

Beveridgean Phillips Curve

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This paper proposes a new, Beveridgean model of the Phillips curve. While the New Keynesian Phillips Curve is based on monopolistic pricing under price-adjustment costs, the Beveridgean Phillips curve is based on directed-search pricing under price-adjustment costs. Under directed-search pricing, prices respond to slack instead of marginal costs. The Beveridgean Phillips curve links the inflation gap to the unemployment gap, with the following properties. First, it produces the divine coincidence: it guarantees that the rate of inflation is on target whenever the rate of unemployment is efficient. Second, whenever the Beveridge curve shifts, the Phillips curve shifts if it is formulated with inflation and unemployment, but it remains unaffected if it is formulated with inflation and labor-market tightness. Third, the Phillips curve displays a kink at the point of divine coincidence if we assume that wage decreases—which reduce workers' morale—are more costly to producers than price increases—which upset customers. These three properties describe recent US data well.

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1. Introduction

The post-pandemic spike in inflation in the United States, which coincided with wide labor-market fluctuations, revealed new facts about the Phillips curve. First, the Phillips curve ensures divine coincidence: inflation is on target whenever the labor market is efficient; inflation rises above target whenever the labor market is inefficiently tight; and inflation falls below target whenever the labor market is inefficiently slack (Benigno and Eggertsson 2023, 2024a; Gitti 2024). Second, tightness is a better measure of slack than unemployment in the Phillips curve (Barnichon, Oliveira, and Shapiro 2021; Barnichon and Shapiro 2022, 2024). Third, the Phillips curve is kinked at the point of divine coincidence: the Phillips curve is steeper when the labor market is inefficiently tight than when the labor market is inefficiently slack (Babb and Detmeister 2017; Smith, Timmermann, and Wright 2023; Benigno and Eggertsson 2023, 2024a). All of these empirical findings are summarized by figure 1.

To explain these facts, we propose a Beveridgean model of the Phillips curve. We start from the structure of the matching model of business cycles developed by Michaillat and Saez (2022). In that model, however, inflation is constant. To generate price dynamics, we introduce price competition through directed search (Moen 1997). Furthermore, to ensure that unemployment fluctuates, we introduce price rigidity through quadratic price-adjustment costs (Rotemberg 1982). In the model, directed search produces a Phillips curve linking labor-market slack to inflation, and the price-adjustment costs ensure that the Phillips curve is nonvertical. The Phillips curve guarantees that inflation is on target whenever the labor market is operating efficiently, generating the divine coincidence. Indeed, when the labor market is operating efficiently, the surplus from each seller-buyer relationship is maximized, so price-setters have no incentive to change prices.

Second, whenever the Beveridge curve shifts, it shifts if it is formulated with inflation and unemployment but not if it is formulated with inflation and labor-market tightness. After the Great Recession, but especially after the pandemic, the US Beveridge curve shifted drastically (Michaillat and Saez 2024, figure 10). This led to shifts of efficient unemployment rate, and therefore to the point of divine coincidence on the unemployment-inflation Phillips curve. Hence, the model offers a microfounded source of Phillips curve shifts—the Beveridge shocks—improving upon the reduced-form “cost-push shocks” used in the New Keynesian literature. Since the efficient tightness does not respond to shifts in the Beveridge curve—it remains equal to 1—the tightness-inflation Phillips curve is unaffected by Beveridge shocks.

Finally, by moving away from symmetric price-adjustment costs, the model can produce a kink in the Phillips curve at the point of divine coincidence. If we assume that price decreases—which are also wage decreases and reduce workers’ morale—are more costly to producers than price increases—which only upset customers—the model generates a

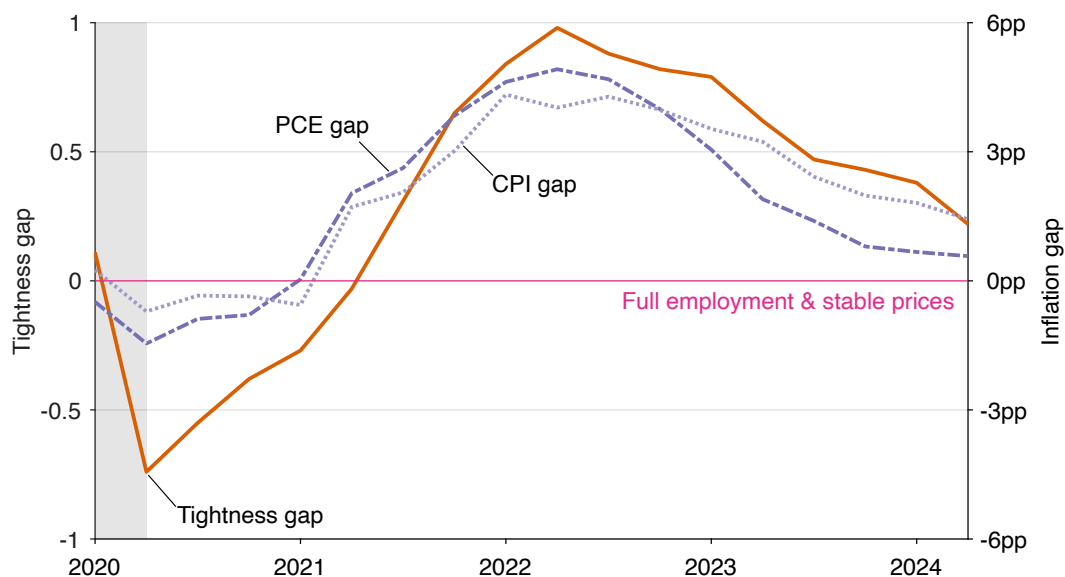


FIGURE 1. Phillips curve in the United States, 2020–2024

Note: The tightness gap is the labor-market tightness produced by Michaillat and Saez (2024, figure 8B), minus 1. The CPI gap is the percent change from a year ago of the consumer price index (CPI) for all urban consumers less food and energy, as measured by the US Bureau of Labor Statistics (2024a), minus 2%. The PCE gap is the percent change from a year ago of the personal consumption expenditures (PCE) price index, as measured by the US Bureau of Labor Statistics (2024b), minus 2%.

kinked Phillips curve. Producing a kink does not require a constraint on wage or price movements (as assumed in models of with downward wage rigidity); it only requires that the price-adjustment cost is larger for price decreases than for price increases. With a kink, the model predicts that macroeconomic shocks generate larger movements in inflation when the economy is inefficiently tight, and larger movements in unemployment when the economy is inefficiently slack.

The divine coincidence has important implications. First, it helps explain the behavior of inflation. It can therefore explain part of the flare-up in inflation in 2021–2023, since the US labor market has been inefficiently tight in the aftermath of the coronavirus pandemic, from the middle of 2021 to today (Michaillat and Saez 2024). Second, it is important for policy. It implies that the full-employment and price-stability mandates of the Federal Reserve coincide. It means that by maintaining the economy at full employment, the Fed can be sure that inflation will also be on target.

The model demonstrates that the divine coincidence arises in theory under common assumptions (directed search with price-adjustment costs). So it might not be as surprising as it first seemed that the divine coincidence appears in the data. More generally, the model developed here shows how the joint movements of inflation, unemployment, and

tightness can be studied via the Euler, Phillips, and Beveridge curves.

2. Three facts about the Phillips curve

This section reviews recent US evidence that the divine coincidence holds, the tightness-inflation Phillips curve is stabler than the unemployment-inflation Phillips curve, and the Phillips curve is kinked at the point of divine coincidence.

2.1. Divine coincidence

First, inflation rose above its 2% target in early 2021 just as the labor-market tightness rose above its efficient value of 1 (Michaillat and Saez 2024). Both inflation and tightness peaked in the middle of 2022, and they have been declining since then, getting ever closer to their target values in the middle of 2024. Figure 1 illustrates the parallel trajectories followed by inflation (whether measured with the core consumer price index or the personal consumption expenditures price index) and labor-market tightness (the number of vacant jobs per unemployed workers). This parallel pattern suggests that the divine coincidence prevails in the United States: inflation is on target whenever the labor market is efficient; inflation rises above target whenever the labor market is inefficiently tight; and inflation falls below target whenever the labor market is inefficiently tight.

The basic evidence of divine coincidence presented in figure 1 is buttressed by other work. Benigno and Eggertsson (2023) use aggregate data for inflation and labor market tightness in the United States. They find clear evidence of divine coincidence in the 2008–2022 period. When the labor market is efficient, which corresponds to a tightness of 1, inflation is on target at 2% (Benigno and Eggertsson 2023, figure 4). Gitti (2024) uses metropolitan-level data for inflation and labor market tightness in the United States, 2001–2022. The divine coincidence appears again at the metropolitan level, although in a more noisy fashion: when labor market tightness is 1, inflation is around 2% (Gitti 2024, figure 1).

Before the pandemic, Carney (2017) also argued that the divine coincidence held in the United States. Before that, in the 1960s, the Council of Economic Advisors believed in the divine coincidence since they thought that at full employment the economy was not subject to inflationary pressures (Bernanke 2022, p. 18). And over the years the Fed through its FOMC statements often indicated that it believed in the divine coincidence, because it believed that price stability necessarily guaranteed full employment, so that its two mandates were equivalent (Thornton 2012, pp. 119, 120, 121, 130).

2.2. The tightness-inflation Phillips curve is stabler than the unemployment-inflation Phillips curve

In recent work, Barnichon, Oliveira, and Shapiro (2021) and Barnichon and Shapiro (2022, 2024) argue that the Phillips curve formulated with labor-market tightness (the vacancy-to-unemployment ratio) and inflation performs better than a Phillips curve formulated with unemployment rate and inflation—which is more traditional. They argue that the labor-market tightness is a superior indicator of inflationary pressures because it captures both labor demand and supply. In particular, they demonstrate that incorporating labor-market tightness improves inflation forecasts compared to using unemployment alone. The superior performance of the tightness-inflation Phillips curve over the unemployment-inflation Phillips curve is especially visible in the pandemic period, when the Beveridge curve has been unstable.

The close connection between labor-market tightness and inflation is visible on figure 1. Tightness and inflation, whether measured from the CPI or from the PCE price index, evolve in tandem between 2020 and 2024. They all are at their efficient levels at the beginning of 2020: 2% for inflation and 1 for tightness. Then they all fall sharply between 2020Q1 and 2020Q2. After that they all recover, first returning to their efficient levels in 2021Q1–2021Q2 and then peaking well above their efficient levels in 2022Q2. Between 2022Q2 and 2024Q4, tightness and inflation have been falling, almost returning to efficiency in 2024Q2.

These findings are also consistent with those by Ball, Leigh, and Mishra (2022). They argue that the spike in inflation was so severe during the pandemic because labor-market tightness was so elevated.

2.3. The Phillips curve is kinked at the point of divine coincidence

Finally, it seems not only that the divine coincidence holds in the United States, but also that the Phillips curve has a kink at the point of divine coincidence.

First, the Phillips curve appears steeper when the labor market is inefficiently tight and flatter when the labor market is inefficiently slack. Such kink has been observed not only in aggregate US data (Benigno and Eggertsson 2023, 2024a), but also in metropolitan US data (Babb and Detmeister 2017; Smith, Timmermann, and Wright 2023; Gitti 2024), and in international data (Smith, Timmermann, and Wright 2023; Benigno and Eggertsson 2024b).

Second, it seems that the kink is located right at the point of divine coincidence. Benigno and Eggertsson (2023, figure 4) find a kink where inflation is 2% and tightness is 1, so right at the point of divine coincidence. Benigno and Eggertsson (2024a, figure 4) report the same results. Babb and Detmeister (2017, table 4) and Smith, Timmermann,

and Wright (2023, table 5) find a kink in the US Phillips curve at an unemployment rate of 4.2%. In the postwar period in the United States, the efficient unemployment rate is just 4.2% (Michaillat and Saez 2024, section 3A). So the kink identified by Babb and Detmeister (2017) and Smith, Timmermann, and Wright (2023) occurs at the efficient unemployment rate.

The Phillips curve kink can also be seen in figure 1. The drop in inflation between 2020Q1 and 2021Q2 is much more subdued than the coinciding drop in tightness. By contrast, the spikes in inflation and tightness between 2021Q2 and 2024Q2 have exactly the same magnitude. This indicates that the tightness-inflation Phillips curve is much flatter when tightness is inefficiently slack than when it is inefficiently high.

