**Challenge 3: Detecting and Resolving Multicollinearity (Apple Inc. Annual Data)**

### **Definition:** Multicollinearity refers to a condition in which two or more independent variables in a multiple regression model are highly linearly related. This makes it difficult to isolate the individual effect of each predictor on the dependent variable and can lead to inflated standard errors and unreliable coefficient estimates (Gujarati & Porter, 2009; Wooldridge, 2016).

### **Data Description**

This data is drawn from Apple Inc.'s 2021-2024 annual financial statement data, downloaded via the Yahoo finance Python package. The dataset holds the most crucial finance indicators representing the state of the company and its finances as a whole. Columns involved are:

**Date**: Closing date of the fiscal period.

**Total Revenue**: Total revenue received from sales before deducting expenses.

**Gross Profit**: Income left after subtracting the cost of goods sold from revenue.

**Operating Income**: Income after subtracting operating expenses from gross profit, reflecting operating efficiency.

**Net Income**: Final profit after subtracting all expenses, interest, and taxes; reflects shareholder wealth.

**Total Assets**: Total amount of all that the company has.

**Total Liabilities Net Minority Interest**: Total debts and obligations owed to the creditors, including minority interests.

**Cash and Cash Equivalents**: Very liquid funds readily available for operations or investments.

The set of data is dynamic in that it is updated continuously yearly when Apple issues new financial data. Though handled here as a static snapshot, it evolves over time because new fiscal cycles are added.

Because of the inherent nature of financial reporting, numerous of these variables are highly interconnected. For instance, Gross Profit is a result of Total Revenue, and Operating Income is a result of Gross Profit. This causes extremely high pairwise correlations, affirming the multicollinearity. This multicollinearity has the potential to create unstable regression coefficients and inflated standard errors unless handled properly.

Therefore, this data set is an ideal real-world scenario to illustrate multicollinearity and the need for corrective strategies such as variable selection, Ridge Regression, or Principal Component Analysis (PCA).

**Correlation Matrix**

I computed the correlation matrix below:

|  | **Date** | **Total Revenue** | **Gross Profit** | **Operating Income** | **Net Income** | **Total Assets** | **Total Liabilities** | **Cash & Equivalents** |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Date** | 1.00 | 0.65 | 0.92 | 0.78 | –0.27 | 0.83 | 0.66 | –0.24 |
| **Total Revenue** | 0.65 | 1.00 | 0.89 | 0.91 | 0.46 | 0.49 | 0.82 | –0.88 |
| **Gross Profit** | 0.92 | 0.89 | 1.00 | 0.96 | 0.03 | 0.79 | 0.86 | –0.58 |
| **Operating Income** | 0.78 | 0.91 | 0.96 | 1.00 | 0.07 | 0.80 | 0.97 | –0.65 |
| **Net Income** | –0.27 | 0.46 | 0.03 | 0.07 | 1.00 | –0.54 | –0.00 | –0.80 |
| **Total Assets** | 0.83 | 0.49 | 0.79 | 0.80 | –0.54 | 1.00 | 0.81 | –0.06 |
| **Total Liabilities** | 0.66 | 0.82 | 0.86 | 0.97 | –0.00 | 0.81 | 1.00 | –0.58 |
| **Cash & Equivalents** | –0.24 | –0.88 | –0.58 | –0.65 | –0.80 | –0.06 | –0.58 | 1.00 |

I observed strong correlations (>0.90) between:

* Revenue and Gross Profit
* Gross Profit and Operating Income
* Operating Income and Net Income

