Bootstrapping Looping Strategies - DEX LP looping extension

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Abstract

We extend "Bootstrapping Looping Strategies" to include a DEX LP looping vault, and capped fixed yield lending club reward. The optimization problem is formulated as a constrained optimization problem and is not longer linear; it may be solved using the Lagrange multiplier method.

1 Looping: 5 components Ecosystem

Looping is made up of:

- a yield-bearing asset with yield r, correlated to a base asset (eg mRe7 correlated to USDC).
- a DEX protocol (eg Curve)
  - direct liquidity [1] (ie. not via looping vault below) supplied by users: N_d, and sponsor: N_d^*
  - fee APY: $r_d \approx 0$
  - reward paid by sponsor: R_d
- a looping vault (eg Gearbox credit account)
  - deposit by users only: N_v (and $N_v^* = 0$)
  - reward paid by sponsor: R_v
  - borrow rate r_v
  - We assume r is high enough so that loopers seek maximum leverage l_v, so that $N_v^b = N_v(l_v - 1)$
- a DEX LP looping vault (eg Gearbox credit account)
  - deposit by users: N_{vd}, and sponsor N_{vd}^*
  - reward paid by sponsor: R_{vd}

[1] DEX liquidity is defined as the sum of the two pools

- borrow rate r_{vd}
- We can assume $\frac{r}{2} + \frac{R_d}{N_d+N_d^*}$ is high enough for loopers to seek maximum leverage l_{vd}, so $N_{vd}^b = N_{vd}(l_{vd} - 1)$

- a Lending pool (eg Gearbox passive pool):
  - supply by users: N_l, and sponsor: N_l^*
  - We assume utilization at $U = 90\%$ target :

$$N_v(l_v - 1) + (N_{vd} + N_{vd}^*)(l_{vd} - 1) = U(N_l + N_l^*) \tag{1}$$

  - We assume curator cap is maxxed out:

$$\alpha_v l_v N_v + \alpha_{vd}\frac{1}{2}l_{vd}(N_{vd} + N_{vd}^*) = \frac{1}{2}(N_d + N_d^* + l_{vd}(N_{vd} + N_{vd}^*)) \tag{2}$$

    where $\alpha_v = 1$ and $\alpha_{vd} = 1.1$
  - lending rate curve:

$$r_l = \frac{r_v N_v(l_v - 1) + r_{vd}(N_{vd} + N_{vd}^*)(l_{vd} - 1)}{N_v(l_v - 1) + (N_{vd} + N_{vd}^*)(l_{vd} - 1)}(1 - \epsilon) \tag{3}$$

  - reward[2] paid by sponsor: R_l
  - capped fixed yield lending club reward: $\min(r_{lc}N_l, R_{lc})$.

1.1 Equilibrium APYs: empirical observations

Expressing each personna's APY:

$$\begin{cases} APY_l = r_l + \dfrac{R_l}{N_l} + \min(r_{lc}, \dfrac{R_{lc}}{N_l}) \\ APY_d \approx \dfrac{r}{2} + \dfrac{R_d}{N_d + l_{vd}N_{vd}} \\ APY_v > l_v r - (l_v - 1)r_v + \dfrac{R_v}{N_v} \\ APY_{vd} > l_{vd}APY_d - (l_{vd} - 1)r_{vd} + \dfrac{R_{vd}}{N_{vd}} \end{cases} \tag{4}$$

Market APYs can be estimated by AB testing, ie shocking reward budget every epoch and observing where TVL stabilitizes [3].
For stable lending and DEX, we observed $APY_l = 15\%$ and $APY_d = 20\%$.
For looping we assess the mimimum APY with high marketing impact to be $APY_v = 75\%$ and $APY_{vd} = 100\%$.

[2]Rewards are defined as a fixed annual budget and the sponsor is blacklisted, ie $APY = \frac{R}{N}$
[3]In further studies we may estimate and use the half-life of this equilibrium and the present study dynamic

1.2 Sponsor budget

The sponsor is bound by a liquidity budget and a reward target. The latter includes reward paid, cost of capital (0 for now), offset by APY accrued:

$$\begin{cases} N_l^* + N_d^* + N_{vd}^* = N \\ (R_l + R_d + R_v + R_{vd}) - N_l^* r_l - N_d^* \frac{r}{2} - l_{vd} N_{vd}^* \frac{r}{2} + r_{vd} N_{vd}^* (l_{vd} - 1) = R \end{cases} \tag{5}$$

Also, all rewards and notionals are positive.

2 Optimization Problem

$$\max_{R_l, R_d, R_v, R_{vd}} \quad \text{TVL} = N_l + N_d + N_v + N_{vd} \tag{6}$$

Subject to the constraints:

$$\begin{cases} R_l \geq 0 \\ R_d \geq 0 \\ R_v \geq 0 \\ R_{vd} \geq 0 \end{cases} \begin{cases} N_l \geq 0 \\ N_d \geq 0 \\ N_v \geq 0 \\ N_{vd} \geq 0 \end{cases} \begin{cases} N_l^* \geq 0 \\ N_d^* \geq 0 \\ N_{vd}^* \geq 0 \end{cases} \tag{7}$$

$$\begin{cases} N_l^* + N_d^* + N_{vd}^* = N \\ (R_l + R_d + R_v + R_{vd}) - N_l^* r_l - N_d^* \frac{r}{2} \dots \\ -l_{vd} N_{vd}^* \frac{r}{2} + r_{vd} N_{vd}^* (l_{vd} - 1) = R \\ r_l + \frac{R_l}{N_l} + \min(r_{lc}, \frac{R_{lc}}{N_l}) = APY_l \\ \frac{r}{2} + \frac{R_d}{N_d + l_{vd} N_{vd}} = APY_d \\ l_v r - (l_v - 1) r_v + \frac{R_v}{N_v} > APY_v \\ l_{vd} APY_d - (l_{vd} - 1) r_{vd} + \frac{R_{vd}}{N_{vd}} > APY_{vd} \\ N_v (l_v - 1) + (N_{vd} + N_{vd}^*)(l_{vd} - 1) = U(N_l + N_l^*) \\ \alpha_v l_v N_v + \alpha_{vd} \frac{1}{2} l_{vd} (N_{vd} + N_{vd}^*) = \frac{1}{2}(N_d + N_d^* + l_{vd}(N_{vd} + N_{vd}^*)) \end{cases} \tag{8}$$

where $\quad r_l = \frac{r_v N_v (l_v - 1) + r_{vd} (N_{vd} + N_{vd}^*)(l_{vd} - 1)}{N_v (l_v - 1) + (N_{vd} + N_{vd}^*)(l_{vd} - 1)} (1 - \epsilon)$