

COMP319 Algorithms 1

Lecture 6

ADT, Lists, Stacks, Queues

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Abstract data type (ADT) / Array Lists / Linked Lists
Stacks / Queues

Textbook Chapter 11

Slide credits: 홍석원, 명지대학교, Discrete Mathematics, Spring 2013

김한준, 서울시립대학교, 자료구조 및 실습, Fall 2016

J. Lillis, UIC's CS 201 Data Structures and Discrete Mathematics I

Abstract data type

ADT

ABSTRACT DATA TYPE

Data Abstraction

- 컴퓨터를 이용한 문제해결에서의 추상화
 - 크고 복잡한 문제를 단순화시켜 쉽게 해결하기 위한 방법
- 자료 추상화(Data Abstraction)
 - 처리할 자료, 연산, 자료형에 대한 추상화 표현
 - 자료(data): 프로그램의 처리 대상이 되는 모든 것을 의미
- 연산(Operations)
 - 어떤 일을 처리하는 과정. 연산자에 의해 수행
 - 예) 더하기 연산은 +연산자에 의해 수행

Data Abstraction

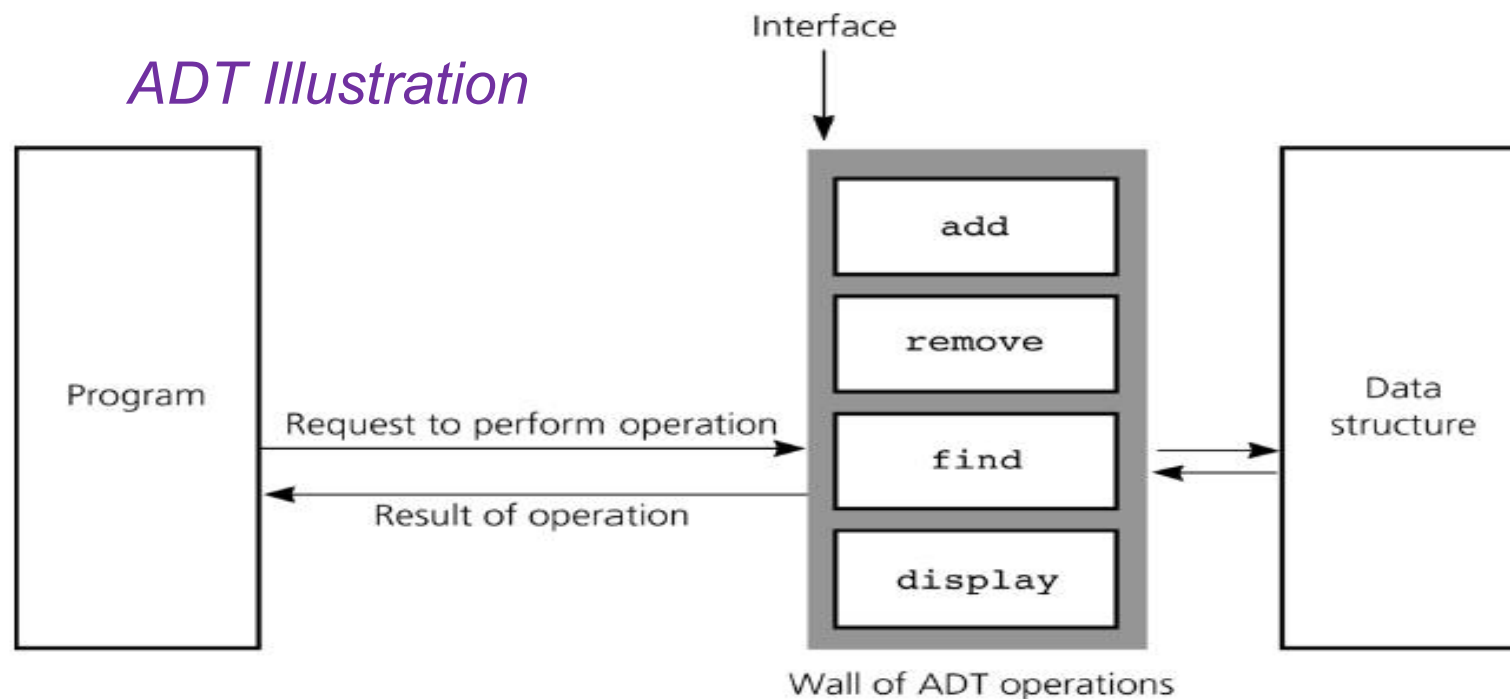
- 추상자료형(ADT, abstract data type)
 - 처리할 자료의 집합과 자료에 대해 수행할 연산자의 집합
 - 예) 정수 자료형
 - 자료 : 정수의 집합. $\{..., -1, 0, 1, ...\}$
 - 연산자 : 정수에 대한 연산자 집합. $\{+, -, \times, \div, \text{modulus}\}$
- 추상화와 구체화
 - 추상화 – “무엇(what)인가?”를 논리적으로 정의
 - 구체화 – “어떻게(how) 할 것인가?”를 실제적으로 표현

Abstract Data Type (ADT)

- Definition: a collection of *data* together with a set of *operations* on that data
 - specifications indicate *what* ADT operations do, but not *how* to implement them
 - data structures are part of an ADT's implementation
- Programmer can use an ADT without knowing its implementation.

Typical operations on Data

- Add data to a data collection
- Remove data from a data collection
- Ask questions about the data in a data collection
 - (Retrieval) What is the value at a particular location
 - (Search) Is x in the collection?



Why ADT?

- Hide the unnecessary details
- Help manage software complexity
- Easier software maintenance
- Functionalities are less likely to change
- Localised rather than global changes

LISTS IMPLEMENTATION BY ARRAYS

An ADT Interface for Array List

- Functions

- isEmpty
- getLength
- Insert
- Delete
- Lookup
- ...

- Data Members

- head
- Size

Lists

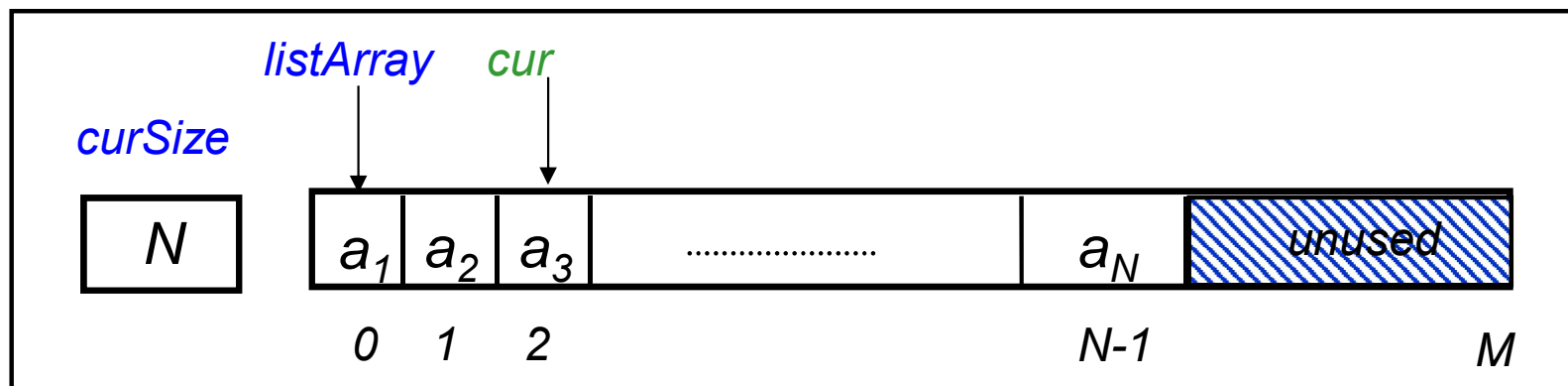
- List: a finite sequence of data items

$$[a_1, a_2, a_3, \dots, a_N]$$
$$[5, 6, 2, 8, \dots, 6]$$

- Lists are pervasive in computing
 - e.g. class list, list of characters (string), list of events
- Typical operations:
 1. Create a list
 2. Insert / Remove an element
 3. Test for emptiness
 4. Find an item / value / k -th element
 5. Retrieve current element / next / previous
 6. Print the entire list

Array-Based List Implementation

- An array is usually a sequence of N -elements stored in consecutive memory locations
 - Maximum (allocation) size (M) is anticipated *a priori* (should be determined before allocated) and larger than array size (N)
- Internal variables:
 - Maximum size *maxSize* (M)
 - Current size *curSize* (N)
 - Current index *cur*
 - Array of elements *listArray*



Coding: ArrayList Definition

```
#define MAX 100    // in C, array size is unknown
                  // so programmers usually predict
                  // the maximum list size

struct ArrayList {
    int arr[MAX];
    int size;
};

void initialize(struct ArrayList *pl)
// start from empty list
{
    pl->size = 0;
}

void print(struct ArrayList *pl)
{
    int i;
    printf("[");
    for (i=0; i<pl->size; i++) printf("%d ", pl->arr[i]);
    printf("]\n");
}
```

Inserting Into an Array

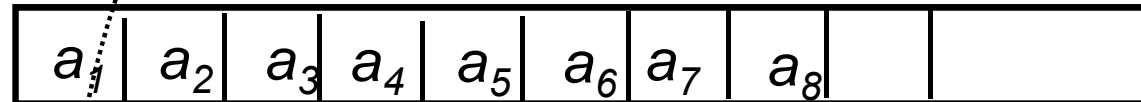
- While retrieval is very fast, insertion and deletion are very slow
 - Function *insert()* has to shift upwards to create gap

