \frac{G}{\gamma_d}\right)$ (if $\gamma_d > 0$).
2. Intersections of two distinct constraint lines among $\{C0, C1, C2\}$.
3. Intersections of one line from $\{C0, C1, C2\}$ with a coordinate axis.

Below we give closed-form expressions for every nontrivial candidate vertex that can be optimal; for each candidate you must (i) check that it is feasible (satisfies all constraints and nonnegativity), and (ii) compute the objective value and select the largest.

Notation

Denote the determinant of a 2×2 coefficient matrix by

$$\Delta\big((a_1, b_1), (a_2, b_2)\big) := a_1 b_2 - a_2 b_1.$$

If $\Delta \neq 0$, the unique solution to $\begin{cases} a_1 x + b_1 y = c_1 \\ a_2 x + b_2 y = c_2 \end{cases}$ is

$$x = \frac{c_1 b_2 - c_2 b_1}{\Delta}, \qquad y = \frac{a_1 c_2 - a_2 c_1}{\Delta}.$$

3.1 Axis candidates

Vertex A: origin. $(R_l, R_d) = (0, 0)$. Objective $TVL_A = 0$.

Vertex B: $R_d = 0$, $C0$ active. (Requires $\gamma_l > 0$.)

$$(R_l, R_d) = \Big(\frac{G}{\gamma_l}, 0\Big), \qquad TVL_B = \alpha_l \frac{G}{\gamma_l},$$

valid only if this point satisfies $C1, C2$ and $R_l \geq 0$.

Vertex C: $R_l = 0$, $C0$ active. (Requires $\gamma_d > 0$.)

$$(R_l, R_d) = \Big(0, \frac{G}{\gamma_d}\Big), \qquad TVL_C = \alpha_d \frac{G}{\gamma_d},$$

valid only if this point satisfies $C1, C2$ and $R_d \geq 0$.

3.2 Intersection of two constraint lines

Solve any pair of two active constraints among $C0, C1, C2$. We denote by (i, j) the system formed by constraints C_i and C_j, where $C_0 \equiv C0,\ C_1 \equiv C1,\ C_2 \equiv C2$.

Vertex V_{01}: intersection of $C0$ and $C1$. Solve

$$\begin{cases} \gamma_l R_l + \gamma_d R_d = G, \\ \delta_l R_l + \delta_d R_d = \delta_R R. \end{cases}$$

Set $\Delta_{01} := \Delta\big((\gamma_l, \gamma_d), (\delta_l, \delta_d)\big)$. If $\Delta_{01} \neq 0$ then

$$\boxed{R_l = \frac{G\,\delta_d - \delta_R R\,\gamma_d}{\Delta_{01}}, \qquad R_d = \frac{\gamma_l\,\delta_R R - \delta_l\,G}{\Delta_{01}}.}$$

This candidate is admissible only when $R_l \geq 0,\ R_d \geq 0$ and it satisfies $C2$.

Vertex V_{02}: intersection of $C0$ and $C2$. Let $\Delta_{02} := \Delta\big((\gamma_l, \gamma_d), (\epsilon_l, \epsilon_d)\big)$. If $\Delta_{02} \neq 0$,

$$\boxed{R_l = \frac{G\,\epsilon_d - \epsilon_R R\,\gamma_d}{\Delta_{02}}, \qquad R_d = \frac{\gamma_l\,\epsilon_R R - \epsilon_l\,G}{\Delta_{02}}.}$$

Feasibility requires nonnegativity and satisfaction of $C1$.

Vertex V_{12}: intersection of $C1$ and $C2$. Let $\Delta_{12} := \Delta\big((\delta_l, \delta_d), (\epsilon_l, \epsilon_d)\big)$. If $\Delta_{12} \neq 0$,

$$\boxed{R_l = \frac{\delta_R R\,\epsilon_d - \epsilon_R R\,\delta_d}{\Delta_{12}}, \qquad R_d = \frac{\delta_l\,\epsilon_R R - \epsilon_l\,\delta_R R}{\Delta_{12}}.}$$

Feasibility requires that this point also satisfies $C0$ and nonnegativity.

3.3 Intersection of a line with one axis (other than C0 axis cases)

Vertex V_{10}: $C1$ and $R_d = 0$. Solve $\delta_l R_l = \delta_R R$. If $\delta_l > 0$,

$$(R_l, R_d) = \Big(\tfrac{\delta_R R}{\delta_l},\, 0\Big), \qquad TVL = \alpha_l\,\tfrac{\delta_R R}{\delta_l},$$

valid if it satisfies $C0, C2$.

Vertex V_{20}: $C2$ and $R_d = 0$. If $\epsilon_l > 0$,

$$(R_l, R_d) = \Big(\tfrac{\epsilon_R R}{\epsilon_l},\, 0\Big), \qquad TVL = \alpha_l\,\tfrac{\epsilon_R R}{\epsilon_l},$$

valid if it satisfies $C0, C1$.

Vertex V_{11}: $C1$ and $R_l = 0$. If $\delta_d > 0$,

$$(R_l, R_d) = \Big(0,\, \tfrac{\delta_R R}{\delta_d}\Big), \qquad TVL = \alpha_d\,\tfrac{\delta_R R}{\delta_d},$$

valid if it satisfies $C0, C2$.

Vertex V_{21}: $C2$ and $R_l = 0$. If $\epsilon_d > 0$,

$$(R_l, R_d) = \Big(0,\, \tfrac{\epsilon_R R}{\epsilon_d}\Big), \qquad TVL = \alpha_d\,\tfrac{\epsilon_R R}{\epsilon_d},$$

valid if it satisfies $C0, C1$.

3.4 Selection rule (algorithm)

To obtain the optimum (R_l^*, R_d^*) and optimal value TVL^*:

1. Compute all candidate vertices listed above for which the corresponding determinant is nonzero or the division is well-defined.
2. For each candidate, check primal feasibility: all three inequalities $C0, C1, C2$ must hold (with equality for the two used in the intersection), and $R_l, R_d \geq 0$.
3. Evaluate $TVL = \alpha_l R_l + \alpha_d R_d$ at every feasible candidate.
4. The maximiser is the feasible candidate with maximal TVL. If two candidates tie, any convex combination on the connecting edge is also optimal.

3.5 Special remarks and degenerate cases

- If $\gamma_l \leq 0$ and $\gamma_d \leq 0$, constraint $C0$ does not limit growth in the positive quadrant; feasibility is then determined by $C1, C2$ (or unbounded if both $C1$ and $C2$ allow unbounded rays). Always check signs before using axis formulas $\frac{G}{\gamma_l}$ etc.
- If any determinant $\Delta_{ij} = 0$, the two lines are parallel; then either they do not intersect (no vertex) or they coincide (infinite intersection). Treat these as degenerate cases: reduce the candidate set accordingly and revert to adjacent vertices on the feasible polygon.
- If coefficients are such that multiple vertices are feasible, a simple numeric evaluation completes the selection; the algebraic formulas above provide exact rational expressions when coefficients are rational.