3. Beveridgean model of the Phillips curve

This section develops our Beveridgean model of the Phillips curve. Unlike in a New Keynesian model, where price dynamics are driven by marginal costs, in the Beveridgean model, price dynamics are driven by the amount of slack in the economy. When the economy is inefficiently slack, sellers are pushed to reduce their prices. Conversely, when the economy is inefficiently tight, sellers are pushed to raise their prices.

3.1. People

The size of the population is normalized to 1. People are organized in large households. The households are all initially identical and indexed by $j \in [0, 1]$. Household $j \in [0, 1]$ has l_j workers. The aggregate labor force is $l = \int_0^1 l_j(t) dj$.

3.2. Matching between workers and customers

Of the l_k workers of household k , y_{jk} work for household j , and a total $y_k = \int_0^1 y_{jk}(t) dk$ are employed across all households. Not all workers are employed, however. $U_k = l_k - y_k$ workers remain unemployed.

Services are sold through long-term worker-household relationships. Once a worker has matched with a household, she becomes a full-time employee of the household. She remains so until they separate, which occurs at rate $s > 0$.

To recruit workers from household k , household j sends V_{jk} of their own employees to visit household k 's shop. These V_{jk} employees advertise the vacancies open by their employer, read applications from household k 's workers, and interview and select suitable candidates. A total $V_k = \int_0^1 V_{jk}(t) dj$ employed workers are at shop k to recruit unemployed workers from household k .

A matching function determines the flow of new matches at shop k based on the number of unemployed workers and recruiters: $h_k = h(U_k, V_k)$ where

$$(1) \quad h(U_k, V_k) = \omega \cdot \sqrt{U_k \cdot V_k} - s \cdot U_k.$$

The matching function h satisfies standard assumptions (Petrongolo and Pissarides 2001): it is 0 when $U = 0$ and $V = 0$, it has constant returns to scale, is increasing in V , and it is increasing in U as long as the market is not too slack—a condition that will be satisfied as long the unemployment rate is below 50%.¹

The partial derivative of the matching function with respect to U_k is

$$\frac{\partial h}{\partial U_k} = \frac{\omega}{2} \cdot \sqrt{\frac{V_k}{U_k}} - s = \frac{\omega}{2} \cdot \sqrt{\theta_k} - s,$$

where

$$\theta_k = \frac{V_k}{U_k}$$

is the tightness of market k . The partial derivative is positive for any $\theta_k \geq \underline{\theta}$ where the lower bound $\underline{\theta}$ is given by

$$(2) \quad \underline{\theta} = 4 \left(\frac{s}{\omega} \right)^2.$$

We impose

$$\omega > 2s$$

so $\underline{\theta} < 1$. In the model the condition $\theta_k \geq \underline{\theta}$ is verified for any $u_k \in [0, 1/2]$. So as long the local unemployment rate is below 50%, the matching function will be increasing in the unemployment rate. In the United States, the efficient unemployment rate averages 4% (Michaillat and Saez 2024). This fact implies that $\omega = 25s$, and thus that the assumption $\omega > 2s$ is easily satisfied in practice. It also implies that $\underline{\theta}$ is very small: $\underline{\theta} \approx 0.006$. Lastly, the matching function is concave in U and concave in V .

Although it satisfies all the standard properties, the matching function h takes an unusual form. We specify the function as such to obtain an hyperbolic Beveridge curve in the model, and therefore be consistent with the empirical Beveridge curve observed in US data (Michaillat and Saez 2024, section 2E).

The tightness on market k is the ratio of the number of recruiters (buyers) and unemployed workers (sellers):

$$\theta_k = \frac{V_k}{U_k}.$$

¹Shimer (2010) also assumes that recruiters enter the matching function directly.

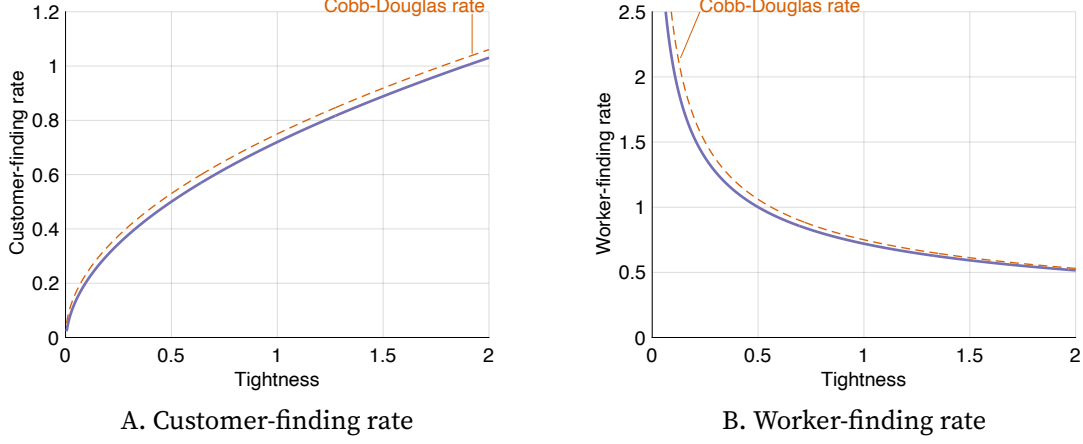


FIGURE 2. Matching rates between workers and customers

Panel A compares the customer-finding rate under the matching function (1) and under a traditional Cobb-Douglas matching function. Panel B compares the worker-finding rate under the matching function (1) and under a traditional Cobb-Douglas matching function.

Equivalently, tightness on market k is the ratio of the numbers of recruiters and unemployment workers: $\theta_k = V_k/U_k$. That is, tightness on market k is the number of recruiters per unemployed workers. We impose $\theta_k \geq \underline{\theta}$ to ensure that the matching function is increasing in U , which also ensures that the number of matches is positive.

Tightness determines all trading rates. The customer-finding rate is

$$(3) \quad f(\theta_k) = \frac{h_k}{U_k} = \omega \cdot \sqrt{\theta_k} - s.$$

For $\theta_k \in [\underline{\theta}, \infty)$, the customer-finding rate f is positive since $\sqrt{\theta_k} \geq 2s/\omega$ so $\omega\sqrt{\theta_k} - s \geq s > 0$. The customer-finding rate is also increasing in θ_k . In fact, $f(\underline{\theta}) = s$ and $f(\infty) = \infty$. Hence, when tightness is higher, it is easier to find a job and sell services. Although the customer-finding rate does not take a usual form (the $-s$ term is unusual), it behaves almost exactly like the rate given by a Cobb-Douglas matching function (figure 2A).

The worker-finding rate is

$$(4) \quad q(\theta_k) = \frac{h_k}{V_k} = \frac{\omega}{\sqrt{\theta_k}} - \frac{s}{\theta_k}.$$

For $\theta_k \in [\underline{\theta}, \infty)$, the worker-finding rate is positive since $\sqrt{\theta_k} \geq 2s/\omega$ so $s/\sqrt{\theta_k} \leq \omega/2$ and $\omega - s/\sqrt{\theta_k} \geq \omega/2 > 0$. The worker-finding rate is also decreasing in θ_k (appendix A.1). This means that when tightness is higher, it is harder to find a worker and buy services. Although the worker-finding rate does not take a usual form (the $-s/\theta_k$ term is unusual), it behaves almost exactly like the rate given by a Cobb-Douglas matching function (figure 2B).

As usual, the customer-finding and worker-finding rates are related by $f(\theta_k) = \theta_k \cdot q(\theta_k)$.

3.3. Cost of unemployment and hiring

Unemployed workers wait in their shop to be hired. During that time, they do not receive any income and cannot engage in home production, which explains why unemployment is individually and socially costly.

There are costs not only on the selling side of the market, but also on the buying side. Hiring workers is indeed costly. Each recruiter looking to hire a worker on behalf of their employer cannot produce utility-providing services. Because the recruiters employed by household j to hire workers from household k do not provide direct utility but are used in the process of hiring other workers, consumption c_{jk} is less than output y_{jk} . Consumption c_{jk} is the number of workers from household k working for household j , minus the number of workers employed by household j to recruit workers from household k :

$$c_{jk} = y_{jk} - V_{jk}.$$

3.4. Balanced flows and unemployment

We assume that flows on each individual market are balanced. This assumption is motivated by the fact that on the US labor market, flows are always approximately balanced (Michaillat and Saez 2021a, p. 7).

At this stage it is convenient to introduce the unemployment rate in shop k , which is

$$u_k = \frac{U_k}{l_k},$$

and the recruiting rate at shop k , which is

$$v_k = \frac{V_k}{l_k}.$$

The number of workers employed in household k is given by a differential equation:

$$\dot{y}_k = f(\theta_k) \cdot U_k - s \cdot y_k = f(\theta_k) \cdot U_k - s \cdot [l_k - U_k] = l_k \cdot [f(\theta_k) \cdot u_k - s \cdot [1 - u_k]].$$

We assume that flows are balanced in all (j, k) cells. In particular flows are balanced in household k : $\dot{y}_k = 0$. This assumption implies that the local unemployment rate is a function of local tightness: $u_k = u(\theta_k)$, where

$$(5) \quad u(\theta_k) = \frac{s}{s + f(\theta_k)} = \frac{s/\omega}{\sqrt{\theta_k}}.$$

From this expression we see that for any $\theta_k \geq \underline{\theta} = 4(s/\omega)^2$, $u(\theta_k) \leq 1/2$. The analysis focuses on this range of tightness—which is without real loss of generality since in practice unemployment rates are not above 50%.

The unemployment function (5) has the following properties when $\theta_k \in [\underline{\theta}, \infty)$: $u(\underline{\theta}) = 1/2$, $u(\infty) = 0$, and u is decreasing in θ_k . When the market is tighter, workers find jobs more rapidly, so the unemployment rate is lower.

Thanks to the shape of the matching function (1), the unemployment rate is an isoelastic function of tightness, and the Beveridge curve is isoelastic as well. In fact the Beveridge curve is a rectangular hyperbola, just like in US data (Michaillat and Saez 2024):

$$(6) \quad v(u_k) = \frac{(s/\omega)^2}{u_k}.$$

From the Beveridge curve we can also express the recruiting rate as a function of local tightness:

$$(7) \quad v(\theta_k) = \frac{s}{\omega} \cdot \sqrt{\theta_k}.$$

The recruiting function (7) has the following properties when $\theta_k \in [\underline{\theta}, \infty)$: $v(\underline{\theta}) = 2(s/\omega)^2$, $v(\infty) = \infty$, and v is increasing in θ_k . When the market is tighter, it takes longer to hire workers, so the recruiting rate is higher.

3.5. Balanced flows and recruiter-producer ratio

Next we compute the recruiter-producer ratio for household k . The number of workers employed by household j in household k follows a differential equation:

$$\dot{y}_{jk} = q(\theta_k) \cdot V_{jk} - s \cdot y_{jk} = q(\theta_k) \cdot [y_{jk} - c_{jk}] - s \cdot y_{jk}.$$

We assume that flows are balanced in all (j, k) cells, so $\dot{y}_{jk} = 0$. This means that tightness determines the gap between consumption and output:

$$y_{jk} = [1 + \tau(\theta_k)] \cdot c_{jk},$$

where the recruiter-producer ratio is a function of tightness:

$$(8) \quad \tau(\theta_k) = \frac{s}{q(\theta_k) - s}.$$

By definition, $\tau(\theta_k) = (y_{jk} - c_{jk})/c_{jk}$. The quantity $y_{jk} - c_{jk}$ is the number of workers hired by household j to recruit new workers from household k , while c_{jk} is the number of workers from household k hired by household j for producing services. Hence the

function τ gives the recruiter-producer ratio required for any local tightness θ_k .