Example : *insert(2, it, arr)*

Size

8

arr

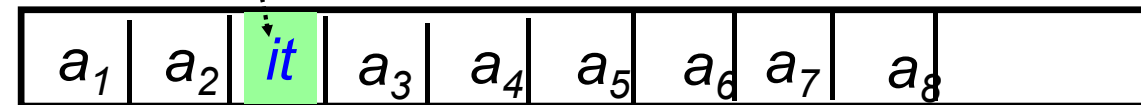


Step 2 : Write into gap

Size

9

arr



Step 1 : Shift upwards

Step 3 : Update Size

Coding: insert()

```
void insert(int j, int it, struct ArrayList *pl)
{ // precondition : 0<=j<=size
  // in this code, checking max is missing

  int i;

  for (i=pl->size-1; i>=j; i=i-1)
      // Step 1: Create gap
      { pl->arr[i+1]= pl->arr[i]; }

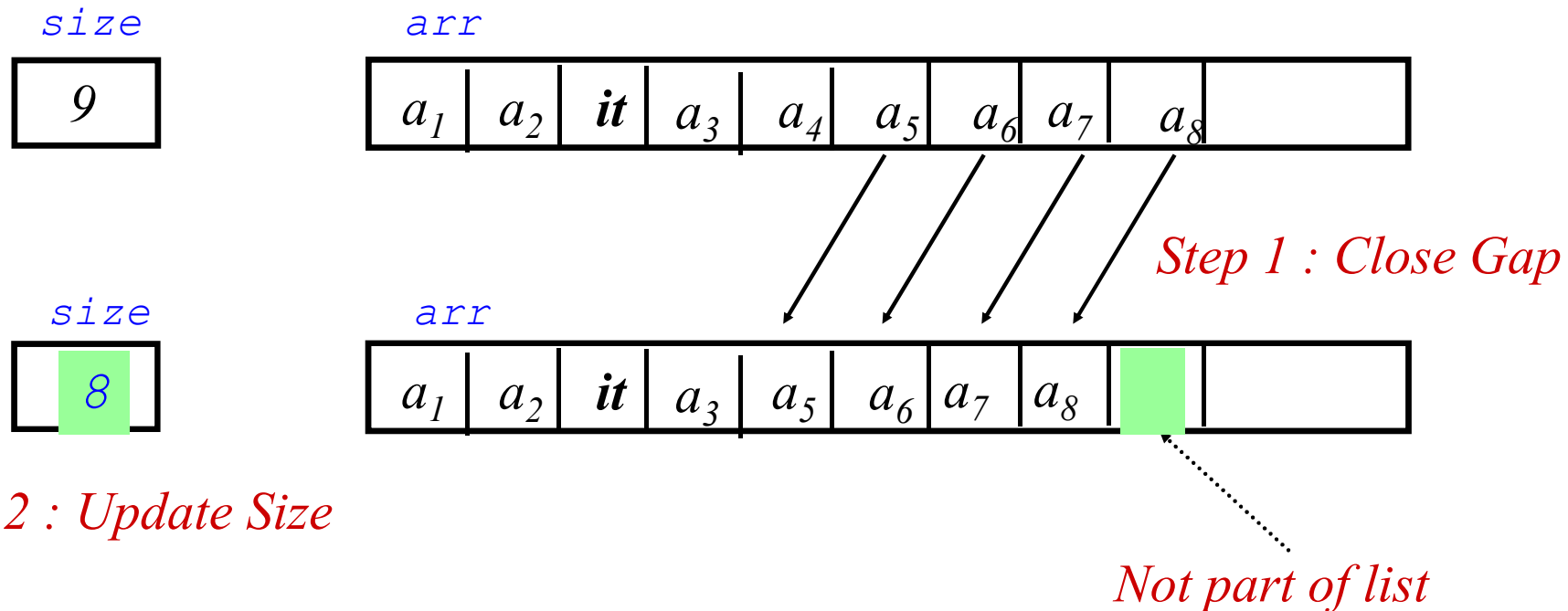
  pl->arr[j]= it;          // Step 2: Write to gap

  pl->size = pl->size + 1; // Step 3: Update size
}
```

Deleting from an Array

- Delete has to shift downwards to close gap of deleted item

Example: `deleteItem(4, arr)`



Coding: delete()

```
void delete(int j, struct ArrayList *pl)
{
    // precondition : 0<=j<size
    int i;

    for (i=j+1; i<pl->size; i=i+1)
        // Step1: Close gap
        { pl->arr[i-1]=pl->arr[i]; }
        // Step 2: Update size
    pl->size = pl->size - 1;
}
```


Coding: lookup()

```
#define TRUE 1
#define FALSE 0
typedef int BOOLEAN;
BOOLEAN lookup (int x,
                struct ArrayList *pl)
{
    int i;
    for (i=0; i<pl->size; i=i+1) {
        if (x == pl->arr[i]) return TRUE;
    }
    return FALSE;
}
```

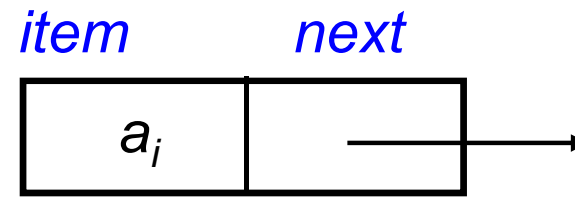
LINKED LISTS

An ADT Interface for Linked List

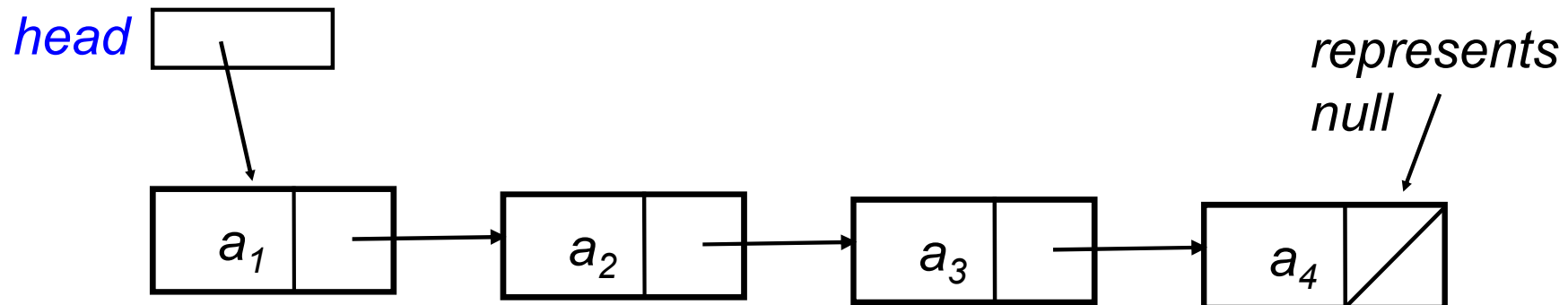
- Functions
 - isEmpty
 - getLength
 - Insert
 - Delete
 - Lookup
 - ...
- Data Members
 - head
 - Size
- **Local variables to member functions**
 - cur
 - prev

Linked List Approach

- Main problem of array is the slow deletion/insertion since it has to shift items in its *contiguous* memory
- **Solution:** linked list where items need *not be contiguous* with nodes of the form



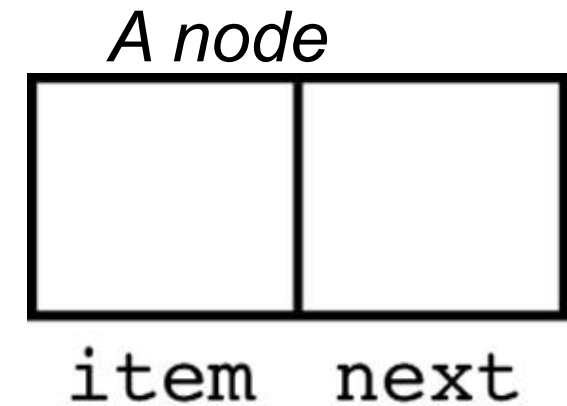
- Sequence (list) of four items $\langle a_1, a_2, a_3, a_4 \rangle$ can be represented by:



Pointer-Based Linked Lists

- A node in a linked list is usually a struct

```
struct Node
{ int item
  struct Node *next;
}; //end struct
```

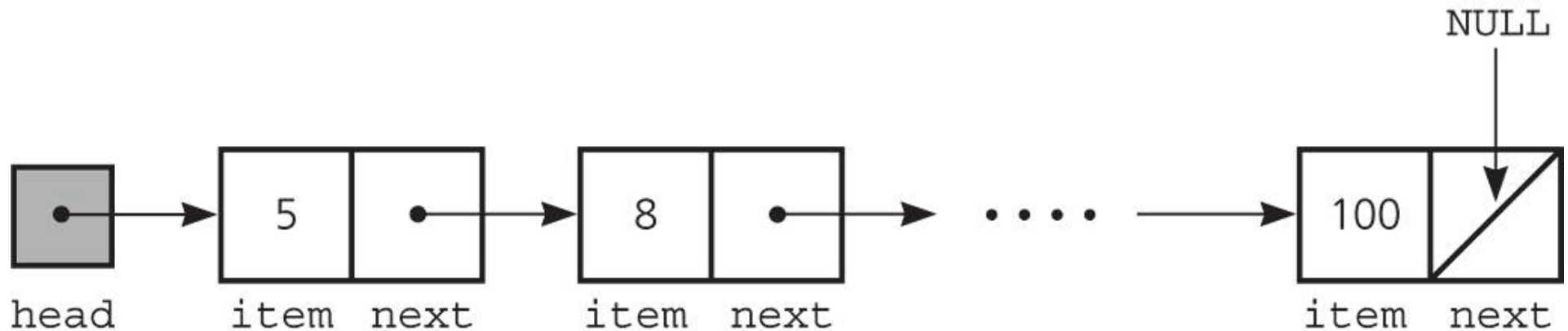


- A node is dynamically allocated

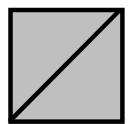
```
struct Node *p;
p = malloc(sizeof(struct Node));
```

A Sample Linked List

- The head pointer points to the first node



- If head is *NULL*, the linked list is empty
 - head=NULL



head

represents empty list

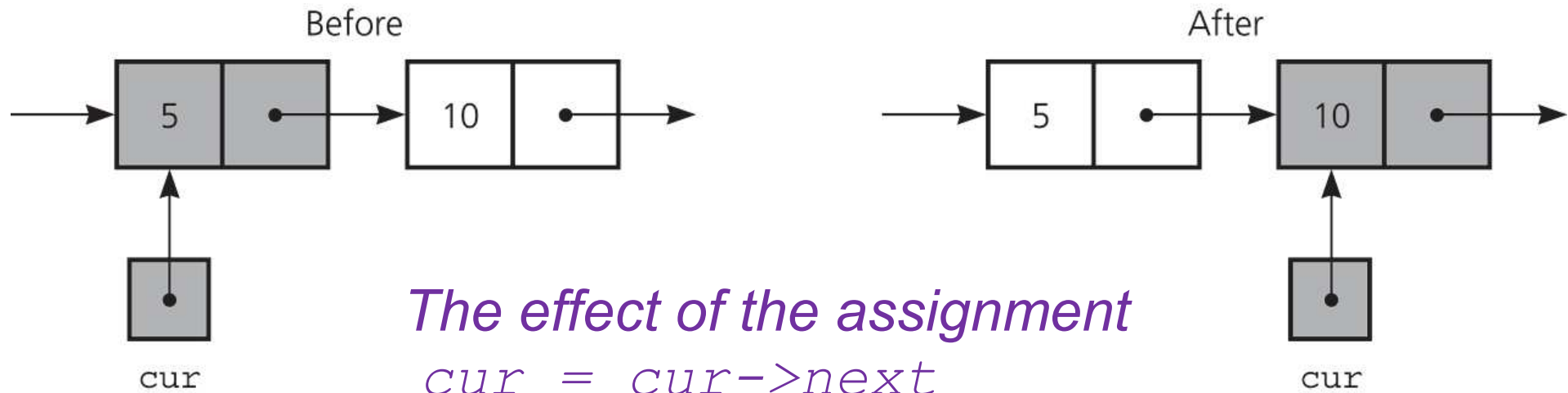
Traverse a Linked List

- Reference a node member with the `->` operator
- A traverse operation visits each node in the linked list

- A pointer variable `cur` keeps track of the current node

```
struct Node *cur;
```

```
for (cur=head; cur!=NULL; cur = cur->next)  
    printf("%d ", cur->item);
```

