AN EARTHLY MODEL OF THE DIVINE COINCIDENCE

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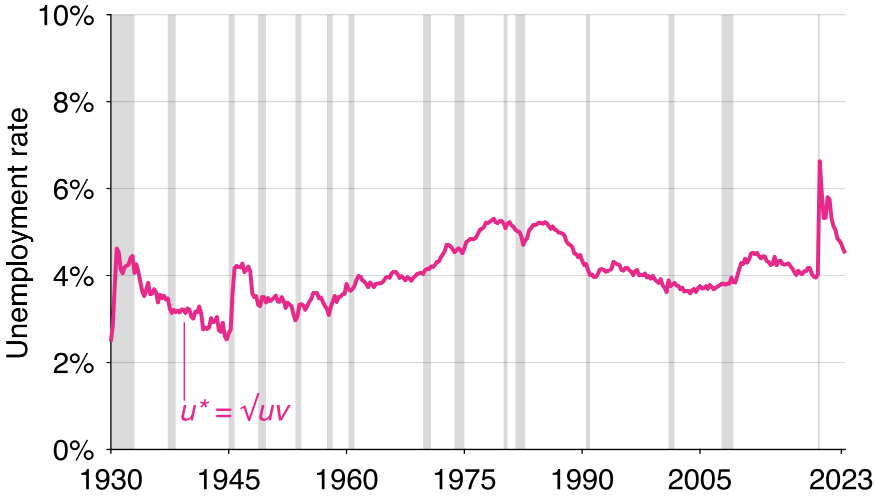
Available at https://pascalmichaillat.org/15/

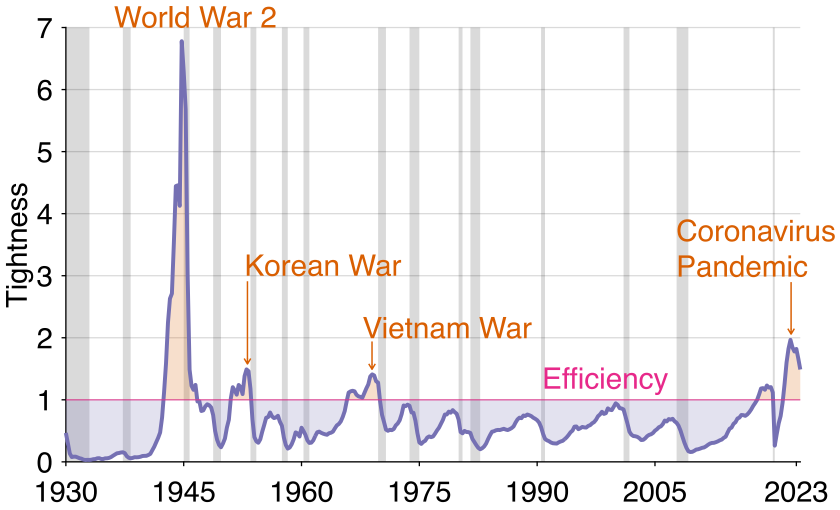
THE FED'S DUAL MANDATE

- responsibility of the Federal Reserve "to promote effectively the goals of maximum employment, stable prices"
  - Federal Reserve Reform Act of 1977
- stable prices: $\pi^* = 2\%$
  - Statement on Longer-Run Goals & Monetary Policy Strategy (2012)
- maximum employment: $u^* = \sqrt{uv}$, $\theta^* = 1$
  - proposal by Michaillat, Saez (2023)
  - u^*, θ^* maximize social welfare

$u^* = \sqrt{uv}$ AVERAGES 4.1% OVER 1930–2023



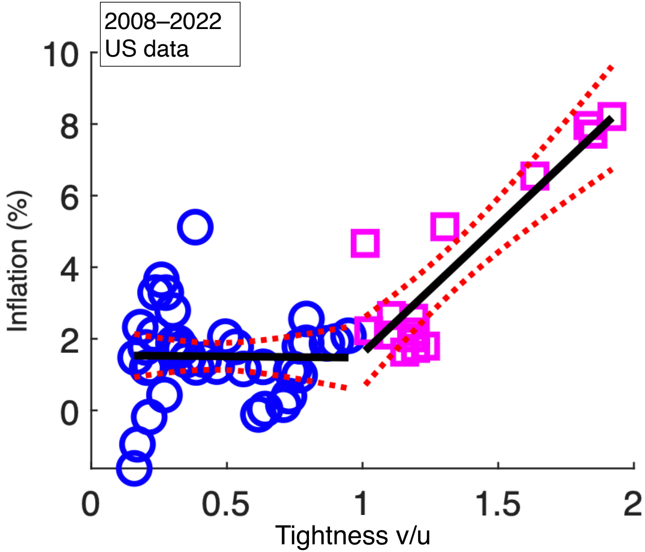
US LABOR MARKET IS GENERALLY INEFFICIENTLY SLACK



TRADITIONALLY, THE TWO MANDATES ARE NOT CONSISTENT

- under traditional Phillips curve: no guarantee that (u^*, π^*) is on curve
- under accelerationist Phillips curve: no guarantee that the NAIRU maximize social welfare
  - all other unemployment rate are inconsistent with stable inflation
- in New Keynesian model with unemployment fluctuations: wage rigidity breaks down divine coincidence
  - Blanchard, Gali (2010) see divine coincidence as unrealistic

BUT: BENIGNO, EGGERTSSON (2023)