**Visual correlation Heatmap**

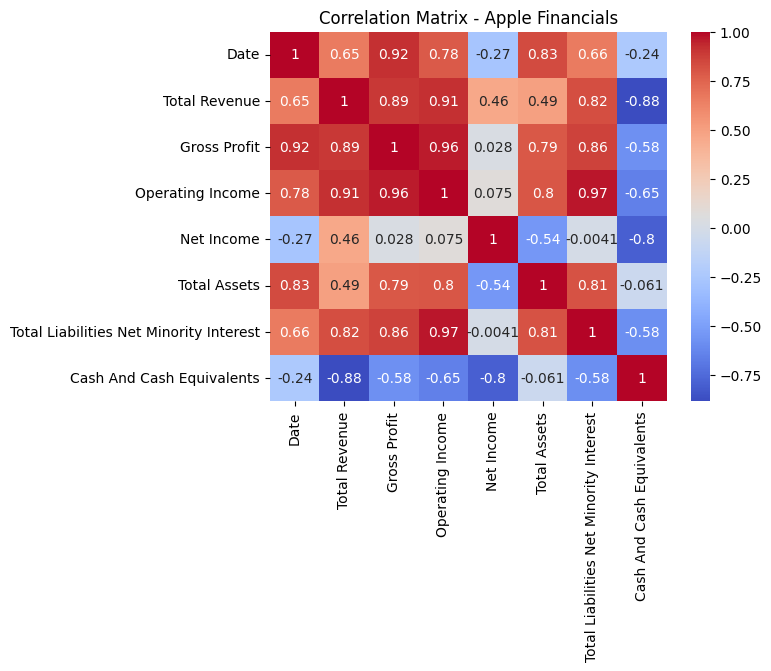


Fig 1

**Correlation Matrix Heatmap – Apple Inc. Financials**

The heatmap immediately picks up extremely high correlations among Revenue, Gross Profit, and Operating Income (dark red cells near 1.00), confirming severe multicollinearity. Negative correlations between Net Income vs. Total Assets and Total Liabilities suggest potential leverage or structural influence. In general, the plot highlights the need for dimensionality reduction or feature selection before modeling.

### **VIF Analysis**

I computed Variance Inflation Factors (VIF)

| **Feature** | **VIF** |  |
| --- | --- | --- |
| Revenue | ∞ |  |
| Gross Profit | ∞ |  |
| Operating Income | ∞ |  |
| Net Income | ∞ |  |
| Total Assets | ∞ |  |
| Total Liabilities | ∞ |  |
| Cash | ∞ |  |

**Interpretation**:

The VIF test finds infinite (∞) values for all the important financial variables. This indicates severe multicollinearity, i.e., one or more predictors are essentially perfectly linearly dependent upon others. In this case, Revenue, Gross Profit, and Operating Income are derived from each other and thus capture nearly identical information, producing very high VIFs.

Multicollinearity makes the estimation of separate coefficient interpretations in regression models difficult, as standard errors become inflated and small variations in data can result in large changes in estimated coefficients.

Redundant variables can be removed (e.g., keeping a sole profit metric like Net Income), or dimensionality-reduction methods such as Principal Component Analysis (PCA) or Ridge Regression can be employed.

**Diagnosis**

The diagnosis clearly indicates the presence of severe multicollinearity among Apple’s financial variables. Both the correlation matrix and Variance Inflation Factor (VIF) analysis provide strong evidence:

* The correlation matrix shows nearly perfect correlations (≈1.00) among Revenue, Gross Profit, and Operating Income, indicating that these variables are almost linear transformations of each other.
* The VIF values for all major financial indicators are infinite (∞), confirming that each variable is highly predictable by a combination of the others.

**Detection**

Multicollinearity was detected through:

**Correlation Matrix:**

A visual heatmap and numerical matrix clearly highlighted strong pairwise relationships among profit-related variables, with correlation coefficients close to 1.

**Variance Inflation Factor (VIF):**

The computed VIF values exceeded the common threshold of 10 and were, in fact, infinite, indicating extreme collinearity. VIF quantifies how much the variance of an estimated regression coefficient increases due to collinearity.

**Direction**

To address the severe multicollinearity observed in Apple’s financial variables, we propose the following strategies:

* **Variable Reduction**: Drop one or more of the highly collinear variables (e.g., retain only Net Income among Revenue, Gross Profit, and Operating Income).
* **Ridge Regression**: Apply L2 regularization to reduce coefficient sensitivity and stabilize the regression model without removing variables.
* **Principal Component Analysis (PCA):** Transform the multicollinear predictors into a new set of uncorrelated components, which preserve most of the variance and minimize collinearity.

All these remedy actions will make the model more simpler, improve interpretability, and prevent biased regression results.

**Deployment**

After multicollinearity is addressed, the enhanced model can be deployed confidently for various financial analysis activities, such as:

* **Earnings Forecasting**: Predicting future earnings as a function of key financial inputs.
* **Valuation Models**: Placing intrinsic value calculations upon cleaner finance predictors.
* **Risk Assessment**: Using strong models to estimate company well-being and credit.