Given the properties of the worker-finding rate q , we infer the properties the properties of the recruiter-producer ratio τ . Since $q(\underline{\theta}) = \omega^2/4s$, $\tau(\underline{\theta}) = 1/[(\omega/2s)^2 - 1] = 1/(1/\underline{\theta} - 1) > 0$. Since q is decreasing in θ for $\theta \geq \underline{\theta}$, τ is increasing in θ for $\theta \geq \underline{\theta}$. Furthermore, $\tau \rightarrow \infty$ when $\theta \rightarrow \bar{\theta}$ when $\bar{\theta} > \underline{\theta}$ is defined by $q(\bar{\theta}) = s$. The upper tightness bound is well defined because $q(\underline{\theta}) > s$, q is decreasing in θ , and $q(\infty) \rightarrow 0$. In fact it is possible to express $\bar{\theta}$ as a function of $\underline{\theta}$ alone (appendix A.2):

$$\bar{\theta} = \frac{\underline{\theta}}{[1 - \sqrt{1 - \underline{\theta}}]^2} > \underline{\theta}.$$

It is also helpful to write the recruiter-producer ratio as a function of the unemployment rate. In a local market k , the recruiter-producer $\tau(\theta_k)$ ratio is the same in all households that hire workers there. So the recruiter-producer ratio is all the ratio between all the recruiters hired from household k and all the producers from household k :

$$\tau_k = \frac{y_k - c_k}{c_k}, \quad y_k - c_k = v_k, \quad c_k = l_k - u_k - v_k.$$

Therefore we can write the recruiter-producer as a function of the unemployment rate:

$$(9) \quad \tau(u_k) = \frac{v(u_k)}{1 - [u_k + v(u_k)]}.$$

3.6. Some elasticities

We now compute a few elasticities that will be important when we solve the model.

The elasticity of the customer-finding rate f given by (3) is

$$\frac{d \ln(f)}{d \ln(\theta)} = \frac{\omega \sqrt{\theta}}{\omega \sqrt{\theta} - s} \cdot \frac{1}{2} = \frac{1}{1 - (s/\omega)/\sqrt{\theta}} \cdot \frac{1}{2} = \frac{1/2}{1 - u},$$

where the unemployment rate is a function of tightness given by (5). Since $1 - u \approx 1$, the elasticity is never far from $1/2$, as it would be with a more common Cobb-Douglas matching function.

Since $q(\theta) = f(\theta)/\theta$, we infer the elasticity of the worker-finding rate q :

$$\frac{d \ln(q)}{d \ln(\theta)} = \frac{d \ln(f)}{d \ln(\theta)} - 1 = -\frac{1/2 - u}{1 - u},$$

where the unemployment rate is a function of tightness given by (5). Since $1 - u \approx 1$ and $1/2 - u \approx 1/2$, the elasticity is never far from $-1/2$, as it would be with a more common Cobb-Douglas matching function.

The elasticity of the unemployment rate (5) simply is

$$(10) \quad \frac{d \ln(u)}{d \ln(\theta_k)} = -(1 - u) \cdot \frac{d \ln(f)}{d \ln(\theta)} = -\frac{1}{2}.$$

The elasticity of the recruiter-producer ratio (8) is given by

$$(11) \quad \frac{d \ln(\tau)}{d \ln(\theta_k)} = -(1 + \tau) \cdot \frac{d \ln(q)}{d \ln(\theta)} = \frac{(1 + \tau) \cdot (1/2 - u)}{1 - u}.$$

From (9), we have

$$1 + \tau(\theta_k) = \frac{1 - u(\theta_k)}{1 - u(\theta_k) - v(\theta_k)},$$

so we can simplify the elasticity of the recruiter-producer ratio:

$$(12) \quad \frac{d \ln(\tau)}{d \ln(\theta_k)} = \frac{1}{2} \cdot \frac{1 - 2u}{1 - u - v}.$$

3.7. Productive efficiency at shop k

What is the efficient allocation of labor at shop k ? We are interested in productive efficiency, that is the allocation of labor that maximizes the amount of services from household k that are consumed. The amount of services consumed is

$$c_k = y_k - V_k = l_k - U_k - V_k = l_k \cdot [1 - u_k - v_k].$$

Maximizing that amount is equivalent to minimizing the sum of the unemployment and recruiting rates, $u_k + v_k$, subject to the Beveridge curve (6). This is exactly the problem studied by Michaillat and Saez (2024). The solution is

$$(13) \quad u_k^* = \sqrt{u_k v_k} = s/\omega, \quad v_k^* = u_k^*, \quad \theta_k^* = 1.$$

Furthermore, the economy is inefficiently tight when there are more recruiters than jobseekers, $u_k < v_k$, inefficiently slack when there are more jobseekers than recruiters, $u_k > v_k$, and efficient when there are as many jobseekers as recruiters, $u_k = v_k$.

3.8. Directed search and price-tightness competition

All workers from household k charge a price p_k per unit time. The expenditure by household j on workers k therefore is

$$p_k \cdot y_{jk} = p_k \cdot [1 + \tau(\theta_k)] \cdot c_{jk}.$$

The relevant price of services is not just p_k but $p_k \cdot [1 + \tau(\theta_k)]$. The price involves the price per unit time as well as the time it takes to replace a worker.

All workers are perfectly substitutable, so households hire workers from the household that offers the cheapest consumption. All households are aware of this fact, so all households price their services to compete with other households:

$$p_k \cdot [1 + \tau(\theta_k)]$$

must be the same across all households k . Just as in Moen (1997), buyers direct their search toward the most attractive sellers, which induces competition across all sellers. Through competition, sellers set prices so buyers are indifferent across all sellers. If sellers set a higher price, then cheaper workers would be available, or workers could be hired with less wait, so they would not get any customers.

Accordingly, there is a price level p such that for all k ,

$$(14) \quad p_k \cdot [1 + \tau(\theta_k)] = p \cdot [1 + \tau(\theta)],$$

where the aggregate market tightness is the ratio of the aggregate number of recruiters to the aggregate number of unemployed workers, given by

$$\theta = \frac{\sum_k V_k}{\sum_k U_k}.$$

3.9. Effect of local price on local tightness

The price chosen by household j determines the tightness θ_j it faces, and therefore the pace at which workers from the household find employment. From (14), we see that the local tightness is given by

$$\theta_j(p_j) = \tau^{-1} \left(\frac{p}{p_j} [1 + \tau(\theta)] - 1 \right).$$

The function τ^{-1} is increasing, so the local tightness $\theta_j(p_j)$ is decreasing in the local price p_j . A high price leads to low tightness and high unemployment. A low price leads to high tightness and low unemployment. In that way, households face downward-sloping demand curves in a price-tightness plane.

In fact, the demand curve $\theta_j(p_j)$ has the following properties for $p_j \in (0, p[1 + \tau(\theta)])$: $\theta_j(0) = \bar{\theta}$, $\theta_j(p) = \theta$, $\theta_j(p[1 + \tau(\theta)]) = 0$. The derivative and elasticity of the demand curve

are:

$$\begin{aligned}\frac{d\theta_j}{dp_j} &= -\frac{p}{p_j^2} \cdot [1 + \tau(\theta)] \cdot \frac{1}{\tau'(\theta_j)} = -\frac{1 + \tau(\theta_j)}{p_j \cdot \tau'(\theta_j)} \\ \frac{d \ln(\theta_j)}{d \ln(p_j)} &= -\frac{1 + \tau(\theta_j)}{\theta_j \cdot \tau'(\theta_j)} = \frac{-1}{d \ln(1 + \tau(\theta_j)) / d \ln(\theta_j)}.\end{aligned}$$

From (11), we infer that

$$\frac{d \ln(1 + \tau)}{d \ln(\theta)} = \frac{\tau}{1 + \tau} \frac{d \ln(\tau)}{d \ln(\theta)} = \frac{\tau \cdot (1/2 - u)}{1 - u}.$$

Hence the elasticity of the demand curve is

$$(15) \quad \frac{d \ln(\theta_j)}{d \ln(p_j)} = -\frac{1 - u(\theta_j)}{\tau(\theta_j) \cdot [1/2 - u(\theta_j)]}.$$

3.10. Efficiency without price-adjustment costs

To provide a benchmark, we describe the case without any price-adjustment cost. Seller k is free to set any price she wants to maximize labor income. That is, she chooses p_k to maximize $p_k \cdot y_k$ subject to the demand constraint (14). Because of the demand constraint, labor income can be written

$$p_k \cdot y_k = p \cdot [1 + \tau(\theta)] \cdot \frac{y_k}{1 + \tau(\theta_k)} = p \cdot [1 + \tau(\theta)] \cdot \frac{1 - u(\theta_k)}{1 + \tau(\theta_k)} \cdot l_k.$$

The variables τ , u , and v are linked by (9), so

$$\frac{1 - u(\theta_k)}{1 + \tau(\theta_k)} = 1 - u(\theta_k) - v(\theta_k).$$

Accordingly, seller k sets local tightness θ_k to minimize $u(\theta_k) + v(\theta_k)$. This is equivalent to choosing the unemployment rate u_k to minimize $u_k + v(u_k)$, where the unemployment and recruiting rates are related by the Beveridge curve (6). The local tightness θ_k and unemployment rate u_k are therefore chosen efficiently: (13) holds, so that $\theta_k = 1$. Here we have just recovered the central efficiency result of Moen (1997).

3.11. Price rigidity

Generally, tightness and unemployment rate are not efficient because prices are somewhat rigid. The local inflation for household k is

$$(16) \quad \pi_k(t) = \frac{\dot{p}_k(t)}{p_k(t)}.$$

Changing prices is costly. As in Rotemberg (1982), households incur a quadratic price-adjustment cost when local inflation departs from normal inflation π^* . The flow disutility caused by prices deviating from the norm is

$$\rho(\pi_k) = \frac{\kappa}{2} \cdot (\pi_k - \pi^*)^2.$$

This quadratic cost appears in the household's utility function.

3.12. People's preferences

People care about two things: their consumption of services and their social status, measured by their relative wealth. In addition people incur a cost from price changes. Each household maximizes the discounted sum of flow utilities,

$$\int_0^\infty e^{-\delta t} \left\{ \ln(c_j(t)) + \sigma \cdot \left[\frac{b_j(t)}{p(t)} - \frac{b(t)}{p(t)} \right] - \frac{\kappa}{2} \cdot [\pi_j - \pi^*]^2 \right\} dt,$$

where $\delta > 0$ is the time discount rate, $\sigma > 0$ indicates concerns for social status, $c_j(t) = \int_0^1 c_{jk}(t) dk$ is total consumption of services, $b_j(t)$ is saving in government bonds, and $b(t) = \int_0^1 b_j(t) dj$ is aggregate wealth in the economy.²

3.13. People's budget constraint

People are subject to a budget constraint. This constraint takes the form of a law of motion of government bond holdings. For household j , the law of motion is

$$\dot{b}_j = i \cdot b_j - \int_0^1 p_k y_{jk} dk + p_j y_j.$$

Because of the matching process and the equalization of prices achieved through directed search, the household's expenditure on services can be rewritten as follows:

$$\int_0^1 p_k y_{jk} dk = \int_0^1 p_k [1 + \tau(\theta_k)] c_{jk} dk = p \cdot [1 + \tau(\theta)] \cdot \int_0^1 c_{jk} dk = p \cdot [1 + \tau(\theta)] \cdot c_j.$$

²In their work on secular stagnation, Michau (2018) and Inagaki, Ono, and Tsuruga (2023) also make the simplifying assumption that the marginal utility of wealth is constant.

Then, because of the matching process, the household's income becomes

$$p_j \cdot y_j = p_j \cdot [1 - u(\theta_j(p_j))] \cdot l_j.$$

Accordingly, the law of motion can be written as

$$(17) \quad \dot{b}_j = i \cdot b_j - p \cdot [1 + \tau(\theta)] \cdot c_j + p_j \cdot [1 - u(\theta_j(p_j))] \cdot l_j.$$

4. Solution of the model

We now solve the Beveridgean model. The main step is to solve household j 's maximization problem, which we do using a Hamiltonian.