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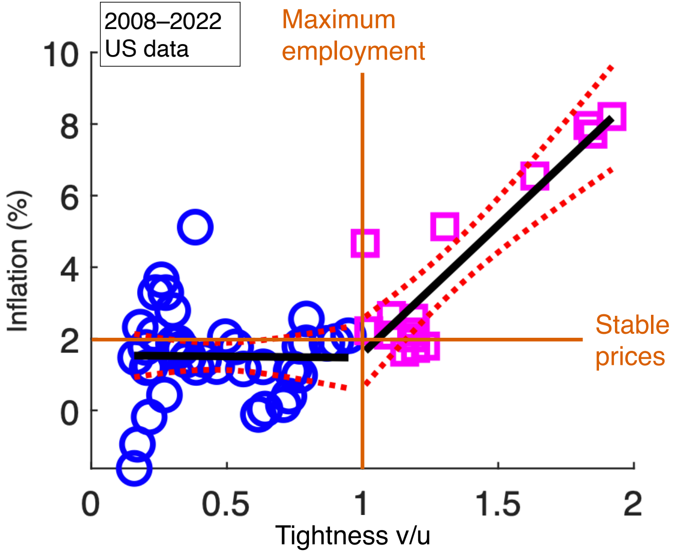
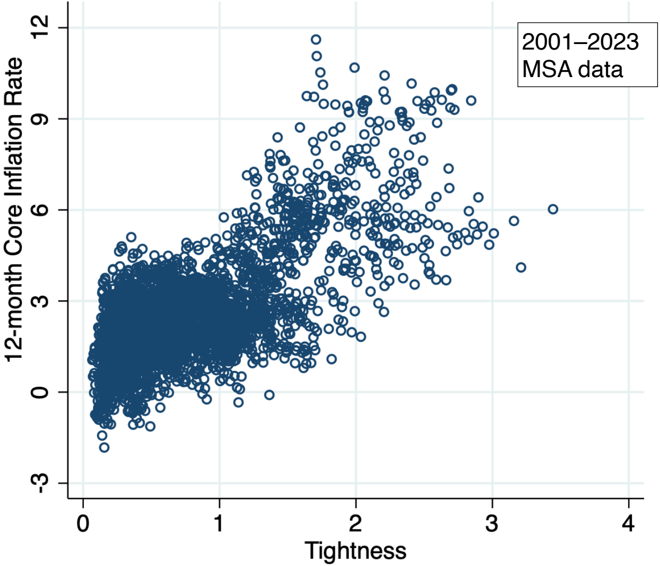


BUT: GITTI (2023)



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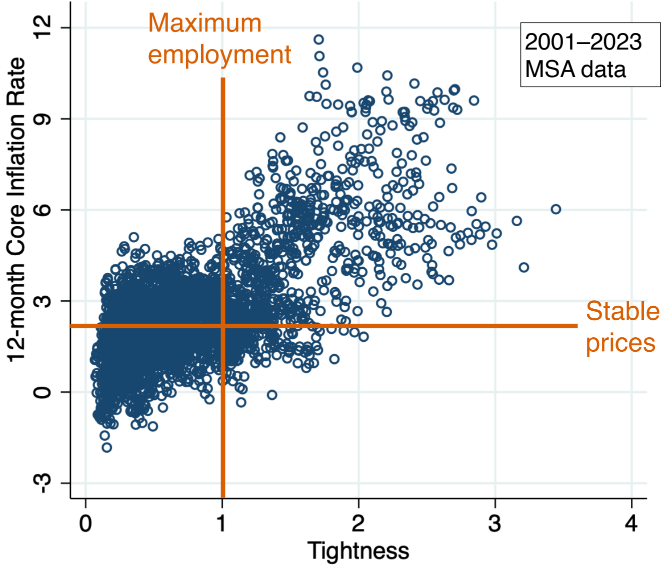


DIVINE COINCIDENCE APPEARS QUITE NATURALLY IN A MATCHING MODEL

- business-cycle model from Michaillat, Saez (2022)
  - sellers find customers through matching $\Rightarrow$unemployment
  - utility from wealth $\Rightarrow$nondegenerate aggregate demand
- price competition through directed search (Moen 1997)
- price rigidity from quadratic price-adjustment costs (Rotemberg 1982)

$\rightsquigarrow$ divine coincidence appears: $\pi = \pi^*$ iff $u = u^*$

- other properties of the model:
  - permanent zero-lower-bound episodes
  - fluctuations in unemployment & inflation
  - with kink in Phillips curve: fluctuations in unemployment in bad times but fluctuations in inflation in good times

DESCRIPTION OF THE MODEL

UNEMPLOYED WORKERS AND RECRUITERS

- people are organized in large households
- services are traded through long-term relationships
  - people are full-time employees in other households
  - employment relationships separate at rate $s > 0$
- household k has l_k workers producing services
  - y_{jk} workers work for household j
  - $y_k = \int_0^1 y_{jk}(t)\, dk$ workers are employed
  - $U_k = l_k - y_k$ unemployed workers are at shop k
- household j sends V_{jk} employees from household k to recruit workers at shop k
  - $V_k = \int_0^1 V_{jk}(t)\, dj$ recruiters are at shop k

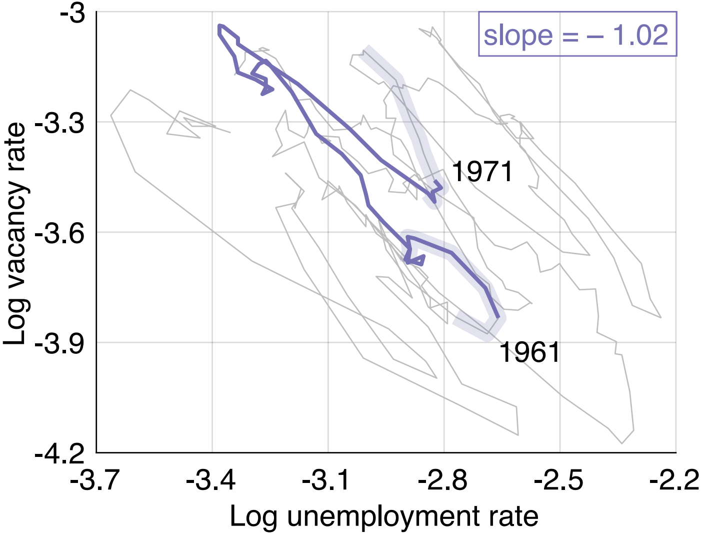
MATCHING BETWEEN WORKERS AND EMPLOYERS

- matching function determines flow of hires at shop k:

$$h_k = h(U_k, V_k) = \omega \cdot \sqrt{U_k \cdot V_k} - s \cdot U_k$$

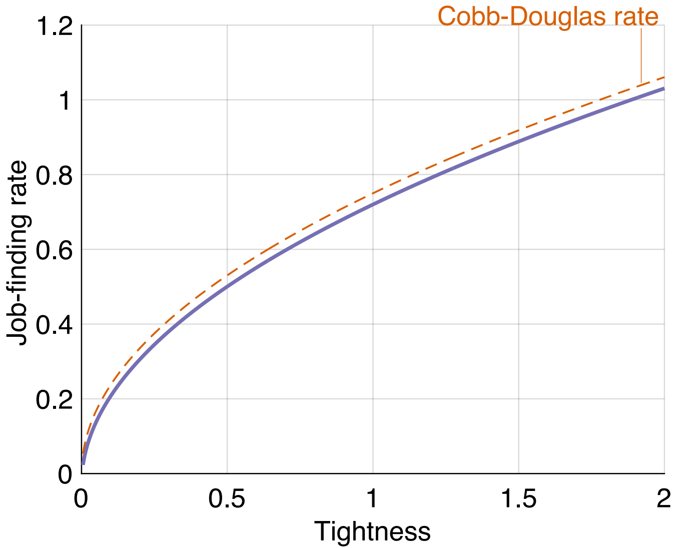
- matching function has standard properties
  - $h = 0$ when $U = 0$ and $V = 0$
  - constant returns to scale
  - increasing in V and U (as long as unemployment $< 50\%$)
  - concave in U and V
- market tightness $\theta_k = V_k/U_k$ determines trading rates
  - job-finding rate: $f(\theta_k) = h_k/U_k = \omega \cdot \sqrt{\theta_k} - s$
  - recruiting rate: $q(\theta_k) = h_k/V_k = \omega/\sqrt{\theta_k} - s/\theta_k$

US BEVERIDGE CURVE ≈ HYPERBOLA

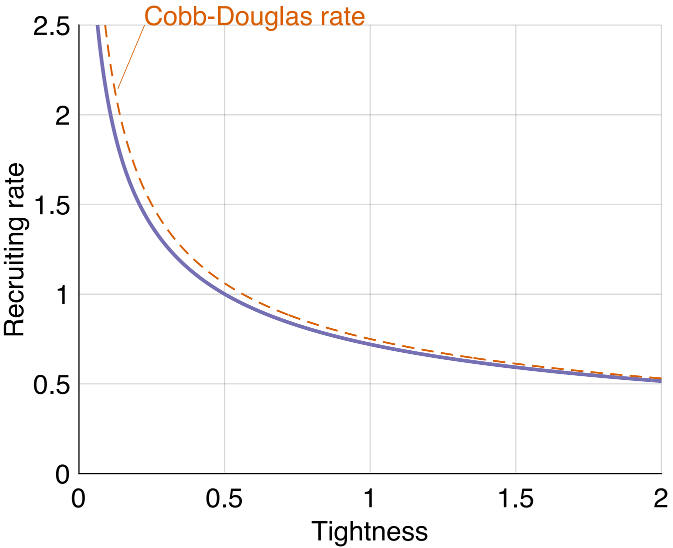


US BEVERIDGE CURVE ≈ HYPERBOLA

MATCHING RATES BETWEEN WORKERS AND EMPLOYERS



MATCHING RATES BETWEEN WORKERS AND EMPLOYERS



BALANCED FLOWS AND UNEMPLOYMENT RATE

- number of employed workers in household k:

$$\dot{y}_k = f(\theta_k) \cdot U_k - s \cdot y_k = f(\theta_k) \cdot U_k - s \cdot [l_k - U_k]$$

- US labor-market flows are balanced (Michaillat, Saez 2021)
  - assume that flows are balanced in all (j, k) cells
  - in particular flows are balanced in household k: $\dot{y}_k = 0$
- local tightness and local unemployment rate are directly related:

$$u(\theta_k) \equiv \frac{U_k}{l_k} = \frac{s}{s + f(\theta_k)}$$

MODEL BEVERIDGE CURVE IS AN HYPERBOLA

- balanced flows: $u_k = s/\left[s + f(\theta_k)\right]$
- matching function: $f(\theta_k) = \omega \cdot \sqrt{\theta_k} - s$

$\leadsto$ $u_k = (s/\omega)/\sqrt{v_k/u_k}$

- Beveridge curve is a rectangular hyperbola, just like in the US:

$$v_k \times u_k = (s/\omega)^2$$

- s/ω: location of the Beveridge curve

BALANCED FLOWS AND RECRUITER-PRODUCER RATIO

- recruiters from household k employed by household j: V_{jk}
  - their services do not deliver direct utility
- producers from household k employed by household j: $c_{jk} = y_{jk} - V_{jk}$
  - their services deliver direct utility
- workers from household k employed by household j:

$$\dot{y}_{jk} = q(\theta_k) \cdot V_{jk} - s \cdot y_{jk} = q(\theta_k) \cdot V_{jk} - s \cdot [c_{jk} + V_{jk}]$$

- flows are balanced in all (j, k) cells: $\dot{y}_{jk} = 0$
- local tightness determines the local recruiter-producer ratio:

$$\tau(\theta_k) \equiv \frac{V_{jk}}{c_{jk}} = \frac{s}{q(\theta_k) - s}$$

PRODUCTIVE EFFICIENCY AT SHOP k

- amount of services consumed:

$$c_k = y_k - V_k = l_k - U_k - V_k = l_k \cdot \left[1 - u_k - v_k\right]$$

- maximizing c_k is equivalent to minimizing $u_k + v_k$
- subject to the Beveridge curve $v_k \times u_k = (s/\omega)^2$
- from Michaillat, Saez (2023), the solution to the maximization is

$$u_k^* = \sqrt{u_k v_k} = s/\omega, \qquad \theta_k^* = 1$$

DIRECTED SEARCH AND PRICE/TIGHTNESS COMPETITION

- all workers from household k charge price p_k per unit time
- expenditure by household j on workers k is

$$p_k \cdot y_{jk} = p_k \cdot [c_{jk} + V_{jk}] = p_k \cdot [1 + \tau(\theta_k)] \cdot c_{jk}$$