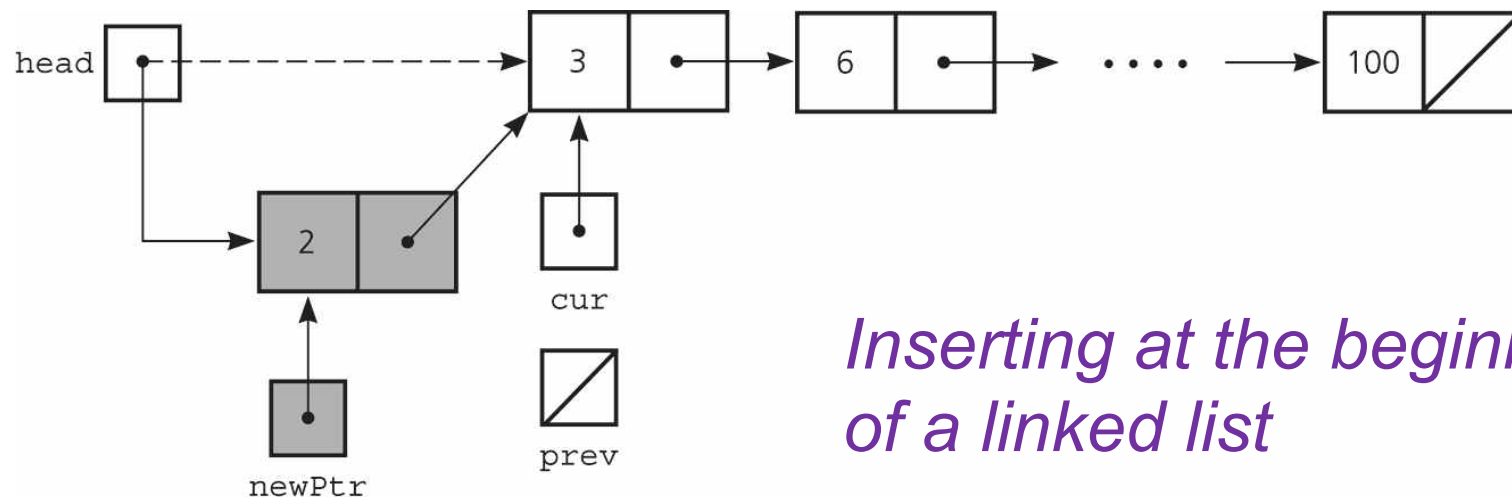


Insert a Node into a Linked List

- To insert a node at the beginning of a linked list

```
newPtr->next = head;
```

```
head = newPtr;
```



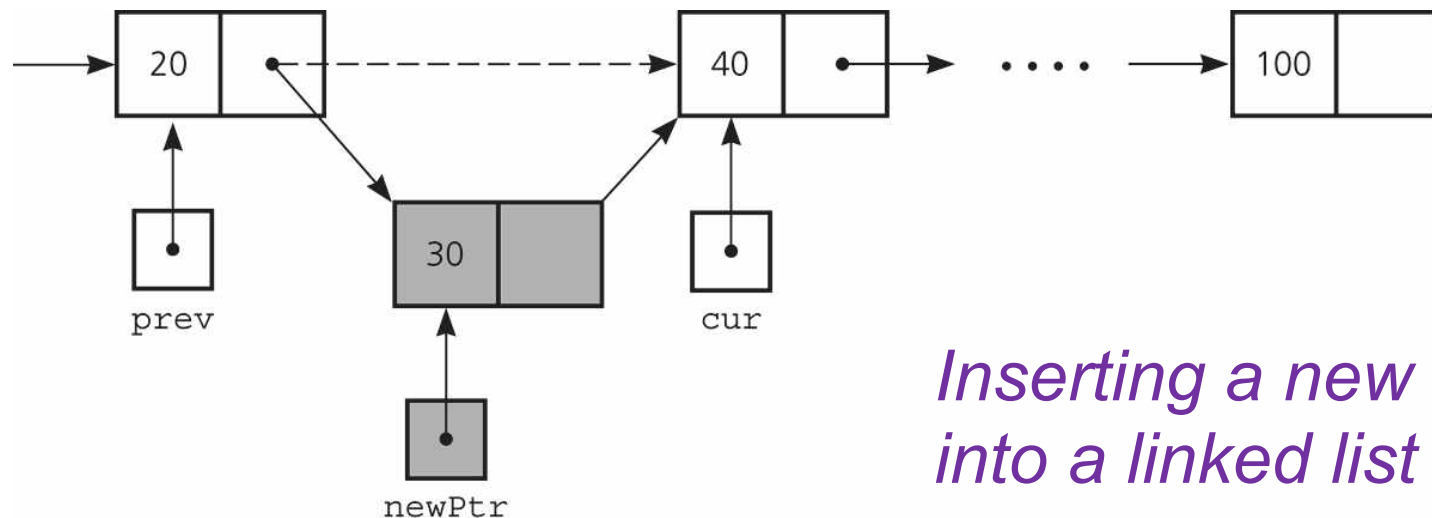
Inserting at the beginning of a linked list

Insert a Node into a Linked List

- To insert a node **between two nodes**

```
newPtr->next = cur;
```

```
prev->next = newPtr;
```



*Inserting a new node
into a linked list*

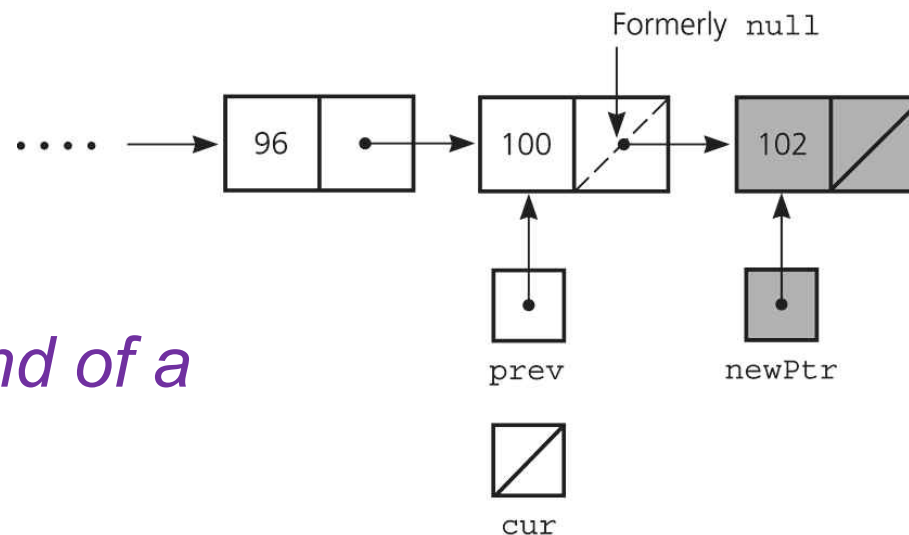
Insert a Node into a Linked List

- Inserting at the end of a linked list (if `cur` is `NULL`) is not a special case

```
newPtr->next = cur;
```

```
prev->next = newPtr;
```

Inserting at the end of a linked list



Delete a Node from a Linked List

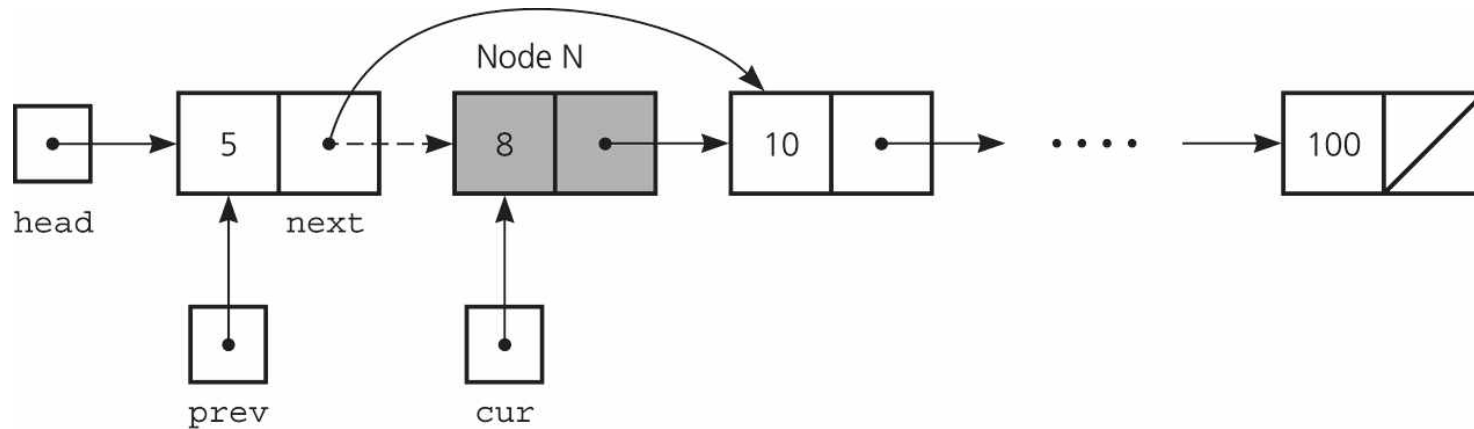
- Deleting an interior/last node with returning the deleted node to system

```
prev->next=cur->next;  
cur->next = NULL;           // not necessary  
free(cur);  
cur = prev->next;           // only if keep cur
```

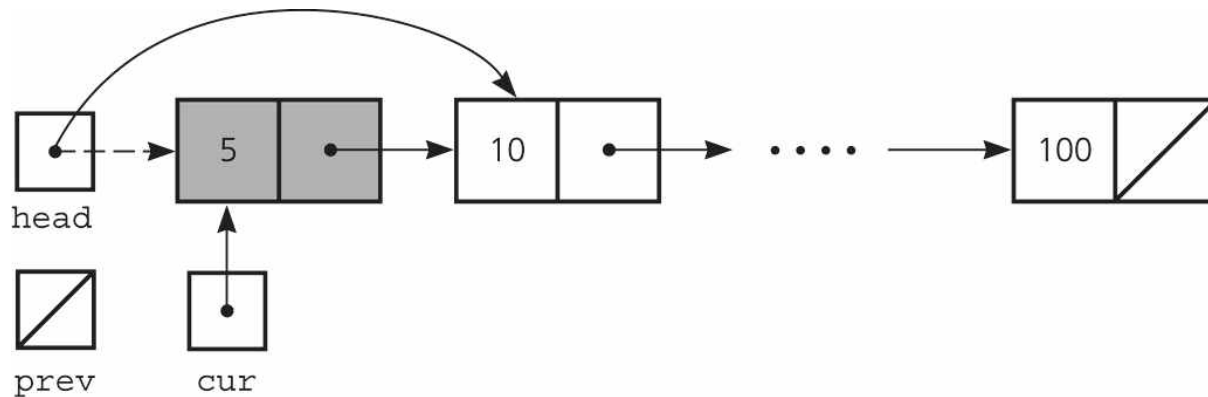
- Deleting the first node with returning the deleted node to system

```
cur = head;  
head = head->next;  
cur->next = NULL;           // not necessary  
free(cur);
```