4.1. Construction of the Hamiltonian

The Hamiltonian of household j 's problem is

$$\begin{aligned} \mathcal{H}_j = & \ln(c_j) + \sigma \cdot \left[\frac{b_j}{p} - \frac{b}{p} \right] - \frac{\kappa}{2} \cdot [\pi_j - \pi^*]^2 \\ & + \mathcal{A}_j \cdot [i \cdot b_j - p \cdot [1 + \tau] \cdot c_j + p_j \cdot [1 - u(\theta_j(p_j))] \cdot l_j] \\ & + \mathcal{B}_j \cdot \pi_j \cdot p_j. \end{aligned}$$

The control variables are consumption c_j and inflation π_j . The state variables are bond holdings b_j and price level p_j . The costate variables are \mathcal{A}_j , which applies to the law of motion of bond holdings (17), and \mathcal{B}_j , which applies to the law of motion of the price level (16).

We focus on a symmetric solution of model, in which all households behave the same. In this symmetric situation, we can drop the index j .

4.2. First-order condition with respect to consumption

We begin with the first-order condition $d\mathcal{H}_j/dc_j = 0$. It gives

$$\begin{aligned} 1/c_j &= \mathcal{A}_j \cdot p \cdot [1 + \tau] \\ 1/\mathcal{A} &= p \cdot [1 + \tau] \cdot c \\ (18) \quad 1/\mathcal{A} &= p \cdot y. \end{aligned}$$

Taking the log and then time derivative of this last equation yields

$$(19) \quad \begin{aligned} -\ln(\mathcal{A}) &= \ln(p) + \ln(y) \\ -\frac{\dot{\mathcal{A}}}{\mathcal{A}} &= \pi + \frac{\dot{y}}{y}. \end{aligned}$$

4.3. First-order condition with respect to inflation

Next we turn to the first-order condition $d\mathcal{H}_j/d\pi_j = 0$. It yields

$$(20) \quad \begin{aligned} \mathcal{B}_j \cdot p_j &= \kappa \cdot (\pi_j - \pi^*) \\ \mathcal{B} &= \frac{\kappa}{p} \cdot (\pi - \pi^*). \end{aligned}$$

Taking the log and then time derivative of the last equation yields

$$(21) \quad \begin{aligned} \ln(\mathcal{B}) &= \ln(\kappa) - \ln(p) + \ln(\pi - \pi^*). \\ \frac{\dot{\mathcal{B}}}{\mathcal{B}} &= -\pi + \frac{\dot{\pi}}{\pi - \pi^*}. \end{aligned}$$

4.4. First-order condition with respect to saving

The next first-order condition is $d\mathcal{H}_j/db_j = \delta \cdot \mathcal{A}_j - \dot{\mathcal{A}}_j$. It gives

$$\frac{\sigma}{p} + \mathcal{A}_j \cdot i = \delta \cdot \mathcal{A}_j - \dot{\mathcal{A}}_j.$$

Reshuffling the terms yields

$$\frac{\dot{\mathcal{A}}}{\mathcal{A}} = \delta - i - \frac{\sigma}{p \cdot \mathcal{A}}$$

Using $1/(p \cdot \mathcal{A}) = y$, which comes from (18), we obtain

$$(22) \quad \frac{\dot{\mathcal{A}}}{\mathcal{A}} = \delta - (i + \sigma \cdot y)$$

4.5. First-order condition with respect to price

The final first-order condition is $d\mathcal{H}_j/dp_j = \delta \cdot \mathcal{B}_j - \dot{\mathcal{B}}_j$. This condition becomes

$$\mathcal{A}_j \cdot (1 - u_j) \cdot l_j - \mathcal{A}_j \cdot p_j \cdot l_j \cdot u'(\theta_j) \cdot \theta'(p_j) + \mathcal{B}_j \cdot \pi_j = \delta \cdot \mathcal{B}_j - \dot{\mathcal{B}}_j.$$

From the elasticity (10), we have the following derivative:

$$u'(\theta_j) = -\frac{u(\theta_j)}{2 \cdot \theta_j}.$$

And from the elasticity (15), we have

$$\theta'(p_j) = -\frac{\theta_j[1 - u(\theta_j)]}{\tau(\theta_j) \cdot p_j \cdot [1/2 - u(\theta_j)]}.$$

Hence,

$$p_j \cdot u'(\theta_j) \cdot \theta'(p_j) = \frac{1 - u(\theta_j)}{1 - 2u(\theta_j)} \cdot \frac{u(\theta_j)}{\tau(\theta_j)}.$$

Reshuffling terms gives:

$$\begin{aligned} (\delta - \pi_j) \cdot \mathcal{B}_j - \dot{\mathcal{B}}_j &= \mathcal{A}_j \cdot y_j \cdot \left[1 - \frac{u(\theta_j)}{\tau(\theta_j)[1 - 2u(\theta_j)]} \right] \\ -\frac{\dot{\mathcal{B}}}{\mathcal{B}} &= \pi - \delta + \frac{\mathcal{A} \cdot y}{\mathcal{B}} \cdot \left[1 - \frac{u(\theta)}{\tau[1 - 2u(\theta)]} \right]. \end{aligned}$$

Using $y \cdot \mathcal{A} = 1/p$, which comes from (18), and $\mathcal{B} = \kappa(\pi_j - \pi^*)/p$, which comes from (20), we link inflation to the unemployment rate:

$$-\frac{\dot{\mathcal{B}}}{\mathcal{B}} = \pi - \delta + \frac{1}{\kappa} \cdot \frac{1}{\pi - \pi^*} \cdot \left[1 - \frac{u}{\tau[1 - 2u]} \right].$$

Then, using the expression for τ given by (9), we conclude that

$$(23) \quad -\frac{\dot{\mathcal{B}}}{\mathcal{B}} = \pi - \delta + \frac{1}{\kappa} \cdot \frac{1}{\pi - \pi^*} \cdot \left[1 - \frac{u}{v(u)} \cdot \frac{1 - u - v(u)}{1 - 2u} \right],$$

where u is the unemployment rate and $v(u)$ is the recruiting rate, given by (6).

4.6. Aggregate demand: Euler equation

We now derive the aggregate demand from optimal consumption and saving. Combining the first-order conditions (19) and (22), we obtain an Euler equation:

$$(24) \quad \frac{\dot{y}}{y} = (i - \pi + \sigma \cdot y) - \delta.$$

In the Euler equation, $i - \pi$ is the real interest rate, which gives the financial return on saving, while $\sigma \cdot y$ is the marginal rate of substitution between wealth and consumption, which gives the hedonic return on saving. Just as in the New Keynesian model developed

by Michaillat and Saez (2021b), the presence of wealth in the utility function produces a discounted Euler equation (McKay, Nakamura, and Steinsson 2017).

In steady state ($\dot{y} = 0$), equation (24) yields the Euler curve:

$$(25) \quad y = \frac{\delta - i + \pi}{\sigma}$$

The Euler curve gives the steady-state amount of output demanded by households when they optimally save over time. The preference over social status and wealth, σ , determines the slope of the curve.

When we solve the model, we focus on inflation π and unemployment u , so we rewrite the Euler equation in terms of the unemployment rate u instead of output y . Since $y = (1 - u)l$, we have $\dot{y} = -\dot{u} \cdot l$ and

$$\frac{\dot{y}}{y} = -\frac{\dot{u}}{1 - u}.$$

Accordingly, the Euler equation (24) becomes

$$(26) \quad \frac{\dot{u}}{1 - u} = \delta - [i - \pi + \sigma \cdot (1 - u) \cdot l]$$

The Euler curve (25) becomes

$$(27) \quad 1 - u = \frac{\delta - i + \pi}{\sigma \cdot l}$$

4.7. Aggregate supply: Phillips equation

Next we derive the aggregate supply from optimal pricing. Combining the first-order conditions (21) and (23), we obtain a Phillips equation linking inflation to unemployment:

$$(28) \quad \dot{\pi} = \delta \cdot (\pi - \pi^*) - \frac{1}{\kappa} \cdot \left[1 - \frac{u}{v(u)} \cdot \frac{1 - u - v(u)}{1 - 2u} \right].$$

In the Phillips equation, the parameter κ is the price-adjustment cost. The term in square bracket measures the inefficiency of the labor market. When the economy is inefficiently tight, $v > u$ so the term is positive. When the economy is efficient, $v = u$ so the term is zero. When the economy is inefficiently slack, $v < u$ so the term is negative.

In steady state ($\dot{\pi} = 0$), equation (28) yields the Phillips curve:

$$(29) \quad \kappa \cdot \delta \cdot (\pi - \pi^*) = 1 - \frac{u}{v(u)} \cdot \frac{1 - u - v(u)}{1 - 2u}$$

The Phillips curve gives the steady-state inflation chosen by households given the competition they face from other households, and the cost they face in changing prices. The

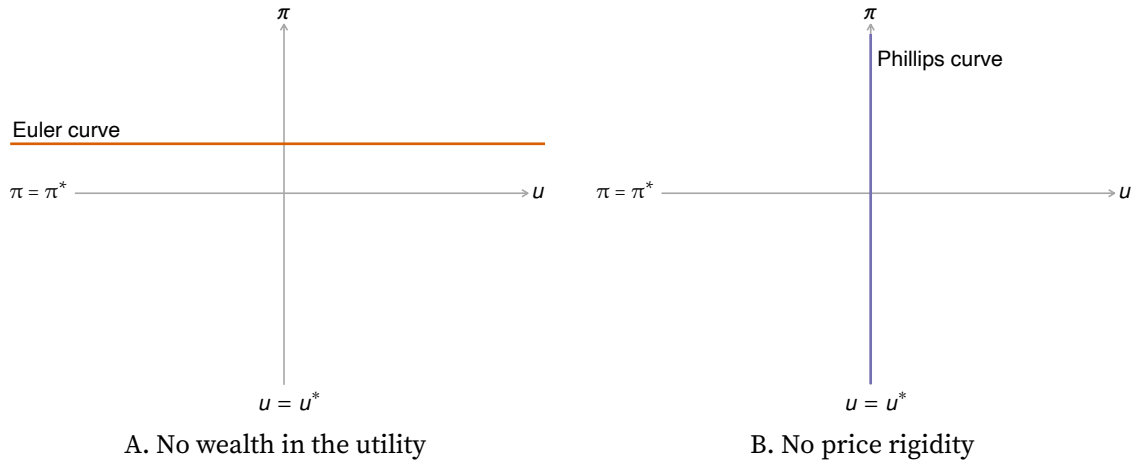


FIGURE 3. Euler and Phillips curves in special cases

A: The Euler curve without wealth in the utility is given by (30). B: The Phillips curve without price rigidity is given by (31).

price-adjustment cost, κ , determines the slope of curve.

4.8. Special cases

Before moving forward, let's pause to examine a few special cases. Consider the Euler and Phillips curves in a standard unemployment-inflation (u, π) plane.

Without wealth in the utility ($\sigma = 0$), the Euler curve (25) would be horizontal:

$$(30) \quad \pi = i - \delta.$$

This curve just imposes that the real interest rate equals the discount rate. Then inflation π is determined one-for-one by the nominal interest rate i . This is the Fisher effect: a higher interest rate leads to higher inflation. The curve is degenerate because it does not involve unemployment u . The curve would also be horizontal in an output-inflation plane, since it does not involve output.

Without price rigidity ($\kappa = 0$), the Phillips curve (29) would be vertical:

$$(31) \quad u = u^*.$$

Indeed, without price rigidity, unemployment and recruiting rates must be equal, so the unemployment rate is efficient ($u = u^*$). The curve would also be vertical in an output-inflation plane, since the unemployment rate pins down the level of output irrespective of inflation: $y = (1 - u^*)l$.

4.9. Divine coincidence

The divine coincidence directly appears in the Phillips equation (29). The equation shows that inflation is on target ($\pi = \pi^*$) if and only if the right-hand side is zero, which happens if and only if unemployment is efficient ($u = v$ so $u = u^*$). Therefore, if the government can bring unemployment to its efficient level, it will also automatically ensure that inflation is on target. In other words, when the government achieves its employment mandate, it also automatically achieves its price mandate.

