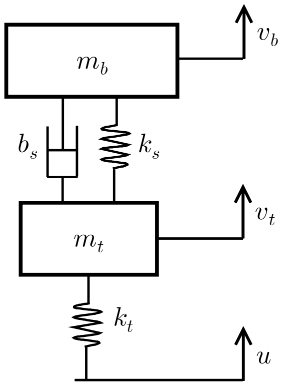
Linear Control System HW 1

[1] Sketched on the right is a model of a vehicle suspension. m_b represents the mass of the vehicle body, m_t is the mass of the unsprung mass, k_s and b_s are the spring constant and dashpot coefficient of the suspension, respectively, and k_t represents the tire stiffness. Differential equations to describe the motion are

$$m_b \frac{dv_b(t)}{dt} = f_s(t) + b_s[v_t(t) - v_b(t)]$$
$$\frac{df_s(t)}{dt} = k_s[v_t(t) - v_b(t)]$$
$$m_t \frac{dv_t(t)}{dt} = -f_s(t) - b_s[v_t(t) - v_b(t)] + f_t(t)$$
$$\frac{df_t(t)}{dt} = k_t[u(t) - v_t(t)]$$



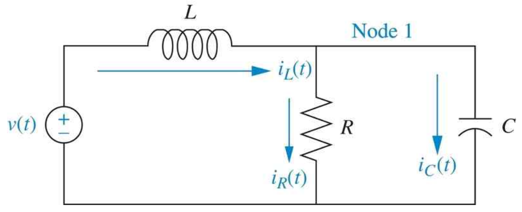
where v_b and v_t are the vertical velocities of the body and of the unsprung mass, respectively, f_s and f_t are the forces stored in the spring and the tire, respectively, and u is the (vertical velocity) excitation from the road.

a. Find the transfer function from $U(s)$ to $V_b(s)$(Note: Use Laplace Transform to the differential equation instead of using the formula, $G(s) = C(sI - A)^{-1}B + D$.)

b. Obtain the state space realizations in controllable and observable canonical forms.

c. Assuming $b_s = 0$, find the poles and zeros of the transfer function.

[2] GIven the electrical network of Figure below, find a state space representation if the output is the current through the resistor.



[3] Consider the following system.

$$\frac{dx(t)}{dt} = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [1, 2]\, x(t)$$

(a) Obtain the transformation matrix G(s) using $G(s) = C(sI - A)^{-1}B + D$.

(b) Convert G(s) into state space model using observable canonical form.

(c) Convert G(s) into diagonal canonical form.

[4] Consider a second order system below,

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t)$$

$$y = [1,\ 0,] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \mathbf{C}\mathbf{x}(t),$$

(a) Obtain the transfer function G(s) from the above state space model.

(b) Find a matrix J in the Jordan form which is related to A by

$J = T^{-1}AT$

where T is a transformation matrix. Obtain T as well.

(c)Obtain the free response x(t), given the initial condition x(0) = $[1,\ 1]^T$.

[5] Find the eigen values and eigen vectors of the following matrices.

i) A = $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$

ii) A = $\begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$

iii) A = $\begin{bmatrix} 3 & 0 & 0 \\ 5 & 4 & 0 \\ 3 & 6 & 1 \end{bmatrix}$