- workers are perfectly substitutable
  - only $c_j = \int_0^1 c_{jk}(t)\, dk$ enters the utility function
- $p_k \cdot [1 + \tau(\theta_k)]$ must be the same across sellers (Moen 1997)
  - if not, there are cheaper workers available (lower p_k)
  - or workers that can be hired more easily (lower τ_k)
- there is a price level p so $p_k \cdot [1 + \tau(\theta_k)] = p \cdot [1 + \tau(\theta)]$ for all k

EFFECT OF LOCAL PRICE ON LOCAL TIGHTNESS

- price chosen by household k determines the tightness it faces:

$$\theta_k = \tau^{-1}\left(\frac{p}{p_k}[1+\tau(\theta)]-1\right)$$

- the function τ^{-1} is increasing, so θ_k is decreasing in p_k

⇝ a high price leads to low tightness, high unemployment

⇝ a low price leads to high tightness, low unemployment

EFFICIENCY WITHOUT PRICE-ADJUSTMENT COSTS

- seller chooses price to maximize income subject to demand curve
- subject to demand $\theta_k(p_k)$, seller chooses p_k to maximize:

$$p_k \cdot y_k = p \cdot [1 + \tau(\theta)] \cdot \frac{y_k}{1 + \tau(\theta_k)} = p \cdot [1 + \tau(\theta)] \cdot \frac{1 - u(\theta_k)}{1 + \tau(\theta_k)} \cdot l_k$$

- $\tau(\theta), u(\theta), v(\theta)$ are linked by

$$\frac{1 - u(\theta_k)}{1 + \tau(\theta_k)} = 1 - u(\theta_k) - v(\theta_k)$$

- seller sets local tightness θ_k to minimize $u(\theta_k) + v(\theta_k)$

$\Leftrightarrow$ sets unemployment rate u_k to minimize $u_k + v(u_k)$

$\Leftrightarrow$ unemployment rate u_k is efficient (Moen 1997)

PRICE RIGIDITY

- unexpected price/wage changes upset customers/workers
  - Shiller (1996): higher-than-normal price inflation upsets customers, who feel unfairly treated when they go to the store
  - Bewley (1999): lower-than-normal wages damage workers' morale, who feel unfairly treated
- inflation chosen by household k: $\pi_k = \dot{p}_k / p_k$
- flow disutility when inflation deviates from norm (Rotemberg 1982):

$$\frac{\kappa}{2} \cdot \left[\pi_k - \pi^*\right]^2$$

PEOPLE'S PREFERENCES

- household j maximizes utility

$$\int_0^\infty e^{-\delta t} \left\{ \ln\big(c_j(t)\big) + \sigma \cdot \left[\frac{b_j(t)}{p(t)} - \frac{b(t)}{p(t)}\right] - \frac{\kappa}{2} \cdot [\pi_j - \pi^*]^2 \right\} dt$$

- $\delta > 0$: time discount rate
- $\sigma > 0$: status concerns
- $c_j(t) = \int_0^1 c_{jk}(t)\, dk$: total consumption of services
- $b_j(t)$: saving in government bonds
- $b(t) = \int_0^1 b_j(t)\, dj$: aggregate wealth

PEOPLE'S BUDGET CONSTRAINT

- law of motion of government bond holdings for household j:

$$\dot{b}_j = i \cdot b_j - \int_0^1 p_k y_{jk} \, dk + p_j y_j$$

- because of matching and directed search, expenditure becomes:

$$\begin{aligned} \int_0^1 p_k y_{jk} \, dk &= \int_0^1 p_k [1 + \tau(\theta_k)] c_{jk} \, dk \\ &= p \cdot [1 + \tau(\theta)] \cdot \int_0^1 c_{jk} \, dk \\ &= p \cdot [1 + \tau(\theta)] \cdot c_j \end{aligned}$$

- because of matching and directed search, income becomes:

$$p_j \cdot y_j = p_j \cdot [1 - u(\theta_j(p_j))] \cdot l_j$$

SOLUTION OF THE MODEL

AGGREGATE SUPPLY: PHILLIPS EQUATION

- from optimal pricing by households:

$$\dot{\pi} = \delta \cdot (\pi - \bar{\pi}) - \frac{1}{\kappa} \cdot \left[1 - \frac{u}{v(u)} \cdot \frac{1 - u - v(u)}{1 - 2u}\right]$$

- κ: price-adjustment cost
- $1 - \frac{u \cdot (1-u-v)}{v \cdot (1-2u)}$: inefficiency of the economy
  - zero $\Leftrightarrow u = v \Leftrightarrow \theta = 1 \Leftrightarrow$ efficiency
  - positive $\Leftrightarrow v > u \Leftrightarrow \theta > 1 \Leftrightarrow$ inefficiently tight
  - negative $\Leftrightarrow u > v \Leftrightarrow \theta < 1 \Leftrightarrow$ inefficiently slack
- in steady state ($\dot{\pi} = 0$), Phillips curve:

$$\kappa \cdot \delta \cdot (\pi - \pi^*) = 1 - \frac{u}{v(u)} \cdot \frac{1 - u - v(u)}{1 - 2u}$$

AGGREGATE DEMAND: EULER EQUATION

- from optimal consumption and saving by households:

$$\frac{\dot{u}}{1-u} = \delta - \left[i(\pi) - \pi + \sigma \cdot (1-u) \cdot l\right]$$