Both mandates are achieved by moving the Euler curve along the Phillips curve to arrive at the point where $u = u^*$ and $\pi = \pi^*$. This can be done for instance through monetary policy, which affects the nominal interest rate i and therefore the location of the Euler curve (25). The efficient nominal interest rate i^* ensures that inflation is on target ($\pi = \pi^*$) and unemployment is efficient ($u = u^*$). From the Euler curve (25), we obtain an expression for the efficient nominal interest rate i^* :

$$1 - u^* = \frac{\delta - i^* + \pi^*}{\sigma \cdot l}$$

so that the efficient nominal interest rate is

$$(32) \quad i^* = \pi^* + \delta - \sigma \cdot (1 - u^*) \cdot l.$$

When the nominal interest rate is set to i^* , the model admits a steady-state solution in which the divine coincidence prevails: $(\pi, u) = (\pi^*, u^*)$ satisfies both (29) and (25) when i is given by (32).

Such divine steady state exists only when $i^* \geq 0$. If $i^* < 0$, then the divine steady state is not a solution of the model since it would violate the zero lower bound constraint that $i \geq 0$. In that case the central bank would resort to setting $i = 0$.

5. Dynamics around the divine steady state

The model admits a steady-state solution in which the divine coincidence prevails. At that point, inflation is on target and the unemployment rate is efficient. To investigate the dynamics around that steady state, we now linearize the differential equations (26) and (28) around the divine steady state (π^*, u^*) .

5.1. Linearized model around the divine steady state

We begin by introducing the deviations from efficient steady state: $\hat{u} = u - u^*$ and $\hat{\pi} = \pi - \pi^*$. We also allow the nominal interest rate to follow a Taylor rule:

$$(33) \quad i = i^* + \phi(\pi - \pi^*),$$

where $i^* \geq 0$ is the efficient nominal interest rate and $\phi \geq 0$ is the automatic response of the nominal interest rate to inflation. With the Taylor rule, the Euler equation (26) becomes

$$\frac{\dot{u}}{1-u} = \delta + \phi\pi^* - [i^* + (\phi - 1)\pi + \sigma \cdot (1-u) \cdot l]$$

Using the value of i^* from (32), we can simplify the Euler equation:

$$(34) \quad \frac{\dot{u}}{1-u} = \sigma \cdot (u - u^*) \cdot l - (\phi - 1)(\pi - \pi^*).$$

We start by linearizing differential equation (34) around (u^*, π^*) . The linearized version is easy to derive since the differential equation is almost linear:

$$(35) \quad \dot{u} = (1 - u^*) \cdot [\sigma \cdot l \cdot \hat{u} - (\phi - 1)\hat{\pi}].$$

The linearized version of differential equation (28) around (u^*, π^*) is a little bit more complicated to derive. The key is to find the partial derivative of

$$\mathcal{P}(u) = -\frac{1}{\kappa} \cdot \left[1 - \frac{u}{v(u)} \cdot \frac{1-u-v(u)}{1-2u} \right]$$

with respect to u at u^* . This will be the coefficient in front of \hat{u} in the linearized equation. To do that, we need the derivative of

$$\mathcal{Q}(u) = \frac{u}{v(u)} \cdot \frac{1-u-v(u)}{1-2u}$$

with respect to u at u^* . From (6), we know that the elasticity of $v(u)$ with respect to u is $d\ln(v)/d\ln(u) = -1$, so we have

$$\frac{d\ln(\mathcal{Q})}{d\ln(u)} = 1 + 1 + \frac{-u+v}{1-u-v} + \frac{2u}{1-2u}.$$

When $u = u^*$, we also have $u = v$, so the elasticity simplifies to

$$\frac{d\ln(\mathcal{Q})}{d\ln(u)} = 2 \left[1 + \frac{u^*}{1-2u^*} \right].$$

Moreover, $Q(u^*) = 1$ so we have the following derivative:

$$\frac{dQ}{du} = \frac{2}{u^*} \left[1 + \frac{u^*}{1 - 2u^*} \right].$$

Since $\mathcal{P}'(u) = \mathcal{Q}'(u)/\kappa$, we finally get

$$\frac{dQ}{du} = \frac{2}{\kappa u^*} \cdot \frac{1 - u^*}{1 - 2u^*}.$$

Accordingly, the linearized Phillips curve is

$$(36) \quad \dot{\pi} = \delta \hat{\pi} + \frac{2}{\kappa} \cdot \frac{1 - u^*}{(1 - 2u^*)u^*} \cdot \hat{u}$$

5.2. Classification of the linearized model

The Euler-Phillips system (26)-(28) is nonlinear, but we can determine its properties around the divine steady state from its linearized form. Combining (35) and (36), we find that around the divine steady state $[u, \pi] = [u^*, \pi^*]$, the linearized Euler-Phillips system is

$$(37) \quad \begin{bmatrix} \dot{u}(t) \\ \dot{\pi}(t) \end{bmatrix} = \begin{bmatrix} \sigma y^* & -(\phi - 1)(1 - u^*) \\ 2(1 - u^*)/[\kappa u^*(1 - 2u^*)] & \delta \end{bmatrix} \begin{bmatrix} \hat{u}(t) \\ \hat{\pi}(t) \end{bmatrix}.$$

We denote by \mathbf{M} the matrix in (37), and by $\mu_1 \in \mathbb{C}$ and $\mu_2 \in \mathbb{C}$ the two eigenvalues of \mathbf{M} , assumed to be distinct.

We classify the Euler-Phillips system from the trace and determinant of \mathbf{M} (Hirsch, Smale, and Devaney 2013, pp. 61–64). The classification relies on the property that $\text{tr}(\mathbf{M}) = \mu_1 + \mu_2$ and $\det(\mathbf{M}) = \mu_1 \mu_2$. Using (37), we compute the trace and determinant of \mathbf{M} :

$$\begin{aligned} \text{tr}(\mathbf{M}) &= \delta + \sigma y^* \\ \det(\mathbf{M}) &= \delta \sigma y^* + \frac{2(\phi - 1)}{\kappa} \cdot \frac{(1 - u^*)^2}{u^*(1 - 2u^*)}. \end{aligned}$$

Clearly, $\text{tr}(\mathbf{M}) > \delta > 0$. Further, since $\phi \geq 0$, we have $\phi - 1 \geq -1$, so that

$$\det(\mathbf{M}) \geq \delta \sigma y^* - \frac{2}{\kappa} \frac{(1 - u^*)^2}{u^*(1 - 2u^*)}.$$

Just as in Michailat and Saez (2021b), we assume that the marginal utility of wealth is large enough to ensure that the determinant is positive:

$$(38) \quad \sigma \geq \frac{2}{\kappa \delta l} \cdot \frac{1 - u^*}{u^*(1 - 2u^*)}.$$

Under this assumption, $\text{tr}(\mathbf{M}) > 0$ and $\det(\mathbf{M}) > 0$, so the Euler-Phillips system is a source for any $\phi \geq 0$. When prices are more flexible (lower κ), the marginal utility of wealth needs to be larger to ensure that the determinant is positive and the system is a source.

Indeed $\det(\mathbf{M}) > 0$ indicates that μ_1 and μ_2 are either real, nonzero, and of the same sign; or complex conjugates. Since in addition $\text{tr}(\mathbf{M}) > 0$, μ_1 and μ_2 must be either real and positive, or complex with a positive real part. Indeed, if μ_1 and μ_2 were real and negative, $\text{tr}(\mathbf{M}) = \mu_1 + \mu_2 < 0$. If they were complex with a negative real part, $\text{tr}(\mathbf{M}) = \mu_1 + \bar{\mu}_1 = 2 \text{Re}(\mu_1) < 0$.

When μ_1 and μ_2 are real and positive, the solution of the linearized system is

$$(39) \quad [\hat{u}(t), \hat{\pi}(t)] = x_1 e^{\mu_1 t} \mathbf{v}_1 + x_2 e^{\mu_2 t} \mathbf{v}_2,$$

where $\mathbf{v}_1 \in \mathbb{R}^2$ and $\mathbf{v}_2 \in \mathbb{R}^2$ are the linearly independent eigenvectors respectively associated with the eigenvalues μ_1 and μ_2 , and $x_1 \in \mathbb{R}$ and $x_2 \in \mathbb{R}$ are constants determined by the terminal condition (Hirsch, Smale, and Devaney 2013, p. 35). From (39), we see that the Euler-Phillips system is a source when $\mu_1 > 0$ and $\mu_2 > 0$. The solutions start at 0 when $t \rightarrow -\infty$ and go to infinity parallel to \mathbf{v}_2 when $t \rightarrow +\infty$.

When μ_1 and μ_2 are complex conjugates with a positive real part, we write the eigenvalues as $\mu_1 = \mu + i\beta$ and $\mu_2 = \mu - i\beta$ with $\mu > 0$. We also write the eigenvector associated with μ_1 as $\mathbf{v}_1 + i\mathbf{v}_2$, where the vectors $\mathbf{v}_1 \in \mathbb{R}^2$ and $\mathbf{v}_2 \in \mathbb{R}^2$ are linearly independent. Then the solution takes a more complicated form:

$$\begin{bmatrix} \hat{u}(t) \\ \hat{\pi}(t) \end{bmatrix} = e^{\mu t} [\mathbf{v}_1, \mathbf{v}_2] \begin{bmatrix} \cos(\beta t) & \sin(\beta t) \\ -\sin(\beta t) & \cos(\beta t) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix},$$

where $[\mathbf{v}_1, \mathbf{v}_2] \in \mathbb{R}^{2 \times 2}$ is a 2×2 matrix, and $x_1 \in \mathbb{R}$ and $x_2 \in \mathbb{R}$ are constants determined by the terminal condition (Hirsch, Smale, and Devaney 2013, pp. 44–55). These solutions wind periodically around the steady state, moving away from it as $t \rightarrow +\infty$. Hence, the Euler-Phillips system is a spiral source.

Overall, when the marginal utility of wealth is large enough (equation (38)), the linearized model is source whether monetary policy is active ($\phi > 1$) or passive ($0 \geq \phi \geq 1$). This is just as in the New Keynesian model. That model is a source irrespective of monetary policy when the marginal utility of wealth is large enough (Michaillat and Saez 2021b, proposition 1).

5.3. Local uniqueness of the solution

We assume that the marginal utility of wealth is large enough: equation (38) holds. Therefore, the Euler-Phillips system is a source, which implies that the solution of the model is always locally unique—even when monetary policy is passive. The only solution in

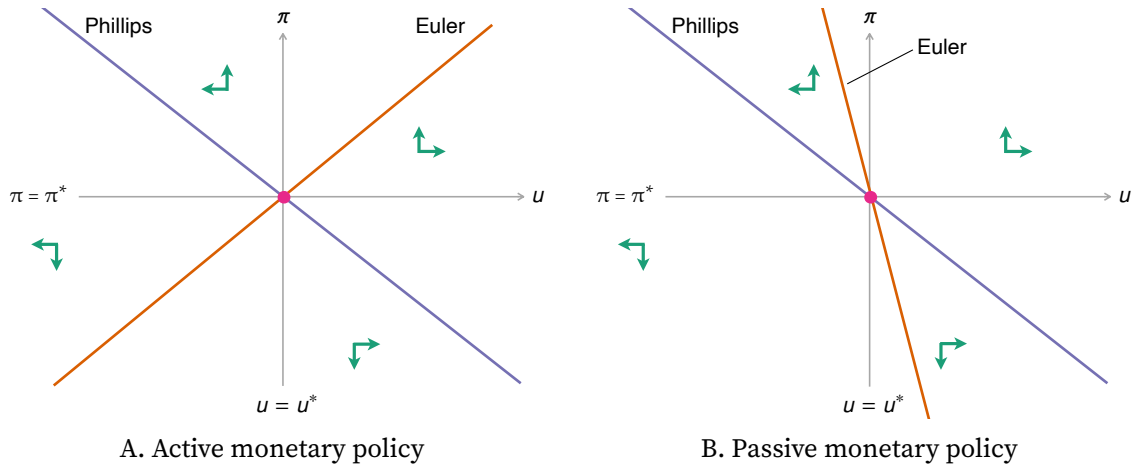


FIGURE 4. Phase diagrams of the linearized model

The figure displays phase diagrams for the dynamical system generated by the linearized Euler equation (35) and Phillips equation (36). The variable u is unemployment; u^* is the efficient rate of unemployment; π is inflation; π^* is the inflation target. The Euler curve is the locus $\dot{u} = 0$, given by (41). The Phillips curve is the locus $\dot{\pi} = 0$, given by (40). The monetary-policy rate is given by $i = i^* + \phi(\pi - \pi^*)$: when monetary policy is active, $1 < \phi$; when monetary policy is passive, $0 \leq \phi \leq 1$. The figure shows that the linearized model is a source whether monetary policy is active or passive.

the vicinity of the divine steady state is to jump to the steady state and stay there. If the economy jumped somewhere else, unemployment or inflation would diverge away from the steady state.