Delete a Node from a Linked List



Deleting a node from a linked list



Deleting the first node

Look up

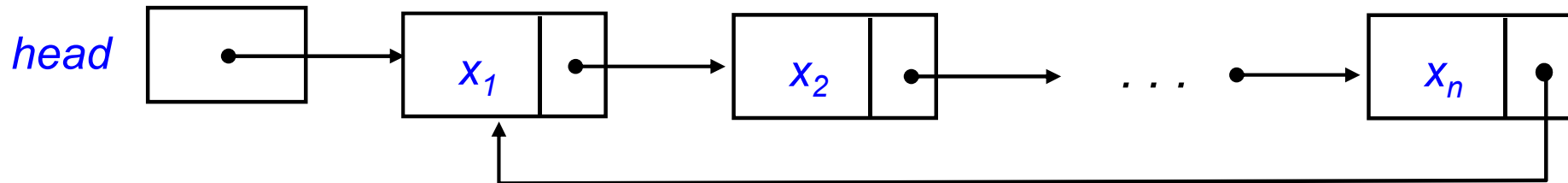
```
struct Node* lookup_recursive (int x, struct Node *L)
{ if (L == NULL) return NULL;
  else if (x == L->item) return L;
  else return lookup_recursive (x, L->next);
}
```

```
Struct Node* lookup_iterative (int x, struct Node *L)
{ struct Node *cur;
  for (cur=head; cur!=NULL; cur = cur->next)
    if ( x == cur->item ) return cur;
  return NULL; // not found
}
```

Circular Linked Lists

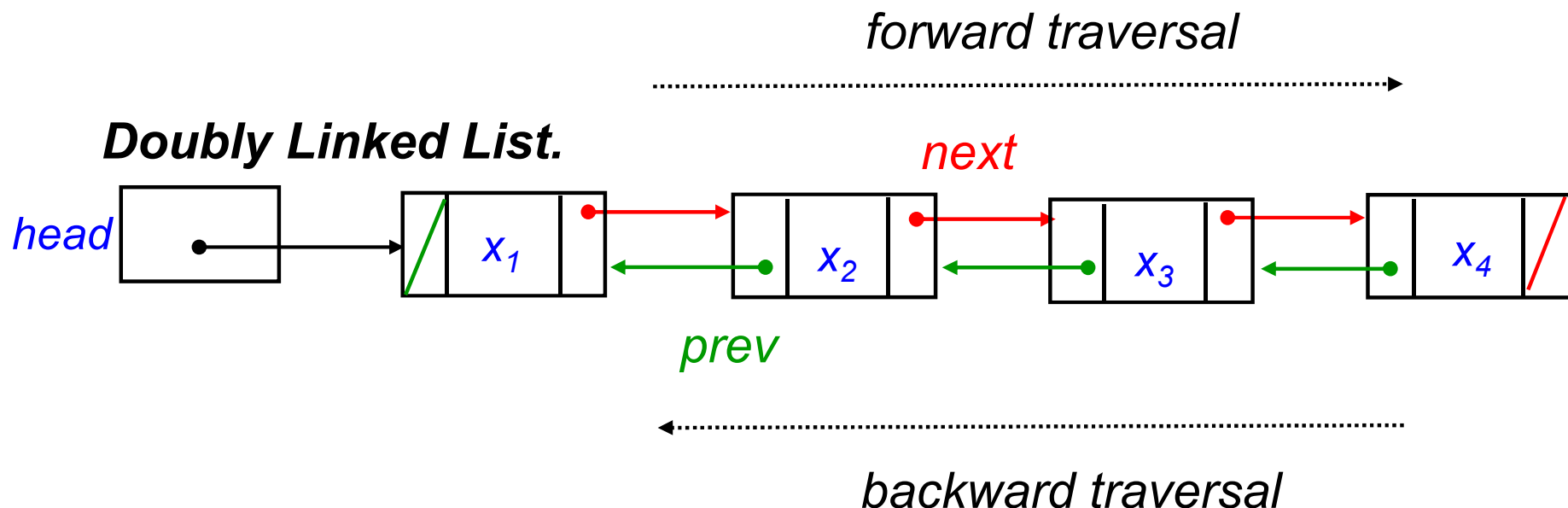
- May need to cycle through a list repeatedly, e.g. round robin system for a shared resource
- *Solution* : Have the last node point to the first node

Circular Linked List.



Doubly Liked Lists

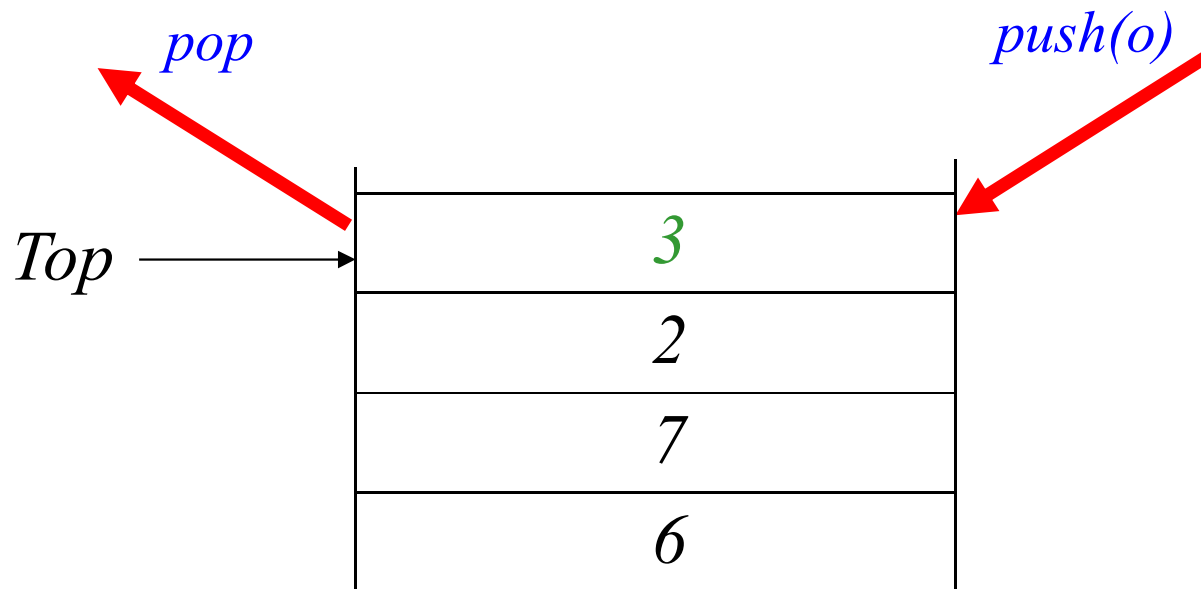
- Frequently, we need to traverse a sequence in BOTH directions efficiently
- *Solution* : Use doubly-linked list where each node has two pointers



STACKS

What is a Stack?

- A *stack* is a list with the restriction that insertions and deletions can be performed in only one position, namely, the end of the list, called the *top*.
- The operations: **push** (insert) and **pop** (delete)
 - Other operations: top, clear, etc.



Stack ADT Interface

- The main functions in the Stack ADT are (S is the stack)

boolean isEmpty(S); // return true if empty

boolean isFull(S); // return true if full

void push(S, item); // insert *item* into stack

Item pop(S); // return and remove most recent item

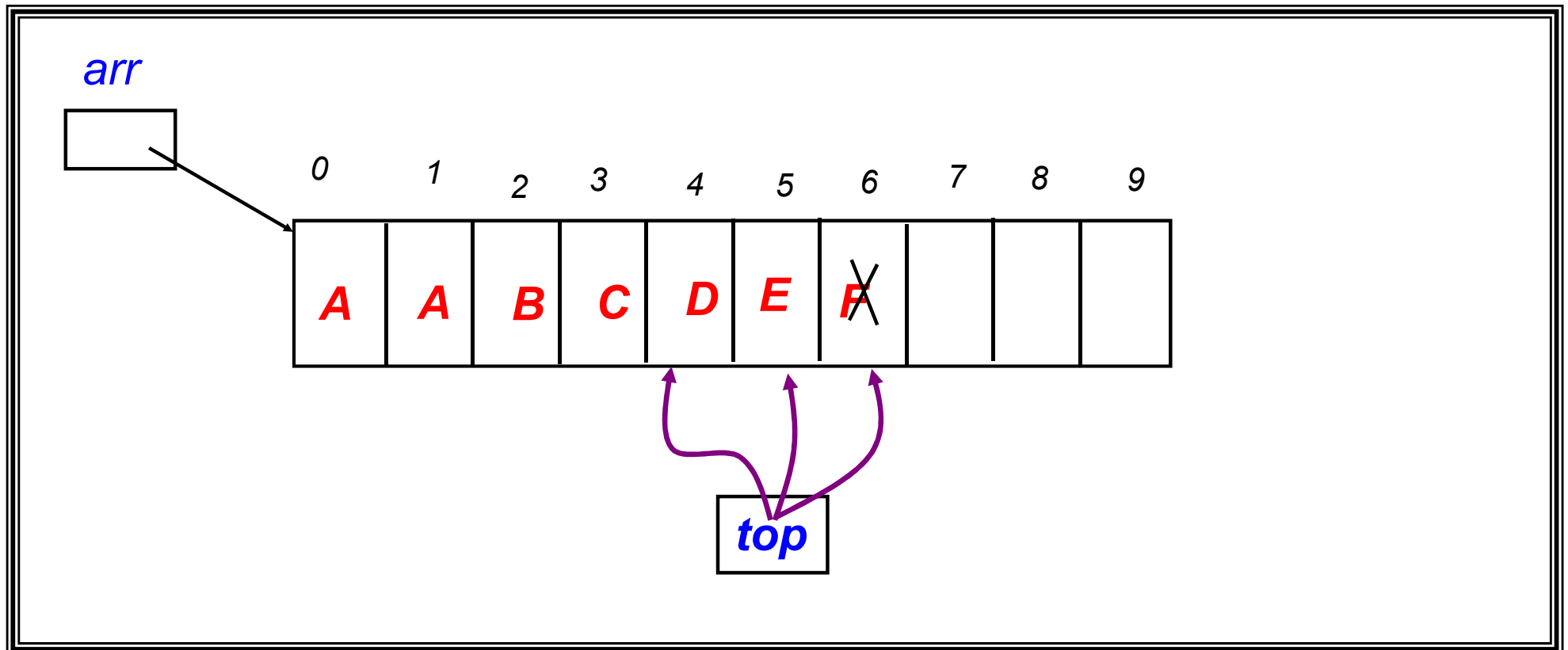
void clear(S); // remove all items from stack

