The Bessel differential equation is

$$x^2y'' + xy' + (x^2 - v^2)y = 0$$

For integer values of ν (ν = n), the solution gives rise to a special function called the Bessel function. $J_n(x)$, which is the Bessel function of the first kind of order n, can be shown to have the below power series representation:

$$J_n(x) = x^n \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m}}{2^{2m+n} m!\,(n+m)!}$$

Bessel functions are widely used in science and engineering. Therefore, we need to have a way of generating the values of the function given n and x. We use the equation above to do that.

Write a program that takes two inputs; n and x, and returns the value of the Bessel function, $J_n(x)$.

Use a loop to sum up to the first 21 elements of the power series (m = 0 to m = 20). Better accuracy will be achieved with more iterations, but we wish to make the program run in the least time possible.

A table of Bessel functions can be found in the Project 1 folder, to test your program once it is done.

Hint:

Create two functions:

- bessel(n, x) for the primary computation
- fact(k) for finding factorials of numbers, such as those present in the denominator. This function would be called in bessel(n, x)

Get values of n and x as input. n is an integer, x is a float.

Pass these values into bessel(n, x), and print the result.

Make sure you test the program by referring to the table of Bessel functions present in the folder.