Unlike in the New Keynesian model, indeterminacy is never a risk, so the central bank does not need to worry about how strongly its policy rate responds to inflation. The central bank can even follow an interest-rate peg without creating indeterminacy.

5.4. Phase diagram

We now construct the phase diagrams of the linearized model to understand its dynamics better.³ The diagrams are displayed in figure 4.

We begin with the linearized Phillips equation (36), which gives $\dot{\pi}$. First, we plot the locus $\dot{\pi} = 0$, which is the linearized Phillips curve. The locus is given by

$$(40) \quad \hat{\pi} = -\frac{2}{\delta \kappa u^*} \cdot \frac{1 - u^*}{1 - 2u^*} \cdot \hat{u}.$$

The Phillips curve is downward sloping, and goes through the point $[u = u^*, \pi = \pi^*]$. Second, we plot the arrows giving the directions of the trajectories solving the Euler-Phillips system. The sign of $\dot{\pi}$ is given by (36): any point above the Phillips curve (where

³Michaillat and Saez (2021b) show for instance how to use the phases diagrams to study ZLB episodes of finite duration and forward guidance. They also show how to construct sample solutions to the Euler-Phillips system using the phase diagrams.

$\dot{\pi} = 0$) has $\dot{\pi} > 0$, and any point below the curve has $\dot{\pi} < 0$. So inflation is rising above the Phillips curve and falling below it.

We next turn to the linearized Euler equation (35), which gives \dot{u} . We plot the locus $\dot{u} = 0$, which is the linearized Euler curve. The locus is given by

$$(41) \quad \hat{u} = \frac{\phi - 1}{\sigma \cdot l} \cdot \hat{\pi}.$$

The Euler curve goes through the point $[u = u^*, \pi = \pi^*]$. It is downward sloping if $\phi < 1$, vertical if $\phi = 1$, and upward sloping if $\phi > 1$.

Next we use the Euler equation (35) to determine the sign of \dot{u} . We first consider an active monetary policy ($\phi > 1$), as showed in figure 4A. Any point above the Euler curve has $\dot{u} < 0$, and any point below it has $\dot{u} > 0$. Hence, in all four quadrants of the phase diagram, the trajectories move away from the steady state. We conclude that the Euler-Phillips system is a source when monetary policy is active.

Second, we consider a passive monetary policy ($\phi \in [0, 1)$), as showed in figure 4B. Now any point above the Euler curve has $\dot{u} > 0$, and any point below it has $\dot{u} < 0$. Nevertheless, in all four quadrants of the phase diagram, the trajectories move away from the steady state. We conclude that the Euler-Phillips system remains a source when monetary policy is passive.

The phase diagrams also illustrate the origin of the condition (38) on the marginal utility of wealth. The Euler-Phillips system remains a source with passive monetary policy as long as the Euler curve is steeper than the Phillips curve in figure 4B. The Euler curve is the most flat with an interest-rate peg ($\phi = 0$), and then its slope is just the marginal utility of wealth. Thus, the marginal utility is required to be above a certain level—which is given by (38).

6. Response to shocks and application to the pandemic

Next, we use the linearized model to study business-cycle shocks. Since the linearized model is a source, the solution of the model is given by the intersection of the linearized Euler curve and linearized Phillips curve. In response to unexpected, permanent shocks, the solution just jumps from the old intersection to the new intersection. Although the model is dynamic, we can therefore study the response to shocks by comparative statics.⁴

6.1. Intuitions behind the Euler and Phillips curves

In this section we use comparative statics to study the effect of macroeconomic shocks in the model. Since we rely on movements of and along the Euler and Phillips curves, it is

⁴The same is true in the New Keynesian model (Michaillat and Saez 2021b, section 5).

key to understand what these curves represent—to understand the trade-offs they capture.

The Euler curve (41) imposes that the rate of return on wealth equals the time discount rate—otherwise households would not keep their consumption constant. With wealth in the utility function, the returns on wealth are not only financial but also hedonic. As showed by (24), the total rate of return is the real interest rate $r = i - \pi = i_0 + (\phi - 1)\pi$ plus the hedonic rate of return $\sigma y = \sigma(1 - u)l$.

The Euler curve imposes that the real interest rate, which depends on inflation, plus the hedonic rate of return on wealth, which depends on unemployment, equal the time discount rate. It thus links inflation to unemployment. When the real interest rate is higher, people have a financial incentive to save more and postpone consumption. They keep consumption constant only if the hedonic returns on wealth fall enough to offset the increase in financial returns: this requires the unemployment rate to rise. As a result, the Euler curve describes unemployment as an increasing function of the real interest rate. When monetary policy is active, the real rate is an increasing function of inflation. Then, the Euler curve describes unemployment as an increasing function of inflation. On the other hand, when monetary policy is passive, the real rate is a decreasing function of inflation. Then, the Euler curve describes unemployment as a decreasing function of inflation.

The Phillips curve (40) explains why inflation is above target whenever the unemployment rate is inefficiently low, and below target whenever the unemployment rate is inefficiently high. When inflation is above target, a seller can reduce its price-adjustment cost by lowering its rate of inflation. Since pricing is optimal, however, there cannot exist any profitable deviation from the current situation. This means that the seller must incur a commensurate cost when it lowers its rate of inflation. With lower inflation, the price charged by the seller drops relative to the prices of other sellers. The absence of profitable deviation imposes that the price reduction must be costly, so the price must already be below the profit-maximizing price. And since local tightness is inversely related to the seller's price, local tightness must be above the profit-maximizing tightness, which is just the efficient tightness of 1 (section 3.10). In other words, the Phillips curve says that when inflation is above target, tightness must be inefficiently high and the unemployment rate must be inefficiently low.

The same logic holds when inflation is below target. Then a seller can reduce its price-adjustment cost by raising its rate of inflation. With higher inflation, the price charged by the seller rises relative to the prices of other sellers. The absence of profitable deviation imposes that the price increase must be costly, so the price must already be above the profit-maximizing price. And since local tightness is inversely related to the seller's price, local tightness must be below the profit-maximizing tightness, which is the efficient tightness of 1. So the Phillips curve says that when inflation is below target,

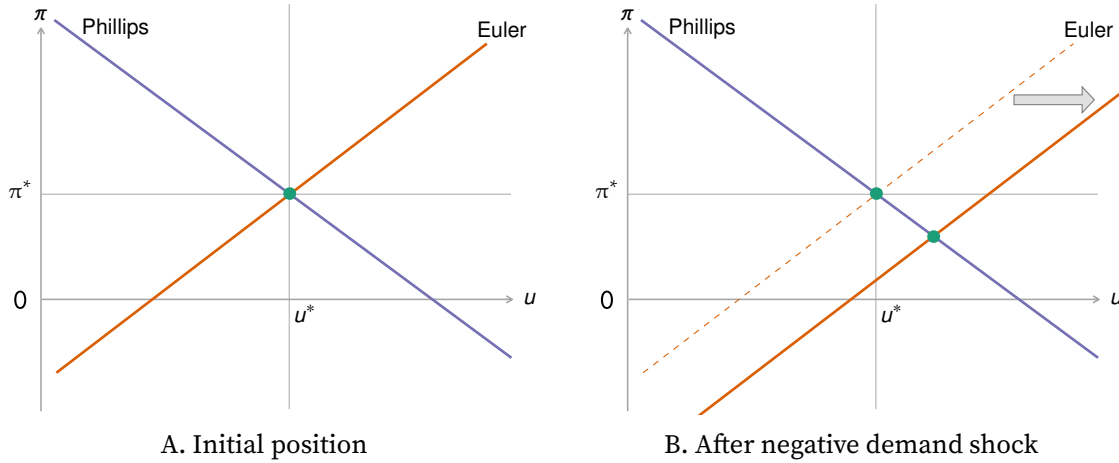


FIGURE 5. Response of the linearized model to a negative demand shock

A: The Phillips curve is given by equation (40). The Euler curve is given by equation (41). The variable u is unemployment; u^* is the efficient rate of unemployment; π is inflation; π^* is the inflation target. The intersection of the Phillips and Euler curves gives the solution of the model when monetary policy is given by (33) with $\phi > 1$. B: The shock is an unexpected permanent decrease in the discount rate (δ), an unexpected permanent increase in the marginal utility of wealth (σ), or an unexpected permanent increase in the nominal interest rate (i). The graph shows that unemployment increases while inflation decreases after the shock.

tightness must be inefficiently low and the unemployment rate must be inefficiently high.

6.2. Typical recession: negative demand shock

We consider first a traditional business-cycle shock: an aggregate-demand shock, which shifts the Euler curve. Such shock could be caused by a change in sentiment, reflected in a different marginal utility of wealth σ or different discount rate δ . A high σ for example indicates a low desire for consumption and therefore produces a low aggregate demand. The shock could also be a change in monetary policy, affecting the nominal interest rate i . A high i makes it more appealing to save and therefore dampens aggregate demand.

We begin by looking at the effect of a negative aggregate-demand shock under the standard assumption that monetary policy is active. In the linearized model, the negative demand shock leads to an outward shift of the Euler curve (figure 5). In response to the negative shock, unemployment is higher, the unemployment gap is also higher, and inflation is lower. The economy is moving along the Phillips curve so there is a negative correlation between unemployment and inflation.

If monetary policy is passive instead of active, the effects of the negative aggregate-demand shock remain the same (figure 6A). The negative demand shock leads to an outward shift of the Euler curve, which raises unemployment and the unemployment gap, and lowers inflation.

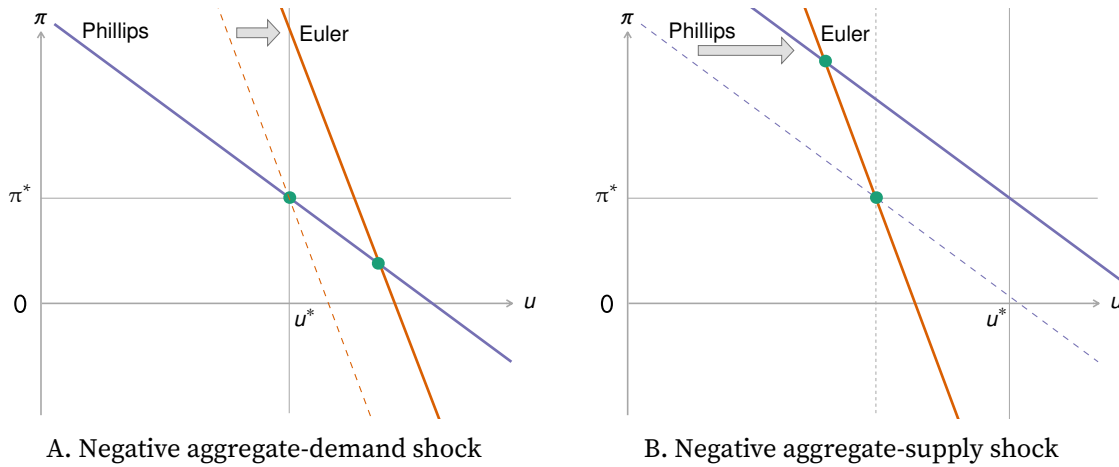


FIGURE 6. Response of the linearized model to shocks under passive monetary policy

A: The panel reproduces figure 5B when monetary policy is given by (33) with $\phi \in [0, 1]$ instead of $\phi > 1$. The responses of unemployment and inflation to the negative aggregate-demand shock remain the same. B: The panel reproduces figure 7B when monetary policy is given by (33) with $\phi \in [0, 1]$ instead of $\phi > 1$. The response of inflation to the negative aggregate-supply shock remains the same, but the response of unemployment changes. Unemployment falls after the shock instead of rising.