Item top(S); // retrieve most recent item

Implementation by Array

- use Array with a **top** index pointer as an implementation of stack

StackAr



Sample Operation

➔ `Stack S = malloc(sizeof(stack));`

➔ `push(S, "a");`

➔ `push(S, "b");`

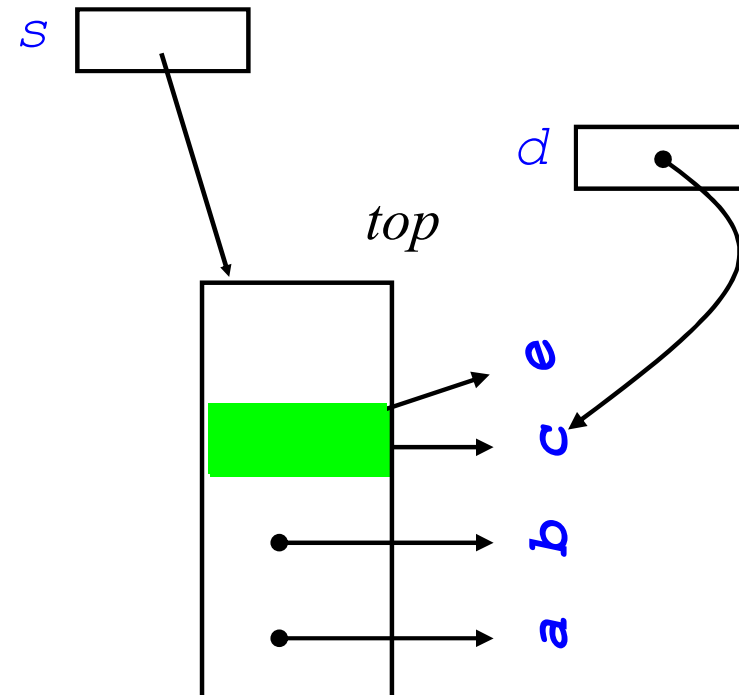
➔ `push(S, "c");`

➔ `d=top(S);`

➔ `pop(S);`

➔ `push(S, "e");`

➔ `pop(S);`



Stack Implementation in C

```
typedef short BOOLEAN;
#define TRUE ((short) (1))
#define FALSE ((short) (0))

/* maximum stack size */
#define MAX (1024)

typedef struct {
    int A[MAX];
    int top;
} STACK;
```

```
void clear(STACK *pS)
{ pS->top = -1; }
```

```
BOOLEAN isEmpty(STACK *pS)
{ return (pS->top < 0); }
```

```
BOOLEAN isFull(STACK *pS)
{ return
    (ps->top >= MAX-1);
}
```

Stack in C: push and pop

```
BOOLEAN push(int x, STACK *pS) {  
    if ( isFull(pS) ) return FALSE;  
    else { ps->A[++(pS->top)] = x;  
          return TRUE; }  
}
```

```
BOOLEAN pop(STACK *pS, int *px) { /* (*px) for return */  
    if ( isEmpty(pS) ) return FALSE;  
    else { (*px) = pS->A[(pS->top)--];  
          return TRUE;  
    }  
}
```

Applications

- Many application areas use stacks:
 - line editing
 - bracket matching
 - postfix calculation
 - function call stack

Stack Example: Line Editing

- A line editor would place characters that are read into a buffer but may use a backspace symbol (denoted by ←) to correct typing errors
- *Refined Task*
 - read in a line
 - correct the errors via backspace
 - print the corrected line in reverse

Input : `abc_defgh←2klp←←←wxyz`

Corrected Input : `abc_defg2klpwxzy`

Reversed Output : `zyxwplk2gfed_cba`

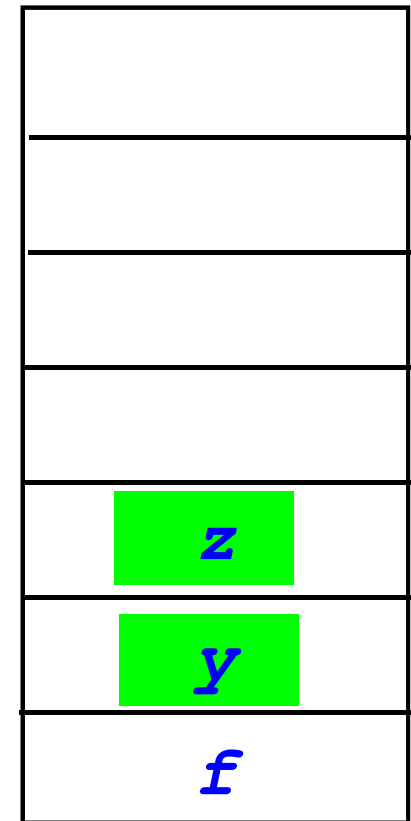
The Procedure

- Initialize a new stack
- For each character read:
 - if not a backspace, *push the char into stack*
 - if it is a backspace, *pop out last char entered*
- To print in reverse, *pop out each char for output*

Input : *fgh←r←←yz*

Corrected Input : *fyz*

Reversed Output : *zyf*



Stack

Bracket Matching Problem

- Ensures that pairs of brackets are properly matched

- An Example:*

$\{a, (b+f[4]) * 3, d+f[5]\}$

- Bad Examples:*

$(..)..)$ // too many closing brackets

$(..(..)$ // too many open brackets

$[..(..)]..)$ // mismatched brackets

Informal Procedure

Initialize the stack to empty

For every char read

if open bracket then *push onto stack*

if close bracket, then

return & remove most recent item

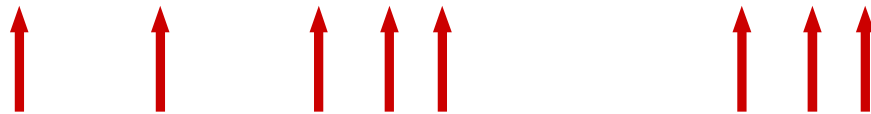
from *the stack*

if doesn't match then *flag error*

if non-bracket, *skip the char read*

Example

$\{a, (b+f[4]) * 3, d+f[5]\}$



Stack

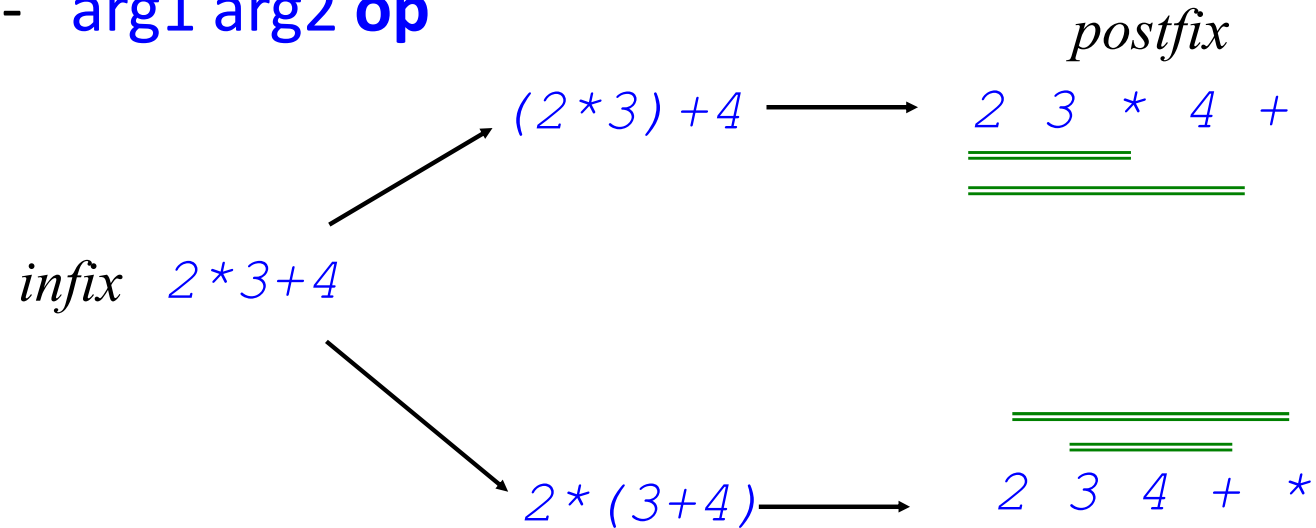
Postfix Calculator

- Computation of arithmetic expressions can be efficiently carried out in Postfix notation with the help of a stack.

Infix - **arg1 op arg2**

Prefix - **op arg1 arg2**

Postfix - **arg1 arg2 op**



Informal Procedure

Initialise stack S

For each item read.

If it is an operand,

push on the stack

If it is an operator,

pop arguments from stack;

perform operation;

push result onto the stack

Expr

2	<i>push</i> (S, 2)
3	<i>push</i> (S, 3)
4	<i>push</i> (S, 4)
+	<i>arg2</i> = <i>pop</i> (S) <i>arg1</i> = <i>pop</i> (S) <i>push</i> (S, <i>arg1</i> + <i>arg2</i>)
*	<i>arg2</i> = <i>pop</i> (S) <i>arg1</i> = <i>pop</i> (S) <i>push</i> (S, <i>arg1</i> * <i>arg2</i>)



Stack

STACKS: LINKED LIST IMPLEMENTATION

Recap: Stack ADT Interface

- The main functions in the Stack ADT are (S is the stack)

boolean isEmpty(S); // return true if empty

boolean isFull(S); // return true if full

void push(S, item); // insert *item* into stack

Item pop(S); // return and remove most recent item

void clear(S); // remove all items from stack

Item top(S); // retrieve most recent item

Implementation by Linked Lists

- Can use a **Linked List** as implementation of stack

StackLL

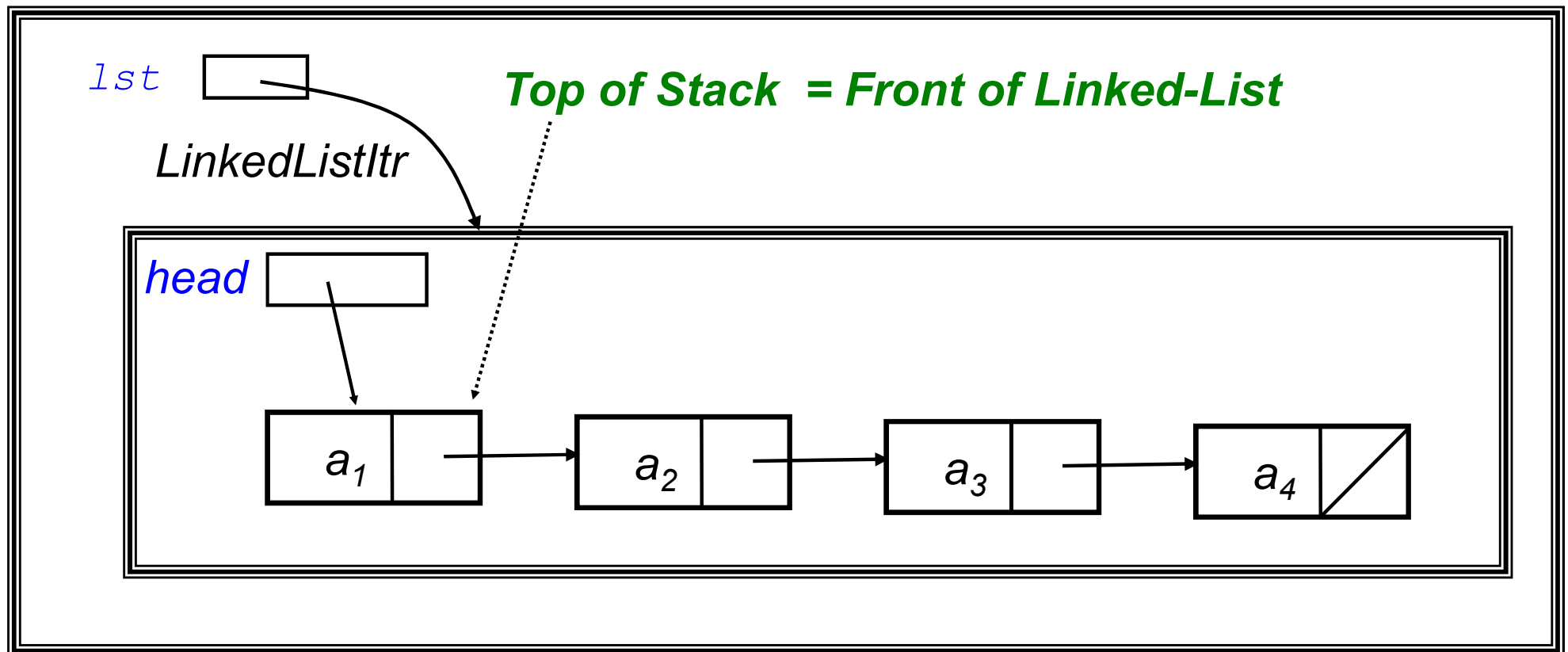
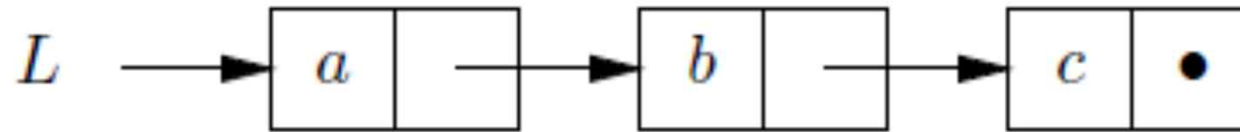
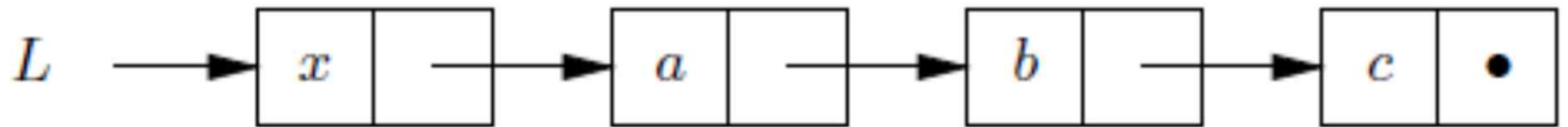