- $i(\pi) - \pi$: real interest rate, financial return on saving
- $\sigma \cdot y$: MRS between wealth & consumption, hedonic return on saving
  - discounted Euler equation (McKay, Nakamura, Steinsson 2017)
- in steady state ($\dot{u} = 0$), Euler curve:

$$\pi = i(\pi) - \delta + \sigma \cdot (1-u) \cdot l$$

DIVINE COINCIDENCE APPEARS IN PHILLIPS CURVE

- Phillips curve is given by

$$\kappa \cdot \delta \cdot (\pi - \pi^*) = 1 - \frac{u}{v} \cdot \frac{1 - u - v}{1 - 2u}$$

- $\pi = \pi^* \Leftrightarrow u = v \Leftrightarrow \theta = 1 \Leftrightarrow u = u^*$
- Phillips curve goes through (u^*, π^*) so divine coincidence holds

⇝ if monetary policy is set appropriately, inflation is on target whenever unemployment is efficient

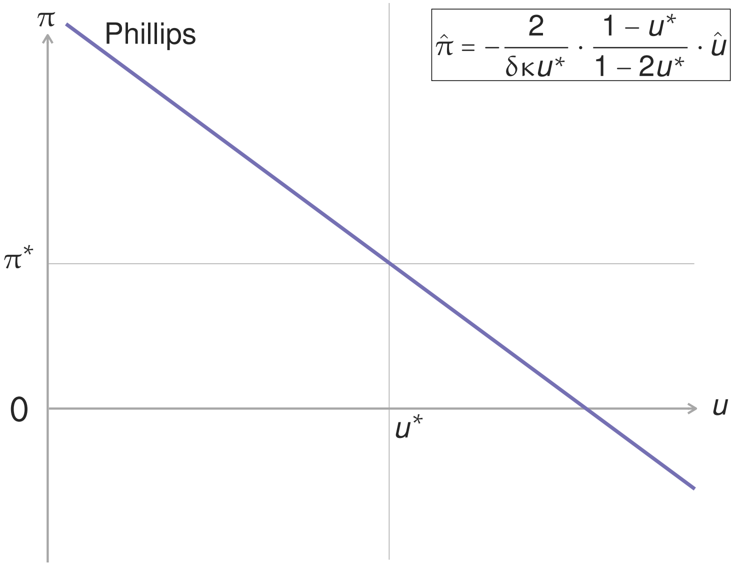
⇝ the price and employment mandates are consistent

MONETARY POLICY SATISFYING THE DUAL MANDATE

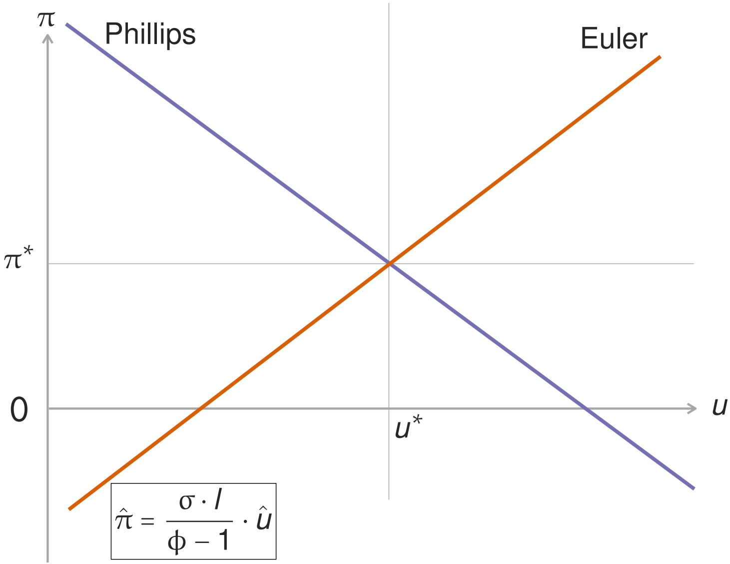
- nominal interest rate i^* ensures:
  - inflation is on target: $\pi = \pi^*$
  - unemployment is efficient: $u = u^*$
- from Euler curve: $i^* = \pi^* + \delta - \sigma \cdot (1 - u^*) \cdot l$
- policy can take different forms:
  - interest-rate peg: $i(\pi) = i^*$
  - Taylor rule with $\phi > 0$: $i(\pi) = i^* + \phi \cdot (\pi - \pi^*)$
- dual-mandate policy also maximizes social welfare