6.3. Pandemic recession: negative supply shock

We consider next an unusual business-cycle shock: an aggregate-supply shock, which shifts the Beveridge curve, and then the Phillips curve. Such shock shifts the Beveridge curve (6), so it involves a change in either the job-separation rate s or the matching efficacy ω . Both an increase in job separations and a decrease in matching efficacy shift the Beveridge curve outward, which raises the efficient unemployment rate u^* .

We begin by looking at the effect of a negative aggregate-supply shock under the standard assumption that monetary policy is active. In the linearized model, a negative supply shock leads to an outward shift of the Phillips curve (figure 7). In response to the negative shock, unemployment is higher, and inflation is higher. But the key is that the unemployment gap is lower (it has become negative) and inflation is higher. Indeed the efficient unemployment rate has increased more than actual unemployment, so the unemployment rate is now inefficiently low. Such excessive tightness leads to higher inflation.

If monetary policy is passive instead of active, the effects of the negative aggregate-supply shock are mostly but not entirely the same (figure 6A). The negative supply shock leads to an outward shift of the Euler curve, which raises inflation. Unlike with active monetary policy, here the unemployment rate falls after the shift of the Euler curve. Since the unemployment rate decreases but the efficient unemployment rate increases, the unemployment gap unambiguously increases. Just like with active monetary policy, the flare-up in inflation is associated with an inefficiently tight labor market.

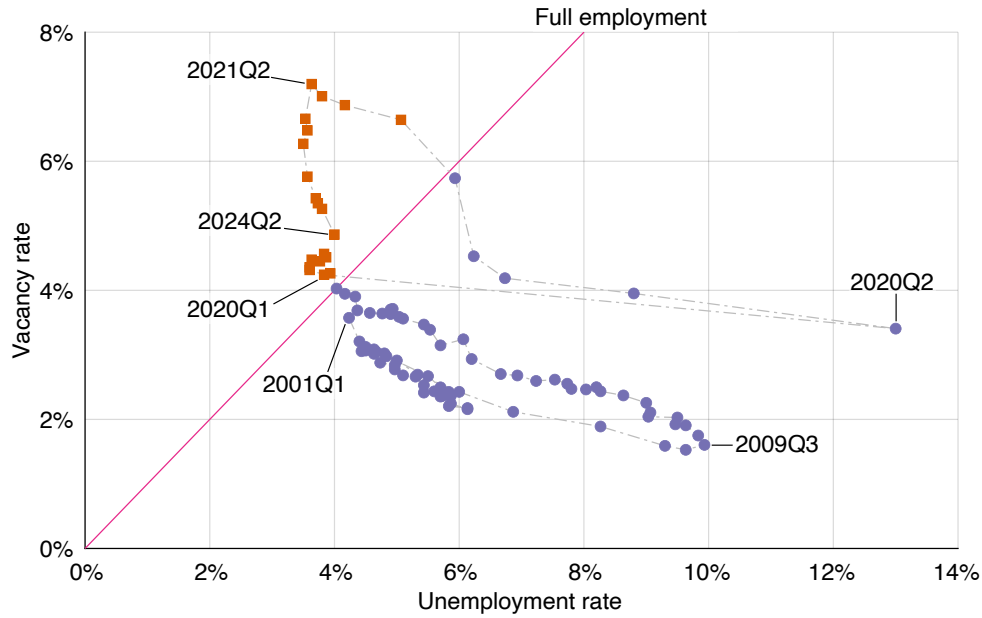
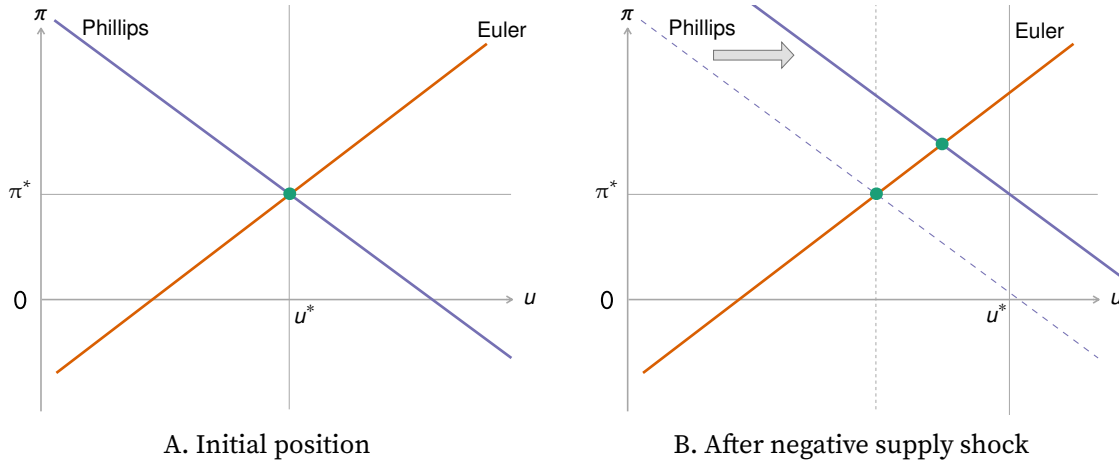


FIGURE 7. Response of the linearized model to a negative supply shock

A: The Phillips curve is given by equation (40). The Euler curve is given by equation (41). The variable u is unemployment; u^* is the efficient rate of unemployment; π is inflation; π^* is the inflation target. The intersection of the Phillips and Euler curves gives the solution of the model when monetary policy is given by (33) with $\phi > 1$. B: The shock is an unexpected permanent decrease in the matching efficacy (ω) or an unexpected permanent increase in the job-separation rate (s). The graph shows that both unemployment and inflation increase after the shock. C: The US Beveridge curve was produced by Michailat and Saez (2024, figure 10).

If the Euler curve remains the same after the outward shift of the Phillips curve—for example because the central bank is not aware of it—then we obtain a burst of inflation after the adverse shock to the Phillips curve (figure 7B). Since the US Beveridge curve has shifted dramatically outward in the aftermath of the coronavirus pandemic (figure 7C), the flare-up in inflation in 2021–2023 might partly result from this dramatic shift.

7. Adding a kink to the Phillips curve

This section shows how a kink can be added to the Phillips curve. In the model, the kink appears because wage cuts are more painful to workers than price increases are to consumers.

7.1. Asymmetric price-adjustment cost

So far we have assumed that the price-adjustment cost is symmetric, just as in the original work by Rotemberg (1982). It is not unreasonable to assume that the price-adjustment cost is asymmetric, however, because in the model the cost of a price increase and the cost of a price decrease capture different phenomena, affecting different people.

When prices fall, or increase less than normal, workers in household k feel short-changed. Indeed, the price p_k is their hourly salary. And Bewley (1999, 2005) has shown that workers' morale dips when their wages do not grow as expected. So here we assume that workers incur a quadratic cost when wage growth $\pi_k(t)$ falls short of the normal growth π^* .

When prices rise, or increase more than normal, it is the customers of household k that are unhappy. Shiller (1997) shows that higher-than-normal inflation upsets customers, who feel unfairly treated when they go to the store. In fact, such inflation makes customers angry at sellers. Here we assume that sellers internalize the anger of customers directed at them and incur another quadratic cost when wage growth $\pi_k(t)$ is above the normal growth π^* .⁵

Formally, if $\pi_k > \pi^*$, the flow disutility caused by prices deviating from the norm is

$$\rho(\pi_k) = \frac{\kappa^+}{2} \cdot (\pi_k - \pi^*)^2.$$

This cost reflects the fact that higher-than-normal prices upset customers. If $\pi_k < \pi^*$, the flow disutility caused by prices deviating from the norm is

$$\rho(\pi_k) = \frac{\kappa^-}{2} \cdot (\pi_k - \pi^*)^2.$$

⁵This is a reduced-form way to capture how sellers internalize customers' anger at price increases. Eyster, Madarasz, and Michaillat (2021) provide a complete model of why price increases anger customers, and how firms internalize such anger.

This cost reflects the fact that lower-than-normal wages damage workers' morale.

The parameters $\kappa^+ > 0$ and $\kappa^- > 0$ govern the price-adjustment costs. Since the costs come from different sources when inflation is too high and too low, we allow the parameters κ^+ and κ^- to be different. We postulate that workers' anger at wage cuts is stronger than customers' anger at price increases, so we assume

$$(42) \quad \kappa^- > \kappa^+.$$

This gap between κ^+ and κ^- will generate a kink in the Phillips curve.⁶

Even with $\kappa^- > \kappa^+$, the cost function $\rho(\pi)$ is continuous and differentiable at π^* since

$$\begin{aligned} \lim_{\pi \rightarrow (\pi^*)^+} \rho(\pi) &= \lim_{\pi \rightarrow (\pi^*)^-} \rho(\pi) = 0 \\ \lim_{\pi \rightarrow (\pi^*)^+} \rho'(\pi) &= \lim_{\pi \rightarrow (\pi^*)^-} \rho'(\pi) = 0, \end{aligned}$$

so that we can complete the definition of the cost function at π^* by $\rho(\pi^*) = 0$ and $\rho'(\pi^*) = 0$.

7.2. Kink in the Phillips curve

Although we assume that the price-adjustment cost is asymmetric, all the derivations remain the same. As a result, the Phillips equation (28) is now defined piecewise. For $\pi < \pi^*$, the Phillips equation is

$$(43) \quad \dot{\pi} = \delta \cdot (\pi - \pi^*) - \frac{1}{\kappa^-} \cdot \left[1 - \frac{u}{v(u)} \cdot \frac{1 - u - v(u)}{1 - 2u} \right].$$

And for $\pi > \pi^*$, the Phillips equation is

$$(44) \quad \dot{\pi} = \delta \cdot (\pi - \pi^*) - \frac{1}{\kappa^+} \cdot \left[1 - \frac{u}{v(u)} \cdot \frac{1 - u - v(u)}{1 - 2u} \right].$$

Accordingly, the linearized Phillips equation (36) is also defined piecewise. For $\pi < \pi^*$, the linearized Phillips equation is

$$(45) \quad \dot{\pi} = \delta \hat{\pi} + \frac{2}{\kappa^-} \cdot \frac{1 - u^*}{(1 - 2u^*)u^*} \cdot \hat{u}.$$

⁶The assumption of symmetric, quadratic price-adjustment costs is extremely popular in the New Keynesian literature, but a few papers use asymmetric price-adjustment costs—albeit with a different functional form than here. For instance, Cao, Luo, and Nie (2023, equation (14)) assume an asymmetric price-adjustment cost to remove episodes of extreme deflation from their model. Cao, Diba, and Lee (2023, p. 6) use the same asymmetric price-adjustment cost to resolve the anomalies of the New Keynesian model at the zero lower bound.

And for $\pi > \pi^*$, the linearized Phillips equation is

$$(46) \quad \dot{\pi} = \delta \hat{\pi} + \frac{2}{\kappa^+} \cdot \frac{1 - u^*}{(1 - 2u^*)u^*} \cdot \hat{u}.$$

Because of the shape of the linearized Phillips equation, the linearized Phillips curve, obtained by setting $\dot{\pi} = 0$ in (45) and (46), has a kink at the point $\pi = \pi^*$ and $u = u^*$. For $\hat{\pi} < 0$ and $\hat{u} > 0$, the linearized Phillips curve is

$$(47) \quad \hat{\pi} = -\frac{2}{\delta \kappa^-} \cdot \frac{1 - u^*}{(1 - 2u^*)u^*} \cdot \hat{u}.$$

And for $\hat{\pi} > 0$ and $\hat{u} < 0$, the linearized Phillips curve is

$$(48) \quad \hat{\pi} = -\frac{2}{\delta \kappa^+} \cdot \frac{1 - u^*}{(1 - 2u^*)u^*} \cdot \hat{u}.$$

Since $\kappa^- > \kappa^+$, the branch (48) of the linearized Phillips curve is steeper than the branch (47). The linearized Phillips curve is therefore downward sloping with a kink at the point $[u = u^*, \pi = \pi^*]$.