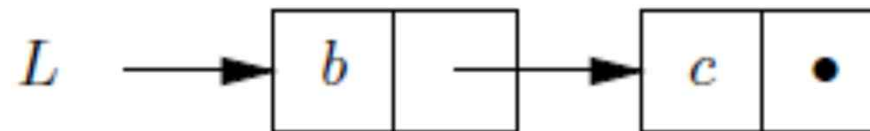
Illustration of push and pop



(a) List L .



(b) After executing $push(x, L)$.



(c) After executing $pop(L, x)$ on list L of (a).

Stack by Linked Lists in C

```
typedef short BOOLEAN;
#define TRUE ((short) (1))
#define FALSE ((short) (0))

struct NODE {
    int element;
    struct NODE *next;
};

typedef struct {
    struct NODE *head;
} STACK;

void initialize(STACK *pS)
{ pS->head = NULL; }

void clear(STACK *pS) {
    struct NODE *n, *pre;
    n = pS->head;
    while ( n ) {
        pre = n;
        n = n->next;
        free(pre);
    }
    initialize(pS);
}
```

Stack by Linked Lists in C: pop

```
BOOLEAN isEmpty(STACK *pS) { return (pS->head == NULL); }  
BOOLEAN isFull(STACK *pS) {  
    // list grows as much as memory size, so no full stack  
    return FALSE;  
}
```

```
BOOLEAN pop(STACK *pS, int *px) {  
    struct NODE *n;  
    if ( isEmpty(pS) ) return FALSE;  
    else {  
        n = pS->head; pS->head = n->next;  
        (*px) = n->element; free(n);  
        return TRUE;  
    }  
}
```

Stack by Linked Lists in C: push

```

BOOLEAN push(int x, STACK *pS) {
    struct NODE *n;
    n = (struct NODE*) malloc(sizeof(struct NODE));
    n->element = x;
    n->next = pS->head;
    pS->head = n;
    return TRUE;
}

```

/ EXAMPLE*

[37 75 45 80 24 13 94 42 21 74]

37 [75 45 80 24 13 94 42 21 74]

75 [45 80 24 13 94 42 21 74]

[102 45 80 24 13 94 42 21 74]

[106 102 45 80 24 13 94 42 21 74]

106 [102 45 80 24 13 94 42 21 74]

**/*

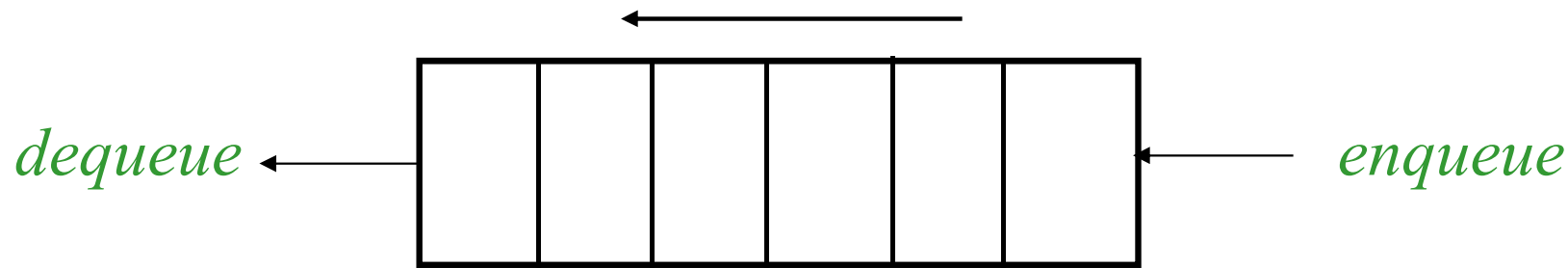
Summary

- The ADT stack operations have a last-in, first-out (LIFO) behavior
- Stack has many applications
 - algorithms that operate on algebraic expressions
 - a strong relationship between recursion and stacks exists
- Stack can be implemented using arrays or linked lists

QUEUES

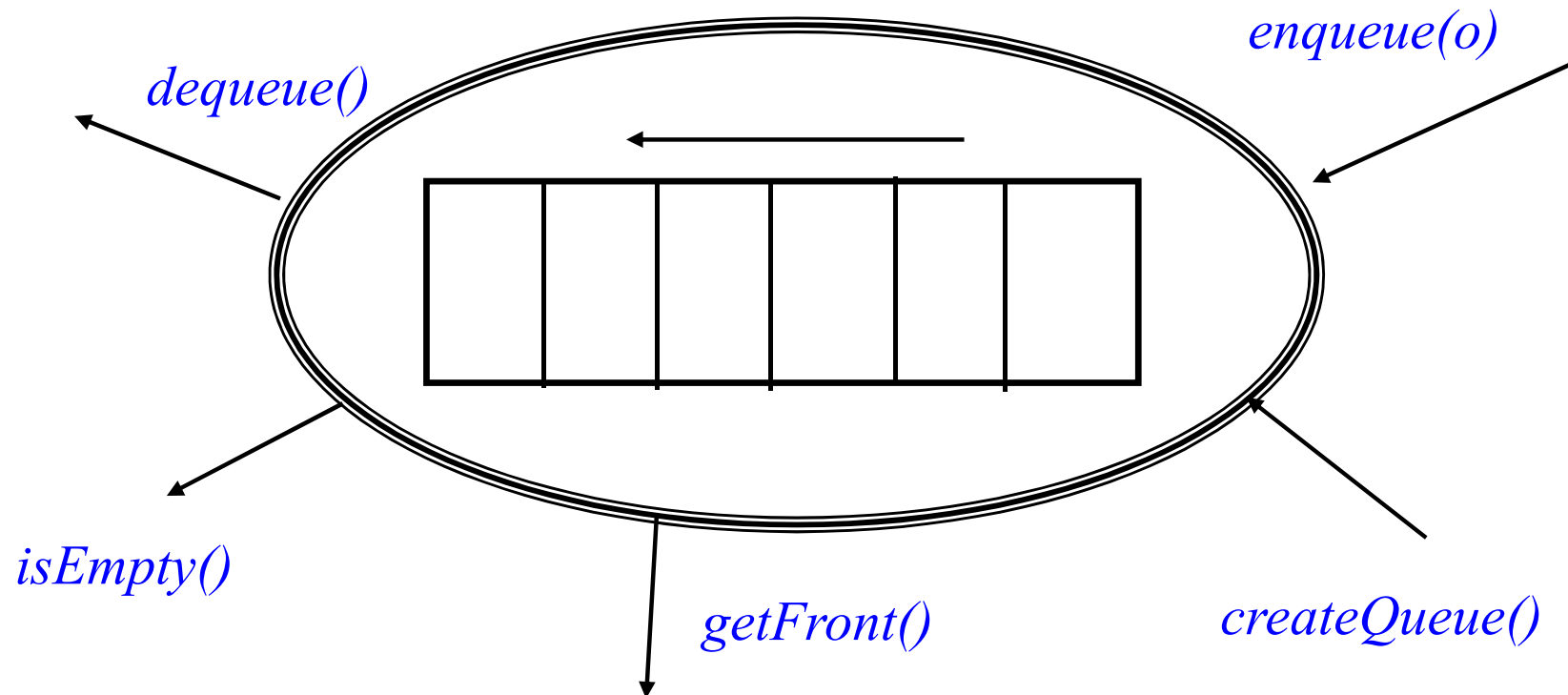
What is a Queue?

- Like stacks, queues are lists. With a queue, however, insertion is done at one end whereas deletion is done at the other end.
- Queues implement the FIFO (first-in first-out) policy. E.g., a printer/job queue!
- Two basic operations of queues:
 - **dequeue**: remove an item/element from front
 - **enqueue**: add an item/element at the back



Queue ADT

- Queues implement the FIFO (first-in first-out) policy
 - An example is the printer/job queue!