DYNAMICS OF THE MODEL

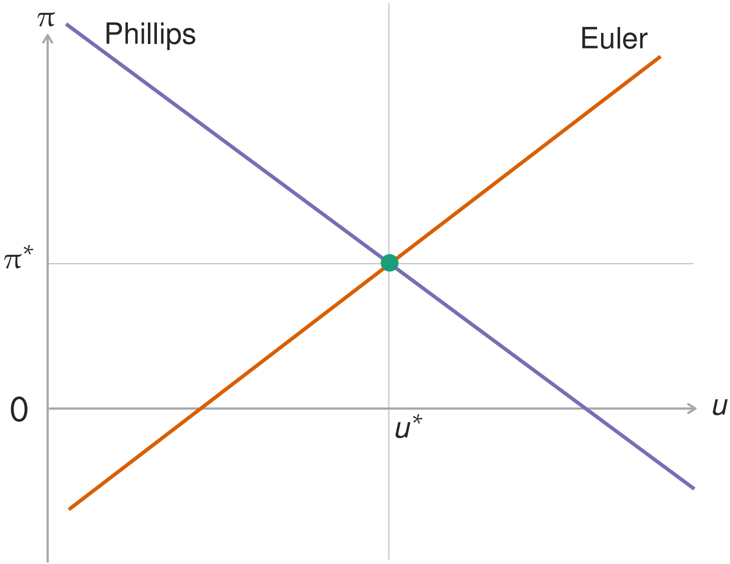
LINEARIZED PHILLIPS CURVE



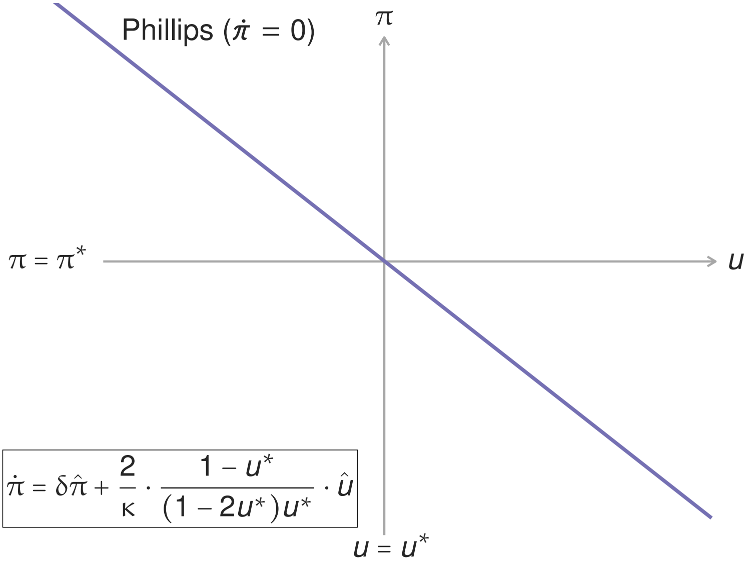
LINEARIZED EULER CURVE



DIVINE COINCIDENCE IN THE EARTHLY MODEL

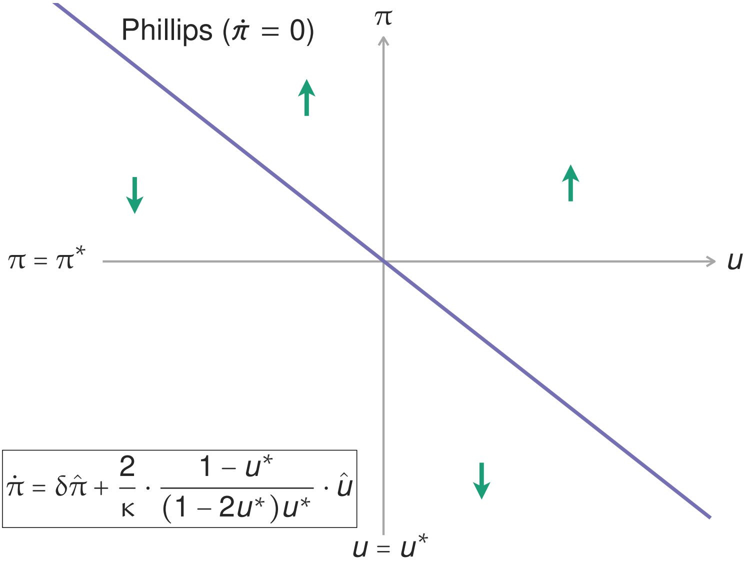


PHASE DIAGRAM OF THE EARTHLY MODEL



$$\boxed{\dot{\pi} = \delta\hat{\pi} + \frac{2}{\kappa} \cdot \frac{1 - u^*}{(1 - 2u^*)u^*} \cdot \hat{u}}$$

PHASE DIAGRAM OF THE EARTHLY MODEL



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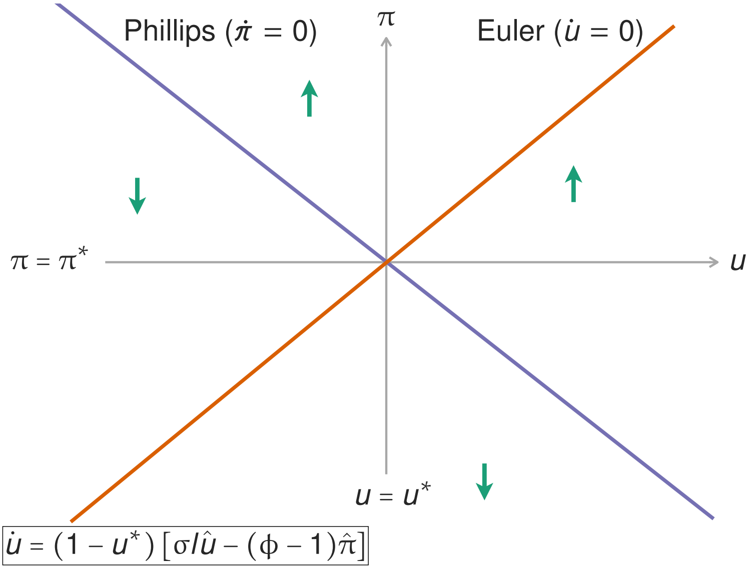
PHASE DIAGRAM OF THE EARTHLY MODEL (TAYLOR RULE)