Because $\kappa^- > \kappa^+$, the Phillips curve is steeper when $u < u^*$ and flatter when $u > u^*$. Previous papers have proposed models of a kinked Phillips curve. Benigno and Eggertsson (2023) and Gitti (2024) develop New Keynesian models in which the kink arises from differential wage rigidity: wages are rigid when tightness is below 1 but flexible when tightness is above 1. In an old-fashioned Keynesian model, Chiarella et al. (2003) obtain a kink from a related assumption: that nominal wages are rigid downward. Here the kink comes from a different assumption: that wage decreases are more costly to producers than price increases. The novelty of our model is that it guarantees that the kink occurs at the efficient unemployment rate—it guarantees that the divine coincidence holds.

Despite the kink, the phase diagram of the linearized model retains the same properties (figure 8). Indeed, even with the kink, the arrows giving the directions of the trajectories solving the Euler-Phillips system are the same. The sign of $\dot{\pi}$ is given by (46) and (45). It remains the case that any point above the Phillips curve has $\dot{\pi} > 0$, and any point below the curve has $\dot{\pi} < 0$. So inflation is rising above the Phillips curve and falling below it, even with the kink. As a result, the linearized model remains a source, whether monetary policy is active (figure 8A) or passive (figure 8B).

7.3. Implications of the kink

Since the linearized model remains a source with the kink, the solution of the linearized model remains given by the intersection of the linearized Euler curve and linearized Phillips curve. In response to unexpected, permanent shocks, the solution just jumps

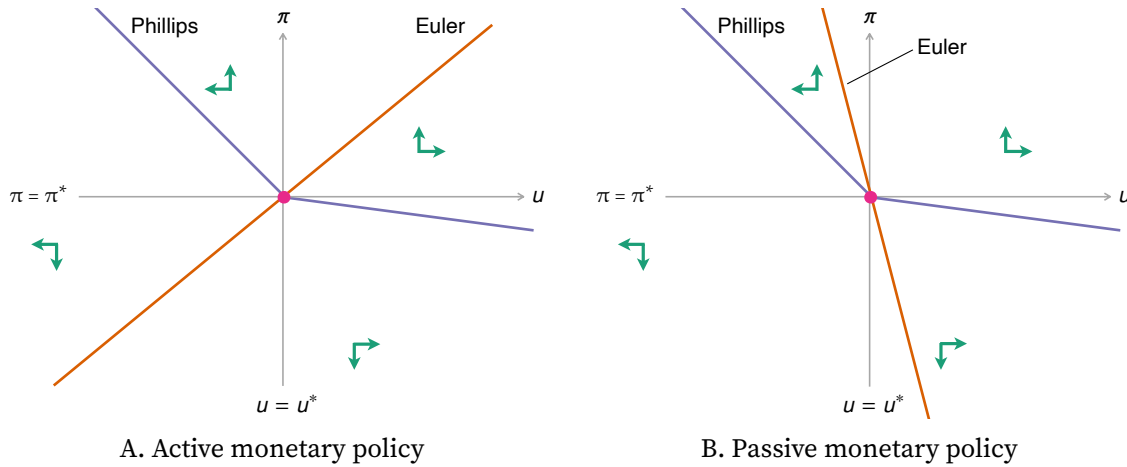


FIGURE 8. Phase diagrams of the linearized model with a kinked Phillips curve

This figure reproduces figure 4 when the Phillips equation has a kink. Instead of being given by (36), the linearized Phillips equation is given by (45) when $\pi < \pi^*$ and by (46) when $\pi > \pi^*$. Even with a kinked Phillips curve, the linearized model is a source whether monetary policy is active or passive.

from the old intersection to the new intersection. We can therefore continue to study the response to shocks by comparative statics.

While the kink in the Phillips curve does not change the comparative statics qualitatively, it does have quantitative consequences. It implies that, starting from a divine situation, a negative aggregate-demand shock will only have a small negative effect on inflation. This is because the economy is moving along the flat branch of the Phillips curve (figure 9A. By contrast, a negative aggregate-supply shock will have a large positive effect on inflation. This is because then the economy is moving along the steep branch of the Phillips curve (figure 9B.

More generally, when the economy is inefficiently tight ($\hat{u} < 0$), any shock tends to generate larger movements in inflation and smaller movements in unemployment. when the economy is inefficiently slack ($\hat{u} > 0$), any shock tends to generate smaller movements in inflation and larger movements in unemployment.

8. Conclusion

8.1. Summary

The divine coincidence has recently appeared in US data. That is, inflation appears on target whenever the labor market is efficient (figure 1). To explain this phenomenon, we propose a model of the divine coincidence. The key is to combine Moen (1997)'s directed search with Rotemberg (1982)'s price-adjustment costs. These assumptions generate a nonvertical Phillips curve that goes through the point of divine coincidence—where

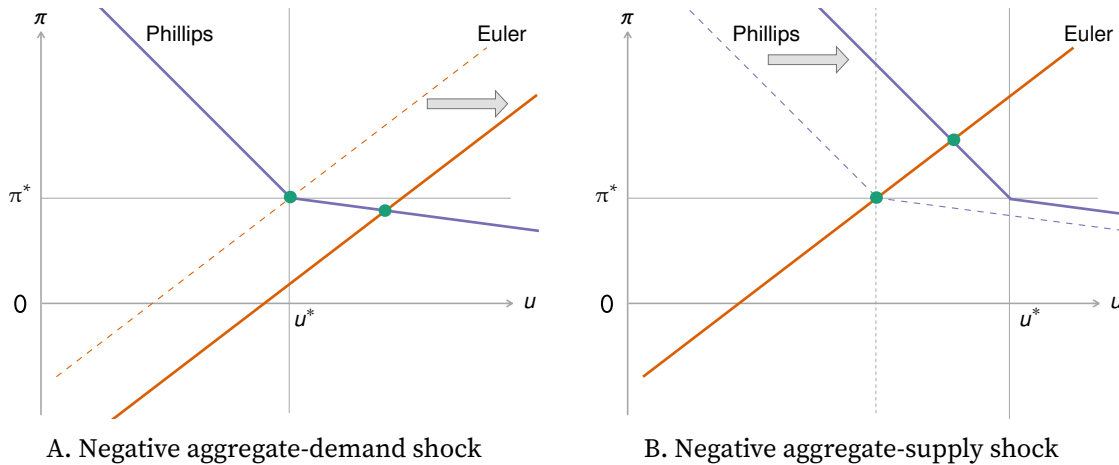


FIGURE 9. Response of the linearized model to shocks with a kinked Phillips curve

This figure reproduces figure 5B and figure 7B when the Phillips equation has a kink. Instead of being given by (40), the linearized Phillips curve is given by (47) when $\pi < \pi^*$ and by (48) when $\pi > \pi^*$. Despite the kinked Phillips curve, unemployment and inflation have the same qualitative responses to shocks.

unemployment is efficient and inflation is on target.

8.2. Positive implications

The model predicts that inflation rises above target whenever the labor market is inefficiently tight. The model therefore provides an explanation for the flare-up in inflation in 2021–2023. Indeed, the US labor market has been inefficiently tight in the aftermath of the coronavirus pandemic—in fact tighter than at any point since the end of World War 2 (Michaillat and Saez 2024). Such excessive tightness must have fueled the flare-up in inflation, especially if the Phillips curve is kinked: steeper when the economy is inefficiently tight and flatter when it is inefficiently slack. The model produces a kinked Phillips curve when we assume that wage cuts are more costly to producers than price hikes.

8.3. Normative implications

The divine coincidence matters a great deal for policy because it implies that the full-employment and price-stability mandates of the Federal Reserve coincide. According to the model, if the Fed can maintain the economy at full employment, it will also maintain inflation on target, thus satisfying its dual mandate. And to maintain the economy at full employment, the Fed can rely on the sufficient-statistic formula developed by Michaillat and Saez (2022, equation (31)). From any initial nominal interest rate and unemployment gap, the formula gives the change in nominal interest rate required to bring the unemployment gap to zero.

8.4. Choice of target variables

In the model, since the full-employment and price-stability mandates of the Fed coincide, aiming for the inflation target or the efficient unemployment rate is completely equivalent. In practice, however, the Fed should target the variable that is the most volatile—to observe more clearly departures from the dual mandate.

The slope of the Phillips curve in turn determines which of inflation and unemployment is the most volatile. If the Phillips curve is steep, aggregate-demand shocks will mostly generate movements in inflation, and targeting inflation will be easier. By contrast, if the Phillips curve is flat, aggregate-demand shocks will mostly generate movements in unemployment, and targeting unemployment or tightness will be easier.

In the United States, the Phillips curve appears kinked: flat when unemployment is inefficiently high, and steep when unemployment is inefficiently low. The implication is that the Fed should target the efficient unemployment rate u^* when the economy is inefficiently slack; and it should target an inflation rate of 2% when the economy is inefficiently hot.

8.5. Other applications of Moen-Rotemberg pricing

In this paper we combine Moen (1997)’s directed search with Rotemberg (1982)’s price-adjustment costs to generate a nonvertical Phillips curve. Such Moen-Rotemberg pricing could have applications in other models as well. Matching models—whether of the labor market, of the product market, or of the entire economy—require some price rigidity to generate realistic fluctuations in market tightness (Shimer 2005; Michaillat and Saez 2015). Various pricing mechanisms have been developed to generate such rigidity: fixed price norms (Shimer 2004; Hall 2005); credible bargaining (Hall and Milgrom 2008; Petrosky-Nadeau and Zhang 2021); staggered Nash bargaining (Gertler and Trigari 2009); and price norms that are rigid functions of the parameters (Blanchard and Gali 2010; Michaillat 2012, 2024; Michaillat and Saez 2015, 2019). The Moen-Rotemberg pricing mechanism could also be used in any matching model that requires some price rigidity. A distinctive feature of Moen-Rotemberg pricing is that it relies on directed search, while the other rigid pricing mechanisms rely on random search.

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Appendix A. Proofs

A.1. Proof that the worker-finding rate is decreasing in tightness

This appendix proves that the worker-finding rate $q(\theta)$ is decreasing in tightness θ . The derivative of the worker-finding rate with respect to θ_k is

$$\frac{dq}{d\theta_k} = -\frac{\omega}{2} \cdot \theta_k^{-3/2} + s\theta_k^{-2} = -\theta_k^{-3/2} \cdot \left[\frac{\omega}{2} - \frac{s}{\sqrt{\theta_k}} \right] < 0.$$

Indeed, we showed that for $\theta_k \in [\underline{\theta}, \infty)$,

$$\omega - \frac{s}{\sqrt{\theta_k}} \geq \frac{\omega}{2},$$

so that

$$\frac{\omega}{2} - \frac{s}{\sqrt{\theta_k}} \geq 0,$$

which implies that $dq/d\theta_k < 0$. In fact, $q(\underline{\theta}) = \omega^2/4s > s$ and $q(\infty) = 0$.

A.2. Expression for the upper bound on tightness

This appendix expresses the upper bound on tightness, $\bar{\theta}$, as a function of the lower bound on tightness, $\underline{\theta}$. The upper bound on tightness is implicitly defined by

$$q(\bar{\theta}) = s,$$

where $q(\theta)$ is the worker-finding rate and s is the job-separation rate.

Indeed, using (4), we write $q(\bar{\theta}) = s$ as

$$\frac{\omega}{\sqrt{\bar{\theta}}} - \frac{s}{\bar{\theta}} - s = 0.$$

With a change of variable $x = 1/\sqrt{\bar{\theta}}$, this is equivalent to solving the second-order polynomial equation

$$-sx^2 + \omega x - s = 0.$$

The determinant of the equation is $\Delta = \omega^2 - 4s^2 > 0$. The two solutions of the equation are

$$x' = \frac{\omega \pm \sqrt{\Delta}}{2s} = \frac{\omega}{2s} \cdot \left[1 \pm \sqrt{1 - \frac{4s^2}{\omega^2}} \right] = \frac{1 \pm \sqrt{1 - \underline{\theta}}}{\sqrt{\underline{\theta}}}.$$

Accordingly, the tightnesses that solve $q(\theta) = s$ are given by $1/(x')^2$ or

$$\theta' = \frac{\underline{\theta}}{[1 \pm \sqrt{1 - \underline{\theta}}]^2}$$

The only solution that is larger than $\underline{\theta}$ is

$$\bar{\theta} = \frac{\underline{\theta}}{[1 - \sqrt{1 - \underline{\theta}}]^2}.$$