Bootstrapping Looping Strategies

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Abstract

Although very profitable for users, looping strategies are resource-intensive and require skillful management at chain level. This paper presents a formalization of the bootstrapping optimization of a looping ecosystem, which allows to optimize the chain sponsor's resource consumption.

We only formalize the equilibrium state of the ecosystem, with strong but intuitive assumptions that lead to a closed form solution.

The optimization problem is formulated as a constrained optimization problem and solved using the Lagrange multiplier method.

1 Looping: a 4 Dapp Ecosystem

Looping is made up of components:

- a yield-bearing asset with yield r, correlated to a base asset (eg mRe7 correlated to USDC).
- a Lending protocol (eg Gearbox passive pool):
  - supply by users: N_l, and sponsor: N_l^*
  - borrowed amount: $N_b <= (N_l + N_l^*)$
  - the protocol's risk manager sets the borrow cap proportional to dex liquidity: $N_b <= g(N_d + N_d^*)$
  - borrow rate curve: $r_b = IR(\frac{N_b}{N_l+N_l^*})$
  - lending rate curve: $r_b(1-\epsilon)$
  - reward[1] paid by sponsor: R_l
- a DEX protocol (eg Curve)
  - liquidity[2] supplied by users: N_d, and sponsor: N_d^*
  - fee APY: $r_d \approx 0$

[1] Rewards are defined as a fixed annual budget and the sponsor is blacklisted, ie $APY = \frac{R}{N}$

[2] DEX liquidity is defined as the sum of the two pools

- reward paid by sponsor: R_d

- a looping vault (eg Gearbox credit account)
  - deposit by users only: N_v (and $N_v^* = 0$)
  - We assume r is high enough so that loopers seek maximum leverage l, so that $N_b = N_v(l-1)$
  - reward paid by sponsor: R_v

1.1 Equilibrium APYs: empirical observations

Expressing each personna's APY:

$$\begin{cases} APY_l = r_b(1-\epsilon) + \dfrac{R_l}{N_l} \\ APY_d \approx \dfrac{r}{2} + \dfrac{R_d}{N_d} \\ APY_v = lr - (l-1)r_b + \dfrac{R_v}{N_v} \end{cases}$$

Market APYs can be estimated by AB testing, ie shocking reward budget every epoch and observing where TVL stabilizes [3].
For stable lending and DEX, we observed $APY_l = 15\%$ and $APY_d = 20\%$.
For looping we assess the mimimum APY with high marketing impact to be $APY_v = 75\%$.
Given assumptions about user TVL's, we can now solve for user TVL:

$$\begin{cases} N_l = \dfrac{R_l}{APY_l - r_b(1-\epsilon)} \\ N_d = \dfrac{R_d}{APY_d - \frac{r}{2}} \\ N_v \leq \dfrac{R_v}{APY_v - lr + (l-1)r_b} \end{cases} \tag{1}$$

We use the last constraint to top up looping yield if necessary:

$$R_v = N_v \max(0, APY_v - lr + (l-1)r_b) \tag{2}$$

1.2 Looped amount: Lending constraint

The interest rate curve is parametrized by:

$$IR(U) = \begin{cases} r_0 + s_0 U & \text{if } U <= U_1 \\ r_1 + s_1(U - U_1) & \text{if } U_1 < U <= U_2 \\ r_2 + s_2(U - U_2) & \text{if } U_2 < U <= 1 \end{cases}$$

[3] In further studies we may estimate and use the half-life of this equilibrium and the present study dynamic

In a properly configured lending market, borrow rate naturally hovers around the kink[4] , which we set at a typical value of 90%. [5]. Assuming equilibrium is achieved at the kink greatly simplifies the problem: we can parametrize the IR curve only by $IR(90\%)$. [6].

$$\begin{cases} U = 0.9 \\ r_b = IR(U) \\ N_b \leq U(N_l + N_l^*) \end{cases}$$

1.3 Sponsor budget

The sponsor is bound by a liquidity budget and a reward target. The latter includes reward paid, cost of capital (0 for now), offset by APY accrued:

$$\begin{cases} N_l^* + N_d^* = N \\ (R_l + R_d + R_v) - N_l^* r_b(1-\epsilon) - N_d^* \frac{r}{2} = R \end{cases}$$

Of course, all rewards and notional are positive.

1.4 Looped amount: DEX liquidity constraint

Curator sets cap according to the DEX liquidity:

$$N_d^* \leq \alpha(N_d + N_d^*), \text{ where } \alpha = \frac{1}{2} \text{ is setup by curator} \tag{3}$$

1.5 Looped amount

Since lending and DEX liquidity are expensive for the sponsor, we assume both contraints are active:

$$\begin{cases} U(N_l + N_l^*) = \alpha(N_d + N_d^*) \\ N_v(l-1) = U(N_l + N_l^*) \end{cases} \tag{4}$$

Solving for N_l^* and N_d^* and using liquidity budget constraint $N_l^* + N_d^* = N$:

$$\begin{cases} N_l^* = \dfrac{\alpha(N_d + N) - UN_l}{U + \alpha} \\ N_d^* = \dfrac{U(N_l + N) - \alpha N_d}{U + \alpha} \\ N_v = \dfrac{1}{l-1} \dfrac{1}{\frac{1}{\alpha} + \frac{1}{U}}(N_d + N_l + N) \end{cases} \tag{5}$$

[4]Gearbox has two kinks, we focus on the second kink
[5]See Morpho blog post
[6]keeping a piecewise linear IR curve leads to a partitioned quadratic problem, which is still solvable but more complex

2 2D Linear Programming Problem

$$\max_{R_l, R_d} \quad \text{TVL} = N_l + N_d + N_v \tag{6}$$

Where:

$$\begin{cases} N_l = \dfrac{R_l}{APY_l - r_b(1-\epsilon)} \\ N_d = \dfrac{R_d}{APY_d - \frac{r}{2}} \\ N_v = \dfrac{1}{l-1}\dfrac{1}{\frac{1}{\alpha}+\frac{1}{U}}(N_d + N_l + N) \\ N_l^* = \dfrac{\alpha(N_d + N) - UN_l}{U+\alpha} \\ N_d^* = \dfrac{U(N_l + N) - \alpha N_d}{U+\alpha} \\ R_v = N_v \max(0, APY_v - lr + (l-1)r_b) \end{cases} \tag{7}$$

Subject to the constraints:

$$\begin{cases} R_l \geq 0 \\ R_d \geq 0 \\ N_l^* \geq 0 \\ N_d^* \geq 0 \\ (R_l + R_d + R_v) - N_l^* r_b(1-\epsilon) - N_d^* \dfrac{r}{2} = R \end{cases} \tag{8}$$

Note that input parameters must satisfy:

$$\begin{cases} \dfrac{APY_v - lr}{l-1} \leq r_b \leq \dfrac{APY_l}{1-\epsilon} \\ \dfrac{r}{2} \leq APY_d \end{cases} \tag{9}$$