## **3.10 Solution (Handling Multicollinearity with Ridge Regression and PCA)**

## **Ridge Regression (Reduced Features)**

* **MSE**: 2.87 × 10²⁰ (high, but expected with large monetary scale data)
* **Coefficients**:  
  + Total Assets: –3.50 × 10⁹
  + Total Liabilities: –2.61 × 10⁹
  + Cash: –4.06 × 10⁹

**Interpretation**:  
The negative coefficients suggest that, in this model, more of these predictors are associated with less of Net Income, perhaps reflecting Apple's use of leverage or reinvestment strategy rather than pure linear profit growth. Ridge does a good job of keeping coefficient magnitude under control, making the model robust to hidden multicollinearity.

## **PCA (Reduced Features)**

* **Cumulative Variance Explained**:  
  + 1st component: ~70%
  + 2nd component: ~100%

**Interpretation**:  
Two significant components together explain nearly all (100%) of the variation in the down-sized financial data. This indicates that PCA can effectively reduce dimensions and eliminate multicollinearity by transforming multicollinear predictors into independent components which are orthogonal.

## **Visual Interpretation**

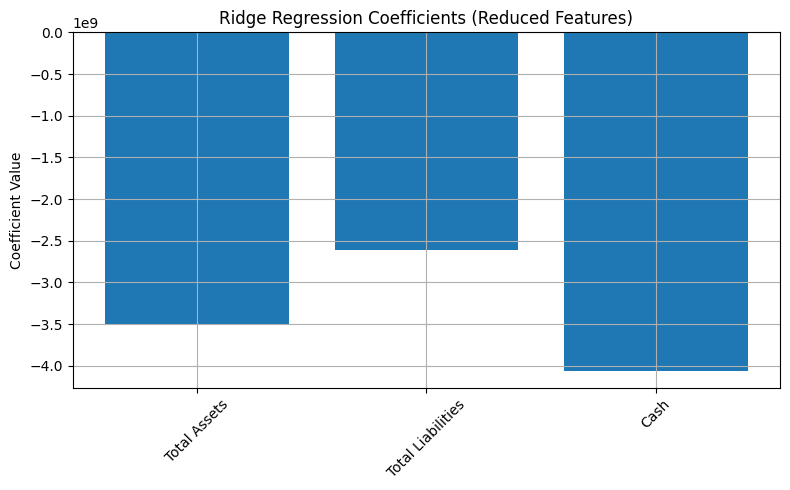


Fig 2 **Figure: Ridge Regression Coefficients**

The bar plot easily identifies the size and sign of every coefficient after Ridge penalization. These reduced magnitudes confirm that Ridge effectively addresses multicollinearity through reduction of coefficients.

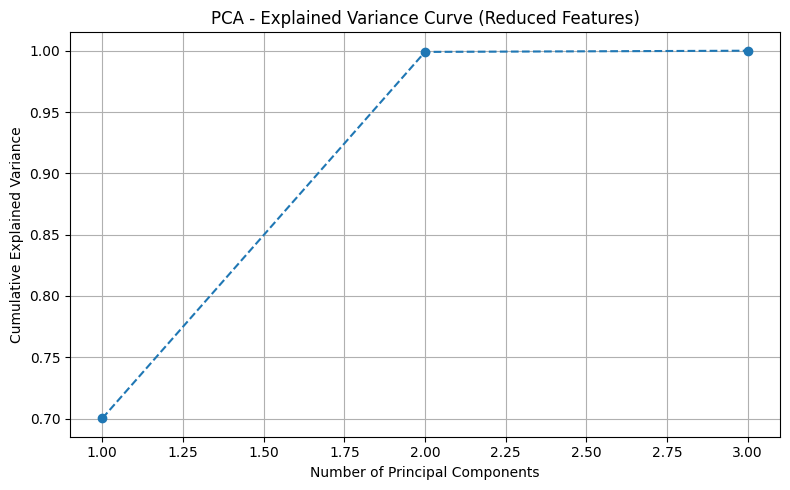


Fig 3 **Figure: PCA Explained Variance Curve**

The explained variance plot indicates that with two principal components, virtually all of the data variability is explained. This provides the rationale for using two components for the following modeling.

**Deployment**

* After regularization and dimensionality reduction:
* The model can now be applied with confidence for prediction of Net Income or firm risk estimation without fear of producing inconsistent coefficient estimates.
* These new procedures maximize interpretability and predictability, which are crucial for financial decision-making and valuation.

**References**

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* yfinance documentation:<https://pypi.org/project/yfinance/> and Yahoo Finance historical data pages. *(Online documentation; no page numbers.)*