Sample Operation

→ *Queue *Q;*

→ *enqueue(Q, "a");*

→ *enqueue(Q, "b");*

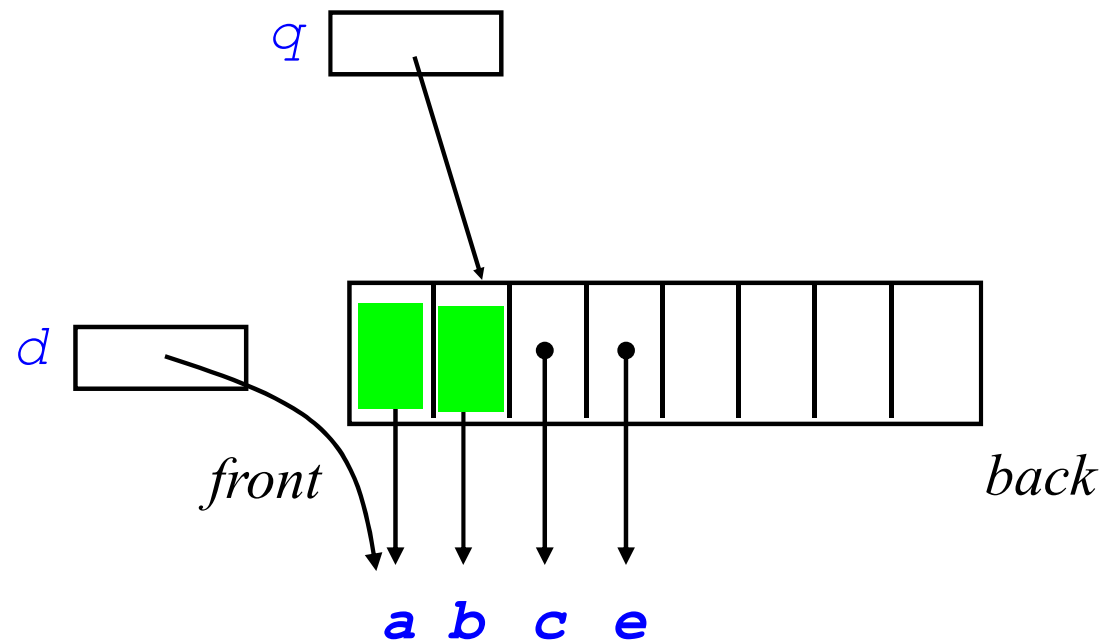
→ *enqueue(Q, "c");*

→ *d=getFront(Q);*

→ *dequeue(Q);*

→ *enqueue(Q, "e");*

→ *dequeue(Q);*



Queue ADT interface

- The main functions in the Queue ADT are (Q is the queue)

```
void enqueue(o, Q)           // insert o to back of Q

void dequeue(Q);             // remove oldest item

item getFront(Q);           // retrieve oldest item

boolean isEmpty(Q);          // checks if Q is empty

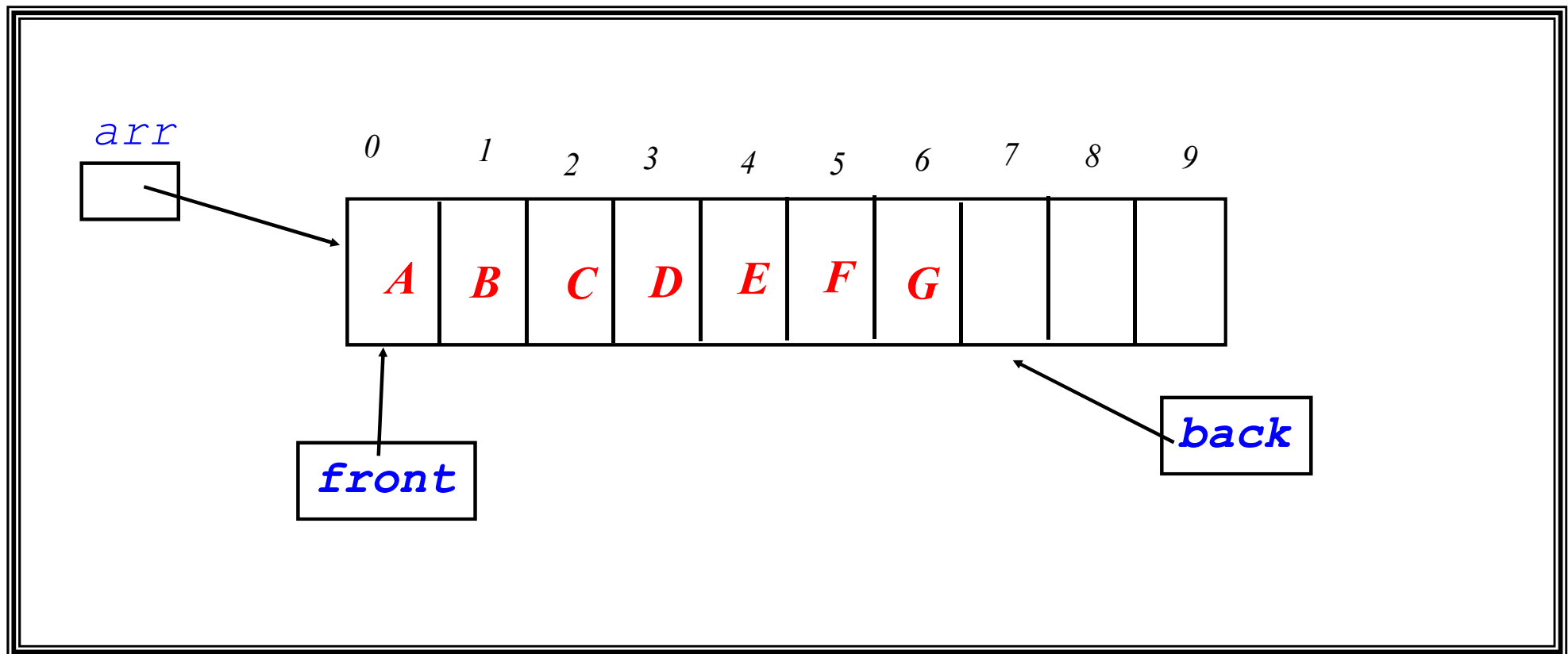
boolean isFull(Q);           // checks if Q is full

void clear(Q);               // make Q empty
```

Implementation of Queue (Array)

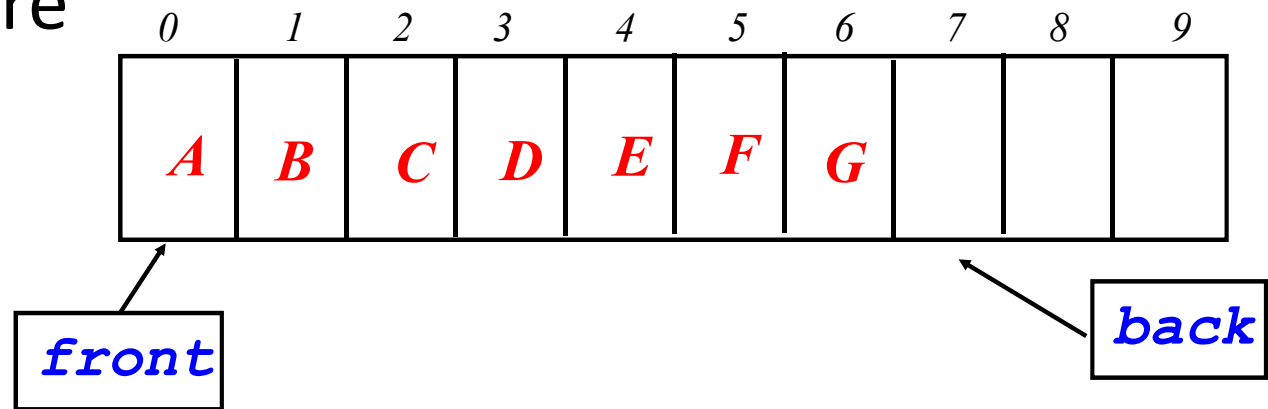
- use Array with **front** and **back** pointers as implementation of queue

Queue

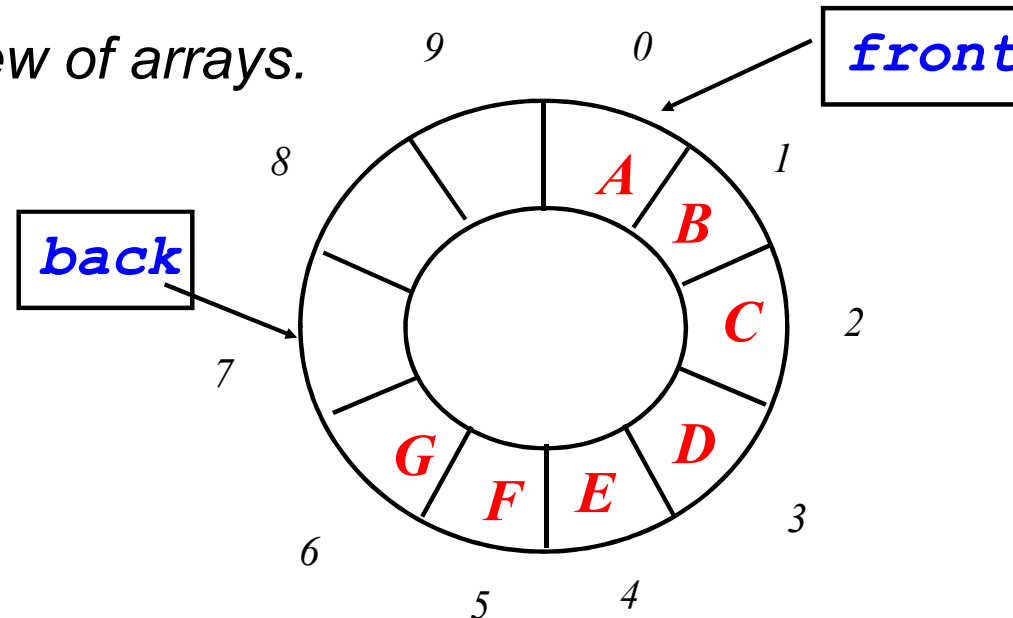


Circular Array

- To implement queue, it is best to view arrays as circular structure



Circular view of arrays.



How to Advance

- Both front & back pointers should make advancement until they reach end of the array. Then, they should re-point to beginning of the array

front = adv(front);
back = adv(back);

```
int adv(int p)
{ int r = p+1;
  if (r<maxsize) return r;
  else return 0;
}
```

upper bound of the array

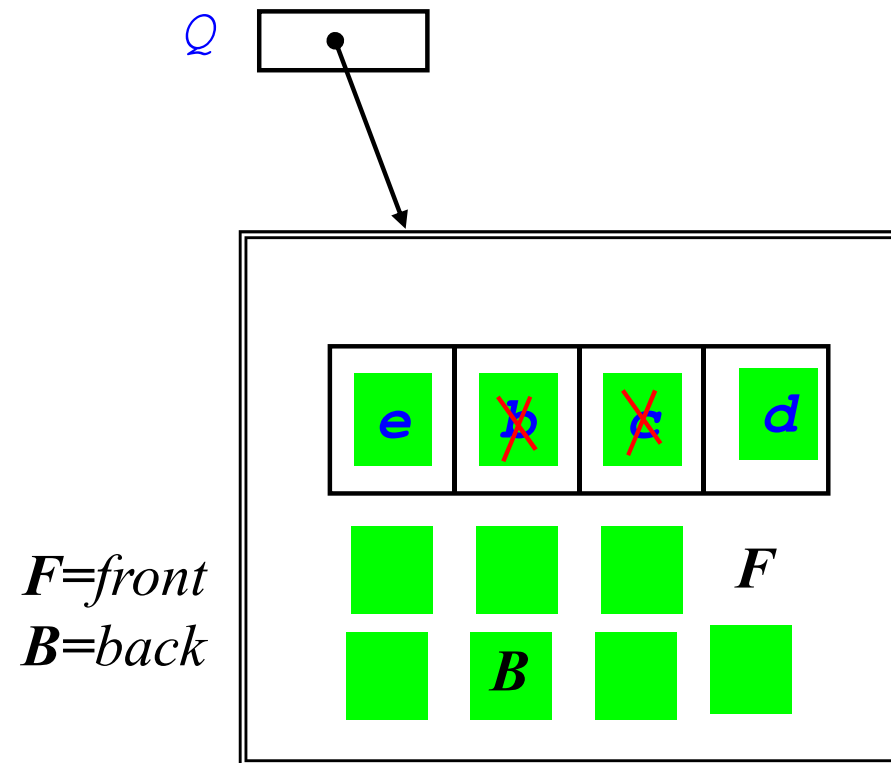
Alternatively, use modular arithmetic:

```
int adv(int p)
{ return ((p+1) % maxsize);
}
```

mod operator

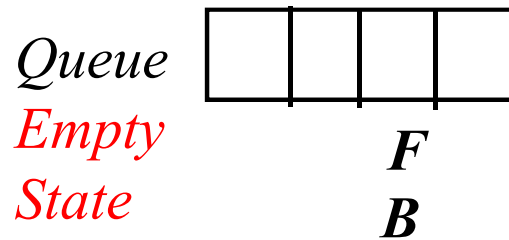
Sample

→ `Queue *Q;`
 → `enqueue(Q, "a");`
 → `enqueue(Q, "b");`
 → `enqueue(Q, "c");`
 → `dequeue(Q);`
 → `dequeue(Q);`
 → `enqueue(Q, "d");`
 → `enqueue(Q, "e");`
 → `dequeue(Q);`

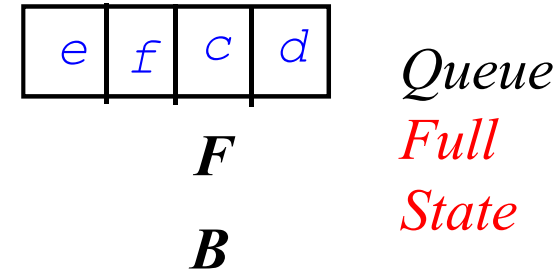


Checking for Full/Empty State

What does $(F==B)$ denote?