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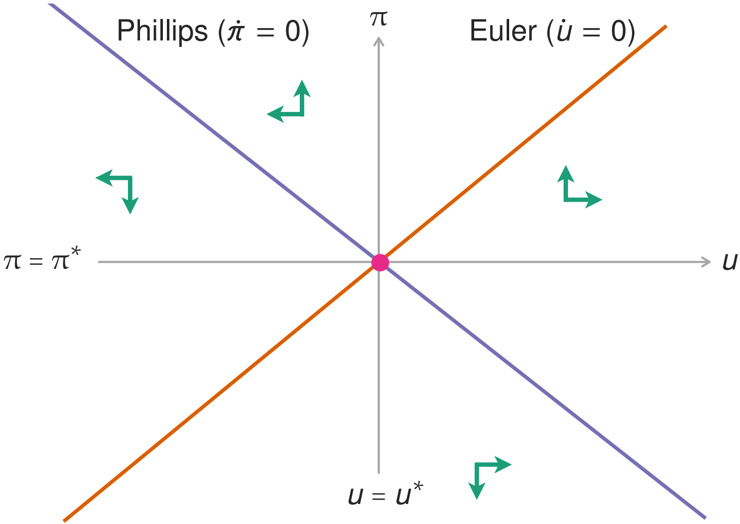
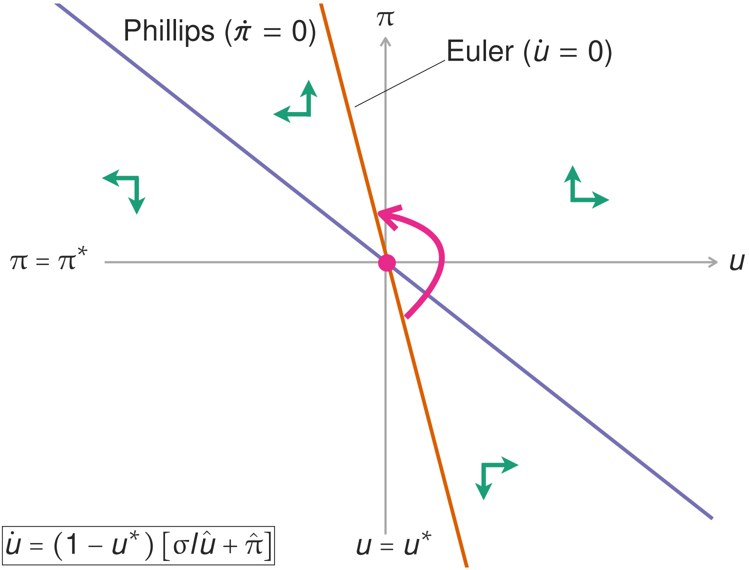


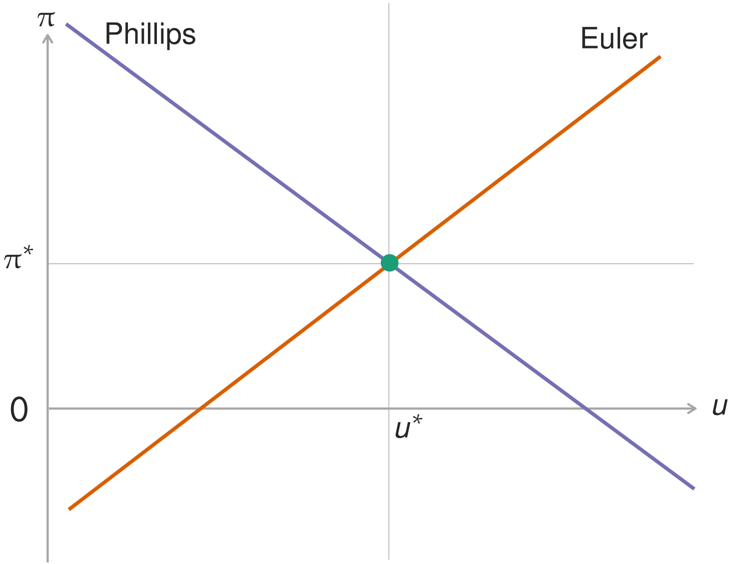
$$\boxed{\dot{u} = (1 - u^*)\left[\sigma / \hat{u} - (\phi - 1)\hat{\pi}\right]}$$

PHASE DIAGRAM OF THE EARTHLY MODEL (PEG)



RESPONSE TO DEMAND AND SUPPLY SHOCKS

NEGATIVE DEMAND OR MONETARY SHOCK



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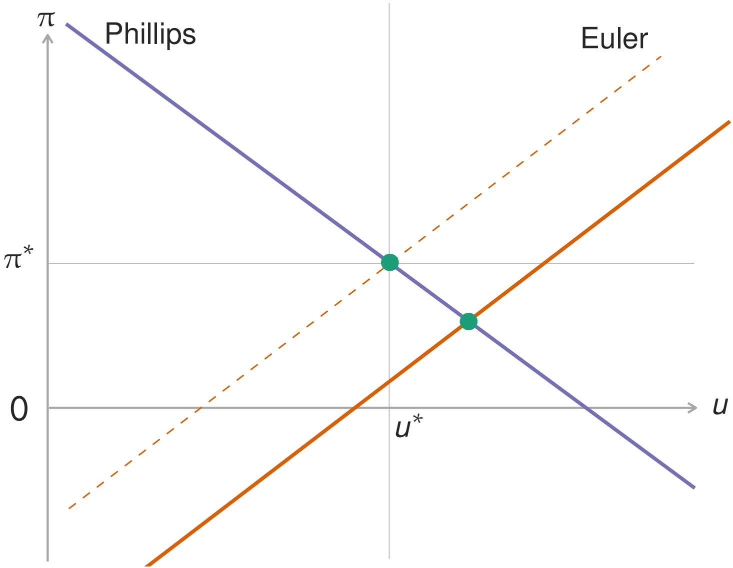
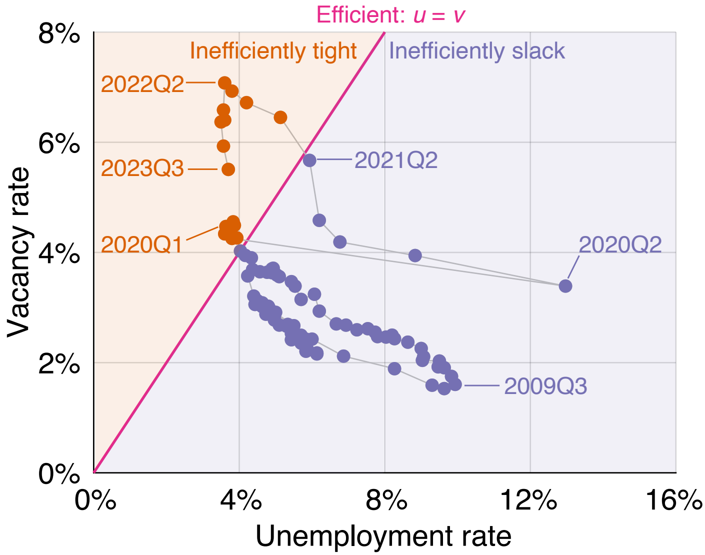


PANDEMIC SHIFT OF THE BEVERIDGE CURVE



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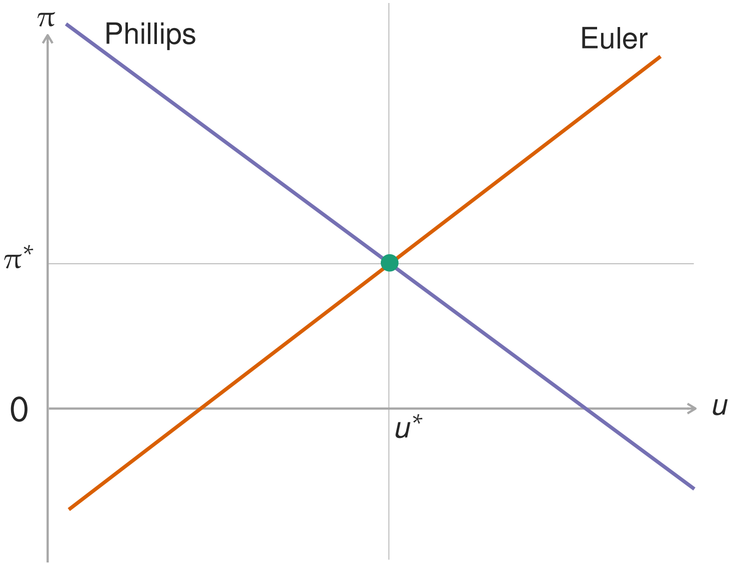

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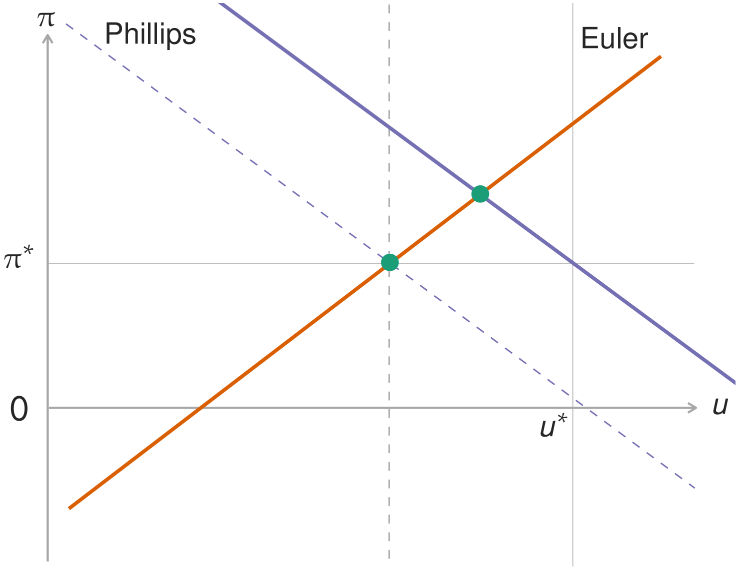
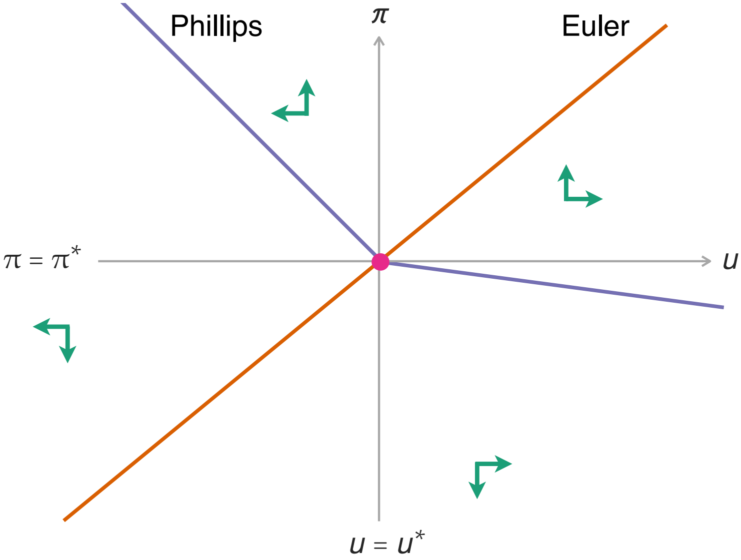
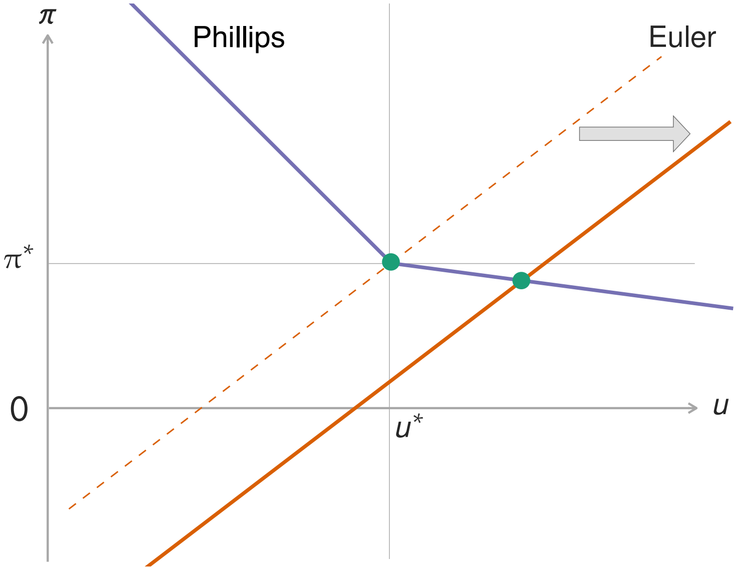


KINK IN THE PHILLIPS CURVE

DOWNWARD WAGE RIGIDITY > UPWARD PRICE RIGIDITY



NEGATIVE DEMAND SHOCK: UNEMPLOYMENT GAP ↑



NEGATIVE SUPPLY SHOCK: INFLATION ↑