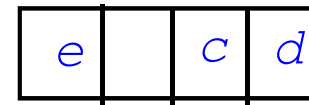
size 0



size 4

Alternative - Leave a Deliberate Gap!

No need for *size* field.



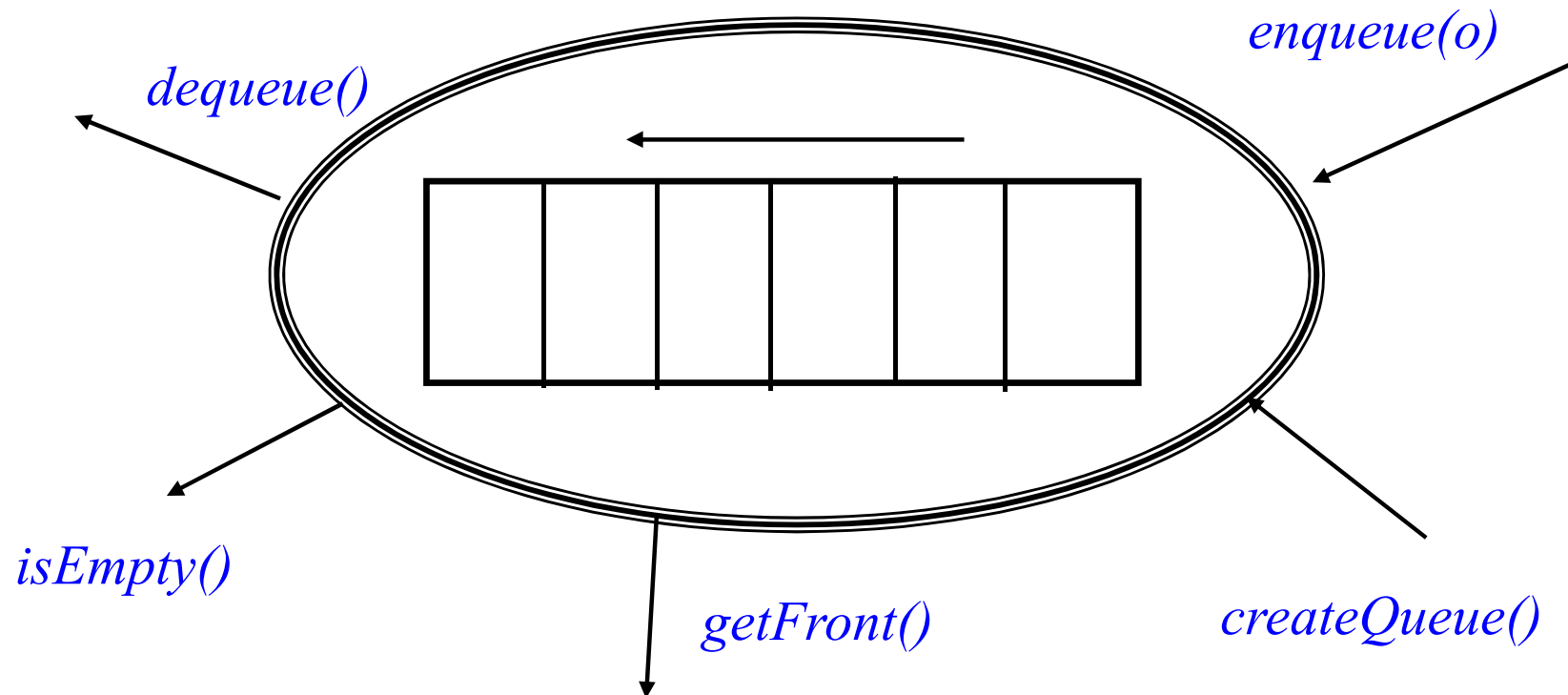
Full Case : $(adv(B) == F)$

$B \quad F$

QUEUES: LINKED LIST IMPLEMENTATION

Recap: Queue ADT

- Queues implement the FIFO (first-in first-out) policy
 - An example is the printer/job queue!



Recap: Queue ADT interface

- The main functions in the Queue ADT are
(Q is the queue)

```
void enqueue(o, Q)           // insert o to back of Q

void dequeue(Q);            // remove oldest item

Item getFront(Q);           // retrieve oldest item

boolean isEmpty(Q);         // checks if Q is empty

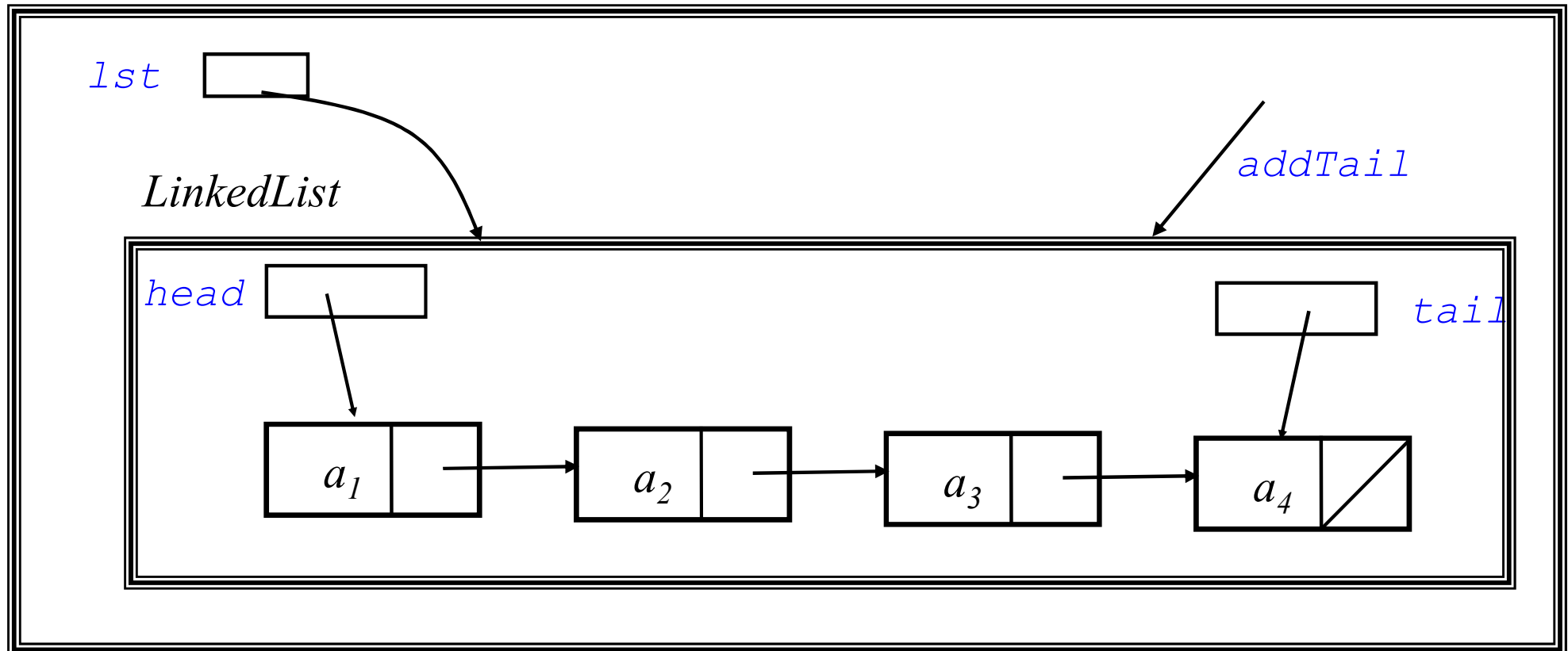
boolean isFull(Q);          // checks if Q is full

void clear(Q);              // make Q empty
```

Queue by Linked List

- Can use LinkedList as underlying implementation of Queues

Queue



Queue by Linked Lists in C

```
typedef short BOOLEAN;
#define TRUE ((short) (1))
#define FALSE ((short) (0))
```

```
struct NODE {
    int element;
    struct NODE *next;
};
```

```
typedef struct {
    struct NODE
        *front, *rear;
} QUEUE;
```

```
void initialize(QUEUE *pQ)
{ pQ->front = NULL; }
```

```
void clear(QUEUE *pQ) {
    struct NODE *n, *pre;
    n = pQ->front;
    while ( n ) {
        pre = n;
        n = n->next;
        free(pre);
    }
    initialize(pQ);
}
```

Queue by Linked Lists: dequeue

```
BOOLEAN isEmpty(Queue *pQ) {return (pQ->front == NULL); }
```

```
BOOLEAN isFull(Queue *pQ) { return FALSE; }
```

```
BOOLEAN dequeue(Queue *pQ, int *px) {  
    struct NODE *n;  
    if ( isEmpty(pQ) ) return FALSE;  
    else {  
        n = pQ->front;  
        pQ->front = n->next;  
        (*px) = n->element;  
        free(n);  
        return TRUE;  
    }  
}
```

Queue by Linked Lists: enqueue

```

BOOLEAN enqueue(int x, QUEUE *pQ) {
    struct NODE *n;
    n = (struct NODE*) malloc(sizeof(struct NODE));
    n->element = x;
    n->next = NULL;
    if ( isEmpty(pQ) ) {
        pQ->front = pQ->rear = n;
    }
    else {
        pQ->rear->next = n;
        pQ->rear = n;
    }
    return TRUE;
}

```

/* EXAMPLE

[74	21	42	94	13	24	80	45	75	37]	
	<u>74</u>	[21	42	94	13	24	80	45	75	37]
	<u>21</u>	[42	94	13	24	80	45	75	37]	
	[42	94	13	24	80	45	75	37	<u>102</u>]	
	[42	94	13	24	80	45	75	37	102	<u>106</u>]
	<u>42</u>	[94	13	24	80	45	75	37	102	106]
	*/										

Summary

- The definition of the queue operations gives the ADT queue first-in, first-out (FIFO) behavior
- The queue can be implemented by linked lists or by arrays
- There are many applications
 - Printer queues,
 - Telecommunication queues,
 - Simulations,
 - Etc.

END OF LECTURE 6